Discussion Paper No. 543

TESTING MULTIPLE NON-NESTED FACTOR DEMAND SYSTEMS

Matteo Manera
and
Michael McAleer

June 2001

The Institute of Social and Economic Research
Osaka University
6-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

Testing Multiple Non-nested Factor Demand Systems

Matteo Manera

Department of Economics
Bocconi University, Milan, Italy

Michael McAleer

Department of Economics
University of Western Australia

Revised: June 2001

Keywords. Systems of equations; Factor demands; Flexible functional forms; Paired non-nested tests; Joint non-nested tests.

JEL classifications. C52; C30.

Corresponding author. Matteo Manera – Istituto di Economia Politica – Università Commerciale "L. Bocconi" – Via R. Sarfatti, 25 – 20136 Milano – Italia – office (direct): +39-02-58365413 – fax: +39-02-58365439 – e-mail: matteo.manera@uni-bocconi.it

Acknowledgements. Seminar participants at the Agricultural and Resource Economics (ARE) Group and the Department of Economics at the University of Western Australia (UWA) offered helpful comments. The first author would like to thank the ARE Group and the Department of Economics of UWA for financial support. The second author wishes to acknowledge the financial support of the Australian Research Council and the Institute of Social and Economic Research at Osaka University.

Abstract. Empirical factor demand analysis typically involves making a choice from among several competing non-nested functional forms. Each of the commonly used factor demand systems, such as Translog, Generalized Leontief, Quadratic, and Generalized McFadden, can provide a valid and useful empirical description of the underlying production structure of the firm. As there is no theoretical guidance on selecting the model which is best able to capture the relevant features of the data, formal testing procedures can provide additional information. Paired and joint univariate non-nested tests of a null model against both single and multiple alternatives have been discussed at length in the literature, whereas virtually no attention has been paid to either paired or joint multivariate non-nested tests. This paper shows how some multivariate non-nested tests can be derived from their univariate counterparts, and examines how to use these tests empirically to compare alternative factor demand systems. The empirical application involves the classical Berndt-Khaled annual data set for the U.S. manufacturing sector over the period 1947-1971. A statistically adequate empirical specification is determined for each competing factor demand system. The empirical results are interpreted for each system, and the models are compared on the basis of multivariate paired and joint non-nested procedures.

1. Introduction

Empirical factor demand analysis typically involves making a choice from among several competing non-nested functional forms. Each of the commonly used factor demand systems, such as Translog, Generalized Leontief, Quadratic and Symmetric Generalized McFadden, can provide a valid and useful empirical description of the underlying production structure of the multi-input neoclassical firm. A common feature of flexible functional forms is that they are non-nested (or separate). Thus, given two or more systems of factor demands, it is not possible to obtain one system by imposing suitable parametric restrictions on the other(s). Moreover, as there is no a priori theory suggesting that the specification of one system should be preferred over another, it is necessary to choose from among the competing models using empirical considerations.

The important task of model determination can be accomplished using a formal non-nested testing procedure. Paired and joint univariate non-nested tests of a null model against both single and multiple alternatives have been discussed at length in the literature. However, virtually no attention has been paid to either paired or joint multivariate non-nested tests. This paper shows how some multivariate non-nested tests can be derived from their univariate counterparts, and examines how to use these tests empirically to compare alternative factor demand systems.

As the outcome of a non-nested test is influenced by the type of misspecification affecting the competing models, it is essential to investigate the performance of each factor demand system against real data. The empirical application presented is very popular in the applied production literature, and contains annual data on aggregate output of U.S. manufacturing industries, and prices and quantities for a capital-labour-energy-materials (KLEM) technology over the period 1947-1971 (see Berndt and Khaled, 1979). A statistically adequate empirical specification is determined for each competing factor demand system. Estimation results and some diagnostic statistics are presented for each factor demand system, and each is used to calculate some classical indicators of the production structure of an economic sector, such as price and output elasticities. The systems are then compared on the basis of multivariate paired and joint non-nested testing procedures. Finally, the empirical results are interpreted for each system, and some practical issues regarding model selection and testing of systems of equations in applied research are discussed.

This paper is organized as follows. In Section 2, alternative factor demand systems are presented using some of the most popular flexible functional forms, namely Translog, Quadratic, Generalized Leontief and Symmetric Generalized McFadden (for further details, see Diewert and Wales, 1987). In Section 3, multivariate extensions of some well-known and pedagogically appealing univariate paired non-nested tests (namely, the J, P0 and P1 tests of Davidson and MacKinnon, 1981; the PE statistic of MacKinnon, White and Davidson, 1983; and the Bera and McAleer (1989) test, hereafter the BEM test), and the multivariate joint non-nested test of Barten and McAleer (1997), hereafter the BAM test, are described. Section 4 reports the main empirical results from estimation and diagnostic testing of the competing factor demand systems. Estimated price and output elasticities from each model are also presented. In Section 5, multivariate paired and joint non-nested tests are applied to compare the alternative factor demand systems. The empirical results are interpreted for each system. Section 6 provides some concluding comments.

2. Alternative factor demand systems

In this section attention is focused on the four most widely used flexible functional forms in the context of cost function estimation: Translog, Quadratic, Symmetric Generalized McFadden and Generalized Leontief. As is customary in applied factor demand analysis, the cross-equation symmetry restrictions are maintained for each model. Several techniques are available for imposing the appropriate curvature conditions on the cost function (see Morey, 1986). In this paper, we have left unconstrained the matrix of second-order partial derivatives of the cost function with respect to factor prices in each model, and have checked ex post if the negative semi-definiteness of the Hessian of the cost function is satisfied over the sample period.

Consider the following specification for the logarithm of the firm's cost function C(·):

$$\ln C(\cdot) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln P_i + \alpha_y \ln Y + \alpha_t t + \alpha_{yt} t \ln Y \qquad (2.1)$$

$$+ \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij} \ln P_i \ln P_j + \sum_{i=1}^{n} \alpha_{iy} \ln P_i \ln Y$$

$$+ \sum_{i=1}^{n} \alpha_{it} t \ln P_i + \frac{1}{2} \alpha_{yy} (\ln Y)^2 + \frac{1}{2} \alpha_{tt} t^2$$

where P_i indicates the price of the i-th input (i,j=1,...,n), Y is output, t=1,...,T is a time trend, and the symmetry condition $\alpha_{ij}=\alpha_{ji}$ for all i,j, is imposed. Necessary and sufficient conditions for C(·) to be linearly homogeneous in input prices are given by:

$$\sum_{i=1}^{n}\alpha_i = 1; \sum_{i=1}^{n}\alpha_{ij} = 0; \sum_{i=1}^{n}\alpha_{iy} = 0; \sum_{i=1}^{n}\alpha_{it} = 0 \text{ (i=1,...,n).} \tag{2.2}$$

By using the Translog specification (2.1) and Shephard's lemma, the following expressions for the input shares are obtained (see Berndt and Christensen, 1973, p. 85):

$$Sh_i = \alpha_i + \sum_{i=1}^{n}\alpha_{ij}\ln P_j + \alpha_{iy}\ln Y + \alpha_{it}t \text{ (i=1,...,n)} \tag{2.3}$$

with $\alpha_{ij}=\alpha_{ji}$ for all i,j=1,…,n.

The Quadratic second-order approximation to the firm's true cost function can be defined as follows:

$$\begin{aligned} C(\cdot) &= \alpha_0 + \sum_{i=1}^{n}\alpha_i P_i + \alpha_y Y + \alpha_t t + \alpha_{yt} Yt \\ &+ \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_{ij}P_i P_j + \sum_{i=1}^{n}\alpha_{iy}P_i Y \\ &+ \sum_{i=1}^{n}\alpha_{it}P_i t + \frac{1}{2}\alpha_{yy}Y^2 + \frac{1}{2}\alpha_{tt}t^2 \end{aligned} \tag{2.4}$$

with $\alpha_{ij}=\alpha_{ji}$ for all i,j=1,...,n. A direct application of Shephard's lemma yields the system of factor demands:

$$X_i = \alpha_i + \sum_{i=1}^{n}\alpha_{ij}P_j + \alpha_{iy}Y + \alpha_{it}t \text{ (i=1,...,n).} \tag{2.5}$$

The major drawback of the Quadratic functional form is that linear homogeneity restrictions in prices cannot be imposed parametrically, which means that the cost function under linear homogeneity in factor prices has to be respecified and a new separate demand system obtained.

When linear homogeneity in prices is imposed, the cost function and the related factor demands have the following forms:

$$\frac{C(\cdot)}{P_1} = \alpha_0 + \sum_{i=2}^{n} \alpha_i \frac{P_i}{P_1} + \alpha_y Y + \alpha_t t + \alpha_{yt} Yt \tag{2.6}$$

$$+ \frac{1}{2}\sum_{i=2}^{n}\sum_{i=2}^{n} \alpha_{ij} \frac{P_i}{P_1}\frac{P_j}{P_1} + \sum_{i=2}^{n} \alpha_{iy} \frac{P_i}{P_1} Y$$

$$+ \sum_{i=2}^{n} \alpha_{it} \frac{P_i}{P_1} t + \frac{1}{2}\alpha_{yy} Y^2 + \frac{1}{2}\alpha_{tt} t^2$$

and

$$\frac{X_i}{P_1} = \alpha_i + \sum_{j=2}^{n} \alpha_{ij} \frac{P_i}{P_1} + \alpha_{iy} Y + \alpha_{it} t \quad \text{(i=2,...,n).} \tag{2.7}$$

Consider the following functional form for a cost function, with the usual symmetry conditions imposed:

$$C(\cdot) = Y \frac{\frac{1}{2}\sum_{i=1}^{n} \sigma_{ij} P_i P_j}{\sum_{i=1}^{n} \theta_i P_i} + \sum_{i=1}^{n} \alpha_i P_i \tag{2.8}$$

$$+ \sum_{i=1}^{n} \alpha_{ii} P_i Y + \sum_{i=1}^{n} \alpha_{it} P_i t$$

The cost function defined by expression (2.8) is a generalization of the functional form due to McFadden (1978, p. 279), as suggested by Diewert and Wales (1987, p. 53). It should be noted that the σ_{ij} parameters have to sum to zero in order to identify all the coefficients in expression (2.8). The related system of Symmetric Generalized McFadden factor demands can be derived:

$$\frac{X_i}{Y} = \frac{\sum_{j=1}^{n} \sigma_{ij} P_j}{\sum_{i=1}^{n} \theta_i P_i} - \theta_i \frac{\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} P_i P_j}{\left(\sum_{i=1}^{n} \theta_i P_i\right)^2} \tag{2.9}$$

$$+\frac{\alpha_i}{Y}+b_{ii}+\frac{\alpha_{it}t}{Y}.$$

The traditional Generalized Leontief cost function is a functional form in the square roots of input prices. In this paper, we consider the following version:

$$C(\cdot)=Y\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_{ij}\sqrt{P_iP_j}\right)+\sum_{i=1}^{n}\alpha_iP_i+\sum_{i=1}^{n}\alpha_{it}P_it \tag{2.10}$$

where $\alpha_{ij}=\alpha_{ji}$ for all i,j is a maintained hypothesis. The system of factor demands is derived from (2.10), as follows:

$$\frac{X_i}{Y}=\sum_{i=1}^{n}b_{ij}\sqrt{\frac{P_j}{P_i}}+\frac{\alpha_i}{Y}+\frac{\alpha_{it}t}{Y}\quad \text{(i=1,...,n).} \tag{2.11}$$

3. Alternative non-nested testing procedures

This section presents some alternative non-nested testing procedures for competing systems of equations. In particular, modifications of the paired J, P0, P1, PE and BEM tests are presented as extensions of the corresponding univariate statistics.

3.1. *Multivariate paired non-nested tests*

Some variants of both the J and the P tests (see Davidson and MacKinnon, 1981; MacKinnon, White and Davidson, 1983) may be applied to the case of multivariate regression models. For this purpose, the null and the alternative hypotheses are given as:

$$H_0\text{: } y_i = f_i(X_i,a_i) + u_{0i} \tag{3.1}$$

and

$$H_1\text{: } y_i = g_i(Z_i,b_i) + u_{1i} \tag{3.2}$$

where i denotes equation i=1,...,n, in a system of equations, u_0 and u_1 are distributed as $N(0,\Omega_0)$ and $N(0,\Omega_1)$, respectively, and Ω_j is the contemporaneous covariance matrix of the error term corresponding to hypothesis H_j, j=0,1. Equations (3.1) and (3.2) are multivariate, non-simultaneous models, such as factor demand systems or systems of cost share equations.

A straightforward extension of the J test to a multivariate context is based on the following auxiliary equation:

$$H_c : y_i = f_i(\cdot) + \lambda_i \hat{g}_i(\cdot) + u_i \quad \text{(i=1,...,n)} \tag{3.3}$$

which is obtained by combining linearly equations (3.19) and (3.20) and replacing $g_i(\cdot)$ with the fitted values $\hat{g}_i(\cdot)$. Alternatively, H_c in (3.3), which is a composite model, is equivalent to H_0 in (3.1) with the addition of $\hat{g}_i(\cdot)$. The null hypothesis H_0 is not rejected if the parameters λ_i are zero for all i. A Wald-type statistic enables testing of the null hypothesis H_0 against the alternative H_1 that the λ_i are all jointly zero. If the roles of H_0 and H_1 are reversed, it is possible to test the validity of H_1 against H_0.

Davidson and MacKinnon (1982, p. 555) discuss two multivariate versions of the P test. The first test is based on the following composite model:

$$H_c : y_i - \hat{f}_i(\cdot) = \lambda_i \left[\hat{g}_i(\cdot) - \hat{f}_i(\cdot)\right] + \hat{F}_i(\cdot)d_i + u_i \tag{3.4}$$

where $\hat{F}_i(\cdot)$ is the row vector of derivatives of $f_i(\cdot)$ with respect to a_i evaluated at $\hat{a}_i$ and $d_i = a_i - \hat{a}_i$. Under H_0 the vector u_i is distributed as $N(0,\Omega_0)$, so that model (3.4) must be estimated by a systems generalized least squares procedure using a covariance matrix which is proportional to $\hat{\Omega}_0$. The Wald test of $\lambda_1=\ldots=\lambda_n=0$ is the multivariate extension of Davidson and MacKinnon's P0 test. They also propose the following auxiliary regression equation:

$$H_c : y_i - \hat{f}_i(\cdot) = \hat{F}_i(\cdot)d_i + \lambda_i \left(\hat{\Omega}_0 \hat{\Omega}_1^{-1}\right)\left[\hat{g}_i(\cdot) - \hat{f}_i(\cdot)\right] + u_i \tag{3.5}$$

where the variables are defined using the same notation as above. The Wald test of $\lambda_1=\ldots=\lambda_n=0$ is the multivariate extension of Davidson and MacKinnon's P1 test.

The multivariate analogue of the PE test for models with dependent variables subject to different transformations is given by the following auxiliary regression equation:

$$H_c : y_i - \hat{f}_i(\cdot) = \lambda_i \left[\hat{g}_i(\cdot) - h_i(\hat{f}_i)\right] + \hat{F}_i d_i + u_i \tag{3.6}$$

where the variables are defined as in the extension of the P test.

The BEM approach (see Bera and McAleer, 1989) can be readily adapted to the multivariate case. Let the null and the alternative hypotheses be given by expressions (3.1) and

$$H_1: h_i(y_i) = g_i(Z_i, b_i) + u_{1i} \tag{3.2'}$$

where $h_i(\cdot)$ is a known transformation of y_i, and u_{0i} and u_{1i} are NID($0,\sigma^2_0 I_T$) and NID($0,\sigma^2_1 I_T$), respectively. Combining the disturbances u_{0i} and u_{1i} linearly, with weights $(1-\lambda_i)$ and λ_i, yields:

$$(1-\lambda_i)[y_i - f_i(\cdot)] + \lambda_i[h_i(y_i) - g_i(\cdot)] = u_i \tag{3.7}$$

where u_i is NID under both H_0: $\lambda_i=0$ and H_1: $\lambda_i=1$. Dividing expression (3.7) by $(1-\lambda_i)$ gives:

$$y_i = f_i(\cdot) + \theta_{0i} u_{1i} + u_i/(1-\lambda_i) \tag{3.8}$$

where $\theta_{0i} = -\lambda_i/(1-\lambda_i)$, while dividing (3.7) by λ_i gives:

$$h_i(y_i) = g_i(\cdot) + \theta_{1i} u_{0i} + u_i/\lambda_i \tag{3.9}$$

where $\theta_{1i} = -(1-\lambda_i)/\lambda_i$. A test of $\theta_{0i}=0$ ($i=1,\ldots,n$) in (3.8) corresponds to a test of $\lambda_i=0$ in (3.7), so that if $\theta_{0i}=0$ is not rejected, $\lambda_i=0$ is not rejected. In a similar manner, a test of $\theta_{1i}=0$ in (3.9) is equivalent to a test of $\lambda_i=1$ in (3.7), so that if $\theta_{1i}=0$ is not rejected, $\lambda_i=1$ is not rejected. A serious problem is that H_0 and H_1 are not testable because u_{0i} and u_{1i} in (3.8) and (3.9) are not observable. It is possible to replace the disturbances from H_0 and H_1 in (3.8) and (3.9), respectively, with some estimated residuals as follows. First, systems (3.1) and (3.2') are estimated to obtain the fitted values $\hat{y}_i$ and $\hat{h}_i(y_i)$, respectively. Second, $\hat{y}_i$ and $\hat{h}_i(y_i)$ are transformed as $h_i(\hat{y}_i)$ and $h_i^{-1}\left\lfloor \hat{h}_i(y_i) \right\rfloor$, respectively. Third, the models given by:

$$h_i(\hat{y}_i) = g_i(Z_i, b_i) + \eta_{1i} \tag{3.10}$$

and

$$h_i^{-1}\left\lfloor \hat{h}_i(y_i) \right\rfloor = f_i(X_i, a_i) + \eta_{0i} \tag{3.11}$$

are estimated to obtain the residuals $\hat{\eta}_{1i}$ and $\hat{\eta}_{0i}$, respectively. Finally, the models given by:

$$y_i = f_i(X_i, a_i) + \theta_{0i}\hat{\eta}_{i1} + \varepsilon_i \tag{3.12}$$

and

$$h_i(y_i) = g_i(Z_i, b_i) + \theta_{1i}\hat{\eta}_{0i} + \varepsilon_i \tag{3.13}$$

are estimated. The BEM test is a Wald-type test of $\theta_{01} = \ldots = \theta_{0n} = 0$ for H_0 in (3.12) and $\theta_{11} = \ldots = \theta_{1n} = 0$ for H_1 in (3.13), respectively.

3.2. *Multivariate joint non-nested tests*

The test procedures considered so far belong to the class of multivariate paired non-nested tests, whose common feature is that the null hypothesis is tested against a specific non-nested alternative, after which the roles of null and alternative are reversed. Recently, some univariate tests have been proposed in the literature in which the null hypothesis is tested against several alternatives simultaneously, leading to joint tests. This section concentrates on the multivariate version of one of these procedures, namely the BAM test of Barten and McAleer (1997).

Consider m non-nested non-linear systems of equations with different non-linear data transformations on the dependent variable y_i, i=1,…n, as follows:

H_1: $h_{1i}(y_i) = g_{1i}(X_{1i},b_{1i}) + u_{1i}$ (3.14)

H_2: $h_{2i}(y_i) = g_{2i}(X_{2i},b_{2i}) + u_{2i}$

•

•

•

H_m: $h_{mi}(y_i) = g_{mi}(X_{mi},b_{mi}) + u_{mi}$

where u_{ji} is NID$(0,\sigma^2_{ji})$ for j=1,…m. Suppose that H_1 is, in the first instance, chosen as the null hypothesis. It is then possible to combine the disturbances in (3.14) as follows:

$$\left(1-\sum_{j=2}^{m}\lambda_{ji}\right)u_{1i}+\lambda_{2i}u_{2i}+\ldots+\lambda_{mi}u_{mi}=u_i\,. \qquad (3.15)$$

Under the null hypothesis H_1: $\lambda_{21}=\ldots=\lambda_{2n}=\lambda_{31}=\ldots=\lambda_{3n}=\ldots=\lambda_{m1}=\ldots=\lambda_{mn}=0$, $u_{1i}=u_i$ so that the first model given by H_1 is valid. Setting $\mu_{ji}=-\lambda_{ji}\Big/\left(1-\sum_{j=2}^{m}\lambda_{ji}\right)$, (3.15) can be rewritten as:

$$h_{1i}(y_i)=g_{i1}(X_{i1},b_{i1})+\sum_{j=2}^{m}\mu_{ji}\left[h_{ji}(\cdot)-g_{ji}(\cdot)\right]+v_i \qquad (3.16)$$

where $v_i=u_i\Big/\left(1-\sum_{j=2}^{m}\lambda_{ji}\right)$. Testing the null hypothesis (3.16) involves verifying if the μ_{ji} are jointly zero. Unfortunately, a problem with this procedure is that the parameters μ_{ji} are not identified. It is possible to resolve the problem by extending McAleer's (1983) univariate joint testing procedure in the following manner. First, since $h_{1i}(y_i) = g_{1i}(X_{1i},b_{1i}) + u_{1i}$ under the null hypothesis H_1, replace y_i in $h_{ji}(y_i)$, j=2,...,m, with:

$$\hat{y}_{1i}=h_{1i}^{-1}\left[g_{1i}\left(X_{1i},\hat{b}_{1i}\right)\right] \qquad (3.17)$$

where $\hat{b}_{1i}$ is the generalized least squares estimate of b_{1i} under the null hypothesis H_1. It is worth noting that $\hat{y}_{1i}$ under H_1 is asymptotically uncorrelated with u_{1i}, and hence with v_i. Second, estimate the auxiliary system:

$$h_{ji}(\hat{y}_{i1}) = g_{ji}(X_{ji}, b_{ji}) + \eta_{1ji} \quad \text{(j=2,...m, i=1,…,n)} \tag{3.18}$$

to obtain the residuals

$$\hat{\eta}_{1ji} = h_{ji}(\hat{y}_{i1}) - g_{ji}\left(X_{ji}, \hat{b}_{1ji}\right) \tag{3.19}$$

where $\hat{b}_{1ji}$ are the generalized least squares estimates of b_{ji} from (3.18) and $g_{ji}(\cdot)$ includes an intercept term. Third, use the residuals in (3.19) to compute the following modification of (3.16):

$$h_{1i}(y_i) = g_{1i}(X_{1i}, b_{1i}) + \sum_{j=2}^{m} \mu_{ji}\hat{\eta}_{1ji} + v_{i1}\,. \tag{3.20}$$

Finally, upon estimating the parameters in (3.20), test the extent to which the residuals $\hat{\eta}_{1ji}$ in (3.20) contribute to the empirical performance of H_1 through a standard Wald-type test.

4. Empirical evidence on factor demand systems

In order to illustrate the usefulness of the non-nested tests presented above, the systems of factor demands introduced in Section 2 are estimated using Berndt and Khaled's (1979) classical annual data set for the U.S. manufacturing sector over the period 1947-1971. It is assumed that U.S. manufacturing can be described by a regular aggregate production function relating the flows of gross output Y to the services of four inputs, namely capital (K), labour (L), energy (E) and materials (M). Corresponding to such a production function, there exists a dual cost function summarizing all the characteristics of the representative firm's technology.

When output quantity and input prices are exogenous, the dual cost function can be written as:

$$C = C(Y, P_k, P_l, P_e, P_m, t) \tag{4.1}$$

where $C(\cdot)$ represents total input costs, P_i, i=K,L,E,M, are the factor prices, and t is an index of technical progress.

For purposes of empirical implementation, the existence of random errors in the cost minimizing behaviour of the firm is such that each equation in each demand system has an additive disturbance term which reflects the firm's errors in deciding the optimal level of inputs. First-order serial correlation for each system is accommodated using a Cochrane-Orcutt transformation for each equation (see, e.g., Berndt, 1991, pp. 476-9). The estimated single-equation serial correlation coefficients have been used to estimate the system.

The Translog system (TLG) comprises the cost equation (4.1) specified by the functional form (2.1), and the three share equations (2.3) for labour, energy and materials, in addition to the linear homogeneity restrictions (2.2) and symmetry conditions. It is well known that only n-1 share equations are estimated because the four cost shares (2.3) sum to unity, so that the sum of the disturbances across the four equations is zero for each observation. Consequently, the covariance matrix is singular and non-diagonal (Berndt and Wood, 1975, p. 261). The disturbance from the Sh_k equation is omitted and the vector *u* comprising the disturbances of the remaining share equations and the cost function is specified as a multivariate normal distribution with $E(u)=0$ and $E(uu')=\Omega$, where Ω is constant over time (Diewert and Wales, 1987, p. 58).

The Quadratic demand system is given by the cost equation (4.1), specified by equation (2.6) and the three demand equations (2.7) for labour, energy and materials. As already noted, the reason for excluding the capital equation is that, since linear homogeneity in prices cannot be imposed parametrically, a normalization with respect to an arbitrarily chosen factor price is required. QDR denotes the Quadratic demand system with linear homogeneity in prices imposed, NHQDR1 denotes the same system with non-homogeneity, that is, when linear homogeneity in prices is not imposed, and NHQDR denotes the Quadratic demand system formed from the four demand equations for capital, labour, energy and materials without imposing linear homogeneity in prices.

Finally, the Symmetric Generalized McFadden demand system (SGM) is formed from the four equations (2.9), and the Generalized Leontief model (GLT) is given by equation (2.11). In both the

SGM and GLT specifications, the dependent variables are input levels divided by output, as this makes the assumption of homoskedasticity of the disturbances more plausible. The cost function is not estimated since it does not contain any additional information.

The main characteristics of the factor demand systems used in the empirical application are summarized in Table 1. All systems are estimated with the multivariate least squares routine Lsq implemented in Tsp 4.4 (for details, see Hall, Cummins and Schnake, 1997). Linear disembodied technical change is accommodated by the presence of linear and quadratic trends in the estimated equations. The estimated parameters are reported in Appendix 1, and Appendix 2 shows the results of some diagnostic statistics for the estimated models.

[Table 1, Appendix 1 and Appendix 2]

In particular, Appendix 2 indicates the absence of both first-order serial correlation and heteroskedasticity, and that the curvature properties of the firm's cost function are satisfied for each functional form.

Factor demand systems are typically used to calculate indicators which can be useful for describing the production structure of an economic sector. For each estimated model, Appendix 3 reports the mean values of input demand elasticities with respect to input prices and output over the period 1948-1971.

[Appendix 3]

The magnitudes and signs of these elasticities depend crucially on the selected model. This is particularly true for the price elasticities of capital, which are roughly comparable for GLT, SGM, NHQDR and TLG, but appear quite different for NHQDR1 and QDR, that is, for the Quadratic functional form where the demand for capital is not directly estimated. In general, direct price elasticities are negative and output elasticities are positive, as suggested by theory, and the cross-price elasticities are all below one in absolute value. From their signs, it is possible to obtain information about factor substitution and complementarity, which is also not independent of the functional form. For example, capital and energy are complements, according to GLT, NHQDR1, SGM and TLG, but substitutes on the basis of NHQDR and QDR. Capital and labour are substitutes for all models, except for NHQDR and NHQDR1. Labour and materials are complements according to only GLT, SGM and NHQDR1. Energy and materials are substitutes in all systems, with the exception of NHQDR1. Finally, materials and labour are complements for GLT, NHQDR1, SGM and TLG, whereas they are substitutes for NHQDR and QDR. In summary, all estimated systems seem to offer plausible interpretations of the production structure of the US manufacturing sector over the period 1948-1971, but the interpretations depend on the chosen specification. Systems QDR and NHQDR suggest the existence of substitution and complementarity relationships among the factors, which do not agree, in general, with the indications of the other models.

5. Empirical evidence on multivariate non-nested tests

Alternative factor demand systems based on competing non-nested flexible functional forms were estimated in Section 4. Each model was shown to be statistically adequate and to capture the relevant features of the data. In addition, the conclusions drawn in terms of price and output elasticities were not unique, and depended crucially on the chosen model. Thus, economic theory is of little assistance in discriminating among the competing models, and the empirical evidence suggests that the in-sample performance of each model is acceptable. Moreover, the choice of model has important implications for economic analysis. In this case, non-nested testing procedures can provide useful additional information. The factor demand systems of Sections 2 and 4 are compared in this section on the basis of the multivariate paired and joint non-nested tests illustrated in Section 3.

In Table 2 the results of preliminary systems RESET tests for each competing model are reported. These tests are calculated by adding the corresponding squared fitted values to each equation of the system and by testing their joint significance using a Wald statistic. Two versions of the systems RESET test are presented. The first test is calculated under the condition that the coefficients of the squared fitted values in each equation are different, leading to a Wald statistic with a $\chi^2(n)$ distribution, where n is the number of equations in the system (in our case, n=4). The second version of the test is based on the condition that the coefficients of the squared fitted values in each equation are identical, and is a Wald test with a $\chi^2(1)$ distribution. Rejection of the null hypothesis of correct model specification by the $\chi^2(n)$-RESET test is interpreted as misspecification of at least one equation of the system, but not necessarily of the whole system, whereas rejection of the null hypothesis by the $\chi^2(1)$-RESET test indicates that the system itself is misspecified.

[Table 2]

The results show that NHQDR and NHQDR1 are rejected at the 1% significance level by both the $\chi^2(4)$- and $\chi^2(1)$-RESET tests, and QDR is rejected by both versions of the test, but at different levels of significance. Specification TLG is rejected at the 1% significance level by the $\chi^2(4)$-RESET test, but is not rejected by the $\chi^2(1)$-RESET test. This is a contradiction, since the test indicates that functional form misspecification affects a sub-set of the system, but that the system itself does not suffer from misspecification. Finally, GLT and SGM are rejected only at the 5% significance level and only by the $\chi^2(4)$-RESET test. Thus, the systems RESET tests are incapable of determining a single model which performs best, although NHQDR and NHQDR1 appear to be more problematic than the others. This last evidence is in line with the empirical results of Section 4.

Table 3a shows the results obtained by comparing non-nested systems of equations with the same dependent variables, namely SGM and GLT on the one hand, and QDR and NHQDR1 on the other. The three different paired non-nested tests used in the empirical application are the J, P0 and P1 tests, as discussed in Section 3, which are implemented according to equations (3.3), (3.4) and (3.5), respectively. Each test is presented in two versions, with the $\chi^2(4)$-version being based on the condition that the coefficients λ_i (i=1,…,4) are different from each other, whereas the $\chi^2(1)$-version imposes the condition that $\lambda_1=\ldots=\lambda_4=\lambda$. The interpretation of the non-nested tests is analogous to those of the systems RESET tests. Rejection of the null hypothesis of correct specification by the $\chi^2(4)$-test is interpreted as misspecification of at least one equation of the system, but not necessarily of the whole system, against the chosen non-nested alternative. Conversely, a rejection of the null hypothesis by the $\chi^2(1)$- test indicates that the system itself is rejected against its non-nested counterpart.

[Table 3a]

The SGM system is rejected against GLT at the 1% significance level by the $\chi^2(4)$-version of each of the J, P0 and P1 tests. Each non-nested test suggests that there is a problem in at least one equation of the SGM system when compared with its GLT counterpart. The empirical evidence is mixed when the $\chi^2(1)$-version of the test is considered as, in this case, SGM is rejected at the 1%

level only by the P0 test, whereas the J test rejects SGM at the 5% level, and the P1 test suggests non-rejection of SGM. In particular, the P1 test denotes an inconsistency as the $\chi^2(1)$-version implies that SGM as a whole is correctly specified as compared with its GLT counterpart. When the roles of the null and alternative are reversed, GLT is rejected at the 5% level by the $\chi^2(4)$-version of the J and P0 tests, whereas the P1 test rejects GLT at the 1% level. The behaviour of the J and P1 tests is contradictory, as the $\chi^2(1)$-version of these tests suggests that there are no problems with the specification of the GLT system as a whole, whereas the $\chi^2(1)$-version of the P0 test rejects the null at the 5% level. In summary, the multivariate paired non-nested tests suggest that both the SGM and GLT specifications suffer from problems of misspecification.

In Table 3a, the second pair of competing models is given by QDR and NHQDR1. When QDR is the null hypothesis, it is strongly rejected by all three non-nested tests, under both the $\chi^2(4)$- and $\chi^2(1)$ versions. However, when NHQDR1 is the null, only the P1 test strongly rejects NHQDR1, regardless of which version of the test is used. The J test marginally rejects NHQDR1 with the $\chi^2(4)$-version, whereas it does not reject the null with the $\chi^2(1)$-version. The P0 test does not reject NHQDR1 at all. In this case, the results from these multivariate paired non-nested tests are interpreted as indirect evidence against the hypothesis of linear homogeneity in input prices.

Table 3b reports the results from the comparison of pairs of systems of equations with different dependent variables, where the competing pairs of systems are TLG and QDR, SGM and NHQDR, GLT and NHQDR, and TLG and NHQDR1. The multivariate non-nested tests used are the PE test (see equation (3.6)) and the BEM test (see equations (3.12) and (3.13)).

[Table 3b]

Both the $\chi^2(4)$- and $\chi^2(1)$-versions of the tests are presented. In general, all competing models are rejected by all tests, although there are a few interesting cases. When TLG and QDR are compared using the $\chi^2(1)$-version of the PE test neither is rejected. Moreover, the $\chi^2(1)$-version of both the PE and BEM tests is unable to reject TLG against the alternative system NHQDR1. One possible interpretation is that the $\chi^2(1)$-version of the test is too restrictive to detect specification problems which are likely to affect only a sub-set of each system. A similar comment applies in testing TLG against NHQDR1 using the $\chi^2(1)$-version of both the PE and BEM tests.

Tables 4a-4b present the empirical results obtained by using the multivariate joint BAM test (see equation (3.20)). The three competing systems are NHQDR, SGM and GLT, and each null model is tested jointly against two alternatives. Four versions of the BAM test are available, according to the restrictions placed on the λ_{ji} (j=1,2; i=1,…,4) in equation (3.20). If the λ_{ij} are unrestricted, this yields the $\chi^2(8)$-version of the BAM test, with four parameters across each of two alternative systems. The $\chi^2(4)$-version is obtained when the following restrictions hold: $\lambda_{11}=\lambda_{21}$, $\lambda_{12}=\lambda_{22}$, $\lambda_{13}=\lambda_{23}$, $\lambda_{14}=\lambda_{24}$, with the same coefficient for the two non-nested alternatives across each of the four equations. The $\chi^2(2)$-version is given by the following restrictions: $\lambda_{11}=\lambda_{12}=\lambda_{13}=\lambda_{14}$, $\lambda_{21}=\lambda_{22}=\lambda_{23}=\lambda_{24}$, with the same coefficient for each equation across each of two non-nested alternatives. Finally, the $\chi^2(1)$-version of the BAM test is obtained by imposing the following restrictions: $\lambda_{11}=\lambda_{21}=\lambda_{12}=\lambda_{22}=\lambda_{13}=\lambda_{23}=\lambda_{14}=\lambda_{24}=\lambda$, with the same coefficient across four equations and two alternatives. Notice that the $\chi^2(8)$- and $\chi^2(2)$-versions do not impose restrictions across the alternative models, whereas the $\chi^2(4)$- and $\chi^2(1)$-versions do impose cross-alternative restrictions.

[Tables 4a and 4b]

In general, all three models are strongly rejected, regardless of which version of the test is used. As before, there are a few cases worth highlighting. When the $\chi^2(2)$-version of the BAM test is used, TLG is not rejected against NHQDR1 and QDR jointly. Thus, the two alternative systems, each considered as a whole, do not add useful information to the TLG null model as a whole. When the $\chi^2(1)$-version of the test is used, neither QDR nor TLG is rejected against the other two models jointly. In this case, the added information given by a linear combination of both equations and alternatives is empirically irrelevant. Finally, if the $\chi^2(4)$-version of the test is considered, the TLG system is rejected only at the 5% significance level, whereas all other models are rejected at the 1% level. In summary, these results suggest that the information contained in a linear combination of the corresponding equations across the two alternatives is statistically important in rejecting each model against two non-nested alternatives jointly, at least at the 5% level.

6. Conclusion

The key points of this paper can be summarized as follows. Alternative factor demand systems have been presented using some of the most popular flexible functional forms, namely Translog, Quadratic, Generalized Leontief and Symmetric Generalized McFadden. Each system has been estimated using Berndt and Khaled's (1979) classical annual data set for the U.S. manufacturing sector over the period 1947-1971. The important task of model determination has been accomplished using a formal non-nested testing procedure. Multivariate extensions of some well-known and pedagogically appealing univariate paired non-nested tests (namely, the J, P0 and P1 tests of Davidson and MacKinnon, 1981; the PE statistic of MacKinnon, White and Davidson, 1983; and the Bera and McAleer (1989)), and the multivariate joint non-nested test of Barten and McAleer (1997), have been applied to compare alternative factor demand systems. Preliminary systems RESET tests for each competing model have also been reported. Systems RESET tests and multivariate paired non-nested tests were each presented in two versions, namely $\chi^2(4)$ and $\chi^2(1)$. Four versions of the multivariate joint non-nested test were developed, namely $\chi^2(8)$, $\chi^2(4)$, $\chi^2(2)$ and $\chi^2(1)$.

The main results are as follows. Each model has been shown to be statistically adequate and to capture the relevant features of the data. In addition, the conclusions drawn in terms of price and output elasticities were not unique, and depended crucially on the chosen model. Systems RESET tests were incapable of determining a single model which performs best, although NHQDR and NHQDR1 appeared to be more problematic than the others. The multivariate paired extensions of the J, P0 and P1 tests suggested that both the SGM and GLT specifications suffered from problems of misspecification. When the same tests were used to compare QDR and NHQDR1, the results were interpreted as indirect evidence against the hypothesis of linear homogeneity in input prices. Systems involving different dependent variables were compared using the multivariate non-nested PE and BEM tests. In general, all competing models were rejected by all tests, although there were a few interesting cases. When TLG and QDR were compared using the $\chi^2(1)$-version of the PE test, neither was rejected. Moreover, the $\chi^2(1)$-version of both the PE and BEM tests was unable to reject TLG against the alternative system NHQDR1. Finally, the multivariate joint BAM test was used to compare systems NHQDR, SGM and GLT. In general, the results suggested that the information contained in a linear combination of the corresponding equations across the two alternatives was statistically important in rejecting each model against two non-nested alternatives jointly.

References

Barten, A.P. and M. McAleer (1997), "Comparing the empirical performance of alternative demand systems", *L'Actualité Économique*, **73**, 27-45, reprinted in *Économétrie Appliquée*, ed. by C. Gouriéroux and C. Montmarquette, Paris, Éditions Economica, 27-45.

Bera, A.K. and M. McAleer (1989), "Nested and non-nested procedures for testing linear and log-linear regression models", *Sankhya* B, **51**, 212-224.

Berndt, E.R. (1991), *The Practice of Econometrics: Classic and Contemporary*, Reading, Addison-Wesley, 449-506.

Berndt, E.R. and L.R. Christensen (1973), "The translog function and the substitution of equipment, structures and labor in U.S. manufacturing 1929-68", *Journal of Econometrics*, **1**, 81-114.

Berndt, E.R. and M.S. Khaled (1979), "Parametric productivity measurement and choice among flexible functional forms", *Journal of Political Economy*, **87**, 1220-1245.

Berndt, E.R. and D.O. Wood (1975), "Technology, prices and the derived demand for energy", *Review of Economics and Statistics*, **57**, 259-268.

Davidson, R. and J.G. MacKinnon (1981), "Several tests for model specification in the presence of alternative hypotheses", *Econometrica*, **49**, 781-793.

Davidson, R. and J.G. MacKinnon (1982), "Some non-nested hypothesis tests and the relations among them", *Review of Economic Studies*, **49**, 551-565.

Diewert, W.E. and T.J. Wales (1987), "Flexible functional forms and global curvature conditions", *Econometrica*, **55**, 43-68.

Hall, B.H., Cummins C. and R. Schnake (1997), *Time Series Processor. Version 4.4. User's Guide*, Stanford, TSP International.

McAleer, M. (1983), "Exact tests of a model against non-nested alternatives", *Biometrika*, **70**, 285-288.

McFadden, D. (1978), "The general linear profit function", in *Production Economics: a Dual Approach to Theory and Applications*, ed. by M.A. Fuss and D.L. McFadden, Amsterdam, North-Holland, 409-453.

MacKinnon, J.G., White, H. and R. Davidson (1983), "Tests for model specification in the presence of alternative hypotheses: Some further results", *Journal of Econometrics*, **21**, 53-70.

Morey, E.R. (1986), "An introduction to checking, testing and imposing curvature properties: The true function and the estimated function", *Canadian Journal of Economics*, **19**, 207-235.

Table 1. Factor demand systems

System	Equations	Transformations on the dependent variables
GLT	4 factor demands (K,L,E,M)	Ratios of levels (K/Y,L/Y,E/Y,M/Y)
NHQDR	4 factor demands (K,L,E,M)	Levels (K,L,E,M)
NHQDR1	1 cost function (C) 3 factor demands (L,E,M)	Levels (C) Levels (L,E,M)
QDR	1 cost function (C) 3 factor demands (L,E,M)	Levels (C) Levels (L,E,M)
SGM	4 factor demands (K,L,E,M)	Ratios of levels (K/Y,L/Y,E/Y,M/Y)
TLG	1 cost function (C) 3 factor demands (L,E,M)	Logarithms (C) Shares ($P_l L/C, P_e E/C, P_m M/C$)

Table 2. Systems RESET tests

System	$\chi^2(4)$	$\chi^2(1)$
GLT	11.427*	3.395
	(0.022)	(0.065)
NHQDR	85.996**	32.553**
	(0.000)	(0.000)
NHQDR1	160.788**	116.171**
	(0.000)	(0.000)
QDR	20.202**	5.915*
	(0.000)	(0.015)
SGM	11.647*	3.425
	(0.020)	(0.064)
TLG	15.676**	0.915
	(0.003)	(0.339)

Notes: The systems RESET test is calculated by adding the squared fitted values to each equation of the system and by testing their joint significance under the assumptions that each coefficient attached to the fitted values is different ($\chi^2(4)$) or identical ($\chi^2(1)$) across all equations; * denotes rejection of the null hypothesis of correct functional form at the 5% significance level; ** denotes rejection of the null hypothesis of correct functional form at the 1% significance level; P-values are given in parentheses.

Table 3a. Multivariate *paired* non-nested tests: J, P0 and P1

Test	H_0	H_1	$\chi^2(4)$	$\chi^2(1)$
J	SGM	GLT	16.103**	6.157*
			(0.003)	(0.013)
	GLT	SGM	12.818*	3.0005
			(0.012)	(0.083)
	QDR	NHQDR1	254.161**	250.495**
			(0.000)	(0.000)
	NHQDR1	QDR	9.802*	1.452
			(0.044)	(0.228)
P0	*SGM*	GLT	25.140**	7.089**
			(0.000)	(0.008)
	GLT	SGM	11.843*	3.904*
			(0.019)	(0.048)
	QDR	NHQDR1	201.006**	196.511**
			(0.000)	(0.000)
	NHQDR1	QDR	8.223	1.513
			(0.084)	(0.219)
P1	SGM	GLT	34.884**	3.167
			(0.000)	(0.075)
	GLT	SGM	23.564**	1.457
			(0.000)	(0.227)
	QDR	NHQDR1	2191.842**	2172.844**
			(0.000)	(0.000)
	NHQDR1	QDR	207.978**	203.181**
			(0.000)	(0.000)

Notes: * denotes rejection of the null hypothesis H_0 against the paired alternative H_1 at the 5% significance level; ** denotes rejection of the null hypothesis H_0 against the paired alternative H_1 at the 1% significance level; P-values are given in parentheses.

Table 3b. Multivariate *paired* non-nested tests: PE and BEM

Test	H_0	H_1	$\chi^2(4)$	$\chi^2(1)$
PE	TLG	QDR	43.533**	0.134
			(0.000)	(0.714)
	QDR	TLG	22.394**	1.486
			(0.000)	(0.223)
	SGM	NHQDR	114.973**	103.424**
			(0.000)	(0.000)
	NHQDR	SGM	21.583**	9.397*
			(0.000)	(0.002)
	NHQDR	GLT	40.532**	13.489**
			(0.000)	(0.000)
	GLT	NHQDR	130.980**	112.255**
			(0.000)	(0.000)
	TLG	NHQDR1	23.299**	0.488
			(0.000)	(0.485)
	NHQDR1	TLG	34.810**	8.908**
			(0.000)	(0.003)
BEM	TLG	QDR	10.419*	2.880
			(0.034)	(0.090)
	QDR	TLG	44.825**	43.371**
			(0.000)	(0.000)
	SGM	NHQDR	74.299**	71.625**
			(0.000)	(0.000)
	NHQDR	SGM	30.161**	14.297**
			(0.000)	(0.000)
	NHQDR	GLT	45.783**	17.621**
			(0.000)	(0.000)
	GLT	NHQDR	61.417**	52.231**
			(0.000)	(0.000)
	TLG	NHQDR1	51.168**	0.144
			(0.000)	(0.705)
	NHQDR1	TLG	307.541**	50.291**
			(0.000)	(0.000)

Notes: * denotes rejection of the null hypothesis H_0 against the paired alternative H_1 at the 5% significance level; ** denotes rejection of the null hypothesis H_0 against the paired alternative H_1 at the 1% significance level; P-values are given in parentheses.

Table 4a. Multivariate *joint* non-nested test: BAM

H_0	H_1	H_2	$\chi^2(8)$	$\chi^2(2)$
NHQDR	SGM	GLT	136.063**	22.387**
			(0.000)	(0.000)
SGM	NHQDR	GLT	102.234**	71.896**
			(0.000)	(0.000)
GLT	NHQDR	SGM	74.306**	52.565**
			(0.000)	(0.000)
QDR	NHQDR1	TLG	52.809**	42.687**
			(0.000)	(0.000)
TLG	NHQDR1	QDR	72.120**	3.712
			(0.000)	(0.156)
NHQDR1	TLG	QDR	377.083**	51.890**
			(0.000)	(0.000)

Notes: * denotes rejection of the null hypothesis H_0 against the multiple alternatives H_1 and H_2 at the 5% significance level; ** denotes rejection of the null hypothesis H_0 against the multiple alternatives H_1 and H_2 at the 1% significance level; P-values are given in parentheses.

Table 4b. Multivariate *joint* non-nested test: BAM

H_0	H_1	H_2	$\chi^2(4)$	$\chi^2(1)$
NHQDR	SGM	GLT	37.245**	15.928**
			(0.000)	(0.000)
SGM	NHQDR	GLT	74.236**	71.601**
			(0.000)	(0.000)
GLT	NHQDR	SGM	61.389**	52.239**
			(0.000)	(0.000)
QDR	NHQDR1	TLG	26.982**	0.365
			(0.000)	(0.546)
TLG	NHQDR1	QDR	11.393*	1.121
			(0.022)	(0.290)
NHQDR1	TLG	QDR	23.186**	9.582**
			(0.000)	(0.002)

Notes: * denotes rejection of the null hypothesis H_0 against the multiple alternatives H_1 and H_2 at the 5% significance level; ** denotes rejection of the null hypothesis H_0 against the multiple alternatives H_1 and H_2 at the 1% significance level; P-values are given in parentheses.

Appendix 1. Factor demand systems: Estimation

Parameters	GLT	NHQDR	NHQDR1	QDR	SGM	TLG
α_0	-	-	45.882 (29.815)	-18.090 (59.415)	-	0.748** (0.088)
α_k	0.619** (0.110)	-0.288 (0.353)	241.786** (62.258)	-	0.652** (0.111)	-
α_l	0.196* (0.077)		0.983** (0.157)	0.575** (0.125)	0.209** (0.078)	0.411** (0.078)
α_e	0.761** (0.093)	-0.033 (0.140)	1.893** (0.218)	1.014** (0.129)	0.790** (0.092)	0.198** (0.016)
α_m	0.012 (0.043)	-0.058 (0.104)	1.150** (0.130)	-0.008 (0.058)	-0.007 (0.04)	0.127 (0.098)
α_y	-	-	0.045 (0.168)	0.960** (0.147)	-	0.840** (0.015)
α_t	-	-	-	-	-	-
α_{kk}	0.003** (0.0007)	-0.037 (0.075)	-566.152** (86.485)	-	0.003** (0.0004)	-
α_{kl}	0.002** (0.0003)	0.211** (0.047)	-0.063 (0.073)	-	-	-
α_{ke}	-0.0007 (0.0004)	0.088 (0.050)	-0.382** (0.106)	-	-	-
α_{km}	-0.0007** (0.0002)	-0.075* (0.034)	-0.251** (0.071)	-	-	-
α_{ky}	-	0.001** (0.0003)	0.777** (0.183)	-	-	-
α_{kt}	-	-0.038* (0.017)	8.055** (1.379)	-	-	-
α_{ll}	-0.00002 (0.0007)	-0.391** (0.058)	-0.635** (0.091)	-0.739** (0.073)	0.004** (0.0004)	0.116* (0.046)
α_{le}	0.003** (0.0008)	0.662** (0.063)	0.195* (0.079)	0.303** (0.064)	-	0.007 (0.010)
α_{lm}	-0.0006 (0.0004)	0.026 (0.050)	-0.162** (0.054)	0.242** (0.052)	-	-0.126* (0.050)
α_{ly}	-	0.003** (0.0003)	0.003** (0.0004)	0.003** (0.0004)	-	-0.030* (0.015)
α_{lt}	-0.003 (0.004)	-	0.030** (0.008)	0.012* (0.005)	-0.005 (0.004)	0.0002 (0.001)
α_{ee}	-0.003** (0.001)	-0.195 (0.120)	-0.770** (0.114)	-0.625** (0.118)	0.001* (0.0005)	0.011* (0.005)
α_{em}	0.002** (0.0006)	0.236** (0.083)	-0.110* (0.055)	0.104 (0.089)	-	-0.013 (0.008)
α_{ey}	-	0.0009** (0.0003)	0.0008 (0.0005)	0.001* (0.0005)	-	-0.029** (0.003)

Appendix 1. Factor demand systems: Estimation (continued)

Parameters	GLT	NHQDR	NHQDR1	QDR	SGM	TLG
α_{et}	0.023** (0.006)	-	0.054** (0.008)	0.026** (0.006)	0.021** (0.006)	0.0007** (0.0002)
α_{mm}	0.004** (0.0006)	-0.046 (0.106)	-0.533** (0.083)	-0.235* (0.099)	0.005** (0.0002)	0.180** (0.057)
α_{my}	-	0.004** (0.0002)	0.005** (0.0004)	0.004** (0.0001)	-	0.098** (0.019)
α_{mt}	0.002 (0.002)	-	0.025** (0.005)	-0.004 (0.002)	0.003 (0.002)	-0.002 (0.001)
α_{yy}	-	-	-	-	-	-
α_{yt}	-	-	-	-	-	-
α_{tt}	-	-	-	-	-	-
σ_{ll}	-	-	-	-	-0.009** (0.002)	-
σ_{le}	-	-	-	-	0.008** (0.002)	-
σ_{lm}	-	-	-	-	-0.004** (0.001)	-
σ_{ee}	-	-	-	-	-0.015** (0.002)	-
σ_{em}	-	-	-	-	0.008** (0.002)	-
σ_{mm}	-	-	-	-	-0.001 (0.002)	-
ρ_c	-	-	0.471	0.855	-	0.537
ρ_k	0.468	0.728	-	-	0.486	-
ρ_l	0.405	0.092	0.492	0.717	0.458	-0.097
ρ_e	0.464	-0.140	0.227	0.405	0.456	0.203
ρ_m	-0.134	-0.149	0.260	-0.162	-0.112	0.014

Notes: * denotes rejection of the null hypothesis of a zero coefficient at the 5% significance level; ** denotes rejection of the null hypothesis of a zero coefficient at the 1% significance level; ρ_i (i=c,k,l,e,m) are the single-equation serial correlation coefficients; standard errors are given in parentheses.

Appendix 2. Factor demand systems: Diagnostic statistics

Statistic	GLT	NHQDR	NHQDR1	QDR	SGM	TLG
R^2_c	-	-	0.993	0.955	-	0.999
R^2_k	0.676	0.994	-	-	0.691	-
R^2_l	0.963	0.977	0.972	0.956	0.961	0.832
R^2_e	0.820	0.992	0.985	0.984	0.830	0.947
R^2_m	0.806	0.998	0.991	0.996	0.793	0.815
HET_c	-	-	0.012 [0.914]	5.989 [0.014]	-	0.746 [0.388]
HET_k	1.029 [0.310]	0.029 [0.864]	-	-	1.078 [0.299]	-
HET_l	0.350 [0.554]	1.175 [0.278]	0.424 [0.515]	0.632 [0.427]	0.070 [0.791]	0.283 [0.594]
HET_e	0.004 [0.952]	3.034 [0.082]	0.012 [0.914]	2.420 [0.120]	0.007 [0.934]	0.613 [0.433]
HET_m	0.166 [0.684]	1.055 [0.304]	0.127 [0.721]	2.034 [0.154]	0.023 [0.880]	0.142 [0.706]
AR_c	-	-	0.182 [0.435]	0.419 [0.091]	-	0.182 [0.410]
AR_k	0.354 [0.095]	0.326 [0.111]	-	-	0.354 [0.096]	-
AR_l	0.084 [0.697]	-0.122 [0.571]	0.182 [0.388]	0.281 [0.210]	0.068 [0.753]	0.068 [0.749]
AR_e	-0.006 [0.977]	-0.204 [0.337]	0.092 [0.660]	0.280 [0.190]	-0.019 [0.928]	0.078 [0.719]
AR_m	0.062 [0.771]	-0.160 [0.449]	0.164 [0.505]	0.371 [0.089]	0.022 [0.918]	0.220 [0.314]
NOB	24	24	24	24	24	24
Concavity violations	0	0	0	0	0	0

Notes: R^2_i (i=c,k,l,e,m) values are computed as the squared correlation coefficients of actual and fitted values of the dependent variables for each equation; HET_i are LM-type heteroskedasticity tests calculated by regressing the squared residuals on a constant term and the squared fitted values of the dependent variables for each equation; AR_i are the serial correlation coefficients for each equation, namely the estimated coefficients of the regression of the residuals from each equation on their one-period lagged counterparts; NOB indicates the total number of observations; Concavity violations refer to the number of principal minors which do not satisfy the conditions for the Hessian matrix of the cost function to be negative semi-definite; P-values are given in parentheses.

Appendix 3. Factor demand systems: Price and output elasticities (mean values)

Elasticities	GLT	NHQDR	NHQDR1	QDR	SGM	TLG
ε_{kk}	-0.071 (0.018)	-0.025 (0.003)	-2.222 (0.434)	-0.002 (0.0006)	-0.062 (0.017)	-0.120 (0.067)
ε_{kl}	0.196 (0.022)	0.207 (0.005)	-0.0004 (0.00004)	0.002 (0.0007)	0.219 (0.043)	0.331 (0.010)
ε_{ke}	-0.064 (0.003)	0.068 (0.013)	-0.002 (0.0003)	0.0008 (0.0008)	-0.068 (0.025)	-0.062 (0.009)
ε_{km}	-0.062 (0.003)	-0.055 (0.008)	-0.001 (0.0002)	-0.0008 (0.0002)	-0.090 (0.005)	-0.148 (0.060)
ε_{lk}	0.184 (0.015)	0.195 (0.015)	-0.059 (0.005)	0.374 (0.091)	0.195 (0.036)	0.064 (0.004)
ε_{ll}	-0.441 (0.019)	-0.541 (0.060)	-0.879 (0.101)	-0.882 (0.135)	-0.389 (0.055)	-0.301 (0.006)
ε_{le}	0.317 (0.009)	0.713 (0.079)	0.210 (0.025)	0.289 (0.085)	0.315 (0.018)	0.069 (0.004)
ε_{lm}	-0.060 (0.003)	0.027 (0.002)	-0.167 (0.013)	0.219 (0.054)	-0.122 (0.012)	0.167 (0.009)
ε_{ek}	-0.064 (0.006)	0.067 (0.009)	-0.293 (0.046)	0.125 (0.115)	-0.054 (0.025)	-0.074 (0.009)
ε_{el}	0.337 (0.040)	0.748 (0.014)	0.221 (0.011)	0.299 (0.064)	0.361 (0.045)	0.427 (0.019)
ε_{ee}	-0.495 (0.047)	-0.175 (0.036)	-0.692 (0.152)	-0.504 (0.202)	-0.523 (0.024)	-0.696 (0.014)
ε_{em}	0.222 (0.013)	0.201 (0.031)	-0.094 (0.016)	0.079 (0.028)	0.215 (0.018)	0.343 (0.025)
ε_{mk}	-0.068 (0.005)	-0.061 (0.008)	-0.205 (0.031)	-0.151 (0.020)	-0.078 (0.003)	-0.012 (0.004)
ε_{ml}	-0.070 (0.007)	0.032 (0.001)	-0.196 (0.012)	0.250 (0.045)	-0.110 (0.011)	0.074 (0.008)
ε_{me}	0.246 (0.003)	0.225 (0.045)	-0.105 (0.022)	0.088 (0.032)	0.246 (0.027)	0.025 (0.003)
ε_{mm}	-0.107 (0.011)	-0.042 (0.006)	-0.485 (0.077)	-0.188 (0.059)	-0.058 (0.016)	-0.086 (0.006)
ε_{ky}	0.656 (0.065)	0.220 (0.015)	0.787 (0.088)	0.994 (0.075)	0.638 (0.067)	0.100 (0.061)
ε_{ly}	0.877 (0.038)	0.796 (0.096)	0.816 (0.099)	0.783 (0.101)	0.884 (0.046)	0.724 (0.003)
ε_{ey}	0.314 (0.075)	0.181 (0.009)	0.168 (0.010)	0.236 (0.012)	0.306 (0.081)	0.186 (0.048)
ε_{my}	0.977 (0.005)	0.887 (0.024)	0.967 (0.031)	0.886 (0.028)	0.980 (0.011)	0.991 (0.004)

Notes: ε_{sw} denotes the demand elasticity of input s to the price of input w; ε_{sy} indicates the demand elasticity of input s with respect to output y; mean values are calculated over the period 1948-1971; standard deviations are reported in parentheses.