Market Power and Growth in a Schumpeterian Framework of Innovation.

Alberto Bucci[*]
University of Ancona (Ancona, Italy)
and
Catholic University of Louvain (Louvain, Belgium)
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[*] IRES - Department of Economics, Catholic University of Louvain, Place Montesquieu, 3 - 1348 Louvain la Neuve, (Belgium).
Fax: + 32 10 473945 - e-mail:bucci@ires.ucl.ac.be

Introduction

Can market power be really considered as *"the price"* that a society as a whole is called to pay in order to have a more dynamically efficient economic system?

The schumpeterian answer to this question would be certainly positive, the *"monopoly power"* being seen as the reward accruing to the successful innovator from his/her innovative activity (Aghion, Dewatripont and Rey, 1997). More precisely, in the schumpeterian tradition (J. Schumpeter, 1942), the bigger this reward is, the larger the incentives to innovate will also be, with the consequence that, within this particular research line, (ex-post) market power is generally considered as an important stimulus to the R&D-activity *(first schumpeterian hypothesis)* and, as such, it should importantly contribute to increase output over time[1].

Obviously, the major premise to this point of view is that the knowledge/R&D capital is really the main "engine of growth" of a country and nowadays such a premise seems to be fully confirmed by the majority of empirical evidence[2]. Lichtenberg (1992), for instance, studies the role that R&D plays in accounting for the international differences in the productivity levels and growth rates among countries at *different development stages* and, at this purpose, he estimates a model (derived from the Mankiw, Romer and Weil paper of 1992) in which the aggregate production function for each country is a Cobb-Douglas with physical, human and R&D capital as inputs[3]. Using non-linear econometric techniques, the author finds, for the entire sample, that the coefficient on the *"R&D intensity"* variable (the ratio between the total R&D expenses and GDP) is both positive and statistically significative in accounting for the international differences in the *GDP growth rate per adult*. The papers by Gittleman and Wolff (1995) and Verspagen (1996) reach a similar conclusion, highlighting that in the long-run there exists a significative correlation between technological variables and real per-capita GDP growth[4].

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[1]many recent studies in the field of the *Economics of Innovation* have shown that a higher concentration in the product market makes the results of the innovative activity more easily *appropriable* by private agents, so fostering their decisions of investment in R&D capital. In particular, Aghion e Howitt (1997) write:*"...(product market competition)...reduces the size of monopoly rents that can be appropriated by successful innovators, and therefore diminishes the incentive to innovate"* (pag. 284).

[2]the empirical literature analysing the relationship between innovative activity and productivity growth (not only at the macroeconomic level) is practically boundless. However, important synthesis of it can be found in Griliches (1979) and, more recently, Monhen (1992).

[3]Lichtenberg's sample consists of 74 different countries and the estimation period is 1960 through 1985.

[4]another very important result stemming from the work by Gittleman and Wolff (1995) is that the R&D activity allows to account for the cross-country differences in the aggregate productivity growth rates *only* when the analysed sample consists of the more (industrially) advanced economies. As far as the medium/low income countries are concerned, the effect of R&D on the aggregate growth rate is almost negligible, the development of these countries being a process mainly based on variables other than the "technological" ones. However, another branch of applied literature (in the field of "technological diffusion") convincingly shows either that the technological spillovers are quantitatively very relevant and that they positively affect (above all) the developing countries. One of the more recent and significative contributions in this area is certainly that by Coe, Helpman and Hoffmaister (1997), showing that: i) on average, a one percent point increase in R&D expenses of the more advanced economies determines a

Briefly, on the empirical side, the innovative activity seems to play a central role in boosting the long-run wealth of nations (not only directly, but also, we would say, indirectly, thanks to the diffusion of the so-called technological spillovers).

On the theoretical side, instead, though the idea (based on Schumpeter's original message) of a potentially positive relationship between monopoly power and aggregate growth is both simple and clear in itself, it is not universally confirmed by those models which are generally defined as *"R&D-Based Growth Models"*.

At this aim, however, it is useful to recall the following two important things:

1) in the New Growth Theory there exist different *"schumpeterian perspectives"*. On the one hand there are models (such as Aghion and Howitt, 1992 and Grossman and Helpman, 1991, chapter 4) in which the growth process is stochastic, reflecting the uncertainty of the innovative activity (from which it derives); on the other hand, there are also models in which the link between growth and innovation is deterministic (e.g. P. Romer, 1990 and Grossman and Helpman, 1991, chapter 3);

2) in general, the first-type models (the stochastic ones, with vertical innovation) predict unambiguously the existence of a negative relationship between (product) market competition and aggregate economic growth (see Aghion and Howitt, 1997 and 1998), whereas the second-type models (the deterministic ones, with horizontal innovation), and in particular the two models cited above, are less definitive on this point.

In order to illustrate this, we analyse, in the next section, the 1990 Romer's and the 1991 Grossman and Helpman's models in detail. The main conclusion we reach is that only the second one accurately respects the message of the first *"schumpeterian hypothesis"* (stating that "ex-post" market power represents a necessary condition in order to stimulate the innovative activity of the industrial enterprises and the technical progress of an economic system as a whole).

However, one important limit of the G-H model is that it does not allow us to calculate an optimal *finite* mark-up over the marginal costs (within the industry producing technologically advanced goods), given the aim of the aggregate economic growth rate maximisation (actually, in that model the "growth-maximising-mark up" would be infinite). In other words, in the context of the G-H model, the relationship between β - the mark-up - and $\gamma(\beta)$ - the aggregate growth rate - approaches a positively sloping straight line, for β going to infinite.

0.06% increase in the output of the developing countries; ii) the technological spillovers coming from the USA are, in absolute terms, the largest either because the USA are the major commercial partner of many developing countries and because the total American R&D expenses are definitely greater than those of any other industrialised country; iii) the main *"propagation channel"* of technological spillovers is international trade. This, in fact, allows the developing countries to use a broader variety of intermediate goods, to imitate more easily the new technologies incorporated in them and, eventually, to increase more rapidly the productivity of their own resources.

On the other hand, in Romer's (1990) model, such a relationship is always negative (meaning that higher levels of market power in the imperfectly competitive industry depress the growth rate of the whole economic system).

On the basis of such very general considerations, we build, in sections 2 and 3 rispectively, two models, taking explicitly into account the theoretical framework suggested by Paul Romer.

In the first one, according to the G-H approach, we introduce in the intermediate industry a "one-to-one" technology employing only human capital, while leaving unchanged (in comparison with the original Romer's paper) the technology of the research sector (the production function of the final good is assumed to be in this first model a Cobb Douglas employing only unskilled labour and N different varieties of capital goods). With these changes, it is possible to note that, in the steady-state, the relationship between β and $\gamma(\beta)$ (which is always negative in Romer) becomes positive for a given range of values of β (for β going to infinity, $\gamma(\beta)$ approaches a positive concave function).

In the second model (in comparison with the previous one), we let the technology of the final output industry employ human capital, instead of unskilled labour. By this assumption we allow for the case of an increased *"inter-sectoral competition"* for the acquisition of the scarce resource (the human capital), now being used in each activity of the economic system. The main conclusion we shall reach through this second model is that $\gamma(\beta)$ is now characterised by a well-defined point of maximum [there exists a finite β^* (the mark-up over the marginal costs in the un-competitive industry) such that $\gamma(\beta^*)$ – the aggregate growth rate – is maximised].

(At the end of the description of each model, we shall give the economic intuition of the different results we get).

In general, what we show in sections 2 and 3 is that in the most familiar "deterministic", neo-schumpeterian models of innovation and economic development, the relationship between (some) measure of market power and the aggregate growth rate is not robust at all. Such a relationship, indeed, depends on variables such as the kind of inputs each industry employs in order to obtain its own output and the way in which these inputs are combined (Cobb-Douglas versus C.E.S., for instance).

In the last two sections we respectively make very general comments on the policy implications of the two models and summarise the most important conclusions of the paper.

1. The role the monopoly power plays in the original papers by P. Romer (1990) and Grossman and Helpman (G-H, Ch.3).

In the original model by P. Romer (1990), the economy aggregate growth rate is equal to $\gamma = \dfrac{\frac{\sigma L}{\eta} - \rho}{\sigma + \alpha}$ ([5]), where $\rho(>0)$ is the individual discount rate, L is the aggregate (fixed) supply of human capital, $\eta(>0)$ is the inverse of the productivity parameter of the "research human capital", $\alpha(0<\alpha<1)$ is a technological parameter and, finally, σ is the inverse of the intertemporal substitution elasticity[6]. (See Appendix A for the derivation of a simple necessary condition that assures that γ be upper-bounded in this model).

Moreover, the price each producer of intermediate goods sets for his own output turns out to be equal to a (fixed) mark-up $(1/\alpha)$ over the marginal cost of production.

If we define this mark-up as $\beta \equiv \dfrac{1}{\alpha}$ (>1), then the growth rate (γ) can also be written as: $\gamma = \dfrac{L - \beta\rho\eta}{\eta(\sigma\beta + 1)}$, from which it is easy to get: $\dfrac{\partial\gamma}{\partial\beta} = \dfrac{-\eta(\eta\rho + \sigma L)}{\eta^2(\sigma\beta+1)^2}$ (<0).

In words, β (the mark-up charged over the marginal cost in the intermediate, monopolistically competitive sector) always affects γ negatively.

In addition to this, it is worth pointing out that in this model the human capital (L) is exclusively used in the final output sector and in the research one and that the market allocates the quantity $L_Y = \dfrac{\sigma L + \eta\rho}{\sigma + \alpha} = \dfrac{\beta(\sigma L + \eta\rho)}{(\sigma\beta + 1)}$ of skilled labour to the homogeneous final good sector and the quantity $L_R = L - L_Y = \dfrac{L\alpha - \eta\rho}{\sigma + \alpha} = \dfrac{L - \beta\eta\rho}{\sigma\beta + 1}$ to research.

Given these values (L_Y and L_R), we can verify that:

- $\dfrac{\partial L_Y}{\partial\beta} = \dfrac{\sigma L + \eta\rho}{(\sigma\beta+1)^2}$ >0, whereas
- $\dfrac{\partial L_R}{\partial\beta} = -\left[\dfrac{\eta\rho + \sigma L}{(\sigma\beta+1)^2}\right]$<0.

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[5]for a simplified version of the Romer's model of Technological Change, see Barro and Sala-i-Martin (1995, pp. 226-230), from which the present formula is taken, and Aghion and Howitt (1998, page 39).

[6]the presence of this term (σ) in the formula of the aggregate growth rate is justified by the fact that in the original Romer's model the istantaneous utility function of the representative agent is assumed to be isoelastic.

The economic intuition behind these results is quite simple: an increase in the mark-ups (and, in this way, in the prices) of all the intermediate inputs, *ceteris paribus* makes it more profitable for the final good producers to replace the capital inputs with human capital. As a consequence, the demand for this factor continuously increases in the final output sector (L_Y increases), to the research sector's detriment (L_R decreases). At the end (when the mark-ups are very high), the whole human capital will likely be allocated to the final good sector instead of the research sector (which is the true engine of growth of Romer's economy).

(For a formal treatment of the relationship between mark-up and growth as stemming from the 1990 Romer's model, see Appendix 1).

The G-H's model (1991, Chapter 3), instead, is completely different. In this particular case, the scarce resource (human capital) is used in both the intermediate and research sectors[7] and in equilibrium each producer of capital goods produces the same quantity of output $\left(x_j = x = \frac{1}{Np}, \forall j \in [1, N] \right)$ at the same price $\left(p_j = p = \frac{1}{\alpha} \cdot w, \forall j \in [1, N] \right)$. So, an increase in the mark-ups of all the intermediates reduces the total output of this sector $\left(X = Nx = \frac{1}{p} \right)$ and, as a consequence, its human capital needs. In this way, the resources that are released by the imperfectly competitive sector are necessarily allocated to the competitive research sector, being (also in this model) the only engine of growth (the steady-state growth rate is $\gamma = \left(\frac{1-\alpha}{\alpha} \right) \frac{\dot{N}}{N} = (\beta - 1) \frac{\dot{N}}{N}$, $\beta \equiv \frac{1}{\alpha} > 1$).

In addition to this, in the G-H original model the steady-state innovation rate is

$$\gamma_N = \left(\frac{\dot{N}}{N} \right)^s = \left(\frac{1-\alpha}{a} \right) L - \rho\alpha = \left(\frac{\beta - 1}{a\beta} \right) L - \frac{\rho}{\beta}.$$

Therefore, we can easily see that the intensity of the innovative effort ($\dot{N}/N$) depends not only on the mark-up ($\partial \gamma_N / \partial \beta$ is, indeed, always positive), but also on L (a measure of the size of the market in which the new variety of capital goods can potentially be sold)[8]. In particular, $\dot{N}/N$ is positive only for certain values of L $\left(L > \frac{a\rho}{\beta - 1} = \frac{a\rho\alpha}{1-\alpha} \right)$.

[7] in particular, the technology used for producing capital goods is a *"one-to-one technology"*, with human capital as the only input.

[8] obviously, this result is strictly linked to the "sunk" nature of the research outlays.

This is why the relationship between γ and β is positive only when β is very large. On the contrary, when β is slightly larger than one, such a relationship can be also negative (see Appendix 2 for major details on this point). Intuitively, when $1 < \beta < A\left(= \sqrt{\frac{a\rho + L}{L}} > 1\right)$, the mark-ups (charged over marginal costs in the monopolistically competitive sector) are yet too small to let the constraint on L (a function of β) be not binding (and, then, $\dot{N}/N$ be positive), so that γ could even be negative. Instead, when β>A (going to infinite), the constraint on L will certainly cease to be binding, $\frac{\dot{N}}{N}$ will tend to L/a and γ to infinite, too[9]. Finally, when β is equal to 1 (as it will be the case in a perfect competition situation), γ will be equal to zero.

In summary, it turns out that:

a) in a dynamic context, as far as the contribution of market power to the growth process of a country is concerned, the G-H's model seems to be much more in line with the original schumpeterian message than Romer's model;
b) whenever the link between γ and β is analysed in a neo-schumpeterian framework of innovation and growth, it is fundamental to examine the way the scarce resources (those productive factors being available in fixed supply) are allocated and employed within the economic system as a whole.

At this purpose, we present below two different models. Such models encompass both the Romer and G-H ones and, through them, we show that it is possible to reach different conclusions about the relationship between monopoly power and economic development, depending on the particular way one models the structure of the economy.

2. Model 1

Following Romer (1990), we imagine an economy which is composed by three sectors, producing respectively an homogeneous final good, N different varieties of technologically advanced goods and knowledge. The total supply of skilled (H) and unskilled (L) labour is exogenously fixed. However, unlike Romer's model, we shall assume in this section that the final output sector uses just unskilled labour and

[9]when $\beta \to +\infty, \gamma(\beta) \to (\beta - 1) \cdot \frac{L}{a}$ (this means that $\gamma(\beta)$ approaches a positively sloping straight line).

intermediate goods, human capital being employed to produce capital goods and "research output" (patents) only. In particular, we shall suppose, following G-H (1991, chapter 3), that the production technology in the intermediate sector is of the "one-to-one" type in the skilled labour.

• *The production of the final output*.

The final output sector is **competitive** and is characterised by the following Cobb-Douglas technology:

(1) $$Y_t = A \cdot L^{1-\alpha} \cdot \sum_{j=1}^{N_t} \left(x_{jt}\right)^{\alpha}, \qquad 0<\alpha<1,$$

where Y is total output, L is unskilled labour, x_j is the quantity of the j-th variety of intermediate inputs which is used for producing Y and A and N represent respectively a productivity parameter and the total number of capital goods invented up to *t* . Unskilled labour is exclusively employed in the final output sector in this model. Moreover, contrary to Romer (1990), in (1) the homogeneous consumer good is obtained without human capital (in the next section we shall analyse the consequences of introducing human capital in (1) as well).

(From now on, in order to ease the notation, the index t, near the variables depending on time, will be omitted).

The representative firm maximises its own instantaneous profits, taking prices as given. Thus, its objective function is:

(2) $$\pi_Y = A \cdot L^{1-\alpha} \cdot \sum_{j=1}^{N} \left(x_j\right)^{\alpha} - \overline{w_L} \cdot L - \sum_{j=1}^{N} p_j \cdot x_j ,$$

where $\overline{w_L}$ is the wage earned (at time t) by a unit of unskilled labour and p_j is the price of a unit of the j-th variety of capital goods. In (2), Y has been taken as the numeraire ($P_Y = 1$).

Differentiating π_Y with respect to x_j and equating the result to zero, we get:

(3) $$\frac{\partial \pi_Y}{\partial x_j} = 0 \Rightarrow A \cdot \alpha \cdot L^{1-\alpha} \cdot \left(x_j\right)^{\alpha-1} = p_j .$$

p_j, in (3), is the j-th intermediate input inverse demand function. Moreover, as L is used only in the final output sector and is in fixed supply, without loss of generality we can set L=1 and re-write (3) as:

(3bis) $A \cdot \alpha \cdot (x_j)^{\alpha-1} = p_j$, which implies

(4) $$x_j = \left(\frac{A \cdot \alpha}{p_j} \right)^{\frac{1}{1-\alpha}}.$$

• <u>*The intermediate goods sector.*</u>

In this sector, each firm produces a (horizontally) differentiated intermediate good which is used by the final good firms as an input. Each intermediate firm has access to the same (one-to-one) technology, employing skilled labour only:

(5) $$x_j = h_j , \qquad \forall j \in [1, N].$$

Given N (the number of technologically advanced goods invented up to t), (5) implies that the total quantity of human capital allocated at time t to this sector (H_j) is equal to:

(5') $$\sum_{j=1}^{N} x_j = \sum_{j=1}^{N} h_j = H_j .$$

The firm producing the j-th variety, at each point in time seeks to maximise her own instantaneous profit, subject to the demand constraint (3bis):

(6) $$\underset{x_j}{Max}\, \pi_j = p_j \cdot x_j - w \cdot x_j = A \cdot \alpha \cdot (x_j)^{\alpha} - w \cdot x_j .$$

The solution to this problem is:

(6') $$\frac{\partial \pi_j}{\partial x_j} = 0 \Rightarrow A \cdot \alpha^2 \cdot (x_j)^{\alpha-1} = w ,$$

where w is the wage paid to a unit of human capital in the intermediate sector (at time t).

Substituting (4) in (6'), we obtain:

(6'bis) $$A \cdot \alpha^2 \cdot \left(\frac{A \cdot \alpha}{p_j} \right)^{-1} = w \Rightarrow p_j = \frac{1}{\alpha} w = p , \qquad \forall j \in [1, N].$$

Given (6'bis), (4) implies that:

(4') $$x_j = \left(\frac{A\cdot\alpha}{p}\right)^{\frac{1}{1-\alpha}} = x, \qquad \forall j \in [1, N].$$

(6'bis) and (4'), taken together, say that each producer of intermediate goods will decide (at time t) to produce the same quantity of output at the same price. In particular, making use of (5'), (4') and (3bis), respectively, it is possible to write:

(5'') $$N\cdot x = H_j \Rightarrow x = \frac{H_j}{N};$$

(3') $$p = A\cdot\alpha\cdot(H_j)^{\alpha-1}\cdot N^{1-\alpha},$$

so that (6) becomes:

$$\pi_j = (p-w)\cdot x = (p-\alpha\cdot p)\cdot x = (1-\alpha)\cdot px =$$

(6'') $$= (1-\alpha)\cdot A\cdot\alpha\cdot(H_j)^{\alpha-1}\cdot N^{1-\alpha}\cdot H_j\cdot N^{-1} =$$

$$= A\cdot\alpha\cdot(1-\alpha)\cdot(H_j)^{\alpha}\cdot N^{-\alpha}, \forall j\in[1,N].$$

This means that the profit each producer of technologically advanced capital goods is able to obtain will be equal for each of them and decreasing in the number of varieties existing in t (N).

• *<u>The research sector.</u>*

Producing the generic j-th variety of capital goods entails the purchase of a specific blueprint (the j-th one) from the **competitive** research sector, being characterised by the following technology:

(7) $$\dot{N} = \frac{1}{\eta}\cdot N\cdot H_N$$

where $\frac{1}{\eta}$ ($\eta > 0$) is the productivity parameter of the human capital employed in the sector (H_N) and N is the number of horizontally differentiated intermediate goods existing at time t.

As this sector is competitive, the price of the j-th blueprint will be equal, at time t, to the discounted value of the profits that can be made, from t onwards, by the j-th firm of intermediate goods.[10]

In other words, it will be the case that:

(8) $$P_{Nt} = \int_t^{\infty} e^{-r(\tau-t)} \cdot \pi_\tau d\tau = A \cdot \alpha \cdot (1-\alpha) \cdot \int_t^{\infty} (H_{j\tau})^{\alpha} \cdot (N_\tau)^{-\alpha} \cdot e^{-r(\tau-t)} d\tau \quad , \qquad \tau > t.$$

In (8), P_{Nt} is the price (at time t) of the generic j-th blueprint, π is the profit of the j-th intermediate firm and r is the (exogenous) interest rate.

From (7), it is clear that producing a new blueprint requires a quantity of human capital equal to $\frac{\eta}{N}$, for a total cost of $w \cdot \frac{\eta}{N}$. As a consequence, the (static) free-entry condition can be stated as:

(9) $$P_N = \frac{\eta}{N} \cdot w \Rightarrow w = \frac{P_N \cdot N}{\eta} \quad ,$$

where P_N assumes the value indicated in (8) and w is the wage paid to a unit of human capital in the research sectot at time t.

In words, (9) simply states that the entry of new firms into the sector will continue until the price that one can obtain from the sale of an additional blueprint equals the production marginal cost of the blueprint itself $\left(\frac{\eta}{N} \cdot w\right)$.

• <u>*Consumers.*</u>

An infinite-lived representative consumer solves the following dynamic problem:

(10) $$\begin{cases} \underset{Y_t}{Max} U_0 = \int_0^{\infty} e^{-\rho t} \cdot \log(Y_t) dt \\ s.t.: \dot{W}_t = w_t + r_t \cdot W_t - Y_t \end{cases}$$

and the transversality condition, stating that $\lim_{t \to \infty} \lambda_t \cdot W_t = 0$ (λ_t being the so-called *"co-state variable"*)[11].

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[10] in fact, there exists a 1:1 relationship among the number of blueprints (produced in the research sector), the number of firms operating in the intermediate sector and the number of capital goods.

[11] the reason why a transversality condition is considered is that the value of the consumer wealth (W) must be asymptotically equal to zero, otherwise something valuable would be left over.

In (10), U_0 is the intertemporal utility function, $\log(Y_t)$ is the instantaneous utility function, $\rho(>0)$ is the individual discount factor and W, w and r are, respectively, the total wealth of our consumer, his/her wage and the (exogenous) interest rate (at time t).

Applying the Maximum Principle to (10) yields:

$$(11) \qquad \gamma_Y = \frac{\dot{Y}_t}{Y_t} = r_t - \rho \ .$$

In steady-state, when the left hand side is constant, r will be constant, too.

• *The equilibrium in the Human Capital Market and the Steady State.*

In order to determine the optimal allocation of the (fixed) total stock of human capital to the two sectors using this resource (respectively the intermediate sector and the research one), in what follows we shall employ the same Romer's methodology consisting in equating the equilibrium wage rates of the two sectors. To do so, we have just to imagine that the human capital is an homogeneous input within this economy and that the same skills actually being used to produce, say, technologically advanced goods can be put into use to produce, with the same productivity level, also knowledge.

If this is the case, then two conditions must simultaneously be met:

$$(12) \qquad w_j = w_N$$

$$(13) \qquad H = H_j + H_N, \ \forall t.$$

The first condition (12) says that the wage earned by a unit of human capital in the capital goods sector (w_j) must be equal to the wage earned by the same unit of human capital in the case this one were used to produce blueprints (w_N).

The second condition, instead, is a simple constraint, in accordance to which, at any point in time, the sum of the human capital stocks employed in both the intermediate sector (H_j) and the research one (H_N) must exactly be equal to the fixed supply (H).

Given this, let's begin considering the condition stated in (12). The value of w_j is expressed by (6'); in order to determine w_N, it is worth noticing that in steady state H_N is constant and, according to (13), H_j will be constant, too. So, on the balanced growth path, each sector employing human capital will be able to use a constant amount of this input. As a consequence, (8) can be written as:

(8') $$P_{Nt} = A \cdot \alpha \cdot (1-\alpha) \cdot \left(H_j\right)^{\alpha} \cdot \int_t^{\infty} \left(N_\tau\right)^{-\alpha} \cdot e^{-r(\tau - t)} d\tau \quad , \qquad \tau > t.$$

Since $\frac{\dot{N}_t}{N_t} = \frac{1}{\eta} \cdot H_N$ (=constant), we also have that $N_t = N_0 \cdot e^{\left(\frac{1}{\eta} H_N\right)t}$, which implies that $\left(N_\tau\right)^{-\alpha} = \left(N_t\right)^{-\alpha} \cdot e^{-\left(\frac{\alpha}{\eta} H_N\right)(\tau - t)}$.

In conclusion:

$$P_{Nt} = A \cdot \alpha \cdot (1-\alpha) \cdot \left(H_j\right)^{\alpha} \cdot \int_t^{\infty} \left(N_t\right)^{-\alpha} \cdot e^{-\left(\frac{\alpha}{\eta} H_N\right)\cdot(\tau - t)} \cdot e^{-r(\tau - t)} d\tau =$$

(8'') $$= A \cdot \alpha \cdot (1-\alpha) \cdot \left(H_j\right)^{\alpha} \cdot \left(N_t\right)^{-\alpha} \cdot \int_t^{\infty} e^{-\left(\frac{\alpha}{\eta} H_N + r\right)(\tau - t)} d\tau =$$

$$= A \cdot \alpha \cdot (1-\alpha) \left(H_j\right)^{\alpha} \cdot \left(N_t\right)^{-\alpha} \cdot \frac{\eta}{r \cdot \eta + \alpha \cdot H_N}.$$

Given P_{Nt}, it is easy (from (9)) to get w_N. Further, equating w_j with w_N, we can write an expression for H_j, as a function of α, r, η and H_N [12]. Finally, from (13), we can first of all calculate H_N:

(13') $$H_N = H - H_j = H - \left(\frac{\alpha}{1-\alpha}\right) \cdot \left(r \cdot \eta + \alpha \cdot H_N\right) = \left(\frac{1-\alpha}{1-\alpha+\alpha^2}\right) H - \left(\frac{\alpha}{1-\alpha+\alpha^2}\right) \cdot r\eta \quad ,$$

and, then, write H_j as:

(14) $$H_j = \left(\frac{\alpha}{1-\alpha+\alpha^2}\right) \cdot \left(r\eta + \alpha H\right).$$

Briefly, in steady state:

a) r is constant (and exogenously given);

b) H_j and H_N are both constant;

c) $\frac{\dot{N}_t}{N_t} = \frac{1}{\eta} H_N = \frac{1}{\eta}\left(\frac{1-\alpha}{1-\alpha+\alpha^2} H - \frac{\alpha}{1-\alpha+\alpha^2} r\eta\right)$ is constant;

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[12]this is: $H_j = \left(\frac{\alpha}{1-\alpha}\right) \cdot \left(r\eta + \alpha H_N\right)$.

d) $\frac{\dot{Y}_t}{Y_t} = (1-\alpha)\frac{\dot{N}_t}{N_t} = \left(\frac{1-\alpha}{\eta}\right)\cdot\left(\frac{1-\alpha}{1-\alpha+\alpha^2}H - \frac{\alpha}{1-\alpha+\alpha^2}r\eta\right)$ is also constant.

In order to find the steady-state value of r, from (11):

$$(11')\; r = \frac{\dot{Y}_t}{Y_t} + \rho = \left(\frac{1-\alpha}{\eta}\right)\cdot\left(\frac{1-\alpha}{1-\alpha+\alpha^2}H - \frac{\alpha}{1-\alpha+\alpha^2}r\eta\right) + \rho \Rightarrow r = \frac{(1-\alpha)^2}{\eta}\cdot H + \left(1-\alpha+\alpha^2\right)\cdot\rho .$$

Thus, given r, we can write the following relationships:

$$(15\text{-}1)\quad \gamma_Y = \frac{\dot{Y}_t}{Y_t} = r - \rho = (1-\alpha)\cdot\left[\left(\frac{1-\alpha}{\eta}\right)H - \alpha\rho\right];$$

$$(15\text{-}2)\quad H_N = \left(\frac{1-\alpha}{1-\alpha+\alpha^2}\right)H - \left(\frac{\alpha}{1-\alpha+\alpha^2}\right)\cdot r\eta = (1-\alpha)H - \alpha\eta\rho;$$

$$(15\text{-}3)\quad H_j = \left(\frac{\alpha}{1-\alpha+\alpha^2}\right)\cdot\left(r\eta + \alpha H\right) = \alpha\left(H + \eta\rho\right);$$

$$(15\text{-}4)\quad \gamma_N = \frac{\dot{N}_t}{N_t} = \frac{1}{\eta}H_N = \left(\frac{1-\alpha}{\eta}\right)H - \alpha\rho .$$

In particular, (15-1) can also be written as:

$$(15.5)\quad \gamma_Y(\beta) = \frac{(\beta-1)^2 H - (\beta-1)\eta\rho}{\eta\beta^2}, \qquad \beta \equiv \frac{1}{\alpha} > 1 .$$

(15.5) states that the output growth rate of this economy (γ_Y) is a function of the given total stock of human capital (H), the model parameters (η and ρ) and, above all, β (the mark-up charged over the marginal cost of production by the monopolistically competitive producers of intermediate goods).

A qualitative graph of $\gamma_Y(\beta)$ is the following (see Appendix 3 for its derivation):

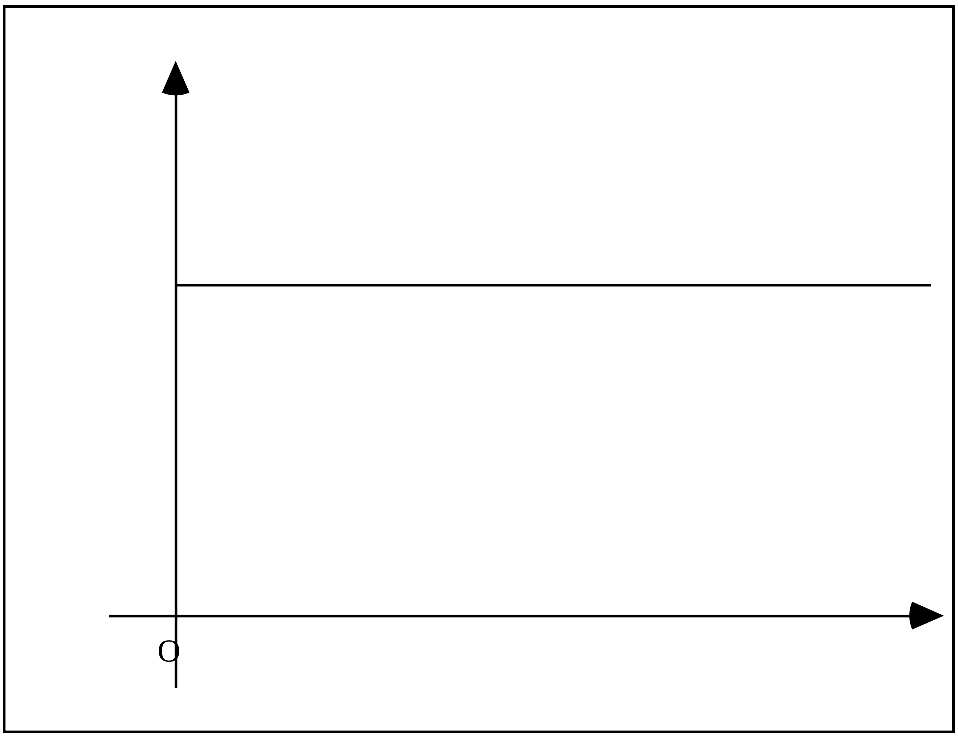

Fig. 1

In order to try to explain economically why Fig. 1 displays right that particular shape, we re-write (15-4) as:

(15-4 bis) $\gamma_N = \frac{1}{\eta}\left(\frac{\beta-1}{\beta}\right)H - \frac{\rho}{\beta}$, $\beta \equiv \frac{1}{\alpha} > 1$.

From (15-4 bis) it is clear that, given η and ρ, γ_N depends both on β ($\partial\gamma_N / \partial\beta$ is always positive) and H. So, γ_N will be positive if and only if $H > \frac{\eta\rho}{\beta-1}$.

Moreover, according to (15-1):

$$\gamma_Y = \left(\frac{\beta-1}{\beta}\right)\cdot\gamma_N.$$

Let's now consider three different ranges of β:

1) when β is slightly larger than 1, the constraint on H comes out to be binding and γ_N (and, as a consequence, γ_Y) may easily be negative. Basically, in this first range of β values, the lack of a "strong" market power enjoyed by the innovating firms requires a market large enough to make the innovative activity profitable;

2) when β takes on sufficiently high values, the constraint on H is likely to be no more operative, so that both γ_N and γ_Y may be positive. In this case, the monopoly rents accruing to the successful innovator are so high that, in order to stimulate firms to innovate, there is no need for a large-sized potential market;

3) finally, when β tends to infinite, both γ_N and γ_Y will tend to $\frac{H}{\eta}$ (the highest aggregate growth rate this economy may reach).

At this point, it is useful to make two further remarks.

The first one is that (15-4) exactly corresponds to the steady state innovation rate found by G-H (1991, chapter 3, page 61), whereas this is not true for (15-1).

This is due to the fact that in this model we have assumed that the technology being used to produce the homogeneous final output is Cobb-Douglas (and not C.E.S., as in the G-H's model)[13].

Therefore, the main conclusions we can draw so far are the following:

1) in order to have a positive relationship between β and γ (at least in any relevant range of β), it is fundamental to create a sort of *competition* between the intermediate sector and the research one for the acquisition of the scarce resource (human capital)[14];

2) in addition to this, the "type" of production function which is employed in the final output sector matters since it is potentially able (for high values of β) to modify the shape of the above-mentioned relationship;

3) finally, the final output sector technology itself influences the level of the steady-state aggregate growth rate of the economy.

A second observation we would like to make about the model outlined in this section concerns the *"behaviour"* over time of x (the quantity of each intermediate input which is produced in equilibrium).

(5'') implies that $x_t = \frac{H_{jt}}{N_t}$. In steady-state:

a) $H_{jt} = H_j, \quad \forall t$;

---

[13] in fact, it can be checked that, given the production function:

[A] $$Y = \left[\sum_{j=1}^{N} \left(x_j \right)^{\alpha} \right]^{\frac{1}{\alpha}}, \quad 0 < \alpha < 1,$$

in the steady state (where $x_j = x \ \forall j \in [1, N]$ and X=Nx=constant), [A] can also be written as:

[A'] $$Y = \left(N \cdot x^{\alpha} \right)^{\frac{1}{\alpha}} = N^{\frac{1-\alpha}{\alpha}} \cdot X \Rightarrow \frac{\dot{Y}}{Y} = \left(\frac{1-\alpha}{\alpha} \right) \frac{\dot{N}}{N}.$$

In our model, instead, final output sector technology is Cobb-Douglas and the growth rate is equal to:

(15-1) $$\frac{\dot{Y}}{Y} = (1-\alpha) \frac{\dot{N}}{N}.$$

Comparing [A'] with (15-1), it is evident that, with the same intensity of innovative effort ($\dot{N}/N$) and with identical technologies in the intermediate and research sectors, the higher value of γ_Y in [A'] can only be imputed to having used (in the G-H's model) a C.E.S. production function.

[14] this is what we shall call, below, *"factor competition"*.

b) $\frac{\dot{N}_t}{N_t} = \frac{1}{\eta} H_{Nt} = \frac{1}{\eta} H_N$ = constant.

From (b), we have that $N_t = N_0 \cdot e^{\left(\frac{1}{\eta}H_N\right)t}$ (where N_0 is the number of capital goods varieties existing at t=0). So, on the equilibrium growth path:

(5''bis) $$x_t = \frac{H_j}{N_0 \cdot e^{\left(\frac{1}{\eta}H_N\right)t}} = \frac{H_j}{N_0} \cdot e^{-\left(\frac{1}{\eta}H_N\right)t} = K \cdot e^{-\left(\frac{1}{\eta}H_N\right)t}, \quad K \equiv \frac{H_j}{N_0} > 0.$$

A qualitative graph of the x_t function is the following:

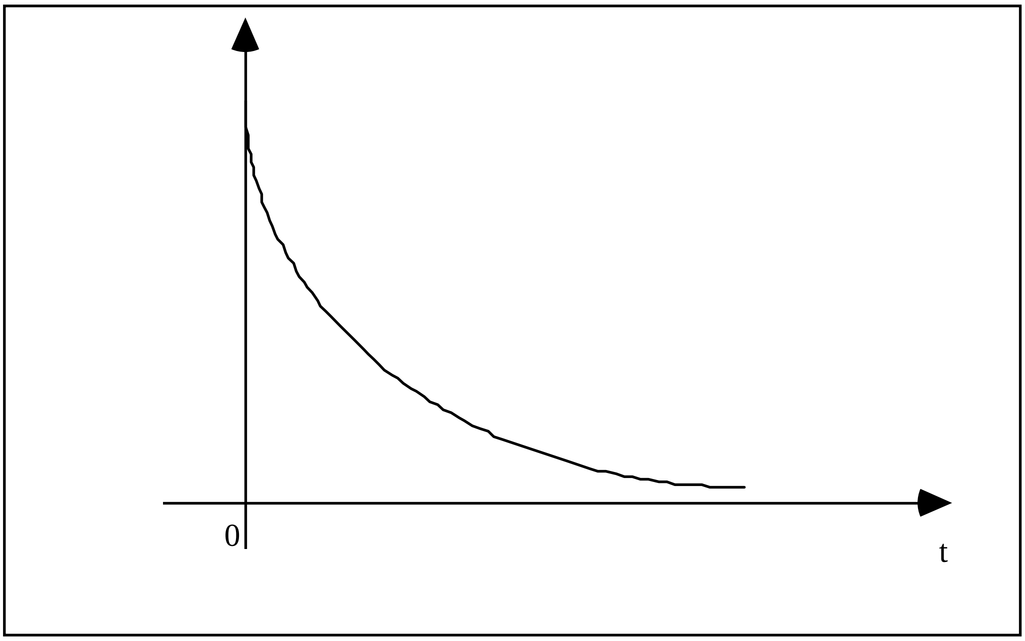

Fig. 2

Fig. 2 implies that x_t goes to zero, but only in infinite time: each producer of intermediates reduces his/her output over time, whereas the total quantity of the sector's output ($N_t \cdot x_t$) remains always constant (and equal to H_j). The fact that the equilibrium growth rate of output, given certain values of H, is positive (even though the total production of capital goods is constant) depends on the **endogenous increase of N** over time[15] and confirms, once again, that in such schumpeterian-flavour models the real engine of growth is represented by the industrial innovation .

Briefly, what the analysis outlined in this section confirms is that market power (β) and market size (H) represent two basic*"ingredients"* for economic development in a general equilibrium deterministic schumpeterian framework of innovation and growth[16]. At the same time, it seems also to emphasise that the way the (scarce)

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[15] a similar result can also be found in the original model by G-H.

[16]Jones (1995) shows that in most OECD countries, the number of engineers and scientists engaged in the R&D activity has risen dramatically over the past half century, with no tendency for per-capita growth

resources are distributed and employed in each economic sector is of paramount importance in changing the shape of the monopoly power - growth relationship.

3. What happens to the relationship between β and γ if the "inter-sectoral-factor-competition" increases?

The economic intuition we can draw from the previous section's model is quite simple: if the fixed-supply input (H) is "contended" by the (monopolistically competitive) intermediate sector and the research sector, then an increase in the monopoly power enjoyed by each producer of capital goods $\left(\frac{1}{\alpha}\right)$ reduces, *ceteris paribus*, the total intermediates output ($N \cdot x = H_j$, see (15-3)). As a consequence, the total stock of human capital that may be used in the research sector (H_N) increases, to the advantage of the aggregate growth rate.

But what happens if the human capital is also used in the final output sector? The answer to this question represents the core of the next section.

3.1 Model 2: An extension of the previous one.

In comparison with the model of the previous section, the present one differs only for the final output technology, being now the following:

(16) $$Y_t = A \cdot H_{Yt}^{1-\alpha} \cdot \sum_{j=1}^{N_t} x_{jt}^{\alpha}, \qquad 0 < \alpha < 1,$$

where H_Y is the quantity of skilled labour used for producing Y.

If we retrace the same steps we have seen in detail in the previous section, then it is possible to write the following main relationships:

- *The final output sector.*

(17) $$p_j = A \cdot \alpha \cdot H_Y^{1-\alpha} \cdot (x_j)^{\alpha-1} \Rightarrow$$

---

rate to increase proportionally. Obviously, this evidence contradicts one of the main conclusions of this model (and the other most famous R&D-based growth models). Recent attempts to model the growth process of a country as a process not depending (or only asymptotically not depending) on the so-called "scale-effects" include, besides Jones (1995), Alwyn Young (1998), Peretto and Smulders (1998), Jones (1997) and Aghion and Howitt (1998, pp.407-415). In the last two models, the steady state growth rate of the economy turns out to depend positively upon the population growth rate.

(17’) $$x_j = H_Y \cdot \left(\frac{A\cdot\alpha}{p_j}\right)^{\frac{1}{1-\alpha}}.$$

(17) represents the j-th variety capital good (inverse) demand coming from the representative firm producing the homogeneous final output.

• *The technologically advanced goods sector.*

(18) $$w = A\cdot\alpha^2 \cdot H_Y^{1-\alpha} \cdot (x_j)^{\alpha-1};$$

(19) $$p_j = \frac{1}{\alpha}\cdot w = p, \qquad \forall j \in [1; N];$$

(20) $$x_j = H_Y \cdot \left(\frac{A\cdot\alpha}{p}\right)^{\frac{1}{1-\alpha}} = x = \frac{H_j}{N}, \qquad \forall j \in [1; N].$$

(18) shows the wage paid to human capital in this sector at time t, whereas (19) and (20) represent, respectively, the price and output of each variety of capital goods.

Finally, given (18), (19) and (20), it is easy to obtain the profit of the j-th intermediate producer:

(21) $$\pi_j = A\cdot\alpha\cdot(1-\alpha)\cdot H_Y^{1-\alpha}\cdot H_j^{\alpha}\cdot N^{-\alpha}, \qquad \forall j \in [1; N].$$

• *The research sector.*

(22) $$\dot{N} = \frac{1}{\eta}\cdot N \cdot H_N, \quad \eta > 0;$$

(23) $$P_{Nt} = \int_t^{\infty} e^{-r(\tau-t)}\cdot\pi_\tau\, d\tau = \int_t^{\infty} e^{-r(\tau-t)}\cdot\left[A\cdot\alpha\cdot(1-\alpha)\cdot(H_{Y\tau})^{1-\alpha}\cdot(H_{j\tau})^{\alpha}\cdot(N_\tau)^{-\alpha}\right]d\tau, \tau > t;$$

(24) $$P_{Nt} = \frac{\eta\cdot w_t}{N_t} \Rightarrow w_t = \frac{P_{Nt}\cdot N_t}{\eta}.$$

(22) is the "production function" of blueprints; (23) is the price of a generic blueprint at time t and (24) is the (static) free-entry condition.

• *Consumers.*

According to what was stated in the previous section (as far as the consumers' side is concerned), the main relationship that will be considered in this section is:

(25) $$\gamma_Y = \frac{\dot{Y}_t}{Y_t} = r_t - \rho$$.

• *The human capital market equilibrium and the steady state*

In order to determine the optimal allocation of the (fixed) total stock of human capital to the three sectors using this resource, the conditions we shall use are:

(26) $H = H_{jt} + H_{Nt} + H_{Yt}$, $\forall t$;

(27) $w_Y = w_j$;

(12) $w_j = w_N$.

In particular, (26) states that now H must be "distributed" to all of the three sectors composing the economic system.

Needless to say that (26), (27) and (12) must be simultaneously met.

Following a procedure similar to the one used in the last section, it is possible to show that r, γ_Y, H_N, H_Y e H_j assume, respectively, these values (see Appendix B for a formal derivation of this result):

(31) $$r = \frac{\alpha \cdot (1-\alpha)^2}{\eta} \cdot H + \alpha\rho \cdot \left(\alpha^2 + 2 - 2\alpha\right);$$

(25') $$\gamma_Y = r - \rho = \frac{\alpha \cdot (1-\alpha)^2}{\eta} \cdot H + \rho\left(\alpha^3 + 2\alpha - 2\alpha^2 - 1\right);$$

(30') $$H_N = \alpha \cdot (1-\alpha) \cdot H - \eta\rho\left(\alpha^2 + 1 - \alpha\right);$$

(28''') $$H_Y = (1-\alpha) \cdot \left(H + \eta\rho\right);$$

(29'') $$H_j = \alpha^2\left(H + \eta\rho\right).$$

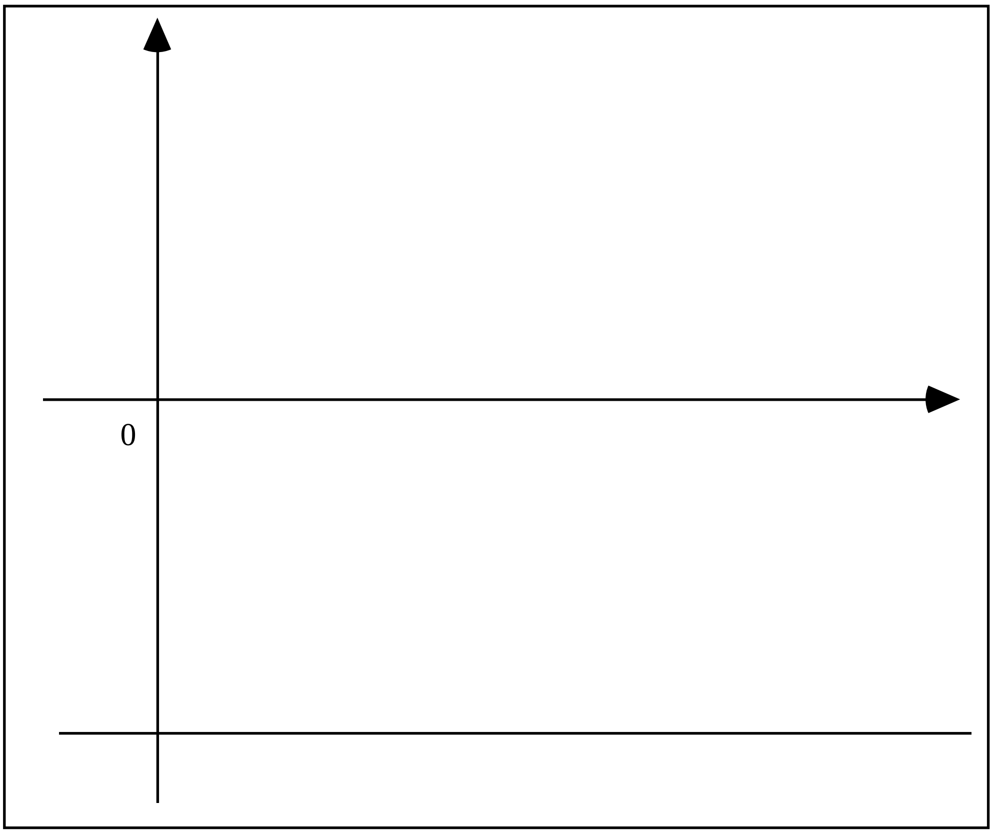
(25') shows, according to the most famous *R&D-based models*, that the aggregate growth rate depends:

• positively on the scale factor (H);

• positively on the productivity parameter of research-human capital $(1/\eta)$;

• negatively on ρ.

As far as the relationship between γ_Y and the mark-up rate $\left(\frac{1}{\alpha} \equiv \beta\right)$ is concerned, its (qualitative) graph corresponds to the following Fig. 3 (see Appendix 4):



<u>Fig. 3</u>

From Fig. 3, it is clear that the higher *"inter-sectoral competition"* for the acquisition of the scarce resource allows us to determine the exact dimension that mark-ups (in the intermediate sector) must have $(\cong 2{,}2)$[17] in order for the maximum growth rate to be reached[18]. In addition to this, in that figure the decreasing length of $\gamma(\beta)$, after point C, depends on the fact that when market power takes on values higher than an upper limit and goes to infinite (α goes to zero), both H_j and H_N go to zero (see (29'') and (30')) and the entire human capital stock is employed only in the final output sector (which is not the "growth-engine-sector").

Finally, Fig. 3 seems to be consistent (when β is included in the E-D range) with the results recently found by **Smulders** and **van de Klundert (1995)** about the linkage between (product) market concentration and growth (although, in their model, the research activity is both deterministic and "*firm-specific*"). In fact, they conclude (page. 150):

[17]as it is explained in Appendix 4, this point of maximum is obtained under the specific assumption that H is sufficiently large (and, in particular, $H = 4\eta\rho(> 3\eta\rho)$).

[18]however, it has to be stressed that, compared with the previous model, γ_Y is now lower. This can be explained by the fact that it is H_N itself to be now lower ((30') is always less than (15-2)).

"[...] increasing concentration is conducive to growth to some critical level. It should be recalled that innovation is an in-house activity. Each firm has to do its own research for the product it produces, which induces fixed costs. With more concentration, aggregate fixed cost is reduced and innovation is enhanced. Excessive concentration, however, depresses innovation because large monopoly power induces firms to aim at high prices and lucrative current production rather than at innovation for high future profits".

At the end of this second *"exercise"*, the following two major ideas seem to be corroborated: within a general equilibrium deterministic schumpeterian framework of innovation and growth:

1) the existence of a positive relationship between β and γ (at least in a relevant range of values of β) is due to the employment of human capital in the intermediate sector;

2) the final output technology (its functional form and the inputs it uses) importantly contributes to influence (especially for high values of β) the shape of the above-mentioned relationship.

4. "...Schumpeter might really be right". Some policy evaluations about the models presented above.

In this paragraph we briefly analyse two major policy implications stemming from the models presented above.

The first one concerns the possible impact of monopoly power on the development level of a nation and can be stated as follows: given the general structure of the economy we have been considering throughout this paper, and under the specific hypothesis that the main goal of the Economic Policy Authorities is the maximisation of the aggregate growth rate, the existence of market power in the intermediate sector should be, at least to a certain degree, tolerated.

In fact, it is clear from the two models we presented that there always exists a range of β values (greater than one) in which $\gamma(\beta)$ is positive and increasing. Therefore, our analysis supports the main conclusion of *"Capitalism, Socialism and Democracy"* (Schumpeter, 1942), according to which the monopolies assessment has to be carried out not only in terms of *static (in)efficiency*, but also (and above all) in terms of *dynamic efficiency*. In this sense, the dynamic gains rising from the continuous technological progress (induced by lower and lower levels of competition in the product market) should be compared with the (static) welfare losses related to a higher (product) market "monopolization". And it is not impossible that, at least for particular values of the mark-up term, the monopoly gains exceed the monopoly losses.

The other related issue concerns the size the mark-up (charged in the intermediate sector) has to take on in order to maximise the aggregate growth rate.

From this point of view, we would suggest the necessity for a public regulator to implement a rule-of-reason-type approach. In fact, since an indiscriminate action against any kind of monopoly might be (according to our models) inappropriate, the choice of the "tolerable", growth-maximising β is tightly linked to the "type" of economy the regulator faces and to his/her knowledge of the main characteristics of this economy (which production processes do the different sectors use? In which sector is monopoly power localized? How is the scarce resource distributed among the different sectors ?). Depending on the answers to these questions, two different situations may arise:

1) in the first one, the "optimal" (growth-maximising) mark-up would be infinite (model 1 of this paper and G-H, 1991, chapter 3);

2) in the second one, it should assume, for sufficiently large H (the total stock of human capital), a finite (greater than one) value (model 2).

Conclusions

One of the main findings of the **New Growth Theory** is to consider Technological Change not simply as a *"manna from heaven"* (J. Fagerberg, 1994), but, on the contrary, as the outcome of some activity intentionally conducted by private, profit seeking agents (and R&D is certainly one of such possible activities).

In order to make formally explicit the positive relationship between the economic growth rate (γ) and the intensity of innovative effort, in the early 1990's several theoretical models [Segerstrom et Alii (1990); Romer, P. (1990); Grossman & Helpman (1991)] have been published. All of them share the common feature that technology is a non-rival and partially excludable good, so that it is a powerful source of positive externalities.

In such a framework, it is not surprising that the only force capable to stimulate private agents to innovate be represented by some measure of *"monopoly power"* (β), accruing to the potential innovator (if successful).

However, the idea to consider the existence of any possible positive linkage among market power, innovation and growth is not unambiguously present in the so-called *deterministic, neo-schumpeterian R&D-based growth models*, even if all of these approaches imagine a very similar general economic structure (Aghion & Howitt, 1997).

Starting from this point, in sections 2 and 3 of this paper we proposed two models encompassing both the Romer and G-H approaches and showed that, within a more composite schumpeterian framework of deterministic innovation, a positive relationship between market power and growth can stem only under particular hypothesis concerning the kind of technology and inputs which are employed in each sector composing the economic system. In other words, depending on the way the structure of the economy is modelled, increasing market power may or may not stimulate R&D, hence innovation and growth.

We think that this result is particularly relevant in terms of public policies towards the intermediate monopolies, suggesting the necessity for a growth-maximising regulator to assess case-by-case (depending on the general characteristics of the economic system he or she faces) the type of his/her intervention.

Finally, as far as the future research work is concerned, it could be particularly interesting, from our point of view, to analyse how the monopoly power-growth relationship eventually changes in the case an economic system (with our characteristics) accumulates over time not only "ideas" (in the form of new, horizontally differentiated intermediate goods), but also human capital, through a separate "education sector". More generally, such an extension might be useful not only because it is intended to relax an hypothesis which we find in all the most important models of R&D-driven growth (namely that the supply of human capital is always fixed), but also because it would allow us to verify how the development of an economy is going to be influenced by the simultaneous presence of two different state variables.

Appendix A

In this appendix we derive the condition under which the steady-state growth rate of Romer's (1990) model is bounded.

If we assume that the growth rate of population (n) is equal to zero, that L(0) is equal to one and that δ (the capital depreciation rate) is also equal to zero, then the consumer problem can be expressed as:

$$\begin{cases} \underset{\{c_t\}_{t=0}^{\infty}}{Max} U_0 \equiv \int_0^{\infty} e^{-\rho t} \cdot \left[\dfrac{c_t^{1-\sigma} - 1}{1-\sigma} \right] dt \\ s.t.: \dot{k}_t = r_t \cdot k_t + w_t - c_t \end{cases}$$

From the First Order Conditions of this problem, we get:

(a) $\dfrac{\dot{c}_t}{c_t} = \dfrac{1}{\sigma} \cdot \left[-\dfrac{\dot{\lambda}_t}{\lambda_t} - \rho \right]$, where λ_t is the "Hamiltonian multiplier";

(b) $-\dfrac{\dot{\lambda}_t}{\lambda_t} = r_t$;

(c) $\lim_{t \to +\infty} \lambda_t \cdot k_t = 0$

Given (b), from (a), we get:

(a') $\dfrac{\dot{c}_t}{c_t} = \dfrac{1}{\sigma} \cdot [r_t - \rho] = \dfrac{1}{\sigma} \cdot (r - \rho)$

(in steady-state r is constant).

In steady state we also know that all the variables depending on time grow at the same constant rate. Therefore, we have that:

(1) $\dfrac{\dot{k}_t}{k_t} = \dfrac{\dot{c}_t}{c_t} = \dfrac{1}{\sigma} \cdot (r - \rho)$, from which it turns out that

(2) $k_t = k_0 \cdot e^{\frac{1}{\sigma}(r-\rho) t}$.

In addition to this, from (b), we obtain:

(b') $\lambda_t = \lambda_0 \cdot e^{-rt}$

As a consequence, (c) becomes:

(c')
$$\lim_{t\to+\infty}\lambda_t\cdot k_t=\lim_{t\to+\infty}\lambda_0\cdot k_0\cdot e^{-rt}\cdot e^{\frac{1}{\sigma}(r-\rho)t}=$$
$$=\lim_{t\to+\infty}\lambda_0\cdot k_0\cdot e^{-\left[r-\left(\frac{r-\rho}{\sigma}\right)\right]\cdot t}=0\Rightarrow\left[r-\left(\frac{r-\rho}{\sigma}\right)\right]>0\Rightarrow$$
$$\Rightarrow r<\frac{\rho}{1-\sigma}$$

In the simplified version of the Romer's model of endogenous technological change we have been considering throughout the paper, r is equal to (see Barro and Sala-i-Martin, 1995, page 229):

$$r=\frac{\sigma\alpha L+\eta\rho\alpha}{\eta(\sigma+\alpha)}.$$

So, from (c'), it must be true that:

- $L < \frac{\eta\rho}{(1-\sigma)} \cdot \left(\frac{1+\alpha}{\alpha}\right)$.

This condition assures, in the Romer's (1990) model that the aggregate growth rate be bounded.

Appendix B.

In this Appendix, we show that r, γ_Y, H_N, H_Y and H_j take on, respectively, those values indicated in the main text of this paper (section 3.1).

From (16), we get:

$$w_Y = \frac{\partial Y}{\partial H_Y} \cdot P_Y(=1) = A \cdot (1-\alpha) \cdot H_Y^{-\alpha} \cdot \sum_{j=1}^{N} x_j^{\alpha} = A \cdot (1-\alpha) \cdot H_Y^{-\alpha} \cdot N \cdot x^{\alpha} =$$

(16')
$$= A \cdot (1-\alpha) \cdot H_Y^{-\alpha} \cdot N \cdot \left(\frac{H_j}{N}\right)^{\alpha} = A \cdot (1-\alpha) \cdot H_Y^{-\alpha} \cdot N^{1-\alpha} \cdot H_j^{\alpha}$$

The value of w_j is given by (18). Equating (16') to (18), we have:

(28)
$$A \cdot (1-\alpha) \cdot H_Y^{-\alpha} \cdot H_j^{\alpha} \cdot N^{1-\alpha} = A \cdot \alpha^2 \cdot H_Y^{1-\alpha} \cdot H_j^{\alpha-1} \cdot N^{1-\alpha} \Rightarrow H_Y = \left(\frac{1-\alpha}{\alpha^2}\right) \cdot H_j.$$

Substituting (28) into (26), we get the following expression for H_j:

(29)
$$H_j = \left(\frac{\alpha^2}{1-\alpha+\alpha^2}\right) \cdot (H - H_N).$$

In order to find H_Y, we plug (29) into (28) and get:

(28')
$$H_Y = \left(\frac{1-\alpha}{1-\alpha+\alpha^2}\right) \cdot (H - H_N).$$

In steady state, H_N, H_j and H_Y are constant. As a consequence:

$$P_{Nt} = A \cdot \alpha \cdot (1-\alpha) \cdot (H_Y)^{1-\alpha} \cdot (H_j)^{\alpha} \cdot \int_t^{\infty} (N_\tau)^{-\alpha} \cdot e^{-r(\tau - t)} d\tau =$$

(23')
$$= A \cdot \alpha \cdot (1-\alpha) \cdot (H_Y)^{1-\alpha} \cdot (H_j)^{\alpha} \cdot N_t^{-\alpha} \cdot \frac{\eta}{\alpha \cdot H_N + r \cdot \eta}$$

This follows from the fact that, in steady state, $N_t = N_0 \cdot e^{\left(\frac{1}{\eta} H_N\right) \cdot t}$, so that $N_\tau^{-\alpha} = N_t^{-\alpha} \cdot e^{\frac{-\alpha}{\eta} \cdot H_N \cdot (\tau - t)}$ and $\int_t^{\infty} e^{-\left(\frac{\alpha}{\eta} \cdot H_N + r\right) \cdot (\tau - t)} d\tau = \frac{\eta}{\alpha \cdot H_N + r \cdot \eta}$.

Substituting (23') into (24) yields:

(24’) $$w_N = \frac{A \cdot \alpha \cdot (1-\alpha) \cdot H_Y^{1-\alpha} \cdot H_j^{\alpha} \cdot N_t^{1-\alpha}}{\alpha \cdot H_N + r \cdot \eta}.$$

Finally, equating (24') to (18), we have:

(30) $$H_N = \frac{(\alpha - \alpha^2) \cdot H - (\alpha^2 + 1 - \alpha) \cdot r\eta}{\alpha(\alpha^2 + 2 - 2\alpha)}.$$

So, (29) and (28') can be re-written as:

(29’) $$H_j = \left(\frac{\alpha}{\alpha^2 + 2 - 2\alpha}\right) \cdot (\alpha H + r\eta);$$

(28’’) $$H_Y = \left[\frac{1-\alpha}{\alpha(\alpha^2 + 2 - 2\alpha)} \cdot (\alpha H + r\eta)\right],$$

whereas (22) becomes:

(22’) $$\frac{\dot{N}}{N} = \frac{1}{\eta} H_N = \frac{1}{\eta}\left[\frac{(\alpha - \alpha^2) \cdot H - (\alpha^2 + 1 - \alpha) \cdot r\eta}{\alpha(\alpha^2 + 2 - 2\alpha)}\right] = \text{constant.}$$

In addition to this, in steady state we know that:

a) $r = \gamma_Y + \rho$ = constant (from (25);

b) $\gamma_Y = \frac{\dot{Y}}{Y} = (1-\alpha) \cdot \frac{\dot{N}}{N}$ (from (16) and (20)).

As a consequence:

(31) $$r = (1-\alpha) \cdot \frac{1}{\eta} \cdot \left[\frac{(\alpha - \alpha^2) \cdot H - (\alpha^2 + 1 - \alpha) \cdot r\eta}{\alpha(\alpha^2 + 2 - 2\alpha)}\right] + \rho \Rightarrow$$
$$\Rightarrow r = \frac{\alpha(1-\alpha)^2}{\eta} \cdot H + \alpha\rho(\alpha^2 + 2 - 2\alpha)$$

and:

(25’) $$\gamma_Y = r - \rho = \frac{\alpha \cdot (1-\alpha)^2}{\eta} \cdot H + \rho(\alpha^3 + 2\alpha - 2\alpha^2 - 1);$$

(30’) $$H_N = \alpha \cdot (1-\alpha) \cdot H - \eta\rho(\alpha^2 + 1 - \alpha);$$

(28’’’) $$H_Y = (1-\alpha) \cdot (H + \eta\rho);$$

(29'') $H_j = \alpha^2 (H + \eta\rho)$.

Appendix 1: the relationship between aggregate growth rate (γ) and mark-up (β) in the P. Romer's 1990 original model.

$$\gamma = \frac{L - \beta\rho\eta}{\eta(\sigma\beta + 1)}$$ is the overall economy growth rate.

The symbols have the following meaning:

- L(>0) is the (fixed) aggregate supply of human capital;
- $\beta \equiv \frac{1}{\alpha}$ (>1) is the mark-up charged over marginal costs by each intermediate producer;
- $\rho (> 0)$ is the time preference rate (or subjective discount rate);
- $\eta (> 0)$ is the inverse of the productivity parameter of the "research human capital";
- $\sigma (> 0)$ is the inverse of the intertemporal substitution elasticity.

Analysis of the First Derivative

$$\frac{\partial\gamma}{\partial\beta} = -\frac{\eta(\rho\eta + \sigma L)}{\eta^2 (\sigma\beta + 1)^2}$$ (<0)

Analysis of the Second Derivative

$$\frac{\partial^2\gamma}{\partial\beta^2} = \frac{2\sigma(\rho\eta + \sigma L)}{\eta(\sigma\beta + 1)^3}$$ (>0)

Horizontal Asymptote

$$\lim_{\beta \to +\infty} \gamma(\beta) = -\frac{\rho}{\sigma}$$ ($\gamma = -\frac{\rho}{\sigma}$ is an horizontal asymptote).

Vertical Asymptote

$$\lim_{\beta \to -\frac{1}{\sigma}} \gamma(\beta) = +\infty$$ ($\beta = -\frac{1}{\sigma}$ is a vertical asymptote)

Intersections with the γ-axis

$$\gamma = \frac{L}{\eta}$$

Intersections with the β -axis

$$\frac{L - \beta\rho\eta}{\eta(\sigma\beta + 1)} = 0 \Rightarrow \beta\rho\eta = L \Rightarrow \beta = \frac{L}{\rho\eta} > 1 \text{ for } L > \rho\eta.$$

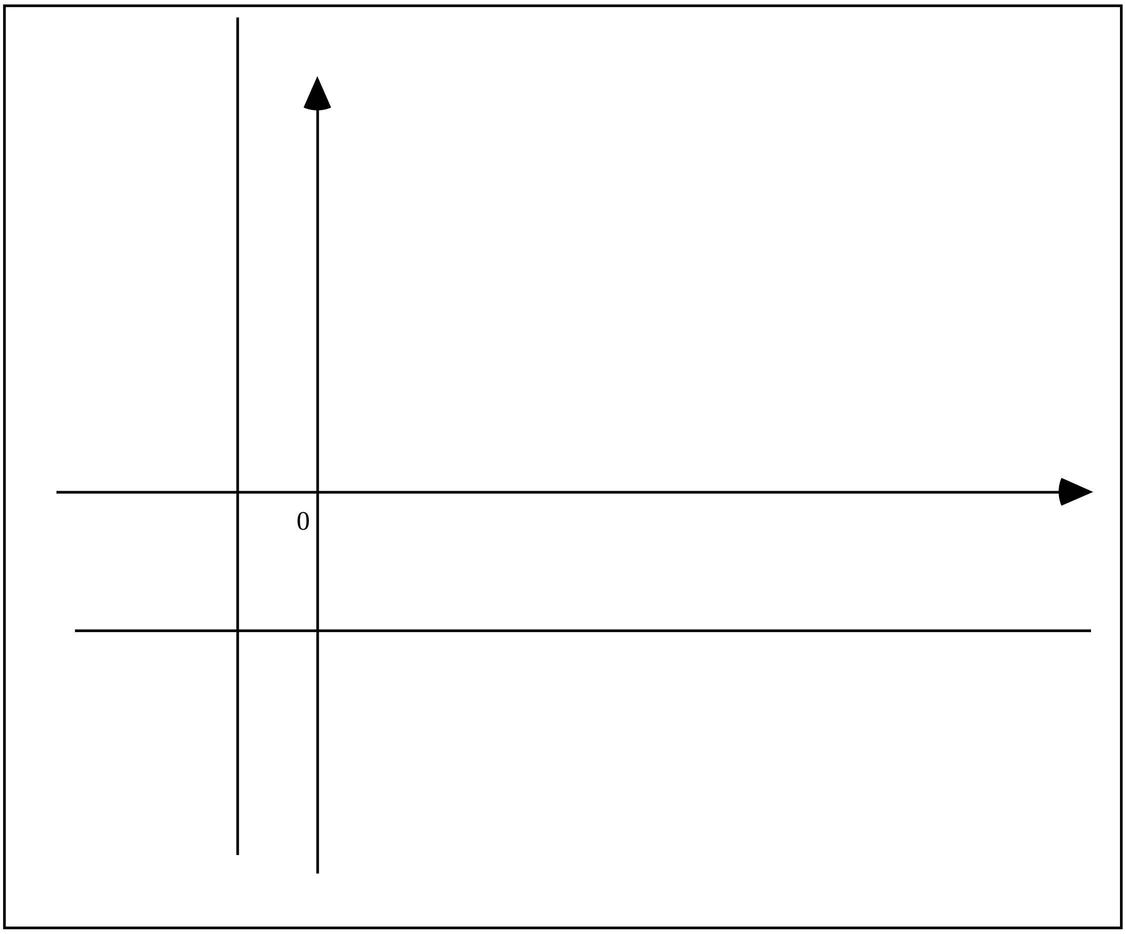
In addition to this, $\beta = 1 \Rightarrow \gamma = \dfrac{L - \rho\eta}{\eta(\sigma + 1)} > 0$ for $L > \rho\eta$.

Finally:

$$\lim_{\beta \to 0} \frac{\partial\gamma}{\partial\beta} = -\frac{(\rho\eta + \sigma L)}{\eta} < 0$$

$$\lim_{\beta \to +\infty} \frac{\partial\gamma}{\partial\beta} = 0^-$$

Qualitative Graph of $\boxed{\gamma(\beta)}$:



Notice that when $\beta \in [1;+\infty)$, $\gamma(\beta)$ is always decreasing in β.

Appendix 2: the relationship between aggregate growth rate (γ) and mark-up (β) in the Grossman and Helpman's model (1991, chapter 3).

$\gamma = \frac{(\beta-1)^2}{\beta} \cdot \frac{L}{a} - \rho\left(\frac{\beta-1}{\beta}\right)$ is the overall economy growth rate.

The symbols have the following meaning:

- L(>0) is the (fixed) aggregate supply of human capital;
- $\beta \equiv \frac{1}{\alpha}$ (>1) is the mark-up charged over marginal costs by each intermediate producer;
- $\rho(>0)$ is the time preference rate (or subjective discount rate);
- a(>0) is the inverse of the productivity parameter of the "research human capital".

Analysis of the First Derivative

$\frac{\partial\gamma}{\partial\beta} = \frac{L(\beta^2-1)-a\rho}{a\beta^2} = 0$ iff $\beta = \pm\sqrt{\frac{a\rho+L}{L}}$, with $\frac{a\rho+L}{L} > 1$.

(We indicate with A the root $\beta = +\sqrt{\frac{a\rho+L}{L}}$ >1).

Analysis of the Second Derivative

$\frac{\partial^2\gamma}{\partial\beta^2} = \frac{2(L+a\rho)}{a\beta^3}$ >0.

Thus, when:

- $0 < \beta < \sqrt{\frac{a\rho+L}{L}}$, $\gamma(\beta)$ decreases;
- $\beta = \sqrt{\frac{a\rho+L}{L}}$, $\gamma(\beta)$ reaches a minimum;
- $\beta > \sqrt{\frac{a\rho+L}{L}}$, $\gamma(\beta)$ increases.

Horizontal Asymptote.

$$\lim_{\beta \to +\infty} \gamma(\beta) = +\infty$$ (there is no horizontal asymptote)

Vertical Asymptote.

$$\lim_{\beta \to +0^+} \gamma(\beta) = +\infty$$ (there is a vertical asymptote in $\beta = 0$).

Intersections with the β -axis:

$$\frac{(\beta - 1)^2}{\beta} \cdot \frac{L}{a} - \rho\left(\frac{\beta - 1}{\beta}\right)_{=0}$$

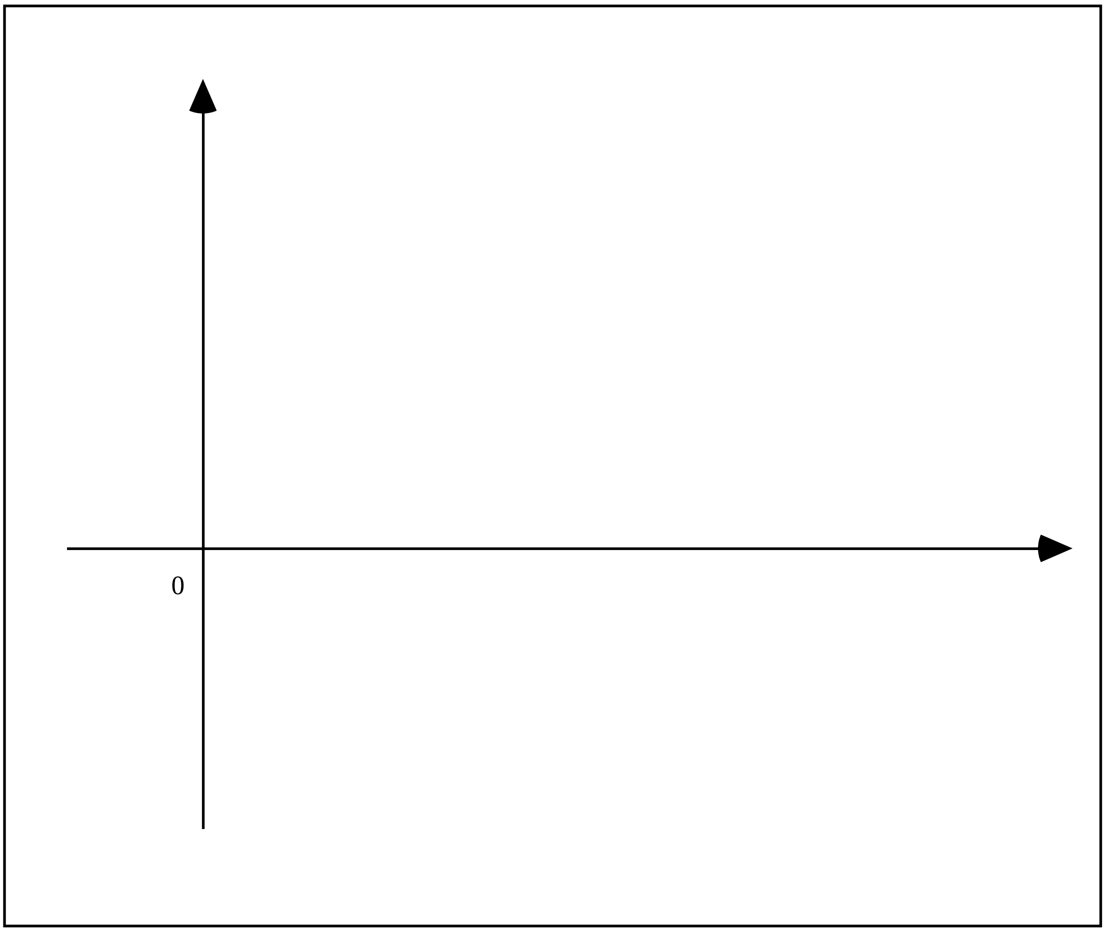
$$\Rightarrow \frac{(\beta - 1)^2 L - a\rho(\beta - 1)}{a\beta} = 0 \Rightarrow (\beta - 1)[L(\beta - 1) - a\rho] = 0 \Rightarrow \beta_1 = 1$$ and

$$\beta_2 = \frac{a\rho + L}{L} > \sqrt{\frac{a\rho + L}{L}}.$$

Finally:

- $$\lim_{\beta \to 0^+} \frac{\partial\gamma}{\partial\beta} = -\infty$$
- $$\lim_{\beta \to +\infty} \frac{\partial\gamma}{\partial\beta} = \frac{L}{a} > 0$$
- $$\lim_{\beta \to +1} \frac{\partial\gamma}{\partial\beta} = -\rho < 0.$$

Qualitative graph of $\boxed{\gamma(\beta)}$:



Obviously, the only relevant range of the function is, for our purposes, that being at the right of $\beta = 1$.

Appendix 3: the relationship between aggregate growth rate (γ) and mark-up (β) as stemming from the Section 2 model.

In that section we found that the relationship linking together β and γ is:

$$\gamma(\beta) = \frac{(\beta-1)^2 \cdot H - (\beta-1)\eta\rho}{\eta\beta^2}, \qquad \beta \equiv \frac{1}{\alpha} > 1.$$

The symbols have the (usual) following meaning:

- H(>0) is the (fixed) aggregate supply of human capital;
- $\beta \equiv \frac{1}{\alpha}$ (>1) is the mark-up charged over marginal costs by each intermediate producer;
- $\rho(>0)$ is the time preference rate (or subjective discount rate);
- $\eta(>0)$ is the inverse of the productivity parameter of the "research human capital".

Analysis of the First Derivative

$$\frac{\partial\gamma}{\partial\beta} = \frac{\beta(2H+\eta\rho) - 2(H+\eta\rho)}{\eta\beta^3} = 0 \quad \text{iff} \quad \beta = \frac{2(H+\eta\rho)}{2H+\eta\rho} > 1.$$

Analysis of the Second Derivative

$$\frac{\partial^2\gamma}{\partial\beta^2} = \frac{-2\beta(2H+\eta\rho)+6(H+\eta\rho)}{\eta\beta^4} > 0 \ \text{ iff } \ \beta < \frac{3(H+\eta\rho)}{2H+\eta\rho}$$

Horizontal Asymptote.

$$\lim_{\beta\to+\infty} \gamma(\beta) = \frac{H}{\eta}$$

($\gamma(\beta) = \frac{H}{\eta}$ is an horizontal asymptote)

Vertical Asymptote.

$$\lim_{\beta\to 0} \gamma(\beta) = +\infty$$

($\beta = 0$ is a vertical asymptote).

Intersections with the β -axis:

$$(\beta-1)^2 H-(\beta-1)\eta\rho=0 \Rightarrow \beta_1=1 \text{ and } \beta_2=1+\frac{\eta\rho}{H}>1.$$

Notice that $\frac{3(H+\eta\rho)}{2H+\eta\rho}>1+\frac{\eta\rho}{H}$ iff H is greater than $\eta\rho$ (in order to graph $\gamma(\beta)$, from now on we will assume H> $\eta\rho$).

Also, $1+\frac{\eta\rho}{H}$ is always greater than $\frac{2(H+\eta\rho)}{2H+\eta\rho}$.

Intersections with the horizontal asymptote:

$$\frac{H\beta^2-(2H+\eta\rho)\beta+(H+\eta\rho)}{\eta\beta^2}=\frac{H}{\eta} \Rightarrow \beta=\frac{H+\eta\rho}{2H+\eta\rho}<1.$$

Using all of these information, we can obtain the graph showed in section 2.

Appendix 4: the relationship between aggregate growth rate (γ) and mark-up (β) as stemming from the model of Section 3.1 .

In that section we found that:

$$\gamma(\beta) = \frac{-\eta\rho\beta^3 + (H + 2\eta\rho)\cdot\beta^2 - 2(H + \eta\rho)\cdot\beta + (H + \eta\rho)}{\eta\beta^3}, \qquad \beta \equiv \frac{1}{\alpha} > 1 .$$

The meaning of H, η and ρ, respectively, is the same as in the previous appendix.

Analysis of the first derivative

$$\frac{\partial\gamma}{\partial\beta} = \frac{-\beta^2(H + 2\eta\rho) + 4\beta(H + \eta\rho) - (3H + 3\eta\rho)}{\eta\beta^4} = 0 \Rightarrow$$

$$\Rightarrow \beta_{1/2} = \frac{2(H + \eta\rho) \pm \sqrt{H^2 - H\eta\rho - 2(\eta\rho)^2}}{H + 2\eta\rho}$$

The two roots (β_1 and β_2) will be real and distinct iff $H^2 - H\eta\rho - 2(\eta\rho)^2 > 0$, which implies $H > 2\eta\rho$.

If this condition ($H > 2\eta\rho$) is met, then β_1 and β_2 will also be positive, as $2(H + \eta\rho)$ is always greater than $\sqrt{H^2 - H\eta\rho - 2(\eta\rho)^2}$.

Horizontal Asymptote.

$\lim_{\beta \to +\infty} \gamma(\beta) = -\rho$ ($\gamma(\beta) = -\rho$ is an horizontal asymptote)

Vertical Asymptote.

$\lim_{\beta \to 0^+} \gamma(\beta) = +\infty$ ($\beta = 0$ is a vertical asymptote).

Intersections with the β -axis

$$\eta\rho\beta^3 - (H + 2\eta\rho)\cdot\beta^2 + 2(H + \eta\rho)\cdot\beta - (H + \eta\rho) = (\beta - 1)\cdot\left[\beta^2\eta\rho - (H + \eta\rho)\cdot\beta + (H + \eta\rho)\right] = 0 \Rightarrow$$

$\Rightarrow \beta_1 = 1$

and $\beta_{2/3} = \dfrac{(H + \eta\rho) \pm \sqrt{H^2 - 2H\eta\rho - 3(\eta\rho)^2}}{2\eta\rho}$.

β_2 and β_3 will be real and distinct iff $H^2 - 2H\eta\rho - 3(\eta\rho)^2 > 0$, which implies $H > 3\eta\rho$.

Thus, when $H > 3\eta\rho$:

- $\gamma(\beta)$ intersects the β axis in two real and distinct points (and in β=1);
- the solution to $\gamma'(\beta) = 0$ will be represented by two real and distinct roots.

In addition to this, β_2 and β_3 are positive, as $(H + \eta\rho)$ is always greater than $\sqrt{H^2 - 2H\eta\rho - 3(\eta\rho)^2}$.

If we set $H = 4\eta\rho$ $(> 3\eta\rho)$, then it is easy to check that:

1) $\gamma'(\beta) = 0$ in β=1,13962039 and β=2,193712943;
2) $\gamma(\beta)$ intersects the β axis in β=1, β=1,381966011 and β=3,618033989.

Finally, when $\beta \cong 1{,}14$, $\gamma(\beta) = -0{,}06\rho$, whereas when $\beta \cong 2{,}2$, $\gamma(\beta) = 0{,}13\rho$.

A qualitative graph of $\gamma(\beta)$ has been drawn in Section 3.1.

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