

Technical Appendix for “Frictional Wage Dispersion in Search Models: A Quantitative Assessment”

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This Technical Appendix to Hornstein, Krusell, and Violante (2009) (HKV, hereafter) contains a set of derivations that complement some of the results stated without proofs in our main paper. In Section 1, we show that the matching model of Pissarides (1985) in its version with heterogeneous productivities yields the same expression for the Mm ratio as in the sequential search model (Section I.B in HKV). In Section 2, we derive the Mm ratio in the sequential search model with endogenous effort (Section II.A in HKV). Section 3 derives a detailed characterization of the search model with wage shocks during employment but no on-the-job search (Section III in HKV). In Section 3.2, we derive the Mm ratio in a model with returns to labor market experience (Section III in HKV). In Section 4, we show that even if agents are risk-averse the implications for the Mm ratio are limited if agents can self-insure through a risk-free asset (Section IV in HKV). For the model with directed search, in Section 5 we derive an upper bound for the Mm ratio that is close to the Mm ratio in the standard search model (Section V in HKV). In Section 6.1, we present the model with on-the-job search, derive the Mm ratio, and closed-form solutions for the equilibrium separation rate (Section VI.A in HKV). In Section 6.2, we show how to compute bounds for the Mm ratio in the model with endogenous search effort on the job (Section VI.B in HKV). Section 6.3 describes how to derive the Mm ratio in a simple version of search models with counteroffers and wage tenure contracts (Section VI.C in HKV).

Unless noted otherwise all our environments are populated by ex-ante equal, risk-neutral, infinitely lived individuals who discount the future at rate r . Unemployed agents receive a

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utility flow b which includes unemployment benefits and a value of leisure and home production, net of disutility from being jobless and, unless explicitly modelled, net of search effort costs. Unemployed agents also receive job/wage offers at the instantaneous rate λ_u , and wages are drawn from a well-behaved distribution function $F(w)$ with upper support w^{\max} . Draws are i.i.d. over time and across agents. Employed agents become unemployed at the instantaneous rate σ . Our environments differ according to whether wages remain fixed while on the job; whether agents can affect the wage offer arrival rate; whether search is undirected or directed; and whether wage offers arrive also during employment. We only consider steady-state allocations.

1 Matching model

The matching model not only specifies the search behavior of workers, but it also endogenizes the wage offer arrival rate which is taken as given in the search model. A job, corresponding to a firm, can be either vacant or filled. There is free entry of vacant firms. The flow of contacts m between vacant jobs and unemployed workers is governed by an aggregate matching technology $m(u, v)$. Let the workers' contact rate be $\lambda_u = m/u$ and the firm's contact rate be $\lambda_f = m/v$. Upon meeting, worker and firm jointly draw a value p , distributed according to $F(p)$ with upper support p^{\max} , which determines flow output of their potential match. Once p is realized, they bargain over the match surplus in a Nash fashion and determine the wage $w(p)$.

The Nash rule for the wage establishes that

$$w(p) = \beta p + (1 - \beta) rU, \quad (1)$$

where rU is the flow value of unemployment, and β is the worker's bargaining power.¹

From the worker's point of view, it is easy to see that:

$$\begin{aligned} rW(p) &= w(p) - \sigma [W(p) - U] \\ rU &= b + \lambda_u \int_{p^*}^{p^{\max}} [W(p) - U] dF(p), \end{aligned}$$

i.e., the value of employment is expressed in terms of the value p of the match drawn; similarly, the optimal search strategy is expressed in terms of a reservation productivity p^* . Rearranging these two expressions, we arrive at an equation for the reservation productivity

$$p^* = b + \frac{\lambda_u \beta}{r + \sigma} \int_{p^*}^{p^{\max}} [p - p^*] dF(p), \quad (2)$$

where we have used the fact that $p^* = rU$. Substituting (1) into (2), we obtain

$$w^* = b + \frac{\lambda_u}{r + \sigma} \int_{p^*}^{p^{\max}} [w(p) - w^*] dF(p)$$

¹This equation uses the free-entry condition of firms that drives the value of a vacant job to zero. See, for example, Pissarides (2000), Section 1.4, for a step-by-step derivation of this wage equation.

Using the definition $b = \rho \bar{w}$ in the last equation, we again obtain the formula for the mean-min ratio in equation (2) of HKV.

2 Endogenous search effort

Consider an extension of the baseline model where search effort is endogenous. Let $c_u(\lambda_u)$, with $c'_u > 0$ and $c''_u > 0$, be the effort cost as a function of the offer arrival rate λ_u , the endogenous variable chosen by the unemployed worker. The flow values of employment and unemployment are

$$\begin{aligned} rW(p) &= w(p) - \sigma [W(p) - U] \\ rU &= \max_{\lambda_u} \left\{ b - c_u(\lambda_u) + \lambda_u \int_{p^*}^{p^{\max}} [W(p) - U] dF(p) \right\}. \end{aligned}$$

The same derivations as in Section 2 in the main text lead to the reservation-wage equation

$$w^* = b - c_u(\lambda_u^o) + \frac{\lambda_u^*}{r + \sigma} (\bar{w} - w^*), \quad (3)$$

where λ_u^o denotes the optimal individual choice, and $\lambda_u^* \equiv \lambda_u^o [1 - F(w^*)]$. The first-order condition for optimal search effort is

$$c'_u(\lambda_u^o) = \frac{1}{r + \delta} \int_{w^*}^{w^{\max}} (w - w^*) dF(z). \quad (4)$$

We follow Christensen et al. (2005) and choose the isoelastic functional form $c_u(\lambda_u) = \kappa_u \lambda_u^{1+1/\gamma}$ for the effort cost, with $\gamma > 0$ denoting the elasticity of the optimal search effort with respect to the expected return from search. Using the relationship between marginal and average search cost which follows from this specification, we arrive at the net return from search relative to the average wage

$$\frac{b - c_u(\lambda_u^o)}{\bar{w}} = \rho - \frac{\lambda_u^*}{r + \sigma} \frac{\gamma}{1 + \gamma} \left(1 - \frac{1}{Mm} \right). \quad (5)$$

Combining (3) and (5), and rearranging, we obtain

$$Mm = \frac{\frac{\lambda_u^*}{r + \sigma} \frac{1}{1 + \gamma} + 1}{\frac{\lambda_u^*}{r + \sigma} \frac{1}{1 + \gamma} + \rho},$$

which is equation (4) in HKV.

3 Imperfect correlation between wage and job value

3.1 Wage shocks during employment

In this section, we characterize the equilibrium of the search model with wage shocks while employed. An employed agent receives a new wage at the instantaneous rate δ and the new

wage comes from the same distribution F that determines wage offers when unemployed. We first state the discrete-time approximation of the search model for a fixed and finite length of the time period. We then derive the continuous-time representation as the limit of the discrete-time approximation when the length of the time period becomes arbitrarily small. We show that the Bellman equations for the employment and unemployment values for the continuous- and discrete-time version are the same. Our derivation of the mean-min ratio for wage inequality is therefore independent of the time representation. We then show that in the discrete-time version the first-order autocorrelation coefficient of wages is one minus the arrival rate of wage changes. Finally, we consider a variation of the baseline model where wage shocks when employed come from a different distribution than wage shocks when unemployed. In particular, we study the Mortensen and Pissarides (1994) environment where unemployed workers on meeting a job always receive the highest wage.

3.1.1 Discrete time versus continuous time

The discrete-time approximations of the Bellman equations for the value of employment, $W(w)$, and unemployment, U , are

$$W(w) = w\Delta + e^{-r\Delta} \left\{ \delta\Delta \int \max\{W(z), U\} dF(z) + (1 - \delta\Delta) W(w) \right\}, \quad (6)$$

$$U = b\Delta + e^{-r\Delta} \left\{ \lambda_u\Delta \int \max\{W(z), U\} dF(z) + (1 - \lambda_u\Delta) U \right\}, \quad (7)$$

where Δ is the length of the time interval, and $\lambda_u\Delta$ ($\delta\Delta$) is the probability that an (un)employed worker receives a wage offer at the end of the interval Δ . Using the definition of the reservation wage, $W(w^*) = U$, and rearranging terms the value equations can be rewritten as

$$(1 - e^{-r\Delta}) W(w) = w\Delta + e^{-r\Delta} \left\{ \delta\Delta \int_{w^*}^{w^{\max}} [W(z) - W(w)] dF(z) - \delta\Delta F(w^*) [W(w) - U] \right\},$$

$$(1 - e^{-r\Delta}) U = b\Delta + e^{-r\Delta} \left\{ \lambda_u\Delta \int_{w^*}^{w^{\max}} [W(z) - U] dF(z) \right\}$$

Dividing by the length of the time interval and taking the limit as $\Delta \rightarrow 0$ we obtain the continuous-time Bellman equations

$$rW(w) = w + \delta \int_{w^*}^{w^{\max}} [W(z) - W(w)] dF(z) - \delta F(w^*) [W(w) - U], \quad (8)$$

$$rU = b + \lambda_u \int_{w^*}^{w^{\max}} [W(z) - U] dF(z). \quad (9)$$

Rather than studying the continuous-time limit of the search model with wage changes, we can just use the discrete-time approximation and consider a unit length interval, $\Delta = 1$. In this case we obtain the following expressions for the value functions of being employed and

unemployed:

$$\begin{aligned} (1 - \beta) W(w) &= w + \beta \left\{ \delta \int_{w^*}^{w^{\max}} [W(z) - W(w)] dF(z) - \delta F(w^*) [W(w) - U] \right\} \\ (1 - \beta) U &= b + \beta \lambda_u \int_{w^*}^{w^{\max}} [W(z) - U] dF(z), \end{aligned}$$

where $\beta \equiv e^{-r} \equiv 1/(1 + \bar{r})$. Note that we can rewrite these discrete-time value equations as

$$\bar{r} \bar{W}(w) = w + \delta \int_{w^*}^{w^{\max}} [\bar{W}(z) - \bar{W}(w)] dF(z) - \delta F(w^*) [\bar{W}(w) - \bar{U}] \quad (10)$$

$$\bar{r} \bar{U} = b + \lambda_u \int_{w^*}^{w^{\max}} [\bar{W}(z) - \bar{U}] dF(z), \quad (11)$$

where $\bar{W} \equiv W/(1 + \bar{r})$ and $\bar{U} \equiv U/(1 + \bar{r})$. Expressions (10) and (11) are formally equivalent to the expressions (8) and (9) for the continuous-time value functions. Therefore, the results for the mean-min ratio also apply for the discrete-time version of the paper.

3.1.2 The reservation wage

As a first step towards deriving the mean-min wage ratio for the continuous-time model we characterize the reservation wage. For this purpose, evaluate the employment value expression (8) at w^* and use the definition of the reservation wage, $W(w^*) = U$, to deliver

$$rU = w^* + \delta \int_{w^*}^{w^{\max}} [W(z) - W(w^*)] dF(z).$$

Now substitute the unemployment value expression (9) for the left-hand side and solve for the reservation wage

$$w^* = b + (\lambda_u - \delta) \int_{w^*}^{w^{\max}} [W(z) - W(w^*)] dF(z). \quad (12)$$

Integration by parts on the right-hand side yields

$$\begin{aligned} \int_{w^*}^{w^{\max}} [W(z) - W(w^*)] dF(z) &= [(W(z) - W(w^*)) F(z)]_{w^*}^{w^{\max}} - \int_{w^*}^{w^{\max}} W'(z) F(z) dz \\ &= W(w^{\max}) - W(w^*) - \int_{w^*}^{w^{\max}} W'(z) F(z) dz \\ &= \int_{w^*}^{w^{\max}} W'(z) [1 - F(z)] dz. \end{aligned} \quad (13)$$

From the employment value expression (8) it follows that

$$W'(w) = \frac{1}{r + \delta}. \quad (14)$$

Hence the reservation-wage expression is

$$\begin{aligned}
w^* &= b + \frac{\lambda_u - \delta}{r + \delta} \int_{w^*}^{w^{\max}} [1 - F(z)] dz \\
&= b + \frac{(\lambda_u - \delta) [1 - F^*]}{r + \delta} \int_{w^*}^{w^{\max}} \left[\frac{1}{1 - F^*} - \frac{F(z)}{1 - F^*} \right] dz
\end{aligned} \tag{15}$$

with $F^* = F(w^*)$.

3.1.3 The equilibrium wage distribution

We now construct the equilibrium wage distribution $G(w)$ implied by the interaction of the wage-offer distribution and the reservation wage. The measure of agents with wage below w is $(1 - u)G(w)$. Agents leave this stock because their wage changes and their new wage is either less than the reservation wage or higher than the current wage. Agents enter this stock if they were unemployed and receive an acceptable wage offer below w , or if they were employed at wage above w and are forced to accept a lower wage; hence

$$\begin{aligned}
&(1 - u)G(w)\delta\{F(w^*) + [1 - F(w)]\} \\
&= \{u\lambda_u + (1 - u)[1 - G(w)]\delta\}[F(w) - F(w^*)].
\end{aligned} \tag{16}$$

We can solve this expression for the equilibrium wage distribution as a function of the wage-offer distribution:

$$G(w) = \left[\frac{\lambda_u u}{\delta(1 - u)} + 1 \right] [F(w) - F(w^*)]. \tag{17}$$

In steady state, the inflows and outflows from employment balance:

$$(1 - u)\delta F(w^*) = u\lambda_u [1 - F(w^*)].$$

Using the expression for steady-state employment in (17) we obtain

$$G(w) = \frac{F(w) - F(w^*)}{1 - F(w^*)}. \tag{18}$$

Thus, the equilibrium wage distribution with and without wage shocks during employment are the same, namely the wage-offer distribution truncated at the reservation wage.

3.1.4 The mean-min ratio

Based on the equilibrium wage distribution we can calculate the average wage of employed workers as

$$\bar{w} = \int_{w^*}^{w^{\max}} w \frac{dF(w)}{1 - F^*} = \frac{w^{\max} - w^* F^*}{1 - F^*} - \int_{w^*}^{w^{\max}} \frac{F(z)}{1 - F^*} dz. \tag{19}$$

Solving the average-wage expression (19) for the right-hand side integral term and substituting this term for the corresponding integral in the reservation-wage expression (15) yields

$$\begin{aligned}
w^* &= b + \frac{(\lambda_u - \delta) [1 - F^*]}{r + \delta} \left[\int_{w^*}^{w^{\max}} \frac{1}{1 - F^*} dz + \bar{w} - \frac{w^{\max} - w^* F^*}{1 - F^*} \right] \\
&= b + \frac{(\lambda_u - \delta) [1 - F^*]}{r + \delta} [\bar{w} - w^*].
\end{aligned} \tag{20}$$

Using the definition of the replacement rate, $b = \rho\bar{w}$, we can solve equation (20) for the reservation wage and obtain an expression for the mean-min ratio, that is, the ratio of average wages to the reservation wage,

$$Mm = \frac{\frac{(\lambda_u - \delta)[1 - F(w^*)]}{r + \delta} + 1}{\frac{(\lambda_u - \delta)[1 - F(w^*)]}{r + \delta} + \rho} = \frac{\frac{\lambda_u^* - \delta + \sigma^*}{r + \delta} + 1}{\frac{\lambda_u^* - \delta + \sigma^*}{r + \delta} + \rho}, \quad (21)$$

with $\lambda_u^* \equiv (1 - F^*)\lambda_u$ and $\sigma^* \equiv \delta F^*$. This is equation (5) in Section III of HKV. Note that as δ goes to infinity Mm goes to $1/\rho$.

3.1.5 Wage persistence in the discrete-time model

It is straightforward to show that the equilibrium wage distribution for the discrete-time and the continuous-time versions of the model are the same; we now work with the former. We need the expected value of the cross-product of today's and tomorrow's wage, conditional on being employed in both periods, to calculate the autocorrelation coefficient. We proceed in two steps: first, we obtain the value conditional on today's wage, and then we integrate over today's wage to obtain the unconditional expectation. This delivers

$$\begin{aligned} E[w'w|w] &= (1 - \delta)w^2 + \delta w E[\tilde{w}|\tilde{w} \geq w^*] \\ &= (1 - \delta)w^2 + \delta w\bar{w} \\ E[w'w] &= (1 - \delta)E[w^2] + \delta\bar{w}^2. \end{aligned}$$

We can now define the first-order autocorrelation coefficient as

$$\begin{aligned} \rho &= \frac{(1 - \delta)E[w^2] + \delta\bar{w}^2 - \bar{w}^2}{Var(w)} \\ &= \frac{(1 - \delta)(E[w^2] - \bar{w}^2)}{Var(w)} \\ &= 1 - \delta. \end{aligned}$$

3.1.6 The mean-min ratio for a Mortensen-Pissarides environment

Suppose now that an unemployed worker who receives a wage offer always receives the highest wage w^{\max} , whereas wage changes of employed workers continue to be drawn from the distribution F . We will show that the mean-min ratio that we previously derived for our baseline model, equation (21), represents an upper bound for the mean-min ratio in the Mortensen and Pissarides (1994) environment.

The value function equation for an employed worker, (8), remains unchanged, but the value function equation for an unemployed worker is now

$$rU = b + \lambda_u [W(w^{\max}) - U]. \quad (22)$$

Following the same steps as in Section 3.1.2 we derive the modified expression for the reservation wage:

$$w^* = b + \frac{\lambda_u - \delta}{r + \delta} [w^{\max} - w^*] + \frac{\delta}{r + \delta} \int_{w^*}^{w^{\max}} F(z) dz. \quad (23)$$

The modified steady-state expression characterizing the equilibrium wage distribution is now

$$\begin{aligned} & (1-u)G(w)\delta\{F(w^*)+[1-F(w)]\} \\ &= (1-u)[1-G(w)]\delta[F(w)-F(w^*)] \text{ for } w < w^{\max}. \end{aligned} \quad (24)$$

Note that there are no inflows from the pool of unemployed since all unemployed workers who receive wage offers receive the highest wage. Thus, the equilibrium wage distribution for $w < w^{\max}$ is

$$G(w) = F(w) - F(w^*). \quad (25)$$

Since all unemployed workers receive the highest wage with probability one there is now a mass point at $w = w^{\max}$, that is, the cumulative density function is discontinuous at w^{\max} .

Integrating the wage with respect to the equilibrium wage distribution then yields the average wage

$$\bar{w} = \int_{w^*}^{w^{\max}} w dF(w) + w^{\max} F(w^*) = w^{\max} + F(w^*)(w^{\max} - w^*) - \int_{w^*}^{w^{\max}} F(w) dw. \quad (26)$$

Solving the average-wage expression (26) for the right-hand side integral term and substituting this term for the corresponding integral in the reservation-wage expression (23) yields

$$\begin{aligned} \left[1 + \frac{\lambda_u - (1 - F^*)\delta}{r + \delta}\right] w^* &= \left(\rho - \frac{\delta}{r + \delta}\right) \bar{w} + \frac{\lambda_u + \delta F^*}{r + \delta} w^{\max} \\ &> \left[\rho + \frac{\lambda_u - \delta(1 - F^*)}{r + \delta}\right] \bar{w}. \end{aligned} \quad (27)$$

Note that the last inequality implies that the mean-min ratio for this setup is bounded above by that of the baseline economy with wage shocks given in (21).

3.2 Returns to experience

Suppose that workers enter the labor market with a level of human capital (experience) normalized to one. At rate δ , an employed worker with experience level h sees her experience jump to level $h' = h(1 + \bar{g})$. During unemployment experience remains unchanged. In order to keep the stock of experience finite, we also assume that workers exit from the labor force at rate ϕ . Let $F(w)$ be the wage offer distribution, where the wage is “per unit of experience”.

The value of employment and unemployment are:

$$rW(w, h) = wh + \delta[W(w, h') - W(w, h)] - \sigma[W(w, h) - U(h)] - \phi W(w, h) \quad (28)$$

$$rU(h) = bh + \lambda_u \int_{w^*}^{w^{\max}} [W(z, h) - U(h)] dF(z) - \phi U(h). \quad (29)$$

Clearly, these two equations are homogeneous in h , and therefore we can rewrite them as:

$$(r + \phi)W(w) = w + \eta W(w) - \sigma[W(w) - U] \quad (30)$$

$$(r + \phi)U = b + \lambda_u \int_{w^*}^{w^{\max}} [W(z) - U] dF(z), \quad (31)$$

where $\eta \equiv \delta \bar{g}$ is the expected instantaneous growth rate of experience.

From the value of work, we obtain

$$W(w) = \frac{w + \sigma U}{r + \phi + \sigma - \eta}, \quad (32)$$

and since $W(w^*) = U$, we arrive at

$$w^* = (r + \phi - \eta) U. \quad (33)$$

Evaluating (30) at w^* , and equating it to (31), we see that

$$w^* + \eta U = b + \lambda_u \int_{w^*}^{w^{\max}} [W(z) - U] dF(z).$$

Inserting (32) inside the integral into the above equation, and using (33) to substitute out U , we arrive at

$$w^* = b + \lambda_u \int_{w^*}^{w^{\max}} \left[\frac{z - w^*}{r + \phi + \sigma - \eta} \right] dF(z) - \frac{\eta}{r + \phi - \eta} w^*.$$

Integrating and rearranging yields

$$Mm = \frac{\frac{\lambda_u^*}{r + \phi + \sigma - \eta} + \frac{1}{1 - \eta / (r + \phi)}}{\frac{\lambda_u^*}{r + \phi + \sigma - \eta} + \rho},$$

which is the expression for the Mm ratio in equation (8) in the main text.

4 Risk aversion

We study two search models with risk averse agents. In the first model, workers have CRRA utility but no access to storage. In the second model, workers have CARA utility and access to borrowing/saving.

4.1 Constant relative risk aversion

Let $u(c)$ be the utility of consumption, with $u' > 0$, and $u'' < 0$. To make progress analytically, we assume that workers have no access to storage, i.e., $c = w$ when employed, and $c = b$ when unemployed. Then, the reservation-wage equation becomes

$$u(w^*) = u(\rho \bar{w}) + \frac{\lambda_u^*}{r + \sigma} [\mathbb{E}_{w^*} [u(w)] - u(w^*)], \quad (34)$$

with $\mathbb{E}_{w^*} [u(w)] = \mathbb{E} [u(w) | w \geq w^*]$. A second-order Taylor expansion of $u(w)$ around the conditional mean \bar{w} yields

$$u(w) \simeq u(\bar{w}) + u'(\bar{w})(w - \bar{w}) + \frac{1}{2} u''(\bar{w})(w - \bar{w})^2.$$

Take the conditional expectation of both sides of the above equation and arrive at

$$\mathbb{E}_{w^*} [u(w)] \simeq u(\bar{w}) + \frac{1}{2} u''(\bar{w}) \text{var}(w), \quad (35)$$

where $\text{var}(w)$ denotes the wage variance. Let $u(w)$ belong to the CRRA family, with θ representing the coefficient of relative risk aversion. Then, using (35) in (34), and rearranging, we obtain

$$Mm \simeq \left[\frac{\frac{\lambda_u^*}{r+\sigma} \left(1 + \frac{1}{2} (\theta - 1) \theta \text{cv}^2\right) + \rho^{1-\theta}}{\frac{\lambda_u^*}{r+\sigma} + 1} \right]^{\frac{1}{\theta-1}}, \quad (36)$$

which is equation (11) in the main text. It is easy to derive third- and fourth-order approximations of the reservation-wage equation involving the coefficients of skewness and kurtosis. In Hornstein, Krusell, and Violante (2007) we show that our conclusions remain extremely robust to higher-order approximations.

4.2 Constant absolute risk aversion

Even if workers are risk-averse, access to a risk-free asset for self-insurance purposes brings the equilibrium wage distribution quite close to the full-insurance environment that we study in our paper. As an example we use the environment studied by Shimer and Werning (2007) who derive the optimal reservation-wage policies for risk-averse workers that may or may not have access to a risk-free asset.

Preferences are of the constant absolute risk aversion variety

$$E \int_0^\infty e^{-rt} u[c(t)] dt \text{ and } u(c) = -e^{-\theta c} \text{ with } \theta > 0. \quad (37)$$

Unlike all other environments studied in our paper, if workers accept a wage offer w they will receive that wage for a fixed time T . Thus separations are deterministic, not random. Normalizing the average wage rate at one, $\bar{w} = 1$, we can rewrite the reservation wage for workers with access to a risk-free asset with rate of return r , equation (4) in Shimer and Werning (2007), as an expression for the Mm ratio:

$$Mm_{save} = \left[\rho + \frac{\lambda_u [1 - F(w^*)]}{r\theta} \left\{ 1 + \int_{w^*}^\infty u[r\beta_T(w - w^*)] \frac{dF(w)}{1 - F(w^*)} \right\} \right]^{-1}, \quad (38)$$

where $\beta_T = \int_0^T e^{-rs} ds$ denotes the present value of one unit of income with remaining job duration T . Analogously, the Mm ratio for workers who cannot save can be derived from equation (7) of Shimer and Werning (2007):

$$Mm_{aut} = \left[\rho + \frac{1}{\theta} \log \left\{ 1 + \beta_T \lambda_u [1 - F(w^*)] \left[1 + \int_{w^*}^\infty u(w - w^*) \frac{dF(w)}{1 - F(w^*)} \right] \right\} \right]^{-1}. \quad (39)$$

We approximate the accepted wage distribution, $H(dw) = F(dw)[1 - F(w^*)]$, with a gamma distribution with density function

$$h(w; w^*, \alpha, \zeta) = \left(\frac{w - w^*}{\alpha} \right)^{\zeta-1} \exp\left(-\frac{w - w^*}{\alpha}\right) / [\alpha \Gamma(\zeta)] \text{ for } w \geq w^*. \quad (40)$$

The mean and standard deviation of wages for this distribution are

$$\bar{w} = w^* + \alpha\zeta \text{ and } \sigma_w = \alpha\sqrt{\zeta}. \quad (41)$$

This allows us to solve for the integral terms in equations (38) and (39) using

$$\int_{w^*}^{\infty} u[r\beta_T(w - w^*)] dH(w) = -(1 + \alpha\theta r\beta_T)^{-\zeta}.$$

We now derive the Mm ratios that are implied by the following observations: a monthly interest rate $r = 0.0041$, an effective job-finding rate $\lambda_u^* = [1 - F(w^*)]\lambda_u^* = 0.43$, a replacement rate $\rho = 0.4$, and an unemployment rate $u = 0.065$. The job-finding rate and the unemployment rate jointly imply a fixed job duration $T = 33$ months. Given that we have normalized the average wage at one, the coefficient of absolute risk aversion is equal to the coefficient of relative risk aversion at the average wage. We consider the values $\theta = 1$ and $\theta = 10$. For either of the two cases—access or no access to a risk-free asset—conditional on the normalization of average wages we have two expressions for the Mm ratio: equation (41) and either equation (38) with access to a risk-free asset or equation (39) without access to a risk-free asset. Conditional on the parameter α of the gamma distribution we solve these two equations for the mean-min ratio and the implied distribution parameter ζ and coefficient of variation, σ/\bar{w} .

In Figure 1 we plot the implied Mm ratio (with and without savings) and the wage distribution’s coefficient of variation against an assumed value of the gamma distribution parameter α between 0 and 5.

Two results stand out. First, access to a risk-free asset significantly reduces the Mm ratio. Second, observations on the coefficient of variation for wage distributions provide a significant upper bound for wage inequality. Without access to a risk-free asset the implied Mm ratios can be large: up to 1.7 with a risk aversion of $\theta = 10$ and $\alpha = 5$. With access to a risk-free asset the implied Mm ratio drops substantially, even for a high level of risk aversion: from 1.7 to about 1.12. Finally, the Mm ratio is increasing in the parameter α , but so is the coefficient of variation of the underlying wage distribution. Based on evidence provided in Hornstein, Krusell, and Violante (2007) we consider the coefficient of variation for residual wage inequality to be strictly less than 0.5. This significantly limits the magnitude of the parameter α that is consistent with the observed coefficient of variation. For example, with no access to a risk-free asset and a risk aversion of $\theta = 10$, α has to be strictly less than one and the implied Mm ratio is less than 1.6. Alternatively, with the same risk aversion, but access to a risk-free asset, α has to be strictly less than three and the Mm ratio is less than 1.1.

5 Directed search

The description of the directed search model follows Moen (1997) and Rogerson, Shimer and Wright (2005). There is free entry of vacant firms. Firms post wages and workers observe the wage distribution and direct their search to the most attractive firm. A high wage w_i posted

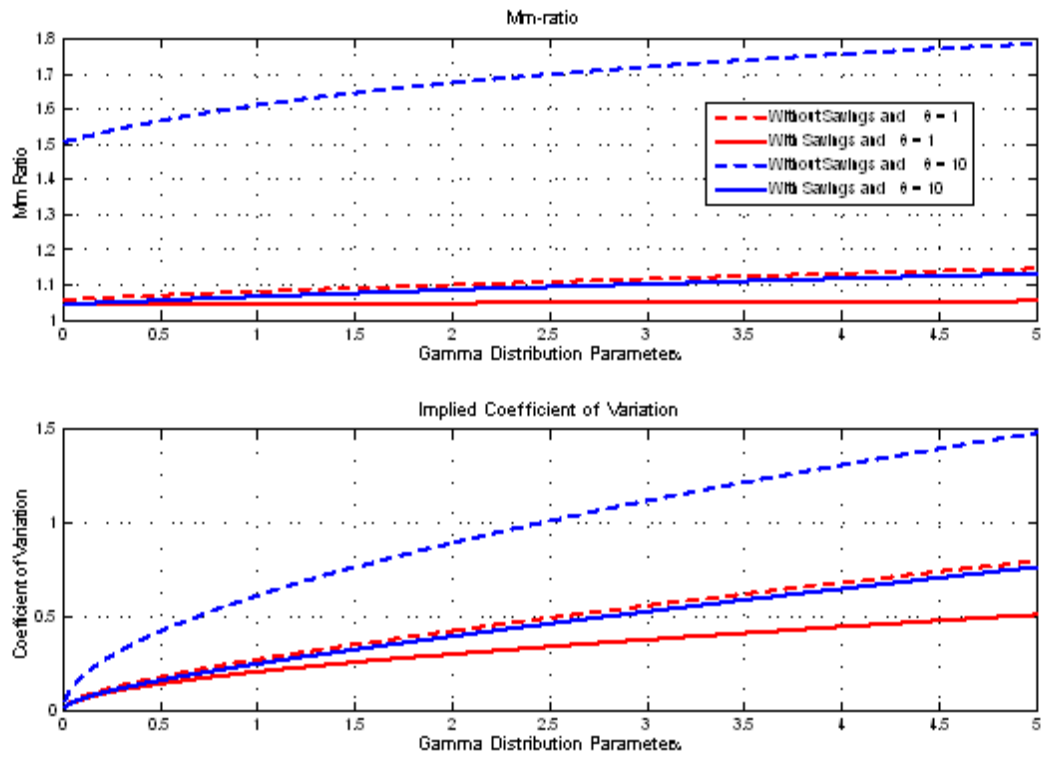


Figure 1: Frictional wage dispersion with CARA utility

by firm i attracts more applicants, which reduces workers' contact rate λ_i . In equilibrium, unemployed workers are indifferent about where to apply, and therefore if we denote by U_i the value of an unemployed worker directing her search to generic firm i , we have $U_i = U$ for all i .

More specifically, we have

$$rU_i = b + \lambda_i [W(w_i) - U_i],$$

where $W(w_i)$ is the value of a worker employed by firm i

$$rW(w_i) = w_i - \sigma [W(w_i) - U].$$

Combining these two equations, and using the equality $U_i = U$ yields

$$rU_i = \frac{b(r + \sigma) + \lambda_i w_i}{r + \sigma + \lambda_i}. \quad (42)$$

From the equilibrium condition it follows that workers are indifferent between applying at firms posting the lowest wage and firms posting the average wage, so we obtain:

$$\frac{b(r + \sigma) + \lambda_{\min} w_{\min}}{r + \sigma + \lambda_{\min}} = \frac{b(r + \sigma) + \bar{\lambda} \bar{w}}{r + \sigma + \bar{\lambda}}. \quad (43)$$

Collecting terms and multiplying through, we arrive at

$$[r + \sigma + p(\bar{\theta})] p(\theta_{\min}) w_{\min} - [r + \sigma + p(\theta_{\min})] p(\bar{\theta}) \bar{w} = (r + \sigma) [p(\theta_{\min}) - p(\bar{\theta})] \rho \bar{w},$$

and then dividing through by w_{\min} yields

$$\begin{aligned} [(r + \sigma + \lambda_{\min}) \bar{\lambda} + \rho(r + \sigma) (\lambda_{\min} - \bar{\lambda})] Mm &= (r + \sigma + \bar{\lambda}) \lambda_{\min} \\ &= (r + \sigma + \lambda_{\min}) \bar{\lambda} + (r + \sigma) (\lambda_{\min} - \bar{\lambda}). \end{aligned}$$

Rearranging, we have

$$\left[\frac{(r + \sigma + \lambda_{\min}) \bar{\lambda}}{(r + \sigma) (\lambda_{\min} - \bar{\lambda})} + \rho \right] Mm = \frac{(r + \sigma + \lambda_{\min}) \bar{\lambda}}{(r + \sigma) (\lambda_{\min} - \bar{\lambda})} + 1,$$

and collecting terms we arrive at

$$Mm = \frac{\mu + 1}{\mu + \rho}, \quad (44)$$

where

$$\mu \equiv \left(1 + \frac{\lambda_{\min}}{r + \sigma} \right) \frac{\bar{\lambda}}{\lambda_{\min} - \bar{\lambda}}. \quad (45)$$

In equilibrium, $\lambda_{\min} \geq \bar{\lambda}$ since λ_{\min} is the job-finding rate associated with searching for jobs with the lowest wage. The mean-min ratio Mm is decreasing in μ , and the function μ is decreasing in λ_{\min} :

$$\frac{\partial \mu}{\partial \lambda_{\min}} = - \frac{\bar{\lambda}}{(\lambda_{\min} - \bar{\lambda})^2} \left(1 + \frac{\bar{\lambda}}{r + \sigma} \right) < 0. \quad (46)$$

Therefore Mm is increasing in λ_{\min} , but note that there is an upper bound to the function $\mu(\lambda_{\min})$

$$\lim_{\lambda_{\min} \rightarrow \infty} \mu(\lambda_{\min}) = \frac{\bar{\lambda}}{r + \sigma}.$$

Therefore

$$Mm \leq \frac{1 + \frac{\bar{\lambda}}{r + \sigma}}{\rho + \frac{\bar{\lambda}}{r + \sigma}}, \quad (47)$$

which is the inequality in equation (10) in HKV.

6 On-the-job search

6.1 The job ladder model

We generalize the canonical search model and turn it into the job ladder model outlined by Burdett (1978). A worker employed with wage \hat{w} encounters new job opportunities w at rate λ_e . These opportunities are randomly drawn from the wage offer distribution $F(w)$ and they are accepted if $w > \hat{w}$. The equations describing the value of employment and unemployment are:

$$\begin{aligned} rW(w) &= w + \lambda_e \int \max\{W(z) - W(w), 0\} dF(z) - \sigma [W(w) - U] \\ rU &= b + \lambda_u \int \max\{W(z) - U, 0\} dF(z). \end{aligned}$$

Without loss of generality, and motivated by what equilibrium firm behavior would dictate, we assume that the wage-offer distribution is such that unemployed workers accept all wage offers: $F(w^*) = 0$. A worker can always reject a wage offer and keep the current wage. The worker may lose the current job at an exogenous separation rate σ .

6.1.1 The reservation wage

Workers continue to follow reservation-wage strategies and the Bellman equations can be rewritten as

$$rW(w) = w + \lambda_e \int_w^{w^{\max}} [W(z) - W(w)] dF(z) - \sigma [W(w) - U] \quad (48)$$

$$rU = b + \lambda_u \int_{w^*}^{w^{\max}} [W(z) - U] dF(z). \quad (49)$$

Evaluate the employment value equation (48) at w^* , using the reservation-wage property, $W(w^*) = U$, and the unemployment value expression (49) to obtain

$$\begin{aligned} rU &= b + \lambda_u \int_{w^*}^{w^{\max}} [W(z) - W(w^*)] dF(z) \\ &= w^* + \lambda_e \int_{w^*}^{w^{\max}} [W(z) - W(w^*)] dF(z). \end{aligned}$$

We can solve this expression for the reservation wage

$$w^* = b + (\lambda_u - \lambda_e) \int_{w^*}^{w^{\max}} [W(z) - W(w^*)] dF(z). \quad (50)$$

As with equation (12), we can integrate the right-hand side integral by parts, as in (13), and obtain the reservation-wage equation

$$w^* = b + (\lambda_u - \lambda_e) \int_{w^*}^{w^{\max}} W'(z) [1 - F(z)] dz. \quad (51)$$

Note that differentiating the employment value equation (48) with respect to the current wage yields

$$W'(w) = \frac{1}{r + \sigma + \lambda_e [1 - F(w)]}. \quad (52)$$

Substituting (52) in (51) we can rewrite the reservation wage as

$$w^* = b + (\lambda_u - \lambda_e) \int_{w^*}^{w^{\max}} \frac{1 - F(z)}{r + \sigma + \lambda_e [1 - F(z)]} dz. \quad (53)$$

6.1.2 The equilibrium wage distribution

We now construct the equilibrium wage distribution $G(w)$ implied by the interaction of the wage-offer distribution and the reservation wage.

The measure of agents with wage below w is $(1 - u)G(w)$. Agents leave this stock because (1) they are separated at rate σ , or (2) they receive an outside offer which they accept at rate $\lambda_e [1 - F(w)]$. Workers enter this stock if they were unemployed and receive a wage offer below w . In a steady state the inflows and outflows balance:

$$(1 - u)G(w) \{\sigma + \lambda_e [1 - F(w)]\} = u\lambda_u F(w).$$

We can solve this expression for the equilibrium wage distribution as a function of the wage-offer distribution:

$$G(w) = \frac{\lambda_u u}{1 - u} \cdot \frac{F(w)}{\sigma + \lambda_e [1 - F(w)]}. \quad (54)$$

In steady state, if all the job offers are above w^* so that $F(w^*) = 0$,

$$u\lambda_u = (1 - u)\sigma.$$

Hence

$$G(w) = \frac{\sigma F(w)}{\sigma + \lambda_e [1 - F(w)]} \quad (55)$$

and

$$\begin{aligned} 1 - G(w) &= \frac{\sigma + \lambda_e}{\sigma + \lambda_e [1 - F(w)]} [1 - F(w)] \\ &\simeq \frac{r + \sigma + \lambda_e}{r + \sigma + \lambda_e [1 - F(w)]} [1 - F(w)], \end{aligned} \quad (56)$$

because r is “second order” compared to σ and λ_e .

6.1.3 The mean-min ratio

The average wage is

$$\begin{aligned}
\bar{w} &= \int_{w^*}^{w^{\max}} w dG(z) = [wG(w)]_{w^*}^{w^{\max}} - \int_{w^*}^{w^{\max}} G(z) dz \\
&= w^{\max} - \int_{w^*}^{w^{\max}} G(z) dz \\
&= [w^{\max} - w^*] + w^* - \int_{w^*}^{w^{\max}} G(z) dz \\
&= w^* + \int_{w^*}^{w^{\max}} [1 - G(z)] dz.
\end{aligned} \tag{57}$$

Solve the wage distribution expression (56) for $1 - F$ and use it in the reservation-wage expression (53) to obtain

$$w^* \simeq b + \frac{\lambda_u - \lambda_e}{r + \sigma + \lambda_e} \int_{w^*}^{w^{\max}} [1 - G(z)] dz.$$

Finally substituting for the integral term from the average-wage equation (57) we can solve for the mean-min ratio:

$$\begin{aligned}
w^* &\simeq \rho \bar{w} + \frac{\lambda_u - \lambda_e}{r + \sigma + \lambda_e} (\bar{w} - w^*) \Rightarrow \\
Mm &\simeq \frac{\frac{\lambda_u - \lambda_e}{r + \sigma + \lambda_e} + 1}{\frac{\lambda_u - \lambda_e}{r + \sigma + \lambda_e} + \rho}.
\end{aligned} \tag{58}$$

Equation (58) corresponds to equations (14) in Section VI.A of HKV.

6.1.4 Turnover rates in the basic on-the-job search model

We show how to derive closed-form solutions for the average tenure and separation rate in the equilibrium of the standard model with on the job search.

The job-to-job transition rate for employed workers is

$$\chi = \lambda_e \int_{w^*}^{w^{\max}} [1 - F(w)] dG(w). \tag{59}$$

Following Nagypál (2005), and integrating the right hand side by parts, yields

$$\begin{aligned}
\chi &= \lambda_e - \lambda_e \int_{w^*}^{w^{\max}} F(w) dG(w) \\
&= \lambda_e - \lambda_e [F(w)G(w)]_{w^*}^{w^{\max}} + \lambda_e \int_{w^*}^{w^{\max}} G(w) dF(w) \\
&= \lambda_e \int_{w^*}^{w^{\max}} G(w) dF(w) \\
&= \lambda_e \sigma \int_{w^*}^{w^{\max}} \frac{F(w)}{\sigma + \lambda_e [1 - F(w)]} dF(w).
\end{aligned} \tag{60}$$

The last step uses equation (55) for G . Now change the variable of integration to $z = F(w)$, and we obtain

$$\begin{aligned} \int_0^1 \frac{z}{\sigma + \lambda_e [1 - z]} dz &= -\frac{\lambda_e \{z\}_0^1 + (\lambda_e + \sigma) \{\log[\sigma + \lambda_e(1 - z)]\}_0^1}{\lambda_e^2} \\ &= -\frac{\lambda_e + (\lambda_e + \sigma) (\log \sigma - \log(\sigma + \lambda_e))}{\lambda_e^2} \\ &= \frac{(\lambda_e + \sigma) \log\left(\frac{\sigma + \lambda_e}{\sigma}\right)}{\lambda_e^2} - \frac{1}{\lambda_e}. \end{aligned} \quad (61)$$

Hence, the job-to-job separation rate is

$$\chi = \frac{\sigma (\lambda_e + \sigma) \log\left(\frac{\sigma + \lambda_e}{\sigma}\right)}{\lambda_e} - \sigma \quad (62)$$

which is equation (15) in HKV.

6.2 On-the-job search with endogenous effort choice

We now derive the results stated in Section 7.2 formally. Based on the derivations in Mortensen (2007), we generalize the Christensen et al. (2005) model in order to allow for asymmetric search cost functions off and on the job. Let the utility cost of attaining a contact rate λ_i in employment state $i \in \{u, e\}$ be $c_i(\lambda_i)$, with $c'_i > 0$ and $c''_i > 0$. In particular, we will assume that the search cost function is isoelastic: $c_i(\lambda_i) = \kappa_i \lambda_i^{1+1/\gamma}$, with $\gamma > 0$. Optimal search effort choice will deliver a scalar λ_u^o for the unemployed worker and a policy function $\lambda_e^o(w)$ for the employed worker.

A worker employed at a job with wage w accepts any job offer with a higher wage. The flow values of unemployment and employment are, respectively,

$$rU = b - c_u(\lambda_u^o) + \lambda_u^o \int_{w^*}^{w^{\max}} [W(z) - U] dF(z), \quad (63)$$

$$rW(w) = w - c_w[\lambda_e^o(w)] + \lambda_e^o(w) \int_w^{w^{\max}} [W(z) - W(w)] dF(z) - \sigma [W(w) - U]. \quad (64)$$

The first-order condition for optimal search effort for an unemployed worker and for a worker employed at wage w that characterize the search effort choices $\{\lambda_u^o, \lambda_e^o(w)\}$ are, respectively,

$$c'_e[\lambda_e^o(w)] = \int_w^{w^{\max}} [W(z) - W(w)] dF(z) \quad (65)$$

$$c'_u(\lambda_u^o) = \int_{w^*}^{w^{\max}} [W(z) - U] dF(z). \quad (66)$$

By simple inspection of (65), it is immediate that $\lambda_e^o(w)$ is decreasing in w .

In steady state, flows for the workers employed at wage w or lower satisfy

$$(1 - u) \left\{ G(w)\sigma + [1 - F(w)] \int_{w^*}^w \lambda_e^o(z) dG(z) \right\} = u\lambda_u^o F(w), \quad (67)$$

and flows in and out of unemployment satisfy the relation

$$u\lambda_u^o = (1 - u)\sigma.$$

Combining the last two equations we obtain

$$\sigma F(w) = G(w)\sigma + [1 - F(w)]G(w)\bar{\lambda}_e(w),$$

where

$$\bar{\lambda}_e(w) \equiv \frac{\int_{w^*}^w \lambda_e^o(w) dG(w)}{G(w)}$$

is the average search effort for workers with wages less than w . From (67) it is easy to see that the equilibrium wage distribution $G(w)$ can be expressed as a function of the wage-offer distribution $F(w)$, i.e., through

$$1 - G(w) = \frac{[\sigma + \bar{\lambda}_e(w)][1 - F(w)]}{\sigma + \bar{\lambda}_e(w)[1 - F(w)]} \simeq \frac{[r + \sigma + \bar{\lambda}_e(w)][1 - F(w)]}{r + \sigma + \bar{\lambda}_e(w)[1 - F(w)]}, \quad (68)$$

where the approximation sign comes from $r \simeq 0$. This is exactly the same approximation as the one we used in deriving equation (56).

Note that the right-hand side of (68) is increasing in $\bar{\lambda}_e(w)$. Since we have derived that search effort is decreasing in w , we can establish the following ranking:

$$\lambda_{w^*} \geq \bar{\lambda}_e(w) \geq \lambda_w \geq \lambda_{w^{\max}} = 0,$$

where, for notational simplicity, we have denoted $\lambda_e^o(w)$ by λ_w . The above inequalities, in conjunction with (68), imply

$$\begin{aligned} \frac{(r + \sigma + \lambda_{w^*})[1 - F(w)]}{r + \sigma + \lambda_w[1 - F(w)]} &\geq 1 - G(w) \geq \frac{(r + \sigma + \lambda_w)[1 - F(w)]}{r + \sigma + \lambda_w[1 - F(w)]} \\ &\geq \frac{(r + \sigma + \lambda_{w^{\max}})[1 - F(w)]}{r + \sigma + \lambda_w[1 - F(w)]}. \end{aligned} \quad (69)$$

Now we derive the reservation-wage equation for this model. Going back to the value function in (64), the derivative of the worker value function (using the Envelope Theorem) is

$$W'(w) = \frac{1}{r + \sigma + \lambda_w[1 - F(w)]}.$$

Integrating by parts and using the derivative above yields

$$\int_w^{w^{\max}} [W(z) - W(w)] dF(z) = \int_w^{w^{\max}} \frac{[1 - F(z)]}{r + \sigma + \lambda_z[1 - F(z)]} dz. \quad (70)$$

The definition of the reservation wage, $W(w^*) = U$, implies that

$$\begin{aligned} w^* - c_e(\lambda_{w^*}) + \lambda_{w^*} \int_{w^*}^{w^{\max}} [W(z) - W(w^*)] dF(z) \\ = b - c_u(\lambda_u) + \lambda_u \int_{w^*}^{w^{\max}} [W(z) - W(w^*)] dF(z), \end{aligned} \quad (71)$$

where for notational simplicity we denoted λ_u^o by λ_u . Since the search cost functions are isoelastic, from (65) we obtain

$$c_e(\lambda_{w^*}) = \kappa_e \lambda_{w^*}^{1+1/\gamma} = \lambda_{w^*} \left(\frac{\gamma}{1+\gamma} \right) c'_e(\lambda_{w^*}) = \lambda_{w^*} \left(\frac{\gamma}{1+\gamma} \right) \int_{w^*}^{w^{\max}} [W(z) - W(w^*)] dF(z) \quad (72)$$

for the optimal search cost when employed; similarly, from (66) we arrive at

$$c_u(\lambda_u) = \lambda_u \left(\frac{\gamma}{1+\gamma} \right) \int_{w^*}^{w^{\max}} [W(z) - W(w^*)] dF(z) \quad (73)$$

for the optimal search cost when unemployed. Substituting (73) and (72) into (71) and using (70) we arrive at

$$w^* = b + \frac{\lambda_u - \lambda_{w^*}}{1+\gamma} \int_{w^*}^{w^{\max}} \frac{1 - F(z)}{r + \sigma + \lambda_z [1 - F(z)]} dz. \quad (74)$$

Now we use the inequalities in (69) to construct bounds for the Mm ratio. Putting together (74) and (69) we obtain

$$\begin{aligned} w^* &\geq \rho \bar{w} + \frac{\lambda_u - \lambda_{w^*}}{(1+\gamma)(r + \sigma + \lambda_{w^*})} \int_{w^*}^{w^{\max}} [1 - G(z)] dz \\ &= \rho \bar{w} + \frac{\lambda_u - \lambda_{w^*}}{(1+\gamma)(r + \sigma + \lambda_{w^*})} [\bar{w} - w^*] \end{aligned} \quad (75)$$

and, similarly,

$$\begin{aligned} w^* &\leq \rho \bar{w} + \frac{\lambda_u - \lambda_{w^*}}{(1+\gamma)(r + \sigma + \lambda_{w^{\max}})} \int_{w^*}^{w^{\max}} [1 - G(z)] dz \\ &= \rho \bar{w} + \frac{\lambda_u - \lambda_{w^*}}{(1+\gamma)(r + \sigma)} [\bar{w} - w^*]. \end{aligned} \quad (76)$$

Inequalities (75) and (76) yield the following bounds for the Mm ratio

$$\frac{\frac{\lambda_u - \lambda_{w^*}}{(1+\gamma)(r + \sigma)} + 1}{\frac{\lambda_u - \lambda_{w^*}}{(1+\gamma)(r + \sigma)} + \rho} \leq Mm \leq \frac{\frac{\lambda_u - \lambda_{w^*}}{(1+\gamma)(r + \sigma + \lambda_{w^*})} + 1}{\frac{\lambda_u - \lambda_{w^*}}{(1+\gamma)(r + \sigma + \lambda_{w^*})} + \rho},$$

as in equation (16) in HKV.

We can proceed in a similar manner to construct bounds on search costs. Combining (73) with (70), and using the inequalities in (69), we arrive at

$$\frac{\lambda_u}{r + \sigma + \lambda_{w^*}} \left(\frac{\gamma}{1+\gamma} \right) \left(1 - \frac{1}{Mm} \right) \leq \frac{c_u(\lambda_u)}{\bar{w}} \leq \frac{\lambda_u}{r + \sigma} \left(\frac{\gamma}{1+\gamma} \right) \left(1 - \frac{1}{Mm} \right),$$

which is the expression reported in equation (17) in HKV.

6.3 On-the-job search with counteroffers

This model is a simplified version of Cahuc, Postel-Vinay and Robin (2006) without heterogeneous firms' and workers' productivities. See also Mortensen (2005) for a similar presentation.

Consider an economy with ex-ante equal, risk-neutral infinitely lived workers who discount the future at rate r . There is only one type of firm with productivity p . Let U be the value of unemployment and $W(w)$ be the value of employment at wage w . If an unemployed worker is contacted by a firm, Nash bargaining yields a wage w^* which solves

$$W(w^*) = U + \beta [W(p) - U]. \quad (77)$$

However, if an employed worker is contacted by another firm, the two firms compete in a Bertrand fashion and bid up her wage to p .

The flow value of unemployment is

$$rU = b + \lambda_u [W(w^*) - U], \quad (78)$$

where

$$rW(w^*) = w^* + \lambda_e [W(p) - W(w^*)] - \sigma [W(w^*) - U]. \quad (79)$$

The value $W(p)$ is the value of employment after being contacted by a poaching firm. Because of Bertrand competition among firms, after the contact, the wage jumps to p , and hence

$$rW(p) = p - \sigma [W(p) - U]. \quad (80)$$

From equation (77), we obtain

$$(1 - \beta) [W(w^*) - U] = \beta [W(p) - W(w^*)]. \quad (81)$$

In what follows, we obtain closed-form solutions for the RHS and LHS of the above equation.

From equation (80) we arrive at

$$\begin{aligned} (r + \sigma) [W(p) - U] &= p - rU = p - b - \lambda_u [W(w^*) - U] \\ &= p - b - \lambda_u \beta [W(p) - U], \end{aligned}$$

where we first substitute (78) for rU , and then use (77). Collecting terms, we arrive at

$$W(p) - U = \frac{p - b}{r + \sigma + \beta \lambda_u}. \quad (82)$$

Evaluating (80) at $w = w^*$, we have

$$\begin{aligned} (r + \sigma) [W(w^*) - U] &= w^* + \lambda_e [W(p) - W(w^*)] - rU \\ &= w^* + \lambda_e [W(p) - W(w^*)] - b - \lambda_u [W(w^*) - U]. \end{aligned}$$

Using (81), we arrive at

$$W(w^*) - U = \frac{\beta(w^* - b)}{\beta(r + \sigma + \lambda_u) - (1 - \beta)\lambda_e}. \quad (83)$$

Using (82) and (83) into the Nash bargaining relation (77) yields

$$\frac{\beta(w^* - b)}{\beta(r + \sigma + \lambda_u) - (1 - \beta)\lambda_e} = \frac{\beta(p - b)}{r + \sigma + \beta\lambda_u}$$

and through simple algebra one obtains

$$w^* = b + \frac{\beta(r + \sigma + \lambda_u) - (1 - \beta)\lambda_e}{r + \sigma + \beta\lambda_u}(p - b), \quad (84)$$

which is the reservation wage equation (18) of HKV.

We now show that, in a plausible parameterization of the Postel-Vinay and Robin (2002) version of this model with $\beta = 0$, the reservation wage can be negative. Let $\varepsilon(w^*)$ and $\varepsilon(p)$ be the fractions of employment at wage w^* and p , respectively. From the steady-state flows, it is easy to see that

$$\varepsilon(w^*) = \frac{\sigma}{\lambda_e + \sigma} \quad \text{and} \quad \varepsilon(p) = \frac{\lambda_e}{\lambda_e + \sigma}.$$

As a result, the average wage is

$$\bar{w} = \frac{\sigma}{\lambda_e + \sigma}w^* + \frac{\lambda_e}{\lambda_e + \sigma}p.$$

Now set $\beta = 0$ and use the above equation to substitute out p from (84). After simple manipulations, one obtains

$$w^* = \frac{(r + \sigma + \lambda_e)\rho - (\lambda_e + \sigma)}{r}\bar{w} \leq 1 - \frac{\sigma}{r}(1 - \rho).$$

In the baseline parameterization, $r = 0.0041$, $\sigma = 0.03$ and $\rho = 0.4$ which implies that $w^* < 0$ independently of λ_e .

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