# Overcoming Adverse Selection: How Public Intervention Can Restore Market Functioning

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# WEB APPENDIX

## Suboptimality of trivial interventions under buybacks

Suppose that $p_g < B - S$ and so the seller will not be able to finance the new project if she joins the governmental scheme. Let us first look for a pure strategy equilibrium. Either $p_m < B - S$ and then there is no private market as there are no gains from trade. Or $p_m > B - S$ and then no-one joins the governmental scheme.

Let $\underline{\theta}$ denote the lowest value of $\theta_g$ such that the market can be revived when types $\theta \leq \theta_g$ accept the government's offer:[†]

$$\underline{\theta} = \frac{B}{R_0}.$$

Let $\theta^{**}(p_g)$ be defined by:

$$U_0\Big(\theta^{**}(p_g)\Big) = p_g.$$

If $\theta^{**}(p_g) < \underline{\theta}$, the equilibrium involves no rejuvenation. Welfare is then

$$W = p_g F\Big(\theta^{**}(p_g)\Big) + \int_{\theta^{**}(p_g)}^{1} U_0(\theta) dF(\theta) - (1+\lambda)\Big[p_g - m^{-}\big(\theta^{**}(p_g)\big)R_0\Big] F\Big(\theta^{**}(p_g)\Big)$$

$$< E\Big[U_0(\theta)dF(\theta)\Big]$$

unless $p_g = 0$. Offering such a $p_g$ necessarily reduces welfare.

Assume next that $\theta^{**}(p_g) \geq \underline{\theta}$. Then if $\theta_g = \theta^{**}(p_g)$, $p_m + S \geq B$ and so $p_m > p_g$, a contradiction since no-one would join the government's scheme.

[†] An offer at price $p$ that revives the market ($p \geq B - S$) yields net profit

$$\Big[\int_{\theta_g}^{\frac{p-S}{R_0}} \theta dF(\theta)\Big] R_0 - \Big[F\Big(\frac{p-S}{R_0}\Big) - F(\theta_g)\Big] p,$$

whose derivative with respect to $p$ is negative. Hence, a necessary and sufficient condition for market rebound is that at $p = \theta_g R_0 - S$, $p \geq B - S$ or $\theta_g R_0 \geq B$.

So necessarily $\theta_g = \underline{\theta}$ and financing by the market must be random. When refusing to join the government's scheme, the seller is financed by the market at price $p_m = B - S$ with probability $\alpha$ and the market breaks down with probability $1-\alpha$ such that

$$p_g = (1-\alpha)U_0(\theta_g) + \alpha B.$$

Finally,

$$p_m = R_0 H\Big(\theta_g, \frac{p_m + S}{R_0}\Big) \iff B - S = R_0 H\Big(\theta_g, \frac{B}{R_0}\Big).$$

Welfare is

$$\begin{aligned} W \ =& p_g F(\theta_g) - (1+\lambda)\int_0^{\theta_g} \big[p_g - \theta R_0\big] dF(\theta) \\ &+ \alpha\Big[\int_{\theta_g}^{\theta^*} (p_m + S) dF(\theta) + \int_{\theta^*}^1 U_0(\theta) dF(\theta)\Big] \\ &+ (1-\alpha)\Big[\int_{\theta_g}^1 U_0(\theta) dF(\theta)\Big] \end{aligned}$$

$W$ is linear in $\alpha$ ($p_g$ is a function of $\alpha$, whereas all the other variables are being held constant as $\alpha$ varies). If $W$ decreases with $\alpha$, then it is bounded above by

$$p_g F(\theta_g) - (1+\lambda)\int_0^{\theta_g} \big[p_g - \theta R_0\big] dF(\theta) + \int_{\theta_g}^1 U_0(\theta) dF(\theta)$$

which is lower than the laissez-faire welfare $\displaystyle\int_0^1 U_0(\theta) dF(\theta)$.

If $W$ increases with $\alpha$, the maximum is achieved at $\alpha = 1$. The intervention then coincides with the minimum non-trivial intervention, except that there is no investment for types below $\theta_g$; hence this intervention is dominated by doing the minimal non-trivial intervention and investing for all participating sellers.

# Ex-ante moral hazard under buybacks

Let us extend the model by introducing a "stage 0", at which the seller chooses the asset quality. At private and unobserved cost $\Psi(e)$, the seller generates distribution $F(\theta|e)$ such that $\partial(f/F)/\partial e > 0$ and $\partial(f/F)/\partial\theta < 0$.

**Proposition 11.**

*(i) Strategic substitutability. Consider an arbitrary (i.e., possibly out of equilibrium) expectation $e^*$. Under ex-ante moral hazard, the seller chooses a higher effort ($e$) when expected to choose a lower one ($e^*$).*

*(ii) Consequently, there exists a unique equilibrium.*

*(iii) If there is an equilibrium intervention, effort is lower than in the absence of intervention.*

Intuitively, if the equilibrium effort is high, interventions face less adverse selection and are more generous (higher $p$). This implies that the seller expects to be bailed out more often and so puts in less effort.

*Proof*: (i) For conciseness let us restrict our attention to the region of parameters for which an interior solution prevails:

$$\frac{f(\theta^*|e^*)}{F(\theta^*|e^*)} = \frac{\lambda R_0}{(1+\lambda)S}. \tag{A.1}$$

Condition (A.1 ) defines a policy cutoff $\theta^*(e^*)$ as a function of the *equilibrium* value of effort. From $\partial(f|F)/\partial e > 0$, $\theta^*$ is an increasing function of $e^*$.

The seller chooses her effort $e$ so as to maximize:

$$\mathcal{U} \equiv U_0\big(\theta^*(e^*)\big)F\big(\theta^*(e^*)|e\big) + \int_{\theta^*(e^*)}^1 U_0(\theta)dF(\theta|e) - \Psi(e)$$

or, after an integration by parts

$$\mathcal{U} = U_0(1) - \int_{\theta^*(e^*)}^1 R_0 F(\theta|e)d\theta - \Psi(e).$$

And so

$$\frac{\partial^2 \mathcal{U}}{\partial e^* \partial e} = R_0 \frac{d\theta^*}{de^*} F_e\big(\theta^*(e^*)|e\big) < 0.$$

(ii) Uniqueness of equilibrium, if it exists, is a corollary of (i). Consider $e = R(e^*)$ given by $\Psi'(e) = R_0 \int_{\theta^*(e^*)}^1 \big[-F_e(\theta|e)\big]d\theta$. The equilibrium may involve mixed strategies by the government if at the level $\widehat{e}$ at which the government is indifferent between an intervention and laissez-faire, $R(\widehat{e}) < \widehat{e}$. The equilibrium then has $\overline{e} = \widehat{e}$ and randomization by the government between intervention and laissez-faire.

(iii) Under laissez-faire the first-order condition is

$$\Psi'(e) = R_0 \int_0^1 [-F_e(\theta|e)]d\theta.$$

*Commitment.* Let us now assume that the government can commit to a price $p_g$ (and therefore to a cutoff $\theta^*$) *before* effort is chosen. Effort is then chosen so as to maximize

$$\theta^* R_0 F(\theta^*|e) + \int_{\theta^*}^1 \theta R_0 dF(\theta|e) - \psi(e).$$

The cross-partial derivative of this function with respect to $\theta^*$ and $e$ is $R_0 F_e(\theta^*|e) < 0$. So a lower $\theta^*$ induces a higher effort. In turn, the government wants to commit to a price that is lower than that that will prevail under non-commitment.

# Proof of Proposition 6

Consider the upper envelope of the equilibrium utilities offered by the buyers:

$$U(\theta) = \sup_{\{i\}} \{U_i(\theta)\}.$$

As earlier, let

$$y = \sup_{\{i,\theta|x_i(\theta)=1\}} \{y_i(\theta)\} \quad \text{and} \quad \widetilde{\theta} \equiv \sup_{\{\theta,i\}} \{\theta|x_i(\theta) = 1\}.$$

From the proof of Lemma 2, there exists $\check{y} \geq y$, with strict inequality if and only if $z(\widetilde{\theta}) \equiv z > 0$, such that an upper bound on buyer profit is

$$\int_0^{\check{\theta}} \left[\theta R_0 + S - V(\theta\check{y})\right] dF_m(\theta),$$

which, from the definition of the constrained efficient outcome, is strictly negative if $\check{y} > y_m$. If $\check{y} = y_m$, then (a) $z = 0$ (and so $\check{y} = y$) and (b) $x(\theta)$ cannot be equal to 0 on a positive-measure subset of $[0, \check{\theta}]$, otherwise the buyers would make a strictly negative profit. The outcome then coincides with the constrained efficient outcome.

So let us assume that $\check{y} < y_m$ and so, a fortiori, $y < y_m$. Because $x(\theta) = 0$ for $\theta \in [\widetilde{\theta}, \theta^*]$, the profit made by buyers on those types is strictly negative. Furthermore, it must be the case that $\theta R_0 + S - U(\theta) > 0$ on an interval $[\widetilde{\theta} - \varepsilon, \widetilde{\theta}]$ for some $\varepsilon > 0$.[‡]

Suppose first that $z > 0$ and consider an "entering buyer" (by this we mean a buyer with a zero or arbitrarily small equilibrium profit, as we will show that the proposed contract makes a strictly positive profit) offering a single skin-in-the-game contract specifying $\{z - \kappa, y + \eta, x = 1\}$[§] defining a schedule $\widehat{U}(\theta) = \max\{B, z - \kappa + (y + \eta)\theta + b,\, U_0(\theta)\}$, such that $\eta > 0$ and

$$U(\widetilde{\theta} - \varepsilon) = z - \kappa + b + (y + \eta)(\widetilde{\theta} - \varepsilon).$$

The buyer then attracts at least types in $[\widetilde{\theta} - \varepsilon, \widetilde{\theta}]$, which by continuity yields a strictly positive profit for $(\varepsilon, \eta, \kappa)$ small. He may also attract types in $[\widetilde{\theta}, \theta^*]$, which a fortiori are profitable. He does not attract any type below $\widetilde{\theta} - \varepsilon$. Hence the deviation is strictly profitable.

Suppose finally that $z = 0$. Let the deviating buyer make a single skin-in-the-game offer $\{0, y + \eta, x = 1\}$. From robust choice this schedule attracts exactly types in $[\theta^{**}, \theta^*]$ with $\theta^{**} < \widetilde{\theta}$, as well as some (profitable) types above $\theta^*$. But even if $\theta^{**} = 0$, this deviation is strictly profitable since $y + \eta < y_m$ for $\eta$ small.

# Proof of Proposition 7

Only (iv) and (v) require some elaboration.

(iv) Note that

$$\text{sign}\left(\frac{d\overline{W}}{d\theta^*}\right) = \text{sign}\left[\frac{(1+\lambda)S}{\lambda b} - \int_{\theta_0(\theta^*)}^{\theta^*} \frac{\theta}{(\theta^*)^2} \frac{f(\theta)}{f(\theta^*)} d\theta\right].$$

Under good news about the prior distribution, $f(\theta)/f(\theta^*)$ decreases and so $\partial W/\partial \theta^*$ is positive over a wider range of $\theta^*$s.

(v) Recall the first-order condition under pure buybacks:

$$f(\theta^*)(1+\lambda)S = F(\theta^*)\lambda(R_0 + \lambda S).$$

[‡] Recall that by convexity of $U(\cdot)$: $\frac{d}{d\theta}\big(\theta R_0 + S - U(\theta)\big) \geq R_0 - y > 0$.

Furthermore, $\pi(\theta) \leq \theta R_0 + S - U(\theta)$; so if $\theta R_0 + S - U(\widetilde{\theta}) \leq 0$, the buyers' profit is strictly negative.

[§] It is also possible to upset the equilibrium through a contract specifying the same $z$.

To show that $\theta^*$ is higher under a general scheme, we note that

$$F(\theta^*) > \int_{\theta_0(\theta^*)}^{\theta^*} \frac{\theta b}{(\theta^*)^2} f(\theta) d\theta.$$

Indeed, the right-hand side of this inequality is bounded above by $\int_0^{\theta^*} \frac{\theta b}{(\theta^*)^2} f(\theta) d\theta = bm^-(\theta^*)F(\theta^*)/(\theta^*)^2$. Thus, we need to show that at the optimum of the outright sales mechanism:

$$\frac{\theta^* R_0}{b} > \frac{m^-(\theta^*)}{\theta^*};$$

The LHS of this inequality exceeds 1 since $\theta^* R_0 = b + \theta^* y$. The RHS is always smaller than 1.

## Proof of Proposition 10

Let $\xi_g(\theta) = 1$ if $\theta \in \Theta_g$ and $\xi_g(0) = 0$ otherwise. Let

$$m_g = \int_0^1 \xi_g(\theta) dF(\theta)$$

denote the size of government involvement. Welfare can now be rewritten as

$$\widehat{W} \equiv W - \varepsilon m_g = E[Sx(\theta)] + \lambda E[\pi(\theta)] - \varepsilon m_g + \overline{\theta} R_0 \tag{A.2}$$

where $\pi(\theta)$ is the monetary outcome on type $\theta$ ($\pi(\theta)+U(\theta) = \theta R_0 + Sx(\theta)$). The maximization of (A.2 ) subject to the (IC) constraint and

$$E\big[[1 - \xi_g(\theta)]\pi(\theta)\big] \geq 0$$

is a priori complex.

But consider any possible intervention and corresponding $\Theta_g$ and $\Theta_m$. Let $\{x(\cdot), U(\cdot)\}$ be the combined (government plus market) mechanism faced by the seller. Consider having the government deviate to offer the same mechanism $\{x(\cdot), U(\cdot)\}$ and asking precisely the types in $\Theta_g$ to participate in the government's scheme. This is incentive-compatible and produces exactly the same welfare and intervention costs as before. So without loss of generality we can restrict attention to strategy profiles where the government offers the same mechanism as the market (but attracts only a subset of types). So, letting $\widetilde{\pi}(\theta) \equiv \theta R_0 + S - V(\theta y)$ where $y$ is the skin in the game offered by the market. We can now without loss of generality solve:

$$\min_{\{\xi_g(\cdot)\}} \left\{ \varepsilon \int_0^{\theta^*} \xi_g(\theta) dF(\theta) \right\}$$

s.t.

$$\int_0^{\theta^*} [1 - \xi_g(\theta)] \widetilde{\pi}(\theta) dF(\theta) \geq 0 \qquad (\mu)$$

The Lagrangian of this optimization problem is $-\varepsilon - \mu\widetilde{\pi}(\theta)$. Because $\widetilde{\pi}(\theta)$ is strictly increasing (from Lemma 4), there is indeed a cutoff $\theta_g$ such that $\xi_g(\theta) = 1$ if and only if $\theta < \theta_g$. Finally, the theorem of the maximum guarantees that as $\varepsilon$ converges to 0, the optimum converges to the mechanism of subsection B.