Appendix: Not for Publication

Currency Misalignments and Optimal Monetary Policy: A Reexamination

Charles Engel
University of Wisconsin

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Appendix A

Model Equations

A1.a. *Households*

The representative household in the home country maximizes

(A1) $$U_t(h) = \mathrm{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[\frac{1}{1-\sigma} C_{t+j}(h)^{1-\sigma} - \frac{1}{1+\phi} N_{t+j}(h)^{1+\phi} \right] \right\},\ \sigma > 0, \phi \geq 0$$

$C_t(h)$ is the consumption aggregate. We assume Cobb-Douglas preferences:

(A2) $$C_t(h) = \left(C_{Ht}(h)\right)^{\frac{\nu}{2}} \left(C_{Ft}(h)\right)^{1-\frac{\nu}{2}},\ 0 \leq \nu \leq 2 .$$

In turn, $C_{Ht}(h)$ and $C_{Ft}(h)$ are CES aggregates over a continuum of goods produced in each country:

(A3) $$C_{Ht}(h) = \left(\int_0^1 C_{Ht}(h,f)^{\frac{\xi-1}{\xi}} df \right)^{\frac{\xi}{\xi-1}} \quad \text{and} \quad C_{Ft}(h) = \left(\int_0^1 C_{Ft}(h,f)^{\frac{\xi-1}{\xi}} df \right)^{\frac{\xi}{\xi-1}} .$$

$N_t(h)$ is an aggregate of the labor services that the household sells to each of a continuum of firms located in the home country:

(A4) $$N_t(h) = \int_0^1 N_t(h,f) df .$$

Households receive wage income, $W_t(h)N_t(h)$, aggregate profits from home firms, Γ_t. They pay lump-sum taxes each period, T_t. Each household can trade in a complete market in contingent claims (arbitrarily) denominated in the home currency. The budget constraint is given by:

(A5) $$P_t C_t(h) + \sum_{\nabla^{t+1} \in \Omega_{t+1}} Z(\nabla^{t+1} \mid \nabla^t) D(h, \nabla^{t+1}) = W_t(h) N_t(h) + \Gamma_t - T_t + D(h, \nabla^t) ,$$

where $D(h,\nabla^t)$ represents household *h*'s payoffs on state-contingent claims for state ∇^t. $Z(\nabla^{t+1} \mid \nabla^t)$ is the price of a claim that pays one dollar in state ∇^{t+1}, conditional on state ∇^t occurring at time *t*.

In this equation, P_t is the exact price index for consumption, given by:

(A6) $$P_t = k^{-1} P_{Ht}^{\nu/2} P_{Ft}^{1-(\nu/2)},\ \ k = (1-(\nu/2))^{1-(\nu/2)} (\nu/2)^{\nu/2} .$$

P_{Ht} is the Home-currency price of the Home aggregate good and P_{Ft} is the Home currency price of the Foreign aggregate good. Equation (A6) follows from cost minimization. Also, from cost minimization, P_{Ht} and P_{Ft} are the usual CES aggregates over prices of individual varieties, *f*:

(A7) $$P_{Ht} = \left(\int_0^1 P_{Ht}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}}, \text{ and } P_{Ft} = \left(\int_0^1 P_{Ft}(f)^{1-\xi} df \right)^{\frac{1}{1-\xi}} .$$

Foreign households have analogous preferences and face an analogous budget constraint.

Because all Home households are identical, we can drop the index for the household and use the fact that aggregate per capita consumption of each good is equal to the consumption of each good by each household. The first-order conditions for consumption are given by:

(A8) $P_{Ht}C_{Ht} = \frac{\nu}{2}P_tC_t$,

(A9) $P_{Ft}C_{Ft} = \left(1-\frac{\nu}{2}\right)P_tC_t$,

(A10) $C_{Ht}(f) = \left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} C_{Ht}$ and $C_{Ft}(f) = \left(\frac{P_{Ft}(f)}{P_{Ft}}\right)^{-\xi} C_{Ft}$,

(A11) $\beta\left(C(\nabla^{t+1})/C(\nabla^t)\right)^{-\sigma}(P_t/P_{t+1}) = \ddot{Z}(\nabla^{t+1}\,|\,\nabla^t)$.

In equation (A11), we explicitly use an index for the state at time t for the purpose of clarity. $\ddot{Z}(\nabla^{t+1}\,|\,\nabla^t)$ is the normalized price of the state contingent claim. That is, it is defined as $Z(\nabla^{t+1}\,|\,\nabla^t)$ divided by the probability of state ∇^{t+1} conditional on state ∇^t .

Note that the sum of $Z(\nabla^{t+1}\,|\,\nabla^t)$ across all possible states at time $t + 1$ must equal $1/R_t$, where R_t denotes the gross nominal yield on a one-period non-state-contingent bond. Therefore, taking a probability-weighted sum across all states of equation (A11), we have the familiar Euler equation:

(A12) $\beta R_t \mathrm{E}_t\left[\left(C(\nabla^{t+1})/C(\nabla^t)\right)^{-\sigma}(P_t/P_{t+1})\right] = 1$.

Analogous equations hold for Foreign households. Since contingent claims are (arbitrarily) denominated in Home currency, the first-order condition for Foreign households that is analogous to equation (A11) is:

(A13) $\beta\left(C^*(\nabla^{t+1})/C^*(\nabla^t)\right)^{-\sigma}(E_tP_t^*/E_{t+1}P_{t+1}^*) = \ddot{Z}(\nabla^{t+1}\,|\,\nabla^t)$.

Here, E_t refers to the home currency price of foreign currency exchange rate.[1]

As noted above, we will assume at this stage that labor input of all households is the same, so $N_t = N_t(h)$.

A1.b. *Firms*

Each Home good, $Y_t(f)$ is made according to a production function that is linear in the labor input. These are given by:

(A14) $Y_t(f) = A_tN_t(f)$.

Note that the productivity shock, A_t , is common to all firms in the Home country. $N_t(f)$ is a CES composite of individual home-country household labor, given by:

(A15) $N_t(f) = \left(\int_0^1 N_t(h,f)^{\frac{\eta_t-1}{\eta_t}}\,dh\right)^{\frac{\eta_t}{\eta_t-1}}$,

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[1] We offer an apology to the reader here. We want to stick to CGG's notation, who use e_t for the log of the nominal exchange rate. Consistency requires us to use E_t to refer to the level of the nominal exchange rate, so we have used the distinct but similar notation E_t to be the conditional expectation operator.

where the technology parameter, η_t, is stochastic and common to all Home firms.

Profits are given by:

(A16) $\Gamma_t(f) = P_{Ht}(f)C_{Ht}(f) + E_t P^*_{Ht}(f)C^*_{Ht}(f) - (1-\tau_t)W_t N_t(f)$.

In this equation, $P_{Ht}(f)$ is the home-currency price of the good when it is sold in the Home country. $P^*_{Ht}(f)$ is the foreign-currency price of the good when it is sold in the Foreign country. $C_{Ht}(f)$ is aggregate sales of the good in the home country:

(A17) $C_{Ht}(f) = \int_0^1 C_{Ht}(h, f)dh$.

$C^*_{Ht}(f)$ is defined analogously. It follows that $Y_t(f) = C_{Ht}(f) + C^*_{Ht}(f)$.

There are analogous equations for $Y^*_t(f)$, with the foreign productivity shock given by A^*_t, the foreign technology parameter shock given by η^*_t, and foreign subsidy given by τ^*_t.

A1.c *Equilibrium*

Goods market clearing conditions in the Home and Foreign country are given by:

(A18) $Y_t = C_{Ht} + C^*_{Ht} = \frac{\nu}{2}\frac{P_t C_t}{P_{Ht}} + \left(1-\frac{\nu}{2}\right)\frac{P^*_t C^*_t}{P^*_{Ht}} = k^{-1}\left(\frac{\nu}{2}S_t^{1-(\nu/2)}C_t + \left(1-\frac{\nu}{2}\right)(S^*_t)^{-\nu/2}C^*_t\right)$,

(A19) $Y^*_t = C_{Ft} + C^*_{Ft} = \left(1-\frac{\nu}{2}\right)\frac{P_t C_t}{P_{Ft}} + \frac{\nu}{2}\frac{P^*_t C^*_t}{P^*_{Ft}} = k^{-1}\left(\frac{\nu}{2}(S^*_t)^{1-(\nu/2)}C^*_t + \left(1-\frac{\nu}{2}\right)S_t^{-\nu/2}C_t\right)$.

We have used S_t and S^*_t to represent the price of imported to locally-produced goods in the Home and Foreign countries, respectively:

(A20) $S_t = P_{Ft}/P_{Ht}$,

(A21) $S^*_t = P^*_{Ht}/P^*_{Ft}$.

Equations (A11) and (A13) give us the familiar condition that arises in open-economy models with a complete set of state-contingent claims when PPP does not hold:

(A22) $\left(\frac{C_t}{C^*_t}\right)^{\sigma} = \frac{E_t P^*_t}{P_t} = \frac{E_t P^*_{Ht}}{P_{Ht}}(S^*_t)^{-\nu/2}S_t^{(\nu/2)-1}$.

Total employment is determined by output in each industry:

(A23) $N_t = \int_0^1 N_t(f)df = A_t^{-1}\int_0^1 Y_t(f)df = A_t^{-1}\left(C_{Ht}V_{Ht} + C^*_{Ht}V^*_{Ht}\right)$.

where

(A24) $V_{Ht} \equiv \int_0^1\left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} df$, and $V^*_{Ht} \equiv \int_0^1\left(\frac{P^*_{Ht}(f)}{P^*_{Ht}}\right)^{-\xi} df$.

A2 *Price and Wage Setting*

Households are monopolistic suppliers of their unique form of labor services. Household *h* faces demand for its labor services given by:

(A25) $N_t(h) = \left(\frac{W_t(h)}{W_t}\right)^{-\eta_t} N_t$,

where

(A26) $W_t = \left(\int_0^1 W_t(h)^{1-\eta_t} dh\right)^{\frac{1}{1-\eta_t}}$.

The first-order condition for household *h*'s choice of labor supply is given by:

(A27) $\frac{W_t(h)}{P_t} = (1+\mu_t^W)(C_t(h))^{\sigma}(N_t(h))^{\phi}$, where $\mu_t^W \equiv \frac{1}{\eta_t - 1}$.

The optimal wage set by the household is a time-varying mark-up over the marginal disutility of work (expressed in consumption units.)

Because all households are identical, we have $W_t = W_t(h)$ and $N_t = N_t(h)$.

Since all households are identical, we have from equation (A27):

(A28) $W_t / P_{Ht} = (1+\mu_t^W)C_t^{\sigma} N_t^{\phi} S_t^{1-(v/2)}$.

We adopt the following notation. For any variable K_t:

$\dot{K}_t$ is the value under flexible prices.

$\bar{K}_t$ is the value of variables under globally efficient allocations. In other words, this is the value for variables if prices were flexible, and optimal subsidies to monopolistic suppliers of labor and monopolistic producers of goods were in place. This includes a time-varying subsidy to suppliers of labor to offset the time-varying mark-up in wages in equation (A27).

Flexible Prices

Home firms maximize profits given by equation (A16), subject to the demand curve (A10). They optimally set prices as a mark-up over marginal cost:

(A29) $\dot{P}_{Ht}(f) = \dot{E}_t \dot{P}_{Ht}^*(f) = (1-\tau_t)(1+\mu^P)\dot{W}_t / A_t$, where $\mu^P \equiv \frac{1}{\xi - 1}$.

When optimal subsidies are in place:

(A30) $\bar{P}_{Ht}(f) = \bar{E}_t \bar{P}_{Ht}^*(f) = \bar{W}_t / A_t$.

From (A27), (A29), and (A30), it is apparent that the optimal subsidy satisfies

(A31) $(1-\tau_t)(1+\mu^P)(1+\mu_t^W) = 1$.

Note from (A29) that all flexible price firms are identical and set the same price. Because the demand functions of Foreign residents have the same elasticity of demand for Home goods as Home residents, firms set the same price for sale abroad:

(A32) $E_t \dot{P}_{Ht}^* = \dot{P}_{Ht}$ and $E_t \bar{P}_{Ht}^* = \bar{P}_{Ht}$

From (A31), using (A28), we have:

(A33) $\dot{P}_{Ht} = \dot{E}_t \dot{P}_{Ht}^* = \dot{W}_t / ((1+\mu^W)A_t)$ and $\dot{P}_{Ft}^* = \dot{E}_t^{-1} \dot{P}_{Ft} = \dot{W}_t^* / ((1+\mu^{*W})A_t^*)$.

We can conclude:

(A34) $\dot{S}_t^{*-1} = \dot{S}_t$.

Because $\dot{P}_{Ht}(f)$ is identical for all firms, (A23) collapses to

(A35) $\dot{Y}_t = A_t \dot{N}_t$.

PCP

We assume a standard Calvo pricing technology. A given firm may reset its prices with probability $1-\theta$ each period. We assume that when the firm resets its price, it will be able to reset its prices for sales in both markets. We assume the PCP firm sets a single price in its own currency, so the law of one price holds.

The firm's objective is to maximize its value, which is equal to the value at state-contingent prices of its entire stream of dividends. Given equation (A11), it is apparent that the firm that selects its price at time *t*, chooses its reset price, $P^0_{Ht}(f)$, to maximize

$$\text{(A36)}\quad \mathrm{E}_t\sum_{j=0}^{\infty}\theta^j Q_{t,t+j}\left[P^0_{Ht}(f)(C_{Ht+j}(f)+C^*_{Ht+j}(f))-(1-\tau_t)W_{t+j}N_{t+j}(f)\right],$$

subject to the sequence of demand curves given by equation (A10) and the corresponding Foreign demand equation for Home goods. In this equation, we define

$$\text{(A37)}\quad Q_{t,t+j}\equiv\beta^j\left(C_{t+j}/C_t\right)^{-\sigma}(P_t/P_{t+j}).$$

The solution for the optimal price for the Home firm for sale in the Home country is given by:

$$\text{(A38)}\quad P^0_{Ht}(z)=\frac{\xi}{\xi-1}\frac{\mathrm{E}_t\sum_{j=0}^{\infty}\theta^j Q_{t,t+j}(1-\tau_t)W_{t+j}P^{\xi}_{Ht+j}(C_{Ht+j}+C^*_{Ht+j})/A_{t+j}}{\mathrm{E}_t\sum_{j=0}^{\infty}\theta^j Q_{t,t+j}P^{\xi}_{Ht+j}(C_{Ht+j}+C^*_{Ht+j})}.$$

Under the Calvo price setting mechanism, a fraction θ of prices remain unchanged from the previous period. From equation (A7), we can write:

$$\text{(A39)}\quad P_{Ht}=\left[\theta(P_{Ht-1})^{1-\xi}+(1-\theta)(P^0_{Ht})^{1-\xi}\right]^{1/(1-\xi)},$$

LCP

The same environment as the PCP case holds, with the sole exception that the firm sets its price for export in the importer's currency rather than its own currency when it is allowed to reset prices. The Home firm, for example, sets $P^*_{Ht}(f)$ in Foreign currency. The firm that can reset its price at time *t* chooses its reset prices, $P^0_{Ht}(f)$ and $P^{0*}_{Ht}(f)$, to maximize

$$\text{(A40)}\quad \mathrm{E}_t\sum_{j=0}^{\infty}\theta^j Q_{t,t+j}\left[P^0_{Ht}(z)C_{Ht+j}(f)+E_t P^{0*}_{Ht}C^*_{Ht+j}(f)-(1-\tau_t)W_{t+j}N_{t+j}(f)\right].$$

The solution for $P^0_{Ht}(z)$ is given by:

$$\text{(A41)}\quad P^0_{Ht}(z)=\frac{\xi}{\xi-1}\frac{\mathrm{E}_t\sum_{j=0}^{\infty}\theta^j Q_{t,t+j}(1-\tau_t)W_{t+j}P^{\xi}_{Ht+j}C_{Ht+j}/A_{t+j}}{\mathrm{E}_t\sum_{j=0}^{\infty}\theta^j Q_{t,t+j}P^{\xi}_{Ht+j}C_{Ht+j}}$$

We find for export prices,

(A42) $$P_{Ht}^{0*}(z) = \frac{\xi}{\xi - 1} \frac{\mathrm{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} (1-\tau_t) W_{t+j} (P_{Ht+j}^*)^{\xi} C_{Ht+j}^* / A_{t+j}}{\mathrm{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t,t+j} E_{t+j} (P_{Ht+j}^*)^{\xi} C_{Ht+j}^*}.$$

Equation (A39) holds in the LCP case as well. However, the law of one price does not hold. For export prices, we have:

(A43) $$P_{Ht}^* = \left[\theta (P_{Ht-1}^*)^{1-\xi} + (1-\theta)(P_{Ht}^{0*})^{1-\xi} \right]^{1/(1-\xi)}$$

Subsidies

As in CGG, we will assume that subsidies to monopolists are not set at their optimal level except in steady-state. That is, instead of the efficient subsidy given in equation (A31), we have:

(A44) $$(1-\tau)(1+\mu^P)(1+\mu^W) = 1.$$

Here, μ^W is the steady-state level of μ_t^W. We have dropped the time subscript on the subsidy rate τ_t because it is not time-varying.

Appendix B

Log-linearized Model

In this section, we present log-linear approximations to the models presented above.

In this section, we present all of the equations of the log-linearized model, but we separate out those that are used in the derivation of the loss function (which do not involve price setting or wage setting) and those that are not.

Equations used for derivation of loss functions

We define the log of the currency misalignment as the average of the difference between Foreign and Home prices:

(B1) $$m_t \equiv \frac{1}{2}(e_t + p^*_{Ht} - p_{Ht} + e_t + p^*_{Ft} - p_{Ft}).$$

In the flexible-price and PCP models, $m_t = 0$.

We also define the export premium as the average by which consumer prices of imported goods exceeds the average of locally produced goods:

(B2) $$z_t \equiv \frac{1}{2}(p^*_{Ht} + p_{Ft} - p^*_{Ft} - p_{Ht})$$

In all three models, to a first order, $\ln(V_{Ht}) = \ln(V^*_{Ht}) = \ln(V_{Ft}) = \ln(V^*_{Ft}) = 0$. That allows us to approximate equation (A23) and its foreign counterpart as:

(B3) $n_t = y_t - a_t$, and

(B4) $n^*_t = y^*_t - a^*_t$.

The market-clearing conditions, (A18) and (A19) are approximated as:

(B5) $$y_t = \frac{\nu}{2}c_t + \frac{2-\nu}{2}c^*_t + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s^*_t,$$

(B6) $$y^*_t = \frac{\nu}{2}c^*_t + \frac{2-\nu}{2}c_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s^*_t.$$

The condition arising from complete markets that equates the marginal utility of nominal wealth for Home and Foreign households, equation (A22), is given by:

(B7) $$\sigma c_t - \sigma c^*_t = m_t + \frac{\nu-1}{2}s_t - \left(\frac{\nu-1}{2}\right)s^*_t.$$

We define "relative" and "world" log output by:

(B8) $$y^R_t \equiv \frac{1}{2}(y_t - y^*_t)$$

(B9) $$y^W_t \equiv \frac{1}{2}(y_t + y^*_t).$$

For use later, it is helpful to use equations (B5)-(B7) to express c_t, c^*_t, s_t, in terms of y_t and y^*_t and the price deviations, m_t and z_t.

(B10) $$c_t = \frac{\nu-1}{D}y^R_t + y^W_t + \frac{\nu(2-\nu)}{2D}m_t$$

(B11) $c_t^* = -\left(\frac{\nu - 1}{D}\right) y_t^R + y_t^W - \frac{\nu(2-\nu)}{2D} m_t$,

where $D \equiv \sigma\nu(2-\nu) + (\nu - 1)^2$.

We can further simplify these by defining:

(B12) $c_t^R \equiv \frac{1}{2}(c_t - c_t^*)$

(B13) $c_t^W \equiv \frac{1}{2}(c_t + c_t^*)$

Then

(B14) $c_t^R = \frac{\nu - 1}{D} y_t^R + \frac{\nu(2-\nu)}{2D} m_t$

(B15) $c_t^W = y_t^W$.

And, solving for the terms of trade, we find:

(B16) $s_t = \frac{2\sigma}{D} y_t^R + z_t - \frac{(\nu - 1)}{D} m_t$,

(B17) $s_t^* = -\frac{2\sigma}{D} y_t^R + z_t + \frac{(\nu - 1)}{D} m_t$.

Under a globally efficient allocation, the marginal rate of substitution between leisure and aggregate consumption should equal the marginal product of labor times the price of output relative to consumption prices. To see the derivation more cleanly, we insert the shadow real wages in the efficient allocation, $\overline{w}_t - \overline{p}_{Ht}$ and $\overline{w}_t^* - \overline{p}_{Ft}^*$ into equations (B18) and (B19) below. So, the efficient allocation would be achieved in a model with flexible wages and optimal subsidies. These equations then can be understood intuitively by looking at the wage setting equations below ((B22)-(B23), and (B24)-(B25)) assuming the optimal subsidy is in place. But, to emphasize, they do not depend on a particular model of wage setting, and are just the standard efficiency condition equating the marginal rate of substitution between leisure and aggregate consumption to the marginal rate of transformation.

(B18) $a_t = \overline{w}_t - \overline{p}_{Ht} = \left(\frac{\sigma}{D} + \phi\right)\overline{y}_t^R + (\sigma + \phi)\overline{y}_t^W - \phi a_t$,

(B19) $a_t^* = \overline{w}_t^* - \overline{p}_{Ft}^* = -\left(\frac{\sigma}{D} + \phi\right)\overline{y}_t^R + (\sigma + \phi)\overline{y}_t^W - \phi a_t^*$.

Equations of wage and price setting

The real Home and Foreign product wages, from equation (A28), are given by:

(B20) $w_t - p_{Ht} = \sigma c_t + \phi n_t + \frac{2-\nu}{2} s_t + \mu_t^W$,

(B21) $w_t^* - p_{Ft}^* = \sigma c_t^* + \phi n_t^* + \left(\frac{2-\nu}{2}\right) s_t^* + \mu_t^{*W}$.

We can express $w_t - p_{Ht}$, and $w_t^* - p_{Ft}^*$ in terms of y_t and y_t^* and the exogenous disturbances, a_t, a_t^*, μ_t^W, and μ_t^{*W}:

(B22) $w_t - p_{Ht} = \left(\frac{\sigma}{D} + \phi\right) y_t^R + (\sigma + \phi) y_t^W + \frac{D-(\nu-1)}{2D} m_t + \frac{2-\nu}{2} z_t - \phi a_t + \mu_t^W$,

(B23) $w_t^* - p_{Ft}^* = -\left(\frac{\sigma}{D} + \phi\right) y_t^R + (\sigma + \phi) y_t^W - \left(\frac{D-(\nu-1)}{2D}\right) m_t + \frac{2-\nu}{2} z_t - \phi a_t^* + \mu_t^{*W}$,

<u>Flexible Prices</u>

We can solve for the values of all the real variables under flexible prices by using equations (B3), (B4), (B10), (B11), (B16), (B22) and (B23), as well as the price-setting conditions, from (A33):

(B24) $\dot{w}_t - \dot{p}_{Ht} = a_t$,

(B25) $\dot{w}_t^* - \dot{p}_{Ft}^* = a_t^*$.

<u>PCP</u>

Log-linearization of equations (A38) and (A39) gives us the familiar New Keynesian Phillips curve for an open economy:

(B26) $\pi_{Ht} = \delta(w_t - p_{Ht} - a_t) + \beta \mathrm{E}_t \pi_{Ht+1}$,

where $\delta = (1-\theta)(1-\beta\theta)/\theta$.

We can rewrite this equation using (B22) and (B18) as:

(B27) $\pi_{Ht} = \delta\left[\left(\frac{\sigma}{D} + \phi\right)\tilde{y}_t^R + (\sigma+\phi)\tilde{y}_t^W\right] + \beta \mathrm{E}_t \pi_{Ht+1} + u_t$,

where $u_t = \delta \mu_t^W$.

Similarly for foreign producer-price inflation, we have:

(B28) $\pi_{Ft}^* = \delta\left[-\left(\frac{\sigma}{D} + \phi\right)\tilde{y}_t^R + (\sigma+\phi)\tilde{y}_t^W\right] + \beta \mathrm{E}_t \pi_{Ft+1}^* + u_t^*$.

<u>LCP</u>

Equation (B26) holds in the LCP model as well. But in the LCP model, the law of one price deviation is not zero. We have:

(B29) $\pi_{Ht} = \delta\left[\left(\frac{\sigma}{D} + \phi\right)\tilde{y}_t^R + (\sigma+\phi)\tilde{y}_t^W + \frac{D-(\nu-1)}{2D} m_t + \frac{2-\nu}{2} z_t\right] + \beta \mathrm{E}_t \pi_{Ht+1} + u_t$,

(B30) $\pi_{Ft}^* = \delta\left[-\left(\frac{\sigma}{D} + \phi\right)\tilde{y}_t^R + (\sigma+\phi)\tilde{y}_t^W - \left(\frac{D-(\nu-1)}{2D}\right) m_t + \frac{2-\nu}{2} z_t\right] + \beta \mathrm{E}_t \pi_{Ft+1}^* + u_t^*$.

In addition, from (A42) and (A43), we derive:

(B31) $\pi_{Ht}^* = \delta(w_t - p_{Ht}^* - e_t - a_t) + \beta \mathrm{E}_t \pi_{Ht+1}^* = \delta(w_t - p_{Ht} - m_t - z_t - a_t) + \beta \mathrm{E}_t \pi_{Ht+1}^*$

We can rewrite this as

(B32) $\pi_{Ht}^* = \delta\left[\left(\frac{\sigma}{D} + \phi\right)\tilde{y}_t^R + (\sigma+\phi)\tilde{y}_t^W - \left(\frac{D+\nu-1}{2D}\right) m_t - \frac{\nu}{2} z_t\right] + \beta \mathrm{E}_t \pi_{Ht+1}^* + u_t$.

Similarly, we can derive:

(B33) $\pi_{Ft} = \delta\left[-\left(\frac{\sigma}{D} + \phi\right)\tilde{y}_t^R + (\sigma+\phi)\tilde{y}_t^W + \frac{D+\nu-1}{2D} m_t - \frac{\nu}{2} z_t\right] + \beta \mathrm{E}_t \pi_{Ft+1} + u_t^*$.

Consider equations (B29)-(B33). If $z_t = 0$ in these equations, then $\pi_{Ft} - \pi_{Ht} = \pi_{Ft}^* - \pi_{Ht}^*$. This in turn implies $z_{t+1} = 0$. By induction, if the initial condition $z_0 = 0$ holds, it follows that $z_t = 0$ in all periods in the LCP model, or, in other words, $s_t^* = -s_t$. That is, the relative price of Foreign to Home goods is the same in both countries. We emphasize that this is true for a first-order approximation in the LCP model.

So we can simplify equations (B29)-(B33):

$$\text{(B34)} \quad \pi_{Ht} = \delta\left[\left(\frac{\sigma}{D}+\phi\right)\tilde{y}_t^R + (\sigma+\phi)\tilde{y}_t^W + \frac{D-(\nu-1)}{2D}m_t\right] + \beta \mathrm{E}_t\pi_{Ht+1} + u_t\,,$$

$$\text{(B35)} \quad \pi_{Ft}^* = \delta\left[-\left(\frac{\sigma}{D}+\phi\right)\tilde{y}_t^R + (\sigma+\phi)\tilde{y}_t^W - \left(\frac{D-(\nu-1)}{2D}\right)m_t\right] + \beta \mathrm{E}_t\pi_{Ft+1}^* + u_t^*\,.$$

$$\text{(B36)} \quad \pi_{Ht}^* = \delta\left[\left(\frac{\sigma}{D}+\phi\right)\tilde{y}_t^R + (\sigma+\phi)\tilde{y}_t^W - \left(\frac{D+\nu-1}{2D}\right)m_t\right] + \beta \mathrm{E}_t\pi_{Ht+1}^* + u_t\,.$$

$$\text{(B37)} \quad \pi_{Ft} = \delta\left[-\left(\frac{\sigma}{D}+\phi\right)\tilde{y}_t^R + (\sigma+\phi)\tilde{y}_t^W + \frac{D+\nu-1}{2D}m_t\right] + \beta \mathrm{E}_t\pi_{Ft+1} + u_t^*\,.$$

Relationship to CGG's Phillips Curve

The Phillips curve in CGG's PCP model has Home inflation depending only on the Home output gap. Our model should be equivalent to theirs when there is not home bias in preferences, but equation (B27) has π_{Ht} depending on both $\tilde{y}_t^R$ and $\tilde{y}_t^W$. This will not reduce to a function only of $\tilde{y}_t$ except in the case of $\sigma = 1$.

However, it can be seen that in fact (B27) is equivalent to CGG's equation when one recognizes that CGG's definition of the output gap differs from the one used here. CGG define the output gap as the difference between y_t and what I will call $\bar{y}_t^{CGG}$. $\bar{y}_t^{CGG}$ is the efficient level of Home output when the Foreign output level is taken as given. That contrasts to our definition in which $\bar{y}_t$ is the globally efficient output level. CGG's definition is convenient because their analysis focuses on non-cooperative monetary policy, while the definition used here is more convenient because of the focus on cooperative monetary policy. But this is a matter of convenience: algebraically the equations are the same.

To see this, note that equation (B26) is the same as in CGG. When there are no deviations from the law of one price, equation (B22) can be written as:

$$\text{(B38)} \quad \begin{aligned} w_t - p_{Ht} &= \left(\frac{\sigma}{D}+\phi\right)y_t^R + (\sigma+\phi)y_t^W - \phi a_t + \mu_t^W \\ &= \left(\frac{\sigma}{D}+\phi\right)\left(\frac{y_t - y_t^*}{2}\right) + (\sigma+\phi)\left(\frac{y_t+y_t^*}{2}\right) - \phi a_t + \mu_t^W \end{aligned}.$$

When there is no home bias in preferences, this simplifies to:

$$\text{(B39)} \quad w_t - p_{Ht} = \left(\frac{\sigma+1}{2}+\phi\right)y_t + \frac{\sigma-1}{2}y_t^* - \phi a_t + \mu_t^W$$

Then, CGG define:

$$\text{(B40)} \quad \bar{y}_t^{CGG} = \frac{2}{\sigma+1-2\phi}\left[(1+\phi)a_t - \left(\frac{\sigma-1}{2}\right)y_t^*\right].$$

So we can write using (B39) and (B40):

(B41) $$w_t - p_{Ht} - a_t = \left(\frac{\sigma + 1}{2} + \phi\right)(y_t - \overline{y}_t^{CGG}) + \mu_t^W \, .$$

Substituting this expression into equation (B26) gives us CGG's version of the Phillips curve, in which inflation depends only on the Home output gap under their definitions.

Appendix C

C.I Derivation of Welfare Function in Clarida-Gali-Gertler model with Home Bias in preferences

The object is to rewrite the welfare function, which is defined in terms of home and foreign consumption and labor effort into terms of the squared output gap and squared inflation. We derive the joint welfare function of home and foreign households, since we will be examining cooperative monetary policy.

Most of the derivation requires only 1st-order approximations of the equations of the model, but in a few places, 2nd-order approximations are needed. If the approximation is 1st-order, I'll use the notation "$+o\left(\left\|a^2\right\|\right)$" to indicate that there are 2nd-order and higher terms left out, and if the approximation is 2nd-order, I will use "$+o\left(\left\|a^3\right\|\right)$". ($a$ is notation for the log of the productivity shock)

From equation (A1), the period utility of the planner is given by:

(C1) $$\upsilon_t \equiv \frac{1}{1-\sigma}(C_t^{1-\sigma}+C_t^{*1-\sigma})-\frac{1}{1+\phi}(N_t^{1+\phi}+N_t^{*1+\phi}).$$

Take a second-order log approximation around the non-stochastic steady state. We assume allocations are efficient in steady state, so we have $C^{1-\sigma}=C^{*1-\sigma}=N^{1+\phi}=N^{*1+\phi}$. The fact that $C^{1-\sigma}=N^{1+\phi}$ follows from the fact that in steady state, $C=N$ from market clearing and symmetry, and $C^{-\sigma}=N^{\phi}$ from the condition that the marginal rate of substitution between leisure and consumption equals one in an efficient non-stochastic steady state.

We get:

(C2) $$\begin{aligned}\upsilon_t &= 2\left(\frac{1}{1-\sigma}-\frac{1}{1+\phi}\right)C^{1-\sigma}+C^{1-\sigma}\left(c_t+c_t^*\right)+\frac{1-\sigma}{2}C^{1-\sigma}\left((c_t)^2+(c_t^*)^2\right)\\ &\quad -C^{1-\sigma}\left(n_t+n_t^*\right)-\frac{1+\phi}{2}C^{1-\sigma}\left((n_t)^2+(n_t^*)^2\right)+o\left(\left\|a^3\right\|\right)\end{aligned}.$$

Since we can equivalently maximize an affine transformation of (C2), it is convenient to simplify that equation to get:

(C3) $$\upsilon_t = c_t+c_t^*-n_t-n_t^*+\frac{1-\sigma}{2}\left(c_t^2+c_t^{*2}\right)-\frac{1+\phi}{2}\left(n_t^2+n_t^{*2}\right)+o\left(\left\|a^3\right\|\right).$$

Utility is maximized when consumption and employment take on their efficient values:

(C4) $$\upsilon_t^{\max} = \bar{c}_t+\bar{c}_t^*-\bar{n}_t-\bar{n}_t^*+\frac{1-\sigma}{2}\left(\bar{c}_t^2+\bar{c}_t^{*2}\right)-\frac{1+\phi}{2}\left(\bar{n}_t^2+\bar{n}_t^{*2}\right)+o\left(\left\|a^3\right\|\right).$$

In general, this maximum may not be attainable because of distortions. We can write $x_t=\bar{x}_t+\tilde{x}_t$, where $\tilde{x}_t \equiv x_t-\bar{x}_t$. So, we have:

(C5)

$$\upsilon_t = \left[\bar{c}_t + \bar{c}_t^* - \bar{n}_t - \bar{n}_t^* + \frac{1-\sigma}{2}\left(\bar{c}_t^2 + \bar{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\bar{n}_t^2 + \bar{n}_t^{*2}\right) \right]$$
$$+\tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^* + \frac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2} + 2\bar{c}_t\tilde{c}_t + 2\bar{c}_t^*\tilde{c}_t^*\right) - \frac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2} + 2\bar{n}_t\tilde{n}_t + 2\bar{n}_t^*\tilde{n}_t^*\right)$$
$$+o\left(\left\|a^3\right\|\right)$$

or,

(C6) $$\upsilon_t - \upsilon_t^{\max} = \tilde{c}_t + \tilde{c}_t^* - \tilde{n}_t - \tilde{n}_t^* + \frac{1-\sigma}{2}\left(\tilde{c}_t^2 + \tilde{c}_t^{*2}\right) - \frac{1+\phi}{2}\left(\tilde{n}_t^2 + \tilde{n}_t^{*2}\right)$$
$$+(1-\sigma)\left(\bar{c}_t\tilde{c}_t + \bar{c}_t^*\tilde{c}_t^*\right) - (1+\phi)\left(\bar{n}_t\tilde{n}_t + \bar{n}_t^*\tilde{n}_t^*\right) + o\left(\left\|a^3\right\|\right).$$

We can rewrite this as:

(C7) $$\upsilon_t - \upsilon_t^{\max} = 2\tilde{c}_t^W - 2\tilde{n}_t^W + (1-\sigma)\left((\tilde{c}_t^R)^2 + (\tilde{c}_t^W)^2\right) - (1+\phi)\left((\tilde{n}_t^R)^2 + (\tilde{n}_t^W)^2\right)$$
$$+2(1-\sigma)\left(\bar{c}_t^R\tilde{c}_t^R + \bar{c}_t^W\tilde{c}_t^W\right) - 2(1+\phi)\left(\bar{n}_t^R\tilde{n}_t^R + \bar{n}_t^R\tilde{n}_t^R\right) + o\left(\left\|a^3\right\|\right)$$

The object is to write (C7) as a function of squared output gaps and squared inflation if possible. We need a second-order approximation of $2\tilde{c}_t^W - 2\tilde{n}_t^W$. But because the rest of the terms are squares and products, the 1st-order approximations that have already been derived will be sufficient.

Recalling that $m_t = 0$ in the PCP model, we can write equations (B14)-(B15) as:

(C8) $c_t^R = \dfrac{\nu-1}{D} y_t^R + o\left(\left\|a^2\right\|\right)$,

(C9) $c_t^W = y_t^W + o\left(\left\|a^2\right\|\right)$.

It follows from (C8) and (C9) that:

(C10) $\bar{c}_t^R = \dfrac{\nu-1}{D}\bar{y}_t^R + o\left(\left\|a^2\right\|\right)$,

(C11) $\bar{c}_t^W = \bar{y}_t^W + o\left(\left\|a^2\right\|\right)$,

(C12) $\tilde{c}_t^R = \dfrac{\nu-1}{D}\tilde{y}_t^R + o\left(\left\|a^2\right\|\right)$,

(C13) $\tilde{c}_t^W = \tilde{y}_t^W + o\left(\left\|a^2\right\|\right)$.

Next, we can easily derive:

(C14) $\tilde{n}_t^R = \tilde{y}_t^R + o\left(\left\|a^2\right\|\right)$, and

(C15) $\tilde{n}_t^W = \tilde{y}_t^W + o\left(\left\|a^2\right\|\right)$.

These follow as in (B3)-(B4) because $n_t = y_t - a_t + o\left(\left\|a^2\right\|\right)$ and $\bar{n}_t = \bar{y}_t - a_t$ (and similarly in the Foreign country.)

We need expressions for $\bar{n}_t^R$ and $\bar{n}_t^W$. We have, using (B18)-(B19):

$$a_t^R = \left(\frac{\sigma}{D} + \phi\right)\bar{y}_t^R - \phi a_t^R + o\left(\left\|a^2\right\|\right),$$

$$a_t^W = (\sigma + \phi)\bar{y}_t^W - \phi a_t^W + o\left(\left\|a^2\right\|\right).$$

Using $a_t^R = \bar{y}_t^R - \bar{n}_t^R$ and $a_t^W = \bar{y}_t^W - \bar{n}_t^W$, we can write these as

(C16) $\bar{n}_t^R = \dfrac{1-\sigma}{1+\phi}\left(\dfrac{(1-\nu)^2}{D}\right)\bar{y}_t^R + o\left(\left\|a^2\right\|\right)$, and

(C17) $\bar{n}_t^W = \dfrac{1-\sigma}{1+\phi}\bar{y}_t^W + o\left(\left\|a^2\right\|\right)$.

Turning attention back to the loss function in equation (C7), we focus first on the terms

$$(1-\sigma)\left((\tilde{c}_t^R)^2 + (\tilde{c}_t^W)^2\right) - (1+\phi)\left((\tilde{n}_t^R)^2 + (\tilde{n}_t^W)^2\right) + 2(1-\sigma)\left(\bar{c}_t^R\tilde{c}_t^R + \bar{c}_t^W\tilde{c}_t^W\right) - 2(1+\phi)\left(\bar{n}_t^R\tilde{n}_t^R + \bar{n}_t^R\tilde{n}_t^R\right)$$

. These involve only squares and cross-products of $\tilde{c}_t^R$, $\tilde{c}_t^W$, $\bar{c}_t^R$, $\bar{c}_t^W$, $\tilde{n}_t^R$, $\tilde{n}_t^W$, $\bar{n}_t^R$, and $\bar{n}_t^W$. We can substitute from equations (C10)-(C17) into this expression. It is useful provide a few lines of algebra since it is a bit messy:

$$(1-\sigma)\left((\tilde{c}_t^R)^2 + (\tilde{c}_t^W)^2\right) - (1+\phi)\left((\tilde{n}_t^R)^2 + (\tilde{n}_t^W)^2\right)$$

(C18) $$+2(1-\sigma)\left(\bar{c}_t^R\tilde{c}_t^R + \bar{c}_t^W\tilde{c}_t^W\right) - 2(1+\phi)\left(\bar{n}_t^R\tilde{n}_t^R + \bar{n}_t^R\tilde{n}_t^R\right)$$

$$= \left[(1-\sigma)\left(\frac{\nu-1}{D}\right)^2 - (1+\phi)\right](\tilde{y}_t^R)^2 - (\sigma+\phi)(\tilde{y}_t^W)^2 + 2\nu(2-\nu)\left(\frac{(1-\sigma)(\nu-1)}{D}\right)^2 \bar{y}_t^R\tilde{y}_t^R$$

Now return to the $2\tilde{c}_t^W - 2\tilde{n}_t^W$ term in equation (C7) and do a 2nd-order approximation. Start with equation (A18), dropping the k^{-1} term because it will not affect the approximation, and noting that in the PCP model, $S_t^* = S_t^{-1}$:

(C19) $Y_t = \dfrac{\nu}{2}S_t^{(2-\nu)/2}C_t + \left(\dfrac{2-\nu}{2}\right)S_t^{\nu/2}C_t^*$

Then use equation (A22), but using the fact that $S_t^* = S_t^{-1}$ and there are no deviations from the law of one price:

(C20) $C_t^* = C_t S_t^{\frac{1-\nu}{\sigma}}$.

Substitute in to get:

(C21) $Y_t = \dfrac{\nu}{2}S_t^{\frac{2-\nu}{2}}C_t + \left(\dfrac{2-\nu}{2}\right)S_t^{\frac{\nu}{2}+\frac{1-\nu}{\sigma}}C_t$.

Solve for C_t:

(C22) $C_t = Y_t\left(\dfrac{\nu}{2}S_t^{\frac{2-\nu}{2}} + \left(\dfrac{2-\nu}{2}\right)S_t^{\frac{\nu}{2}+\frac{1-\nu}{\sigma}}\right)^{-1}$.

Then we get this 2nd-order approximation:

(C23) $c_t = y_t - \left(\frac{2-\nu}{2}\right)\left(\nu + \frac{1-\nu}{\sigma}\right)s_t - \frac{1}{2}\left(\frac{2-\nu}{2}\right)\frac{\nu}{2}(\nu-1)^2\left(\frac{\sigma-1}{\sigma}\right)^2 s_t^2 + o\left(\left\|a^3\right\|\right)$.

Symmetrically,

(C24) $c_t^* = y_t^* + \left(\frac{2-\nu}{2}\right)\left(\nu + \frac{1-\nu}{\sigma}\right)s_t - \frac{1}{2}\left(\frac{2-\nu}{2}\right)\frac{\nu}{2}(\nu-1)^2\left(\frac{\sigma-1}{\sigma}\right)^2 s_t^2 + o\left(\left\|a^3\right\|\right)$.

Averaging these two equations, we get:

(C25) $c_t^W = y_t^W - \frac{1}{2}\left(\frac{2-\nu}{2}\right)\frac{\nu}{2}(\nu-1)^2\left(\frac{\sigma-1}{\sigma}\right)^2 s_t^2 + o\left(\left\|a^3\right\|\right)$.

Now we can take a 1st-order approximation for s_t to substitute out for s_t^2. From equation (B16), setting $m_t = 0$ and $z_t = 0$, we have:

(C26) $s_t^2 = \frac{4\sigma^2}{D^2}(y_t^R)^2 + o\left(\left\|a^3\right\|\right)$.

Substituting into equation (C26), we can write:

(C27) $c_t^W = y_t^W - \frac{\nu(2-\nu)}{2}\left(\frac{(\nu-1)(\sigma-1)}{D}\right)^2 (y_t^R)^2 + o\left(\left\|a^3\right\|\right)$.

Evaluating (C27) at flexible prices, we have:

(C28) $\overline{c}_t^W = \overline{y}_t^W - \frac{\nu(2-\nu)}{2}\left(\frac{(\nu-1)(\sigma-1)}{D}\right)^2 (\overline{y}_t^R)^2 + o\left(\left\|a^3\right\|\right)$.

It follows from the fact that $\tilde{c}_t^W = c_t^W - \overline{c}_t^W$ that

(C29) $\tilde{c}_t^W = \tilde{y}_t^W - \frac{\nu(2-\nu)}{2}\left(\frac{(\nu-1)(\sigma-1)}{D}\right)^2 \left((\tilde{y}_t^R)^2 + 2\overline{y}_t^R\tilde{y}_t^R\right) + o\left(\left\|a^3\right\|\right)$.

See section C.3 below for the second-order approximations for $\tilde{n}_t$ and $\tilde{n}_t^*$:

(C30) $\tilde{n}_t = \tilde{y}_t + \frac{\xi}{2}\sigma_{p_H t}^2 + o\left(\left\|a^3\right\|\right)$

(C31) $\tilde{n}_t^* = \tilde{y}_t^* + \frac{\xi}{2}\sigma_{p_F{}^* t}^2 + o\left(\left\|a^3\right\|\right)$.

Adding these two equations together gives us:

(C32) $2\tilde{n}_t^W = 2\tilde{y}_t^W + \frac{\xi}{2}\sigma_{p_H t}^2 + \frac{\xi}{2}\sigma_{p_F{}^* t}^2 + o\left(\left\|a^3\right\|\right)$

Substitute expressions (C29)-(C31) along with (C18) into the loss function (C7):

$$\upsilon_t - \upsilon_t^{\max} = -\nu(2-\nu)\left(\frac{(\nu-1)(\sigma-1)}{D}\right)^2\left((\tilde{y}_t^R)^2 + 2\overline{y}_t^R\tilde{y}_t^R\right) - \frac{\xi}{2}(\sigma_{p_H t}^2 + \sigma_{p_F{}^* t}^2)$$

(C33) $$+\left[(1-\sigma)\left(\frac{\nu-1}{D}\right)^2 - (1+\phi)\right](\tilde{y}_t^R)^2 - (\sigma+\phi)(\tilde{y}_t^W)^2 + 2\nu(2-\nu)\left(\frac{(1-\sigma)(\nu-1)}{D}\right)^2 \overline{y}_t^R\tilde{y}_t^R$$

$$= -\left[\frac{\sigma}{D} + \phi\right](\tilde{y}_t^R)^2 - (\sigma+\phi)(\tilde{y}_t^W)^2 - \frac{\xi}{2}(\sigma_{p_H t}^2 + \sigma_{p_F{}^* t}^2)$$

This expression reduces to CGG's when there is no home bias ($\nu = 1$). To see this from their expression at the top of p. 903, multiply their utility by 2 (since they take average utility), and set their γ equal to ½ (so their country sizes are equal.)

C.2 Derivation of Welfare Function under LCP with Home Bias in Preferences

The second-order approximation to welfare in terms of logs of consumption and employment of course does not change, so equation (C6) still holds. As before, we break down the derivation into two parts. We use first-order approximations to structural equations to derive an approximation to the quadratic term

$$(1-\sigma)\left((\tilde{c}_t^R)^2+(\tilde{c}_t^W)^2\right)-(1+\phi)\left((\tilde{n}_t^R)^2+(\tilde{n}_t^W)^2\right)+2(1-\sigma)\left(\bar{c}_t^R\tilde{c}_t^R+\bar{c}_t^W\tilde{c}_t^W\right)-2(1+\phi)\left(\bar{n}_t^R\tilde{n}_t^R+\bar{n}_t^R\tilde{n}_t^R\right)$$
.

Then we use second order approximations to the structural equations to derive an expression for $2\tilde{c}_t^W - 2\tilde{n}_t^W$.

The quadratic term involves squares and cross-products of $\tilde{c}_t^R$, $\tilde{c}_t^W$, $\bar{c}_t^R$, $\bar{c}_t^W$, $\tilde{n}_t^R$, $\tilde{n}_t^W$, $\bar{n}_t^R$, and $\bar{n}_t^W$. Expressions (C10)-(C11) still gives us first-order approximations for $\bar{c}_t^R$ and $\bar{c}_t^W$; equations (C14)-(C15) are first-order approximations for $\tilde{n}_t^R$ and $\tilde{n}_t^W$; and, (C16)-(C17) are first-order approximations for $\bar{n}_t^R$ and $\bar{n}_t^W$. But we need to use equations (B14)-(B15) and (C10)-(C11) to derive:

(C34) $\tilde{c}_t^R = \dfrac{\nu-1}{D}\tilde{y}_t^R + \dfrac{\nu(2-\nu)}{2D}m_t + o\left(\left\|a^2\right\|\right)$,

(C35) $\tilde{c}_t^W = \tilde{y}_t^W + o\left(\left\|a^2\right\|\right)$.

With these equations, we can follow the derivation as in equation (C18). After tedious algebra, we arrive at the same result, with the addition of the terms $\dfrac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2}m_t^2$ and $\dfrac{(1-\sigma)\nu(2-\nu)(\nu-1)}{D^2}m_t y_t^R$. Note that the last term involves output levels, not output gaps. That is, we have:

(C36)

$$(1-\sigma)\left((\tilde{c}_t^R)^2+(\tilde{c}_t^W)^2\right)-(1+\phi)\left((\tilde{n}_t^R)^2+(\tilde{n}_t^W)^2\right)$$
$$+2(1-\sigma)\left(\bar{c}_t^R\tilde{c}_t^R+\bar{c}_t^W\tilde{c}_t^W\right)-2(1+\phi)\left(\bar{n}_t^R\tilde{n}_t^R+\bar{n}_t^R\tilde{n}_t^R\right)$$
$$=\left[(1-\sigma)\left(\frac{\nu-1}{D}\right)^2-(1+\phi)\right](\tilde{y}_t^R)^2-(\sigma+\phi)(\tilde{y}_t^W)^2+2\nu(2-\nu)\left(\frac{(1-\sigma)(\nu-1)}{D}\right)^2\bar{y}_t^R\tilde{y}_t^R\cdot$$
$$+\frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2}m_t^2+\frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{D^2}m_t y_t^R$$

The derivation of $2\tilde{c}_t^W - 2\tilde{n}_t^W$ is similar to the PCP model. However, one tedious aspect of the derivation is that we cannot make use of the equality $S_t^* = S_t^{-1}$ that holds under PCP and flexible prices. We write out the equilibrium conditions for home output, and its foreign equivalent, from equations (A18) and (A19):

(C37) $Y_t = \frac{\nu}{2} S_t^{1-(\nu/2)} C_t + \left(1-\frac{\nu}{2}\right)(S_t^*)^{-\nu/2} C_t^*$,

(C38) $Y_t^* = \frac{\nu}{2} (S_t^*)^{1-(\nu/2)} C_t^* + \left(1-\frac{\nu}{2}\right) S_t^{-\nu/2} C_t$.

We directly take second-order approximations of these equations around the efficient non-stochastic steady state:

(C39)
$$\begin{aligned} y_t + \frac{1}{2} y_t^2 = {} & \frac{\nu}{2} c_t + \left(\frac{2-\nu}{2}\right) c_t^* + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t^* \\ & + \frac{1}{2}\left\{ \frac{\nu}{2} c_t^2 + \left(\frac{2-\nu}{2}\right) c_t^{*2} + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)^2 s_t^2 + \left(\frac{\nu}{2}\right)^2 \left(\frac{2-\nu}{2}\right) s_t^{*2} \right\} \\ & + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t c_t - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t^* c_t^* + o\left(\left\| a^3 \right\|\right) \end{aligned}$$

(C40)
$$\begin{aligned} y_t^* + \frac{1}{2} y_t^{*2} = {} & \frac{\nu}{2} c_t^* + \left(\frac{2-\nu}{2}\right) c_t + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t^* - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t \\ & + \frac{1}{2}\left\{ \frac{\nu}{2} c_t^{*2} + \left(\frac{2-\nu}{2}\right) c_t^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)^2 s_t^{*2} + \left(\frac{\nu}{2}\right)^2 \left(\frac{2-\nu}{2}\right) s_t^2 \right\}. \\ & + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t^* c_t^* - \frac{\nu}{2}\left(\frac{2-\nu}{2}\right) s_t c_t + o\left(\left\| a^3 \right\|\right) \end{aligned}$$

Averaging (C39) and (C40), we find:

(C41) $y_t^W + \frac{1}{2}\left((y_t^R)^2 + (y_t^W)^2\right) = c_t^W + \frac{1}{2}\left((c_t^R)^2 + (c_t^W)^2\right) + \frac{\nu(2-\nu)}{16}(s_t^2 + s_t^{*2}) + o\left(\left\| a^3 \right\|\right)$.

Next, we can use equations (B14)-(B17) to get approximations for $(c_t^R)^2$, $(c_t^W)^2$, s_t^2, and s_t^{*2}. These equations are linear approximations for c_t^R, c_t^W, s_t, and s_t^*, but since we are looking to approximate the squares of these variables, that is sufficient. With some algebra, we find:

(C42)

$$\begin{aligned} c_t^W = y_t^W - \frac{\nu(2-\nu)}{2}\left(\frac{(\nu-1)(1-\sigma)}{D}\right)^2 (y_t^R)^2 - \frac{\nu(2-\nu)}{8D^2} m_t^2 - \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{2D^2} m_t y_t^R \\ - \frac{\nu(2-\nu)}{8} z_t^2 + o\left(\left\| a^3 \right\|\right) \end{aligned}.$$

Note that if set $m_t = 0$ and $z_t = 0$ in (C42), we would arrive at the second-order approximation for c_t^W from the PCP model.

Then following the derivations as in the PCP model derivation of (C29), we can write:

(C43)
$$\tilde{c}_t^W = \tilde{y}_t^W - \frac{\nu(2-\nu)}{2}\left(\frac{(\nu-1)(\sigma-1)}{D}\right)^2\left((\tilde{y}_t^R)^2 + 2\bar{y}_t^R\tilde{y}_t^R\right) - \frac{\nu(2-\nu)}{8D^2}m_t^2 - \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{2D^2}m_t y_t^R - \frac{\nu(2-\nu)}{8}z_t^2 + +o\left(\left\|a^3\right\|\right)$$

As shown in section B.3, we can make the following second-order approximation:

(C44)
$$2\tilde{n}_t^W = 2\tilde{y}_t^W + \frac{\xi}{2}\left[\frac{\nu}{2}\sigma_{p_H t}^2 + \frac{2-\nu}{2}\sigma_{p_H * t}^2 + \frac{\nu}{2}\sigma_{p_F * t}^2 + \frac{2-\nu}{2}\sigma_{p_F t}^2\right] + o\left(\left\|a^3\right\|\right).$$

We then can substitute (C43), and (C44), along with (C36) into the loss function (C7). We find:

(C45)
$$\begin{aligned}
v_t - v_t^{\max} &= 2\tilde{y}_t^W - \nu(2-\nu)\left(\frac{(\nu-1)(\sigma-1)^2}{D}\right)\left((\tilde{y}_t^R)^2 + 2\bar{y}_t^R\tilde{y}_t^R\right) - \frac{\nu(2-\nu)}{4D}m_t^2 \\
&- \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{D^2}m_t y_t^R - \frac{\nu(2-\nu)}{4}z_t^2 \\
&- 2\tilde{y}_t^W - \frac{\xi}{2}\left[\frac{\nu}{2}\sigma_{p_H t}^2 + \frac{2-\nu}{2}\sigma_{p_{H*} t}^2 + \frac{\nu}{2}\sigma_{p_{F*} t}^2 + \frac{2-\nu}{2}\sigma_{p_F t}^2\right] \\
&+ \left[(1-\sigma)\left(\frac{\nu-1}{D}\right)^2 - (1+\phi)\right](\tilde{y}_t^R)^2 - (\sigma+\phi)(\tilde{y}_t^W)^2 + 2\nu(2-\nu)\left(\frac{(1-\sigma)(\nu-1)}{D}\right)^2\bar{y}_t^R\tilde{y}_t^R \\
&+ \frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2}m_t^2 + \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{D^2}m_t y_t^R \\
&= -\left[\frac{\sigma}{D} + \phi\right](\tilde{y}_t^R)^2 - (\sigma+\phi)(\tilde{y}_t^W)^2 - \frac{\nu(2-\nu)}{4D}m_t^2 - \frac{\nu(2-\nu)}{4}z_t^2 \\
&- \frac{\xi}{2}\left[\frac{\nu}{2}\sigma_{p_H t}^2 + \frac{2-\nu}{2}\sigma_{p_{H*} t}^2 + \frac{\nu}{2}\sigma_{p_{F*} t}^2 + \frac{2-\nu}{2}\sigma_{p_F t}^2\right] + o(\left\|a^3\right\|)
\end{aligned}$$

C.3 Derivations of Price Dispersion Terms in Loss Functions

In the PCP case, we can write

(C46)
$$A_t N_t = A_t \int_0^1 N_t(f)df = Y_t \int_0^1 \left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} df = Y_t V_t,$$

where $V_t \equiv \int_0^1 \left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi} df$. Taking logs, we can write:

(C47) $a_t + n_t = y_t + v_t$.

We have $v_t \equiv \ln\left(\int_0^1 e^{-\xi\hat{p}_{Ht}(f)}df\right)$,

where we define

(C48) $\hat{p}_{Ht}(f) \equiv p_{Ht}(f) - p_{Ht}$.

Following Gali (2008), we note

(C49)
$$e^{(1-\xi)\hat{p}_{Ht}(f)} = 1 + (1-\xi)\hat{p}_{Ht}(f) + \frac{(1-\xi)^2}{2}\hat{p}_{Ht}(f)^2 + o\left(\left\|a^3\right\|\right).$$

By the definition of the price index P_{Ht}, we have $\int_0^1 e^{(1-\xi)\hat{p}_{Ht}(f)}df = 1$. Hence, from (C49),

(C50) $\int_0^1 \hat{p}_{Ht}(f)df = \frac{\xi - 1}{2}\int_0^1 \hat{p}_{Ht}(f)^2 df + o\left(\left\|a^3\right\|\right)$.

We also have

(C51) $e^{-\xi \hat{p}_{Ht}(f)} = 1 - \xi \hat{p}_{Ht}(f) + \frac{\xi^2}{2}\hat{p}_{Ht}(f)^2 + o\left(\left\|a^3\right\|\right)$

It follows, using (C50):

(C52)

$$\int_0^1 e^{-\xi \hat{p}_{Ht}(f)}df = 1 - \xi\int_0^1 \hat{p}_{Ht}(f)df + \frac{\xi^2}{2}\int_0^1 \hat{p}_{Ht}(f)^2 df + o\left(\left\|a^3\right\|\right) = 1 + \frac{\xi}{2}\int_0^1 \hat{p}_{Ht}(f)^2 df + o\left(\left\|a^3\right\|\right)$$

Note the following relationship:

(C53) $\int_0^1 \hat{p}_{Ht}(f)^2 df = \int_0^1 (p_{Ht}(f) - \mathrm{E}_f(p_{Ht}(f))^2 df + o\left(\left\|a^3\right\|\right) = \mathrm{var}(p_{Ht}) + o\left(\left\|a^3\right\|\right)$.

Using our notation for variances, $\sigma^2_{p_H t} \equiv \mathrm{var}(p_{Ht})$, and taking the log of (C52) we arrive at

(C54) $v_t = \frac{\xi}{2}\sigma^2_{p_H t} + o\left(\left\|a^3\right\|\right)$.

Substituting this into equation (C47), and recalling that $\bar{y}_t = \bar{n}_t + a_t$, we arrive at equation (C30). The derivation of (C31) for the Foreign country proceeds identically.

For the LCP model, we will make use of the following second-order approximation to the equation $Y_t = C_{Ht} + C^*_{Ht}$:

(C55) $y_t = \frac{\nu}{2}c_{Ht} + \left(\frac{2-\nu}{2}\right)c^*_{Ht} + \frac{1}{2}\left(\frac{\nu}{2}\right)\left(\frac{2-\nu}{2}\right)(c^2_{Ht} + 2c_{Ht}c^*_{Ht} + c^{*2}_{Ht}) + o\left(\left\|a^3\right\|\right)$.

In the LCP model, we can write:

(C56)

$$A_t N_t = A_t\int_0^1 N_t(f)df = C_{Ht}\int_0^1\left(\frac{P_{Ht}(f)}{P_{Ht}}\right)^{-\xi}df + C^*_{Ht}\int_0^1\left(\frac{P^*_{Ht}(f)}{P^*_{Ht}}\right)^{-\xi}df = C_{Ht}V_{Ht} + C^*_{Ht}V^*_{Ht},$$

where the definitions of V_{Ht} and V^*_{Ht} are analogous to that of V_t in the PCP model. Taking a second-order log approximation to (C56), we have:

(C57)
$$\begin{aligned} a_t + n_t = {} & \frac{\nu}{2}(c_{Ht} + v_{Ht}) + \left(\frac{2-\nu}{2}\right)(c^*_{Ht} + v^*_{Ht}) \\ & + \frac{1}{2}\left(\frac{\nu}{2}\right)\left(\frac{2-\nu}{2}\right)\left((c_{Ht} + v_{Ht})^2 + 2(c_{Ht} + v_{Ht})(c^*_{Ht} + v^*_{Ht}) + (c^*_{Ht} + v^*_{Ht})^2\right) + o\left(\left\|a^3\right\|\right) \end{aligned}$$

We can follow the same steps as in the PCP model to conclude:

(C58) $v_{Ht} = \frac{\xi}{2}\sigma^2_{p_H t} + o\left(\left\|a^3\right\|\right)$

(C59) $v^*_{Ht} = \frac{\xi}{2}\sigma^2_{p_H{}^* t} + o\left(\left\|a^3\right\|\right)$

Substituting these expressions into (C57) and cancelling higher order terms, we find:

(C60)

$$a_t + n_t = \frac{\nu}{2}(c_{Ht} + \frac{\xi}{2}\sigma^2_{p_H t}) + \left(\frac{2-\nu}{2}\right)(c^*_{Ht} + \frac{\xi}{2}\sigma^2_{p_H{}^* t}) + \frac{1}{2}\left(\frac{\nu}{2}\right)\left(\frac{2-\nu}{2}\right)(c^2_{Ht} + 2c_{Ht}c^*_{Ht} + c^{*2}_{Ht}) + o\left(\left\|a^3\right\|\right)$$

Then using equation (C55), we can write:

(C61) $$a_t + n_t = y_t + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma^2_{p_H t} + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma^2_{p_H{}^* t}\right) + o\left(\left\|a^3\right\|\right).$$

Keeping in mind that $\bar{y}_t = \bar{n}_t + a_t$, we can write:

(C62) $$\tilde{n}_t = \tilde{y}_t + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma^2_{p_H t} + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma^2_{p_H{}^* t}\right) + o\left(\left\|a^3\right\|\right).$$

Following analogous steps for the Foreign country,

(C63) $$\tilde{n}^*_t = \tilde{y}^*_t + \frac{\xi}{2}\left(\frac{\nu}{2}\sigma^2_{p_F{}^* t} + \left(\frac{2-\nu}{2}\right)\frac{\xi}{2}\sigma^2_{p_F t}\right) + o\left(\left\|a^3\right\|\right).$$

Adding (C62) and (C63) gives us equation (C44).

Finally, to derive the loss functions for policymakers, we note that the loss function is the present expected discounted value of the period loss functions derived here (equation (C33) for the PCP model and (C45) for the LCP model.) That is, the policymaker seeks to minimize $-\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j (u_{t+j} - u^{\max}_{t+j})$.

Following Woodford (2003, chapter 6), we can see that, in the PCP model, if prices are adjusted according to the Calvo price mechanism given by equation (A39) for P_{Ht} that

(C64) $$\sum_{j=0}^{\infty} \beta^j \sigma^2_{p_H t+j} = \frac{\theta}{(1-\beta\theta)(1-\theta)} \sum_{j=0}^{\infty} \beta^j \pi^2_{Ht+j}.$$

Analogous relationships hold for P^*_{Ft} in the PCP model, and for P_{Ht}, P^*_{Ft}, P_{Ft}, and P^*_{Ht} in the LCP model. We can then substitute this relationship into the present value loss function, $-\mathrm{E}_t \sum_{j=0}^{\infty} \beta^j (u_{t+j} - u^{\max}_{t+j})$, to derive the loss functions of the two models presented in the text.

The text writes the loss function as:

(C65)
$$\Psi_t \propto \left[\frac{\sigma}{D} + \phi\right](\tilde{y}^R_t)^2 + (\sigma+\phi)(\tilde{y}^W_t)^2 + \frac{\nu(2-\nu)}{4D}m^2_t$$
$$+ \frac{\xi}{2\delta}\left(\frac{\nu}{2}(\pi_{Ht})^2 + \frac{2-\nu}{2}(\pi_{Ft})^2 + \frac{\nu}{2}(\pi^*_{Ft})^2 + \frac{2-\nu}{2}(\pi^*_{Ht})^2\right)$$

We can use the fact that if $\frac{\nu}{2}\pi_{Ht} + \frac{2-\nu}{2}\pi_{Ft} = \pi_t$, then

$$\frac{\nu}{2}(\pi_{Ht})^2 + \frac{2-\nu}{2}(\pi_{Ft})^2 = \pi_t^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)(\pi_{Ft} - \pi_{Ht})^2 .$$

This follows because for any *a*, *x*, and *y*, if $ax + (1-a)y = z$, then $ax^2 + (1-a)y^2 = z^2 + a(1-a)(x-y)^2$.

Likewise,

$$\frac{\nu}{2}(\pi_{Ft}^*)^2 + \frac{2-\nu}{2}(\pi_{Ht}^*)^2 = (\pi_t^*)^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)(\pi_{Ht}^* - \pi_{Ft}^*)^2 .$$

It follows that

$$\begin{aligned}
&\frac{\nu}{2}(\pi_{Ht})^2 + \frac{2-\nu}{2}(\pi_{Ft})^2 + \frac{\nu}{2}(\pi_{Ft}^*)^2 + \frac{2-\nu}{2}(\pi_{Ht}^*)^2 \\
&= \pi_t^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)(\pi_{Ft} - \pi_{Ht})^2 + (\pi_t^*)^2 + \frac{\nu}{2}\left(\frac{2-\nu}{2}\right)(\pi_{Ht}^* - \pi_{Ft}^*)^2 \\
&= \pi_t^2 + (\pi_t^*)^2 + \left(\frac{\nu(2-\nu)}{2}\right)(s_t - s_{t-1}) \\
&= 2(\pi_t^R)^2 + 2(\pi_t^W)^2 + \left(\frac{\nu(2-\nu)}{2}\right)(s_t - s_{t-1})
\end{aligned} \tag{C66}$$

The second equality follows because, under Calvo pricing, to a first order $\pi_{Ft} - \pi_{Ht} = \pi_{Ft}^* - \pi_{Ht}^* = s_t - s_{t-1}$. The third equality follows because $(\pi_t^R)^2 + (\pi_t^W)^2 = \frac{(\pi_t - \pi_t^*)^2 + (\pi_t - \pi_t^*)^2}{4} = \frac{\pi_t^2 + (\pi_t^*)^2}{2}$. Then substitute (C66) into (C65) to get the simplified loss function presented in the paper.

Appendix D

The model is closed with equations for monetary policy. This appendix solves the model algebraically when there are no cost push shocks and labor supply elasticity parameter, ϕ, is set to zero. These solutions can be used to derive the impulse response functions in Figure 1 of the paper.

We also assume for simplicity that the foreign productivity shock is zero. (Since the model is symmetric, the solution for the response to foreign productivity shocks is straightforward.)

We assume the Home productivity shock follows the AR1 process given by:

$$a_t = \rho a_{t-1} + \varepsilon_t, \qquad E_{t-1}\varepsilon_t = 0.$$

D.1 PCP model

With no mark-up shocks, the Phillips curves, (B27) and (B28) simplify to:

(D1) $$\pi_{Ht} = \delta\left[\tilde{y}_t^R + \sigma\tilde{y}_t^W\right] + \beta \mathrm{E}_t \pi_{Ht+1}$$

(D2) $$\pi_{Ft}^* = \delta\left[-\tilde{y}_t^R + \sigma\tilde{y}_t^W\right] + \beta \mathrm{E}_t \pi_{Ft+1}^*$$

The optimal targeting rules can be written as:

(D3) $$\tilde{y}_t - \tilde{y}_{t-1} + \xi\pi_{Ht} = 0$$

(D4) $$\tilde{y}_t^* - \tilde{y}_{t-1}^* + \xi\pi_{Ft}^* = 0.$$

We will assume $\tilde{y}_{-1} = \tilde{y}_{-1}^* = 0$. It follows immediately from these equations that under the optimal policy, $\tilde{y}_t = \tilde{y}_t^* = \pi_{Ht} = \pi_{Ft}^* = 0$.

From equations (B18) and (B19) we have

(D5) $$a_t = \frac{\sigma}{D}\bar{y}_t^R + \sigma\bar{y}_t^W$$

(D6) $$a_t^* = -\left(\frac{\sigma}{D}\right)\bar{y}_t^R + \sigma\bar{y}_t^W$$

It follows that :

(D7) $$\bar{y}_t = \frac{1+D}{2\sigma}a_t + \frac{1-D}{2\sigma}a_t^*$$

(D8) $$\bar{y}_t^* = \frac{1+D}{2\sigma}a_t^* + \frac{1-D}{2\sigma}a_t$$

Since $\tilde{y}_t = \tilde{y}_t^* = 0$, these two equations solve for actual Home and Foreign output.

Since $\tilde{s}_t = 2\tilde{y}_t^R$, we have $\tilde{s}_t = 0$, or $s_t = \bar{s}_t = a_t$. Since $q_t = 2(\nu - 1)s_t$, we have $\tilde{q}_t = 0$, which implies $q_t = (\nu - 1)a_t$.

Then $\pi_t = \frac{\nu}{2}\pi_{Ht} + \frac{2-\nu}{2}(e_t - e_{t-1} + \pi_{Ft}^*) = \frac{2-\nu}{2}(e_t - e_{t-1})$. Assuming $p_{H,-1} = p_{F,-1}^* = 0$, we have $p_{Ht} = p_{Ft}^* = 0$, so $s_t = e_t + p_{Ft}^* - p_{Ht} = e_t$.

Therefore, $\pi_t = \frac{2-\nu}{2}(a_t - a_{t-1}) = \frac{2-\nu}{2}((\rho - 1)a_{t-1} + \varepsilon_t)$.

To calculate impulse responses for the exchange rate, we have, setting $a_{t-1} = 0$, $E_t e_{t+k} = E_t s_{t+k} = \rho^k \varepsilon_t$ for $k > 0$. For impulse responses for consumer price inflation, we have $E_t \pi_{t+k} = \frac{2-\nu}{2} \rho^{k-1}(\rho - 1)\varepsilon_t$ for $k > 0$.

D.2 LCP Model

Under LCP, we can write the Phillips curves as:

(D9) $\pi_t^R = \frac{\delta}{2}\tilde{q}_t + \beta \mathrm{E}_t \pi_{t+1}^R$

(D10) $\pi_t^W = \delta\sigma\tilde{y}_t^W + \beta \mathrm{E}_t \pi_{t+1}^W$.

The targeting rules are:

(D11) $\tilde{q}_t - \tilde{q}_{t-1} + 2\sigma\xi\pi_t^R = 0$

(D12) $\tilde{y}_t^W - \tilde{y}_{t-1}^W + \xi\pi_t^W = 0$.

Assuming at time period -1 all variables are at their efficient levels, these equations immediately imply that $\pi_t^R = \pi_t^W = 0$, which imply $\pi_t = \pi_t^* = 0$. Also, $\tilde{y}_t^W = 0$ and $\tilde{q}_t = 0$.

Since $\tilde{q}_t = 2(\nu - 1)\tilde{y}_t^R + \nu(2-\nu)m_t = 0$, we have $m_t = \frac{-2(\nu-1)}{\nu(2-\nu)}\tilde{y}_t^R$. Then, because $\tilde{s}_t = \frac{2\sigma}{D}\tilde{y}_t^R - \frac{(\nu-1)}{D}m_t$, we find

(D13) $\tilde{y}_t^R = \frac{\nu(2-\nu)}{2}\tilde{s}_t$.

Note also that since $\tilde{y}_t^W = 0$, we have $\tilde{y}_t = \tilde{y}_t^R$

The deviation of the relative price of imports from the efficient level is given by $\tilde{s}_t = s_t - a_t$. The evolution of s_t in turn is determined by the expectational difference equation:

(D14) $s_t - s_{t-1} = -\delta s_t + \beta \mathrm{E}_t(s_{t+1} - s_t) + \delta a_t$.

This equation has the solution

(D15) $s_t = \gamma_1 s_{t-1} + \gamma_2 a_t$, where

$$\gamma_1 = \frac{1+\beta+\delta - \left((1+\beta+\delta)^2 - 4\beta\right)}{2\beta}, \quad \text{and} \quad \gamma_2 = \frac{\delta}{1+\beta(1-\gamma_1-\rho)+\delta}.$$

However, recognizing that $\delta \equiv (1-\theta)(1-\beta\theta)/\theta$, we see that these solutions simplify to:

$$\gamma_1 = \theta \qquad \text{and} \qquad \gamma_2 = \frac{(1-\theta)(1-\beta\theta)}{1-\beta\rho\theta}.$$

It follows that $\tilde{s}_t = \theta\tilde{s}_{t-1} + (\gamma_2 - 1)a_t + \theta a_{t-1}$.

To get impulse responses, with some algebra, we can show, setting $a_{t-1} = 0$ and $\tilde{s}_{t-1} = 0$,

(D16) $E_t\tilde{s}_{t+k} = \left[\rho^k\left(\frac{(\gamma_2 - 1)+\theta}{\rho-\theta}\right) - \theta^{k+1}\frac{\gamma_2}{\rho-\theta}\right]\varepsilon_t$.

It follows immediately from (D13) that

(D17) $$E_t\tilde{y}_{t+k} = \frac{\nu(2-\nu)}{2}\left[\rho^k\left(\frac{(\gamma_2-1)+\theta}{\rho-\theta}\right)-\theta^{k+1}\frac{\gamma_2}{\rho-\theta}\right]\varepsilon_t.$$

From the Phillips curve (B29), we have

(D18) $$\pi_{Ht} = \delta\left[\left(\frac{\sigma}{D}+\phi\right)\tilde{y}_t^R + \frac{D-(\nu-1)}{2D}m_t\right]+\beta\mathrm{E}_t\pi_{Ht+1}.$$

Substituting in the relationships $m_t = \frac{-2(\nu-1)}{\nu(2-\nu)}\tilde{y}_t^R$ and (D13), we get:

(D19) $$\pi_{Ht} = \frac{\delta(2-\nu)}{2}\tilde{s}_t + \beta\mathrm{E}_t\pi_{Ht+1}.$$

The forward solution to this equation is given by:

(D20) $$\pi_{Ht} = \frac{\delta(2-\nu)}{2}(\tilde{s}_t + \beta\mathrm{E}_t\tilde{s}_{t+1} + \beta^2\beta\mathrm{E}_t\tilde{s}_{t+2} + \ldots).$$

Setting $a_{t-1}=0$ and $\tilde{s}_{t-1}=0$, we can get impulse responses from this equation:

(D21) $$E_t\pi_{Ht+k} = \frac{\delta(2-\nu)}{2}\left[\frac{\rho^k}{1-\beta\rho}\left(\frac{(\gamma_2-1)+\theta}{\rho-\theta}\right)-\frac{\theta^{k+1}}{1-\beta\theta}\frac{\gamma_2}{\rho-\theta}\right]\varepsilon_t.$$