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ABSTRACT

Data on contestants' choices in Italian Game Show *Affari Tuoi* are analysed in a way that separates the effect of risk attitude (preferences) from that of beliefs concerning the amount of money that will be offered to contestants in future rounds. The most important issue addressed in the paper is what belief function is actually being used by contestants. The parameters of this function are estimated freely along with the parameters of a choice model. Separate identification of the belief function and preferences is possible by virtue of the fact that at a certain stage of the game, beliefs are not relevant, and risk attitude is the sole determinant of choice. The rational expectations hypothesis is tested by comparing the estimated belief function with the "true" offer function which is estimated using data on offers actually made to contestants. We find that there is a significant difference between these two functions, and hence we reject the rational expectations hypothesis. However, when a simpler "rule-of-thumb" structure is assumed for the belief function, we find a correspondence to the function obtained from data on actual offers. Our overall conclusion is that contestants are rational to the extent that they make use of all available relevant information, but are not fully rational because they are not processing the information in an optimal way. The importance of belief-formation is confirmed by the estimation of a mixture model which establishes that the vast majority of contestants are forward-looking as opposed to myopic.

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1. Introduction

When an individual is faced with a choice problem under risk, that individual's risk attitude is the principal determinant of their choice. However, this assumes that the payoff from the choice is instantaneous. If the payoff is made at some future date, and the eventual payoff is dependent on the state of the world at that future date, or on some intervention by others, then the individual's beliefs of what will pertain at that future date must enter the decision making process. Obvious examples are found in economics: schooling decisions depend on individuals' beliefs about the structure of the labour market that will pertain several years in the future; firms base their investment and production plans on beliefs about the future evolution of consumer preferences.

Researchers analysing such decisions clearly cannot rely solely on choice data, because any observed choice is usually compatible with many different combinations of risk attitudes and beliefs. This is a problem that is frequently encountered in experimental economics, although not all researchers appear to recognise it or to pay due attention to it.

One notable exception is Manski (2002), who analyses the problem in the context of an ultimatum game. He emphasises that the proposer's decision depends on her subjective probability distribution of the respondent's possible reactions, and that knowledge of the observed decision alone is insufficient to identify the proposer's decision-making process. In contexts such as this, researchers have tended to appeal to rational expectations theory, and to assume that agents form the same beliefs as would a researcher with access to econometric estimation facilities. This, of course, places structure on agent's beliefs, allowing the researcher to isolate risk attitudes (henceforth "preferences"). However, although many tests of the rational expectations hypothesis appear in the macroeconomics literature (see Attfield et al., 1991), there is little evidence of whether agents behave according to the rational expectations hypothesis in micro-economic contexts.

An alternative approach, reviewed by Manski (2004), is to ask individuals directly about their beliefs. Bellemare et al. (2005) follow this approach, again in the context of an ultimatum game. They find that a model estimated with this information on beliefs incorporated, has higher predictive power than a model based on the assumption of rational expectations. However, belief elicitation can itself cause problems: Rutström and Wilcox (2006), in an experiment on the repeated matching pennies game, find that agents tend to alter their strategies when asked to state their beliefs about opponents' behaviour.

In recent years, the analysis of data from television game shows has become a popular means of analysing individuals' behaviour under risk. It is obvious why researchers favour this sort of data. Game shows provide a good natural context in which contestants face well-defined decision problems in a *ceteris paribus* environment. Furthermore, it cannot be denied that contestants have salient incentives, allowing studies using such data to overcome both the Harrison and List (2004) and the Rabin (2000) critiques.[1] One game show that scores particularly highly on these criteria is the Italian show *Affari Tuoi*, data from which is analysed in this paper. This game is played in many different countries under different names and with slightly different rules. Researchers seem to be in agreement on the usefulness of the resulting data: Bombardini and Trebbi (2007) assert that *Affari Tuoi* "presents several features that we would have chosen, were we to design such an experiment"; Post et al (2008) describe the Dutch version of the same game as having "such desirable features that it almost appears to be designed to be an economics experiment rather than a TV show".

The rules of *Affari Tuoi* will be explained in detail in Section 2. For the time being, let us simply recognise that the contestant faces a sequence of choice problems, in which the choice is between an uncertain lottery, and a certain amount offered by the "Banker", and that if the

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[1] See, among the others, Friend and Blume (1975), Gertner (1993), Metrick (1995), Beetma and Shotman (2001) and Hartley et al. (2005) for an analysis of individual risk attitude through TV games.

Banker's offer is accepted, the game ends. Many researchers have now realised that, at each stage of the game, a typical contestant is not treating the choice problem as a single isolated task, but is instead forming beliefs of what will happen in future rounds, and using these beliefs in making their decision in the current round. In particular, they are forming an expectation of what the Banker's offer will be in future rounds, should they stay in the game. It is clear that such beliefs have the potential to influence the current decision. It is also clear that this setting bears similar features to that of the ultimatum game considered earlier, with the attendant difficulties in separately identifying preferences and beliefs.

There has been a large volume of recent research analysing data from the various different versions of this game show (see the survey of Andersen at al., 2007). Much of this research has focused on the search for the best characterisation of behaviour under risk (e.g., Botti et al., 2008). Typically, the assumption of rational expectations is implicitly adopted, whereby the inferential problem of predicting the Banker's offer is solved outside the choice model. This is done in a variety of ways, usually based on parametric characterisations (e.g., de Roos and Sarafidis (2006), Deck et al. (2008) and Mulino et al. (2006)).

In this paper, we treat the formation of beliefs about the Banker's offer as the central focus. In particular, by incorporating a predicted Banker's offer equation into the choice model, we are able to estimate the equation that contestants actually use to form beliefs. We are then able to compare this true belief equation with the equation that would be used under the assumption of rational expectations, hence enabling a formal test of the rational expectations hypothesis.

With these objectives in mind, we note that the game show has a peculiar structure: in the final round of the game, contestants' choices unequivocally reveal information on their risk aversion (since there is no contamination from beliefs about future rounds). The information on risk attitude extracted from the choice made in the final round is combined with information on choices from earlier rounds in order to identify the parameters of the belief function that is used by contestants in these earlier rounds.

In addition to testing for rational expectations, we provide an assessment of the validity of another assumption that has been made in previous work: that *all* contestants are forward-looking. While it may seem natural for contestants to base their decisions on their beliefs of what will unfold in future rounds, it is doubtful that every contestant behaves in this way. We would therefore like to allow for a proportion of the population to be forward-looking, and for the remainder to be "myopic", that is, to base their choice solely on the possible outcomes from the current round. This leads us to a "mixture model", of the type estimated in very similar contexts by Conte et al. (2008) and Harrison and Rutström (2008). One of the parameters in the mixture model is the "mixing proportion" which represents the proportion of the population who are forward-looking. This parameter is estimated along with the preference estimates for both models and the belief estimates for the forward-looking model.

In Section 2, the rules of *Affari Tuoi* are explained in detail. Section 3 provides a theoretical analysis of the choice problem perceived by contestants, and a discussion of the identification problem. In Section 4, we construct the log-likelihood function for the choice model, and describe our chosen method for maximising it. Section 5 presents the results from the choice model, and also reports the result of a test of the rational expectations hypothesis. Section 6 reports on the estimation of the choice model with an alternative belief function, to which we refer as the "rule-of-thumb" belief function, and which appears to fit the choice data better than the function assumed in Section 5. Section 7 estimates a model that assumes that the belief function for future offers includes a stochastic component. Section 8 reports on the results of a mixture model which allows the co-existence of myopic and forward-looking contestants. Section 9 concludes. Appendix 1 contains the results of a Monte Carlo study whose purpose is to verify the identification of the choice model. Appendix 2 reports on our analysis of the data on Banker's offers, and obtains the functional forms and fitted equations that are used in the test of the rational expectations hypothesis in Section 5.

2. The game

Affari Tuoi is a 5-round stop-and-go game between a contestant and a Banker. The game starts with 20 contestants, one from each of the 20 Italian regions. They are each randomly assigned a sealed box, containing one of the 20 prizes displayed in Table 1. The show begins by contestants answering a general knowledge question. The first contestant to answer correctly is selected to play the game.

€ 0.01	€ 5,000
€ 0.20	€ 10,000
€ 0.50	€ 15,000
€ 1	€ 20,000
€ 5	€ 25,000
€ 10	€ 50,000
€ 50	€ 75,000
€ 100	€ 100,000
€ 250	€ 250,000
€ 500	€ 500,000

Table 1: List of prizes as displayed to contestants.

In each of the 5 rounds, the selected contestant opens a fixed number of boxes (6 in the first round, then groups of 3 boxes); on each occasion that a box is opened, the cash value of that box is revealed, indicating a sum of money which is no longer available to the contestant.

At the end of each round, the Banker makes a proposal: he either offers "the swap", that is, the opportunity to change her box with one of the remaining boxes of her own choosing; or, he offers a definite amount of money to her to quit the game. Throughout the paper, we refer to this definite money amount as the *Banker's offer*. If the contestant accepts the Banker's offer, the game ends; otherwise she proceeds to the next round. If the contestant reaches the final round, and rejects the Banker's offer in this round, she receives the content of the box in her possession at that time.

Our sample consists of 298 showings, and therefore contains data on 298 contestants' decisions[2]. Figure 1 shows, for each round, the number of contestants receiving a Banker's offer and the number of contestants accepting that offer. Figure 2 shows the proportion of contestants accepting the Banker's offer by round. It is calculated by dividing the number of contestant accepting an offer in any round by the number of contestants receiving an offer in that round. This proportion is seen to rise dramatically in the course of the game.

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[2] For a more detailed description of the data set used in this paper, see Botti et al. (2007, 2008).
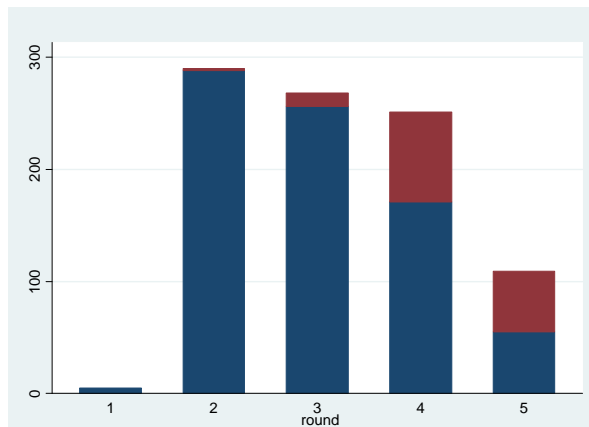
Figure 1: The number of contestants receiving a Banker's offer in each round (column height) and the number of contestants accepting the offer (red section)
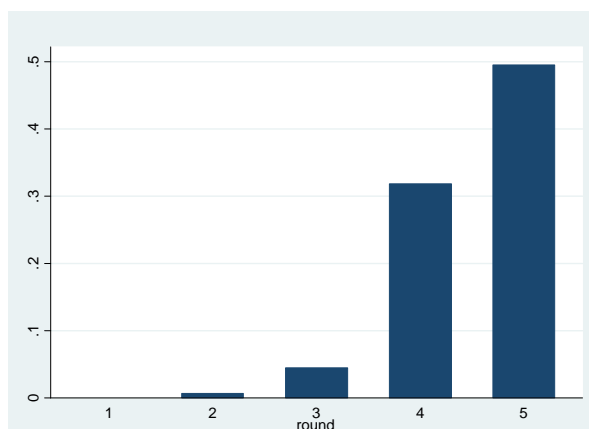

Figure 2: The number of contestants accepting an offer in each round as a proportion of the number receiving an offer in that round.

In the analysis that follows, we restrict attention to decisions made in rounds 3, 4 and 5, so we lose the four contestants who accept the Banker's offer in round 2, leaving a sample of 294. Our principal reason for restricting attention to the last 3 rounds is that, since offers are rarely accepted in rounds 1 and 2, there is insufficient variability in the data to explain the choice process. We also focus attention solely on the instances when the Banker makes a monetary offer; we do not analyse the behaviour of contestants when offered the "swap", previously mentioned. The reason is that we do not consider the "swap" decision to be informative: a rational contestant must be indifferent between swapping and not swapping.[3] Of course, this assumes that the contestant has no information whatsoever about the content of their own box. It is a simple matter to test this assumption econometrically. To such a test we now turn.

It is often claimed (e.g. de Roos and Sarafidis, 2006; Mulino et al., 2006) that, in the Italian game, since the Banker knows the contents of the contestant's box, he tends to base his offer on this information. This could raise the possibility of strategic behaviour by contestants: that they base their decision on information extracted from the Banker's offer. Let us briefly investigate this claim. The following table shows the correlation coefficient between the offer measured as a proportion of the expected value of the lottery, and the content of the contestant's box. These correlations are computed separately for the three rounds.

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[3] Blvatskyy and Pogrebna (2006) focus attention on the *swap* in *Affari Tuoi*.

	correlation	*p*-value
Round 3	0.086	0.160
Round 4	0.092	0.146
Round 5	-0.045	0.636

Table 2: Correlation per round between the Banker's offer measured as a proportion of the expected value of the lottery, and the content of the contestant's box.

We see that these correlations are small in magnitude, and none of them is significantly different from zero. Hence we are *not* able to corroborate the claim that the Banker's offer contains information about the content of the contestant's box.

Regardless of this finding, let us investigate whether the contestant is able to extract information, from any source, about the contents of their box. To this end, we simply investigate the determinants of the contestant's decision to accept the "deal". We estimate a random effects probit model of "deal" acceptance, with, as explanatory variables, the expected value of the lottery (EV), the offer as a proportion of EV, the standard deviation of the lottery, and the content of the contestant's box.

Decision to accept the Banker's offer ("deal")	
EV	-0.00090 (0.00223)
Banker's offer ÷ EV	2.90676 (0.30181)
Std. Dev. lottery	0.00777 (0.00201)
Content of contestant's box	-0.00045 (0.00065)
intercept	-2.66549 (0.22340)
Number of contestants(n)	631
Average number of choices per contestant	2.1
Log-likelihood	-278.26633

Table 3: Results from a random effect probit regression of the contestants' decision to accept "deal".

The most important conclusion from the results in Table 3 is that the content of the contestant's box appears to have no effect on the contestant's decision. From this we infer that, even if it were possible for the contestant to glean information from the Banker's offer or from another source, there is no evidence that such information is being used by contestants in forming their decisions.

3. Modelling choice rules

In the final stage of the game, the contestant is offered a one-shot choice between participating in a lottery with two equi-probable outcomes (the remaining two prizes), or accepting the final offer made by the Banker. In any given round other than the final round, contestants are not evaluating a one-shot lottery, but a sequence of nested lotteries, and the offer in later rounds might be higher (or lower) than the one they currently face. Every possible lottery that might be encountered in future rounds needs to be considered. For each of these possible lotteries, an expectation must be formed of the Banker's offer that might be made. Let $\tilde{off}_t(X)$ be the random variable representing the Banker's offer in future round t, given the set of remaining prizes X.

Let $G\left(\tilde{off}_t\left(X\right)|X\right)$ be its probability distribution function, conditional on the set of prizes X. Further, let $off_t^e\left(X\right)$ be the mean of this distribution, so that we may write $\tilde{off}_t\left(X\right)=off_t^e\left(X\right)+\upsilon$ where υ is a mean-zero random term.

Contestant i accepts the Banker's offer in round t if:

$$U_i\left(off_{it}\right)>EU_{it}\left(X_{it}\right),\quad t=3,4,5,\qquad(1)$$

where

$$
\begin{aligned}
EU_{i3}\left(X_{i3}\right)&=\frac{1}{56}\sum_{j=1}^{56}\max\left\{EU\left[\tilde{off}_4\left(X_{i3j}\right)\right],\frac{1}{10}\sum_{k=1}^{10}\max\left[EU\left[\tilde{off}_5\left(X_{i3jk}\right)\right],EU\left(X_{i3jk}\right)\right]\right\}\\
EU_{i4}\left(X_{i4}\right)&=\frac{1}{10}\sum_{j=1}^{10}\max\left\{EU\left[\tilde{off}_5\left(X_{i4j}\right)\right],EU\left(X_{i4j}\right)\right\}\\
EU_{i5}\left(X_{i5}\right)&=EU\left(X_{i5}\right)
\end{aligned}
\qquad(2)
$$

$EU_{it}\left(X_{it}\right)$ is the value of continuing with the game beyond round t; $EU\left(\tilde{off}_t\left(.\right)\right)=\int_{-\infty}^{+\infty}U\left(\tilde{off}_t\left(.\right)\right)dG\left(\tilde{off}_t\right)$ is the expected utility of an offer received in a future round t. X_{i3j}, $j=1,\ldots,56$, represents one of the $\binom{8}{5}=56$ possible lotteries deriving from the third-round lottery X_{i3}, that contestant i might face in round 4 by opening 3 of the remaining 8 boxes. *Conditional* on lottery X_{i3j} being realised in the fourth round, there are $\binom{5}{2}=10$ possible lotteries in the fifth round: X_{i3jk}, $k=1,\ldots,10$. Similarly X_{i4j}, $j=1,\ldots,10$, represents one of the $\binom{5}{2}=10$ possible lotteries deriving from the fourth-round lottery X_{i4}, that might be confronted in round 5 by opening 3 of the remaining 5 boxes. Finally, $EU_{i5}\left(X_{i5}\right)$ is simply the expected utility of the lottery consisting of the two outcomes remaining in round 5. Note that, if it is assumed that there is no random element in the future offer (i.e. that $\upsilon=0$), then $EU\left(\tilde{off}_t\left(.\right)\right)$ simply reduces to $U\left(\tilde{off}_t\left(.\right)\right)$. This is the case we start with in Section 4. We then introduce the possibility of a positive variance of υ in Section 7.

The problem of separating beliefs from preferences has been raised in Section 1. We remarked there that, as a way of overcoming this problem, researchers commonly rely on the hypothesis of rational expectations. For example, de Roos and Sarafidis (2006) and Mulino et al. (2006) use data on offers made in all showings in order to form predictive equations for the Banker's offer in each round. Then, they use the prediction thus obtained as the contestant's belief. Andersen et al (2006, 2007) and Post et al. (2008) do similar.

In a departure from this convention, this paper recognises that *Affari tuoi* provides a suitable environment to estimate *both* preferences *and* beliefs in the absence of any restrictive assumptions about the way that beliefs are formed, and hence to test whether such beliefs are in fact formed according to rational expectations theory. This is possible because, in round 5, contestants' choices do not involve any belief formation. The contestant's problem in round 5 is just a straightforward choice between two lotteries: one with two equi-probable prizes; the other being a certainty of the Banker's offer. It is only in rounds 3 and 4 that beliefs are formed. Under the reasonable assumption of invariance over time of contestants' preferences, we are there-

fore able to combine the data from round 5 choices with that from rounds 3 and 4 in order to estimate preferences and beliefs jointly. Of course, in doing this we are making the further identifying assumption that the belief functions are the same for all contestants. In making such an assumption, we are simply following the conventions of those who have *assumed* rational expectations (e.g. Post et al., 2008).

An important point is that the distribution of the risk aversion parameter in round 5 is truncated from above, for the obvious reason that the most risk-averse contestants are likely leave the game in earlier rounds. This might raise concerns of attrition or selection bias in estimation. This would indeed be a problem if we were estimating preferences using data from *only* round 5. But the simultaneous use of data from all three rounds enables us to estimate the complete distribution of preferences over the population. It is intuitively helpful to imagine the following sequence being repeated until convergence: first, the preferences of the contestants reaching round 5 are estimated; then these contestants' choices in earlier rounds are used to deduce their beliefs; the beliefs thus estimated are extended to contestants who left the game before round 5; finally, the preferences of these other contestants are deduced, filling in the truncated upper tail of the risk attitude distribution.

For readers not satisfied with this explanation, we have also carried out Monte-Carlo simulations in order to confirm the validity of our estimation procedure. For the purpose of the simulation, we assume that the game consists of only two rounds, corresponding to the "fourth" and "fifth" rounds of the real game, and that there are 300 contestants. We have used actual lotteries and offers from the fourth and fifth rounds of the real game to generate the situations assumed in the simulated game. All that is simulated is the contestants' decisions. For each of 1000 replications, two models are estimated using the method of maximum simulated likelihood (see Section 4). The first model is one that is applicable to a "selection" sample similar to the real data. That is, individuals who accept the deal in the "fourth" round are not observed in the "fifth" round. The second model is one that uses information on *both* decisions by every contestant, whether or not they accept the deal in the "fourth" round. Note that this second model is indisputably free of any selection problems. Both models are estimated using the same simulated data.

Further details of the simulation are presented in Appendix 1. The important conclusion from the simulation is that both beliefs and preferences appear to be consistently estimated in both models. Hence it is established that consistency is achieved even with the most risk-averse contestants being selected out in early rounds.

4. The econometric choice model

In this section we introduce the econometric model that simultaneously estimates preferences and beliefs. We assume throughout that contestants are expected utility maximisers. We further assume that contestant i has the Constant Absolute Risk Aversion (CARA) utility function, given by:

$$U_i(x) = \frac{1 - \exp(-r_i x)}{1 - \exp(-r_i x_{\max})}, \qquad (3)$$

where x is the outcome[4], x_{max} is the highest possible outcome (500 thousand euros). Note that the functional form of (3) results from a normalisation that ensures that $U(0)=0$ and $U(x_{max})=1$. r_i is the coefficient of absolute risk aversion for contestant i. We assume, in the spirit of Holt and Laury (2002), that this coefficient is distributed across the population according to:

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[4] Here, and throughout the paper, we measure money amounts in units of €1,000.

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$$r_i \sim N\left(\mu_r, \sigma_r^2\right). \tag{4}$$

Let us first assume that the future offer function has a degenerate distribution, so that the error term υ introduced in Section 3 is identically zero, and contestants place a probability of 1 on the event that the future offer equals the expected value of the offer, $\tilde{off}_t = off_t^e$.[5] It follows that the expected utility of the future offer in (3) collapses simply to the utility of the expected offer:

$$EU\left(\tilde{off}_t(.)\right) = U\left(off_t^e(.)\right). \tag{5}$$

Given this assumption, the latent variable underlying a contestant's choice is:

$$y_{it}^* = U_i\left(off_t\right) - EU_{it}\left(X_{it}\right) + \varepsilon_{it}. \tag{6}$$

Here ε_{it} is a Fechner-type error term (Hey and Orme, 1994), with $\varepsilon_{it} \sim N\left(0, \sigma_\varepsilon^2\right)$. It has the interpretation of a computational error in the calculation of utilities and expected utilities. This error is assumed to be homoscedastic and uncorrelated with all other variables in the model. Let us define the binary variable y_{it} to take the value 1 if contestant i accepts the Banker's offer in round t, and the value -1 otherwise. The relationship between the observable variable y_{it} and the latent variable y_{it}^* is then given by:

$$\begin{aligned} y_{it} &= 1 &&\text{if} && y_{it}^* > 0 \\ y_{it} &= -1 &&\text{if} && y_{it}^* \le 0. \end{aligned} \tag{7}$$

It is clear that for each contestant, we observe either a sequence of minus-ones (if the contestant never accepts the money offer) or a sequence of minus-ones followed by a plus-one (if the contestant accepts an offer). Let $f\left(y_{it} \mid \boldsymbol{\theta}\right)$ be the probability of the choice observed for contestant i in round t, conditional on the values of the model's parameters which are assembled in the vector $\boldsymbol{\theta}$. From (14) and (15), this probability is given by:

$$f\left(y_{it} \mid \boldsymbol{\theta}\right) = \Phi\left(y_{it} \times \frac{U_i\left(off_t\right) - EU_{it}\left(X_{it}\right)}{\sigma_\varepsilon}\right). \tag{8}$$

Contestant i's likelihood contribution is the joint probability of observing the sequence of outcomes $\left(y_{i3}, \cdots, y_{iT_i}\right)$, where T_i is the round in which the game ends for contestant i. Given the assumption of independence between rounds, this is given by:

$$L_i\left(\boldsymbol{\theta}\right) = \prod_{t=3}^{T_i} f\left(y_{it} \mid \boldsymbol{\theta}\right). \tag{9}$$

We also allow for the possibility of sub-optimal behaviour, by introducing a tremble parameter, ω ($0 \le \omega \le 1$) (Moffatt and Peters, 2001). This represents the probability that contestants lose concentration and choose completely at random between the two alternatives. With this additional parameter, contestant i's likelihood contribution becomes:

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[5] This assumption is relaxed in Section 7 below, where we assume that the stochastic term υ has a positive variance.

$$L_i(\boldsymbol{\theta},\omega) = \prod_{t=3}^{T_i} \left\{ (1-\omega) f\left(y_{it} \mid \boldsymbol{\theta}\right) + \frac{\omega}{2} \right\} \tag{10}$$

In view of the constraint $0 \leq \omega \leq 1$, the parameter that is in fact estimated is ψ where $\omega = \dfrac{\exp(\psi)}{1+\exp(\psi)}$. After estimation of ψ, an estimate of ω is deduced, and a standard error is found using the delta method (Oehlert, 1992). Other parameters that are constrained, such as $\sigma_r (\geq 0)$ and $\sigma_\varepsilon (\geq 0)$ are estimated using similar techniques.

The full sample log-likelihood is given by:

$$LogL(\boldsymbol{\theta},\omega) = \sum_{i=1}^{n} \prod_{t=3}^{T_i} \left\{ (1-\omega) f\left(y_{it} \mid \boldsymbol{\theta}\right) + \frac{\omega}{2} \right\}. \tag{11}$$

To understand how the maximum simulated likelihood technique has been applied to this problem, we simply note that $r_i = \mu_r + \eta_i$, with $\eta_i \sim N(0,\sigma_r^2)$. Note also that $\boldsymbol{\theta}$ in (11) includes μ_r, but not η_i. We can integrate η_i out to obtain

$$L_i(\boldsymbol{\theta},\omega) = \int_{-\infty}^{+\infty} \prod_{t=3}^{T_i} \left\{ (1-\omega) f\left(y_{it} \mid \boldsymbol{\theta},\eta_i\right) + \frac{\omega}{2} \right\} \frac{1}{\sigma_r} \phi\left(\frac{\eta_i}{\sigma_r}\right) d\eta_i. \tag{12}$$

Following Lerman and Manski (1981), this integral can be approximated by a sample average of the integrand computed drawing R numbers from a standard normal distribution. This way we obtain an unbiased estimator of the integral in (12), with a variance that goes to zero as R increases.

Note that (12) is the log-likelihood function for the straightforward random-effects probit model (Avery et al., 1983) except for the fact that the unobserved heterogeneity term (η_i defined above (12)) enters the model in a highly non-linear way (see (3)).

5. Estimation of the choice model and a test of rational expectations

In this Section, we estimate the model constructed in the previous section, using the method of maximum simulated likelihood. Our sample consists of 294 players observed making 2.15 choices on average. In each model, integration over η_i is performed by simulation using 100 draws for each contestant based on Halton sequences (Train, 2003). We adopt this procedure in preference to the more commonly used Gauss-Hermite quadrature, since, given the complexity of the model, the computational burden is considerably lower for the former than for the latter.

Table 4 presents estimates of the model, with a CARA specification.[6] The results are presented in two columns. The first column contains the results of the model estimated with all parameters unconstrained. The second shows the results obtained with the parameters of the belief functions constrained by the assumption of rational expectations. These constrained parameters, derived in detail in Appendix 2, are as follows. The rational expectation of the fourth-round offer, given information on fourth-round offers made in all showings, depends on the Expected Value of the lottery (EV) in the following way:

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[6] We have also used a CRRA and an Expo-power specification, assuming non-zero lifetime wealth as a parameter to be estimated, along similar lines to Andersen et al. (2006). We find that the estimate of the lifetime wealth parameter is not significantly different from zero. However, the use of these specifications never significantly improves the fit over the CARA specification (cf Andersen et al, 2006; de Roos and Sarafidis, 2006; Post et al, 2008). The various results are available from the authors upon request.

$$\tilde{off}_4^{RE} = \exp\left[-0.460 + 0.857\log\left(EV\right)\right]. \tag{13}$$

The rational expectation of the fifth-round offer is:

$$\tilde{off}_5^{RE} = \min\left[EV, \exp\left(0.665 + 1.305\log\left(EV\right)\right)\right]. \tag{14}$$

Note that (14) incorporates the fact, clearly evident in the data, that fifth-round offers are "upper-censored" at EV. That is, in round 5, the Banker's offer is frequently exactly equal to EV, but very rarely above EV (see Figure A2.4 in the Appendix).

The unconstrained model is estimated assuming the same functional forms as (13) and (14) for the two offer functions, respectively:

$$\begin{aligned} \tilde{off}_4 &= \exp\left[\gamma_4 + \beta_4 \log\left(EV\right)\right] \\ \tilde{off}_5 &= \min\left[EV, \exp\left(\gamma_5 + \beta_5 \log\left(EV\right)\right)\right] \end{aligned}. \tag{15}$$

The difference is that in (15) the four parameters are treated as unknowns, and estimated freely within the choice model.

Constraining the parameters of the belief function, as done in the second column of Table 4, is in line with the approach of other researchers cited in Sections 1 and 3. Here, we note that on the evidence of a likelihood ratio test comparing the two columns ($\chi^2(4) = 14.24$; $p = 0.008$), there is a significant difference between the parameters actually used by contestants to form beliefs (column 1) and those estimated using all the available information (column 2). This test result amounts to a strong rejection of the rational expectations hypothesis. In particular, the negative estimate of γ_5 in the first column indicates that contestants do not recognise the fact that fifth-round offers are upper-censored.

Given this rejection of the rational expectations hypothesis, it is important to consider what the consequences are of incorrectly assuming it. Firstly, we note that the estimate of the risk-attitude parameter (μ_r) is noticeably larger when rational expectations is assumed, than when estimation is free. This implies that the assumption of rational expectations is causing a bias in the estimation of risk attitude, such that contestants appear to be more risk averse than they truly are.

The significantly positive estimates of σ_r vindicate the assumption of varying risk attitude over the population[7]. The Fechner error parameter, σ_ε, is significantly different from zero in the constrained model, but not the unconstrained model. This suggests that, after allowing for heterogeneity in the risk aversion parameter, very little measurement error remains to be explained. Finally, the tremble parameter is statistically significant. Its magnitude (in the unconstrained model) indicates that contestants lose concentration on around 4% of occasions. This is in line with estimates obtained elsewhere in the literature (see, for example, Loomes et al., 2002).

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[7] Parameters representing standard deviations of error terms, and tremble probabilities, are clearly constrained to be non-negative, and the hypothesis under test is one for which the parameter of interest is on the boundary of the parameter space. Many authors have acknowledged the problems that arise in this situation (see, for example, Hey and Orme, 1994, footnote 17, Moffatt and Peters, 2001). There is usually a simple remedy: for a test with size 0.05, the 0.10 column from tables should be consulted to obtain the appropriate critical value. This remedy has been prescribed in a more general context by Godfrey (1988, p.94).

	Estimation with be-liefs unconstrained	Beliefs formed under rational expectations
μ_r	0.02453 (0.00358)	0.03140 (0.00393)
σ_r	0.02004 (0.00308)	0.02627 (0.00342)
σ_ε	0.01225 (0.00973)	0.02884 (0.01279)
ω	0.04275 (0.02350)	0.03663 (0.03007)
4th round beliefs equation: $\tilde{off}_4(.)=\exp\left(\gamma_4+\beta_4\ln EV(.)\right)$		
γ_4	0.52476 (0.14716)	-0.46000
β_4	0.64452 (0.03881)	0.85700
5th round beliefs equation: $\tilde{off}_5(.)=\min\left[EV(.),\exp\left(\gamma_5+\beta_5\ln EV(.)\right)\right]$		
γ_5	-1.11559 (0.80626)	0.44800
β_5	1.12931 (0.21191)	1.30479
Number of contestants (n)	294	294
Average number of choices per contestant	2.15	2.15
Log-likelihood	-232.54020	-239.65688

Table 4: The table contains estimates from the model defined in (6) and (7). Belief functions are defined in the table. The assumed utility functional is CARA. The estimation technique used is maximum simulated likelihood. Standard errors are shown in parentheses. Some of the standard errors have been obtained using the delta method. The results are divided into two columns: the first is unconstrained; in the second, the parameters $\gamma_4, \beta_4, \gamma_5$ and β_5 are constrained to satisfy the rational expectations hypothesis, according to (13) and (14).

This result has important implications, namely, that it is invalid to assume that beliefs are formed as if all available information is being efficiently processed, and any model that does so is likely to suffer from bias in the estimation of the risk attitude parameter.

In unreported estimation, we also take into account the possibility that contestants believe they will get a *swap* (instead of a Banker's offer) in round 4 and/or 5. The results so obtained do not differ significantly from those in Table 3, for the reason that the estimated probability that contestants assign to a *swap* is not significantly greater than zero.

6. A "rule of thumb" belief function

In Section 5, we presented the offer functions that best explain the data on all offers. It was with these offer functions in mind that we approached the estimation of our choice model: we assumed that contestants formed beliefs using a function with the same structure as the estimated offer function. However, the belief function that best explains the choices of contestants is not necessarily the same as the belief function that best explains actual offers. In this Section, we address the question of what belief function is actually used by contestants in making their choices.

Our approach to addressing this question consists of trying out different belief functions within the choice model, and using nested and non-nested tests to determine which is best able to explain the choice data. To this end, many different specifications of the belief function have

been investigated. We do not report the results from all of these, but we focus on the one that appears to represent actual beliefs most closely. For reasons that will become clear, we refer to this specification as the "rule of thumb" belief function.

Under the "rule of thumb" belief function, contestants use the following very simple formulae to compute their beliefs about future offers:

$$
\begin{aligned}
off_4^e(.) &= \beta_4 EV(.) \quad \beta_4 > 0 \\
off_5^e(.) &= \beta_5 EV(.) \quad \beta_5 > 0
\end{aligned}
\tag{16}
$$

That is, they predict the offer in round 4 to be some fixed multiple β_4 of the expected value of the prizes remaining in that round; and they similarly predict the offer in round 5 to be a multiple β_5 of the expected value of prizes remaining in that round.

Table 5 contains results from a model based on this assumption. As in Table 4, the results are divided into two columns: unconstrained estimates; and estimates constrained according to the rational expectations hypothesis. In the first column, we see that β_4 and β_5 are both estimated with high precision, and the estimate of β_5 is significantly larger in magnitude than that of β_4. This simply confirms that contestants correctly expect the Banker to become more generous as the game progresses.

The Vuong test-statistic (Vuong, 1989) for the null hypothesis that the models in the first columns of Tables 4 and 5, are equally close to the true model, against the alternative that the first is closer to the true model, is 7.56 ($N(0,1)$, p-value = 0.00000).[8] Hence we have overwhelming evidence that contestants are forming beliefs using the "rule-of-thumb" belief functions, in preference to the "superior" functions given by (13) and (14), that make optimal use of the data.

The second column of the table shows the estimates of the model with the parameters β_4 and β_5 constrained to equal the values obtained by performing (external) regressions of actual offer on EV, using the entire data set. We firstly see that the estimates obtained in the unconstrained model are not significantly different from those obtained in the external regressions, which is loosely consistent with the rational expectations hypothesis. We further note that a likelihood ratio test of the hypothesis that the parameters exactly equal their external estimates gives the $\chi^2(2)$ statistic $2 \times (236.53 - 235.54) = 1.97$, which is not significant. We therefore conclude that, under the auxiliary hypothesis of the "rule-of-thumb" belief function, we have no evidence to reject the hypothesis of rational expectations.

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[8] For the Vuong tests performed in this work, we always use a correction for the degrees of freedom that corresponds to the Schwarz's Bayesian Information Criterion.

	Model with "rule of thumb" beliefs	
	Estimation with beliefs un-constrained	Beliefs formed under "rule-of-thumb" rational expecta-tions
μ_r	0.02410 (0.00216)	0.02335 (0.00172)
σ_r	0.01778 (0.00196)	0.01633 (0.00142)
σ_ε	0.01284 (0.01099)	0.01956 (0.00788)
ω	0.06185 (0.02701)	0.05910 (0.02516)
	4th round beliefs equation: $\tilde{off}_4\left(.\right)=\beta_4 EV\left(.\right)$	
β_4	0.38340 (0.02901)	0.33170
	5th round beliefs equation: $\tilde{off}_5\left(.\right)=\beta_5 EV\left(.\right)$	
β_5	0.54627 (0.04672)	0.56674
Number of contest-ants (n)	294	294
Average number of choices per contest-ant	2.15	2.15
Log-likelihood	-235.54346	-236.52784

Table 5: Estimates from the model defined in (6), (7) and (16). The assumed util-ity functional is CARA. The estimation technique used is maximum simulated like-lihood. Standard errors are shown in parentheses. Some of the standard errors have been obtained using the delta method. The first column presents uncon-strained estimates; in the second, the parameters β_4 and β_5 are constrained to satisfy the (rule-of-thumb) rational expectations hypothesis.

Figures 3 and 4 present, for rounds 4 and 5 respectively, graphical comparisons between the "rule-of-thumb" belief functions estimated using the model, and those estimated externally using actual offers, data on which is also shown in the graphs. In each case, we see that the es-timated belief function is impressively close to the external estimate, providing further evidence in favour of rational expectations with "rule-of-thumb" beliefs.
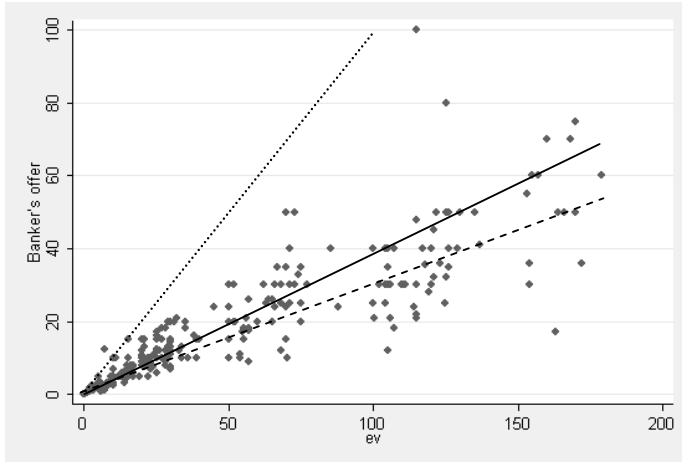
Figure 3: Banker's offer against *EV* in round 4. The dotted line is a 45^0-line. The superimposed continuous line represents the prediction according to the (unconstrained) estimated belief function in Table 3. The dashed line represents the OLS prediction of the Banker's offer using the "rule-of-thumb" belief function.
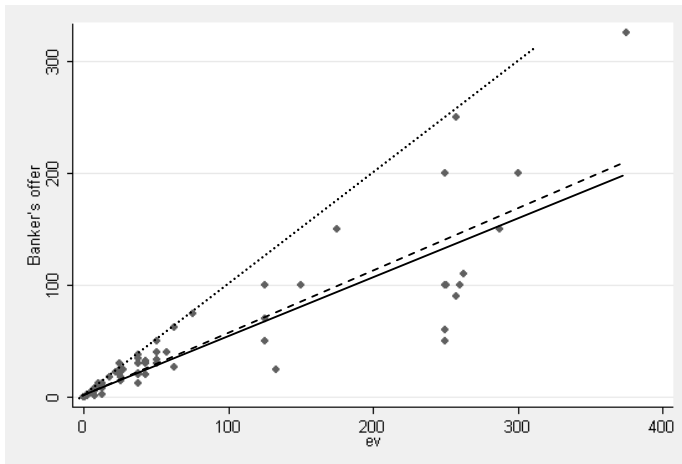

Figure 4: Banker's offer against *EV* in the round 5. The dotted line is a 45^0-line. The superimposed continuous line represents the prediction according to the (unconstrained) estimated belief function in Table 3. The dashed line represents the OLS prediction of the Banker's offer using the "rule-of-thumb" belief function.

7. Introducing randomness in contestants' beliefs

The models estimated in previous Sections are based on the assumption that the future offer function has a degenerate distribution, that is, that the contestant assigns a probability of one to their prediction of the Banker's offer in future rounds. In this Section, we relax this assumption, by allowing contestants to form a subjective non-degenerate probability distribution of what the offer may be in future rounds.[9] We need to modify the model describing contestants' forward-looking behaviour to introduce this component of uncertainty in their expectations.

Suppose, as at the start of Section 3, that the offer function in round *t* is

$$\tilde{off}_t\ (.) = off_t^e(.) + \upsilon.$$
(17)

and assume that the error term υ has a normal distribution with mean zero and variance σ_υ^2, so that $\tilde{off}_t \sim N\left(off_t^e, \sigma_\upsilon^2\right)$.

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[9] See also Mulino et al. (2006).

Given the assumption of normality, we can appeal to a well-known result[10] to deduce the expected utility of the offer as:

$$EU_i\left(\widetilde{off}_t\left(.\right)\right) = -\frac{\exp\left[-r_i off_t^e\left(.\right)\right]}{r_i}\exp\left(\frac{\sigma_v^2 r_i^2}{2}\right). \tag{18}$$

We note that the presence of the stochastic term υ in the belief function has the effect of increasing the expected utility of future offers. (25) is simply used in place of (13) in Section 5,[11] and estimation proceeds as normal, with the additional parameter υ.

Model with randomness in contestants' beliefs	
μ_r	0.02382 (0.00256)
σ_r	0.01945 (0.00217)
σ_ε	0.01101 (0.01063)
ω	0.05398 (0.02618)
σ_υ	2.70866 (14.22065)
4th round beliefs equation: $\widetilde{off}_4\left(.\right) = \beta_4 EV\left(.\right) + \upsilon$	
β_4	0.38833 (0.03861)
5th round beliefs equation: $\widetilde{off}_5\left(.\right) = \beta_5 EV\left(.\right) + \upsilon$	
β_5	0.55632 (0.06474)
Number of contestants (n)	294
Average number of choices per contestant	2.15
Log-likelihood	-235.49498

Table 6: Estimates from the choice model defined in (14) and (15), with the offer function subject to randomness (24). Belief functions are reported above. The assumed utility functional is CARA The estimation technique used is maximum simulated likelihood. Standard errors are shown in parentheses. Some of the standard errors have been obtained using the delta method.

The results are presented in Table 6. These simply show that the introduction of the additional element of randomness has a negligible impact on previous results (cf. Table 5, Column 1). Moreover, the standard deviation of the error term in the offer function, σ_υ, is not significantly different from zero. This suggests that contestants do indeed assign a probability of one to the event that the Banker's offer will exactly equal their expected offer. If we were asked to make a rough intuitive interpretation of these findings, it would be that there is already enough uncertainty in the following rounds due to the fact that contestants do not know which lottery they will face, that they are reluctant to introduce another dimension of uncertainty to their own decision problem. This idea is also consistent with the finding, discussed at the end of Section 6, of contestants neglecting the probability of being offered a *swap* in later rounds.

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[10] See Yassour, Zilberman, and Rausser (1981).

[11] The derivation of the formula for the normalized utility function used in previous models is straightforward.

8. A mixture model of myopic and forward-looking behaviour

Throughout the analysis so far, we have assumed that contestants are forward-looking. That is, they consider the possible outcomes in future rounds when forming a decision during the current round. However, it is not obvious that all contestants are forward-looking. Some might be "myopic", and treat each round as an isolated risky choice problem. A model that allows for the co-existence of both types is a finite mixture model. Such a model is estimated in this Section.

One reason for estimating the mixture model is that it is important to know whether a significant proportion of subjects are myopic. At the end of Section 3, it was explained that self-selection is not an issue since, although the most risk-averse tend to leave the game early, this does not bias either the risk attitude parameter or the estimates of contestants' beliefs. However, this reasoning only applies if contestants actually have a forward-looking perspective. The presence of myopic contestants in the sample can introduce an attrition bias in the estimation of the forward-looking model, since these contestants have a higher probability of leaving the game early, since they neglect the prospect of generous offers in later rounds. Ideally, therefore, we hope for the proportion of myopic subjects to be zero.

We continue to denote as L_i contestant i's likelihood contribution (12) under the assumption of forward-looking behaviour. We also introduce L_i^{myopic} to be contestant i's likelihood contribution under the hypothesis that she behaves myopically. L_i^{myopic} is the same as L_i in (12), except that the expected utility term $EU_{it}(X_{it})$, introduced in (1) and (2) for forward-looking contestants, no longer takes account of the outcomes of future rounds; for myopic contestants, $EU_{it}(X_{it})$ is simply the expected utility of the lottery consisting of the prizes remaining in round t.

Let $\pi(0 \leq \pi \leq 1)$ be the mixing proportion, which is the proportion of the population who are forward-looking. Contestant i's likelihood contribution in the mixture model is then:

$$L_i^{mixture} = (1-\pi)L_i^{myopic} + \pi L_i. \qquad (19)$$

Table 7 reports the parameter estimates for the mixture model, alongside estimates from a pure-myopic model and a pure-forward looking model. The estimates from the pure-foreward-looking model are the same as those from the first column of Table 5, and are reproduced here in order to facilitate comparison. Firstly and most importantly, the mixing proportion is estimated, with reasonable precision, to be 0.911 (with 95% confidence interval $0.856 < \pi < 0.966$). This implies that around 91% of the population are forward-looking and only around 9% are myopic.

Let us now focus on the estimates of the mixture model (third and fouth columns of Table 7). Comparing the estimates of mean risk attitude (μ_r) for the two types, we note first that myopic types are more risk averse than forward-looking types. We further note that the estimate of μ_r for myopic contestants (0.068) is almost six times as large as that estimated on the assumption that *all* subjects are myopic (0.01236; first column). The explanation for this difference is straightforward: when the myopic model is forced to explain the behaviour of forward-looking contestants, their tendency to reject Banker's offers is interpreted as straightforward risk-lovingness, and it is inevitable that the estimate of the risk aversion parameter will be very low.[12]

The estimated standard deviation of risk attitude, σ_r, for myopic types, is much smaller in the mixture model (0.00074), than in the model assuming all individuals are myopic (0.0064). This difference simply reflects the fact that the two types differ quite markedly in their risk attitude, so a model that assumes all individuals are of one type will inevitably over-estimate the

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[12] See de Roos and Sarafidis (2006) and Mulino et al. (2006) for a more detailed explanation.

spread of risk attitude. The estimates obtained in the mixture model of μ_r and σ_r for forward-looking types are very close to the estimates obtained in the model that assumes that all individuals are forward-looking with "rule-of-thumb" beliefs (Table 5, first column). This is for the simple reason that the majority of individuals are forward-looking, as indicated by our high estimate of π in the mixture model. The same reasoning applies to the two parameters β_4 and β_5, whose estimates under the mixture model agree closely with those in the first column of Table 5.

	All contestants Myopic	*All contestants Forward-looking*	*Mixture model*	
			Myopic type	*Forward-looking type*
μ_r	0.01236 (0.00090)	0.02410 (0.00216)	0.06827 (0.00031)	0.02188 (0.00183)
σ_r	0.00639 (0.00156)	0.01778 (0.00196)	0.00074 (0.00025)	0.01687 (0.00176)
σ_ε	0.03692 (0.00882)	0.01284 (0.01099)	0.00004 (0.00047)	0.01585 (0.00834)
		4th round beliefs equ.: $\widetilde{off}_4(.)=\beta_4 EV(.)$		*4th round beliefs equ.:* $\widetilde{off}_4(.)=\beta_4 EV(.)$
β_4	-	0.38340 (0.02901)	-	0.39053 (0.03053)
		5th round beliefs equ.: $\widetilde{off}_5(.)=\beta_5 EV(.)$		*5th round beliefs equ.:* $\widetilde{off}_5(.)=\beta_5 EV(.)$
β_5	-	0.54627 (0.04672)	-	0.55738 (0.04995)
ω	0.07275 (0.03687)	0.06185 (0.02701)	0.00087 (0.00402)	
π	-	-	0.91112 (0.02810)	
Number of contestants (n)	294	294	294	
Average number of choices per contestant	2.15	2.15	2.15	
Log-likelihood	-261.97520	-232.54020	-227.41220	

Table 7: Estimates from the myopic model, the forward looking model, and the mixture model defined in (19). Belief functions are defined in the Table. The assumed utility functional is CARA. Maximum simulated likelihood used in estimation. Standard errors are shown in parentheses. Some of the standard errors have been obtained using the delta method. The parameter π represents the proportion of the population who are of the forward-looking type.

The magnitude of the computational error, as represented by σ_ε, is still small in the mixture model, but is significantly smaller for myopic types (0.00004), than in the model that assumes all individuals are myopic (0.037; Table 2, first column). The reason for this is similar to that advanced above in the context of the parameter σ_r.

The tremble probability, ω, is assumed to be the same for both types in the mixture model[13], and we see that its estimate is small and insignificant.

---

[13] We know of no reason to presume that one type of individual trembles more than the other.

The posterior probability of each contestant being of the forward-looking type can be computed using Bayes' rule, as follows:

$$P\left(i = forward \mid y_{i3}, \cdots, y_{iT_i}\right) = \frac{\pi L_i}{L_i^{mixture}}, \tag{20}$$

Its distribution over the 294 contestants is shown in Figure 5. As expected, the majority of contestants have a high posterior probability of being of the forward-looking type.

In unreported work, we also estimate the choice model under the hypothesis that contestants are not fully forward-looking, but look just one-step-ahead. That is, when in round 3 they only consider all the possible lotteries and the consequent offers they might get in round 4. Unfortunately, *Affari Tuoi* is such that the sequence of rounds is not sufficiently long to capture the difference between the one-step-ahead model and the forward-looking one; these two models predict different behaviour only in round 3. A mixture model has been estimated with these two forward-looking types, and another model estimated with myopic behaviour as a third type. These mixture models fail to converge, and we attribute this to the inability of the data to distinguish between the two types of forward-looking behaviour.
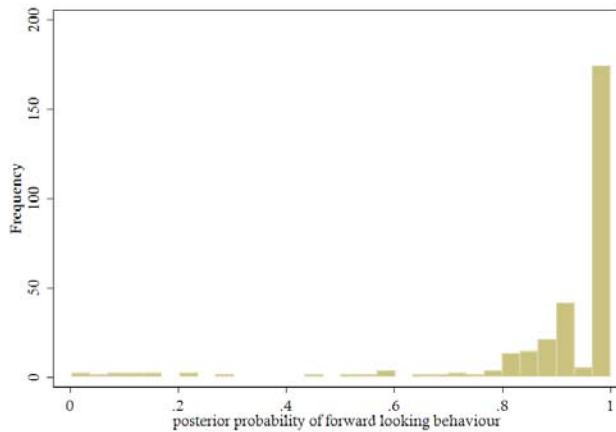

Figure 5: A histogram of the posterior probabilities of being a forward looking type over the sample of 294 contestants.

This is not to say that this is not an interesting research question. We have firmly established that the vast majority of contestants are forward-looking, but it is not clear how far forward-looking agents are looking ahead. We therefore encourage researchers with access to data from games with more rounds to perform similar analysis to that carried out here, in order to address this important question.

The key conclusion from the analysis in this Section is that it is safe to assume that forward-looking behaviour prevails. This is a welcome result, since it means that the attrition bias resulting from the presence of myopic contestants in the sample is not likely to be severe.

9. Conclusion

The econometric problem of estimating the belief function has been the principal focus of the paper. The separate identification of beliefs and preferences in this choice model was given an intuitive rationalization in Section 3, where we also justify the assumption using a Monte Carlo study. One result (established in Section 7) is that beliefs are deterministic: once a belief has been formed about the Banker's offer in a particular round, the contestant assigns a probability of one to this value. More importantly, we have been interested in whether such beliefs are formed in accordance with the rational expectations hypothesis. In order to test the rational ex-

pectations hypothesis, we have compared the belief function estimated within the choice model to the "true" offer function estimated using the complete data set of Banker's offers.

A problem with this approach that was not raised earlier is that, in order to implement the assumption of rational expectations, we are implicitly assuming that each contestant has access to the complete set of Banker's offers. The obvious logical problem with this assumption is that contestants cannot possibly know what offers are made in future games; they can only know about offers that have been made previously to their own participation. However, we are following other researchers (e.g. Andersen et al., 2006, 2007; Deck et al., 2008; Mulino et al., 2006; Post et al., 2008) in assuming that all information, including future offers, is available.

In any case, the principal objective of this paper has not been to *estimate* the rational expectations model, but rather to *test* the rational expectations hypothesis using a more general model. Our unconstrained model is fully flexible in terms of the parameter values in the belief function, which is estimated within the choice model, and therefore should tell us how beliefs are actually formed, on the basis of information which is actually available to the contestant at the time decisions are made. One of our principal findings has been that, at least in the context of *Affari Tuoi*, the estimates of the belief function in the unconstrained model do not closely match the estimates of the offer function obtained using sophisticated processing of the offer data. Therefore, constraining the choice model to incorporate the estimated offer function results in biased estimation of the preference parameters.

However, the situation in which we found the belief function to differ from the true offer function was under the assumption of an optimal structure of the offer function, including all of the features, for example upper-censoring, that are apparent in the offer data. When a less elaborate structure is assumed for the belief function, in which it is simply assumed that contestants believe that the offer at a given stage of the game will be a fixed multiple of the expected prize, we find that the belief function estimated within the choice model is *not* significantly different from that estimated from data on actual offers. We have referred to such beliefs as being determined by the "rule-of-thumb" belief function.

Hence we are led to the conclusion that, again in the context of *Affari Tuoi*, contestants are rational up to a point. They appear to be using all of the information that is available (i.e. all of the offer data), but they do not appear to be using it in an optimal way.

Of course, a different conclusion may be reached in the analysis of data from other game shows. Our recommendation to other researchers is that it is always desirable to estimate the belief function in an unconstrained way as a component of the choice model, rather than to rely on prior assumptions about the manner in which such beliefs are formed.

Another important conclusion of the paper, established forcefully through the mixture model estimated in Section 8, is that the vast majority of subjects are forward-looking, meaning that they do indeed take into account expectations of the Banker's offer in future rounds when making their choice in the current round. This finding confirms the importance of the role of contestants' beliefs, and of discovering how these beliefs are formed. It also confirms the validity of the forward-looking choice models, estimated in Sections 5 and 6, that are the focal point of the paper.

Appendix 1: Monte Carlo Simulation

As explained at the end of Section 3, the purpose of the simulation is to confirm that the belief functions are separately identifiable from the preference parameters, with choice data only. For the purpose of the simulation, we assume that the game consists of only two rounds, corresponding to the "fourth" and "fifth" rounds of the real game, and that there are 300 contestants. The experiment uses 1000 replications. At each replication, two models are estimated (using MSL): "selection" and "no-selection". The "selection" model assumes (as in the real game) that contestants accepting "deal" in the fourth round do not make a decision in the fifth round. The "no-selection" model assumes that decisions are made in both rounds, regardless of the decision in the fourth. The "no-selection" model is clearly not subject to any sort of bias, and so provides a useful benchmark for evaluating the performance of the "selection" model.

Model with selection; sample size = 300			
	True value	*Mean*	*Std. Err.*	*p-value of t-test*
Mean risk attitude	0.02000	0.02000	0.00006	0.9576
Log std. dev. risk attitude	-4.42285	-4.46636	0.00623	0.0000
β_5	0.55000	0.54908	0.00283	0.7447
Log. Std. Dev. Fechner-error	-2.99573	-3.00695	0.00675	0.0969

Table A1.1: The table reports summary statistics of the Monte Carlo study. The assumed utility functional is CARA. The beliefs equation assumed is $o\tilde{f\!f_5}(.) = \beta_5 EV(.)$. Number of replications: 1000

Model with no-selection; sample size = 300			
	True value	*Mean*	*Std. Err.*	*p-value of t-test*
Mean risk attitude	0.02000	0.02003	0.00005	0.5702
Log std. dev. risk attitude	-4.42285	-4.45692	0.00544	0.0000
β_5	0.55000	0.55036	0.00263	0.8910
Log. Std. Dev. Fechner-error	-2.99573	-3.00652	0.00617	0.0806

Table A1.2: The table reports summary statistics of the Monte Carlo study. The assumed utility functional is CARA. The beliefs equation assumed is $o\tilde{f\!f_5}(.) = \beta_5 EV(.)$. Number of replications: 1000

Tables A1.1 and A1.2 contain the results from the "selection" and "no-selection" models respectively. Each model has four parameters. For each parameter, the mean estimate is taken over replications, and this mean compared to the true value of the parameter using a *t*-test. The *p*-values for each *t*-test are shown in the final column of the tables. We see that three of the four parameters appear to be "correctly" estimated in both models. Most importantly, the (mean) risk attitude parameter and the fifth-round belief parameter (β_5), are both correctly estimated, even in the situation in which risk-averse contestants are being "selected out" of round 5. The only parameter that is not correctly estimated is the log of the standard deviation of the risk attitude parameter. This parameter represents between-individual heterogeneity. The slight departure of its estimate from its true value is a consequence of the fact that there are only two rounds in this game; at most two observations are available for each contestant. The inconsistency in the estimation of this parameter can of course be avoided by increasing the sample size and/or the number of rounds in the game. Table A1.3 shows results of the same simulation of

the "selection" model, but with a larger sample size of 2000. Here, we see, as expected, that all four of the parameters are estimated without bias.

Model with selection; sample size = 2000				
	True value	*Mean*	*Std. Err.*	*p-value of t-test*
Mean risk attitude	0.02000	0.02001	0.00002	0.7967
Log std. dev. risk attitude	-4.42285	-4.42612	0.00204	0.1091
β_5	0.55000	0.54866	0.00127	0.2909
Log. Std. Dev. Fechner-error	-2.99573	-2.99734	0.00228	0.4805

Table A1.3: The table reports summary statistics of the Monte Carlo study. The assumed utility functional is CARA. The beliefs equation assumed is $\tilde{off_5}(.) = \beta_5 EV(.)$. Number of replications: 1000

Appendix 2: Modelling the Banker's Offer function

The hypothesis of rational expectations implies that contestants are capable of forecasting the Banker's offer in future rounds in the same way as an econometrician with access to estimation algorithms. We therefore use data on Banker's offers from all showings, to estimate the true offer function in each round. The rational expectations hypothesis can then be taken to imply that contestants form beliefs according to the estimated true offer functions.

The key explanatory variable in the determination of the offer is the expected value (*EV*) of the remaining prizes.

Banker's offer in Round 4

Figure A2.1 shows a scatter of the round 4 offers against *EV*. A 45^0-line is super-imposed. The most obvious feature of the scatter is that the offer rises with *EV*. It is also noticed that the offer is nearly always far below *EV*. A fanning out effect is also evident, with the variance of offers rising as *EV* rises. In view of the strong positive skew seen in both variables, it is appropriate to take logarithms before analysing the relationship. Figure A2.2 shows a scatter of the logged variables. Here we see that the relationship is clearly log-linear. The plot also appears homoscedastic. We therefore estimate the straightforward log-linear regression:

$$\log(offer) = \gamma_4 + \beta_4 \log(EV) + u, \qquad (A2.1)$$

where the 4 subscripts are present since we are restricting attention to round 4. The results from estimation of (4) are:

$$\hat{\log}(offer) = -0.460 + 0.857 \log(EV)$$
$$(0.068) \quad (0.018) \qquad . \qquad (A2.2)$$
$$n = 252$$

We may deduce from this estimated regression equation that contestants who form rational expectations use the following formula for the expected offer in round 4:

$$offer = \exp\left[-0.460 + 0.857 \log(EV)\right]. \qquad (A2.3)$$
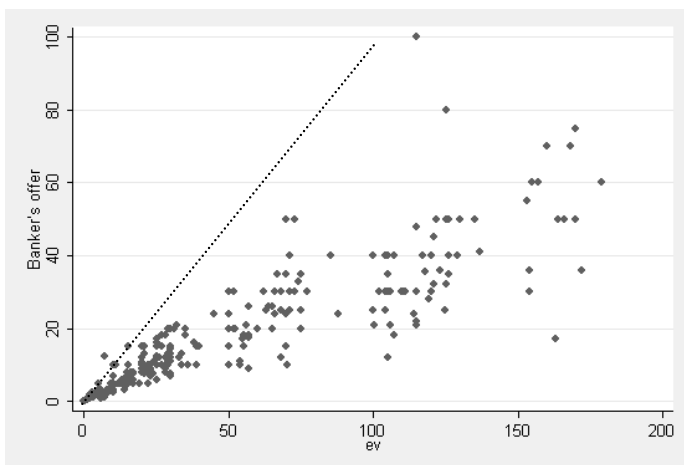
Figure A2.1: Offer against expected value in round 4. 45°-line superimposed.
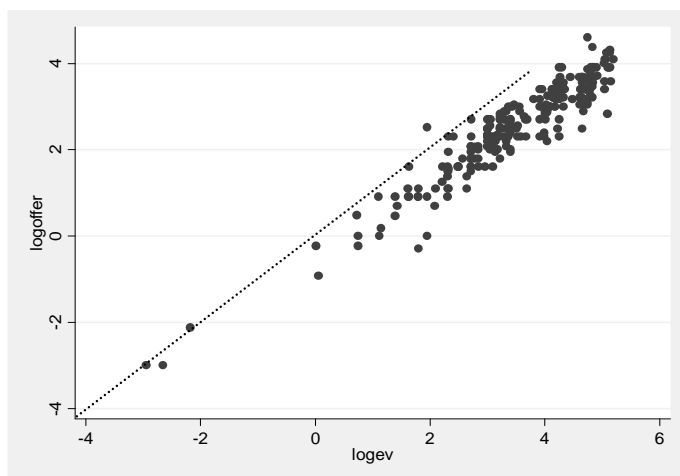

Figure A2.2: Logarithm of offer against logarithm of expected value in round 4. 45°-line superimposed.

Banker's Offer in Round 5

Figure A2.3 shows offer against expected value for all offers made in round 5. A 45^0-line is again super-imposed. Again it is also clear that the offer rarely exceeds the expected value. However, a difference from round 4 is that some points appear to be on (or very close to) the 45^0-line, implying that the offer exactly equals the expected value. We shall treat these observations as upper-censored observations. Note that this censoring tends to arise when the expected value is comparatively low. When the expected value is high, the offer is usually below the expected value. Note also that there is one observation *above* the 45^0-line. This observation is interpreted as errant behaviour on the part of the Banker, and is omitted from the censored regression reported below.

Figure A2.4 shows the scatter of the logged variables. The censoring that was apparent in figure A2.3 is even more clearly apparent in figure A2.4, with a sizable portion of the sample appearing to be on the 45^0-line.
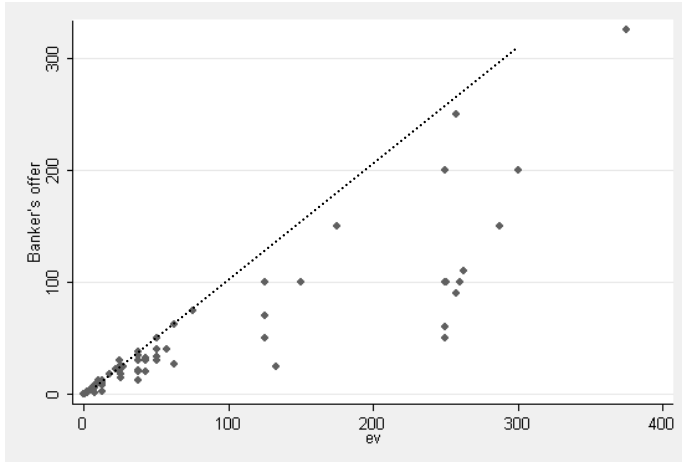
Figure A2.3: Offer against expected value in round 5. 45°-line superimposed.
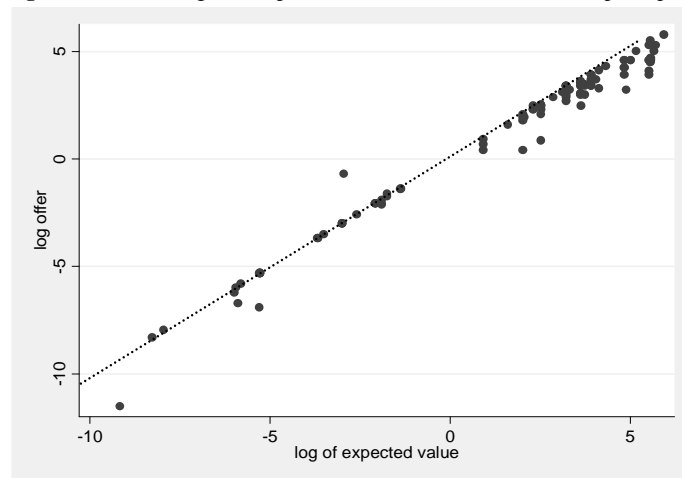

Figure A2.4: Logarithm of offer against logarithm of expected value in round 5. 45°-line superimposed.

We estimate the following censored regression model (omitting the one observation that is above the 45^0 line):

$$\log(\mathit{offer}) = \gamma_5 + \beta_5 \log(EV) + u \quad if \quad \gamma_5 + \beta_5 \log(EV) + u < \log(EV)$$
$$\log(\mathit{offer}) = \log(EV) \quad if \quad \gamma_5 + \beta_5 \log(EV) + u \geq \log(EV) \qquad (A2.4)$$
$$u \sim N(0, \sigma_u^2)$$

where the 5 subscripts are present since we are restricting attention to round 5. The results from this censored regression model are:

$$\hat{\mathit{log}}(\mathit{offer}) = 0.665 + 1.305 \log(EV)$$
$$(0.081) \ (0.358)$$

$$(A2.5)$$

$$n = 106$$
34 uncensored observations
72 right-censored observations

We deduce the following formula for predicted offer in round 5:

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$$offer = \min\left[EV, \exp\left(0.665 + 1.305\log\left(EV\right)\right)\right]. \qquad \text{(A2.6)}$$

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