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ON TESTING THE SIGNIFICANCE OF A SUBSET OF COEFFICIENTS IN A SET OF SEEMINGLY UNRELATED REGRESSIONS USING MIXED ESTIMATION

by

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On Testing the Significance of a Subset of Coefficients
in a Set of Seemingly Unrelated Regressions Using Mixed Estimation

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Theil's (1971), p. 314 F-test for linear restrictions in the context of joint GLS is generalized in this note to the case in which the mixed estimator is used.

Consider the stacked linear model, in which observations for each equation are adjacent,

(1) $$\underline{y} = X\underline{\beta} + \underline{u}$$

where $\underline{y}$ is an nT vector of T observations for n dependent variables, X is an nTxK matrix of observations on K independent variables, $\underline{\beta}$ is a Kx1 coefficient vector, and $\underline{u}$ is an nTx1 vector of errors such that $E(\underline{u}) = 0$, $E(\underline{uu}') = \sigma^2\Omega$, an nTxnT covariance matrix defined as

(2) $$\sigma^2\Omega = \sigma^2\Sigma\otimes I,$$

so that $\sigma^2\Sigma$ is nxn, the contemporaneous covariance matrix of the errors.

Suppose that the analyst has prior information concerning some of the $\underline{\beta}$'s formulated as

(3) $$\underline{r}_1 = R_1\underline{\beta} + \underline{v}_1, \quad E(\underline{v}_1) = 0, \quad E(\underline{v}_1\underline{v}_1') = V_0$$

where $\underline{r}_1$ is a qx1 vector of unbiased estimators of linear combinations of the $\underline{\beta}$, R_1 is a known qxK matrix (of rank q), $\underline{v}_1$ is a qx1 vector of random errors with known (and uncorrelated with $\underline{u}$) covariance matrix V_0. As is well known, the mixed estimator of $\underline{\beta}$ under 1-3 is (see Theil (1971), p. 346-352):

(4) $$\underline{b}_m = \left(\frac{1}{\hat{\sigma}^2} X'\Omega^{-1}X + R_1'V_0^{-1}R_1\right)^{-1}\left(\frac{1}{\hat{\sigma}^2}X'\Omega^{-1}\underline{y} + R_1'V_0^{-1}\underline{r}_1\right),$$

where $\hat{\sigma}^2$ is defined by

(5) $$\hat{\sigma}^2 = \frac{1}{nT-K}(\underline{y}-X\hat{\underline{\beta}})'\Omega^{-1}(\underline{y}-X\hat{\underline{\beta}}),$$

and $\hat{\underline{\beta}}$ is the GLS or Aitken estimator of $\underline{\beta}$ in (1).

The mixed estimator is obtained by applying Aitken estimation to

(6) $$\underline{w} = Z\underline{\beta} + \underline{\varepsilon},$$

where $\underline{w} = \begin{bmatrix} \underline{y} \\ \underline{r}_1 \end{bmatrix}$, $\quad Z = \begin{bmatrix} X \\ R_1 \end{bmatrix}$, $\quad \underline{\varepsilon} = \begin{bmatrix} \underline{u} \\ \underline{v}_1 \end{bmatrix}$,

so that $E(\underline{\varepsilon}) = 0$, $\quad E(\underline{\varepsilon\varepsilon}') = \sigma^2 \begin{bmatrix} \Omega & 0 \\ 0 & \frac{1}{\sigma^2}V_0 \end{bmatrix} = \sigma^2 \Sigma_m$

By analogy to Theil (1971), p. 314, it is evident that, under the null hypothesis $\underline{r}_2 = R_2\underline{\beta}$,

$$\text{(7)} \qquad \frac{(nT+q-K)}{p} \times \frac{(\underline{r}_2 - R_2\underline{b}_m)'[R_2(Z'\Sigma_m^{-1}Z)^{-1}R_2']^{-1}(\underline{r}_2 - R_2\underline{b}_m)}{(\underline{w} - Z\underline{b}_m)'\Sigma_m^{-1}(\underline{w} - Z\underline{b}_m)}$$

is distributed as $F(p,\ nT+q-K)$ where p is the full row rank of R_2, assuming that $\underline{\varepsilon} \sim N(0,\ \sigma^2\Sigma_m)$.

Generalized least squares estimation subject to the constraint $r_2 = R_2\underline{\beta}$ yields

$$\text{(8)} \qquad \dot{\underline{b}}_m = \underline{b}_m + (X'\Omega^{-1}X + \sigma^2R_1'V_0^{-1}R_1)^{-1}R_2'[R_2(X'\Omega^{-1}X + \sigma^2R_1'V_0^{-1}R_1)^{-1}R_2']^{-1}(\underline{r}_2 - R_2\underline{b}_m)$$

$$= \underline{b}_m + (Z'\Sigma_m^{-1}Z)^{-1}R_2'[R_2(Z'\Sigma_m^{-1}Z)^{-1}R_2']^{-1}(\underline{r}_2 - R_2\underline{b}_m).$$

From (8) it follows that

$$\text{(9)} \quad (\dot{\underline{b}}_m - \underline{b}_m)'Z'\Sigma_m^{-1}Z(\dot{\underline{b}}_m - \underline{b}_m) = (\underline{r}_2 - R_2\underline{b}_m)'[R_2(Z'\Sigma_m^{-1}Z)^{-1}R_2']^{-1}$$

$$R_2(Z'\Sigma_m^{-1}Z)^{-1}R_2'[R_2(Z'\Sigma_m^{-1}Z)^{-1}R_2']^{-1}(\underline{r}_2 - R_2\underline{b}_m)$$

$$= (\underline{r}_2 - R_2\underline{b}_m)'[R_2(Z'\Sigma_m^{-1}Z)^{-1}R_2']^{-1}(\underline{r}_2 - R_2\underline{b}_m).$$

Inserting (9) into (7) gives

$$\text{(10)} \qquad \frac{(nT+q-K)}{p} \times \frac{(\dot{\underline{b}}_m - \underline{b}_m)'Z'\Sigma_m^{-1}Z(\dot{\underline{b}}_m - \underline{b}_m)}{(\underline{w} - Z\underline{b}_m)'\Sigma_m^{-1}(\underline{w} - Z\underline{b}_m)}$$

which is F distributed with p and nT+q-K degrees of freedom.

We may use either (7) or (10) to test the hypothesis $\underline{r}_2 = R_2\underline{\beta}$. In applying the above test it should be remembered that the null hypothesis, $\underline{r}_2 = R_2\underline{\beta}$ should not contradict the a priori assumptions $\underline{r}_1 = R_1\underline{\beta}+\underline{v}_1$. If the statements $\underline{r}_2 = R_2\underline{\beta}$ and $\underline{r}_1 = R_1\underline{\beta}+\underline{v}_1$ refer to the same linear combination of the elements of $\underline{\beta}$, then they are contradictory.

An example of the application of this test is in estimation of demand systems as in Paulus (1975) and Berner (1975), in which off-diagonal price coefficients are constrained to be zero in some variants.

Berner and Paulus consider the relative price version of the Rotterdam demand model,

$$(11) \qquad w^*_{it}Dq_{it} = \mu_i Dq_t + \phi A_{it}(\mu) + \sum_{j\neq i}\nu_{ij}(Dp_{jt}-Dp_{it}) + \varepsilon_{it}, \qquad i=1,\dots,n,$$

where the μ_i are marginal budget shares and the ν_{ij} are price coefficients.[1] In Berner (1975), the fifteen goods are divided into five groups with three geographic origins for goods in each group. The system is thus a complete system of consumer import and domestic demand equations. Imposing block-additivity across the groups means that for two groups r and s, $\nu_{ij} = 0$ for $i\varepsilon r$ and $j\varepsilon s$, but $\nu_{ij} \neq 0$ for $i,j\varepsilon r$ or s.

Paulus' results indicate that, at the grouping level used here, the block-additivity assumption may be inappropriate. A simple extension of the model involves adding off-diagonal blocks of price coefficients to allow for specific substitution or complementarity among all the goods

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[1]For details, see Theil (1975), Volume I.

in the pairwise groupings clothing/other and durables/shelter. Nine coefficients are added for each block, for a total of eighteen. The block-additive model thus has 30 free parameters, while the extended model has 48.

Block-additive sample and mixed results for the Netherlands for the period 1954-70 are presented in Table I. Table II presents the prior marginal shares and their standard errors used in estimation. The fact that some of the marginal shares for the sample estimates are negative is disturbing, and indicates a possible misspecification.

Table III presents the estimated parameters of the extended model. Fourteen of the eighteen additional price coefficients are more than twice their standard errors, and many are five to ten times the standard errors. However, the F-test developed here has a value of 1.18 with 18 and 204 degrees of freedom, which is less than the critical value of 1.93 at the 5% level of significance. Hence, block-additivity cannot be rejected in favor of the extended model. It is clear, of course, that other extensions of the model may dominate the block-additive version.[1]

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[1]Theil (1975) in Volume II, Chapter 8, suggests weak separability rather than the strong separability of the block-additive model. This would be stronger than the specification used here for the extended portion of the model, and yet would allow for off-diagonal price action among more groups than the present extended model does.

TABLE I

Rotterdam Model - NETHERLANDS, 1954-1970

Income and Price Coefficients - Sample and Mixed Estimates

Product by Origin	Marginal Shares (μ_i)		Price Coefficients (ν_{ij})					
			Sample			Mixed		
	Sample (1)	Mixed (2)	Domestic (3)	EEC (4)	ROW (5)	Domestic (6)	EEC (7)	ROW (8)
FOOD								
1. Domestic	.0873	.1543	-4.20	1.52	1.24	-5.29	1.53	1.24
	(.0204)	(.0169)	(2.92)	(.015)	(.044)	(1.23)	(.015)	(.044)
2. EEC	.0147	.0192		-1.80	.036		-1.88	.039
	(.0054)	(.0032)		(2.24)	(.023)		(.171)	(.023)
3. ROW	-.0016	.0077			-1.25			-1.41
	(.0039)	(.0014)			(1.24)			(1.01)
CLOTHING								
4. Domestic	-.0120	.0695	.755	-.587	.030	-.587	-.580	.031
	(.0476)	(.0087)	(1.90)	(.063)	(.012)	(.516)	(.063)	(.012)
5. EEC	.0275	.0195		-.098	.231		.031	.231
	(.0203)	(.0043)		(.703)	(.010)		(.226)	(.010)
6. ROW	.0073	.0053			-.381			-.349
	(.0029)	(.0010)			(1.04)			(.408)
SHELTER								
7. Domestic	.2932	.0266	-16.9	12.7	-.632	-12.4	12.6	-.638
	(.0277)	(.0100)	(4.54)	(.147)	(.070)	(2.91)	(1.44)	(.067)
8. EEC	-.2104	.0563		-9.63	.400		-13.9	.401
	(.0223)	(.0013)		(1.00)	(.043)		(.520)	(.041)
9. ROW	.0128	$.89 \times 10^{-4}$			.021			.235
	(.0028)	(.0004)			(4.92)			(3.23)
DURABLES								
10. Domestic	.0141	.0374	-.990	.459	.299	-1.37	.462	.299
	(.0126)	(.0043)	(.246)	(.018)	(.009)	(.150)	(.018)	(.009)
11. EEC	.0052	.0077		-.575	.027		.614	.027
	(.0038)	(.0019)		(.151)	(.006)		(.141)	(.007)
12. ROW	.0031	.0066			-.377			-.434
	(.0038)	(.0016)			(.065)			(.039)
OTHER								
13. Domestic	.6949	.5500	-11.9	.324	.159	-9.48	.334	.163
	(.0036)	(.0308)	(.757)	(.026)	(.009)	(.506)	(.026)	(.009)
14. EEC	.0405	.0223		-1.11	.120		-.818	.119
	(.0458)	(.0048)		(.141)	(.007)		(.079)	(.007)
15. ROW	.0233	.0176			-.664			-.569
	(.0049)	(.0022)			(.705)			(.427)
ϕ	-.1648	-.1634						
	(.0013)	(.0013)						

Notes: Figures in Columns 3-8 are to be divided by 10^2.
Figures in parentheses are standard errors.

TABLE II

Netherlands, 1954-1970

Average Value Shares and Prior Estimates of
Marginal Shares for 15 Products

Product by Orgin (1)	Average Value Shares (2)	Prior Estimate and Standard Deviation of: Income Elasticity (3)	Marginal Share (4)
FOOD			
1. Domestic	.334	.300(.192)	.100(.064)
2. EEC	.008	1.88(.750)	.015(.006)
3. Rest of World	.013	.462(.154)	.006(.002)
CLOTHING			
4. Domestic	.132	.758(.380)	.100(.050)
5. EEC	.016	.313(3.13)	.005(.050)
6. Rest of World	.005	1.20(.400)	.006(.002)
SHELTER			
7. Domestic	.127	.394(.787)	.050(.10)
8. EEC	.003	3.33(3.33)	.010(.010)
9. Rest of world	.002	5.00(5.00)	.010(.010)
DURABLES			
10. Domestic	.115	.522(.235)	.060(.027)
11. EEC	.008	.625(.625)	.005(.005)
12. Rest of World	.005	1.60(1.00)	.008(.005)
OTHER			
13. Domestic	.215	2.33(.700)	.500(.150)
14. EEC	.010	.500(.500)	.050(.050)
15 Rest of World	.006	--	--

TABLE III

Extended Rotterdam Model - NETHERLANDS, 1954-70

Income and Price Coefficients - Sample and Mixed Estimates

Product by Origin	Marginal Shares (μ_i)		Price Coefficients (ν_{ij})						Off-Diagonal Price Coefficients (ν_{ij})					
	Sample (1)	Mixed (2)	Sample: Domestic (3)	Sample: EEC (4)	Sample: ROW (5)	Mixed: Domestic (6)	Mixed: EEC (7)	Mixed: ROW (8)	Sample: Domestic (9)	Sample: EEC (10)	Sample: ROW (11)	Mixed: Domestic (12)	Mixed: EEC (13)	Mixed: ROW (14)
FOOD														
1. Domestic	.1310	.1453	-5.54	.700	1.51	-5.90	.700	1.51						
	(.0164)	(.0140)	(.415)	(10^{-4})	(10^{-4})	(.355)	(10^{-4})	(10^{-4})						
2. EEC	.0188	.0152		-1.19	.013		-1.10	.013						
	(.0050)	(.0033)		(.127)	(10^{-4})		(.085)	(10^{-4})						
3. ROW	-.0045	.0052			-1.41			-1.66						
	(.0040)	(.0014)			(.102)			(.036)		OTHER			OTHER	
CLOTHING														
4. Domestic	.1373	.1199	-6.78	2.69	.331	-6.29	2.66	.328	.878	-.691	.090	.857	-.692	.098
	(.0161)	(.0120)	(.527)	(.165)	(.013)	(.431)	(.163)	(.013)	(.103)	(.020)	(.038)	(.101)	(.020)	(.037)
5. EEC	-.0294	-.0223		1.34	.011		1.18	.011	-.251	-2.75	-.300	-.229	-2.75	-.307
	(.0113)	(.0089)		(.298)	(.015)		(.247)	(.015)	(.080)	(.029)	(.027)	(.079)	(.028)	(.026)
6. ROW	-.0001	.0055			-.351			-.493	-.080	.326	-.234	-.077	.326	-.234
	(.0025)	(.0012)			(.063)			(.031)	(.010)	(.005)	(.002)	(.010)	(.005)	(.002)
SHELTER														
7. Domestic	.0198	.0323	-7.81	-.514	-.400	-1.10	-.510	-.397						
	(.0121)	(.0103)	(.416)	(.113)	(.078)	(.378)	(.112)	(.076)						
8. EEC	.0180	.0132		-.025	.212		.091	.207						
	(.0032)	(.0024)		(.085)	(.048)		(.070)	(.047)						
9. ROW	.0100	.0091			-.226			-.210						
	(.0027)	(.0021)			(.066)			(.056)		SHELTER			SHELTER	
DURABLES														
10. Domestic	.0473	.0454	-3.99	.879	.323	-3.96	.877	.328	1.55	-.018	.055	1.55	-.009	.063
	(.0079)	(.0063)	(.293)	(.043)	(.036)	(.264)	(.043)	(.035)	(.187)	(.071)	(.054)	(.187)	(.071)	(.054)
11. EEC	.0032	.0032		-.982	.409		-.982	.410	-.273	-.009	-.105	-.275	-.008	-.104
	(.0035)	(.0025)		(.091)	(.014)		(.066)	(.014)	(.061)	(.029)	(.020)	(.061)	(.029)	(.020)
12. ROW	.0013	.0067			-.790			-.928	-.085	-.103	.213	-.083	-.106	.210
	(.0033)	(.0019)			(.081)			(.048)	(.054)	(.033)	(.025)	(.053)	(.032)	(.024)
OTHER														
13. Domestic	.5733	.5470	-15.3	.182	.013	-14.6	.179	.011						
	(.0248)	(.0217)	(.623)	(.025)	(.019)	(.545)	(.024)	(.018)						
14. EEC	.0597	.0550		.921	.500		1.04	.500						
	(.0081)	(.0065)		(.215)	(.005)		(.163)	(.005)						
15. ROW	.0142	.0191			-.428			-.553						
	(.0044)	(.0032)			(.113)			(.083)						
ϕ	-.2537	-.2537												
	(3×10^{-5})	(3×10^{-5})												

Notes: Figures in Columns 3-14 are to be divided by 10^2.
Figures in parentheses are standard errors.

Compatibility test statistic ($\chi^2_{(14)}$): 9.82

Sample share: .94 Prior share .06

References

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