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Equilibrium Mergers in a Composite Good Industry*

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Abstract

This industry is formed by single-component producers whose components are combined to create composite goods. When a given firm has the possibility of merging with either a complement or a substitute component producer, its equilibrium choice depends on the degree of product differentiation in the composite good market. A merger between complements, which allows for mixed bundling, only happens when composite goods are very differentiated. Private incentives do not always go along with social interests and the equilibrium merger can differ from the socially optimal merger. After a merger, outsiders have also the opportunity to react and merge to other outsiders or to join the previous merger.

JEL codes: L13, L41.

Keywords: merger, composite goods, substitutes, complements, pricing strategies, countermerger.

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1 Introduction

In nowadays economy, globalization and international considerations leave no room for discussion. New challenges arise, as the information technologies, cost reductions and market liberalization and integration. To adapt to these new conditions, industry structure varies. Mergers, takeovers or joint ventures between different firms are useful devices to restructure an industry. According to ZHEPHIR 2008 report, the global number of mergers and acquisitions increased from more than 42,000 in 2003 to more than 72,000 in 2007, staying close to 54,000 in 2008. The 2008 biggest deal was the acquisition of Wachovia Corp. by Wells Fargo & Company Inc., which was finally taken by the US Government.

Basic motivations for a merger are to strengthen market power, to become more efficient by exploiting economies of scale or streamlining the production process, to reduce management inefficiencies, or to obtain synergies. Besides, there are other reasons related with managerial goals, as to increase the firm size to improve their level or to keep the firm independent. Also to influence the regulation in the sector, to obtain subsidies or to have the Government protecting their interests abroad. In this paper we will focus on the strategic effects of a merger.

Mergers and acquisitions are some of the most regulated firms' decisions by antitrust authorities. They decide if deals are accepted or not, but mergers arise conditioned by merging incentives of individual firms. Authorities must take decisions considering the affected market, competitors and effects in society welfare. This is not easy, specially if the merger implies bundling between complementary products, as in the well-known example of General Electric-Honeywell merger.[1] This last example shows us that some markets are closely linked to others, so all the relevant markets need to be considered.

It is then important for the analysis whether goods are used together or not. Consumer's utility comes from different sort of goods, some ones consumed individually or others used in combinations instead. It is not difficult to find products consumed together, for example think of a mobile phone and mobile phone services supplied by a

[1] The European Commission claimed that the proposed merger will let the new firm to sell complementary goods in a bundle. This pricing strategy would imply price discounts that would give the firm an unbeatable advantage over its rivals. This advantage will lead to the exit of rivals and therefore eventually to strengthening the dominant position of GE. (Nalebuff, 2002).

telecom company. Also think of a car and a car insurance, tennis balls and a racket, or ski boots and skies. Traditionally, a flight and a hotel booking were sold together by travel agencies, but nowadays consumers book by the internet and pick up their package. Then, it is natural to raise the following question: if you are a component producer, will you merge to a substitute or a complement component producer?

Our aim is to focus in these composite goods, which are formed by two types of components, x and y. Consumers can choose between several varieties available for each one, so they design their own composite good, since there is no restriction among the combinations and all components are fully *compatible*.[2] We initially consider an industry where every firm produces a single component to create composite goods and competition is in price. It is assumed that consumers obtain some utility only when they consume the composite good, while consumption of separate components does not provide any utility. Therefore, there is competition among composite goods which are imperfect substitutes, but at the same time a complementarity relationship is established among components of different type and also components of the same type are perfect substitutes.

Two alternative types of merger will be analyzed: a *same type component* merger (ST merger), between any component producer and another firm producing the same component type; and a *different type component* merger (DT merger), between any producer and a firm producing a different component type. Both mergers have completely different implications on post merger prices and nonparticipants' outsiders reaction. Next, we also find which one is the most suitable, both privately and socially. We will focus on strategic effects, so productive efficiencies are left out.

We firstly analyze a general setting with n_x and n_y compatible varieties of each component. After a ST merger, and due to the internalization of competition among both substitute components, equilibrium prices for the merged firm increase, equilibrium prices of the remaining x-type components increase by strategic complementarity, while equilibrium prices of y-type components decrease by strategic substitutability. Some outsiders are better off while others are worse off. However, after a DT merger, the new entity fully controls the pricing of one composite good (the *insider* composite good), partially controls

[2] One can also think of a situation where complements are different inputs in a product chain where complements are assambled by the producers thus possibly leading to vertically related relationships. In our case such a vertical relation is not considered.

the pricing of $n_x + n_y - 2$ of them, (*mix-and-match* composite goods), and has no control on the others (*outsider* composite goods*).* Mixed bundling is the pricing strategy that yields the highest payoffs to the merged firm, as compared with pure bundling and pure component pricing. With mixed bundling, the new entity will set a bundle price smaller than the sum of the component prices. These component prices will increase compared to the pre-merger situation to divert consumption from the mix-and-match composite goods to the insider one. Finally, outsiders will reduce their prices.

Next, we resort to the two by two case to find out which merger will endogenously arise and their welfare implications. Both ST and DT mergers are profitable, but the *equilibrium* one is the DT merger when composite goods are very differentiated, being the ST merger otherwise. This result is not specific to the two by two case, as we get the same qualitative results for n smaller than ten, but only the DT merger will endogenously arise for greater number of components. Further note that for the asymmetric situations, that is when the number of components of each type is different, the ST merger is the equilibrium one when the merger is proposed by a firm belonging to the component type with less substitutes, otherwise, the DT merger arises. Regarding welfare and based on pure strategic effects, we find that a privately profitable DT merger is always welfare improving under the Consumer Surplus rule, then it will always be permitted by an antitrust authority. On the contrary, authorities' decision when the ST is proposed is to forbid that merger.

One of the stylized facts about mergers is that they are not isolated events, they usually happen in waves. For the two by two case, the *continuation merger* is an approach to describe the merging path in the industry. Hence, once a merger of either type is produced, it is interesting to find out how the outsiders' reaction evolves, either adhering to the initial merger and becoming an *extended merger,* or forming a *countermerger* with the other outsider. We find that only an extended merger will be formed at equilibrium after an initial merger of either type. Finally, if we consider that antitrust authorities should be far-sighted in their merger policy, we identify conditions where authorities block a merger that will trigger a subsequent merger that would result in a welfare improving market structure. This is the case when a ST merger, that implies an Extended Merger response, is blocked.

Related Literature

Merging incentives of substitute products firms have been analyzed since the seminal paper by Salant, Switzer and Reynolds (SSR) (1983). They consider quantity competition and point out that exogenous mergers could be unprofitable for the participants. In equilibrium, a merger only arise if it involves the 80% of the initial independent firms. An interesting result is that outsider firms are always better after the merger, due to the strategic substitutability in quantity competition among substitute goods. One of the main critics is that they consider a symmetric industry after the merger, while it would be an asymmetric one, like in Perry and Porter (1985), who conclude that there are more profitable mergers than in SSR, though the qualitative results are kept. Deneckere and Davidson (1985) find that every merger is profitable, although this is due to upward-sloping reaction functions, the ones which correspond to substitute goods and price competition. Finally, Kamien and Zang (1990) develop a two-stage merger game to evaluate endogenous mergers in a market setting with homogenous products, finding that full monopolization of an industry is not the usual result.

Another strand in the literature considers mergers of firms producing complementary goods. We will distinguish between, models where consumers assemble the components of composite goods and only joint consumption yields utility, from those where there are consumers that also obtain utility by consuming separately some complementary goods. In the former type of models, Gaudet and Salant (1992) extend SSR's analysis to an industry of complementary goods and price competition. They get the same conclusions about merger profitability, although with opposite welfare outcome. Beggs (1994), in a price competition environment, analyzes merging decisions in a setting of two groups with two firms each, where products are complements within the group but substitutes across groups. He assumes no compatibility since consumers cannot choose one good from each group.[3] The conclusion is that complementary product firms usually prefer acting independently instead of being involved in a merger. Economides and Salop (1992) analyze

[3] The compatibility feature of the components can be endogenously obtained. Matutes and Regibeau (1988) design an addressed model where consumers choose between complete systems or mix-and-match systems. In their two stage game, compatibility or incompatibility is decided endogenously by the firms before competing in prices. Matutes and Regibeau (1992) extends the previous model to let firms set a price of separate components and also a price for the composite good (a mixed bundling price strategy). Equilibrium marketing choices depend crucially on compatibility decisions.

competition and integration in different market structures, focusing on two brands of each type of compatible components, bundling strategies are not considered. Apparently, independent firms would react to a merger creating a countermerger. Choi (2007, 2008) takes Economides and Salop's framework with four firms producing components, but let them choose mixed bundling as its optimal profit maximizing strategy after a merger. Bundle price is lower while component prices are higher after the merger.

In the latter type of models, Flores-Fillol and Moner-Colonques (2007) analyze strategic formation of airline alliances when two complementary alliances, following different paths, may be formed to serve a certain city-pair market. So interline service flights in a path can be seen as complementary products, though in a no compatible component's specification. The main results are that alliances hurt rivals and prices of interline fares decrease. Unexpectedly, alliance formation can be unprofitable, specially when the degree of competition is significant and economies of traffic density are low. In another paper, Flores-Fillol and Moner-Colonques (2009) study endogenous mergers in complementary markets with mixed bundling. Here also joint and separate consumption of components is allowed. It is a two stage game and there are three different market structures: independent ownership, single integration and parallel integration. When individual component consumption is allowed, post-merger effect of the merged firm increasing single component prices to make mix-and-match composite goods less attractive to consumers is not so strong. Increases or decreases in prices are intensified depending on the parameter that measures demand asymmetry between composite goods and individual components. To merge is a dominant strategy when competition is soft, remaining independent when competition is strong. They found that incentives to merge are higher if components demands are not so important.

Our contribution to this literature is to consider that any firm producing a component in a composite good industry has potentially open the option to choose a merger with a producer of a substitute component or with a producer of a complementary component. Then, to find which one endogenously arise and its welfare implications. A firm could prefer being independent if the alternative is to merge with a complement producer, but since we consider mergers between substitutes a firm will always prefer to merge with a substitute firm instead of being independent.

In Section 2 we present the general model with n different components for both type

x and type y which defines a system of $n_x \times n_y$ composite goods. Section 3 focuses in a setting with two substitute component firms of each type, all of them fully compatible between each other. In section 4 some variations in the assumptions are studied, no compatibility between components or another criterion to choose the socially optimal merger. Section 5 concludes.

2 The model

Consider a situation where consumers need to combine several components to form a composite good because they only get utility by consuming the composite good, not from the consumption of the single components. For simplicity, we will consider that composite goods are formed by two types of complementary components which are combined in fixed proportions on a one-to-one basis. Component types are denoted by x and y, and every type has n_x and n_y substitutes respectively. Each component substitute of x-type and of y-type is denoted, respectively, by x_i, where $i = 1, .., n_x$, and y_j where $j = 1, .., n_y$. Each of them is initially produced by an independent firm. In this way, we can say that products x_i and y_j are complement components while x_i and x_l are substitute components (as y_j and y_k are). Denote by q_{ij} the composite good ij where $q_{ij} = \min\{x_i, y_j\}$, $i = 1, .., n_x$, $j = 1, .., n_y$. We are considering that all different type components are compatible in the sense that all composite goods are feasible. Thus a market with $n_x n_y$ substitute composite goods is defined. We assume that there is a representative consumer that maximizes the following separable utility function, $V(\mathbf{q}, I) = U(\mathbf{q}) + Z$, where $U(\mathbf{q})$ is the utility obtained by the consumer when she consumes vector $\mathbf{q}^T = (q_{11}, q_{21}, ..., q_{n_x n_y})$.[4] It is assumed to be the following quadratic and strictly concave function: $U(\mathbf{q}) = \boldsymbol{\alpha}^T \mathbf{q} - \frac{1}{2}\mathbf{q}^T \mathbf{A} \mathbf{q}$, with $\boldsymbol{\alpha}^T = (\alpha_{11}, \alpha_{21}, ..., \alpha_{n_x n_y})$ and $\mathbf{A}$ is the following symmetric $(n_x n_y \times n_x n_y)$ matrix

$$\mathbf{A} = \begin{pmatrix} \beta_{11} & \gamma & \cdots & \gamma \\ \gamma & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \gamma & \cdots & \cdots & \beta_{n_x n_y} \end{pmatrix}.$$

Parameter Z is the utility obtained from consumption of the rest of the goods the

[4] Subscript notation should include both elements separated by commas, $q^T = (q_{(1,1)}, q_{(2,1)}, ..., q_{(n_x, n_y)})$, where the first element corresponds to x_i and the second to y_j. However, for simplicity, we reduce this notation skipping over the commas and parenthesis.

consumer buys. Consumer's income is denoted by R, and the budget constraint is $R = Z + \mathbf{p}^T\mathbf{q}$, where $\mathbf{p}^T = (p_{11}, p_{21}, ..., p_{n_x n_y})$, is the price vector and p_{ij} is the price paid by consumers for composite good ij. For simplicity, we will consider the case where $\alpha_{ij} = \alpha$ and $\beta_{ij} = \beta$ for all ij. Maximization of V subject to the budget constraint yields the following system of inverse demand functions $p_{ij} = \alpha - \beta q_{ij} - \gamma \sum_{\forall rs \neq ij} q_{rs}$, $ij, rs = 11, 21, ...n_x n_y$. Inverting the above system and after normalization we obtain the following system of demand functions $q_{ij} = a - p_{ij} + d \sum_{ij \neq rs} p_{rs}$, $ij, rs = 11, 21, ...n_x n_y$. Parameter $a > 0$ is the highest level of demand. Composite goods are imperfect substitutes being d the product differentiation parameter, with $d \in (0, 1)$. If $d = 1$ composite goods are non-differentiated, if $d = 0$ they are independent goods. Note that p_{ij} is either the sum of the prices of its components, $p_{x_i} + p_{y_j}$, or is the price of the bundle if the composite good was marketed in this way. Given the way different components are combined, demand of x_i is the sum of all composite goods demands that include component x_i, that is $\sum_{\forall j} q_{ij}$, and similarly demand of y_j is $\sum_{\forall i} q_{ij}$. Price marginal effects on the composite good demand are $\frac{\partial q_{ij}}{\partial p_{ij}} = -1$, $\frac{\partial q_{ij}}{\partial p_{rs}} = d$, since composite goods are substitutes. Also price marginal effects on the component demands, for x_i and y_j are, respectively:

a) $\frac{\partial \sum_{\forall j} q_{ij}}{\partial p_{x_i}} = -n_y(1-(n_y-1)d)$; $\frac{\partial \sum_{\forall j} q_{ij}}{\partial p_{x_r}} = n_y^2 d; \forall r \neq i$ and $\frac{\partial \sum_{\forall j} q_{ij}}{\partial p_{y_j}} = -(1-(n_x n_y-1)d)$ $\forall j$.

b) $\frac{\partial \sum_{\forall i} q_{ij}}{\partial p_{y_j}} = -n_x(1-(n_x-1)d)$; $\frac{\partial \sum_{\forall i} q_{ij}}{\partial p_{y_s}} = n_x^2 d; \forall s \neq j$ and $\frac{\partial \sum_{\forall i} q_{ij}}{\partial p_{x_i}} = -(1-(n_x n_y-1)d)$ $\forall i$.

Note that own effects are negative, components of the same type are substitutes and components of different type are complements, then condition $d < \frac{1}{n_x n_y - 1}$ must hold. Regarding costs we assume there are constant and common marginal costs denoted by c for producing any component of a composite good, and there are no fixed costs of production. Further, we assume that marginal costs of production do not change when firms merge. All these assumptions regarding costs (no economies of scale and no possibility of fixed costs savings and synergies by merging) imply that we are focusing only on strategic effects derived from merging activities. Firms profits are then defined by $\pi_{x_i} = (p_{x_i} - c) \sum_{j=1}^{n_y} q_{ij}$ for all $i = 1, 2, ..., n_x$ and $\pi_{y_j} = (p_{y_j} - c) \sum_{i=1}^{n_x} q_{ij}$ for all $j = 1, 2, ..., n_y$. Where it is easy to prove that $\frac{\partial^2 \pi_{x_i}}{\partial p_{x_i} \partial p_{x_r}} > 0$ for $r \neq i$ and $\frac{\partial^2 \pi_{x_i}}{\partial p_{x_i} \partial p_{y_j}} < 0$ for all j, then finding that p_{x_i} and p_{x_r} are strategic complements while p_{x_i} and p_{y_j} are strategic substitutes. We are interested first in characterizing the equilibria in this initial situation when all firms produce only one component. First order conditions in this initial case for any firm of type x and y are, respectively: $\frac{\partial \pi_{x_i}}{\partial p_{x_i}} = \sum_{j=1}^{n_y} q_{ij} + (p_{x_i} - c)\frac{\partial \sum_{\forall j} q_{ij}}{\partial p_{x_i}} = 0$ for all $i = 1, 2, ..., n_x$; and $\frac{\partial \pi_{y_j}}{\partial p_{y_j}} = \sum_{i=1}^{n_x} q_{ij} + (p_{y_j} - c)\frac{\partial \sum_{\forall i} q_{ij}}{\partial p_{y_j}} = 0$ for all $j = 1, 2, ..., n_y$. They implicitly define the equilibrium set of initial prices by $p_{x_i}^{IC}$ $\forall i$ $p_{y_j}^{IC}$ $\forall j$.

We are interested in finding the strategic effects of both possible mergers. The merger of two firms producing a component of the same type which we will call a "same type component" merger denoted by superscripts ST, and the merger of two firms producing components of different type, a "different type component" merger, denoted by superscript DT. Both types of mergers are qualitatively different not only by the component combinations considered, but also because the DT merger allows the merged entity to employ more pricing strategies than the ST one. Another difference is related with the way firms control composite good prices. Initially, each isolated $x-$type firm producer *controls partially* the price of n_y composite goods (similarly the y -type one controls n_x composite goods). After a ST merger the new entity *controls partially* the double of composite goods as compared with the initial situation. However, after a DT merger, the new entity fully controls the price of one composite good and controls partially other $n_x + n_y - 2$ composite goods. It is, then, relevant to introduce notation regarding the way composite good prices are set. The fully controlled by the merger will be called the *insider composite good,* those partially controlled will be named *mix-and-match composite goods* (to keep track with the notation introduced by the other authors), and finally those not controlled will be named *outsider composite goods.*

Consider first the ST merger. Suppose a merger between firm producing x_i and that producing x_l, the analysis is the same for a merger of firms producing y-type components. The new entity will select p_{x_i} and p_{x_l} in order to maximize the following profit function: $\pi_{x_i x_l} = (p_{x_i} - c)\sum_{j=1}^{n_y} q_{ij} + (p_{x_l} - c)\sum_{j=1}^{n_y} q_{lj}$. Two first order conditions are easily obtained and read,

$$\frac{\partial \pi_{x_i x_l}}{\partial p_{x_i}} = \sum_{j=1}^{n_y} q_{ij} + (p_{x_i} - c)\frac{\partial \sum_{\forall j} q_{ij}}{\partial p_{x_i}} + (p_{x_l} - c)\frac{\partial \sum_{\forall j} q_{lj}}{\partial p_{x_i}} = 0 \text{ and}$$
$$\frac{\partial \pi_{x_i x_l}}{\partial p_{x_l}} = \sum_{j=1}^{n_y} q_{lj} + (p_{x_l} - c)\frac{\partial \sum_{\forall j} q_{lj}}{\partial p_{x_l}} + (p_{x_i} - c)\frac{\partial \sum_{\forall j} q_{ij}}{\partial p_{x_l}} = 0.$$

Compared with the first order conditions in the initial situation, a new positive term arises: the third term. It incorporates the internalization of competition among both substitute components. It is positive, since components of the same type are substitutes, and therefore implies a shift in the firm's reaction function with respect to the initial situation. Second order conditions for a maximum are satisfied since $\frac{\partial \sum_{\forall j} q_{ij}}{\partial p_{x_i}} < 0$ and $\left|\frac{\partial \sum_{\forall j} q_{ij}}{\partial p_{x_i}}\right| > \left|\frac{\partial \sum_{\forall j} q_{ij}}{\partial p_{x_l}}\right|$, $l \neq i$. The conclusion is that equilibrium prices of the merged firm increase, equilibrium prices of the remaining x-type components increase by strategic complementarity, while equilibrium prices of y-type components decrease by strategic substitutability. Therefore for both mix-and-match and outsider composite goods one component price increases while the other decreases with respect to the initial situation. Further note that since direct effects are greater than indirect ones, there will be an increase in mix-and-match composite good prices, while total effect for outsider composite good prices is ambiguous.

Next, consider a DT merger, say among the firm producing x_i and the one producing y_j. This merger allows the new entity to introduce a new price strategy called mixed bundling. That is, the new entity will set three prices, one price for each component and a price for the composite good which fully controls.[5] If this is the case, it is needed to introduce the demand of bundle q_b, that accounts for sales of both components together in the same package. In line with this, notation for the bundle price p_b is required, to distinguish from the case where consumers buy the two components separately, that is $p_{ij} = p_{x_i} + p_{y_j}$. Finally, noting that the bundle price will be lower at equilibrium than the sum of component prices, $p_b < p_{ij}$, or put it differently, composite good q_{ij} will not be longer demanded.[6] Demand function for the bundle, the insider composite good, reads $q_b = a - p_b + d\sum_{rs \neq ij} p_{rs}$, and demand functions for the other composite goods are reformulated as $q_{rs} = a - p_{rs} + dp_b + d\sum_{\substack{mn \neq rs \\ mn \neq ij}} p_{mn}$, $\forall rs \neq ij$.

Merged firm's profits are $\pi_{x_i y_j} = (p_b - 2c)q_b + (p_{x_i} - c)\sum_{k \neq j}^{n_y} q_{ik} + (p_{y_j} - c)\sum_{l \neq i}^{n_x} q_{lj}$, which include a first term with profits coming from the bundle selling and a second and third terms that account for profits coming from the selling of separate components x_i and y_j. Therefore, three first order conditions need to be analyzed. The first one corresponds to the bundle price,

[5] The other two price strategies we can consider is either pure bundling, which consists of setting only a price for the bundle and not selling the individual components separately; or pure component pricing which means that the new entity sets the prices of the individual components and the price of the bundle is just the sum of its components.

[6] Although it is possible to demand q_{ij}, it is not rational from the consumer's point of view. They will not pay more for the same.

$\frac{\partial \pi_{x_i y_j}}{\partial p_b} = q_b + (p_b - 2c)\frac{\partial q_b}{\partial p_b} + (p_{x_i} - c)\frac{\partial \sum_{k \neq j}^{n_y} q_{ik}}{\partial p_b} + (p_{y_j} - c)\frac{\partial \sum_{l \neq i}^{n_x} q_{lj}}{\partial p_b} = 0$. The bundle price is a new instrument and has an strategic relationship of complementarity with respect to the prices set by individual firms, that is $\frac{\partial \pi^2_{x_i y_j}}{\partial p_b \partial p_{x_l}} = dn_y > 0$ and $\frac{\partial \pi^2_{x_i y_j}}{\partial p_b \partial p_{y_k}} = dn_x > 0$ for $l \neq i$ and $k \neq j$. The other two first order conditions read:

$\frac{\partial \pi_{x_i y_j}}{\partial p_{x_i}} = (p_b - 2c)\frac{\partial q_b}{\partial p_{x_i}} + \sum_{k \neq j}^{n_y} q_{ik} + (p_{x_i} - c)\frac{\partial \sum_{k \neq j}^{n_y} q_{ik}}{\partial p_{x_i}} + (p_{y_j} - c)\frac{\partial \sum_{l \neq i}^{n_x} q_{lj}}{\partial p_{x_i}} = 0$ and

$\frac{\partial \pi_{x_i y_j}}{\partial p_{y_j}} = (p_b - 2c)\frac{\partial q_b}{\partial p_{y_j}} + (p_{x_i} - c)\frac{\partial \sum_{k \neq j}^{n_y} q_{ik}}{\partial p_{y_j}} + \sum_{l \neq i}^{n_x} q_{lj} + (p_{y_j} - c)\frac{\partial \sum_{l \neq i}^{n_x} q_{lj}}{\partial p_{y_j}} = 0.$

From the analysis of the above conditions and their comparison with the initial situation ones we stay the following two results which are proven in the Appendix:

Lemma 1: *Compared with the corresponding prices in the initial situation, the merged firm resulting from two different type component producers will increase component prices* p_{x_i} *and* p_{y_j}.

The reason is that maintaining individual component prices at the initial situation while setting optimally p_b results in positive marginal profits (i.e. $\frac{\partial \pi_{x_i y_j}}{\partial p_{x_i}} > 0$ and $\frac{\partial \pi_{x_i y_j}}{\partial p_{y_j}} > 0$). Therefore, the merged firm has an incentive to increase both individual component prices. The intuition is clear, by dealing with three instruments the new entity is able to set the price for the bundle high enough, as compared with the pure bundling situation (i.e. when only one instrument is at hand), since it is setting higher individual component prices in an strategic way to make less attractive for consumers mix-and-match composite goods and, therefore, diverting demand to its bundle.

Considering outsiders' reaction, we know that their price is an strategic complement of all same type components and an strategic substitute of all different type ones. We also know that both individual component prices of the merged firm increase implying, initially, an ambiguous effect. However, the next result applies.

Lemma 2: *Compared with the corresponding prices in the initial situation, outsiders to a different type merger will decrease prices at equilibrium.*

As before, keeping prices at the initial level implies that marginal profits are negative thus meaning that outsiders will reduce prices to converge to the equilibrium. The intuition is clear, outsiders sell components for both mix-and-match and outsider composite goods and it is optimal for them to compensate the increase of component prices controlled by the merged entity in mix-and-match composite goods. The conclusion is that outsider composite goods reduce their price with respect to the initial situation, while for the mix-and-mach ones the total effect is initially not clear. However and considering that direct effects, mix-and match composite goods should increase.

3 Model with two *x-type* and two *y-type* components

We have just introduced a general specification of a model with composite good competition where components are substitutes within its type and complements across types. We have assumed that any component of one type is fully compatible with any other component of the different type to form a composite good, it is a compatibility setting. Now, we will focus on a simpler industry, where two firms are producing x-type components and the other two are producing y-type components, the two by two case, $n_x = n_y = n = 2$.[7] This approach will let us check the general model statements and Lemmas and find which of the alternative mergers are chosen by firms and which are welfare improving.

We define q_{ij} as the demand for the composite good created by x_i and y_j. Four composite goods are in the market, $q_{ij} = \min\{x_i, y_j\}, \forall ij = 11, 12, 21, 22$ with the following system of demand functions: $q_{ij} = a - p_{ij} + d\sum_{\forall rs \neq ij} p_{rs}, \forall ij = 11, 12, 21, 22$. Demands for the components can be obtained from composite good's demands. As an example, x_i is included in composite goods $i1$ and $i2$, so demand for x_i is $x_i = q_{i1} + q_{i2}$. Firms profits are $\pi_{x_i} = (p_{x_i} - c)(q_{i1} + q_{i2})$ $i = 1, 2$ and $\pi_{y_j} = (p_{y_j} - c)(q_{1j} + q_{2j})$, $j = 1, 2$.

Initial situation: independent ownership

There are four independent component producers. The composite good price is the sum of its component's prices $p_{ij} = p_{x_i} + p_{y_j}$. Per-unit margins of every component in the market are symmetric, as well as composite goods outputs and firms profits. The initial situation equilibrium (margins, outputs and profits) for the compatibility case, denoted by IC reads, $p_{x_i}^{IC} - c = p_{y_i}^{IC} - c = \frac{A}{3-7d}$; $\forall i$ and $q_{ij}^{IC} = \frac{(1-d)A}{3-7d}; \forall ij$ where $A = (a - 2(1-3d)c)$. Finally, $\pi_{x_i}^{IC} = \pi_{y_i}^{IC} = \frac{2(1-d)A^2}{(3-7d)^2}; i = 1, 2$. Quantities are nonnegative if $A \geq 0$, or for $a \geq 2(1-3d)c$, as the denominator is always positive since the second order condition imposes $d < \frac{1}{3}$.

Mergers between same type component producers: the ST merger.

We analyze the merger between $x-$type component producers, the x_1x_2 merger, but the same analysis applies for the y_1y_2 merger. The new firm is now the only producer of

[7] Expressions for the general case, (margins, outpus and profits) are available from the authors upon request and will be used for the simulations at the end of this section.

one type of component in the market. Equilibrium margins and outputs are the following:

$$\begin{array}{l} p_{x_1}^{ST} - c = p_{x_2}^{ST} - c = \frac{(1-d)A}{(1-3d)(3-5d)} \\ p_{y_1}^{ST} - c = p_{y_2}^{ST} - c = \frac{A}{(3-5d)} \end{array} \qquad q_{ij}^{ST} = \frac{(1-d)A}{3-5d};\ \forall ij.$$

Merged firm components have a higher price than outsiders' components. As expected from the general model analysis, the new firm's price is higher while outsiders' price is lower after the merger. It seems natural that outputs for all composite goods are set at the same level due to the symmetry, since the merged firm produces components for every composite good in the market. Since the increase in the $x-$ type component price is greater than the decrease in the $y-$ type one, mix-and match composite good prices rise, while outputs decrease after the merger. Thus consumers are worse off. Firms profits are:

$$\pi_{x_1x_2}^{ST} = \frac{4(1-d)^2A^2}{(1-3d)(3-5d)^2}; \quad \pi_{y_1}^{ST} = \pi_{y_2}^{ST} = \frac{2(1-d)A^2}{(3-5d)^2}.$$

Several features are worth mentioning.[8]

- *The merged firm prefers selling both components.* As the merged firm is now producing two similar components it is important to find whether the new firm would prefer selling both of them or delist one instead. We prove that producing only one component is not an equilibrium choice, the new firm prefers bringing both components to the market. The reason behind that behaviour is that it is better not to reduce output supplied in the market. Staying with only one component reduces composite goods competition, but it is preferable to have more output in the market maintaining more combinations of composite goods: the expansion output effect dominates in terms of profits.

- *The merger is profitable,* $\pi_{x_1x_2}^{ST} > \pi_{x_1}^{IC} + \pi_{x_2}^{IC}$. In the independent ownership situation, x_1 and x_2 were substitutes and there was competition between them. However, the merger lets the new firm internalize the previous negative externality each single firm was imposing in each other. The new firm is monopolizing this type of component in the market.

- *Outsiders are worse off after the merger,* $\pi_{y_i}^{IC} > \pi_{y_1}^{ST} = \pi_{y_2}^{ST}$. The reason is that now outsiders produce complements to the components produced by the merged firm. This entails strategic complementarity that implies also a reduction in outsiders' output.[9]

[8] All the results are proven in the Appendix.

[9] As a consecuence of the first two statements, the new firm achieves higher profits than outsiders in this market.

To end this section note that the two by two case might be somehow particular since it does not allow for outsider composite goods after a merger. The three by three case has been computed and we find that the price increase in the x-type components that are merged is greater than the increase on the outsider producers of x-type components. Then mix-and-match composite goods prices are higher compared with the initial case and are also higher than those of outsider composite goods. The latter decrease their price with respect to the initial situation as long as $\frac{1}{17} < d < \frac{1}{8}$. The merger is profitable and outsiders that produce complement components are worse off, while those producing substitute components are better off.

Mergers between different type component producers: the DT merger.

This merger is between two different type component producers, say the x_1y_1 merger, but the same analysis applies for x_1y_2, x_2y_2 or x_2y_1 mergers. As we know from the analysis above, the merged firm has three available pricing strategies. The simplest option is charging a price for each single component, *pure component pricing* (PC), the only possible strategy in the ST merger. As components produced by the new entity constitute a composite good, it is possible to sell them together in a bundle and not selling them independently, *pure bundling strategy* (PB). The third option is the combination of the previous ones, which allows selling the bundle and also both single components, *mixed bundling strategy* (MB).

Pure component pricing strategy implies the same system of demands as in the initial situation. If we are considering pure bundling we only take into account the bundle, q_b, where $q_b = q_{11} = \min\{x_1, y_1\}$ and q_{22}, since mix-and-match composite goods 12 and 21 are not longer available for consumers. Noting that p_b is the bundle price and $p_{22} = p_{x_2} + p_{y_2}$ the demand system becomes: $q_b = a - p_b + dp_{22}$ and $q_{22} = a - p_{22} + dp_b$.[10] When mixed bundling is the chosen pricing strategy, we reach a demand system with a bundle and three composite goods q_{22}, q_{12} and q_{21}, and thus demand for q_b is:

$$q_b = a - p_b + d((p_{x_1} + p_{y_2}) + (p_{x_2} + p_{y_1}) + (p_{x_2} + p_{y_2}))$$

In the case of mixed bundling, the composite good q_{11} is feasible but not longer demanded by consumers, as we already know q_b offers the same components with a discount on

[10] Note that pure bundling entails the same composite good variety as had we assumed no compatibility between components.

price.[11] Profits for all possible pricing strategies are, where $A' = (1 - 2c(1-d)) > A$:

	PC	PB	MB
$\pi^{DT}_{x_1 y_1}$	$\frac{128(3-5d)A^2}{(29-78d+21d^2)^2}$	$\frac{(3+2d)^2 A'^2}{4(3-d^2)^2}$	$\frac{(17-38d+9d^2)A^2}{4(3-9d+4d^2)^2}$
$\pi^{DT}_{x_2} = \pi^{DT}_{y_2}$	$\frac{8(1-d)(5-3d)^2 A^2}{(29-78d+21d^2)^2}$	$\frac{(2+d)^2 A'^2}{4(3-d^2)^2}$	$\frac{2(1-d)^3 A^2}{(3-9d+4d^2)^2}$

In view of the above we prove that *Mixed Bundling is the pricing option chosen by the merged firm.* The ranking is unambiguous: MB profits are higher than PC profits, and these are higher than profits in PB. Thus, throughout the remainder of the paper, we will consider that the mixed bundling strategy is at place for the DT merger.[12] Equilibrium prices under mixed bundling are:

$$p^{DT}_b - 2c = \frac{(3-d)A}{2(3-9d+4d^2)}, \quad p^{DT}_{x_1} - c = p^{DT}_{y_1} - c = \frac{A}{3-9d+4d^2}, \quad \text{and } p^{DT}_{x_2} - c = p^{DT}_{y_2} - c = \frac{(1-d)A}{3-9d+4d^2}.$$

All prices of the components produced by outsiders decrease, while prices of the components produced by the new entity are higher as compared with the initial situation, as indicated in Lemmas 1 and 2. Also, $(p^{DT}_{x_1} - c) + (p^{DT}_{y_1} - c) > p^{DT}_b - 2c$, so as already indicated, q_{11} is no longer demanded by consumers. The idea that mergers between complements lead to lower prices is not completely true. The bundle price $p^{DT}_b - 2c$ is lower than a composite good price in the situation prior to the merger, moreover, outsider's composite good prices are also lower than before. However, mix-and-match composite goods increase their price. This happens because the merged firm increases its single components price to benefit its insider composite good demand in detrimental to mix-and-match system's demands. If we compare prices of composite goods in the market after the merger we find that the bundle price is always the lowest one: $(p^{DT}_{12} - 2c) = (p^{DT}_{21} - 2c) > (p^{IC}_{ij} - 2c) > (p^{DT}_{22} - 2c) > (p^{DT}_b - 2c)$. Therefore, output levels are:

$$q^{DT}_b = \frac{(3-5d)A}{2(3-9d+4d^2)} > q^{DT}_{22} = \frac{(2-3d+3d^2)A}{2(3-9d+4d^2)} > q^{DT}_{12} = q^{DT}_{21} = \frac{(2-5d+d^2)A}{2(3-9d+4d^2)}$$

The following features are worth to mention:

- *The merger is profitable,* $\pi^{DT}_{x_1 y_1} > \pi^{IC}_{x_1} + \pi^{IC}_{y_1}$. This merger lets the new firm use a mixed bundling strategy, which gives it the possibility of discriminate prices in a way that the insider composite good is favored.

[11] As in Tirole (2005): "buying the bundle is really the only feasible option if the prices of the individual products are high".

[12] The main variables of the other pricing strategies are in the Appendix.

- *Outsiders are worse off after the merger,* $\pi_{x_2}^{IC} = \pi_{y_2}^{IC} > \pi_{x_2}^{DT} = \pi_{y_2}^{DT}$. Mixed bundling imposes a negative externality on outsiders of either type, as they are compelled to reduce margins without getting a higher market share.

Equilibrium merger

We have just checked two alternative mergers, both profitable. The difference between DT and ST merger is that in the first merger the new firm fully controls one composite good price and discriminate prices, while in the last one the new firm cannot but monopolizes one component type. Which one will be chosen is the content of the next proposition.

Proposition 1. *The equilibrium merger that will arise in this market depends on the differentiation parameter between composite goods as follows. The DT merger will be the equilibrium one if* $0 < d < 0.0804$, *while it is the ST merger for* $0.0804 < d < \frac{1}{3}$.

There is a little range for the differentiation parameter for which a merger between complements is privately preferable to a merger between substitutes. If d is close to zero, composite goods are almost independent, so it is preferable for the merged firm to have a monopoly in one composite good. In this way it benefits from the so called "Cournot effect", the reduction in prices for two complements when they are sold by the same firm rather than by separate monopolists. Integration leads to a reduction in both complements prices since the integrated firm captures the increase in demand for a good's complement when it lowers the price of the other good. On the contrary, as d increases composite goods are less differentiated, and a more intense competition appears. In that case, the merged firm prefers to internalize the competition by monopolizing one component type so as it controls one component in every composite good in the market and therefore increasing its market power. It is also worth noting that *outsiders to the DT merger get higher profits than the outsiders to the ST merger.*

We can solve the model for any number of component producers finding that the result in Proposition 1 can be replicated for a different number of them. However, there is a given number of components for each type, which is $n_x = n_y = n = 11$ that imply that for $n \geqslant 11$ the DT merger is the equilibrium one for the whole range for d. In the next Table we show the interval for d such that a merger among complementary components is chosen for strategic reasons. However, as long as the number of component producers increase the assumption of strategic behavior among firms is less plausible.

	$n=3$	$n=5$	$n=10$	$n=25$
range for d	$d<\frac{1}{8}$	$d<\frac{1}{24}$	$d<\frac{1}{99}$	$d<\frac{1}{624}$
$\pi^{DT}_{x_1y_1} > \pi^{ST}_{x_1x_2}$	$d<0.0520$	$d<0.0274$	$d<0.0097$	$d<0.0020$

After a DT merger, the merged firm controls one composite good while after a ST merger, the merged firm reduces competition in its component. If we have two x-type and two y-type producers ($n=2$), the difference between DT or ST merger is controlling one composite good or creating a monopoly in one type of component. However, as n increases, it is more important to achieve the control of one composite good (through a DT merger) than reducing competition in a small percentage in your component (through a ST merger). In fact, for $n \geq 11$, the DT merger is the only equilibrium option for the whole range for d. Thus, the advantages of a DT merger depend less on the number of component producers than the advantages of a ST merger.

We have also computed firms' incentives to form either type of merger when the number of firms producing each type of components is different, the conclusion is that only ST mergers will occur in the component type with less substitutes, while only DT mergers will occur if we consider the component type with more substitutes. In the initial situation all firms are symmetric, every firm produces only one component. So each firm has a similar part from the sales of its type of component. After a two-firm merger, the merged firm will have one part from the sales of every component it is producing. So it is logical that, if the firm proposing the merger is from the type with fewer substitutes, it will prefer to join a substitute component producer, since its part in the total sales of its component is greater. This will entail a ST merger. If the firm proposing the merger is from the type with more substitutes, it will prefer to join a firm producing a component with fewer substitutes, because its part in total sales of the component is greater. That will imply a DT merger.

Socially optimal merger

Firms could have strong incentives to merge, however, antitrust authorities have much to say in this topic. Obviously, the proposed merger have to be profitable for the firms, but once they have decided to be involved in that merger, the specific case must be approved by antitrust authorities. They decide based on the impact of the merger in Consumer Surplus *(CS)*[13].

[13] Antitrust authorities have looked into the CS rule or the Social Welfare (SW) rule to decide about

Proposition 2. *The highest level of Consumer Surplus appears in the DT merger. The following ranking applies for all the range for the differentiation parameter* $CS^{DT} > CS^{IC} > CS^{ST}$.

In the ST merger the new firm is the only producer of x-type components. Consumers are worse off because they have to pay a higher price for this type of component. The same will happen in the case where the merged firm is partially monopolizing the x -type component, as the same-type outsiders will react increasing prices. Although y-type producers have decreased their price, the total effect is that composite good prices are higher and outputs lower in comparison to the initial situation. On the contrary, the DT merger makes consumers better off. The new firm sets a bundle price lower than the composite good price prior to the merger. In addition, outsider composite good prices are also lower than before. And although mix-and-match composite goods have increased their price, the overall effect is positive. Therefore, the conclusion is that DT mergers will always be allowed, while ST mergers forbidden by antitrust authorities that decide based on the CS rule.

Remind that all the analysis is focused on the profitability of each merger type based on strategic effects, then it is convenient to identify the situations where the firms propose a type of merger that does not correspond with the equilibrium one as the cases where the productive efficiencies obtained by the merger are substantial to make this merger type profitable. For example if for $d = 1/4$ two firms propose a DT merger instead of a ST one is an indication that they obtain greater productive efficiencies with a DT merger.

Continuation merger

We are interested in analyzing first, the equilibrium response to a given merger by outsiders, and secondly, how antitrust authorities will modify their decisions if we allow them to behave in a far-sighted way. That is, they are able to anticipate the equilibrium reply to a merger from the firms in the market. After a merger, there are three firms in the market, one which is the result of a previous merger and two outsiders. Now we want to analyze if it is more profitable for an outsider to create an *Extended Merger,* i. e. join the two-firm merger and leave out the other outsider, or create an outsiders' *Countermerger,* i. e. two merged entities formed from two initially independent firms each.

mergers. We have shown results under the CS rule since, apparently, nowadays antitrust authorities base their decisions on this measure more frequently than on the other one.

Without loss of generality, we arbitrarily choose one of the outsiders to be the firm which will decide whom to merge with, say the firm producing y_2.[14] Since we have considered initially two different types of mergers, the DT and the ST merger, we will reach four continuation mergers, but only three qualitatively different. The symmetric Extended Mergers derived from each initial merger type, denoted by superscript EM, and two different countermergers derived from each initial merger type, denoted respectively by superscripts DTC and STC. Noting that when each countermerger takes place, a market structure with two symmetric firms is obtained. Profits are as follows:

Extended Merger	$\pi^{EM}_{x_1y_1y_2} = \frac{(13-3d)A^2}{18(1-d)(1-3d)}$; $\pi^{EM}_{x_2} = \frac{2A^2}{9(1-d)}$
DT Countermerger	$\pi^{DTC}_{x_1y_1} = \pi^{DTC}_{x_2y_2} = \frac{(17-32d)A^2}{9(2-5d)^2}$
ST Countermerger	$\pi^{STC}_{x_1x_2} = \pi^{STC}_{y_1y_2} = \frac{4A^2}{9(1-3d)}$

The DTC defines a market structure with bundle competition between rival composite goods. Once again, if a given firm fully controls a composite good pricing, it can implement mixed bundling. In order to know which pricing strategy will arise in the market, we define a game in which both merged firms decide simultaneously between pure component pricing or mixed bundling (see the Appendix: Additional information). The conclusion is that mixed bundling is a dominant strategy and the outcome (mixed bundling, mixed bundling) will be the Nash Equilibrium of this game.[15] Alternatively, the STC defines a market structure with component competition. See the Figure 1 below where the solid lines correspond to the equilibrium action in the specified range of values for d. The thicker lines represent an action that is the only equilibrium option. The dotted lines represent an action that will never arise in equilibrium.

First of all note that all of them are always profitable as proven in the Appendix, then there will always be incentives for a continuation merger. The outcome of this continuation merger is in the next Proposition.

[14] Note that if the other outsider decides instead, the same cases are reached, a symmetric extended and a countermerger merger.

[15] This result was already established in Economides (Discussion Paper EC-93-29, New York University, 1993). For a detailed price comparison between both strategies of the game see his paper where mixed bundling is the dominant strategy and the same NE is obtained. Further, if composite goods are not very close substitutes, firms would be better off if mixed bundling was not available.

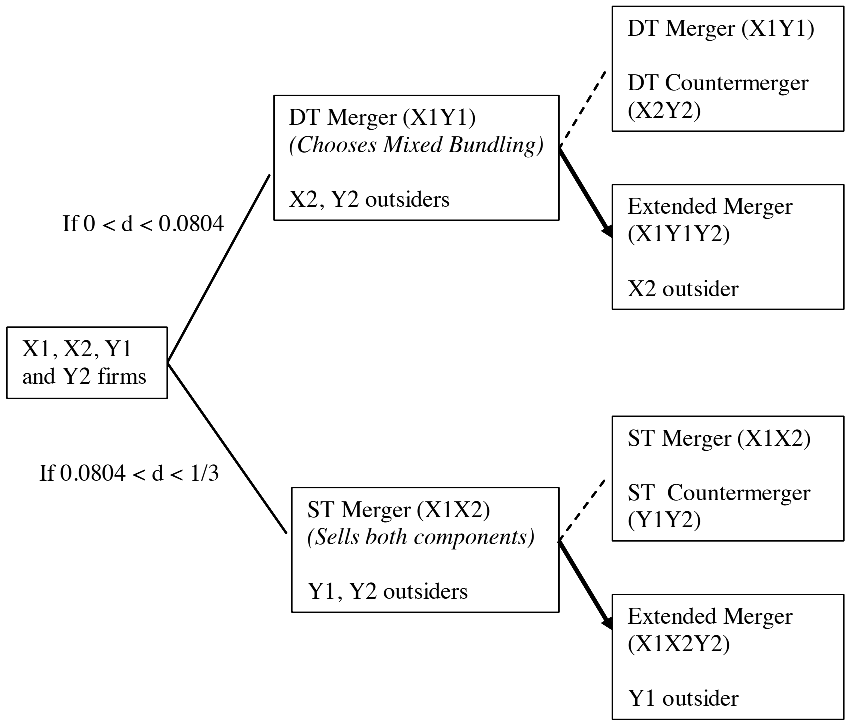

Figure 1: Continuation merger. The compatibility case.

Proposition 3. *Consider,* $0 < d < 0.0804$, *when a DT merger happened. Then any outsider to a DT merger prefers to join the previous merger and become one of the participants in the* EM *rather than creating a DTC* $(\frac{1}{3}\pi^{EM}_{x_1y_1y_2} > \frac{1}{2}\pi^{DTC}_{x_2y_2})$. *This equilibrium merger is also approved by antitrust authorities since CS is greater* $(CS^{EM} > CS^{DT})$.

Private incentives to merge are in the same line as social incentives, antitrust authorities will allow the EM. Also as compared with the initial situation, $CS^{EM} > CS^{DT} > CS^{IC}$ for the range of d under consideration, then we can conclude that the full equilibrium merger path is welfare improving.

Proposition 4. *Consider,* $0.0804 < d < \frac{1}{3}$, *when a ST merger happened, supposing antitrust authorities would have allowed it. Any outsider to a ST merger would prefer to follow the previous merger and become one of the participants in the* EM *rather than creating a STC* $(\frac{1}{3}\pi^{EM}_{x_1x_2y_2} > \frac{1}{2}\pi^{STC}_{y_1y_2})$. *This equilibrium merger would be also approved by antitrust authorities since CS is greater* $(CS^{EM} > CS^{ST})$.

But we already know that when a ST merger is privately profitable it will be forbidden by antitrust authorities under the CS rule. In case this ST merger had been allowed anyway, an EM would improve the situation. This analysis gives us an intuition of which will be the merging path followed by the firms in this industry. Obviously, the last step will be a monopoly, all firms working together, as one can expect from an industry with complementary components. Although a monopoly would be desirable for firms, we wonder whether the society would prefer it or not. CS in monopoly is higher than in EM for $0 < d < 0.2705$, and moreover, it is higher than in the initial situation for $0 < d < 0.2$. [16]So despite being surprising, antitrust authorities should approve a monopoly of all complement and substitute component producers, if composite goods are very differentiated.

What will have happened if antitrust authorities were far-sighted? That is, if they were able to anticipate the whole equilibrium merger path. It is proven in the Appendix that the EM improves upon the initial situation in terms of CS, $CS^{EM} > CS^{IC}$, if $0 < d < 0.1776$. There is then an interesting conclusion: *it is possible that antitrust authorities do not permit a ST merger, although this will be trigger a welfare enhancing merger in the next step of the equilibrium merger path*. For example, for an antitrust authority using the CS decision rule, it happens that for $0.0804 < d < 0.1776$ an equilibrium ST merger will not be permitted, then stopping the welfare improvement derived by the equilibrium EM that will occur had they considered the full equilibrium merger path.

4 Assumptions variations

CS vs SW

So far we had assumed that antitrust authorities follow the CS rule to decide about mergers. In fact, this measure takes into account how consumers' welfare change due to the merger proposed. However, Social Welfare (SW) has been another traditional way to decide if a merger must be allowed or not. It includes CS and also profits of every firm in the market.

We want to check if results would change in Proposition 2 if we assume antitrust authorities use SW instead of CS. The merger that attains the highest level of SW varies as d increases. First is the DT, for $0 < d < 0.1992$, then is preferable not to allow any merger, for $0.1992 < d < 0.2708$, and, finally, the most desirable merger is the

[16]Expressions for variables in monopoly case are in the Appendix, Additional information. Comparisons are in the Appendix, Proofs of 2 by 2 model.

ST one, for $0.2708 < d < \frac{1}{3}$. Interestingly enough, when the DT merger privately occurs, $0 < d < 0.0804$, antitrust authorities using whatsoever rule will undoubtedly allow it, it benefits consumers and firms as well. Private and social incentives go in the same direction. On the contrary, authorities' decision when the ST is the one proposed will be slightly different. We know if they consider CS as decision rule, this merger will never take place. However, if they are looking at SW they will allow the merger only when $0.2708 < d < \frac{1}{3}$. If authorities allow it they will be improving the situation for firms, but not for consumers, based on a transfer from consumers to firm's owners.

Compatibility vs No Compatibility

To see if Compatibility is a key element determining the results we have checked the No Compatibility case. We consider that every component of one type in the market is compatible with only one component of a different type to form a composite good, so only two different combinations are possible, q_{11} and q_{22}.[17] Despite components are not all compatible each other, merging activity is not restricted. There are two alternative mergers: the *compatible component* merger (CC merger) and the *no compatible component* merger (NC merger). The first one involves one firm merging with the firm that produces the corresponding compatible component to its own component. The second one involves this firm merging with any other firm. So complementarity or substitutability among components are not the relevant, compatibility is the important feature since only merging to a compatible component allows to fully control a composite good.

There are some particular features about the CC merger. The new entity has three available pricing strategies, but pure and mixed bundling are equivalent, since mix-and-match composite goods cannot be created. Surprisingly, they are equivalent to pure component pricing as well, since they result in the same composite good prices at equilibrium. It does not discriminate prices offering a discount in the bundle price, p_b exactly matches the sum of the prices for x_1 and y_1. When demand diversion is not possible the strategic incentive to use mixed bundling disappears.[18] This merger is profitable only for

[17] As we already mentioned, this No Compatibility case with two composite goods is a similar demand system as in the pure bundling strategy considered in the DT merger of the Compatibility case. However, pure bundling is never chosen, then it is clear that the merged firm will never prefer to make no compatible its components.

[18] All calculations and proves about the No Compatibility case are in a working paper version of this article.

$0 < d < 0.6628$. As in Flores-Fillol and Moner-Colonques (2007), in a no compatible setting, a merger between complements is unprofitable when competition is high (d is close to 1). Similarly, in Beggs (1994), where mergers are only allowed between complements of the same group, equivalent to a no compatibility assumption. On the contrary, the NC merger is always profitable, $\forall d$. The equilibrium merger follow a similar pattern than the one in the compatibility case. It depends on d, being the CC merger for $0 < d < 0.2611$, and the NC merger for $0.2611 < d < 1$. The socially optimal merger is the CC merger, it is the only one which would be approved by antitrust authorities, since it is desirable for both consumers and society. The ranking of CS is $CS^{CC} > CS^{INC} > CS^{NC}$ regardless of d.

5 Conclusions

We focus on markets characterized by the relationships between two different type of components which define composite goods. We are interested in the incentives and consequences of different types of mergers among firms producing composite goods components. If there is an asymmetric number of producers of each type, a shortage condition makes firms merge between producers of the limited type. We use a symmetric market structure in both type of component producers to study firm's strategic motives to merge. Departing for the received literature, this paper has shown that a complement merger is not the only equilibrium option to be considered. In fact, either a complement component merger or a substitute component merger can be equilibrium outcome depending on the differentiation degree in the composite good market. Consumers always prefer a complement merger, but the merger that attains the highest level of social welfare changes as the differentiation parameter varies.

Different price strategies help us to understand the strategic considerations the merged firm takes into account. Pure component pricing does not let the merged firm to exploit all the merger potential. Pure bundling converts a compatible setting in a no compatible one and although it would be the worst for outsiders profits it is not the equilibrium choice of the merged firm. Mixed bundling leaves open the possibility of discriminate prices with a lower bundle price but it also keeps variety for different consumer preferences. This strategy lets the merged firm to earn the highest profits, and outsiders are better off than if pure bundling was implemented.

In the continuation merger analysis, it is proven that there is always a kind of countermerger that can be privately profitable after a previous merger. Since the privately preferred continuation merger is also socially desirable, a conflict appears in the case of a ST merger. If authorities are far-sighted, a potentially harmful merger could be desirable if the next step is improving today's welfare.

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7 Appendix

7.1 Additional information

DT merger

•Equilibrium prices and outputs for the different strategies:

	PC	PB	MB
$p_b^{DT} - 2c = q_b^{DT}$		$\frac{(3+2d)A'}{2(3-d^2)}$	$\frac{(3-d)A}{2(3-9d+4d^2)}$
$p_{x_1}^{DT} - c = p_{y_1}^{DT} - c$	$\frac{8A}{29-78d+21d^2}$		$\frac{A}{3-9d+4d^2}$
q_{11}^{DT}	$\frac{(13-d(22+3d))A}{29-78d+21d^2}$		
$q_{12}^{DT} = q_{21}^{DT}$	$\frac{(11-d(18-3d))A}{29-78d+21d^2}$		$\frac{(2-5d+d^2)A}{2(3-9d+4d^2)}$
$p_{x_2}^{DT} - c = p_{y_2}^{DT} - c$	$\frac{2(5-3d)A}{29-78d+21d^2}$	$\frac{(2+d)A'}{2(3-d^2)}$	$\frac{(1-d)A}{3-9d+4d^2}$
q_{22}^{DT}	$\frac{(9-d(14-9d))A}{29-78d+21d^2}$	$\frac{(2+d)A'}{2(3-d^2)}$	$\frac{(2-3d+3d^2)A}{2(3-9d+4d^2)}$

where $A = (a - 2(1 - 3d)c)$and $A' = (a - 2(1 - d)c)$.

Socially optimal merger

•CS and SW values for the different mergers:

	IC	ST	DT
CS	$\frac{2(1-d)^2A^2}{(3-7d)^2}$	$\frac{2(1-d)^2A^2}{(3-5d)^2}$	$\frac{(21-82d+104d^2-38d^3+11d^4)A^2}{8(3-9d+4d^2)^2}$
SW	$\frac{2(5-d)(1-d)A^2}{(3-7d)^2}$	$\frac{2(1-d)(5+3d^2-12d)A^2}{(3-5d)^2(1-3d)}$	$\frac{(87-254d+218d^2-70d^3+11d^4)A^2}{8(3-9d+4d^2)^2}$

Continuation merger

• Payoff Matrix for the Pricing Game, all expressions inside the cells multiplied by A^2.

		x_2y_2: PC	x_2y_2: MB
x_1y_1	PC	$\left(\begin{array}{l} \pi_{x_1y_1}^{PC-PC} = \frac{8(3-5d)}{(7-17d)^2} \\ \pi_{x_2y_2}^{PC-PC} = \frac{8(3-5d)}{(7-17d)^2} \end{array}\right)$	$\left(\begin{array}{l} \pi_{x_1y_1}^{PC-MB} = \frac{18(1-d)^2(3-5d)}{(11-37d+24d^2)^2} \\ \pi_{x_2y_2}^{PC-MB} = \frac{3(83-290d+299d^2-96d^3)}{4(11-37d+24d^2)^2} \end{array}\right)$
	MB	$\left(\begin{array}{l} \pi_{x_1y_1}^{MB-PC} = \frac{3(83-290d+299d^2-96d^3)}{4(11-37d+24d^2)^2} \\ \pi_{x_2y_2}^{MB-PC} = \frac{18(1-d)^2(3-5d)}{(11-37d+24d^2)^2} \end{array}\right)$	$\left(\begin{array}{l} \pi_{x_1y_1}^{MB-MB} = \frac{(17-32d)}{9(2-5d)^2} \\ \pi_{x_2y_2}^{MB-MB} = \frac{(17-32d)}{9(2-5d)^2} \end{array}\right)$

• CS and SW values for the different continuation mergers:

	EM	DTC	STC
CS	$\frac{(13-14d+5d^2)A^2}{36(1-d)^2}$	$\frac{(13-44d+41d^2)A^2}{9(2-5d)^2}$	$\frac{2A^2}{9}$
SW	$\frac{(47-117d+77d^2-15d^3)A^2}{36(1-d)^2(1-3d)}$	$\frac{(47-108d+41d^2)A^2}{9(2-5d)^2}$	$\frac{2(5-3d)A^2}{9(1-3d)}$

• Monopoly variables.

Prices and quantities for different composite goods, $\forall i, j = 1, 2; i \neq j$. Industry profits, CS and SW:

$p_{bi}^M - 2c = (p_{x_i}^M + p_{y_j}^M) - 2c$	$\frac{A}{2(1-3d)}$
$q_{bi}^M = q_{ij}^M$	$\frac{A}{2}$
π^M	$\frac{A^2}{(1-3d)}$
CS^M	$\frac{A^2}{2}$
SW^M	$\frac{3(1-d)A^2}{2(1-3d)}$

7.2 Proofs of n_x *by* n_y model

Proofs of Lemmas 1 and 2: Remind that the first order conditions in the initial case for any symmetric firm of type x and of type y, $\frac{\partial \pi_{x_i}}{\partial p_{x_i}} = \sum_{j=1}^{n_y} q_{ij} + (p_{x_i} - c)\frac{\partial \sum_{\forall j} q_{ij}}{\partial p_{x_i}} = 0$ for all $i = 1, 2, ..., n_x$; and $\frac{\partial \pi_{y_j}}{\partial p_{y_j}} = \sum_{i=1}^{n_x} q_{ij} + (p_{y_j} - c)\frac{\partial \sum_{\forall i} q_{ij}}{\partial p_{y_j}} = 0$ for all $j = 1, 2, ..., n_y$, define the equilibrium set of initial prices $p_{x_i}^0$ $\forall i$ $p_{y_j}^0$ $\forall j$. Also note that $\frac{\partial \sum_{\forall j} q_{ij}}{\partial p_{x_i}} = -n_y(1 - (n_y - 1)d)$ $\forall i$ and that $\frac{\partial \sum_{\forall i} q_{ij}}{\partial p_{y_j}} = -n_x(1 - (n_x - 1)d)$ $\forall j$.

- Consider first Lemma 1: as firms producing x_i and y_j merge, they have the following profits: $\pi_{x_i y_j} = (p_b - 2c)q_b + (p_{x_i} - c)\sum_{k \neq j}^{n_y} q_{ik} + (p_{y_j} - c)\sum_{l \neq i}^{n_x} q_{lj}$. Since this new firm has introduced mixed bundling then demand for the bundle is defined by $q_b = a - p_b + d\sum_{ij \neq rs} p_{rs}$, and demands for composite goods are now $q_{rs} = a - p_{rs} + dp_b + d\sum_{\substack{mn \neq rs \\ mn \neq ij}} p_{mn}$, $\forall rs \neq ij$.The marginal profits with respect to p_b, p_{x_i} and p_{y_j} are:

(a) $\frac{\partial \pi_{x_i y_j}}{\partial p_b} = q_b + (p_b - 2c)\frac{\partial q_b}{\partial p_b} + (p_{x_i} - c)\frac{\partial \sum_{k \neq j}^{n_y} q_{ik}}{\partial p_b} + (p_{y_j} - c)\frac{\partial \sum_{l \neq i}^{n_x} q_{lj}}{\partial p_b} = 0$, where $\frac{\partial q_b}{\partial p_b} = -1$, $\frac{\partial \sum_{k \neq j}^{n_y} q_{ik}}{\partial p_b} = d(n_y - 1)$, $\frac{\partial \sum_{l \neq i}^{n_x} q_{lj}}{\partial p_b} = d(n_x - 1)$; also

(b) $\frac{\partial \pi_{x_i y_j}}{\partial p_{x_i}} = (p_b - 2c)\frac{\partial q_b}{\partial p_{x_i}} + \sum_{k \neq j}^{n_y} q_{ik} + (p_{x_i} - c)\frac{\partial \sum_{k \neq j}^{n_y} q_{ik}}{\partial p_{x_i}} + (p_{y_j} - c)\frac{\partial \sum_{l \neq i}^{n_x} q_{lj}}{\partial p_{x_i}} = 0$ where $\frac{\partial q_b}{\partial p_{x_i}} = d(n_y - 1)$, $\frac{\partial \sum_{k \neq j}^{n_y} q_{ik}}{\partial p_{x_i}} = -(n_y - 1)(1 - (n_y - 2)d)$, $\frac{\partial \sum_{l \neq i}^{n_x} q_{lj}}{\partial p_{x_i}} = d(n_x - 1)(n_y - 1)$ and

(c) $\frac{\partial \pi_{x_i y_j}}{\partial p_{y_j}} = (p_b - 2c)\frac{\partial q_b}{\partial p_{y_j}} + (p_{x_i} - c)\frac{\partial \sum_{k \neq j}^{n_y} q_{ik}}{\partial p_{y_j}} + \sum_{l \neq i}^{n_x} q_{lj} + (p_{y_j} - c)\frac{\partial \sum_{l \neq i}^{n_x} q_{lj}}{\partial p_{y_j}} = 0$, where $\frac{\partial q_b}{\partial p_{y_j}} = d(n_x - 1)$, $\frac{\partial \sum_{k \neq j}^{n_y} q_{ik}}{\partial p_{y_j}} = d(n_x - 1)(n_y - 1)$, $\frac{\partial \sum_{l \neq i}^{n_x} q_{lj}}{\partial p_{y_j}} = -(n_y - 1)(1 - (n_y - 2)d)$ for the component prices.

The strategy of the proof is to evaluate the marginal profits of the component prices at the level of the initial equilibrium prices and find that are positive. In such a case the new firm has incentive to increase prices.

Consider the marginal profits for p_{x_i} evaluated at the initial equilibrium prices, $\left(\frac{\partial \pi_{x_i y_j}}{\partial p_{x_i}}\right)^0 = (p_b - 2c)d(n_y - 1) + (\sum_{k \neq j}^{n_y} q_{ik}^0)^{DT} + (p_{x_i}^0 - c)(-(n_y - 1)(1 - (n_y - 2)d)) + (p_{y_j}^0 - c)d(n_x - 1)(n_y - 1)$ where q_{ik}^0 is the demand of composite good ik at the initial equilibrium prices. The first, second and fourth terms are positive while the third one is negative. Also note that the sum of the composite goods produced by the merger firm which include the component x_i is now different as compared with the initial situation since the demand functions are now including p_b. Let us denote this sum as $(\sum_{k\neq j}^{n_y} q_{ik}^0)^{DT} = (n_y-1)a-(1+d)(n_y-1)p_{x_i}^0 - (1+d)\sum_{k\neq j}^{n_y} p_{y_k}^0 + dn_x(n_y-1)\sum_{k=1}^{n_y} p_{y_k}^0 + dn_y(n_y-1)\sum_{k=1}^{n_x} p_{x_k}^0 + d(n_y-1)(p_b - p_{x_i}^0 - p_{y_j}^0)$. Take next the first order condition for p_b and evaluate it at the initial equilibrium prices to find the expression (p_b-2c) that satisfies the first order condition at those prices. Then we obtain:

(1) $(p_b - 2c) = q_b + (p_{x_i}^0 - c)d(n_y-1) + (p_{y_j}^0 - c)d(n_x-1)$.

Next note that $\sum_{l=1}^{n_y} q_{il}^0 = n_y a-(1+d)n_y p_{x_i}^0 - (1+d-dn_x n_y)\sum_{k=1}^{n_y} p_{y_k}^0 + dn_y^2 \sum_{k=1}^{n_x} p_{x_k}^0$ and also $\sum_{l\neq j}^{n_y} q_{il}^0 = (n_y-1)a-(1+d)(n_y-1)p_{x_i}^0 - (1+d)\sum_{k\neq j}^{n_y} p_{y_k}^0 + dn_x(n_y-1)\sum_{k=1}^{n_y} p_{y_k}^0 + dn_y(n_y-1)\sum_{k=1}^{n_x} p_{x_k}^0$. Also note that $q_b - q_{ij}^0 = -p_b + p_{x_i}^0 + p_{y_j}^0$. Now, we can establish the relation between $\sum_{l=1}^{n_y} q_{il}^0$ and $(\sum_{k\neq j}^{n_y} q_{ik}^0)^{DT}$ as follows: $\sum_{l=1}^{n_y} q_{il}^0 = \sum_{l\neq j}^{n_y} q_{il}^0 + q_{ij}^0 = (\sum_{k\neq j}^{n_y} q_{ik}^0)^{DT} + d(n_y-1)(p_{x_i}^0 + p_{y_j}^0 - p_b) + q_{ij}^0 = (\sum_{k\neq j}^{n_y} q_{ik}^0)^{DT} + d(n_y-1)q_b - d(n_y-2)q_{ij}^0$. Finally, by the use of the first order condition for p_{x_i} in the initial case we know that $\sum_{l=1}^{n_y} q_{il}^0 = (p_{x_i}^0 - c)n_y(1-(n_y-1)d)$, and noting that $n_y(1-(n_y-1)d)$ can be rewritten as $((n_y-1)(1-d(n_y-2)+(1-2d(n_y-1))$, we can write that $(\sum_{k\neq j}^{n_y} q_{ik}^0)^{DT} + d(n_y-1)q_b - d(n_y-2)q_{ij}^0 = (p_{x_i}^0 - c)[(n_y-1)(1-d(n_y-2)+(1-2d(n_y-1))]$ or equivalently, we obtain:

(2) $(\sum_{k\neq j}^{n_y} q_{ik}^0)^{DT} - (p_{x_i}^0 - c)((n_y-1)(1-d(n_y-2) = -d(n_y-1)q_b + d(n_y-2)q_{ij}^0 + (p_{x_i}^0 - c)(1-2d(n_y-1))$.

Substituting (1) and (2) in the first order condition for the p_{x_i} of the merged firm and simplifying we obtain:

$\left(\frac{\partial \pi_{x_i y_j}}{\partial p_{x_i}}\right)^0 = (p_{x_i}^0 - c)(1-2d(n_y-1)+d^2(n_x-1)(n_y-1)) + (p_{y_i}^0 - c)d(n_y-1)(d(n_y-1)+(n_x-1)) + d(n_y-2)q_{ij}^0 > 0$.

where all the coefficients for $(p_{x_i}^0 - c), ((p_{x_i}^0 - c)$ and q_{ij}^0 are positives. Just note that$1-2d(n_y-1))$ is positive iff $d < \frac{1}{2(n_y-1)}$ but we know that $d < \frac{1}{n_x n_y - 1} < \frac{1}{2(n_y-1)}$. Going back to expression (b) note that increasing p_{x_i} with respect to $p_{x_i}^0$, other variables kept fixed, implies a decrease in the positive second term and also that the negative third term is now more negative. Thus p_{x_i} increases at equilibrium after the merger. The same reasoning applies, mutandis mutatis to the first order condition for p_{y_i}. This ends the proof of Lemma 1.

- Consider now Lemma 2. We also compare the initial equilibrium prices with the equilibrium prices of the outsider to the merger among firms producing x_i and y_j. Then focusing on outsiders, we know that at the initial equilibrium and for all $l \neq i$, $\frac{\partial \pi_{x_l}}{\partial p_{x_l}} = 0$, that is $\sum_{k=1}^{n_y} q_{lk}^0 = n_y(1-(n_y-1)d)(p_{xl}^0 - c)$. Also for all $k \neq j$ $\frac{\partial \pi_{y_k}}{\partial p_{y_k}} = 0$, that is $\sum_{l=1}^{n_x} q_{lk}^0 = n_x(1-(n_x-1)d)(p_{y_k}^0 - c)$. We use the same strategy in the proof as in Lemma 1, we substitute the equilibrium set of equilibrium prices in the new first order condition and find out its sign. Without loss of generality, consider firm producing x_l and compute its first order condition after the merger. It yields:

(d) $\frac{\partial \pi_{x_l}}{\partial p_{x_l}} = (\sum_{k=1}^{n_y} q_{lk})^{DT} - n_y(1-(n_y-1)d)(p_{x_l} - c)$

As before we must note that $(\sum_{k=1}^{n_y} q_{lk})^{DT}$ is the sum of the output of composite goods produced by firm x_l once we have incorporated to the initial demands the price of the bundle produced by the merged firm, which evaluated at the initial prices yields: $(\sum_{k=1}^{n_y} q_{lk}^0)^{DT} = n_y a - (1+d) n_y p_{x_i}^0 - (1-d(n_x n_y - 1)\sum_{k \neq j}^{n_y} p_{y_k}^0 + d n_y^2 \sum_{k=1}^{n_x} p_{x_k}^0 + d(n_y-1)(p_b - p_{x_i}^0 - p_{y_j}^0)$. But we know that $\sum_{l=1}^{n_y} q_{il}^0 = n_y a - (1+d) n_y p_{x_i}^0 - (1+d-dn_x n_y)\sum_{k=1}^{n_y} p_{y_k}^0 + d n_y^2 \sum_{k=1}^{n_x} p_{x_k}^0$ at the initial situation when there is no bundle price. Then $\sum_{l=1}^{n_y} q_{il}^0 = (\sum_{k=1}^{n_y} q_{lk}^0)^{DT} - d(n_y-1)(p_b - p_{x_i}^0 - p_{y_j}^0)$. An making use of the initial first order condition $n_y(1-(n_y-1)d)(p_{xl}^0 - c) = (\sum_{k=1}^{n_y} q_{lk}^0)^{DT} - d(n_y-1)(p_b - p_{x_i}^0 - p_{y_j}^0)$, then expression (d) evaluated at the initial prices is negative:

$(\frac{\partial \pi_{x_l}}{\partial p_{x_l}})^0 = d(n_y-1)(p_b - p_{x_i}^0 - p_{y_j}^0) < 0$. The reason is that by Lemma 1 we know that $p_{x_i}^0 < p_{x_i}^{DT}$ and $p_{y_j}^0 < p_{y_j}^{DT}$ and also that if the merged firm introduces mixed bundling it must be satisfied that $p_b < p_{x_i} + p_{y_j}$. Therefore the conclusion is that to reach the equilibrium firm producing x_l must reduce it price with respect to the initial situation. The analysis is valid for any outsider producing either type of products.

7.3 Proofs of model with two *x-type* and two *y-type* components

Regarding the ST merger

•Regarding prices, the ranking is $p_{x_i}^{ST} - c > p_{y_i}^{ST} - c$, $p_{x_i}^{ST} - c > p_{x_i}^{IC} - c$, and $p_{y_i}^{IC} - c > p_{y_i}^{ST} - c$; $i = 1, 2$, which is easily proven by inspection since $d < \frac{1}{3}$. Similarly for outputs, $q_{ij}^{IC} > q_{ij}^{ST}; \forall ij$ and profits, $\pi_{x_1 x_2}^{ST} > \pi_{y_1}^{ST} = \pi_{y_2}^{ST}$.

• *The merger is profitable if* $\pi_{x_1 x_2}^{ST} > \pi_{x_1}^{IC} + \pi_{x_2}^{IC}$ which is true if $2(1-d)d(3-12d+13d^2) > 0$ and this always holds for $d < \frac{1}{3}$.

• *Outsiders to the merger are worse off after the merger*, that is, $\pi_{y_i}^{IC} > \pi_{y_1}^{ST} = \pi_{y_2}^{ST}$, by inspection.

•*The merged firm prefers selling both components instead of eliminating one of them.* The profits if the merged firm eliminates one component are the following: $\pi_{x_1x_2}^{STe} = \frac{2(a-2(1-d)c)^2}{(3-d)^2(1-d)}$. As we already know, $\pi_{x_1x_2}^{ST} = \frac{4(1-d)^2(a-2(1-3d)c)^2}{(1-3d)(3-5d)^2}$. We prove that $\pi_{x_1x_2}^{ST} > \pi_{x_1x_2}^{STe}$, knowing that $A = (a-2(1-3d)c)^2 > A' = (a-2(1-d)c)^2$, and $\frac{4(1-d)^2}{(1-3d)(3-5d)^2} > \frac{2}{(3-d)^2(1-d)}$. This last expression is equivalent to $(1+d)^2(9-27d+22d^2-2d^3) > 0$, which is true for $d < \frac{1}{3}$.

Regarding the DT merger

•We prove that *Mixed Bundling is the equilibrium option.*

First, we prove that mixed bundling profits are greater than pure component pricing profits, that is, $\pi_{x_1y_1}^{DT,MB} > \pi_{x_1y_1}^{DT,PC}$. This is true for $(1+d)^2(473-3828d+11278d^2-13220d^3+3969d^4) > 0$, which always holds for $d < \frac{1}{3}$.

Next, we prove that mixed bundling profits are greater than pure bundling profits, that is $\pi_{x_1y_1}^{DT,MB} = \frac{(17-38d+9d^2)(A)^2}{4(3-9d+4d^2)^2} > \pi_{x_1y_1}^{DT,PB} = \frac{(3+2d)^2(A')^2}{4(3-d^2)^2}$. And this inequality holds since:

a) $(A)^2 > (A')^2$ and

b) $4(3-d^2)^2(17-38d+9d^2) > 4(3-9d+4d^2)^2\ (3+2d)^2$ which is equivalent to proving that $(1+d)(72-36d-318d^2+150d^3+113d^4-55d^5) > 0$. This last expression holds for $d < \frac{1}{3}$.

Finally we prove that pure component pricing profits are greater than pure bundling profits, that is, $\pi_{x_1y_1}^{DT,PC} = \frac{128(3-5d)(A)^2}{(29-78d+21d^2)^2} > \pi_{x_1y_1}^{DT,PB} = \frac{(3+2d)^2(A')^2}{4(3-d^2)^2}$. Where we use that $(A)^2 > (A')^2$. And noting that $512(3-5d)(3-d^2)^2-(3+2d)^2(29+3d(-26+7d))^2$ is positive for $d < \frac{1}{3}$.

Then, we know that $\pi_{x_1y_1}^{DT,MB} > \pi_{x_1y_1}^{DT,PC} > \pi_{x_1y_1}^{DT,PB}$.

•Regarding the effect of the different price policies on outsiders, firstly we prove $\pi_{x_2}^{DT,PC} > \pi_{x_2}^{DT,MB}$, since we reach the equivalent expression $(1+d)^2(1-3d)(59-215d+193d^2-45d^3) > 0$, which always holds for $d < \frac{1}{3}$. Then, we prove $\pi_{x_2}^{DT,MB} > \pi_{x_2}^{DT,PB}$, knowing that $A^2 > (A')^2$, as we already said, we have to prove that $\frac{2(1-d)^3}{(3-9d+4d^2)^2} > \frac{(2+d)^2}{4(3-d^2)^2}$, expression which simplifies to $(1+d)(36-72d+27d^2-33d^3+16d^4+16d^5-8d^6) > 0$, which is true for $d < \frac{1}{3}$.We can now present the outsiders' profits ranking for the different price strategies: $\pi_{x_2}^{DT,PC} > \pi_{x_2}^{DT,MB} > \pi_{x_2}^{DT,PB}$. The same ranking for $\pi_{y_2}^{DT}$, by symmetry.

•*The merger is profitable:* we prove that $\pi_{x_1y_1}^{DT,MB} > \pi_{x_1}^{IC} + \pi_{y_1}^{IC}$, this inequality is equivalent to $9-48d-34d^2+592d^3-967d^4+256d^5 > 0$ and it always holds for $d < \frac{1}{3}$.

•*Outsiders are worse off after the merger.* We prove $\pi_{x_2}^{IC} = \pi_{y2}^{IC} > \pi_{x_2}^{DT} = \pi_{y_2}^{DT}$ for

$d < \frac{1}{3}$ by inspection.

•Regarding prices, we prove that $(p_{x_1}^{DT} - c) + (p_{y_1}^{DT} - c) > p_b^{DT} - 2c$ related with single components and bundle prices. We also prove $p_{x_1}^{IC} - c + p_{y_1}^{IC} - c > p_b^{DT} - 2c$, $p_{x_2}^{IC} - c + p_{y_2}^{IC} - c > p_{x_2}^{DT} - c + p_{y_2}^{DT} - c$, for the bundle and outsider composite good. In mix-and-match composite goods, $p_{12}^{DT} - 2c = p_{21}^{DT} - 2c > p_{x_1}^{IC} - c + p_{y_2}^{IC} - c$. All of them hold for $d < \frac{1}{3}$. We prove that $p_{22}^{DT} - 2c > p_b^{DT} - 2c$ and $p_{12}^{DT} - 2c = p_{21}^{DT} - 2c > p_{22}^{DT} - 2c$ for $d < \frac{1}{3}$. The ranking of bundle and composite goods prices is $(p_{12}^{DT} - 2c) = (p_{21}^{DT} - 2c) > (p_{22}^{DT} - 2c) > (p_b^{DT} - 2c)$ for $d < \frac{1}{3}$. All of them are easily proven by inspection.

•Regarding outputs, we prove that $q_b^{DT} > q_{11}^{IC}$ which is equivalent to $8d^3 + 9d^2 - 12d + 3 > 0$, and it is true for $d < \frac{1}{3}$. We also prove the inequality $q_{22}^{DT} > q_{22}^{IC}$, equivalent to $d + 4d^2 - 13d^3 > 0$, which is true for $d < \frac{1}{3}$. In mix-and-match composite goods, we prove that $q_{12}^{IC} = q_{21}^{IC} > q_{12}^{DT} = q_{21}^{DT}$, inequality equivalent to $d(5 - d(12 + d)) > 0$ which holds for $d < \frac{1}{3}$.

•We prove by inspection that $q_b^{DT} > q_{22}^{DT}$, and $q_{22}^{DT} > q_{12}^{DT} = q_{21}^{DT}$. So the ranking for output's levels is $q_b^{DT} > q_{22}^{DT} > q_{12}^{DT} = q_{21}^{DT}$.

Equilibrium merger

•Proof of Proposition 1.

We prove that $\pi_{x_1 y_1}^{DT} > \pi_{x_1 x_2}^{ST}$, this inequality is equivalent to $(1+d)(9 - 168d + 818d^2 - 1600d^3 + 1245d^4 - 256d^5) > 0$. In the domain of d, this expression is positive for $0 < d < 0.0804$, and it is negative for $0.0804 < d < \frac{1}{3}$. So we finally have:

We prove that outsiders to the DT merger get higher profits than outsiders to the ST merger, that is $\pi_{x_2}^{DT} = \pi_{y_2}^{DT} > \pi_{y_2}^{ST}$. The inequality is equivalent to $6d - 11d^2 - 8d^3 + 9d^4 > 0$, which is positive for $d < \frac{1}{3}$.

Socially optimal merger

•Proof of Proposition 2.

Consumer Surplus. We easily prove by inspection that $CS^{IC} > CS^{ST}$. We also prove that $CS^{DT} > CS^{IC}$ arriving to the equivalent expression $45 - 468d + 1857d^2 - 3352d^3 + 2551d^4 - 660d^5 + 283d^6$, which always holds for $d < \frac{1}{3}$. So finally we get $CS^{DT} > CS^{IC} > CS^{ST}$.

Social Welfare. $SW^{IC} > SW^{ST}$ if and only if $8d(3 - 18d + 28d^2 - 9d^3) > 0$ which is positive for $0 < d < 0.2708$, and it is negative for $0.2708 < d < \frac{1}{3}$. We prove as well that $SW^{IC} > SW^{DT}$, since it holds when $-63 + 756d - 3165d^2 + 5528d^3 - 3849d^4 + 1204d^5 -$

$283d^6 > 0$, expression which is negative for $0 < d < 0.1992$, and for $0.2883 < d < \frac{1}{3}$, while it is positive for $0.1992 < d < 0.2883$. To present the complete ranking we prove that $SW^{DT} > SW^{ST}$, which is equivalent to proving that $(1+d)(63 - 540d + 1737d^2 - 2816d^3 + 2369d^4 - 724d^5 - 57d^6) > 0$. This last expression is negative for $0 < d < 0.2686$ and it is positive for $0.2686 < d < \frac{1}{3}$.

Therefore, the complete ranking of SW is the following:

$$\begin{array}{lll} SW^{DT} > SW^{IC} > SW^{ST} & if & 0 < d < 0.1992 \\ SW^{IC} > SW^{DT} > SW^{ST} & if & 0.1992 < d < 0.2686 \\ SW^{IC} > SW^{ST} > SW^{DT} & if & 0.2686 < d < 0.2708 \\ SW^{ST} > SW^{IC} > SW^{DT} & if & 0.2708 < d < 0.2883 \\ SW^{ST} > SW^{DT} > SW^{IC} & if & 0.2883 < d < \frac{1}{3} \end{array}$$

Continuation merger.

A) Upper branch of the tree: Mergers coming from a DT merger ($0 < d < 0.0804$).

• a.1) *Result*: *The Extended merger is profitable.* We prove that $\pi^{EM}_{x_1y_1y_2} > \pi^{DT}_{x_1y_1} + \pi^{DT}_{y_2}$ because it is equivalent to proving that $(13-3d)(3-5d)^2 - 18(1-d)^2(4(1-d) + 2(1-3d)) > 0$, which is always true for $d < \frac{1}{3}$.

• a.2) The game about the pricing strategy. Mixed bundling is the dominant strategy.

Since the game is symmetric we only need to prove that MB is a dominant strategy for one firm, say for the firm producing x_2y_2. First we prove that if x_1y_1 chooses PC, then $\pi^{PC-MB}_{x_2y_2} > \pi^{PC-PC}_{x_2y_2}$, because it is equivalent to proving that $(1+d)(585-4973d+15595d^2-20607d^3 + 8928d^4) > 0$, which is satisfied for $d < \frac{1}{3}$. Therefore, if x_1y_1 chooses PC, then x_2y_2 prefers MB.Similarly, we prove that if x_1y_1 chooses MB, then $\pi^{MB-MB}_{x_2y_2} > \pi^{MB-PC}_{x_2y_2}$. This is equivalent to proving that $(1+d)(113 - 975d + 3058d^2 - 4044d^3 + 1818d^4) > 0$, and it is always satisfied $d < \frac{1}{3}$. So, if x_1y_1 chooses MB then x_2y_2 prefers MB.

• a.3) *Result*: the *DT countermerger is profitable.* We prove that $\pi^{DTC}_{x_2y_2} > \pi^{DT}_{x_2} + \pi^{DT}_{y_2}$. We analyze the equivalent expression $(17-32d)(3-9d+4d^2)^2-36(1-d)^3(2-5d)^2$ and prove that it is always positive for $d < \frac{1}{3}$.

• a.4) *Proof of Proposition 3.*

i) *Firm y_2's decision*: we prove that $\frac{1}{3}\pi^{EM}_{x_1y_1y_2} > \frac{1}{2}\pi^{DTC}_{x_2y_2}$, which is equivalent to proving $(13-3d)(2-5d)^2 - (17-32d)3(1-d)(1-3d) > 0$. and it is is positive for $d < \frac{1}{3}$. Then, the firm producing y_2 chooses the *extended merger.*

ii) *Consumer Surplus*: We prove that $CS^{EM} > CS^{DT}$ which is equivalent to proving that $4(45 - 540d + 1731d^2 - 2400d^3 + 1763d^4 - 628d^5 + 61d^6) > 0$ which is positive for the range of values in which the DT merger occurs, i.e. $0 < d < 0.0804$.

iii) *Social Welfare*: We prove that $SW^{EM} > SW^{DT}$ which is equivalent to proving that $4(63 - 981d + 5019d^2 - 11133d^3 + 12301d^4 - 7191d^5 + 2041d^6 - 183d^7) > 0$. This is positive for the range of values in which the DT merger occurs.

B) Lower branch of the tree: Mergers coming from a ST merger ($0.0804 < d < \frac{1}{3}$)

• b.1) *Result*: the *Extended merger is profitable.* We prove that $\pi^{EM}_{x_1x_2y_2} > \pi^{ST}_{x_1x_2} + \pi^{ST}_{y_2}$ arriving to the equivalent expression $2(3-9d+4d^2)^2(13-3d)-9(1-d)(1-3d)((17-38d+9d^2)+4(1-d)^3)$ and proving that it is always positive for $d < \frac{1}{3}$.

• b.2) *Result*: the *ST countermerger is profitable.* We prove that $\pi^{STC}_{y_1y_2} > \pi^{ST}_{y_1} + \pi^{ST}_{y_2}$ as we arrive to the equivalent expression $6d - 2d^2$ which is always positive for $d < \frac{1}{3}$.

• b.3) *Proof of Proposition 4.*

i) *Firm y_2's decision*: We prove that $\frac{1}{3}\pi^{EM}_{x_1x_2y_2} > \frac{1}{2}\pi^{STC}_{y_1y_2}$. Which is equivalent to proving that $9 + 81d > 0$, which is the case. Then, the firm producing y_2 chooses the *extended merger.*

ii) *Consumer Surplus*: First , we prove that $CS^{EM} > CS^{ST}$, as we know that this inequality is equivalent to $45 - 228d + 358d^2 - 212d^3 + 53d^4 > 0$, which is the case for $d < \frac{1}{3}$. Next, we find when $CS^{EM} > CS^{IC}$. It is equivalent to finding when the expression $45 - 384d + 838d^2 - 608d^3 + 173d^4$ is positive. It is for $0 < d < 0.1776$ and is negative for $0.1776 < d < \frac{1}{3}$.

iii) *Social Welfare*: First, we prove that $SW^{EM} > SW^{ST}$. It is true when $(1-3d)(63-330d + 500d^2 - 270d^3 + 53d^4) > 0$, which always holds for $d < \frac{1}{3}$. Next, we find when $SW^{EM} > SW^{IC}$. This is equivalent to finding when $63 - 795d + 3158d^2 - 4638d^3 + 2603d^4 - 519d^5$ is positive. This expression is positive either for $0 < d < 0.1535$ or for $0.2657 < d < \frac{1}{3}$ while it is negative for $0.1535 < d < 0.2657$.

• *Monopoly comparisons*

We prove by inspection that $\pi^M > \pi^{EM}_{x_1y_1y_2} + \pi^{EM}_{x_2}$.

We prove that $CS^M > CS^{EM}$ since we arrive to the equivalent expression $2(5-22d+13d^2) > 0$, which is positive for $0 < d < 0.0804$ and $0.0804 < d < 0.2705$ while it is negative for $0.2705 < d < \frac{1}{3}$.

We prove that $CS^M > CS^{DT}$ since it is equivalent to $(3-d)(1-d)(5-38d+53d^2) > 0$, expression which is positive for $0 < d < 0.0804$.

Similarly, we also prove that $CS^M > CS^{ST}$ finding an equivalent expression $(1 - 3d)(5 - 7d) > 0$ which is positive $\forall d$.

We finally prove that $CS^M > CS^{IC}$. This is equivalent to proving that $(1 - 5d)(5 - 9d) > 0$. It is positive for $0 < d < 0.0804$ and $0.0804 < d < 0.2$, while it is negative for $0.2 < d < \frac{1}{3}$.