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THE MCKINNON PORTFOLIO BALANCE MODEL

by

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Introduction

In his closing comments at the Chicago Conference on Monetary Problems of the International Economy Harry Johnson expressed the opinion that McKinnon's contribution to the conference was of fundamental importance and would have large impact on future work.<sup>2/</sup> The insights provided by McKinnon are extremely valuable, but I argue there are fundamental problems in the analysis. In this paper I will set out a model that preserves the general tenor of McKinnon's model but that is designed to eliminate internal inconsistencies in McKinnon's treatment. Perhaps one reason why McKinnon's model has not been more widely used is because of the problems that one encounters in attempting to understand and use the model as it now stands. Also, the model will be extended to cover explicitly the effects of policy changes on the employment of labor and capital. It seems rather strange that McKinnon did not attempt to analyze the effects of policy changes on factor utilization. McKinnon's model can best be appreciated perhaps as an attempt to find out where the Keynesian short-run analysis will lead one over the long-run when asset demands are made consistent with portfolio balance considerations.

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\*I have benefited from discussions with Dale Henderson, John Morton, and Don Roper on different points raised in this paper. The opinions expressed in this paper are the author's and cannot be taken as representative of the views of anyone else in the Federal Reserve System.

1/ The model discussed here is set out and analyzed in Section I of Ronald McKinnon "Portfolio Balance and International Payments Adjustment", in Monetary Problems of the International Economy. Edited by R. Mundell and A.K. Swaboda. The University of Chicago Press, 1969. Only the basic closed economy model is discussed here.

2/ Monetary Problems of the International Economy, p. 399.

### The Model

McKinnon uses the following four equations in the closed economy version of his model.<sup>1/</sup>

- |     |                    |           |                                  |
|-----|--------------------|-----------|----------------------------------|
| 1.1 | $E(Y, M, B, K, i)$ | - $Y = 0$ | Commodity market flow condition  |
| 1.2 | $L(Y, i)$          | - $M = 0$ | Money market stock condition     |
| 1.3 | $R(Y, i)$          | - $B = 0$ | Bond market stock condition      |
| 1.4 | $A(Y, i)$          | - $K = 0$ | Commodity market stock condition |

### List of symbols

- E - Total flow magnitude of expenditures
- Y - Income
- M - Money supply
- B - Government bonds
- K - Capital stock
- i - Interest rate on government bonds
- L - Stock demand for money
- R - Stock demand for government securities
- A - Stock demand for goods (capital)

(The symbol A has been substituted for McKinnon's C)

The spirit of McKinnon's analysis is Keynesian in that wages and prices are taken as fixed, people hold stationary expectations regarding income and the interest rate, and the relevant income variable is assumed to be actual measured income. The focus of the analysis is, however, on long-run asset equilibrium, not the short-run asset disequilibrium situation at which Keynesian analysis is usually directed.

<sup>1/</sup> McKinnon, "Portfolio Balance and International Payments Adjustment", p. 210.

In McKinnon's equilibrium the capital stock is in equilibrium so no new investment is occurring. Also, McKinnon makes the demand for securities explicit and consistent with portfolio balance theory. He makes equilibrium asset demands depend only on measured income and the interest rate.

There was apparently some disagreement at the conference on whether the model is long-run or short-run in nature. On the ~~one~~ hand Johnson in his summary discussion said that the model is directed at long-run problems in terms of a long-run stationary state analysis, while Kruger in her comments said that "as the model presently stands, it is manifestly short-run in nature."<sup>1/</sup> The divergent interpretations arise perhaps first, because McKinnon deals with a period of time long enough for asset markets (including the market for physical capital) to clear for asset demands that do not depend on initial wealth levels; but a period of time short enough for prices to be treated as fixed. This period of time may not exist. Second, without proper explanation McKinnon puts together three long-run asset equilibrium equations with one shorter-run adjustment equation for the goods market. The

1/ Monetary Problems of the International Economy, pp. 399 and 249.

short-run adjustment equation does not play any role in the McKinnon's long-run comparative statics analysis.<sup>1/</sup>

McKinnon lumps together firms and households and writes demand functions that include both sectors' demands, which I think introduces some possibility of confusion. Here a distinction will be made between demands by ultimate wealth holders and demands by firms. Firms own physical capital (K) and employ labor (N) to produce a single output (Q). Profit maximizing behavior by firms is assumed. Households consume goods and hold money (M), variable interest rate government bonds (B), and claims on firms (V). To simplify, without

<sup>1/</sup> Perhaps the implications of the asset equations in the model for the consumption function should be spelled out. If ultimate wealth holders long-run demands for money and securities are satisfied, for the existing values of income and the interest rate, then people will be consuming all of their incomes. Savings is done to acquire wealth and once wealth is at desired levels there is no longer any reason to save, i.e., consumption (C) is equal to income. It is important to realize that the consumption equal income relationship is not an identity and does not hold except in full asset equilibrium. A consumption function consistent with the asset relationships is  $C = Y - \alpha[L(Y,r) + R(Y,r) - (M+B+V)]$  where  $\alpha$  is an adjustment coefficient. This consumption function can be written in the more familiar form of  $C = f(Y,r,W)$  where  $W = M+B+V$ . When  $L(Y,r) + R(Y,r) = M+B+V$  then  $C = Y$ . The asset equilibrium conditions and the consumption income equality are interdependent. I drop the latter for the long-run comparative statics analysis done here.

altering the qualitative comparative statics conclusions, it is assumed that the firms hold neither money nor government bonds, and households do not hold stocks of goods.

McKinnon uses only one interest rate explicitly in his model though he considers ~~two separate~~ assets that provide the owner with a yield. His justification for doing this is "The return on the most liquid of these ownership claims... ownership claims on K... which bear interest would be a rate of interest equal to  $i$ ."<sup>1/</sup> Some of the policy shifts that McKinnon contemplates change supplies of government securities relative to ownership claims on firms, without presumably altering the relation between the rates of return. How can this be except if the market treats the assets as perfect substitutes, at least for ranges of relative asset supplies considered? At the risk of possibly misinterpreting the "intent" of McKinnon, I assume that rates of returns on government bonds and ownership claims on firms to be equal because households treat the assets as perfect substitutes.<sup>2/</sup> The demand for government bonds and claims on firms then is written as the demand for the sum of non-monetary assets of households.

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<sup>1/</sup> McKinnon, p. 208.

<sup>2/</sup> Treating all securities as a homogeneous bundle, I think, is a useful and appropriate simplification at this level of abstraction, given the problems that McKinnon's model was built to deal with.

- 1)  $F = F(Y, i)$ , where
- 2)  $F = B + V$ . In equilibrium
- 3)  $B + V = F(Y, i)$ .

Equation (3) replaces McKinnon's equations

1.3  $B = R(Y, i)$  and

1.4  $K = A(Y, i)$ .

The money equilibrium condition is

- 4)  $M = L(Y, i)$ , which is the same as McKinnon's condition.

The fixed output price implies that the typical firm treats the demand for its output as given.<sup>1/</sup> It is assumed that firms are competitive in the market for financial capital. The firm maximizes profits under conditions of fixed wages, elastic supply of funds, and rigid output price. The firm will adjust its use of capital and labor so as to produce the fixed output that it can sell at the going price, for the least cost -- i.e., maximize profits. The cost minimizing condition is for the firm to use capital and labor in amounts so that the ratio of the marginal products is equal to the ratio of the rental rates.<sup>2/</sup> The marginal product of capital here has only a time dimension

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<sup>1/</sup> Mundell, "An Exposition of Some Subtleties of the Keynesian System", *Welwirtschaftliches Archis* (Dec., 1964) p. 220.

<sup>2/</sup> Only fortuitously will the maximum profit position of the firm under these conditions be at a point where the marginal product of labor is equal to the wage rate and the marginal product of capital is equal to the interest rate. The fixed amount of output that the typical firm can sell, given the fixed price of its output, prevents the firm from moving to a position where the marginal products are equal to the rental rates.

since capital is merely accumulated consumption goods. The opportunity cost of holding capital is the interest rate -- the cost of borrowing funds to finance the holding of capital per time period. Algebraically the condition is

5)  $\frac{w}{i} = \frac{MPN(K,N)}{MPK(K,N)}$ , <sup>1/</sup> where  $w$  is the wage rate and  $MPN(K,N)$  and  $MPK(K,N)$  are respectively the marginal products of labor and capital.

If production takes place under conditions of constant returns to scale,  $w$  and  $i$  will determine the capital/labor ratio. The output demand together with equation 5, and the production function,

6)  $Y = Nf(k)$ , where  $k = K/N$ , will determine  $K$  and  $N$  given  $w$  and  $i$ . <sup>2/</sup>

The financial claims on a firm will earn their holders a stream of income that will be the difference between the revenue of the firm and the wage bill. Given the assumptions made here this earnings stream will be expected to continue indefinitely at the present level. The aggregate value of claims on firms will be

7)  $v = \frac{Y - wN}{i}$  .

We have then the following equations in the comparative statics model.

<sup>1/</sup> See Henderson and Quandt, Microeconomic Theory, New York: McGraw-Hill 1958, p. 51.

<sup>2/</sup> Since government spending and taxes are not needed to make the points I want to make here, I assume they are zero except for a lump sum tax on the population to pay for the interest on the government debt. This lump sum tax will change, of course, whenever  $i$  or  $B$  changes. See McKinnon's article for a discussion of the model with positive government spending and taxes. Government debt is treated as outside debt. Private disposable income is equal to the value of output since taxes just match the interest payment on government debt.



$$\text{I } M = L(Y, i) \quad \underline{1/}$$

$$\text{II } B = F(Y, i) - \left(\frac{Y - WN}{i}\right)$$

$$\text{III } Y = Nf(k)$$

$$\text{IV } \frac{w}{i} = \frac{f - kf'}{f'} \quad \underline{2/}$$

These conditions are sufficient to determine the comparative statics equilibrium values of the four variables  $Y$ ,  $i$ ,  $K$  and  $N$  for given levels of  $M$  and  $B$ .

Equations I-IV differ from McKinnon's equations 1.2, 1.3 and 1.4. McKinnon seems to have misspecified the asset equilibrium conditions. It is argued here that the inclusion of only one interest rate means that the demand for non-monetary assets is a demand for their sum. On the demand side McKinnon's  $R(Y, i)$  and the part of  $A(Y, i)$  that represents claims on firms has been aggregated to  $F(Y, i)$ . Firms have a demand for physical capital given by the production function (equation III), the marginal condition (equation IV), and the levels of  $Y, i$ , and  $w$ . In the usual Keynesian analysis the capital stock is taken as fixed at other than its equilibrium value -- i.e., net investment is taking place. Mundell has argued that in the short-run Keynesian model with fixed prices, the marginal condition does not hold because with fixed capital stock the only decision

1/ I am assuming that there are no bank or non-bank financial intermediaries in the system.

2/ For the production function (III)  $Y = Nf(k)$ , the  $MPN = f - kf'$  and the  $MPK = f'$ , where  $f'$  denotes the partial derivative of  $f$  with respect to  $k$  and  $f'' < 0$ .

the firm faces is how much labor to hire in order to produce the output demanded -- assuming this can be done profitably.<sup>1/</sup> However, when the firm can choose its capital stock, and get the capital it wants, then profit maximization will establish the marginal condition (equation IV). Equations (III) and (IV) replace the part of McKinnon's equation 1.4 that represents demand for physical capital by firms. Given that prices are fixed, the value of claims on firms will only fortuitously be equal to the value of the capital owned by firms. The level of  $V = \frac{Y - wN}{i}$  replaces  $K$  as the supply of claims on firms available to ultimate wealth holders.

Using the equations I-IV the model can be analyzed graphically. Equation I is the same as McKinnon's money market equilibrium equation and is plotted as the MM curve in Figure 1. The MM curve has a slope of  $\frac{di}{dY} = \frac{L_Y}{L_i} > 0$ .<sup>2/</sup>

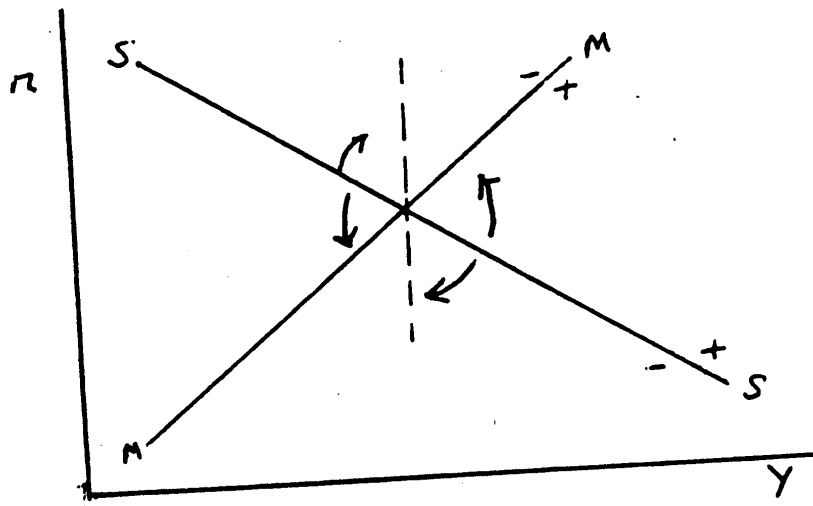


Figure 1

<sup>1/</sup> Mundell, "An Exposition of Some Subtleties of the Keynesian System."

<sup>2/</sup> Subscripts Y and i denote partial derivatives with respect to these variables.

The SS curve is the securities market equilibrium schedule, taking into account the supply of claims on firms in addition to the value of government bonds. Differentiating (II), holding B and w fixed

the following is obtained:

$$8) 0 = (F_Y - 1/i) dY + (F_i + V/i) di + w/i dN.$$

Differentiating the production function and the marginal condition, eliminating dk and solving for dN as a function of dr and dY, then

$$9) dN = 1/f dY - \frac{wf'}{cf''f} di, \text{ where } c = \frac{wf}{(f-kf')^2} > 0. \frac{1/}{}$$

Substituting this expression into equation 8 and solving for the slope in r, Y space, then

$$10) \frac{dr}{dY} = \frac{(F_Y - \frac{V}{Y})}{(F_i + \frac{V}{i} - \frac{w}{i} \frac{Nf'}{c f f''})} \geq 0.$$

To the right of the MM curve the demand for money is greater than the fixed supply of money implicit in the curve and to the left the demand for money is smaller than the supply of money. The + and - signs on the MM curve denote areas of excess demand for and supply of money. The + and - sign on the SS curve denote areas of excess demand

1/ Differentiating the marginal condition (IV)  $\frac{w}{i} = \frac{f - kf'}{f'}$ , holding w constant then  $dk = \frac{1}{cf''} di$ . Differentiating the production function (III)  $Y = Nf(k)$  we get  $dY = Nf' dk + fdN$ . Substituting into the expression for dy the expression for dk obtained by assuming firms are minimizing cost, then  $dY = \frac{Nf'}{cf''} di + fdN$ . Solving for dN one obtains

$$dN = \frac{1}{f} dY - \frac{wf'}{cf''f} di.$$

for and supply of securities. The slope will be negative if the partial effect of an income change on the demand for securities exceeds the value of the partial effect of an income change on the supply of claims on firms, and the slope will be positive if the reverse is the case. <sup>1/</sup>

One cannot be certain of the sign of the slope of the SS curve. It appears that the definite negative sign that McKinnon gave to the slope of his bond curve was the result of misspecification of the asset equilibrium conditions. I will assume

that if the SS curve is positively sloped, its slope is less than the slope of the MM curve. <sup>2/,3/</sup> Along the SS curve the demand for securities is equal to the fixed level of government bonds plus the supply of claims on firms, where the supply of claims on firms is consistent with cost minimizing behavior.

<sup>1/</sup> The denominator of the slope must be negative since each term is negative. The numerator is  $F_Y - \frac{1}{i} \left( \frac{f - w}{f} \right) = F_Y - \frac{1}{i} \left( \frac{Nf - Nw}{Nf} \right) =$

$F_Y - \frac{V}{Y} = F_Y - \left( \frac{V + B}{Y} \right) + \frac{B}{Y}$ . If the marginal effect of the level of income on the demand for securities is not smaller than the ratio of security holdings to income by more than the ratio of government debt to income then the SS curve must have a negative slope. A positive sloped SS curve would not seem to be an unlikely possibility. For latter reference ~~one should note that if the security demand function~~ is homogeneous of degree one in income (here real income) and  $B = 0$  then the SS curve will have a slope of zero.

<sup>2/</sup> The curved arrows are put in to indicate graphically the range of the admitted slopes of the SS curve.

<sup>3/</sup> One would like to set out some simple dynamic postulates and argue that the SS-curve having a slope greater than the MM curve would not be consistent with stability. There are, however, some not so obvious problems in setting out dynamic adjustment equations that are consistent with this type of model. It would seem natural to make  $dY/dt$  depend on the flow excess demand for  $(x_g)$ , which consists of two parts:  $x_g = (c - Y) + dK/dt$ , where C is the consumption demand for goods. There are at least two problems here. First, when the capital stock is not at desired levels the marginal condition as have written it will not hold. Second, the proper expression for flow investment demand  $(dK/dt)$  is difficult to formulate.

One can perform the experiments done by McKinnon but with a clearer understanding of what is happening. The MM curve is identical to his money equilibrium curve, but the SS curve incorporates the ~~iterations~~ iterations made here in McKinnon's model. The qualitative comparative statics conclusions will be the same as those obtained by McKinnon as long as the SS curve is assumed to have a negative slope.

By more clearly specifying the model we can ask a question not answered, or even asked, by McKinnon. Does the long-run level of employment of labor and capital necessarily move in the same direction as income in this model?

Clearly labor employment must increase in the new long-run equilibrium if the shock is an outside injection of government securities, and employment of capital must increase in the new long-run equilibrium if the shock is an outside injection of money.<sup>1/</sup> In each case the effects of the expansionary policy on the level of output and the capital/labor ratio reinforce each other in their effects on labor employment in the first case and capital employment in the second case.

However, an outside increase in the money supply may have opposing influences on the level of employment of labor in the long-run, and an outside increase in the supply of government bonds has contradictory effects on the long-run employment of capital.

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<sup>1/</sup> The outside injection of money or government bonds can be viewed as being made as a transfer payment.

To facilitate the graphical analysis of the question of the effects of policy changes on the long-run utilization of capital and labor, two additional curves are drawn in Figure 2. The  $\bar{N}$  curve represents combinations of  $r$  and  $Y$  that will keep the profit maximizing utilization of labor constant and the  $\bar{K}$  curve represents combinations of  $r$  and  $Y$  that will keep the profit maximizing demand for capital constant. The  $\bar{N}$  curve must have a negative slope and the  $\bar{K}$  curve must have a positive slope. <sup>1/</sup> Policy changes that cause the stock of capital to rise will shift the intersection of the  $MM$  and  $SS$  curves

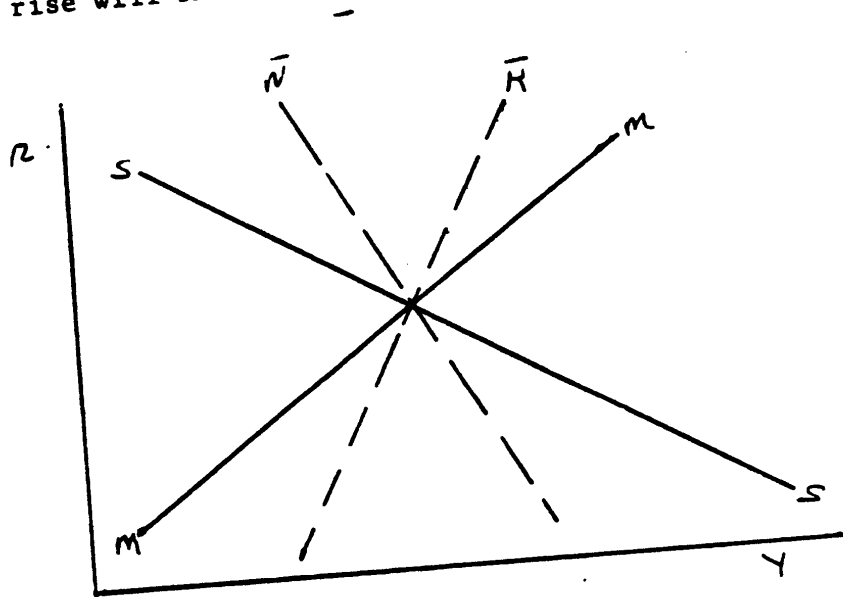


Figure 2

<sup>1/</sup> In footnote 1 page 9  $dN = \frac{1}{f} dy - \frac{Nf'}{cf''f} di$  was derived by assuming the production function and the marginal conditions hold. Solving for the  $dY$  and  $di$  that will make  $dN = 0$ , we get  $\frac{di}{dY} \Big|_{\bar{N}} = \frac{cf''}{Nf'} < 0$ . The definition of  $k$  differentiated gives  $dK = Ndk + kdN$ . Substituting in the  $dN$  that satisfied the production conditions, and  $dk = \frac{1}{cf''} di$  derived footnote 1 p. 9, then  $dK = \frac{N(f-kf')}{cf''f} di + \frac{K}{Y} dY$ . Solving for the ratio of  $di$  to  $dY$  that will keep the desired capital stock constant then  $\frac{di}{dY} \Big|_{\bar{K}} = -\frac{kcf''}{Y-kf'} > 0$ .

into the area to the right of the  $\bar{K}$  curve and policy changes that cause the employment of labor to rise will shift the intersection of the MM and SS curves into the area to the right of the  $\bar{N}$  curve. Areas to the left of the  $\bar{K}$  curve and the  $\bar{N}$  curve are analogously defined. By use of the diagram one can see the effects of policy changes on ~~long-run capital and labor market utilization~~, as well as on income and the interest rate.

Policy: Comparative Statics Analysis

The model can be solved algebraically for Y, i, K and N for given changes in the policy variables by differentiating equations I-IV and using Cramer's rule. Doing this and substituting for dk and dN in the money and securities equations then in matrix form

$$\begin{vmatrix} dM \\ dB \end{vmatrix} = \begin{vmatrix} L_Y & L_i \\ (F_Y - \frac{V}{Y}) & (F_i + \frac{V}{i} - \frac{wNf'}{icff''}) \end{vmatrix} \cdot \begin{vmatrix} dY \\ di \end{vmatrix}.$$

I assume the determinant of the coefficient matrix is positive ( $\Delta > 0$ ). This is the same condition as slope MM - slope SS  $> 0$ .

By using  $dN = \frac{1}{f} dY - \frac{Nf'}{cf''f} di$  and  $dK = \frac{K}{Y} dY + \frac{Nf'}{c f f''} \frac{w}{i} di,$

which are implicit in the solution for di and dY, dN and dK can be solved for explicitly.

An increase in the supply of money by the outside method will shift the MM curve to the right. In the new equilibrium income must be higher than in the initial equilibrium position and interest

rates will be lower if the SS curve is negatively sloped, i.e.,  $F_Y > \frac{V}{Y}$ . If the SS curve is positively sloped, i.e., the supply of securities is more responsive to income changes than is the demand for securities, then an outside increase in the money supply will increase interest rates.

The effect of the outside increase in the money supply on the long-run employment of capital will be positive unless the SS curve is positively sloped, and has a greater slope than the  $\bar{K}$  curve. A necessary though not sufficient condition for  $dK/dM < 0$  is  $F_Y < \frac{V-K}{Y}$ .<sup>1/</sup> While  $dK/dM < 0$  cannot apparently in principle be ruled out it would seem to be an unlikely case.

An outside increase in the money supply will increase employment unless the  $\bar{N}$  curve is flatter than the SS curve (and the SS curve is negatively sloped). A necessary though not sufficient condition for an increase in the money supply to reduce employment is that  $F_Y > \frac{1}{i} = \frac{W^*}{Y}$ , where  $W^* = \frac{Y}{i}$  is totally implicit

<sup>1/</sup> The effects on  $Y$ ,  $i$ ,  $N$ , and  $K$  of a change in money by outside method are

$$dY/dM = \frac{F_i + \frac{V}{i} - \frac{WNf'}{icff''}}{\Delta} > 0,$$

$$di/dM = - \frac{(F_Y - \frac{V}{Y})}{\Delta} \geq 0$$

depending on the slope of the SS curve, and

$$\frac{dK}{dM} = \frac{K}{Y} (F_i + \frac{V}{i}) + \frac{Nf'w}{cff''i} [F_Y - \frac{(V-K)}{Y}]$$



wealth. <sup>1/</sup> One would think it unlikely that the income responsiveness of the demand for financial claims only will exceed the ratio of implicit wealth to income.

It was pointed out previously that if there are no government securities in the system and the demand function for securities is homogeneous in income then the SS curve is flat. In this case an outside injection of money results in an unchanged interest rate, increased income, and higher levels of utilization of both labor and capital. This is a curious result. Outside injections of money do not affect the interest rate but do effect other real magnitudes in the economy. It can be shown that a flexible price version of this model will demonstrate full classical neutrality under the same sort of assumptions. The only difference is the weaker assumption of homogeneity in prices, but not real income, in the demand for securities is needed.

<sup>1/</sup> The expression for the effect of an outside increase in the money supply on the employment of labor is:

$$dN/dM = \frac{N}{Y} \left[ F_i + \frac{V}{i} + \frac{Nf'}{cf''} \left( F_Y - \frac{V}{Y} - \frac{wN}{iY} \right) \right]$$

Substituting  $\frac{wN}{iY} = \frac{Y}{iY} - \frac{wN}{iY} - \frac{Y}{iY} = \frac{V}{Y} - \frac{1}{i}$  into the above expression,

$$dN/dM = \frac{N}{Y} \left[ F_i + \frac{V}{i} + \frac{Nf'}{cf''} \left( F_Y - \frac{1}{Y} \right) \right].$$

Another method of changing the money supply is by an open market operation. This involves both a change in M, shifting the MM curve, and a change in B, shifting the SS curve. An open market operation can be viewed as a combination of an "outside" change in M and an "outside" change in B. Before analyzing this policy combination it is useful to analyze the effect of an outside increase in government bond supplies in isolation.

An outside increase in government bonds shifts the SS curve up increasing the equilibrium level of income and the interest rate. The employment of labor must rise since both income and interest rate changes work in the same positive direction. The increase in government securities drives up interest rates, leading to a shift to more labor intensive production. <sup>1/</sup> The employment of capital in the new equilibrium may increase or decrease depending on the relative slopes of the MM and  $\bar{K}$  curves. If  $dK/di$  is small then the increase in interest rates will have only a small substitution effect and the employment of capital will expand as the demand for output rises. <sup>2/</sup> A decline in the equilibrium capital stock would seem to be a possibility.

<sup>1/</sup> It is assumed that the factor labor is raw labor with no embodied human capital. The source of the labor service flow is not bought or sold and is fixed in total amount.

<sup>2/</sup> The effects on Y, i, N, and K of a change in government bonds supplied to the public by outside methods are

$$\frac{dY}{dB} = \frac{L_i}{\Delta} > 0, \quad \frac{di}{dB} = \frac{L_Y}{\Delta} > 0, \quad \frac{dN}{dB} = -\frac{N}{Y} \frac{\left(L_i + \frac{Nf'}{cf''} L_Y\right)}{\Delta} > 0,$$

and

$$\frac{dK}{dB} = \frac{\frac{Nf'w}{c f f'' i} L_Y - \frac{K}{Y} L_i}{\Delta} \geq 0.$$

An expansionary open market operation will shift the MM curve to the right and the SS curve down. Interest rates will be lower in the new equilibrium if the SS curve is negatively sloped. Only if the SS curve is positively sloped and  $L_Y + F_Y - \frac{V}{Y} < 0$  will interest rates rise. Income will expand if the demand for the total of financial wealth is a ~~positive~~ function of interest rates, which we assume to hold. <sup>1/</sup> The demand for capital in the new equilibrium position will have increased except if  $F_Y + L_Y - \frac{(V-K)}{Y} < 0$ , surely a

remote possibility. <sup>2/</sup> The long run employment of labor will increase unless  $L_Y + F_Y > 1/i$ . The drop in interest rates has a more severe dampening effect on long-run labor employment when the money supply is increased by ~~inside~~ methods than when money is increased by outside methods because the decline in the supply of government securities intensifies the interest rate decline.

1/ Solving the equation system set out previously for  $dB = -dM$  we get

$$\frac{dY}{dM} = \frac{(L_i + F_i + \frac{V}{i} - \frac{wNf'}{icff''})}{\Delta}, \text{ and } L_i + F_i \equiv W_i > 0,$$

where  $W_i$  is the partial effect of the interest rate on the demand for total financial assets. Also,

$$\frac{di}{dM} = \frac{(L_Y + F_Y - \frac{V}{Y})}{\Delta}.$$

2/ The expressions for  $dK$  and  $dN$  are

$$\frac{dK}{dM} = \left[ \frac{K}{Y} (L_i + F_i + \frac{V}{i}) - \frac{Nf'}{c f f''} \frac{w}{i} (L_Y + F_Y - \frac{V-K}{Y}) \right] \text{ and}$$

$$\frac{dN}{dM} = \frac{N}{Y} \left[ (L_i + F_i + \frac{V}{i}) + \frac{Nf'}{c f f''} (L_Y + F_Y - 1/i) \right]$$

-Δ

One cannot carelessly identify the affects of policy changes on income with the effects on long-run labor employment. Policy changes may, in the long-run, alter capital to labor ratios. Inside changes in the supply of money especially may have very different affects on the employment of labor and the level of income.

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