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Sequential vs. Simultaneous Schelling Models: Experimental Evidence

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Sequential vs. Simultaneous Schelling Models:

Experimental Evidence[1]

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Abstract

This paper shows the results of experiments where subjects play the Schelling's spatial proximity model (1969, 1971a). Two types of experiments are conducted; one in which choices are made sequentially, and a variation of the first where the decision-making is simultaneous. The results of the sequential experiments are identical to Schelling's prediction: subjects finish in a segregated equilibrium. Likewise, in the variant of the simultaneous decision experiment the same result is reached: segregation. Subjects' heterogeneity generates a series of focal points in the first round. In order to locate themselves, subjects use these focal points immediately, and as a result, the segregation takes place again. Furthermore, simultaneous experiments with commuting costs allow us to conclude that introducing positive moving costs does not affect segregation.

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1.Introduction

Schelling's model (1969, 1971a) represents a paradigm inside the theory: from a theoretical point of view it is a simple model, laconic in hypothesis and with very powerful results. In addition, it is an empirically relevant model since it offers a clear explanation of the segregation phenomenon; a problem that has worried planners since the second half of the twentieth century[i]. Moreover, segregation has turned out to be one of the most important topics in the socio-political and public economic debate (*The Economist*, 2001).

In general, there are two basic variants of Schelling's "model of spatial proximity"[ii]. The first version is a one-dimensional model and is introduced in Schelling (1969). In Schelling (1971a), a two-dimensional version is presented, which also appears later in Schelling (1971b, 1978). This work experimentally analyzes the one-dimensional model.

In the one-dimensional version of Schelling's spatial proximity model (1969, 1971a) a society is modeled through a sequence of individuals distributed along a line[iii]. Two types of individuals form the society: whites and blacks. The adjacent neighbors to the left and right-hand side define the neighborhood of each individual[iv]. The individuals who compose this society are assumed to be utility maximizers, that is to say, they look for their best interest. The preferences of an agent are marked by his level of tolerance regarding the number of neighbors equal to him. For example, a "slightly" tolerant agent would be one who demands that all his neighbors

next to him are of his same type, while a "moderately" tolerant agent would accept that half of his neighbors were like him[v].

In short, a striking result of Schelling's model is that even when beginning from a society where individuals are moderately tolerant, regarding the number of neighbors of their same type (as defined above), the sum of individual options generates a totally segregated community. Figure 1 illustrates how starting from a situation of complete social integration (circle a), a completely segregated society (circle b) is reached after individuals are allowed to move.

Figure 1:

This equilibrium solution is a very powerful result since it seems to suggest that it is impossible to do anything to counteract segregation because it is simply the equilibrium configuration. This result is surprising and has generated a vast amount of literature from a wide range of scientific disciplines. Why does this occur? Schelling assumes that agents act myopically and sequentially. According to these assumptions, when an agent moves, it is always towards a neighborhood with a higher percentage of members from his own group than in the neighborhood he currently occupies. Further, notice that blocks of homogeneous agents (i.e. groups of agents such that boundary homogenous agents may be defined) will always have interiors where no agents wish to move. When agents move beyond the boundary of a block, this automatically increases the number of spaces where they will be content to be completely segregated. Together, these features mean that individual moves in the Schelling

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model typically increase the level of segregation and that some increases in segregation increase the size of stable segregated groups with complete segregation occurring asymptotically. Schelling's findings turn out to be relatively general and have been theoretically formalized by Granovetter and Soong (1988) and Young (1988). Granovetter and Soong (1988) showed that Schelling's segregation model can be expressed in terms of a system of two coupled difference equations. Likewise, Young (1998) is noteworthy in recasting Schelling's work in the context of evolutionary game theory. In fact, he was the first to point out that techniques developed in evolutionary game theory, and in particular the concept of stochastic stability introduced by Foster and Young (1990), are useful for analyzing Schelling's spatial proximity model. Young (1998, 2001) presents a simple variation of Schelling's one-dimensional model, showing that segregation tends to occur in the long term even if any agent does not prefer a segregated neighborhood. In Young's one-dimensional variant of the Schelling model (1998), he considers that the agents exchange their positions for compensation payments and also that these agents can be wrong. The result is that segregation is the only viable long-term outcome of the dynamics of best response, as long as the preferences of the players on the composition of their neighborhood are biased in favor of their own kind. Pancs and Vriend (2007) obtained similar results, showing that segregation is the only possible long-term outcome when agents with a preference for 50-50 neighborhoods play their best response to the neighborhood's racial composition in a circularly

connected society (a ring). The authors examine the robustness of Schelling's model, focusing on a particular driving force: individual preferences. They also show that even if all individual agents have a strict preference for perfect integration, best response dynamics can lead to segregation. This raises some doubts about the ability of public policies to achieve integration by promoting openness and tolerance for diversity. In a related work, Mobius (2000) shows how random field methods may be applied to a Schelling-type model in ways that allow for richer neighborhood configurations than have previously been studied. This work suggests that statistical mechanics methods may prove to be useful in the modeling of segregation dynamics.

Nevertheless, the original version of Schelling's model has some particular features. One of the most important of these is the fact that subjects move sequentially in order to reach the equilibrium outcome. All the individuals who are not in a situation beyond their threshold of tolerance are organized to carry out their displacement in the society. That is, first an individual decides if he wants to move or not, then the following one decides, and so on until the last one.

From a theoretical point of view, sequentiality is not trivial. This is due to the fact that in the sequential model the k-th subject already views the first $k-1$ decisions as a given and can only decide on the remaining $N-k$. Thus, in the sequential model every subject has different information, which depends on the moment when the individual decides. In a simultaneous model, however, all the subjects would decide on the N possible positions

simultaneously and the amount of information available to each subject would be identical[vi].

The aim of this work is to experimentally test Schelling's model by having the subjects play in both a sequential and a simultaneous way.[vii] This is the first work using a laboratory experiment of Schelling's model in which individuals choose simultaneously[viii].

In order to conduct the experiments, Schelling's model is first designed in its original setting (where subjects make decisions sequentially). A modification is then proposed in which the subjects make decisions simultaneously. The experimental results of Schelling's model when subjects decide sequentially coincide with the theoretical prediction of Schelling's model: subjects end up in equilibrium with total segregation in a single round. Surprisingly, in the experiment using Schelling's model where individuals decide simultaneously, we obtain the total segregation outcome as well. In this second case, subjects' heterogeneity generates a series of focal points in the first round, that are used by the subjects to locate themselves and, as a result, total segregation emerges again[ix].

The rest of the work is structured as follows: the second section shows Schelling's standard model and its equilibrium prediction. Also we introduce a variation of Schelling's model in which subjects make the decisions simultaneously. The third section describes the design of the experiments and how they are performed. The fourth section analyzes the results of the basic experiment. Section fifth shows a variation and the sixth discusses policy implications. The last section concludes.

2. Schelling's spatial proximity model.

2.1. Schelling's sequential model.

In order to represent Schelling's sequential model we start from a circle in which we distribute N subjects of two clearly differentiated types (whites and reds)[x]. The neighborhood of each subject is defined as well as the adjacent neighbors to the left and the right, that is to say, every individual has two neighbors; the first one to his left and the second one to his right. In this way, the number of neighborhoods in the circle is equal to the number of individuals that compose it, N. The model is defined by the following properties: first, subjects are assumed to have a utility function according to which they reach happiness when they have at least a neighbor of their same type[xi]; second, subjects move sequentially and without cost (the first one decides first, then the following one decides, and so on to the N-th agent[xii]).

Though mobility costs do not exist, Schelling imposes that subjects move to the closest place that satisfies their neighbor demand, bearing in mind that moving to the nearest place means being located in the closest space. A space is the distance between two persons. With these minimal requirements the society will change from a situation of complete social integration (Figure 1, circle a) to a situation of absolute segregation (Figure 1, circle b).

How is total segregation reached? Figure 2 illustrates the movements for a society of 8 individuals, ($N=8$).

Figure 2:

In the initial situation (Figure 1, circle a), subjects are completely unhappy given that they do not have any neighbor of their same type. Let's suppose that the white player on the top left side is the first one to make a decision (this player will be called 1). Given that subject 1 is not happy, he moves to the closest place where he is happy. As a consequence, subject 2 is now happy (and so is subject 8) and therefore does not move. Subject 3 is also happy (since subject 1 has moved next to him). Nevertheless, subject 4 continues being an unhappy person since he still does not have any neighbors of his type. Therefore subject 4 moves next to subject 6. This makes both subject 6 (who is of his same color) and subject 5 (who is located next to subject 3) happy. Finally, subject 7, who is not happy, moves next to 2 and by doing so complete segregation is reached (see Figure 2, circle 4).

In short, given the minimal requirements regarding the preferences of the model, full segregation is achieved with only three movements. This is the "magic" of Schelling's model. Nevertheless, the sequential movement makes everything very simple: when the players make decisions they already know what has happened, specifically they know that the previous subjects cannot move anymore, therefore no risk is involved. For example, when subject 4 moves he knows that neither subject 5 nor subject 6 are going to move, (that is, they will not harm him afterwards).

2.2 *Schelling's simultaneous model.*

To describe Schelling's simultaneous model, we assume, as in the previous case, that the society is comprised of *N* subjects of two different

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types distributed along a circle. We define the neighborhood for each one of them in the same way we did in the previous model. In this case the model is characterized by the following properties: first, we continue assuming that subjects have a utility function according to which they reach happiness when they have at least a neighbor of their same type; second, subjects move without costs. In this case, however, all subjects decide whether to move or not at the same time, that is to say, it is a simultaneous decision. Finally, subjects can move to any place they wish along the circle since their movement is not restricted to the most nearby place.

The theoretical model to approach this problem would be a game in a strategic form with *N* players. Every player would have *N-1* pure strategies, which correspond to moving to each of the spaces that the N players form along the circle (these are *N-2* strategies) or remaining still. Therefore, in the case of 8 players we would have the strategy of jumping clockwise into the second space, the third space and so on up to the sixth space. Notice that to move either to the first or seventh space is to remain still. This finite game has, at least, an equilibrium in mixed strategies but in addition, multiple equilibria in pure strategies. With these requirements, the simultaneous model should be completely different from the sequential model. Nevertheless, the equilibrium outcome is also complete segregation, but with two rounds.

More precisely, there are two possible pure equilibria configurations: red-red-red-red-white-white-white-white, (RRRRWWWW) or red-red-white-

white-red-red-white-white, (denoted by RRWWRRWW). The first configuration above may arise from movements by the even numbers (red subjects) together to a common place (independently of the odd numbers or white subjects) or the opposite: joint movement of the odd numbers to the same place. The second configuration arises after movements by two players to the same place and another two players (of the same type) to any other non-clustered space. Notice that none of the players has an incentive to change their strategy in either configuration since they are already happy.

So far, the number of pure strategy equilibria is a large number. Actually it is an exponential number with respect to the size of the game. Hence, the possibility of coordination at any pure equilibrium becomes quite small. Consequently, we will expect a non-pure equilibrium configuration as a result of the first round of the simultaneous game, that is, the most probable configuration is a society with some clusters, but not a happy society.

In fact, the later configurations are also the consequence of mixed strategy equilibria where subjects play a uniform mixed strategy given that information is symmetric but imperfect. Actually, any configuration could be the realization of the uniform mixed strategy. Hernández and Von Stengel (2009) fully characterize the equilibrium set (pure, mixed and correlated) of this "geographical coordination game". Nevertheless, we do not have the same set of equilibria after the first round. Actually, given a new non-symmetric configuration, the number of pure equilibria

decreases, making it easy for players to find a device to coordinate their actions.

To explain the equilibrium outcome, let us suppose that we are in a situation like the one depicted in the second circle of Figure 2 in which individual number 4 is unhappy, as well as individuals 5, 6 and 7. We are going to focus only on the movement of players 4 and 6[xiii]. In this simultaneous game, player 4 does not know *a priori* if player 6 will be waiting for him when he arrives. If he decides to move next to him[xiv], therefore, subject 4 will probably decide to move to the least uncertain place, i.e. he would go to the "group of 2-8". The reason is simple: group 2-8 is more certain than subject 6. There are many reasons why going to the big group is better. First, both subject 2 and 8 are happy next to each other and therefore will not move. Second, subject 4 could "anticipate" that subject 6 might be thinking the same thing he is, that is, subject 6 is thinking about moving to the big group and will not be in his position if subject 4 chooses to move to his side. Figure 3 represents the situation in which only subjects 4 and 6 move and where a situation of absolute segregation is reached (Figure 3, circle 2).

Figure 3:

In a game with these characteristics, subjects can often coordinate their intentions or expectations with others, as each one knows that the other is trying to do the same thing he is doing. Most of the situations provide agents with some hints for coordination. These hints are focal points about what others might expect from them[xv]. It is evident that if there is no

convergence, the process of prediction and interaction turns out to be unsuccessful. The key is that when individuals make their decisions, they try to accomplish a common task, not an individual one[xvi]. Each individual reduces his search space by spontaneously using the hints that have the highest probability of making both of their expectations convergent; what Binmore and Samuelson (2006) call the *exploitation of framing information* (use contextual information).

Finding the hint, or rather searching for this key, involves finding some code that is mutually recognized by all subjects as the key. This search may depend on precedents, accidental agreements, symmetry, geometric configuration, etc. And it is in this way that these keys turn out to be focal points of the game (Schelling, 1960). In sum, it seems that focal points are only needed for the subjects of the simultaneous game in order to generate the full segregation obtained in the sequential model.

3. Design and implementation of the experiment.

The experiment was conducted using an instructions booklet (set) to explain the rules of the game and how subjects could obtain maximum happiness (see a copy of the instructions in the Appendix). In order to ensure that each of the subjects in the experiments had a preference regarding the composition of their neighborhood by which they could achieve happiness, they were paid two euros if at least one of their adjacent neighbors (either to the left or to the right) was of their same type by the end of the experiment. If none of the adjacent neighbors were of their same type, the subjects received zero euros (the individual was unhappy). In both the sequential and the simultaneous versions of the game, the initial configuration was that of maximum unhappiness for all the subjects comprising the society (Figure 1a). Under this initial framework, no one would obtain payment. The only way for subjects to receive payment was for them to move in such a manner as to reach maximum happiness. For the sake of simplicity, 8 subjects were used in all the cases.

The 8 subjects were placed in two rows of 4 subjects each.[xvii] The subjects were given a white or a red scarf to identify them as a *white* typed or a *red* typed subject. The subjects were told to form a circle between both rows and asked to identify the color of the scarves of their adjacent neighbors. This initial position allowed each subject to verify that his neighbors were different from him, and therefore all the subjects were unhappy.

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During the sequential game, subjects had to wait their turn to decide if they were going to move or not (observing what had happened). In the simultaneous game, however, all the subjects made their decisions simultaneously (without knowing the decisions made by the others). The subjects were given a control sheet to inform them about their position and the position of the other players (see an example in Figure 9 of the Appendix). The experiment was run only once (a one-shot game).

The experiments were conducted at the Universidad Pública de Navarra (in Pamplona) and at the Universitat de València as follows:

- Pamplona: 56 subjects were distributed in sequential models (2 groups of 8 subjects) and in simultaneous models (5 groups of 8 subjects)

- Valencia: 40 subjects were distributed in sequential models (2 groups of 8 subjects) and in simultaneous models (3 groups of 8 subjects).

The experiments were run at both universities following a regular class with students who volunteered to participate. Students or subjects were not recruited specifically for the experiment. The task did not last more than 10 minutes and all the subjects earned 2 euros (on average and in mode since all they won, see Tables 1 and 2). The subjects did not receive a show-up fee.

4. Results

4.1 Results of the sequential games

Three out of four of the sequential games worked exactly as the theory predicts. That is to say, subject 1 (who was not happy) moved next to subject 3, making subjects 2, 3 and 8 happy. Subject 4 (who was not happy either) then moved next to subject 6, making subjects 5 and 6 happy. Finally, subject 7 (who was not happy yet) had no other choice but to move next to subject 1 (see Figure 2). Complete segregation was reached as a consequence of the movements of subjects 1, 4 and 7.

Table 1:

Nevertheless, an interesting variation occurred for one of the groups (group A2 of Pamplona, see Figure 4). In this group, in spite of being in a situation in which he was not happy, subject 1 realized (technically "he anticipated") that he could stay still and allow the others to solve the situation later. In other words, subject 1 realized that the other players would have no other choice but to move in order to be happy and that he would end up being happy without having to move. Subject 2, who was not happy given that subject 1 had not moved, fulfilled subject 1's expectation and moved next to subject 4. Thus, he made subjects 1, 3 and 4 happy. As in the previous case, three players moved in the end (subjects 2, 5 and 8, see Figure 4), giving rise to a situation of complete segregation of the society where all subjects are happy.

*Result 1***:** The players of Schelling's sequential model reach the equilibrium of complete segregation with the three movements foreseen in the theory.

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<u>Figure 4:</u>

4.2 Results of the simultaneous games

A priori, and as we anticipated previously, we should expect different results for the simultaneous game, with regard to the sequential game, mainly for two reasons:

1. First, all the subjects in the simultaneous game possess the same information at all times, while in the sequential games we find that subjects have more past information after each decision (since they already know the movements that have happened) and less future information (since fewer movements are left to be solved). Nevertheless, in the simultaneous game all the subjects decide at the same time for each round without knowing the decisions of any of the other subjects as there is no order for decision making.

2. Second, the results in the simultaneous game are only probable (not assured). In other words, a subject can decide to move into a position but when he arrives there, the subjects he expected to find are no longer there because the other subjects have decided to move as well.

Therefore, how should subjects play in this game? The optimal way of playing in the simultaneous game is the following:

- First, every subject generates his own distribution of types of players in which he anticipates who is going to move and who will remain still.

- Second, given this expectation, the subject will decide which is his best response, i.e. he will move to the most convenient place for him based on what he anticipates that the other subjects will do.

As suggested by the literature on "levels of reasoning" (Nagel 1995, Bosch *et al*. 2002) one can expect to find variety in the best responses, that is, we can find subjects that (optimally) do not move, move a little or move a lot. Therefore, given the initial situation of white-red-white-red-white-red-white-red and the heterogeneity of types[xviii], it is very probable that at least two focal points of size 2 would appear: white-white or red-red.

Table 2 shows the number of subjects that moved in every round and the number of happy subjects when each of the rounds was finished. Figures 5, 6, 7 and 8 show the results for the eight simultaneous games: Figure 5 shows the case for two of them (S1 and S3), Figure 6 and Figure 7 show how another two games were played (S2 and S7, respectively), Figure 8 shows how game S6 was played, and Figure 9 shows game S5. Notice that two of the eight games are missing.

Table 2:

In both Pamplona (group S4) and Valencia (group S8), we found that Schelling's simultaneous game was played in such a way that maximum segregation distribution was reached in just one round. The game went from Figure 1a to 1b without intermediate steps. A possible explanation for the results of these two games is that all the subjects made an accurate prediction and reached equilibrium in one movement. The alternative

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explanation is that it occurred accidentally. This is not surprising since according to the theory, this result is relatively probable (20%).

The rest of Schelling's simultaneous games are depicted in the 4 figures mentioned above. In four of them, the two most *a priori* expected focal points were created (white-white and red-red): in two of them the focal points turned out to be located next to each other (Figure 5). In the other two, the focal points were placed at a distance from each other (Figure 6).

Figure 5:

Figure 6:

Figure 7:

Figure 8:

Figure 9:

Regarding the generation of the focal points, there are 4 games (S1, S3, S2 and S7, Figures 5, 6 and 7) in which 2 focal points arise during the first round (close or distant). During the first round of one of the games (S6, Figure 8), a distribution was reached with two focal points comprised of three subjects of every type (white-white-white and red-red-red) and only two individuals, one of each type, were unhappy after making their decision.

Finally, almost perfect segregation was obtained after the first round in one game (S5, Figure 9): only one subject remained outside his desired neighborhood.

It is interesting to highlight that maximum segregation distribution was reached in all the cases; after only one round in two cases and after the

second round in five cases. Session S7 (figure 7) provided the only exception: a not complete segregation outcome (WWRRWWRR).

The simplest explanation for this surprising result (total segregation in the simultaneous games) can be found in the theory (see for example, Reny, 1988, 1993 and Samuelson, 1992, among others) and is sustained upon two principles: rationality and common knowledge.

- From the rationality point of view, all the subjects that were not happy moved to the right place for them to reach happiness.

- From the common knowledge of rationality point of view, subjects who were happy "knew" that the neighbors who comprised the focal point together with them would not move because they were already happy and therefore did not move either[xix]. In addition, those who moved knew they would go to spaces from which - assuming rationality - nobody would move.

Therefore, a requirement as basic as rationality (with common knowledge) is sufficient to make the sequential movement requirement not so important. In other words, rationality with common knowledge generates the same result as that obtained from a sequential model.

Result 2: As a consequence of rationality (together with common knowledge), the subjects of Schelling's simultaneous game reached the completely segregated equilibrium in a maximum of two rounds.

Result 3: The completely segregated equilibrium arises in an almost immediate way with or without sequential movement.

5. Costly & simultaneous moving

We have observed that simultaneity does not make a large difference in the game. However, because moving is costly in real life we cannot provide clear conclusions without analyzing the case of costly moving. We repeated the simultaneous experiment in Pamplona in March 2010 with the only difference being that we included positive costs for commuting. Specifically, subjects have to pay 0.5 euros each time they move (recall that the maximum payoff is 2 euros).[xx] Before presenting the results, let us discuss the expected behavior of subjects in this new set-up. An individual facing this situation may consider two alternatives:

- Moving at a cost of 0.5 euros
- Not moving at a cost of 0 euros

Subjects may compute that (a) if they do not move and nobody moves, the actual configuration at the beginning of round 2 will be identical to round 1. As explained before, no new information is provided (see section 4.2, page 16). (b) If the subject does not move but others do, there is some probability that focal points will be generated. (c) If the subject does move and the others stay, the subject will generate a focal point for sure (and a potential payoff of 1.5 euros); (d) if the subject does move and the others move too, all configurations may arise.

In sum, we may conclude that if any subject believes that the other players are not going to move, then the subject will (be more prone to) move. If the subject believes that the others will move, then the subject will (be more prone to) stay.

Obviously, the higher the cost the lesser the benefits of moving and hence fewer people will move. However, it must be said that the new configuration does not actually depend on the number of subjects moving: a single movement may generate a nearly segregated society (W_R_WRWRWR → WWRRWRWR if subject 2 moves one place), whereas a movement by four subjects at the same time may generate nothing (_W_R_W_R_W_R_W_R → WRWRWRWR if all of them move one place) Once subjects move, a new configuration emerges. This is identical for both costly and costless moving. The configuration obtained is crucial for the final configuration (path dependence).

Additionally, under positive costs, it could be the case that a player anticipates that others will move and thus may avoid the cost of moving. In a situation[xxi] like WWRR_WR_WR, the 5th subject may expect that the 6th subject will move and therefore does not move. An identical argument is also valid for the 6th player. If one of the players decides to stay, then a non-segregated equilibrium is achieved: WWRRWWRR. However, it is worth noting that this is no longer true in many other cases: in a case like WWW_RW_RRR, the final outcome is identical whether player 5 or player 6 moves or if both move.

In sum, given that *i*) the configuration in round 2 does not crucially depend on the number of movements; *ii*) strategic playing after round 2 is risky and *iii*) strategic playing does not univocally affect the final configuration, we do not expect dramatic changes due to the existence of positive costs.

We ran 8 sessions of the simultaneous game with costs and obtained 64 observations. As in previous cases (table 1 & 2), we show the results of the new sessions in Table 3:

Table 3:

Differences in terms of individual movement are very sound: we count a total sum of 33 movements throughout the game. The number of movements for the costless simultaneous game (Table 2) was 61.

As expected, the numbers are very different if we only consider the movements made in the first round: 24 subjects moved at a cost (up to 64, 37.5%) in sharp contrast to the costless case where 48 subjects moved (up to 64, 75.0%). As can be seen, the differences are significant: (t=-4.582, *p-value*=0.000). Movements in the second round are also different: nine (14.06%) subjects moved in the costly case in contrast to the 17 (26.56%) who moved in the costless one. Summarizing,

Result 4: When there are positive costs, subjects are less willing to move. However, the real effect depends on the number of movements. We are much more concerned about the final configuration of the society rather than the number of movements.

Costless simultaneous games achieved the complete segregated society (WWWWRRRR) in 6 out 8 cases and the not complete segregated society (WWRRWWRR) in the remaining 2.

Recall that the costly simultaneous game provided nearly identical numbers: 7 out of 8 cases were WWWWRRRRR and the remaining 1 was WWRRWWRR.

Result 5: Even with positive costs where subjects move less, the Schelling

segregated equilibrium emerges in most of the cases.

Hence, the introduction of costly movement does not make a great

difference in the final configuration of the society.

6. Policy implications

The key question is how relevant these results are (result 2: equilibrium outcome in two steps and result 3: complete segregated configuration) to real life.

It is worth noting that the original model includes two crucial assumptions:

- First, subjects are assumed to have a utility function according to their neighborhood (namely preferences for races).

- Second, subjects move sequentially and without cost: the first one decides first, then the following one decides, and so on to the N-th agent.

This paper shows that a relaxed version of the second assumption does not crucially affect the results. Observe that the second assumption requires that a planner select the first player, then the second, and so on. The planner must also establish the direction of the movement (left/right). The lack of a planner implies that there is no (exogenous) sorting. Hence subjects may move themselves when they want to.

Therefore, when subjects must make a decision to move in real life we see that assumption 2 fails in several domains:

- Subjects' decisions are simultaneous: nobody waits for another subject to decide whether or not to move.

- At the time each subject makes his choice the actual configuration is exogenous, that is, the timing of the configuration is not relevant, only the actual configuration.

Under these circumstances, subjects perceive the decision of moving –

the game - as being simultaneous rather than sequential. The situation

described here is very similar to what we have shown in previous figures.

Second round configurations are formed by a number of non-symmetric

clusters that may be representative of any actual city. For instance (Figure

4; round 2): WWRRWRWR.

In this case, subjects simultaneously decide whether or not to move. This

paper provides empirical (experimental) evidence of how subjects play that

game. They use focal points to make decisions. This is not rare given that

any cluster of players - WW (RR) for whites (reds) - is the safest option for

them.

In the costly simultaneous model, subjects are less prone to move but the

behavior is the same: subjects move to focal points or they do not expect

others to do so.

Finally, there is another issue (not reported in the original model but

largely explored in the literature, see Laurie and Jaggi (2003)[xxii], among

others): subjects might be unable to compute the full configuration of the

city. That is, subjects may use local information only to compute the

solutions and therefore cannot generate a map of the best response paths

for every single player.

In that case, the use of focal points is even truer. The subject is only able

to see the relevant cluster (and not the complete configuration of the city)

For instance, WWRRW…

Hence, the player decides whether to move to that place or not. Obviously this also affects beliefs: players do not need to assume that other participants are able to compute all the possible pure strategies, but only choices over the focal points.

In sum, this paper shows that the requirement of sequential choice, which is hard to believe in real life, is not a limitation for the Schelling model. This is true because the sequential movement with random sorting is not a necessary requirement to reach the equilibrium. The replication with positive commuting costs shows nearly identical results.

7. Conclusions

Schelling's spatial proximity model (1969, 1971a) is established with a series of minimal requirements with regard to the subjects that form society: subjects look for their best interest with some slightly inflexible preferences (a neighbor of their same color). Nevertheless, the equilibrium outcome of this model is very powerful: complete segregation of society. The first aim of this study was to verify by means of experimental analysis if complete segregation takes place. The second aim was to verify if Schelling's model would give the same result of absolute segregation after modifying one of its most important properties, namely substituting sequential movement for simultaneous movement. The results obtained are forceful.

1. First, we obtain that when subjects play Schelling's sequential model, the result is a completely segregated society.

2. Secondly, when subjects play Schelling's simultaneous model nothing changes and complete segregation emerges without difficulty.

Therefore, in spite of the fact that the set of information that players handle between one environment and another is radically different - and in addition uncertainty arises after the movements - the result turns out to be the same: complete segregation of the society.

In Schelling's model, the movements of the individuals are ruled to satisfy their preferences. Individuals seek happiness and whatever others have done or are going to do does not intervene in their decision. This is so

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because of the assumption of the sequential movement, which makes them consider their decision as a problem of individual optimization: they do not need to learn anything nor do they have to signal anything for the future.

In the simultaneous environment, the existence of multiple equilibria makes them look for a coordination system. Therefore, a code is established in the first round, which becomes public knowledge for all the individuals. The first round serves to identify the types of players; it is therefore a learning phase. The second round is a new refinement of equilibria where the selection criterion is given by a follow-up to the focal points that have been created in the previous stage. Once more, it is crucial that individuals behave rationally (a player moves if he is not happy) and rationality is common knowledge (all the players know that happy players will remain still). As a consequence, subjects reach in a simple manner the complete segregation in which they all are happy.

The introduction of commuting costs affects subjects' willingness to move: under the presence of moving costs they move much less. However, the lack of movement does not greatly affect the final configuration of the society. In most cases (87.5%), the complete segregated equilibrium is achieved.

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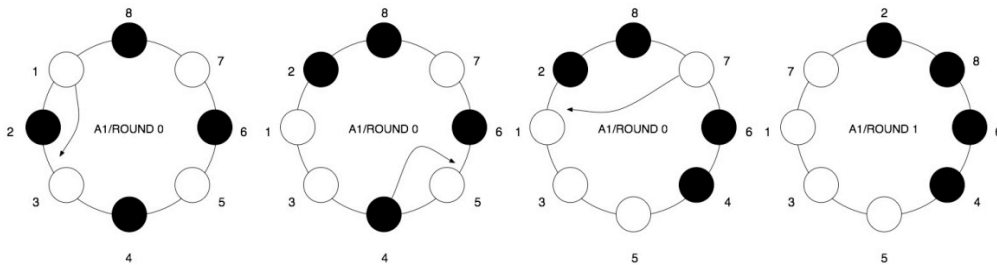
APPENDIX

- a.1) FIGURES

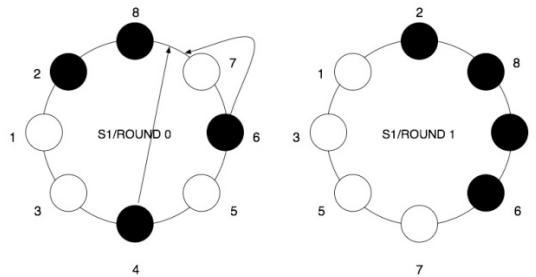- a.2) TABLES

- a.3) EXPERIMENTAL INSTRUCTIONS
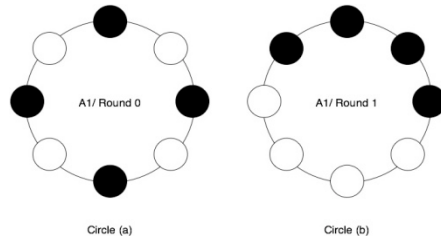
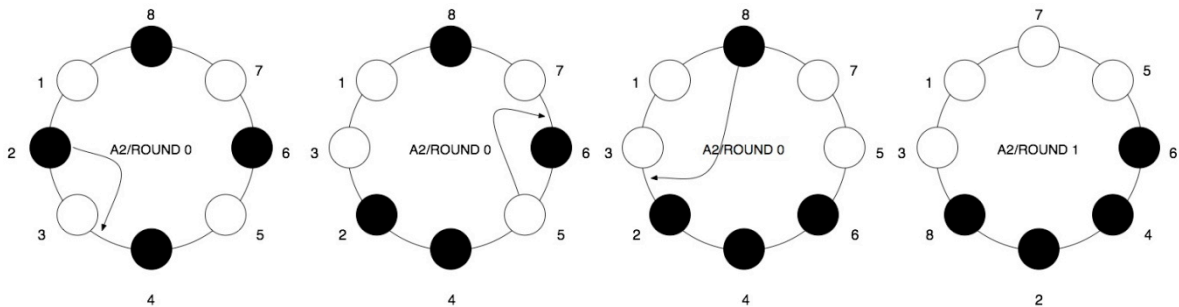Figure 1: Integration (a) versus segregation (b) with N=8 subjects

Figure 2: Movements in the sequential game

Figure 3: Movement of 4 and 6 in the simultaneous game

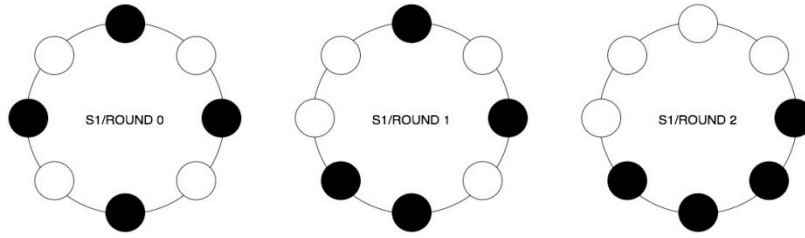Figure 4: Sequential game in which player 1 is unhappy and does not move
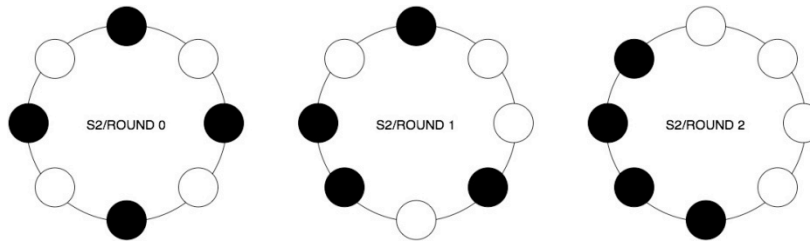
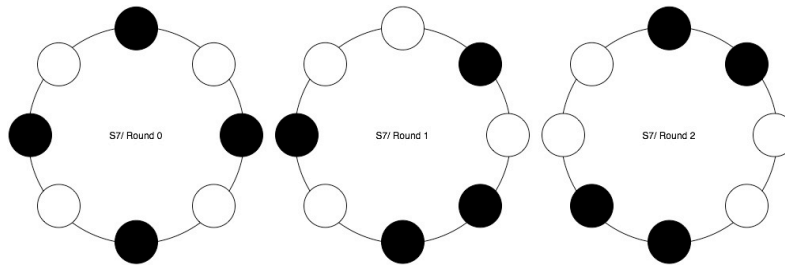Figure 5: Neighbor Focal Points (S1, S3)

Figure 6: Distant Focal Points (S2)

Figure 7: Distant Focal Points (S7)

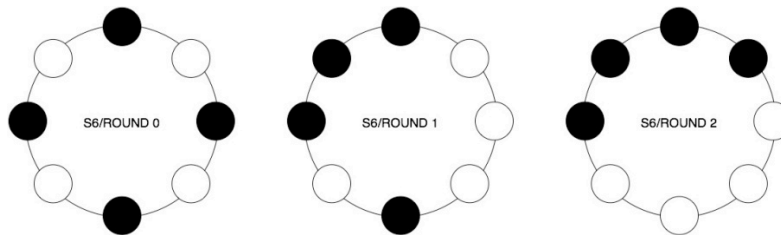Figure 8: Two large focal points (S6)
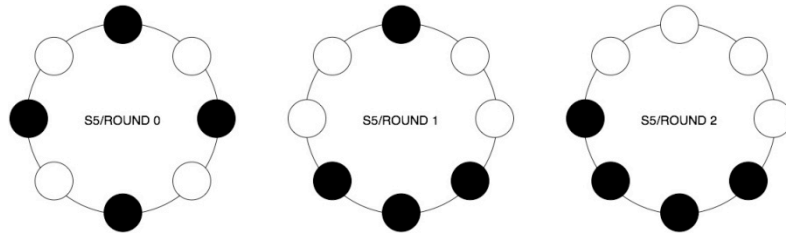
Figure 9: A large focal point and a small focal point (S5)

Group	City	Round	Movers	Non-movers	Happy	Unhappy
A1	Pamplona	0			0	8
		1	1,4,7	2,3,5,6,8	8	0
A2	Pamplona	0			0	8
		1	2,5,8	1,3,4,6	8	0
A3	Valencia	0			0	8
		1	1,4,7	2,3,5,6,8	8	0
A4	Valencia	0			0	8
		1	1,4,7	2,3,5,6,8	8	0

Table 1: Results of the sequential games.

Group	City	Round	Movers	Non-movers	Happy	Unhappy
S1	Pamplona	0			0	8
		1	7	1	4	4
		2	4	4	8	0
S2	Pamplona	0			0	8
		1	5	3	4	4
		2	4	4	8	0
S3	Pamplona	0			0	8
		1	4	4	4	4
		2	3	5	8	0
S4	Pamplona	0			0	8
		1	6	2	8	0
S5	Pamplona	0			0	8
		1	4	4	7	1
		2	1	7	8	0
S6	Valencia	0			0	8
		1	6	2	7	1
		2	1	7	8	0
S7	Valencia	0			0	8
		1	7	1	4	4
		2	4	4	8	0
S8	Valencia	0			0	8
		1	5	3	8	0

Table 2: Results of the simultaneous games

Group	City	Round	Movers	Non-movers	Happy	Unhappy
S1	Pamplona	0			0	8
		1	3	5	4	4
		2	1	7	8	0
S2	Pamplona	0			0	8
		1	2	6	7	1
		2	1	7	8	0
S3	Pamplona	0			0	8
		1	4	4	6	2
		2	1	7	8	0
S4	Pamplona	0			0	8
		1	2	6	8	0
S5	Pamplona	0			0	8
		1	2	6	7	1
		2	1	7	8	0
S6	Pamplona	0			0	8
		1	5	3	6	2
		2	2	6	8	0
S7	Pamplona	0			0	8
		1	2	6	7	1
		2	1	7	8	0
S8	Pamplona	0			0	8
		1	4	4	4	4
		2	2	6	8	0

Table 3: Results of the simultaneous games with costs

EXPERIMENTAL INSTRUCTIONS

The instructions given to the subjects who took part in the experiments are detailed below. Subjects were distributed into groups of eight for both of the treatments: a treatment in which they played Schelling's sequential model (Case A) and another treatment in which they played Schelling's simultaneous model (Case S). The following instructions are for subjects wearing a red scarf in both treatments. The instructions are the same for subjects with a white scarf; the only difference being the color that identifies the subject and the color of the neighbors who make the subjects happy.

Instructions Case A:

1.   Please tie the scarf around your neck.
2.   Eight subjects will participate in the task. Four will sit in one row of desks and another 4 will sit in the row of desks behind them. Please turn around to face all your partners.
3.   There are two types of subjects: those with a white scarf and those with a red one. As you already know, you are Red.

How do I earn money?

4. If at the end of the exercise AT LEAST ONE OF YOUR NEIGHBORS is of your same color, you will earn 2 euros as follows:
    •   If both the neighbor to your right and to your left are white, then you will NOT earn anything.
    •   If the neighbor to your right or to your left (or on both sides) is red, you will earn 2 euros.

5. You are allowed to move (if you want to!). You can seat yourself in the closest space that you wish. A space is the distance between two persons. You can jump as much as you wish (a place, two places, etc). You can only move to your right, that is, counter-clockwise.

6. How can I move? To move, you have to write the place you want to move to on your sheet. Write your current position in blue on your sheet and the position where you want to move to in black. If you do not move, mark your current position in black. Your sheet will be picked up and then you will be told the new set up.

Well, now we are going to play

7. We will now throw a dice. The dice will decide who will be the first person to move. The rest of the players will then move in consecutive order (towards the right). The first player will make his choice (not moving,

moving, jumping one place, jumping two places,…, jumping 6 places). When you are told, you will have to make your choice. Write your current location in blue and the position you are moving to in black on your sheet. If you are not moving, write your current location in black.

8. We will then collect your sheet and tell you your new set up.

9. If at the end of the exercise AT LEAST ONE OF YOUR NEIGHBORS is of your same color then you will earn 2 euros.

Instructions Case S:

1.   Please tie the scarf around your neck.
2.   Eight subjects will participate in the task. Four will sit in one row of desks and another 4 will sit in the row of desks behind them. Please turn around to face all your partners.
3.   There are two types of subjects: those with a white scarf and those with a red one. As you already know, you are Red.

How do I earn money?

4. If at the end of the exercise AT LEAST ONE OF YOUR NEIGHBORS is of your same color, you earn 2 euros as follows:

  •   If both the neighbor to your right and to your left are white, then you will NOT earn anything.
  •   If the neighbor to your right or to your left (or both of them) is red, you will earn 2 euros.

5. You are allowed to move (if you want to!). You can seat yourself in the closest space that you wish. A space is the distance between two persons. You can jump as much as you wish (a place, two places, etc). You can only move to your right, that is, counter-clockwise.

6. How can I move? To move, you have to write the place you want to move to on your sheet. Write your current position in blue on your sheet and the position where you want to move to in black. If you do not move, mark your current position in black. Your sheet will be picked up and then you will be told the new set up.

Well, now we are going to play

7. You will now make your choice (not moving, moving, jumping one place, moving by jumping two places, …, moving by jumping 6 places). When

you are told, you will have to make your choice. Write your current location in blue on your sheet and the position you are moving to in black. If you are not moving, write your current location in black.

8. We will then collect your sheet and tell you your new set up

9. If at the end of the exercise AT LEAST ONE OF YOUR NEIGHBORS is of your same color then you will earn 2 euros.

Figure 9 shows the graph given to the subjects so that they could clearly identify both their position and that of the rest of the individuals in their group. In the simultaneous game, every individual was given a similar graph. The graph was then collected to determine the decision made by each subject. In the sequential game, the same graph was passed from one subject to another, considering the order of movement. The graph was therefore automatically updated with each subjects' annotations.
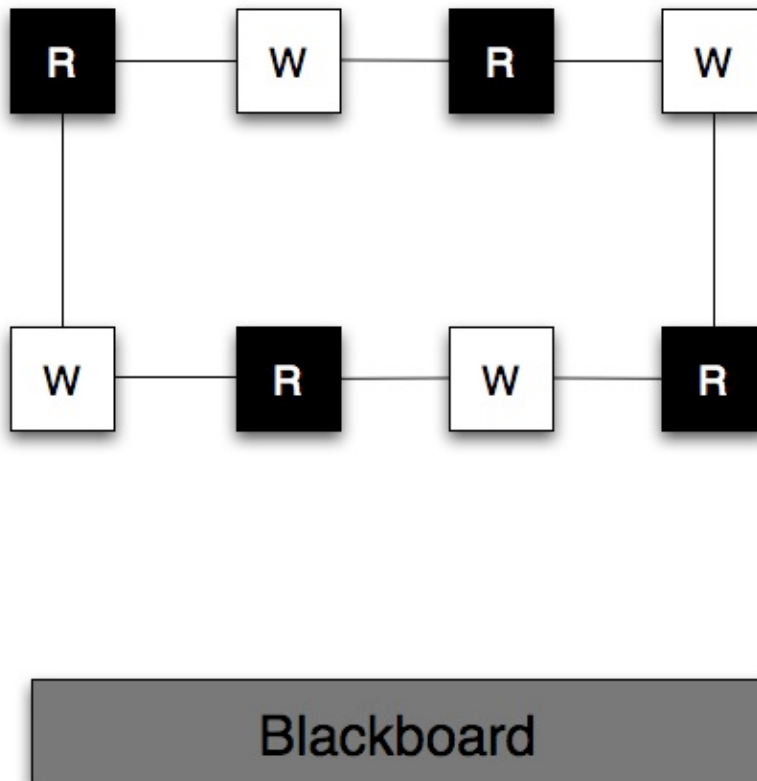

Figure A: Control graph for each group

NOTES

[i] It is interesting to bear in mind that there are many forms of segregation. Segregation can happen in a racial context, but it can also arise for religious reasons, sexual orientation, etc.

[ii] Schelling calls this microeconomic model of neighborhood segregation a *spatial proximity model.* There are other variants of Schelling's model in the literature both in linear and matrix form (see Young, 1998 or Zhang, 2004a, 2004b).

[iii] The number of individuals can be infinite, but Schelling (1971a) refers to the possibility of an infinite continuous line or a circle. The advantage is that in these cases all the individuals have the same number of neighbors.

[iv] Consequently, if we say that each individual has four neighbors, they will be the two on his right-hand side and the two to his left.

[v] In this last frame, together with the assumption that each agent uses the information of the type of neighbors he has to his right and to his left, we could say that agents have very minimum requirements about the composition of their neighborhood.

[vi] In a simultaneous game where all individuals choose at the same time, no one knows *a priori* the choices made by the rest of the agents.

[vii] Dasgupta et al. (2008) also compares a simultaneous and sequential experimental game. They also use colors and subjects have to coordinate. However we require coordination for *n=2* subjects whereas they need full coordination.

[viii] Benito et al. (2009) provide a sequential version of Schelling's experiment in two different settings: with and without moving costs.

[ix] A round is defined in this work as the moment at which all the participants have made their choice. In the simultaneous model a round occurs when everyone has decided, i.e. the first round would be their first choice. In the sequential model a round occurs when all have chosen, that is, when the last agent who is supposed to choose has made his decision.

[x] Even though it is not necessary, symmetry is assumed to mean that the number of subjects *N* is even and that there are *N/2* subjects belonging to each type.

[xi] This should be understood as a not very strict requirement of the model: individuals only need one neighbor equal to them in order to acquire their maximum utility or happiness.

[xii] The decision made by the subject who starts moving is random. But starting from the first one, all the rest move in a consecutive way, for example towards the right. Whether individuals move to the right or to the left hand side is not relevant, what matters is that there is an order of movement and that this movement has to be clear.

[xiii]To simplify the explanation, we assume that subjects 5 and 7 do not move. However, as we will see, in the event that they do move, none of them would choose to locate between subjects 2 and 8 because this position would not be a good strategy for them.

[xiv] It is important to remember that all subjects decide to move (or not move) simultaneously. Therefore, when subject 6 chooses to move, he does not know what subject 4 is going to do.

[xv] Quoting Coricelli and Nagel (2009, p. 9163): "*Psychologists and philosophers define this as theory of mind or mentalizing, the ability to think about others' thoughts and mental states to predict their intentions and actions*".

[xvi] Rizzolatti, Fogassi and Gallese (2006) suggest that in primates and humans there are specific neuronal circuits in order to interiorize the tasks or movements of other human beings or members of the same species or of other species. See also Fogassi et al. (2005) and Gallese, Keysers and Rizzolatti (2004).

[xvii] We keep enough space between subjects (2 meters) to preserve anonymity and communication was strictly forbidden. All the subjects were seated in front of the whiteboard and communication was forbidden. Two monitors were in charge of each session.

[xviii] If all subjects were identical, any movement would generate the initial situation given that all subjects would make the same movement and the only result would be a spin with identical distribution.

[xix] If two people have the same *a prioris*, and their *posterioris* for a given event are common knowledge, then these *posterioris* should also be the same. For a broader discussion about the concept of common knowledge in decision-making in games see Aumann (1976).

[xx] Hence, *final earnings=(2– 0.5*movements)*euros*. In other words, subjects may spend 25% or their potential benefits if they move once, 50% if they move twice, and so on.

[xxi] This is just an example. We should note that the small size (*n=8*) of our game is essential for achieving these results.

[xxii] Laurie and Jaggi (2003) extended the Schelling model of neighborhood racial segregation to include agents who can authentically 'see' their neighbors up to a distance R, which they called 'vision'. They explored how vision interacts with racial preferences and minority concentrations.