Web Appendices to:

Optimal Policy Intervention and the Social Value of Public Information

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Appendix A

Disclosure of the Policymaker's Signal to a Subset of the Private Sector

Here we consider an alternative representation of the extent to which the policymaker reveals its information publicly. Specifically, we follow Cornand and Heinemann (2008) in assuming that the policymaker communicates its private signal, without the introduction of any additional noise, to a fraction of private sector agents: the remainder of the private sector then has access only to their own agent-specific information. The proportion of agents, *Q*, who observe the policymaker's signal which, henceforth, we refer to as the 'public' signal, is viewed as a choice variable of the policymaker. Hence, the approach provides an alternative characterization of transparency to that employed in the paper, with the value of *Q* representing a natural measure of the degree of public disclosure by the policymaker.

The principal features of the model are unchanged from the paper, with the payoff function of individual agents remaining as described by equation (1). The crucial amendment relates to the underlying informational assumptions: now $y \equiv z$, but is only observed by a fraction *Q* of the private sector. Identifying all agents who observe *z* with a superscript *I* (for 'Informed') we have:

$$(A.1) \qquad E_i^I(\theta) = E(\theta \mid x_i, z) = \frac{\sigma_\varepsilon^2 z + \sigma_\phi^2 x_i}{(\sigma_\varepsilon^2 + \sigma_\phi^2)}; \qquad E_i^I(z) = z$$

The optimal action of all agents continues to be determined according to equation (8) of the paper. Hence for an informed agent:

$$(A.2) \qquad a_i^I = (1-r)\left[\frac{\sigma_\varepsilon^2 z + \sigma_\phi^2 x_i}{(\sigma_\varepsilon^2 + \sigma_\phi^2)} + \rho z\right] + rE_i^I(\bar{a})$$

Considering now agents who have access only to their own private information, with such agents identified by a superscript U (Uninformed):

(A.3) $$E_i^U(\theta) = E(\theta \mid x_i) = x_i\,; \qquad E_i^U(z) = E(z \mid x_i) = x_i$$

It follows that actions based only on agents' private signals are described by:

(A.4) $$a_i^U = (1-r)(1+\rho)x_i + rE_i^U(\bar{a})$$

Assuming each agent's action to be a linear function of the signals observed, then:

(A.5a) $$a_i^I = \kappa_1^I x_i + \kappa_2^I z$$

(A.5b) $$a_i^U = \kappa_1^U x_i$$

The average action of all agents, $\bar{a}$, is a weighted average of a_i^I and a_i^U, each aggregated over the respective set of agents. From the properties of x_i:

(A.6) $$\bar{a} = [Q\kappa_1^I + (1-Q)\kappa_1^U]\theta + Q\kappa_2^I z$$

Substituting (A.6) into (A.2) and (A.4), then equating coefficients with (A.5a) and (A.5b) allows us to solve for κ_1^I, κ_2^I and κ_1^U:

$$\kappa_1^I = \frac{[1-rQ+r(1-Q)\rho]\sigma_\phi^2}{[\sigma_\varepsilon^2 + (1-rQ)\sigma_\phi^2]} \qquad \kappa_2^I = \frac{[(1+\rho)\sigma_\varepsilon^2 + (1-r)\rho\sigma_\phi^2]}{[\sigma_\varepsilon^2 + (1-rQ)\sigma_\phi^2]} \qquad \kappa_1^U = 1+\rho$$

Hence:

(A.7a) $$a_i^I = \frac{[(1+\rho)\sigma_\varepsilon^2 + (1-r)\rho\sigma_\phi^2]z + [1-rQ+r(1-Q)\rho]\sigma_\phi^2 x_i}{[\sigma_\varepsilon^2 + (1-rQ)\sigma_\phi^2]}$$

(A.7b) $$a_i^U = (1+\rho)x_i$$

The measure of welfare adopted is a weighted average of the expected payoffs of informed and uninformed agents, with weights Q and $1-Q$ respectively. Substituting (A.5a) and (A.5b), together with the policy rule, into equation (1) of the paper, aggregating with appropriate partitioning of the integral and taking expectations yields:

(A.8) $$E(W \mid \theta) = -Q[(\kappa_1^I)^2\sigma_\varepsilon^2 + (\kappa_2^I - \rho)^2\sigma_\phi^2] - (1-Q)[(\kappa_1^U)^2\sigma_\varepsilon^2 + \rho^2\sigma_\phi^2]$$

Differentiating the above expression with respect to ρ, then substituting for κ_1^I, κ_2^I and κ_1^U, allows us to determine the optimal value of the rule parameter:

(A.9) $$\rho^* = -\frac{[\sigma_\varepsilon^2 + (1-2rQ)\sigma_\phi^2]\sigma_\varepsilon^2}{\{r^2Q(1-Q)\sigma_\phi^4 + [\sigma_\varepsilon^2 + (1-rQ)\sigma_\phi^2]^2\}}$$

Now using our expressions for κ_1^I, κ_2^I, κ_1^U and ρ^* to substitute into (A.8), we find expected welfare with ρ set optimally:

(A.10) $$E(W \mid \theta)\big|_{\rho=\rho^*} = -\frac{\{\sigma_\varepsilon^2 + [1-Q+(1-r)^2Q]\sigma_\phi^2\}\sigma_\varepsilon^2\sigma_\phi^2}{\{[\sigma_\varepsilon^2 + (1-rQ)\sigma_\phi^2]^2 + r^2Q(1-Q)\sigma_\phi^4\}}$$

The central issue is now the relationship between welfare and the proportion of agents, Q, to whom the policymaker's signal is communicated. Differentiating the above expression with respect to Q:

(A.11) $$\frac{\partial E(W \mid \theta)\big|_{\rho=\rho^*}}{\partial Q} = \frac{-r^2(\sigma_\varepsilon^2 + \sigma_\phi^2)\sigma_\varepsilon^4\sigma_\phi^4}{\{[\sigma_\varepsilon^2 + (1-rQ)\sigma_\phi^2]^2 + r^2Q(1-Q)\sigma_\phi^4\}^2}$$

It is directly evident that this expression is strictly negative for $\sigma_\varepsilon^2 > 0$, $\sigma_\phi^2 > 0$, i.e. social welfare is strictly decreasing in Q. It follows that the optimal value of Q is zero and, thus, the policymaker should not share its private signal with any subset of private sector agents.

Appendix B

Equilibrium with Discretionary Policymaking

This Appendix analyzes the scenario in which the policymaker does not/cannot precommit to set its policy instrument g in accordance with a rule: thus equation (9) no longer forms part of the model. Instead, it is assumed that following the realization of the policymaker's private signal z and of the related public signal y, the policymaker has the freedom to set g at any value it sees fit; we refer to this alternative scenario as 'discretion'. This modification amounts to a departure from the paper's implicit assumption regarding the timing of moves. In the rule scenario the policy-response coefficient ρ in (9) is determined in advance of the realization of the model's exogenous stochastic variables, and hence also prior to the setting of g. In contrast, under discretion the policy responses to z and y (which imply a particular choice of g) are determined after the realization of those signals and simultaneously with the selection by private agents of their moves. This apart, our assumptions remain as in the paper: thus equations (1) to (8) describing payoffs, information structure and the representative agent's individually optimal action continue to be of relevance, while the objective of policy is once again the maximization of expected welfare.

In the current scenario, it is appropriate to allow for the possible existence of equilibria which feature a non-zero (unconditional) mean action for each private-sector agent. For this reason, the equivalent of (10) for discretion is:

(B.1) $$a_i = \kappa_0 + \kappa_1 x_i + \kappa_2 y$$

The policymaker's expectation of (normalized) welfare is given by: $E_g(W) = E_g\left[-\int_0^1 (a_i - \theta - g)^2 di\right]$.[1] A straightforward optimization exercise then yields the policymaker's setting of its instrument, for given values of the private-sector coefficients in (B.1):

[1] As in the paper, $E_g(.)$ denotes the expectation $E(.\,|\,z,y)$, while we again use $E_i(.)$ to denote agent i's expectation $E(.\,|\,x_i,y)$.

(B.2) $$g = \kappa_0 + (\kappa_1 - 1)z + \kappa_2 y$$

Since $\int_0^1 \varepsilon_i \, di = 0$, the aggregate private-sector action is now $\bar{a} = \kappa_0 + \kappa_1\theta + \kappa_2 y$. Agent *i*'s expectation of this, and of the setting of *g*, will therefore be:

(B.3a) $$E_i(\bar{a}) = \kappa_0 + \kappa_1 E_i(\theta) + \kappa_2 y$$

(B.3b) $$E_i(g) = \kappa_0 + (\kappa_1 - 1)E_i(z) + \kappa_2 y$$

where $E_i(\theta)$ and $E_i(z)$ are given by equations (5) and (6) of the paper. Substituting (5), (6), (B.3a) and (B.3b) into (8) allows us to express agent *i*'s individually optimal action in terms of a constant and responses to the two signals it observes. Equating this constant and the two response coefficients with their counterparts in (B.1) then yields three simultaneous equations which must hold in any equilibrium. Solving these equations for κ_0, κ_1 and κ_2, we find a unique solution for κ_1: on the other hand, the values of κ_0 and κ_2 are indeterminate. The equilibrium action of agent *i*, and the policymaker's equilibrium choice of instrument setting, are thus described by:

(B.4a) $$a_i = \kappa_0 + \frac{(1-r)\sigma_\phi^2}{[\sigma_\varepsilon^2 + (1-r)\sigma_\phi^2]} x_i + \kappa_2 y$$

(B.4b) $$g = \kappa_0 - \frac{\sigma_\varepsilon^2}{[\sigma_\varepsilon^2 + (1-r)\sigma_\phi^2]} z + \kappa_2 y$$

where κ_0 and κ_2 each take a value common to all agents. An important aspect of discretion, therefore, is that whereas the players' equilibrium responses to their private items of information (i.e. x_i or *z*) are uniquely determined by the model's structural parameters, this is not the case as regards either their equilibrium responses to the signal which is commonly known, *y*, or the mean value of their action (or instrument setting). In formal terms, the model with discretion has an infinite set of equilibria, which share the same private-sector response to private signals, x_i, and also have in common a particular policy response to the private signal *z*. The equilibria differ, however, in respect of both the value of the players' common response to *y*, and the mean value of their action or instrument. These differences across equilibria are not consequential for welfare. This is because the mutual nature of the players' response to *y* is sufficient in itself to neutralize *y*'s impact on each agent's utility and on expected welfare, and this is so regardless of the particular value of the common response to *y*. (A similar comment pertains to the mean action and mean instrument setting.)

The intuition for the indeterminacy of κ_2 (and κ_0) under discretion becomes apparent when we note that agent *i* has a beauty-contest motivation for responding to *y* (and for setting κ_0) in precisely the same way as every other agent. At the same time, the term $a_i - \theta - g$, which is central to welfare, implies that the policymaker will wish to adjust *g* in response to *y* to neutralize fully the potential welfare impact of the representative agent's response to that signal. Hence the players' equilibrium responses to *y* are identical and are payoff-neutral for all parties, implying any common value for κ_2 is consistent with this outcome: a similar logic explains the indeterminacy of κ_0. (Note that it is the heterogeneity of the x_i private signals which ultimately accounts for the uniqueness of κ_1.) Given this identical common response to *y*, the policymaker's expectation of $a_i - \theta - g$ becomes simply $E_g(\kappa_1 x_i - \theta - \rho^D z) = (\kappa_1 - 1 - \rho^D)z$, where κ_1 has its unique equilibrium value, and where ρ^D is the policymaker's response to its private signal *z* under discretion. Clearly, the policymaker's optimal response to *z* is also unique, and equal to $\kappa_1 - 1$.

At this point it is useful to compare the above findings with those of the wider 'global games' literature. The themes present in the preceding analysis are familiar from this literature, which stems from the seminal contribution by Carlsson and VanDamme (*Econometrica*, 1993). The existence of multiple equilibria is jointly attributable to three aspects of the game. First, there is common knowledge regarding the variable *y*. Second, every player, including the policymaker, chooses his or her response to *y* simultaneously. Third, the variable *y* is not merely a signal of the fundamental, but is also itself a component of that fundamental (i.e. of $\theta + g$, the state of the world, as modified by policy intervention), which agents are attempting to estimate. This third feature has a particular importance as regards the multiple equilibria which characterize the game under discretion: note that it is its absence from the original Morris and Shin (2002) model which ensures that that particular global game's equilibrium is unique.

We conclude by briefly commenting on the welfare properties of the equilibria associated with discretion. The variance of the public-signal additional noise term, σ_ξ^2, is notably absent from (B.4a) and (B.4b), and plays no part in determining expected welfare, which is given by:

(B.5) $$E(W \mid \theta) = -\frac{[\sigma_\varepsilon^2 + (1-r)^2 \sigma_\phi^2]\sigma_\varepsilon^2 \sigma_\phi^2}{[\sigma_\varepsilon^2 + (1-r)\sigma_\phi^2]^2}$$

Comparison with (15) reveals that this is the welfare outcome associated with full disclosure of the policymaker's private signal z (in which case rule-based policymaking would be completely ineffective). Notational differences apart, it is also identical to expected welfare in the original Morris and Shin model without policy intervention (as given by equation (17) of their paper). It is clear from this that commitment to the optimal rule is, in welfare terms, unambiguously superior to discretion; all the more so when commitment is accompanied by zero disclosure. In the latter instance, of course, the fact that the rule then replicates the first-best welfare outcome implies that it cannot possibly be bettered by any alternative policy regime.

Appendix C

Persistence of Shocks

Here the model's assumptions are modified to allow the state of the world variable, θ, to follow a first-order autoregressive process. Our aim is to establish whether, and to what extent, this dynamic extension affects the key results of our paper. The framework of Section I is largely retained but, with the state now specified to be AR(1), we have:

(C.1a) $$\theta_t = \mu\theta_{t-1} + \delta_t$$

where the autocorrelation parameter $\mu \in [0,1)$ measures the degree of persistence of the state, and δ_t is a stochastic innovation whose prior distribution is uniform over the real line.

Crucially, we assume that the policymaker's choice of instrument setting in any period is observable with a one-period lag: hence, in making their decisions relating to period *t* actions, for example, private sector agents have exact knowledge of g_{t-1}. This knowledge allows an inference concerning the value of z_{t-1} to be made which, given the autoregressive process which θ follows, provides public information relevant to the realization of θ_t. We further specify that a particular realization of the state becomes known to all parties after a lag of two periods.[2] Thus, at time *t* the most recent θ realization currently observable by all participants is that which occurred in period $t-2$. Consequently, at time *t*, θ_{t-2} (and $\theta_{t-j} \forall j > 2$) is common knowledge among all the game's players (both the policymaker and the private agents), whereas they are heterogeneously and imperfectly informed regarding θ_{t-1} and θ_t.

Since the AR(1) process implies that $\theta_{t-1} = \mu\theta_{t-2} + \delta_{t-1}$, we may combine this with (C.1a) to obtain:

[2] Although the choice of a two-period lag may appear somewhat arbitrary, we note that the assumption of a single-period lag would effectively return us to the framework of the paper, while generalizing to a *k*-period lag leaves our results unaffected in any essential way. This latter generalization is briefly considered following our treatment of the case of a two-period lag.

(C.1b) $$\theta_t = \mu^2\theta_{t-2} + \mu\delta_{t-1} + \delta_t$$

Note that, since the innovations are assumed to be i.i.d., $E(\delta_t\delta_{t-1}) = 0$.

So far as the information structure is concerned, we assume that the signals received by an individual agent in a particular period, $t-j$, relate to that period's innovation δ_{t-j}. Private sector agent *i*'s signals are therefore:

(C.2a) $$x_{i,t-j} = \delta_{t-j} + \varepsilon_{i,t-j}$$

(C.2b) $$y_{t-j} = z_{t-j} + \xi_{t-j}$$

where $\varepsilon_{i,t-j}$ and ξ_{t-j} are independent and serially uncorrelated noise terms with respective known distributions $\varepsilon_{i,t-j} \sim N(0,\sigma_\varepsilon^2)$ and $\xi_{i,t-j} \sim N(0,\sigma_\xi^2)$, and z_{t-j} denotes the policymaker's own noisy signal of δ_{t-j}:

(C.2c) $$z_{t-j} = \delta_{t-j} + \phi_{t-j}$$

where the ϕ_{t-j} noise terms are i.i.d. with known distribution $\phi_{t-j} \sim N(0,\sigma_\phi^2)$.

In the present context, we specify the policy rule to take the form:[3]

(C.3a) $$g_t = \rho_0\theta_{t-2} + \rho_1 z_t + \rho_2 z_{t-1}$$

The setting of the policy instrument in period $t-1$ (i.e. the value of g_{t-1}) is assumed to become common knowledge throughout the economy before agents make their action choices for period *t*. This setting will be given by:

(C.3b) $$g_{t-1} = \rho_0\theta_{t-3} + \rho_1 z_{t-1} + \rho_2 z_{t-2}$$

[3] As in the paper itself, we assume that precommitment to a rule of this kind is possible. Note in addition that, since θ_{t-2} is known to every agent at time *t*, including an additional term in z_{t-2} in the rule (so that $g_t = \rho_0\theta_{t-2} + \rho_1 z_t + \rho_2 z_{t-1} + \rho_3 z_{t-2}$), would in no way affect the reported findings (in other words, the policy coefficient ρ_3 would be redundant).

With θ_{t-3}, z_{t-2} and the rule coefficients known (or precisely inferable) at time *t*, it is clear that z_{t-1} will be inferable from the value of g_{t-1} observed by agents at *t*.[4]

In forming its estimates of θ_t and g_t at time *t*, agent *i* will make optimal use of the items of information θ_{t-2}, $x_{i,t}$, y_t, $x_{i,t-1}$ and z_{t-1}, while the presence of the last of these in *i*'s information set implies that y_{t-1} will be of no informative value in these forecasting exercises. Using our familiar notation $E_i(.)$ for agent *i*'s rational expectation, so that $E_i(\theta_t) \equiv E(\theta_t \mid \theta_{t-2}, x_{i,t}, y_t, x_{i,t-1}, z_{t-1})$, for example, the optimal forecasts are found to be:

(C.4a) $$E_i(\theta_t) = \mu^2\theta_{t-2} + \mu E_i(\delta_{t-1}) + E_i(\delta_t),$$ where:

(C.4b) $$E_i(\delta_{t-1}) = \frac{\sigma_\varepsilon^2 z_{t-1} + \sigma_\phi^2 x_{i,t-1}}{(\sigma_\varepsilon^2 + \sigma_\phi^2)}$$

(C.4c) $$E_i(\delta_t) = \frac{\sigma_\varepsilon^2 y_t + (\sigma_\phi^2 + \sigma_\xi^2) x_{i,t}}{(\sigma_\varepsilon^2 + \sigma_\phi^2 + \sigma_\xi^2)}$$

(C.4d) $$E_i(g_t) = \rho_0\theta_{t-2} + \rho_1 E_i(z_t) + \rho_2 z_{t-1},$$ where:

(C.4e) $$E_i(z_t) = \frac{(\sigma_\varepsilon^2 + \sigma_\phi^2) y_t + \sigma_\xi^2 x_{i,t}}{(\sigma_\varepsilon^2 + \sigma_\phi^2 + \sigma_\xi^2)}$$

As in the paper, we solve for the individual private-sector agent's equilibrium response coefficients using the method of undetermined coefficients. Agent *i*'s individually optimal equation is once again given by equation (8). In a symmetric equilibrium, all agents have identical response coefficients, such that:

[4] The assumption that z_{t-2} is known at time *t* is easily justified here, since with rule-based policymaking z_{t-2} will be precisely inferable at *t* provided at least one antecedent realization of this signal (i.e. some z_{t-j} where $j > 2$) is known to private agents at time *t*. (For example, if z_{t-4} is known at *t*, then z_{t-3} can be inferred exactly from $g_{t-3} = \rho_0\theta_{t-5} + \rho_1 z_{t-3} + \rho_2 z_{t-4}$, which in turn implies that z_{t-2} is perfectly inferable from the counterpart expression for g_{t-2}.)

(C.5a) $$a_i = \kappa_0\theta_{t-2} + \kappa_1 x_{i,t} + \kappa_2 y_t + \kappa_3 x_{i,t-1} + \kappa_4 z_{t-1}$$

where the equilibrium values of the coefficients κ_0, κ_1, κ_2, κ_3 and κ_4, remain to be determined. Given (C.2a) above, it follows that in the current scenario $\int_0^1 x_{i,t}\,di = \delta_t$ and $\int_0^1 x_{i,t-1}\,di = \delta_{t-1}$. Hence, in equilibrium, the average action, $\bar{a} \equiv \int_0^1 a_i di$, and agent *i*'s rational expectation thereof, will respectively be given by:

(C.5b) $$\bar{a} = \kappa_0\theta_{t-2} + \kappa_1\delta_t + \kappa_2 y_t + \kappa_3\delta_{t-1} + \kappa_4 z_{t-1}$$

(C.5c) $$E_i(\bar{a}) = \kappa_0\theta_{t-2} + \kappa_1 E_i(\delta_t) + \kappa_2 y_t + \kappa_3 E_i(\delta_{t-1}) + \kappa_4 z_{t-1}$$

Substituting equations (C.4a) to (C.4e), as well as (C.5c), into (8), and then collecting terms in θ_{t-2}, $x_{i,t}$, y_t, $x_{i,t-1}$ and z_{t-1} yields an equation which, like (C.5a), must hold in equilibrium. Equating the coefficients in this resultant equation with their counterparts in (C.5a) yields five simultaneous equations which can be solved for the unique symmetric Nash equilibrium set of coefficient values:

$$\kappa_0 = (\mu^2 + \rho_0)\,\theta_{t-2} \qquad \kappa_1 = \frac{(1-r)[\sigma_\phi^2 + (1+\rho_1)\sigma_\xi^2]}{[\sigma_\varepsilon^2 + (1-r)(\sigma_\phi^2 + \sigma_\xi^2)]}$$

$$\kappa_2 = \frac{[(1+\rho_1)\sigma_\varepsilon^2 + (1-r)\rho_1\sigma_\phi^2]}{[\sigma_\varepsilon^2 + (1-r)(\sigma_\phi^2 + \sigma_\xi^2)]} \qquad \kappa_3 = \frac{(1-r)\mu\sigma_\phi^2}{[\sigma_\varepsilon^2 + (1-r)\sigma_\phi^2]}$$

$$\kappa_4 = \left\{\frac{\mu\sigma_\varepsilon^2}{[\sigma_\varepsilon^2 + (1-r)\sigma_\phi^2]} + \rho_2\right\}$$

Agent *i*'s equilibrium action is therefore:

(C.6a) $a_i = (\mu^2 + \rho_0)\theta_{t-2} + \dfrac{(1-r)[\sigma_\phi^2 + (1+\rho_1)\sigma_\xi^2]x_{i,t} + [(1+\rho_1)\sigma_\varepsilon^2 + (1-r)\rho_1\sigma_\phi^2]y_t}{[\sigma_\varepsilon^2 + (1-r)(\sigma_\phi^2 + \sigma_\xi^2)]}$

$$+ \frac{(1-r)\mu\sigma_\phi^2}{[\sigma_\varepsilon^2 + (1-r)\sigma_\phi^2]}x_{i,t-1} + \left\{\frac{\mu\sigma_\varepsilon^2}{[\sigma_\varepsilon^2 + (1-r)\sigma_\phi^2]} + \rho_2\right\}z_{t-1}$$

Note that the equilibrium solutions for κ_1 and κ_2 are independent of the autocorrelation parameter μ, and furthermore are identical to the equilibrium values taken by these coefficients in the version of the model in which the state of the world does not exhibit any persistence (i.e. the $\mu \equiv 0$ case considered in the paper). With (normalized) social welfare given by $W = (1-r)^{-1}\int_0^1 u_i di = -\int_0^1 (a_i - \theta - g)^2 di$, it is insightful to consider the equilibrium expression for $a_i - \theta_t - g_t$ for this version of the model. Appropriate substitutions involving (C.1a), (C.3a) and (C.6a) yield $a_i - \theta_t - g_t$ in terms of the noise realizations in periods t and $t-1$:

(C.7a) $a_i - \theta_t - g_t = \dfrac{(1-r)[\sigma_\phi^2 + (1+\rho_1)\sigma_\xi^2]\varepsilon_{i,t} + [\sigma_\varepsilon^2 - (1-r)\rho_1\sigma_\xi^2]\phi_t}{[\sigma_\varepsilon^2 + (1-r)(\sigma_\phi^2 + \sigma_\xi^2)]} +$

$$\frac{[(1+\rho_1)\sigma_\varepsilon^2 + (1-r)\rho_1\sigma_\phi^2]\xi_t}{[\sigma_\varepsilon^2 + (1-r)(\sigma_\phi^2 + \sigma_\xi^2)]} + \mu\frac{[\sigma_\varepsilon^2\phi_{t-1} + (1-r)\sigma_\phi^2\varepsilon_{i,t-1}]}{[\sigma_\varepsilon^2 + (1-r)\sigma_\phi^2]}$$

A noteworthy aspect of (C.7a) is that the autocorrelation parameter μ is absent from the first three terms, and only affects current-period welfare through the earlier-period noise terms (ϕ_{t-1} and $\varepsilon_{i,t-1}$). Even more significantly, the terms in ϕ_{t-1} and $\varepsilon_{i,t-1}$ do not feature the policy-rule coefficients. This implies, of course, that the optimal setting of the policy-rule response to the authorities' current private signal, z_t, is identical to the optimal-rule response to z_t stated in the paper as equation (14).[5] Furthermore, the coefficients on ϕ_{t-1} and $\varepsilon_{i,t-1}$ in (C.7a) are also independent of σ_ξ^2, indicating that the relationship between expected welfare under the optimal rule and

[5] Consequently, with the response coefficients in (C.3a) set optimally, the optimal rule is found to be $g_t = \rho_0\theta_{t-2} + \rho^* z_t + \rho_2 z_{t-1}$, where ρ^* is given by (14), and ρ_0 and ρ_2 may be chosen arbitrarily.

σ_ξ^2 is qualitatively identical to that which holds when θ does not have a persistent component. Thus expected welfare is described by:

(C.8a) $$E(W_t \mid \theta_t)\big|_{\rho_1=\rho^*} = \Psi - \mu^2 \frac{[\sigma_\varepsilon^2 + (1-r)^2\sigma_\phi^2]\sigma_\varepsilon^2\sigma_\phi^2}{[\sigma_\varepsilon^2 + (1-r)\sigma_\phi^2]^2}$$

where ρ^* is given by (14), and Ψ by the right-hand side of (15). It directly follows that $\partial E(W_t \mid \theta_t)\big|_{\rho_1=\rho^*} \big/ \partial\sigma_\xi^2 > 0$, as found in the paper: hence Proposition 1 survives intact when the model is modified to allow the state of the world to exhibit AR(1) persistence.

The sole remaining point of interest is whether this optimal combination of rule and zero disclosure replicates the first-best welfare outcome, i.e. that which results when there is full disclosure of the authorities' information, and agents co-ordinate their actions in a socially efficient manner. In the present context, this would involve an individual action of the following form:

(C.9) $$\tilde{a}_i = \tilde{\kappa}_0\theta_{t-2} + \tilde{\kappa}_1 x_{i,t} + \tilde{\kappa}_2 y_t + \tilde{\kappa}_3 x_{i,t-1} + \tilde{\kappa}_4 z_{t-1}$$

Performing an optimization exercise similar to that described in Section III of the paper reveals the following to be the unique collectively optimal action for agent *i*:

(C.10a) $$\tilde{a}_i = \mu^2\theta_{t-2} + \frac{\sigma_\phi^2 x_{i,t} + \sigma_\varepsilon^2 z_t + \mu(\sigma_\phi^2 x_{i,t-1} + \sigma_\varepsilon^2 z_{t-1})}{(\sigma_\varepsilon^2 + \sigma_\phi^2)}$$

Expected welfare under collectively-optimal private-sector co-ordination is:

(C.11a) $$E(\tilde{W}_t \mid \theta_t) = -(1+\mu^2)\frac{\sigma_\varepsilon^2\sigma_\phi^2}{(\sigma_\varepsilon^2 + \sigma_\phi^2)}$$

Equilibrium expected welfare under the optimal rule combined with zero disclosure is found by taking the limit of (C.8a) as $\sigma_\xi^2 \to \infty$:

(C.12a) $$\lim_{\sigma_\xi^2 \to \infty} E(W_t \mid \theta_t)\Big|_{\rho_1=\rho^*} = -\left\{1+\mu^2+\mu^2 \frac{r^2\sigma_\varepsilon^2\sigma_\phi^2}{[\sigma_\varepsilon^2+(1-r)\sigma_\phi^2]^2}\right\}\frac{\sigma_\varepsilon^2\sigma_\phi^2}{(\sigma_\varepsilon^2+\sigma_\phi^2)}$$

Comparison of (C.11a) and (C.12a) reveals that although equilibrium welfare is maximized by an optimal policy rule combined with zero disclosure, it does not replicate the first-best outcome when $\mu > 0$, i.e. when θ is described by an AR(1) process. Thus Proposition 2 of the paper is not robust to this modification of the basic framework.

Generalizing to the case in which θ_{t-k} (where $k \geq 2$) is the most recent realization of θ known to all agents at time *t*, the counterpart expressions to (C.6a), (C.7a), (C.8a), (C.10a) and (C.11a) are as follows:

(C.6b) $$a_i = (\mu^k+\rho_0)\theta_{t-k} + \frac{(1-r)[\sigma_\phi^2+(1+\rho_1)\sigma_\xi^2]x_{i,t} + [(1+\rho_1)\sigma_\varepsilon^2+(1-r)\rho_1\sigma_\phi^2]y_t}{[\sigma_\varepsilon^2+(1-r)(\sigma_\phi^2+\sigma_\xi^2)]}$$
$$+\sum_{j=1}^{k-1}\frac{(1-r)\mu^j\sigma_\phi^2}{[\sigma_\varepsilon^2+(1-r)\sigma_\phi^2]}x_{i,t-j} + \sum_{j=1}^{k-1}\left\{\frac{\mu^j\sigma_\varepsilon^2}{[\sigma_\varepsilon^2+(1-r)\sigma_\phi^2]}+\rho_{j+1}\right\}z_{t-j}$$

where ρ_j is the coefficient on z_{t-j} in the policy rule $g_t = \rho_0 + \sum_{j=0}^{k-1}\rho_{j+1}z_{t-j}$.

(C.7b) $$a_i - \theta_t - g_t = \frac{(1-r)[\sigma_\phi^2+(1+\rho_1)\sigma_\xi^2]\varepsilon_{i,t} + [\sigma_\varepsilon^2-(1-r)\rho_1\sigma_\xi^2]\phi_t}{[\sigma_\varepsilon^2+(1-r)(\sigma_\phi^2+\sigma_\xi^2)]} +$$
$$\frac{[(1+\rho_1)\sigma_\varepsilon^2+(1-r)\rho_1\sigma_\phi^2]\xi_t}{[\sigma_\varepsilon^2+(1-r)(\sigma_\phi^2+\sigma_\xi^2)]} + \sum_{j=1}^{k-1}\mu^j\frac{[\sigma_\varepsilon^2\phi_{t-j}+(1-r)\sigma_\phi^2\varepsilon_{i,t-j}]}{[\sigma_\varepsilon^2+(1-r)\sigma_\phi^2]}$$

(C.8b)[6] $$E(W_t \mid \theta_t)\Big|_{\rho_1=\rho^*} = \Psi - \frac{(\mu^2-\mu^{2k})[\sigma_\varepsilon^2+(1-r)^2\sigma_\phi^2]\sigma_\varepsilon^2\sigma_\phi^2}{(1-\mu^2)[\sigma_\varepsilon^2+(1-r)\sigma_\phi^2]^2}$$

[6] In deriving (C.8b) use has been made of the fact that $\sum_{j=1}^{k-1}\mu^{2j} = (\mu^2-\mu^{2k})/(1-\mu^2)$

(C.10b) $$\tilde{a}_i = \mu^k \theta_{t-k} + \sum_{j=0}^{k-1} \frac{\mu^j (\sigma_\phi^2 x_{i,t-j} + \sigma_\varepsilon^2 z_{t-j})}{(\sigma_\varepsilon^2 + \sigma_\phi^2)}$$

(C.11b) $$E(\tilde{W}_t \mid \theta_t) = -\left\{1 + \frac{(\mu^2 - \mu^{2k})}{(1-\mu^2)}\right\} \frac{\sigma_\varepsilon^2 \sigma_\phi^2}{(\sigma_\varepsilon^2 + \sigma_\phi^2)}$$

(C.12b) $$\lim_{\sigma_\xi^2 \to \infty} E(W_t \mid \theta_t)\Big|_{\rho_1 = \rho^*} = -\left\{1 + \frac{(\mu^2 - \mu^{2k})}{(1-\mu^2)}\left[1 + \frac{r^2 \sigma_\varepsilon^2 \sigma_\phi^2}{[\sigma_\varepsilon^2 + (1-r)\sigma_\phi^2]^2}\right]\right\} \frac{\sigma_\varepsilon^2 \sigma_\phi^2}{(\sigma_\varepsilon^2 + \sigma_\phi^2)}$$

As a first step towards understanding these results, note that as k increases and θ_{t-k} becomes more remote from the current period (t), the average accuracy of the representative agent's forecasts of θ_t necessarily deteriorates, since additional innovations must be estimated, while the additional signals available to the agent are only of value in forecasting the innovation to which they relate. Because the collectively-optimal strategy requires each agent to set his or her action equal to their private rational expectation of the current state, θ_t, it follows that associated expected welfare (which is simply minus one times the representative agent's mean squared forecast error in respect of θ_t) must decrease as the lag length k increases.[7]

This phenomenon implies equilibrium welfare is also reduced if θ_{t-k} becomes more temporally distant from θ_t. Importantly, however, the presence of the beauty-contest term in (1) implies that equilibrium welfare falls as k increases for a second reason, namely that a greater number of payoff-relevant and commonly-known public signals available to agents at t inevitably exacerbates the externality characterizing their equilibrium actions. In the present context, this externality can be viewed as comprised of a sum of adverse externalities, one arising in relation to each of the innovations that agent I forecasts, or, alternatively, to each of the public signals received. It is intuitively clear that the policymaker has no means of mitigating the externalities which arise in connection with private agents' estimates of the

[7] Differentiation of (C.11b) confirms this reasoning:
$\partial E(\tilde{W}_t \mid \theta_t) / \partial k = 2(\ln \mu)\mu^{2k} \sigma_\varepsilon^2 \sigma_\phi^2 / (1-\mu^2)(\sigma_\varepsilon^2 + \sigma_\phi^2) < 0$ for $\mu \in (0,1)$, since the latter implies that $\ln \mu < 0$.

innovations relating to periods prior to the present. This is because, first, the observability of previous-period instrument settings, and hence the exact predictability of those components of the current setting, g_t, which are responses to $z_{t-1}, z_{t-2}, \ldots, z_{t-k}$, implies a policy-neutrality result obtains in respect of these components. Second, the policymaker has no means of noise-obscuring these public items of information: this is significant, since it is each agent's excessive reaction (motivated by beauty-contest considerations) to these $z_{t-1}, z_{t-2}, \ldots, z_{t-k}$ signals which directly engenders an externality in respect of each of them.

The non-robustness of Proposition 2 to the modifications of our original model considered here is therefore not surprising, since the policymaker has no power whatsoever to affect the externalities originating in current-period private-sector over-reactions to earlier-period public signals. Thus the additional welfare losses arising from these externalities have to be taken as given by the policymaker. Nonetheless, a modified version of Proposition 2 can be seen to obtain, in that optimal policy combined with zero disclosure of the policymaker's private *current* signal can ensure a first-best outcome in relation to *current-period* innovations.