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A MONETARY MODEL OF EXCHANGE MARKET PRESSURE  
APPLIED TO THE POST-WAR CANADIAN EXPERIENCE

by

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# A Monetary Model of Exchange Market Pressure

## Applied to the Post-War Canadian Experience

Lance Girton and Don Roper\*

The monetary approach to the balance of payments has received considerable attention.<sup>1</sup> However, most of the empirical studies employ models of a small country with fixed exchange rates. Without relying on the small-country assumption, we derive a model to explain a measure of exchange market pressure that incorporates both exchange rate movements and official intervention. The model is used to analyze the post-war Canadian experience.

The paper is organized into three sections. The first section contains a discussion of the monetary approach and the problem of determining the degree of independence of a country's monetary policy. In the second section a monetary model that holds for all exchange rate regimes is developed. Empirical estimation of Canadian exchange market pressure and an empirical measure of the autonomy that the Canadian authorities relinquish when pursuing a fixed exchange rate target are presented in the third section.

### I. The Monetary Approach and the Independence Question

If the balance of payments is divided into more than two accounts--e.g., the current, capital, and money accounts--then each account can be explained with a direct or an indirect approach. Using the demands and supplies for the  $k^{\text{th}}$  item as a classification procedure for explaining the  $k^{\text{th}}$  account constitutes a direct approach. An account can be explained indirectly by first explaining the other  $n-1$  accounts and then adding the results. Given that economists are accustomed to explaining the current and capital accounts (and various subaccounts) directly, the argument for the monetary approach can be seen as an argument that the money account be given symmetric treatment.<sup>2</sup>

The argument for symmetry also implies that the traditional view of official intervention (necessary to maintain fixed rates) as accommodating or financing current and capital account transactions should be abandoned.<sup>3</sup> The monetary approach continues to regard the quantity of intervention necessary to achieve a fixed rate target as endogenous, but it shifts the focus for an explanation to the monetary equilibrium condition.

Since, by Walras' Law, the net excess supply of goods and securities by residents<sup>4</sup> of a country represent a net excess demand for money, the traditional approach to the balance of payments, that specified behavioral relationships for the trade and capital accounts, contained an implicit monetary condition. However, the implicit monetary condition was not necessarily one that would have seemed reasonable if money supply and demand functions had been developed explicitly.

The empirical estimation of a monetary model of the balance of payments can be related to, but is not identical with, an empirical estimation of the degree of independence of monetary policy. One can explain an official settlements measure of the balance of payments without testing for independence, and one can test for independence without explaining the balance of payments. In this paper, however, both exercises are undertaken since the monetary approach provides a useful framework within which to estimate the degree of monetary autonomy.<sup>5</sup>

The degree of monetary independence can be measured by the degree to which alterations in the domestic source of the monetary base lead to changes in the demand for domestic base and thereby the total quantity outstanding.<sup>6</sup> If the policy actions used to alter the domestic source of the base fail to influence the demand for base money, then the change in the domestic source will be offset by the official exchange market intervention necessary to achieve a fixed exchange rate target.<sup>7</sup>

## II. The Model

The model developed here, and analyzed further in the appendix, is a monetary model in the sense that it organizes the analysis around the demands and supplies of national monies.

Using an exponential specification of the demand-for-base function, the monetary equilibrium condition for any country  $i$  can be written as<sup>8</sup>

$$(1) \quad H_i = F_i + D_i = P_i Y_i^{\beta_i} \exp(-\alpha_i \rho_i)$$

where

$H_i$  = supply of base money issued by the central bank of country  $i$

$F_i$  = base money created against the purchase of foreign assets

$D_i$  = base money created by domestic credit expansion.

$P_i$  = price level

$Y_i$  = real income

$\rho_i$  = index of interest rates

$\beta_i$  = income elasticity  $> 0$

$\alpha_i$  = interest rate coefficient  $> 0$

The division of  $H$  between its domestic,  $D$ , and foreign,  $F$ , sources is determined by

$$(2) \quad F_i(t) = \int_{-\infty}^t E_i(\tau) R_i'(\tau) d\tau$$

where

$R_i(t)$  = stock of international reserves (primary assets) held by  
the authorities in country  $i$

$R'_i(t)$  = time derivative of  $R_i$  denoting net purchases at time  $t$

$E_i(t)$  = parity or  $i$  currency value of primary reserve assets at  
time  $t$ .

As the formula notes, the country's parity (or price of foreign exchange in the case of foreign exchange reserves)<sup>9</sup> is important only at the time foreign assets are purchased. If, for instance, the monetary authority revalues its currency and acquires a capital gain on their stock of international reserves,  $F$ , as defined by (2), is not affected. As a result of the capital gain the authorities may or may not increase their liabilities outstanding. But any increase in base money related to the capital gain should be treated as an increase in  $D$ , not  $F$ , since the purpose of the model is to explain the quantity of base that the authorities are induced to create or destroy (and the autonomy they sacrifice) in order to stabilize the exchange rate.

Substituting the time derivative of (2), viz.,  $F'_i = E_i R'_i$ , in the differentiated version of (1) and stating the results in percent changes yields

$$(3) \quad h_i = r_i + d_i = \pi_i + \beta_i y_i - \alpha_i \rho_i'$$

where<sup>10</sup>

$$h_i = H'_i/H_i$$

$$d_i = D'_i/H_i$$

$$r_i = E_i R'_i/H_i$$

$$\rho'_i(t) = d\rho_i/dt$$

$$\pi_i = P'_i/P_i$$

$$y_i = Y'_i/Y_i$$

By deflating the rate of change of international reserves, valued in domestic currency,  $E_i R_i'$ , by domestic base money,  $H_i$ , a real measure of the balance of payments,  $r_i$ , is obtained. It is essential to convert the nominal measure of the official intervention into real terms to determine whether the balance of payments is large or small.

To examine the monetary interaction between countries, subtract the monetary equilibrium condition (3) for country  $j$  from the monetary equilibrium condition for country  $i$ :

$$(4) \quad r_i - r_j = -d_i + d_j + \beta_i y_i - \beta_j y_j + \pi_i - \pi_j - \alpha(\rho_i' - \rho_j')$$

where  $\alpha_i$  and  $\alpha_j$  have been assumed equal ( $\alpha = \alpha_i = \alpha_j$ ). Introducing the further notation,

$e_{ij}$  = rate of appreciation of currency  $i$  in terms of currency  $j$

$$\theta_{ij} = \pi_i - \pi_j + e_{ij}$$

= differential inflation rate adjusted for exchange rate changes<sup>11</sup>

$\delta_{ij} = \rho_i' - \rho_j'$  = change in the uncovered interest differential,

equation (4) can be rewritten as

$$(5) \quad r_i - r_j + e_{ij} = -d_i + d_j + \beta_i y_i - \beta_j y_j + \theta_{ij} - \alpha \delta_{ij}$$

The way equation (5) is employed to explain the interaction between two countries depends on whether one of the countries is sufficiently "large" in the sense of being able to pursue an independent monetary policy. Consider first the case of two regions or countries of comparable size,<sup>12</sup> for example, France and Germany. If the mark-franc rate were perfectly fixed

( $e_{ij} = 0$ ), the left-hand side of equation (5) would represent the bilateral (real) balance of payments. If both countries refrained from intervention ( $r_i = 0 = r_j$ ), the left-hand side would reduce to the percent change of the mark-franc rate.<sup>13</sup> If the monetary authorities of the two countries intervened without a commitment to a perfectly constant exchange rate, the composite variable  $r_i - r_j + e_{ij}$ , measures, what we refer to as, exchange market pressure.

We are interested in applying the equation to Canada and the United States. Since the U.S. has been a center or key-currency country, it has had the ability to force most and perhaps all the adjustment burden on those countries who have made efforts to stabilize their exchange rates.<sup>14</sup> This extreme asymmetry in the adjustment burden justifies (as will be explained below) the transference of the center-country's balance of payments from the left to the right-hand side of the equation. If the  $i$  subscripts are changed to  $c$  (for Canada) and the  $j$  subscripts are changed to  $u$  (for the United States), then equation (5) can be rewritten as<sup>15</sup>

$$(6) \quad r_c + e_c = -d_c + h_u + \beta_c y_c - \beta_u y_u + \theta_c - \alpha \delta_c$$

where  $r_u$  has been subsumed under  $h_u$  ( $= d_u + r_u$ ).<sup>16</sup>

The center country's balance of payments,  $r_u$ , can be taken to the right-hand side and used as an independent variable if  $h_u$  is not influenced by the remaining expression,  $r_c + e_c$ . Since  $r_u$  reflects changes in U.S. international reserves, then any official Canadian intervention financed by purchases or sales of U.S. dollars to the U.S. Treasury shows up in both  $r_c$  and  $r_u$ .<sup>17</sup> If U.S. reserve flows are perfectly sterilized,<sup>18</sup> however, then  $h_u (= d_u + r_u)$  is unaffected by  $r_c$ .<sup>19</sup>

The additional observation that  $h_u$  has been managed independently of  $e_c$  implies that  $h_u$  can be taken as an exogenous variable in the equation.

The fact that the U.S. monetary policy has been insulated in the post-war monetary system (such that  $h_u$  in equation (6) can be taken as independent of  $r_c$  and  $e_c$ ) allows further flexibility in the way the model is specified. If U.S. monetary policy had not been independent of the balance of payments, the monetary interaction between the United States and other countries would have been through both the supply and demand sides of base money markets. With  $h_u$  unaffected by exchange market intervention, the link between the United States and the rest of the world is only through the demand side--substitution between securities and commodities. The link is a recursive one that goes from U.S. prices and interest rates, to  $c$ 's prices and interest rates, to the demand for  $c$ 's base, to the induced supply of base,  $r_c$ .

The absence of the supply link (the fact that  $r_c$  does not feed back on  $h_u$ ) means that  $h_u$  need not appear in the equation. Equation (6) can be written to capture the linkages on the demand side by including U.S. prices and interest rates on the right-hand side of (6) while excluding  $h_u$ .<sup>20</sup> Since we want to determine the influence of U.S. monetary policy on Canadian exchange market pressure, however, it is useful to have a one-variable index of U.S. monetary policy rather than the two variables (interest rates and prices) over which the U.S. authorities have less control.

### III. Empirical Investigation

Equation (6) differs from equations in other monetary<sup>21</sup> models of the balance of payments in two ways. First, the dependent variable is exchange market pressure, defined as the sum  $r + e$ , rather than the balance of payments, per se. If the value of the dependent variable,  $r + e$ , is unaffected by its



composition (as will be subsequently measured as  $e/r$ ), then the exchange market pressure is independent of whether the authorities absorb the pressure in their reserves or in their rate. Second, the equation takes account of the fact that, as far as foreign exchange market pressure is concerned, a country's monetary policy can be judged tight or easy only by reference to what is happening in the rest of the world. Consequently, the country's external position is related to foreign monetary conditions. The supply and demand for U.S. money are used to represent world monetary conditions.

The purpose of this section is both to estimate the monetary equation (6) of exchange market pressure and to measure the degree to which the central bank in an open economy can pursue an independent monetary policy. Estimation of equation (6) in its present form would serve the first purpose of explaining exchange market pressure, but it would not provide a measure of monetary independence. If equation (6) were estimated, a minus-one coefficient in front of  $d_c$  would be expected regardless of whether there was multi-collinearity between  $d_c$  and  $\delta_c$  or  $\theta_c$ . The crucial issue in determining the degree that a fixed exchange rate target undermines monetary autonomy is whether the authorities can make their interest rates and prices diverge from U.S. interest rates and prices by the use of monetary policy. In symbols, the measure of independence is the degree to which  $\delta_c$  and  $\theta_c$  depend on the Canadian control variable,  $d_c$ .

To develop an alternative to (6) that will allow one to measure the independence of monetary policy, suppose that  $\delta_c$  and  $\theta_c$  are determined by the reduced form relations:

$$(7) \quad \delta_c = \delta(d_c, h_u, X) \quad \theta_c = \theta(d_c, h_u, X)$$

$$\text{where} \quad \delta_1 = \frac{\partial \delta}{\partial d_c} \leq 0 \quad \delta_2 = \frac{\partial \delta}{\partial h_u} \geq 0$$

$$\theta_1 = \frac{\partial \theta}{\partial d_c} \geq 0 \quad \theta_2 = \frac{\partial \theta}{\partial h_u} \leq 0$$

and X is the set of other variables that influence  $\delta_c$  and  $\theta_c$ .

The set of X variables includes variables other than  $d_c$  and  $h_u$  that might affect  $\delta_c$  and  $\theta_c$ . If Canadian and U.S. securities and goods are not perfect substitutes then anything that affects the supplies and demands for Canadian securities and goods relative to U.S. securities and goods will be in the set of X variables. Changes in monetary conditions outside the U.S. and Canada should be included only if they affect Canadian and U.S. securities and goods markets differentially. Since Canadian and U.S. real incomes might have differential effects on Canadian and U.S. prices and interest rates, the estimated income coefficients could be affected by any imperfect substitutability of Canadian and U.S. goods and securities.

Assuming the expression in (7) are linear, they can be substituted into equation (6) to obtain

$$(8) \quad \begin{aligned} r_c + e_c &= -(1 + \alpha\delta_1 - \theta_1)d_c + (1 - \alpha\delta_2 + \theta_2)h_u + \beta_c y_c - \beta_u y_u + (\theta_x - \alpha\delta_x)X \\ &= -\phi_c d_c + \phi_u h_u + \beta_c y_c - \beta_u y_u + (\theta_x - \alpha\delta_x)X \end{aligned}$$

where

$$\phi_c = 1 + \alpha\delta_1 - \theta_1$$

$$\phi_u = 1 - \alpha\delta_2 + \theta_2$$

To the extent that  $d_c$  affects  $\delta_c$  or  $\theta_c$ , the estimated value of  $\phi_c$  should be less than unity. That is, the Canadian reserve loss or exchange rate depreciation associated with an expansionary monetary policy will be mitigated if the policy can lower Canadian rates relative to U.S. rates, or raise Canadian prices relative to U.S. prices. Assuming that  $\phi_c$  is the same under floating as under fixed rates, it is legitimate to use the data generated under floating rates to estimate the degree of independence the authorities lose by the adoption of a fixed rate target.

The form of the equation to be estimated is

$$(9) \quad r_c + e_c = -\phi_c d_c + \phi_u h_u + \beta_c y_c - \beta_u y_u + v$$

where  $v$  is a random term. The exclusion of the  $X$  variables will not bias the estimated coefficients if the  $X$  variables are uncorrelated with the right-hand variables of equation (9).

To justify an OLS estimation of (9), argument must be provided for the recursiveness of the relationship. Each of the terms on the right-hand side of (9) will be discussed to determine whether they can be regarded as independent of the random term,  $v$ .

Consider first the U.S. monetary policy and income variables. In section II it was argued that U.S. monetary aggregates have been independent of  $r_c$  and this independence also eliminates the only obvious channel through which  $r_c$  might have

influenced U.S. income. It is also unlikely that  $e_c$  would affect  $y_u$  or  $h_u$ . Consequently,  $y_u$  and  $h_u$  can be treated as independent of the error term regardless of whether the Canadian authorities absorb market pressure in their reserves or exchange rate.

If other U.S. monetary aggregates are independent of  $r_c + e_c$ , then the choice of which U.S. monetary aggregate to use depends on which one is the best indicator of U.S. monetary conditions. Since other U.S. monetary aggregate might be good indicators of U.S. monetary conditions, we report results using two additional aggregates, money narrowly defined ( $M1_u$ ) and a broader measure ( $M2_u$ ). Although this freedom of choice exists for the U.S. aggregate, it does not exist for any country whose supply of base money is influenced by exchange market pressure.

The rate of growth of Canadian real income should be independent of  $r_c$  and  $e_c$  to the extent that  $y_c$  is based on past income changes. Since  $y_c$  is measured with a distributed lag in the regressions, all of the past values of  $y_c$  should be independent of the error term, but the possibility of simultaneous equations bias from the current change in real income cannot be ruled out.<sup>22</sup>

Potentially, the most important simultaneity problem occurs in the estimation of  $\theta_c$  under fixed exchange rates.<sup>23</sup> If the Canadian authorities try to sterilize reserve flows,  $\hat{\theta}_c$  will be biased regardless of the success of their sterilization policy. It is generally difficult to determine the direction and amount of bias for the estimated coefficients in a multiple regression. It is useful, however, to consider the bias under the simplifying assumption that  $h_u$ ,  $y_u$ , and  $y_c$  are independent of  $d_c$ . In this case  $\hat{\theta}_c$  can be shown to have an asymptotic bias of  $1/\lambda_c$  (or zero) as the variance of  $d_c$  relative

to the variance of  $v$  approaches zero (or infinity).<sup>24</sup> In summary,  $\hat{\theta}_c$  is biased towards the reciprocal of the sterilization coefficient.<sup>25</sup>

Equation (9) is estimated using annual data for the period 1952 through 1974. During this period, the Canadian dollar floated from 1952 to 1962 and after June of 1970, and was fixed in value to the U.S. dollar in the intervening years. A wide range of domestic policies were pursued, and there were various agreements concerning Canadian-U.S. economic relations.<sup>26</sup>

Two kinds of adjustments were made to the data. First, the series on the stock of Canadian international reserves was adjusted to exclude gold revaluation gains and SDR allocations.<sup>27</sup> Second, the Canadian base money figures were adjusted for the reserve requirement changes that occurred in 1952 and 1967.

In the regressions reported in Table 1, equation (9) is estimated using the percent change in three alternative U.S. monetary aggregates representing U.S. monetary conditions. The first regression uses  $M2_u$ , the second uses money narrowly defined ( $M1_u$ ), and the third uses U.S. base money ( $H_u$ ).

[INSERT TABLE 1]

The estimated coefficients have the correct sign and are significant at the 5% confidence level.<sup>28</sup> The explanatory power of the model is illustrated in Figure 1 where the actual and predicted values of the dependent variable are plotted. The predicted values are those coming from the regression in which a broad U.S. monetary aggregate is used.

Table 1

Aggregate Used	Coefficients of							
	Constant	$d_c$	$h_u$	$y_c$	$y_u$	$\rho$	$R^2$ S.E.	D.W.
$M2_u$	-.04 (1.08)	-.96 (12.74)	1.14 (4.86)	2.80 (3.01)	-2.84 (3.59)	.22 (1.07)	.92 .024	1.80
$M1_u$	-.03 (1.38)	-.96 (16.03)	1.74 (8.37)	2.54 (3.97)	-2.51 (4.83)	-.06 (.28)	.95 .020	2.11
$H_u$	-.03 (1.43)	-.97 (18.53)	1.61 (9.09)	2.63 (4.46)	-2.62 (5.35)	.06 (.30)	.96 .017	2.29

Note: The dependent variable in all the regressions is  $r_c + e_c$ , the measure of exchange market pressure. The values in parentheses below the coefficients indicate t-ratios for the corresponding estimates. S.E. and D.W. denote, respectively, the standard error of the regression and the Durbin-Watson coefficient.  $R^2$  is the coefficient of determination adjusted for degrees of freedom. The income coefficients are the sum of four year distributed lags of real GNP for each country. A second-degree Almon polynomial was used with the far tail tied to zero. All equations were run using the Cochrane-Orcutt technique to adjust for serial correlation.  $\rho$  is the estimated value of the first order autoregression coefficient.

[INSERT FIGURE 1]

To interpret the implication of the estimated value of the  $\phi_c$  coefficient, it is useful to consider the degree that Canadian monetary independence would be curtailed if  $\phi_c$  were .95. Suppose that in a particular year the expression  $\phi_u h_u + \beta_c y_c - \beta_u y_u + v$  is zero so that equation (9) reduces to  $r_c + e_c = -.95d_c$ . Suppose the Canadian authorities initially attempt to increase their money growth rate by 10% by setting  $d_c = .10$ . Then, they must be prepared to either allow their currency to depreciate by 9.5% during the year ( $e_c = -.95(.10) = -9.5\%$ ) or lose reserves at a rate equal to 9.5% of their base ( $r_c = -.95(.10) = -9.5\%$ ) or some combination.

Alternatively, suppose the authorities purchase domestic assets in sufficient quantity to successfully raise their base from C\$10 to C\$11 billion. Then a .95 value of  $\phi_c$  implies that C\$9.5 billion worth of foreign reserves would be required by the Canadian authorities to support their rate. The estimated coefficient on the domestic source of Canadian base money ( $d_c$ ), of  $-.96$  or  $-.97$ , supports the view that the Canadian monetary authorities, when under a fixed exchange rate regime, have little scope for pursuing an independent monetary policy.

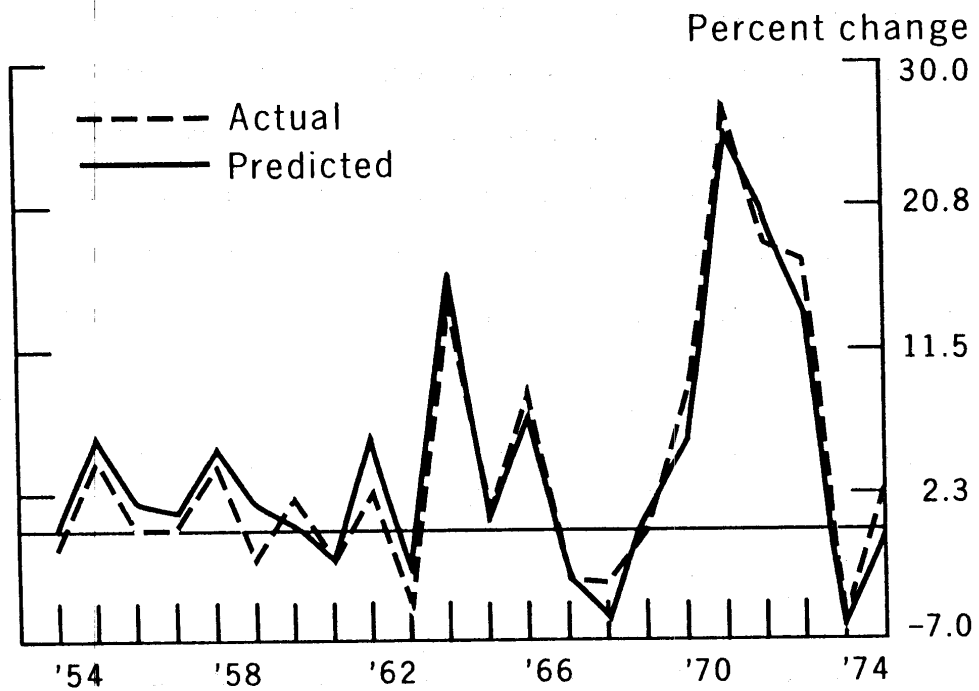
An alternative way of expressing equation (9) that highlights the implications for monetary independence is

$$(9') \quad h_c + e_c = I_c d_c + \phi_u m_u + \beta_c y_c - \beta_u y_u + v.$$

Equation (9') is found by adding  $d_c$  to both sides of equation (9) such that  $I_c = 1 - \phi_c$ . Since  $h_c$  is on the left-hand side of (9'), it is clear that the estimate of  $I_c$  is a measure of the degree to which  $d_c$  influences  $h_c$  when the Canadian authorities keep  $e_c = 0$ . The coefficients obtained from estimating (9') would be identical with those obtained from the estimation of (9), except that  $\hat{I}_c = 1 - \hat{\phi}_c$ . Our

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$$r_c + e_c$$





estimated value of  $\emptyset_c$  above .95 implies a value for  $I_c$  below .05. We use (9) instead of (9') because we are interested in explaining Canadian foreign exchange market pressure as well as testing for the degree of Canadian monetary independence.

The measure of exchange market pressure used as the dependent variable in the regressions can be split into the two components, changes in official reserves, and changes in exchange rates. The assumptions used imply that the total of these two components is not sensitive to the composition. In order to test for the sensitivity of the measure of exchange market pressure to its composition (whether the authorities absorb pressure in international reserves or in the exchange rate), the equations were re-estimated with the ratio  $Q = e_c/r_c$  entered as a separate explanatory variable.<sup>29</sup> The results were the same for all three equations. For that reason, only the  $M2_u$  equation is reported:

$$(10) \quad r_c + e_c = -.04 - .94d_c + 1.12h_u + 2.90y_c - 3.03y_u + .001Q$$

(1.03)(12.21) (4.81) (3.41) (4.15) (1.08)

$$\rho = .08, R^2 = .92, S.E. = 2.025, D.W. = 2.06.$$

(.36)

The coefficient on  $Q$  is not significant and other coefficients are left essentially unchanged indicating that the explained value of exchange market pressure is not sensitive to its composition.

## Appendix

In the text, monetary conditions in the U.S. and Canada were used to develop an equation for estimating a measure of exchange market pressure for Canada. Here we show that the results obtained from focusing on only the two countries are consistent with the limiting case in an explicit multi-country framework. Also, several propositions that were asserted in the text are proved.

The flow monetary equilibrium condition for country  $i$  is

$$A1. \quad h_i = r_i + d_i = \pi_i - \alpha \rho_i' + \beta_i y_i.$$

The policy reaction function for country  $i$  is expressed as

$$A2. \quad d_i = d_i^0 - \lambda_i r_i, \quad \text{or} \quad h_i = d_i^0 + (1 - \lambda_i) r_i.$$

Using the policy reaction function, (A2), to substitute for  $d_i$  in (A1)

and solving for  $r_i$  yields

$$A3. \quad r_i = \frac{\pi_i - \alpha \rho_i' + \beta_i y_i - d_i^0}{(1 - \lambda_i)}$$

The world demand for international reserves is assumed to be equal to the supply. This world reserve equilibrium condition can be expressed as

$$A4. \quad \sum s_i r_i = r_w, \quad \text{where} \quad s_i = \frac{H_i/E_i}{H_w}, \quad H_w = \sum H_i/E_i,$$

and  $r_w$  is the rate of change in the supply of international reserves as a proportion of world base money ( $H_w$ ).

Substituting the demand for international reserves, (A3), into the global reserve equilibrium condition, (A4), yields

$$A5. \quad \sum \frac{s_i}{(1 - \lambda_i)} (\pi_i - \alpha \rho_i' + \beta_i y_i - d_i^0) = r_w$$

Defining  $\theta_i$  and  $\delta_i$  analogous to the definitions of  $\theta_c$  and

$\delta_c$ , i.e., for country  $i$  with respect to the U.S., substituting  $\theta_i$ 's and  $\delta_i$ 's in (A5), and solving for  $n = \pi_u - \alpha\rho'_u$ , yields

$$A6. \quad n = \frac{r_w}{\sum s_i / (1-\lambda_i)} - \sum w_i (\beta_i y_i - e_i + \theta_i - \alpha\delta_i - d_i^0),$$

$$\text{where } w_i = \frac{s_i / (1-\lambda_i)}{\sum s_i / (1-\lambda_i)}.$$

An expression for  $h_c$  in terms of the policy parameters and the other exogenous variables is found by substituting  $\theta_c$ ,  $\delta_c$  and the expression for  $n$  into (A1):

$$A7. \quad h_c = \frac{r_w}{\sum s_i / (1-\lambda_i)} - \sum w_i (\beta_i y_i - e_i + \theta_i - \alpha\delta_i - d_i^0) + (\beta_c y_c - e_c + \theta_c - \alpha\delta_c - e_c).$$

The expression for  $r_c$  is obtained by substituting the expression for  $h_c$  into the policy reaction function, (A2):

$$A8. \quad r_c = \frac{1}{1-\lambda_c} \left[ \frac{r_w}{\sum s_i / (1-\lambda_i)} - \sum w_i (\beta_i y_i - e_i + \theta_i - \alpha\delta_i - d_i^0) + (\beta_c y_c - e_c + \theta_c - \alpha\delta_c - d_c^0) \right].$$

Assuming the other right hand side variables in (A7) are independent of  $d_c^0$ , i.e., the  $e$ 's are fixed and the  $\theta$ 's and  $\delta$ 's are independent of  $d_c^0$ , and that  $\partial\lambda_i/\partial\lambda_j = 0$ , for  $i \neq j$ , then

$$A9. \quad \frac{\partial h_c}{\partial d_c^0} = w_c, \quad \frac{\partial^2 h_c}{\partial d_c^0 \partial \lambda_c} = w_c \sum_{i \neq c} w_i \geq 0, \quad \text{and} \quad \frac{\partial^2 h_c}{\partial d_c^0 \partial \lambda_u} = -\frac{w_c w_u}{1-\lambda_u} < 0.$$

In a highly integrated world, with fixed exchange rate targets, ( $0 \leq w_c \leq 1$ ) determines the power of a country  $c$ , through domestic monetary operations, to influence its own and the world's rate of monetary growth.<sup>30</sup> The second and third expressions in (A9) show the change in the degree of influence over world monetary conditions country  $c$  has with changes

in its own and others sterilization behavior. By inspection of (A9) it can be seen that, assuming no country completely sterilizes,  $w_c$  increases with increases in  $\lambda_c$  and decreases with increases in  $\lambda_u$  ( $u \neq c$ ). In the limit where country  $u$  completely sterilizes ( $\lambda_u = 1$ ), then  $w_c = 0$ , for all  $c \neq u$ , and  $w_u = 1$ .<sup>31</sup>

When  $\lambda_u = 1$  and  $\lambda_c = 0$ , the expressions for  $h_c$  and  $r_c$  reduce to

$$\text{A10.} \quad h_c = h_u + \beta_c y_c - \beta_u y_u - e_c + \theta_c - \alpha \delta_c \quad \text{and}$$

$$\text{A11.} \quad r_c + e_c = -d_c^0 + h_u + \beta_c y_c - \beta_u y_u + \theta_c - \alpha \delta_c,$$

where it is recognized that by definition  $e_u, \theta_u, \delta_u = 0$  and that when  $\lambda_u = 1$ , then  $\sum s_i / (1 - \lambda_i) = 0$  and  $h_u = d_u^0$ .

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## Footnotes

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1/ See Bijan Aghevli and Mohsin Khan, Borts and James Hanson, Connolly and Dean Taylor, Rudiger Dornbusch, Jacob Frenkel and Carlos Rodriguez, Hans Genberg, Harry Johnson, Ryutaro Komiya, Donald McCloskey and Richard Zecher, Norman Miller and Sherry Askin, Robert Mundell (1968)(1971), Michael Mussa, and Zecher. A survey of this literature is given by Marina Whitman.

2/ Surprisingly, the theoretical advantages of the direct over the indirect approach has not, to our knowledge, been demonstrated for any of the accounts. The direct approach may have the practical advantage of increasing the likelihood of correctly specifying the explanatory functions.

3/ The argument that the items placed "below the line" were accommodating other exchange market transactions was emphasized in the discussions over the appropriate definition of the balance of payments in the early 1960's as found, for instance, in the "Bernstein Report" (U.S. Budget Bureau, Balance of Payments Statistics of the U.S., 1965).

4/ "Resident" here means holders of cash balances whose demand is influenced by domestic income. The analysis in this paper will assume that the income variable for one country affects the demand for the liabilities of only the central bank of that country. Implications of relaxing this assumption have been developed in Girton and Roper.



5/ The usefulness of the monetary framework for dealing with the question of monetary independence may explain some of the recent interest in the approach. The growth of financial capital movements in the 1960's made policymakers aware of the conflict between their domestic monetary objectives and the commitment to a fixed exchange rate. One of the early studies concerned with this problem was by Ruth Logue.

6/ The most obvious way the domestic monetary authorities can affect the demand for their liabilities is by changing reserve requirements. In this paper, the impact of reserve requirement changes is subsumed under the supply side by adjusting base money for reserve requirement changes. The independence question concerns the ability to affect this adjusted base.

7/ A more general analysis of the question of autonomy would require that the sources of the base be divided into directly controlled and uncontrolled parts. The purpose would then be to determine the degree that the latter tended to offset the former.

The fact that there may be important uncontrolled elements other than official settlements is highlighted by the Federal Reserve's experience in the early 1920's when, in an effort to increase earnings, they were led to the "discovery" that open market operations were a policy tool equivalent to discounting. According to Ralph Burgess (p. 221): ". . . as fast as the Reserve Banks bought Government securities in the market, the member banks paid off more of their borrowings; and, as a result, earning assets and earnings of the Reserve Bank(s) remained unchanged." Realization that discounting was offsetting open market operations made the Fed aware that the two procedures for purchasing domestic assets were good substitutes in their effect on bank reserves. We assume these other potential offsets to monetary policy are sufficiently under the control of the authorities that intervention in pursuit of an exchange rate target can be usefully isolated as the primary threat to monetary autonomy. This assumption allows us to address the two separate problems--explaining the exchange market pressure and measuring the degree of independence--within the same theoretical framework.

8/ One can specify the demand for base as the product of a money multiplier and the demand for an aggregate (defined as all financial items that absorb base money). Although such a specification may be useful for some purposes, it will not be used here.

9/ International reserves can be expressed to include foreign exchange in which case (2) can be written as

$$F_i = \int_{-\infty}^t E_i R'_i + \int_{-\infty}^t E_{if} R'_{if}$$

where  $E_i$  is the  $i^{\text{th}}$  currency value of the primary asset and  $E_{if}$  is the  $i^{\text{th}}$  currency value of the foreign exchange.

10/ Taking into account foreign exchange, the definition of  $r_i$  is  $r_i = E_i R'_i / H_i + E_{if} R'_{if} / H_i$ . Primes denote derivatives with respect to (the implied argument) time.

11/ Except where relative purchasing power parity (i.e.,  $\theta = 0$ ) is adopted for expository convenience in the appendix, PPP, in either its absolute or relative versions, is not assumed in the paper.  $\theta$  has been introduced for notational convenience only.

12/ From the criterion of monetary autonomy or the distribution of the adjustment burden, the proximate determinates of "size" are the relative magnitudes of base money markets and, especially, the abilities of the monetary authorities to sterilize. Other parameters usually regarded as defining relative country size, especially real national income or wealth, are important only if they allow the authorities to sterilize more or to the degree that they determine the size of a country's base relative to the total base money of those countries with fixed exchange rate targets. This point is demonstrated in the appendix.

13/ In the case of a pure float, the model is similar in spirit to what Frenkel has referred to as a "monetary model of exchange rate determination."

A similar model of the exchange rate is developed by Putnam and Woodbury.

14/ Unless the monetary authority is using exchange market intervention as an instrument to further domestic economic goals, an exchange rate target typically comes at the expense of domestic goals. The problem of monetary autonomy arises as a result of a fixed exchange rate target of which a fixed exchange rate is only one example. It is the rigidity of the target (which may be moving) rather than the rigidity of the rate that matters. A fixed exchange rate target means that the authorities are unwilling to trade this target off against other targets. A model is derived in the appendix in which the monetary authority with a targeted growth path for the exchange rate loses all control over their domestic money growth rate.

15/ Equation (6) can also be derived from a multi-country model as shown in the appendix.

16/ When the U.S. dollar is used as a numeraire, the second subscript, u, has been dropped in order to simplify the notation. In symbols,  $e_c = e_{cu}$ ,  $\theta_c = \theta_{cu}$  and  $\delta_c = \delta_{cu}$ .

17/ The  $h_u$  term, like the growth of base money for other countries, can be found by adding the domestic and foreign sources,  $h_u = d_u + r_u$ . But the international transactions that affect the supply of U.S. base money include official sales and purchases (not to be confused with allocations) of primary assets and not changes in foreign official holdings of U.S. dollar assets (except those that absorb base money). The  $r_u$  term, therefore, does not represent the U.S. official settlements or other measures of the U.S. balance of payments that have been traditionally employed. For non-center countries, of course,  $r_i$ , does represent the (deflated) official settlements balance of payments.

18/ Sterilization can be represented by breaking  $d_i$  into two components, viz.,  $d_i = d_i^0 - \lambda_i r_i$  where  $\lambda_i$  is a sterilization coefficient ranging between unity (for complete sterilization) and a negative number (representing a reinforcement of the balance of payments necessary to play by the rules of the gold standard). Since  $d_i^0$  is exogenous or independent of  $r_i$ , then  $h_u = d_u^0 = -\lambda_u r_u + r_u$  is independent of  $r_u$  and  $r_c$  if  $\lambda_u = 1$ .

19/ The notion of a "dollar standard" -- a phrase developed in the 1920's when the United States abandoned the rules of the gold standard -- is usually taken to mean that other countries adjust to the United States. In an important sense, non-center countries sterilize for the center country and force more of the adjustment burden on themselves when they acquire or lose dollar assets rather than outside reserve assets. Further discussion of the sense in which other countries sterilize for the U.S. and a discussion of their incentives for forcing more of the adjustment burden on themselves is found in Girton and Henderson and in Roper.

20/ Since monetary equilibrium requires  $h_u = \pi_u + \beta_u y_u - \alpha \rho_u'$ ,  $\pi_u - \alpha \rho_u'$  can be substituted for  $h_u - \beta_u y_u$  in equation (6) to obtain

$$r_c + e_c = -d_c + \beta_c y_c + \pi_u - \alpha \rho_u' - \alpha \delta_c + \theta_c.$$

21/ Two possible reasons for referring to equation (6) as a "monetary" model should be sharply distinguished. Suppose that  $d_c$  and  $h_u$  were very stable over time and that  $y_c$  and  $y_u$  were subject to large fluctuations due, say, to earthquakes. If the model were named for the independent variables with the highest variance and greatest potential explanatory power, it would be an "earthquake" model. If it is named for the fact that monetary equilibrium conditions are being used as an organizing framework for explaining  $r + e$ , it would be a "monetary" model. Following this second line of reasoning, equation (6) would still therefore, be a monetary model even if there were no variance, and no explanatory power, in the  $d_c$  and  $h_u$  variables.

22/ One might expect that when the Canadian dollar appreciates ( $e_c > 0$ ), the export industries and the import competing sector might have to contract and this could dampen current real output. If  $y_c$  is independent of the other explanatory variables this would bias  $\hat{\beta}_c$  downwards. A downward bias is consistent with some of the results that show the estimated value of  $\beta_c$  to be slightly lower for current values of  $y_c$  than for lagged values of  $y_c$  when the lags were unconstrained. If the lag between the other part of the dependent variable,  $r_c$ , and current output were sufficiently long, then the impact of  $e_c$  on  $y_c$  would be the only channel for simultaneity bias in the estimate of  $\beta_c$ .

23/ To our knowledge, the study that has come closest to separating the impact of  $r$  on  $d$  (sterilization) from the impact of  $d$  on  $r$  (the offset) is Porter. Porter estimated a monthly model of the German capital account in which his  $d$  variable was primarily a reflection of changes in reserve requirements. Since the Bundesbank alters reserve requirements at the beginning of each month, then that month's capital flow could be taken as depending on the reserve requirement changes at the first of the month. There would be no feedback from the capital movement to the policy variable unless the authorities were anticipating the future capital flows.

24/ If the variance of the random term  $v$  is labeled  $\sigma_v$  and the variance of  $d_c^0$  is  $\sigma^*$ , then the asymptotic bias is given by the formula

$$\text{plim} (\hat{\phi} - \phi) = (1-\lambda\phi) \frac{\lambda}{\sigma^*/\sigma_v + \lambda^2}$$

where the "c" subscripts have been omitted for convenience. If  $\sigma^*/\sigma_v = 0$ ,  $\text{plim}(\hat{\phi}) = 1/\lambda$ . If  $\sigma^*/\sigma_v$  approaches infinity, the bias approaches zero.

In principle,  $\hat{\theta}_c$  might be biased in the opposite direction under floating rates. If, for instance,  $d_c$  were influenced in a linear fashion by  $e_c$  in addition to other variables, then, assuming that  $e_c$  were independent of these other variables,  $\hat{\theta}_c$  would be biased toward  $(\frac{-1}{\lambda_c})$  where  $\lambda_c$  represents the linear impact of  $e_c$  on  $d_c$ . The fact that the Canadian dollar remained around unity vis-à-vis the U.S. dollar is consistent with  $\lambda_c$  being positive such that the bias would be downwards.

25/ Kouri and Porter derive a formula for the asymptotic bias that is correct (if they assume their  $X_1$  variable is independent of their NDA variable), but their verbal discussion of the bias is misleading. They assert (pp. 453-4) that their estimated coefficient (corresponding to our  $\hat{\theta}_c$ ) is biased towards unity rather than towards the reciprocal of the sterilization coefficient. The bias will be towards unity only if the sterilization coefficient is unity. If the authorities only attempt to partially sterilize, then the bias will be towards a number greater than one.

26/ See Robert Dunn for a discussion of the domestic policies and their objectives and the various Canadian-U.S. economic agreements.

27/ In principle, the international reserve figures should also be adjusted for several other types of non-market transactions. We did not do this because of data limitations.

28/ The model has no implication for the sign or significance of the constant term. The constant term is not significant.

29/ Another way of testing for the sensitivity of the dependent variable to its composition would be to split the dependent variable into its components and put one of the components on the right-hand side as an exogenous variable. But doing this would introduce a simultaneity problem since the authorities often react to exchange market pressure by both intervening and allowing the exchange rate to move.

30/ The  $w$ 's, which depend on the  $s$ 's and  $\lambda$ 's, are the relevant measure of country size in the model used in this paper. It should be noted that this measure of size depends on the assumption that domestic open market operations should be scaled by the stock of base money.

31/ If more than one country completely sterilizes the model is over determined.

## DATA APPENDIX

Canadian International Reserve figures were obtained by adjusting "Canadian Official International Reserves-Total" (series B3800 in the Bank of Canada Review (BCR)) for SDR allocations and gold revaluation profits.

These adjustments take out the effect of non-market transactions on the reserve stock figures. There are other types of non-market transactions that we have not adjusted for because of data difficulties.

Non-market transactions in

reserve assets will tend to bias the coefficient on domestic assets toward negative one. The reserve series used consists of end-of-month stock figures.  $r_c$  was calculated by first differencing the assumed average of the end-of-month reserve figures and dividing by the product of lagged values of the exchange rate and Canadian base money.

Canadian Monetary Base was obtained by adding "Total coin outside banks"

(BCR series B2003), "Total notes in circulation (BCR series B51), and chartered bank deposits at the Bank of Canada (BCR series B55). Annual averages of Wednesday figures are used.

Domestic Assets Held by Canadian Monetary Authorities were obtained by subtracting Canadian International Reserves, in Canadian dollar terms, from the Canadian monetary base.

Exchange Rate (U.S. dollar price of Canadian dollars) is the annual average of noon buying rates in New York.

U.S. Monetary Base is the annual average of the weekly figures put out by the St. Louis Federal Reserve Bank.

U.S. and Canadian Real GNP figures are annual averages of quarterly figures put out by the U.S. Commerce Department and Statistics Canada.