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# Exploring the Economic Convergence in the EU New Member States by Using Nonparametric Models[1]


## Abstract

This paper analyzes the process of real economic convergence in the New Member States (NMS) bein g formerly centrally planned economies, using nonparametric methods instead of conventional parametric measurement tools like beta and sigma models. This methodological framework allows the examining of the relative income distribution in different periods of time, the number of modes of the density distribution, the existence of "convergence clubs" in the distribution and the hypothesis of convergence at a single point in time. The modality tests (e.g. the ASH-WARPing procedure) and stochastic kernel are nonparametric techniques used in the empirical part of the study to examine the income distribution in the NMS area. Additionally, random effects panel regressions are used, but only for comparison reasons. The main findings of the paper are the bimodality of the income density distribution over time and across countries, and the presence of convergence clubs in the income distribution from 1995 to 2008. The findings suggest a lack of absolute convergence in the long term (1995-2008) and also when looking only from 2003 onwards. The paper concludes that, in comparison with the parametrical approach, the nonparametric one gives a deeper, real and richer perspective on the process of real convergence in the NMS area.



Keywords: real convergence, nonparametric models, stochastic kernel, modality

JEL classification: C14, F43



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# Die Analyse ökonomischer Konvergenz in neuen EU-Ländern mit nicht-parametrischen Methoden

## Zusammenfassung

Der Beitrag untersucht den realen Konvergenzprozess in den neuen Mitgliedstaaten der EU (NMS), die nach ihrer Transformation von der Plan- zur Marktwirtschaft 2004 bzw. 2007 in die EU aufgenommen wurden. Im Gegensatz zu den üblichen parametrischen Vorgehensweisen wie der Berechnung von beta- bzw. sigma-Konvergenz werden hier nicht-parametrische Ansätze verfolgt. Sie erlauben es, die relative Einkommensverteilung zu unterschiedlichen Zeitpunkten zu untersuchen, die Anzahl der Modalwerte in der Wahrscheinlichkeitsdichte zu bestimmen, das Bestehen von „Konvergenzclubs" innerhalb der Verteilung zu ermitteln und die Aussage zu überprüfen, ob sich der Konvergenzprozess auf einen einzigen Punkt hin bewegt. Die Tests auf Anzahl der Modalwerte (d. h. das ASH-WARPing Verfahren) und die Ermittlung stochastischer Kerndichte-Funktionen sind nicht-parametrische Verfahren, um die Einkommensverteilung in den NMS-Staaten zu untersuchen. Zusätzlich werden Panelregressionen mit Random-Effekten durchgeführt, die jedoch nur zu Vergleichszwecken mit den nicht-parametrischen Ergebnissen dienen.

Die wesentlichen Ergebnisse des Beitrags lauten, dass einerseits eine bimodale Verteilung der Einkommen über die Zeit und unter den Ländern vorliegt, andererseits die Existenz von Konvergenzclubs aus den Einkommensverteilungen von 1995 bis 2008 hergeleitet werden kann. Die Ergebnisse legen den Schluss nahe, dass absolute Konvergenz weder für den längeren Zeitraum 1995 bis 2008 noch für den kürzeren Zeitraum ab 2003 beobachtet werden kann. Der Beitrag schließt mit dem Ergebnis, dass im Vergleich zu parametrischen Methoden nicht-parametrische Ansätze einen tieferen und reichhaltigeren Einblick in den Prozess der realen Konvergenz in den NMS-Ländern vermitteln.

Schlagworte: reale Konvergenz, nicht-parametrische Modelle, stochastische Kerndichte, Modalwerte

JEL-Klassifikation: C14, F43

# Introduction

This paper applies nonparametric techniques to the analysis of real economic convergence in ten New Member States (NMS) area – the former centrally planned economies – in order to provide a broader understanding of this process and different insights than those given by the conventional parametric approach, especially when the available dataset is small. Furthermore, the nonparametric approach to economic convergence is in itself a broader analysis framework in comparison with beta-convergence, for instance. With nonparametric techniques it is possible to derive complex insights to the convergence process, which could not be revealed by parametric models.

The analysis of convergence relies on two fundamental approaches, i.e. the beta- and sigma-convergence models (Barro and Sala-i-Martin, 1992), which are derived from the growth theory (Solow, 1956). Both, but especially the concept of beta-convergence have been criticized in the literature for a number of reasons, such as the assumption of linearity in the growth regressions, the Galton's fallacy problem, the impossibility of detecting convergence clubs etc. (Quah, 1993, 1996; Johnson, 2000; Rassekh, Panik and Kolluri, 2001; Linden, 2002). Nonparametric methods offer alternative approaches to the analysis of economic convergence. They allow data to be modeled without presuming that the data follow a normal distribution and also allow short-term divergent paths, which may occur in a long convergence process, to be captured.

The paper is innovative in two aspects, at both the methodological and empirical levels. First, it provides a tool to analyse the process of real convergence when the available dataset is rather small – a small dataset usually presents problems for regression models. Second, it applies a new measurement tool, i.e. nonparametric techniques, to the analysis of real convergence in the NMS area.

The empirical part of the study is structured as follows. First, the distribution of per capita relative income in the NMS is examined using the Gaussian Kernel density function. The graphical identification of convergence clubs within the period of analysis is confirmed using the ASH-WARPing procedure. The graphical analysis is enriched by adding the stochastic kernel, which illustrates transitions from one year to another, within the NMS area. The first part of the empirical study applies nonparametric models to the analysis of economic convergence, thus relaxing the assumption of linearity specific to the parametric models. It has a strong focus on graphs and aims at identifying the number of modes in the density distribution and whether the NMS converge at a single point in time.

In the second part of the empirical analysis, random effects panel regression models are used to estimate, in a parametric framework, the beta parameter. The results of the parametric regressions will then be compared to the output of the nonparametric analysis in order to see whether the two methodologies lead to the same results and also to find whether the nonparametric models bring new information about the convergence process to light, compared to the standard regression results.

The nonparametric methods applied to the NMS data give insights to the convergence/ divergence patterns and to the existence of convergence clubs in the process of real economic convergence, without making assumptions about the income distribution form. Even though the nonparametric models' empirical results' level of improvement over the parametric results depends on the data used, at a methodological level the nonparametric models represent a step ahead in comparison with the parametric one.

The paper concludes with the modality of income density distribution over time and across countries, states what framework is more appropriate for the analysis of real convergence (the parametric or the nonparametric approach) in the NMS area, analyzes the process of long-term (1995-2008) real convergence in the NMS area and examines the short-term patterns occurring in this process.

# 1 Theoretical Insights

The growth literature provides the basic methodological instruments for the analysis and measurement of economic convergence. Most of the theories of convergence rely on the neoclassical growth model (Solow, 1956), which implies that there is a negative relationship between the initial per capita output and its growth. According to this theory, poorer countries should advance faster than richer ones and will eventually catch up with the latter, when different countries are at different points relative to their balanced growth path and have different initial conditions, but the same steady state. This relationship is referred to as absolute (unconditional) convergence. When the initial capital endowment is not the only difference between economies, but there are also structural differences, then the convergence is referred to as being relative (conditional).

The literature of convergence is based on Barro and Sala-i-Martin's seminal paper (1992), in which they introduced the concept of beta-convergence – the speed of convergence of an economy towards its steady state. The analysis of convergence relies on two fundamental concepts: beta- and sigma-convergence. Beta-convergence occurs when there is a negative correlation between real per capita income growth over time and its initial level, and sigma-convergence occurs when the dispersion of real per capita income across a group of economies falls over time. The two concepts are not similar and beta-convergence is not a sufficient condition for sigma-convergence.

Despite the standard theory that assumes that poorer countries advance faster than richer ones towards a common steady-state or towards their own steady-state, the empirical evidence shows the increase of inequality and income divergence over time (Pritchett, 1997). This paradox is the root of the so-called "convergence clubs" (Baumol, 1986), which comprise a leader and a group of followers. According to the theory of convergence clubs, the leaders preserve their supremacy in terms of development and growth over a long period of time, and only a small number of followers converge with the leader over this time. Quah (1996, 1997), followed by other economists (Galor, 1996; Kumar and Russell, 2002) observed that after 1965 the world became polarized into two categories – rich and poor; this situation is referred to as twin peaks or convergence clubs. In the context of integration in the European Union (EU), the concept of convergence clubs suggests that the achievement of full economic or financial convergence is problematic, and a number of countries will never completely catch up with the leaders. If the polarization phenomenon experienced at the world level also becomes evident at the EU-level, then the achievement of real convergence in the EU space will be problematic.

The concept of beta-convergence has been criticized in the literature for a number of reasons (Quah, 1993, 1996; Johnson, 2000; Rassekh, Panik and Kolluri, 2001; Linden, 2002). The basic criticism of beta-convergence is the possibility of Galton's fallacy, i.e. a negative value of beta may not indicate convergence of growth rates but rather regression toward the mean (Friedman, 1992; Quah, 1993). Another criticism is that the growth regression assumes the condition of homogeneity, i.e. all economies under analysis have the same rate of convergence (Bernard and Durlauf, 1996). Therefore, the process of formation of convergence clubs[2] cannot be identified by the beta-convergence theory. Quah (1993) criticizes the concept of beta-convergence arguing that it brings no information on the way that poor economies are catching up with the richer ones. Friedman (1992) considers that the true test of convergence is a decline in the variance among individual observations. This is in fact the sigma-convergence.

## 2 Data

The empirical research focuses on the NMS and is based on the data collected from the World Economic Outlook Database April 2010 (IMF). The data used here are the NMS' Gross Domestic Products (GDP) at purchasing-power-parity (PPP) per capita, expressed in current US-$, from 1995 to 2008. The NMS considered in the paper are Poland, Hungary, Czech Republic, Slovenia, Slovak Republic, Estonia, Latvia, Lithuania, Romania and Bulgaria.

In table 1 the summary statistics show that the average per capita GDP levels increased in the period of analysis, with a 5-year growth rate of around 42% from 1995 to 2010. From 2006 to 2010, the IMF predicted the slowing down of the 5-year growth rate. Overall, the mean levels of per capita GDP in the NMS are increasing, indicating at a glance that the NMS are in the process of catching up with the Old Member States (OMS).

2 The term "convergence clubs" (*Quah*, 1997) is used to refer to two groups of economies in the analysis of convergence: a group of convergent economies and a group of divergent economies.

Table 1
Summary statistics by sub-periods, 1995-2010

| Sub-intervals | Mean | St. dev. | Min. | Max. |
|---|---|---|---|---|
| 1995-2000 | 9149.36 | 827.41 | 7957.9 | 10435.1 |
| 2001-2005 | 13027.92 | 1486.12 | 11176 | 15292.3 |
| 2006-2010 | 18403.94 | 845.38 | 16977.8 | 19544.2 |

Note. For 2009 and 2010 we have used IMF predictions.

The relative income is the main indicator investigated in the empirical section, in order to ensure the comparability across countries and across years. It is calculated in two ways to facilitate both a cross-sectional and a longitudinal analysis. We are mainly concerned with the cross-sectional representation, however, which requires calculating the relative income by dividing the NMS' GDP per capita levels by their mean in the same year, and then taking the natural logarithm of this value. The longitudinal approach to relative incomes is followed only in section 4c, where we explain the methodology of its construction.

# 3 Multimodality of Income Distribution Density

The traditional parametric models used in the analysis of income convergence are based on the assumption that data follow a certain distribution, e.g., a normal distribution. The *beta* approach relies on another assumption, which does not always hold in practice – the assumption of linearity in the relationship between economic growth and the logarithm of initial income. Due to these assumptions, the parametric models are not able to capture the process of real convergence when this process is characterized by income convergence clubs, short-term divergent paths and, in general, by non-linear dynamics.

This section examines whether the non-parametrical adjusted density is characterized by unimodality or multimodality. This could give insights to the existence of income convergence clubs within the NMS area in the period of analysis. All tests used in this section are applied on the logarithm of relative income per capita.

In the broad framework of the nonparametric models and tests, several procedures have been developed to assess the modality of a univariate distribution (Cox, 1966; Good and Gaskins 1980; Silverman, 1981). While some of the methods depend on the arbitrary choice of the scale of the effects studied (Cox, 1966; Good and Gaskins, 1980), others have incorporated automatic ways of making this choice (Silverman, 1981).

Several tests, all relying on the Gaussian function, have been applied in order to test the multimodality of the relative income in the NMS. The aim of applying several tests was to obtain robust results; this aim was confounded to a degree by the data availability and constraints. For this reason, only the results of two tests are discussed and reported here.

In the broad space of kernel density estimation the number of modes depends on the chosen bandwidth. The bandwidth is a smoothing parameter controlling for variance in the kernel probability density function, which is normally taken as a standard Gaussian function with mean zero and variance 1. For this reason, the first step in the analysis of multimodality was the selection of optimal bandwidths for each year of our analysis, using the bandwidth rules developed by Salgado-Ugarte et al. (1995a). Silverman's Gaussian kernel bandwidths were taken as reference values in the construction of the tests described below. In Annex 1 (table 4), the binwidth/bandwidth rules applied to our 2007 data are presented for exemplification.

## a) Ash-warping

The ASH-WARPing procedure is applied to smooth the histograms used in this paper to estimate the nonparametric univariate density, and also to get information about the modality in the density distribution. This procedure is derived from the general framework called WARP (Weighted Averaging of Rounded Points) developed by Härdle and Scott (1988) and is based on the Averaged Shifted Histogram (ASH) (Scott, 1985).

The theory notwithstanding (Scott, 1992), the empirical evidence has shown that when defining the histogram the choice of origin influences the result (Silverman, 1986). To solve this problem, Scott (1985) proposed averaging several histograms with different origins to produce the ASH.

In the presentation of the ash-warping method, we start by defining first the histogram[3].

If all $n$ observation of a variable belong to the interval [0, Kh) and if the interval is partitioned in $K+1$ bins, with $h$ being the width of bins, then the $k$th bin, $B_k$, is defined as:

$$Bk = [kh, (k+1)h)\ ,\ k = 0,\dots,K \qquad (1)$$

The histogram is defined as:

$$\hat{f}(x) = \frac{v_k}{nh} = \frac{1}{nh}\sum_i I_{(t_k,t_{k+1})}\,(x_i) \qquad (2)$$

Where, $v_k$ is the number of observations in $B_k$, and I is the indicator function, equal to one when $x_i$ lies in the specified interval and zero otherwise.

Let be M a collection of hisograms $\widehat{f_1}$, $\widehat{f_2}$, ..., $\widehat{f_M}$, having the bin width $h$:

$$t_0 = 0, \frac{h}{M}, \frac{2h}{M}, \dots, \frac{(M-1)h}{M} \qquad (3)$$

The following restriction can be applied on the previous relationship:

3 The presentation of ash-warping methodology is based on the *Isaias Hazarmabeth Salgado-Ugarte, Makoto Shimizu*, and *Toru Taniuchi*'s paper „ASH, WARPing, and kernel density estimation for univariate data" (*Stata Technical Bulletin* July 1995).

$$x_i \geq \frac{(M-1)h}{M} \tag{4}$$

With the restriction above, the un-weighted ASH can be expressed as:

$$\hat{f}(\cdot) = \widehat{f_{ASH}}(\cdot) = \frac{1}{M}\sum_{i=1}^{M} \hat{f}_i(\cdot) \tag{5}$$

In a generalized form, ASH can be defined as:

$$f(x;\, M) = \frac{1}{n}\sum_{i=1-M}^{M-1}(1 - \frac{|i|}{M})v_{k+i} \;\; for\; x\epsilon B_k \tag{6}$$

Linear interpolation schemes are sometimes used to make the ASH continuous. They produce the Frequency Polygon of the ASH (FP-ASH)

The ASH is a particular case of the general method WARP, which is defined as:

$$\hat{f}(x; M) = \frac{1}{nh}\sum_{|i|<M}^{M-1} w_{M\,(i)}\, v_{k+1} \qquad for\; x \in B_k \tag{7}$$

Where, $w_{M(i)}$ denote the weighting operation and function, and *M* represents the number of shifted histograms to average.

In fact, the ASH-WARPing procedure involves three steps: (1) binning the data; (2) calculating the weights, and (3) weighting the bins. Different weight functions can be used to approximate the kernel density estimator and, finally, the data are reduced to a list of bin counts along with their midpoints. The density estimate in each bin is computed as the product of the bin count and the weight.

In this paper we have applied the ASH-WARPing procedure on the NMS' relative incomes, using the corresponding Silverman's Gaussian kernel bandwidths for each year of our analysis, as presented in Annex 2 (table 5). The results of this procedure indicate that from 1995 to 2008, the kernel density of the relative income is bimodal in 9 years and unimodal in 5 years. After 2002, the income density is bimodal each year. A detailed situation of the density modality is presented in Annex 2 (table 5). This is a first indication that the NMS do not tend to converge over the long term at a single point, or at least that the NMS convergence cannot be seen as a gradual, continuous process.

The modality of income density distribution can also be analyzed using the ASH-WARPing procedure in a graphical manner. Figures 1, 2 and 3 represent the Gaussian kernel density estimation for the years 1995, 2002 and 2007, using Silverman's optimal bandwidth values (Annex 1, Table 4). Figure 1 indicates the unimodal structure of the distribution function in 1995, while Figures 2 and 3 indicate the bimodality of the density distribution[4] in 2002 and 2008.

[4] Other years have been examined as well, but only the years signifying the beginning, the end and the switching points in the distribution function have been reported in this paper. In any case, the distribution from 2002 to 2008 is bimodal.

Figure 1
Kernel density estimation by using the Silverman's Gaussian kernel bandwidth, 1995

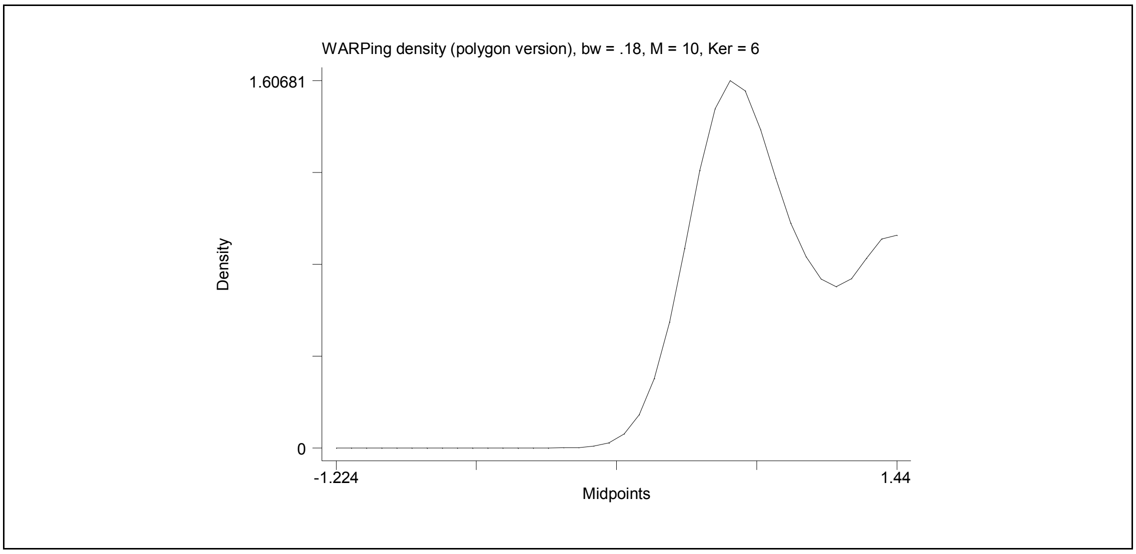



Figure 2
Kernel density estimation by using the Silverman's Gaussian kernel, 2002

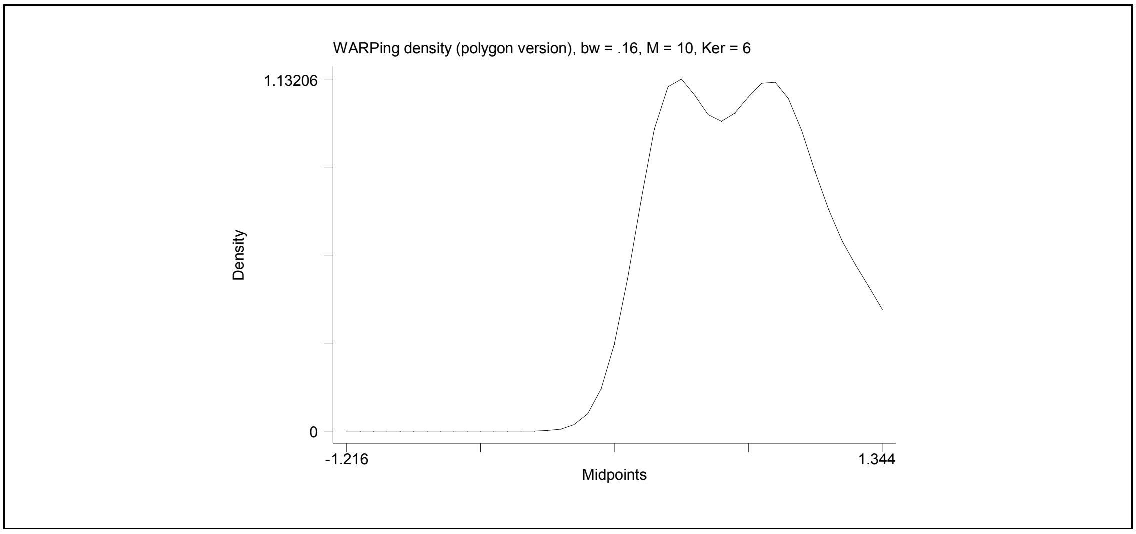
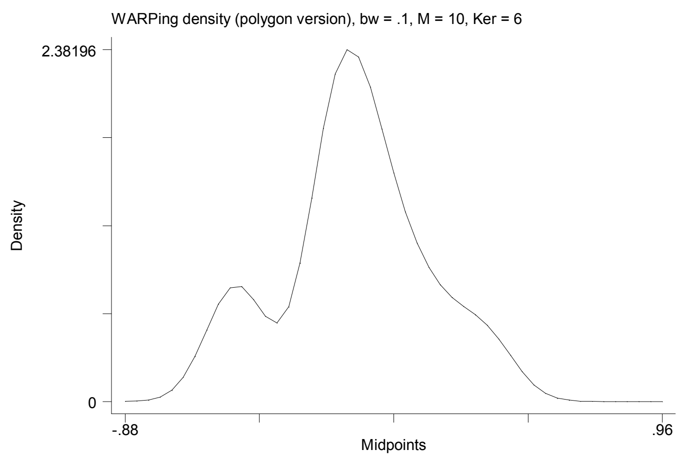


Figure 3
Kernel density estimation by using the Silverman's Gaussian kernel, 2008




## b) Kernel density estimator

The kernel density estimators belong to the class of nonparametric estimators, i.e. they have no fixed structure and depend on all the data points to produce the result. In comparison with the histogram, they smooth out the contribution of each observed data point over the local neighbourhood of that data point. The contribution of data point $x_i$ to the estimate at the arbitrary point $x$ depends on the shape of the kernel function adopted and the width (bandwidth) accorded to it.

A typical form for the kernel density estimator is:

$$\hat{f}(x) = \frac{1}{hn}\sum_{i=1}^{n} K\left(\frac{x-X_i}{h}\right) \tag{8}$$

Where, $\hat{f}(x)$ is the density estimation of the variable $x$, $n$ is the number of observations, $h$ is the bandwidth (smoothing parameter) and $K(\cdot)$ is the smooth and symmetric kernel function integrated to unity.

The bandwidth is very important as the size of the bandwidth chosen for the kernel density estimation determines the degree of smoothing produced. When low values are assigned to $h$, the estimated density for the data is not as smooth as when higher values are assigned. The kernel density estimator uses fixed bandwidths and thus the estimation is sensitive to any low count interval of the distribution. Choosing the best width of the bandwidth $h$ is paramount to an accurate estimation. Several procedures have been proposed in the literature to find the optimal bandwidth. They range from the subjective assessment of a pleasing smoothing of the result (Tarter and Kronmal, 1976) to objective methods that start with the analysis of the shape of the true density distribution. In particular, when a Gaussian kernel is used as the reference function, the minimization of the Mean Integrated Squared Error (MISE) allows $h$ to be derived (Tukey, 1977; Scott, 1979; Silverman, 1978, 1986).

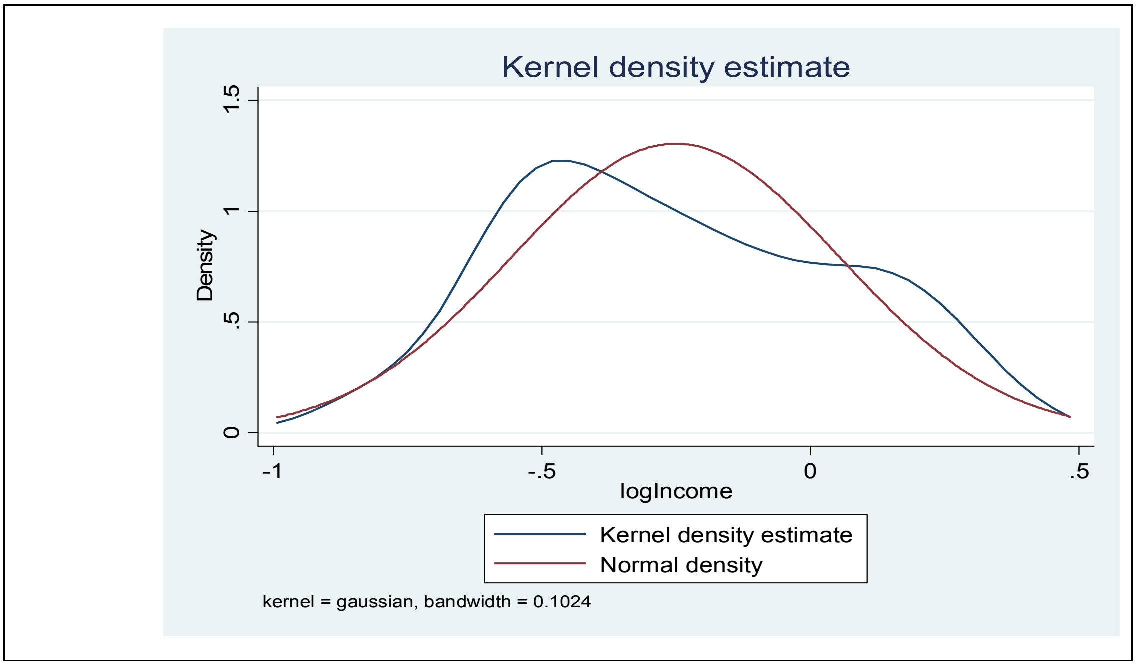
Besides the WARPing method, which was applied at point (*a)* of this section, the univariate kernel density is also used in the paper, with the same aim of providing a graphical representation of the income density distribution. This method also examines the modality of the distribution. The kernel density is estimated using the Gaussian function and the results are presented in Annex 3 (figures 7.1-7.4). To ensure the comparability of the results, the same years have been analyzed here as for the WARPing procedure. Both graphical representations yield the same results, with the exception of the year 2002, for which the WARPing method suggests a bimodal distribution while the kernel density indicates a unimodal distribution.

In a second step, the kernel density was used to examine the modality of the relative income distribution not only among countries, but also over time, within each country. The density of the natural logarithm of relative incomes has been estimated using the Gaussian kernel. In contrast with the previous tests, this time the relative income is constructed to reflect the longitudinal dimension of analysis, i.e. by dividing, for each country, the annual values of the per capita GDP by the related mean. Due to this normalization process, a zero value on the horizontal axis indicates a per capita relative income equal to the national mean of the entire period of analysis.

The results are shown in Annex 4 where, for each graph, the period considered is 1995-2008. For all countries, the kernel density estimates indicate a bimodal distribution. The "twin peaks" shaped in figures 8.1-8.10 are referred to in literature as "convergence clubs" (Quah, 1996). The density shapes give insights to the income polarization in the NMS during the transition period. Although two modes have been identified in the distribution of income densities for each country in part, they reflect different patterns over time. In the case of Bulgaria, Romania, Czech Republic, Slovak Republic and Poland, the relative income densities have two symmetric modes around the national means, while Estonia, Latvia, Lithuania and Hungary have a big mode above the national mean and smaller one below the national mean. This reflects a more favourable income distribution for the first group of NMS from 1995 to 2008. A particular aspect regards Romania which has a bigger mode located below the national mean, and a smaller one above the mean. This suggests a higher concentration of annual incomes in the low income area.

When considering a longitudinal approach not at the country level this time, but at the level of the entire NMS area, a bimodal distribution occurs again. This aspect mainly reflects the bimodality of income distribution among each country in the transition period, and, only to a lesser extent, the bimodality of income distribution across countries (see Figure 4).

Figure 4
Kernel density estimate, NMS area, 1995-2008




## c) Stochastic kernel density

The stochastic kernel density allows the estimation of the conditional density function – a transition function obtained using the kernel density estimation. In contrast with other techniques specific to the measurement of convergence (beta- and sigma-convergence), it uses all the information in the data, i.e. the first period, the last period and the transition process. For instance, beta-convergence considers the transition relative to the first period, but neglects the last period, while sigma-convergence looks at all the observed periods, but only in terms of their standard deviations (Weber, 2009).

In the next paragraphs of this section we introduce the stochastic kernel, starting with the density distribution. The density distribution $\varphi_{t+1}$ of a variable $x$ follows a first order Markov process:

$$\varphi_{t+1} = M \cdot \varphi_t \tag{9}$$

The operator $M$ maps the transition of variable $x$ from its distribution in the state $t$ to its distribution in the state $t+1$. It assumes either a finite number of states in $\varphi_t$ distribution using the Markov Transition Matrix (Shorrocks, 1978) or using a continuous state formulation in the stochastic kernel (Quah, 1996). In a discrete version of the model, the operator $M$ is determined by partitioning the set of possible income values into a finite number of intervals. The properties of $M$ are described by a Markov chain transition matrix whose $(i, j)$ entry is the probability that a country in state $i$ transits to state $j$ in terms of per capita GDP and in one time step. As the per capita GDP is a continuous variable, the transition probability matrix will be a matrix of continuous rows and columns. Therefore, the operator *M* can be seen as a stochastic kernel or a transition function, and real convergence can be seen as the shape of the income distribution at time $t+\tau$ over the range of incomes observed at time *t*.

According to Quah (1996), if *u* and *z* are elements of *B* and also probabilities measures in (R, R), the stochastic kernel is a function relating *u* and *z* by the function $M_{(u,\, z)}$ : (R, R) -> (0, 1), such that:

(i): For each y Є R, $M_{(u,\, z)}\,(y\,,\,\cdot)$ is a probability measure in (R, R);

(ii): For each *A* Є R, $M_{(u,\, z)}\,(\,\cdot\,, A)$ is a measurable function in R;

(iii): For each *A* Є R, it is valid that $u$ (A) = $\int M_{(u,\, z)}\,(y, A)\; dz(y)$

At an initial point in time, for a given *u*, there is some fraction of the economies *dz(u)* with incomes close to *u*. When normalized to a fraction of the total number of economies, the number of economies in that group whose incomes fall in the subset *A* can be written as *M (y, A)*. The integral $\int M_{(u,\, z)}\,(y, A)\; dz(y)$ indicates the number of economies that end up in state *A*, regardless of their initial income levels. Stochastic kernel *M* can therefore be seen as the description of transitions from state *y* to any other portion of the underlying state space R.

According to Arbia et. al. (2005), the Stochastic Kernel can be also written as:

$$\varphi_{t+\tau}(y) = \int_0^{\infty} f_\tau\,(y|x)\varphi_t(x)dx \qquad (10)$$

Where *y* is the relative per capita income in period $t+\tau$, *x* is the relative per capita income in period *t* and $f_\tau\,(y|x)$ is the conditional density given the relative income in period *t*.

One of the most popular kernel functions is the standard Gaussian function with zero mean and 1 variance.

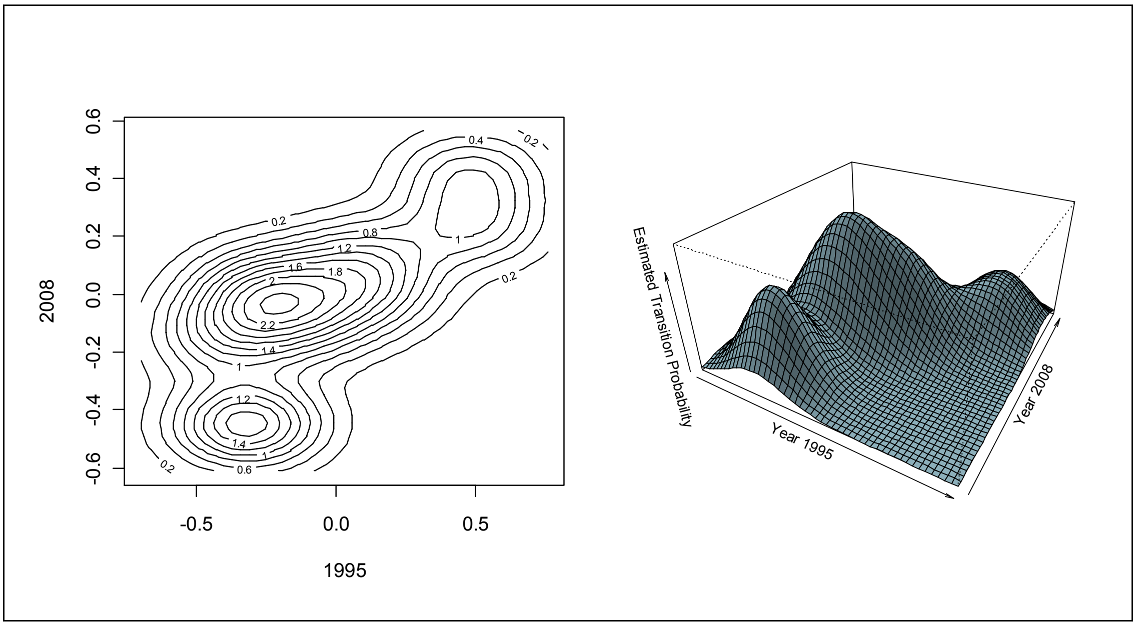
$$f(x) = \int_{-\infty}^{+\infty} f(y,x)dy = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-x_i)^2}{2h^2}} \qquad (11)$$

Where, *x* is a random variable and *h* is the smoothing parameter called bandwidth.

The stochastic kernel, as represented in figures 5 and 6, shows the transition probability associated with the change in the distribution of relative incomes occurring from one period to another. For each transition considered here, two perspectives have been analyzed, one being a two-dimensional representation, and the other, a three-dimensional one. Both indicate the formation of convergence clubs by highlighting "peaks" in the income distribution.

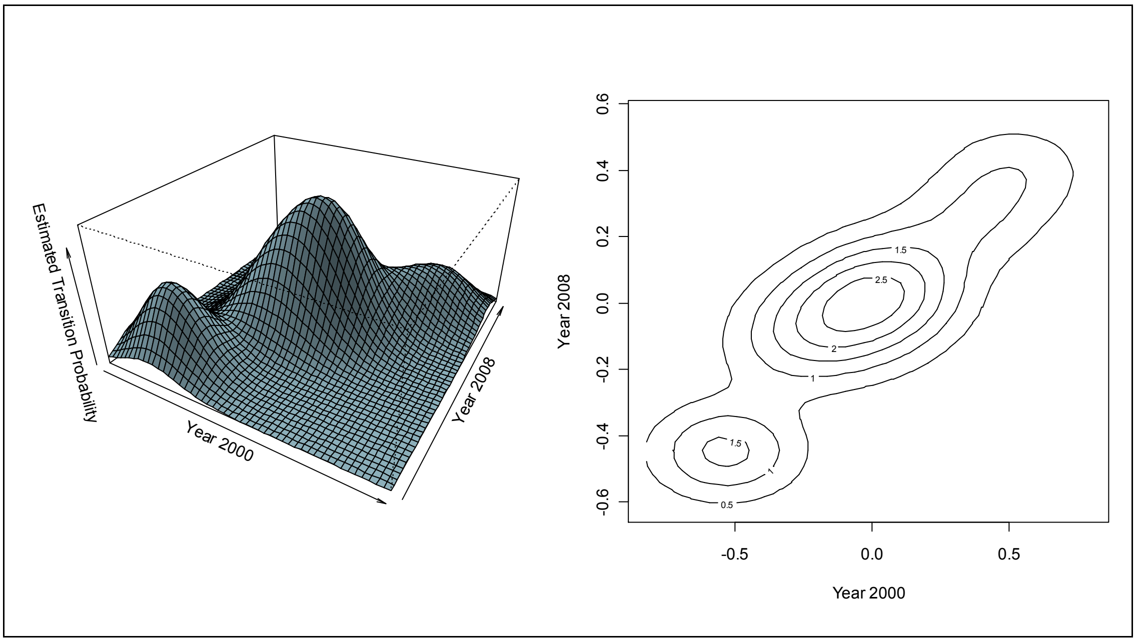
Figure 5 indicates three significant peaks in the stochastic kernel that occurred in the transition from 1995 to 2008. One of them is bigger than the other two and reflects the transition of a sub-group of NMS countries from the poor income category to a new middle income category. This situation reflects an improvement in the relative income distribution among the NMS, since the intermediate income area, which was absent in 1995, becomes the most important category in 2008. The other two categories capture the convergence among the low income countries and, respectively, the convergence towards higher incomes. This picture of the stochastic kernel allows the hypothesis of relative income convergence to a single point from 1995 to 2008 to be rejected.

Figure 5
Stochastic kernel 1995-2008




Even though several sub-periods of time could be examined in detail, the paper analyzes only the sub-interval 2000-2008, in order to capture the changes in the NMS' income density distribution during the global economic crisis. The stochastic kernel for this period of time is presented in Figure 6. In comparison with the period 1995-2008, from 2000 onwards the high income category became smaller because a portion of the countries initially located in this category had moved into the intermediary income category by 2008. This change reduced the number of convergence clubs by two, with the disappearance of the high income category and the stability of the low income category over time.

Figure 6
Stochastic kernel, 2000-2008




In conclusion, the stochastic kernel analysis does not reveal, for any of the cases studied here, convergence to a single point up until 2008. The most significant patterns in the NMS during the transition period as identified by this method are the emerging of a "middle class of the NMS", bipolarization towards the low and intermediate income categories, stability of the small but constant poor income group and shrinking of the high income group, due to the global economic crisis.

The nonparametric analysis detailed in section 4 shows that the density of the income distribution among the NMS cannot be considered as unimodal from 1995 to 2008. The bimodality of the income distribution arises among countries as well as within countries. The bimodal structure of the income distribution and the convergence peaks in the stochastic kernel suggest the lack of real economic convergence within the NMS area, as well as the inappropriateness of the parametric models applied to our dataset.

# 4 A parametric Approach to real Convergence in the New Member States

The convergence literature is based on the seminal work of Barro and Sala-i-Martin (1991) who introduced the concept of beta-convergence. This concept states a negative relationship between growth rate and the initial income per capita, due to the assumption of marginal decreasing productivity. Despite the fact that it is widely used in the empirical work on convergence, the beta-convergence approach has been criticized in the literature, one reason for this being its inability to capture the convergence clubs in the income distribution (Quah, 1996).

In this section we apply random effects panel models to examine the unconditional beta-convergence in the NMS area. These results will then be compared with the output from the nonparametric approach, to get both empirical and methodological insights on the basis of the IMF data and predictions about the NMS. The relevance of alternative spline regression techniques is limited here by the data availability[5].

The estimates of the first random effects panel regression, where the dependant variable is the growth rate between 1995 and 2008 and the independent variable is the logarithm of per capita GDP in 1995, are reported in Table 2.

Table 2
Random-effects panel growth regression (1995-2008)

| **Variable** | **Coefficient** | **St. err.** |
|---|---|---|
| Per capita GDP 1995 (log) | -.0056 | .0056 |
| Constant | .0667 | .0505 |
| rho[6] | .17 | |
| Nr. of obs. | 140 | |

Note. *** Signif. at 1% level, ** signif. at 5% level. –* signif. at 10% level.

When the entire period is considered, i.e. 1995-2008, the beta coefficient is very low, negative and not significant, suggesting that on long term there is no absolute convergence in the NMS area. When looking just at the period of time from 2003 onwards, the regressions still yield negative and low beta coefficients, but which gradually improve in the level of significance[7]. In table 3 we present the estimates of the random effects panel regression which runs between 2003 and 2008. This time, the *beta* coefficient has a low negative value but becomes slightly significant, indicating a slow process of convergence after 2003 in the NMS area.

5 The small working dataset of the NMS' GDP per capita between 1995 and 2008 makes the use of cubic splines inappropriate. If it had been possible to apply this technique, it would have allowed the relationship between the dependant and independent variables on separate income ranges to be estimated. By using the splines, the analysis could have revealed different patterns of convergence or divergence within this period. Without this transformation of this explanatory variable, the whole process of convergence is summarized in the regression analysis by one coefficient, i.e. the beta coefficient.

6 The *rho* statistic indicates the proportion of the total variance attributed to the panel level variance component.

7 A set of regression models, starting from different years after 2003 and ending in 2008 in all cases, are tested, and all of them indicate a slow process of convergence with a gradual improvement in the level of significance after the first year of the regression. From this list, only one regression is reported here (Table 3), as they all lead to the same empirical finding.

Table 3
Random-effects panel growth regression (2003-2008)

| Variable | Coefficient | St. err. |
|---|---|---|
| Per capita GDP 1995 (log) | -.0107* | .0066 |
| Per capita GDP 1995 (log) | -.0107* | .0066 |
| rho | .64 | |
| Nr. of obs. | 60 | |

Note. *** signif. at 1% level, ** signif. at 5% level. – * signif. at 10% level.

In conclusion, the absolute convergence process on long term is not clearly suggested by the conventional beta approach. The evidence of convergence becomes significant, but is still weak when running the analysis only from 2003 onwards. In comparison with the nonparametric models that suggest the absence of absolute convergence over the long term and also after 2002, the parametric regression indicates a lack of absolute convergence in the long term and a weak absolute convergence after 2002. In this light, the nonparametric analysis brings not only new and additional findings about the process of absolute convergence, but also different results. The presence of convergence clubs from 1995 to 2008 as well as the bimodality of the income density distribution each year from 2002 onwards prove that the process of absolute convergence identified in the last six years of our analysis by the linear regression models is not real. In addition, the empirical results obtained in this section show that parametric analysis provides only a little information about the changes occurring over time and the progress towards unconditional convergence – this analysis type is not able to capture the changes in the income density distribution across countries, from one year to another.

# 5 Conclusions

This paper applies several nonparametric techniques to the analysis of absolute convergence in the NMS area, being oriented to provide robust conclusions, at both the methodological and empirical levels. Despite the fact that the methodological orientation is the primary focus of this paper, the conclusions are derived mainly from the empirical findings.

The nonparametric analysis of the income density distribution in the NMS area between 1995 and 2008, as well as the parametric analysis applied to the same period, indicate the lack of real absolute convergence in the long term, instead, there are short periods of convergence and divergence. The divergence represents a yearly characteristic for the NMS area from 2002 to 2008. This short-term characteristic is evident early on in 2000 and onwards. Apart from the nonparametric analysis, the parametric analysis finds evidence of a weak process of absolute convergence in the short term, i.e. from 2002 to

2008. This result, revealed by the random effects regression, should be regarded with caution because of the presence of convergence clubs from 1995 to 2008 and also because of the bimodality of the income density distribution each year after 2002.

During the transition period, the income distribution in the NMS area is of a bimodal structure, which is also graphically illustrated by the convergence clubs (in the stochastic kernel analysis). This aspect is mainly driven by the bimodality that occurs in the income density distribution of each NMS country across the years. Even so, there are years and periods of time when the income distribution among countries also has a bimodal structure. The existence of more convergence clubs in the income density distribution, either in particular years or in the transitions over time, gives insights to the convergence patterns during the period of analysis. In this light, the nonparametric analysis reveals new findings in comparison with the conventional parametric regressions, which in turn reduce the description of the entire convergence process by one coefficient. Even though both the parametric regressions and the nonparametric techniques suggest divergence from 1995 to 2008, the latter provides a broader framework of analysis, and becomes more credible when the number of observations in the dataset is rather small.

At the country level, the density distribution of the per capita GDP is bimodal, which is not surprising as during the transition period these countries have continuously "grown up" and have experienced changes in income distribution. The transitions illustrated by the stochastic kernel show that the NMS exhibit a trend of moving toward the mean income. This could be interpreted as the formation and consolidation of a "middle class" of NMS during the transition period, influenced by the global economic crisis. This consolidation process is mainly and gradually driven by the shrinking of the high-income NMS. Despite the changes seen in the upper middle-income category of the NMS, the "poor countries" category remains stable over time. When we look at the entire period of transition, these changes are not sufficient to sustain the process of real convergence in the NMS area.

In conclusion, when the income density distribution is not normal, or "too non-linear", the nonparametric approach can provide complex, real and "different" information about the salient or hidden aspects of distribution or about the short-term dynamic patterns. In our paper, the nonparametric output reveals more and partially different features of the real convergence process, as compared with the conventional beta approach.

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## ANNEX 1

### Tabel 4. Binwidth-bandwidth rules for the univariate density estimation of relative incomes in the NMS (1980-2015)

Some practical number of bins and binwidth-bandwidth rules for univariate density estimation using histograms, frequency polygons (FP) and kernel estimators (for log of relative income)

| | |
|---|---|
| Sturges' number of bins = | 8.7142 |
| Oversmoothed number of bins <= | 7.4889 |
| FP oversmoothed number of bins <= | 6.8818 |
| Scott's Gaussian binwidth = | 0.2252 |
| Freedman-Diaconis robust binwidth = | 0.2213 |
| Terrell-Scott's oversmoothed binwidth >= | 0.2224 |
| Oversmoothed Homoscedastic binwidth >= | 0.2399 |
| Oversmoothed robust binwidth >= | 0.2880 |
| FP Gaussian binwidth = | 0.2822 |
| FP oversmoothed binwidth >= | 0.3058 |
| Silverman's Gaussian kernel bandwidth = | 0.1181 |
| Haerdle's 'better' Gaussian kernel bandwidth = | 0.1391 |
| Scott's Gaussian kernel oversmoothed bandwidth = | 0.1502 |

## Annex 2

**Table 5. ASH Warping test**

| Year | Number of modes in the non-parametric density | Bandwidth *h* |
|---|---|---|
| 1995 | 1 | 0.18 |
| 1996 | 2 | 0.17 |
| 1997 | 1 | 0.19 |
| 1998 | 1 | 0.19 |
| 1999 | 1 | 0.19 |
| 2000 | 2 | 0.19 |
| 2001 | 1 | 0.17 |
| 2002 | 2 | 0.16 |
| 2003 | 2 | 0.14 |
| 2004 | 2 | 0.13 |
| 2005 | 2 | 0.10 |
| 2006 | 2 | 0.10 |
| 2007 | 2 | 0.10 |
| 2008 | 2 | 0.10 |

*Note.* The number of modes is determined by using the Silverman's Gaussian kernel bandwidth. These bandwidths are reported in the last column.

## Annex 3

**Figure 7:**
**Kernel density estimates by year**

Figure 7.1

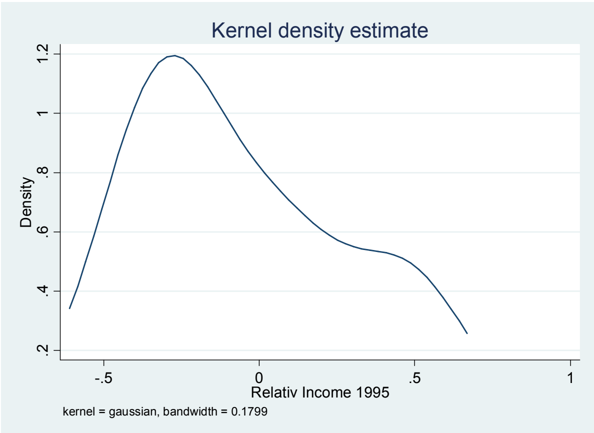



Figure 7.2

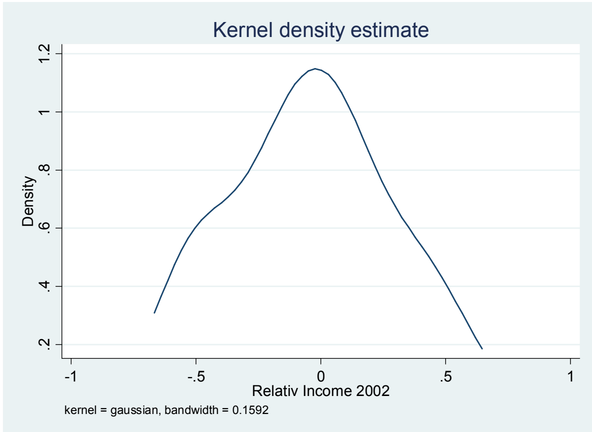



Figure 7.3

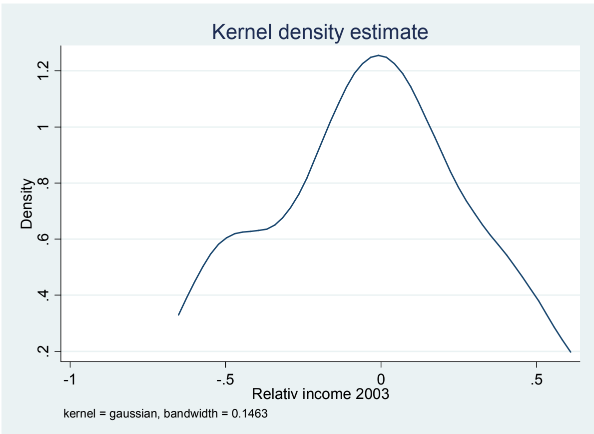
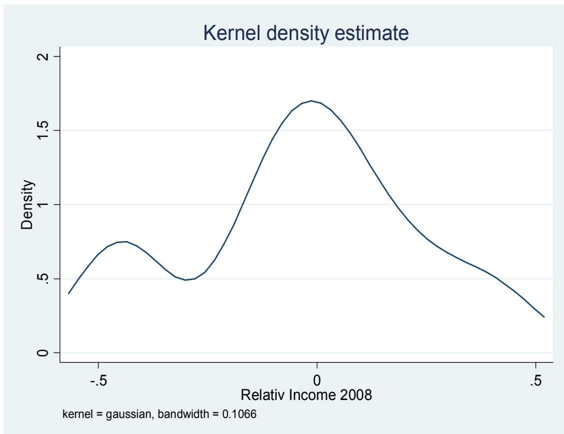



Figure 7.4



## Annex 4

**Figure 8:**
**Kernel density estimates by country**

Figure 8.1

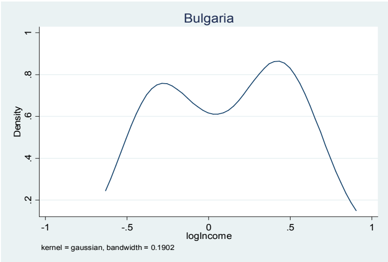



Figure 8.2

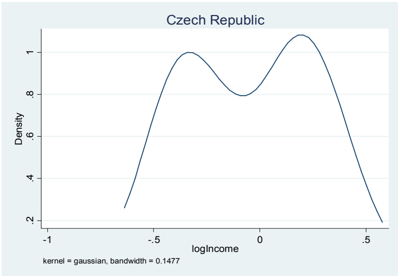



Figure 8.3

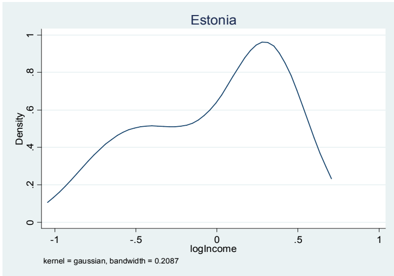



Figure 8.4

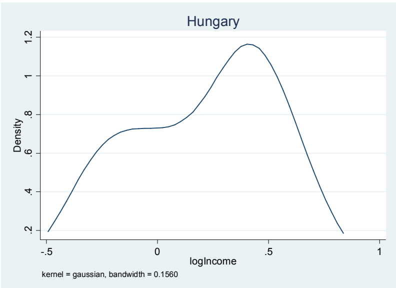



Figure 8.5

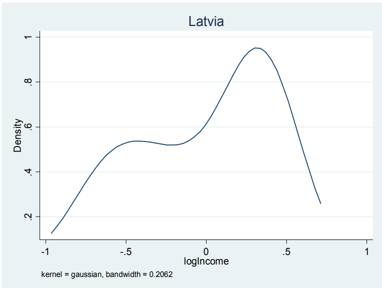



Figure8.6

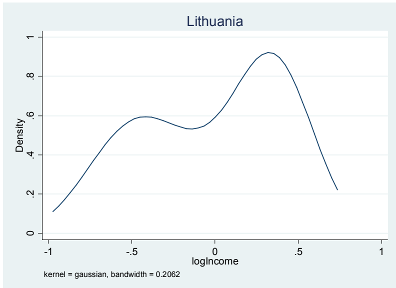



Figure 8.7

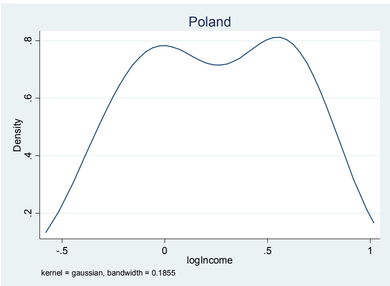



Figure 8.8

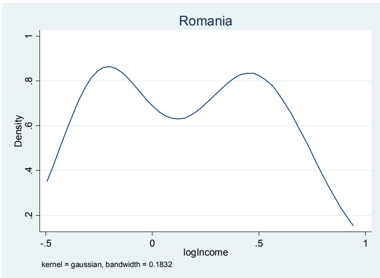



Figure 8.9

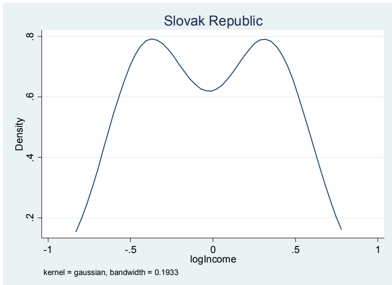



Figure 8.10