Net Foreign Assets and the Exchange Rate:
Redux Revived[*]

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Abstract

We revisit Obstfeld and Rogoff's (1995) results on exchange rate dynamics in a two-country, monetary model with incomplete asset markets, stationary net foreign assets, and endogenous monetary policy. The nominal exchange rate exhibits a unit root. Under flexible prices, it also depends on the stock of real net foreign assets. With sticky prices, the exchange rate depends on the past GDP differential, along with net foreign assets. Endogenous monetary policy and asset dynamics have consequences for exchange rate overshooting. A persistent relative productivity shock results in delayed overshooting under both flexible and sticky prices. A persistent relative interest rate shock generates undershooting under flexible prices.

Keywords: Exchange rate; Monetary policy; Net foreign assets; Overshooting
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1   Introduction

Maurice Obstfeld and Kenneth Rogoff's (1995) "Exchange Rate Dynamics Redux" was originally written to put forth a model of exchange rate determination with an explicit role for current

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account imbalances.[1] The non-stationarity of the model led most of the subsequent literature in the so-called "new open economy macroeconomics" to develop in different directions and "forget" the insights of the model on the dynamic relation between the exchange rate and net foreign asset accumulation by de-emphasizing the role of the latter.[2]

Figure 1 shows two well known stylized facts: the persistent and growing U.S. current account deficit over the 1990s and the likewise persistent appreciation of the dollar.[3] It is a commonly held view that the advent of the "new economy" has been the most significant exogenous shock to affect the position of the U.S. economy relative to the rest of the world in recent years. We can interpret this shock as a (persistent) favorable relative productivity shock. A story that one could tell about the stylized facts in Figure 1 is that the shock caused the U.S. to borrow from the rest of the world and the capital inflow generated exchange rate appreciation. This story could be reconciled with models of exchange rate determination developed in the 1970s and early 1980s.[4] If the shock is taken as permanent, the story can also be reconciled with Obstfeld and Rogoff's original model. Nevertheless, the argument cannot be reconciled with the overwhelming majority of new generation models that followed.

In this paper, we go back to the original intent of Obstfeld and Rogoff's work and develop a two-country model of exchange rate determination in which stationary net foreign asset dynamics play an explicit role.[5] We deal with indeterminacy of the steady state and non-stationarity of the original incomplete markets setup by adopting the overlapping generations framework illustrated in Ghironi (2000). If exogenous shocks are stationary, the departure from Ricardian equivalence generated by the birth of new households with no assets in all periods is sufficient to ensure existence of a determinate steady state and stationarity of real variables. Unexpected temporary shocks cause countries to run current account imbalances, which are re-absorbed over time as the world economy returns to the original steady state.[6]

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[1] See also Obstfeld and Rogoff (1996, Ch. 10).

[2] This is achieved either by assuming unitary intratemporal elasticity of substitution between domestic and foreign goods in consumption as in Corsetti and Pesenti (2001) or by combining the assumptions of complete markets and power utility. (Tille, 2000, provides a clear exposition of the consequences of complete markets.) Kollmann (2001*a*) is a recent exception to the trend, although he uses a non-stationary model. For a survey of the literature, see Lane (2001).

[3] Source: National Accounts and Federal Reserve, respectively. Effective dollar rate: Broad exchange rate weighted average. Current account unit: billions of dollars.

[4] Among others, examples are Dornbusch and Fischer (1980) and Branson and Henderson (1985).

[5] Hau and Rey (2001) explore the relation between capital flows and the exchange rate in a continuous-time model, focusing on the information content of different financial assets.

[6] P. Benigno (2001) achieves stationarity in an incomplete markets, open economy model by introducing costs of bond holdings. See also Kollmann (2001*b*) and Schmitt-Grohé and Uribe (2001*a*). Mendoza (1991) deals with the stationarity issue by assuming an endogenous discount factor as in Uzawa (1968). Schmitt-Grohé and Uribe (2001*b*) compare the quantitative performance of these approaches in a small open economy setup. Ghironi (2000) discusses the approaches to the non-stationarity issue that have been explored thus far. Net foreign asset dynamics

Differently from Obstfeld and Rogoff (1995) (and in line with the more recent literature on monetary policy), we allow for endogenous monetary policy in the form of interest rate reaction functions for the two countries. We consider familiar interest setting rules as in Taylor (1993). Interest rates react to the deviations of CPI inflation and GDP from their steady-state levels. They are also subject to exogenous shocks to allow for the possibility of exogenous changes in monetary policy.[7]

We solve the model with the method of undetermined coefficients illustrated in Campbell (1994). We rely on Uhlig's (1999) implementation of the method when solving the model numerically. The method has the advantage of delivering a process equation for the exchange rate with straightforward quantitative and empirical implications.

We are able to solve a benchmark model with purchasing power parity and flexible prices analytically. The solution for the nominal exchange rate exhibits a unit root, consistent with the empirical findings of Meese and Rogoff (1983). However, today's exchange rate also depends on the stock of real net foreign assets accumulated in the previous period. The intuition is as follows. No-arbitrage ensures that uncovered interest parity holds in our model: expected exchange rate depreciation equals the nominal interest rate differential. To the extent that interest rates react to variables that are affected by net foreign assets, the latter affect the exchange rate too. Thus, the model implies that asset holdings help predict the nominal exchange rate. (We discuss the conditions under which a decrease in asset holdings–a current account deficit/capital inflow– generates an appreciation of the domestic currency.) The response of the exchange rate to shocks is more different from that of a simple random walk the slower the convergence of net foreign assets to the steady state and the higher the degree of substitutability between domestic and foreign goods in consumption.

The exchange rate overshoots its new long-run level following a temporary (relative) productivity shock. If the shock is persistent, endogenous monetary policy and asset dynamics generate delayed overshooting. Endogenous monetary policy is responsible for exchange rate *under*shooting after persistent (relative) interest rate shocks. ("Persistent" does *not* mean "permanent" throughout the paper. When we consider permanent shocks, we say so explicitly.)

Next, we analyze exchange rate and asset dynamics in a sticky-price world. We introduce price stickiness by assuming that it is costly to change output prices over time as in Rotemberg (1982). It is harder to solve the model analytically in this case. We investigate the effect of

nominal and real shocks using a plausible calibration of the model. When prices are sticky, the exchange rate still exhibits a unit root under the Taylor rule.[8] The current level of the exchange rate depends on the past GDP differential, along with net foreign assets. Temporary shocks to relative productivity result in delayed overshooting. So do persistent shocks. Temporary relative interest rate shocks cause immediate overshooting. No overshooting may happen when interest rate shocks are persistent.

Our results on exchange rate overshooting contrast with Obstfeld and Rogoff's (1995), who obtain no overshooting following monetary and/or productivity shocks in their benchmark setup. We show that price stickiness is not necessary to generate overshooting once asset dynamics and endogenous monetary policy are accounted for. This brings a new perspective to bear on a topic that has been at the center of theoretical and empirical research on exchange rates since Dornbusch's (1976) seminal paper. Our model has the potential for reconciling the evidence in favor of delayed overshooting in Clarida and Galí (1994) and Eichenbaum and Evans (1995) with rational behavior and uncovered interest parity.

As far as the empirical performance is concerned, we show that the simple model of this paper delivers exchange rate appreciation following a favorable shock to relative productivity in an environment in which monetary policy follows the Taylor rule. However, the model does not generate accumulation of net foreign debt following the shock, unless the latter is permanent and prices are sticky. The reason is that consumption smoothing is the only motive for asset accumulation in the model. If the relative productivity shock is permanent and prices are sticky, the new long-run level of domestic GDP relative to foreign is above the short-run differential, which causes domestic agents to borrow from abroad to smooth consumption. The stock-market value of the home economy rises permanently above its foreign counterpart.[9] It remains to be seen whether the advent of the "new economy" has shifted U.S. productivity permanently above foreign. If one believes that the shock has been persistent, but not permanent, the model can explain only part of the dynamics in Figure 1. Inclusion of capital accumulation and investment appears a promising way of completing the theory.

On more rigorous grounds, the model yields a set of straightforward empirically testable implications for exchange rate dynamics. Econometric work on these implications will be necessary if we hope to overturn Meese and Rogoff's (1983) dismal conclusions.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 illustrates the log-linear equations that determine exchange rate and asset dynamics. Section 4 discusses the relation between net foreign assets and the exchange rate under flexible prices. Section 5 extends

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[8]Benigno and Benigno (2001) first obtained this result. See also Monacelli (2001).

[9]Also in Obstfeld and Rogoff's original model consumption smoothing causes the domestic economy to run a debt after a permanent favorable shock to relative productivity (which has no short-run effect on GDP in their setup).

the analysis to the case of sticky prices. Section 6 concludes.

2 The Model

The model is a monetary version of the setup in Ghironi (2000). The world consists of two countries, *home* and *foreign*. In each period t, the world economy is populated by a continuum of infinitely lived households between 0 and N_t^W. Each household consumes, supplies labor, and holds financial assets. As in Weil (1989), we assume that households are born on different dates *owning no assets*, but they own the present discounted value of their labor income.[10] The number of households in the home economy, N_t, grows over time at the exogenous rate $n > 0$, *i.e.*, $N_{t+1} = (1 + n)N_t$. We normalize the size of a household to 1, so that the number of households alive at each point in time is the economy's population. Foreign population (N_t^*) grows at the same rate as home population. The world economy has existed since the infinite past. It is useful to normalize world population at time 0 to the continuum between 0 and 1, so that $N_0^W = 1$.

A continuum of goods $i \in [0,1]$ are produced in the world by monopolistically competitive, infinitely lived firms, each producing a single differentiated good. Firms have existed since the infinite past. At time 0, the number of goods that are supplied in the world economy is equal to the number of households. The latter grows over time, but the commodity space remains unchanged. Thus, as time goes, the ownership of firms spreads across a larger number of households. Profits are distributed to consumers via dividends, and the structure of the market for each good is taken as given. We assume that the domestic economy produces goods in the interval $[0, a]$, which is also the size of the home population at time 0, whereas the foreign economy produces goods in the range $(a, 1]$.

The asset menu includes nominal bonds denominated in units of domestic and foreign currency, money balances, and shares in firms. Private agents in both countries trade the bonds domestically and internationally. Shares in home (foreign) firms and domestic (foreign) currency balances are held only by home (foreign) residents.

2.1 Households

Agents have perfect foresight, though they can be surprised by initial unexpected shocks. Consumers have identical preferences over a real consumption index (C), leisure (LE), and real money balances $(\frac{M}{P}$, where M denotes nominal money holdings and P is the consumption-based price index–CPI). At any time t_0, the representative home consumer j born in period $v \in [-\infty, t_0]$

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[10]Blanchard (1985) combines this assumption with a positive probability of not surviving until the next period. This is advantageous for calibration purposes (see below), besides being plausible. We adopt the Weil setup here because it is relatively simpler to illustrate.

maximizes the intertemporal utility function:

$$U_{t_0}^{vj} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\rho \log C_t^{vj} + (1-\rho) \log LE_t^{vj} + \chi \log \frac{M_t^{vj}}{P_t} \right],$$ (1)

with $0 < \rho < 1$.[11]

The consumption index for the representative domestic consumer is:

$$C_t^{vj} = \left[a^{\frac{1}{\omega}} \left(C_{Ht}^{vj} \right)^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}} \left(C_{Ft}^{vj} \right)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}},$$

where $\omega > 0$ is the intratemporal elasticity of substitution between domestic and foreign goods. The consumption sub-indexes that aggregate individual domestic and foreign goods are, respectively:

$$C_{Ht}^{vj} = \left[(\frac{1}{a})^{\frac{1}{\theta}} \int_0^a c_t^{vj}(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \text{ and } C_{Ft}^{vj} = \left[(\frac{1}{1-a})^{\frac{1}{\theta}} \int_a^1 c_{*t}^{vj}(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}},$$

where $c_{*t}^{vj}(i)$ denotes time t consumption of good i produced in the foreign country, and $\theta > 1$ is the elasticity of substitution between goods produced inside each country.

The CPI is:

$$P_t = \left[a P_{Ht}^{1-\omega} + (1-a) P_{Ft}^{1-\omega} \right]^{\frac{1}{1-\omega}}$$

where P_H (P_F) is the price sub-index for home (foreign)-produced goods–both expressed in units of the home currency. Letting $p_t(i)$ be the home currency price of good i, we have:

$$P_{Ht} = \left(\frac{1}{a} \int_0^a p_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}, \quad P_{Ft} = \left(\frac{1}{1-a} \int_a^1 p_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}.$$

We assume that there are no impediments to trade and that firms do not engage in local currency pricing (*i.e.*, pricing in the currency of the economy where goods are sold). Hence, the law of one price holds for each individual good and $p_t(i) = \varepsilon_t p_t^*(i)$, where ε_t is the exchange rate (units of domestic currency per unit of foreign) and $p_t^*(i)$ is the foreign currency price of good i. This hypothesis and identical intratemporal consumer preferences across countries ensure that consumption-based purchasing power parity (PPP) holds, *i.e.*, $P_t = \varepsilon_t P_t^*$.

Workers supply labor (L) in competitive labor markets. The total amount of time available in each period is normalized to 1, so that:[12]

$$LE_t^{vj} = 1 - L_t^{vj}.$$ (2)

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[11] We focus on domestic households. Foreign agents maximize an identical utility function. They consume the same basket of goods as home agents, with identical parameters, and they are subject to similar constraints. We will sometimes refer to the representative consumer of generation v simply as the "representative consumer" below. It is understood that consumers of different generations can behave differently in our model.

[12] A similar constraint holds for foreign agents.

The representative consumer enters a period holding nominal bonds, nominal money balances, and shares purchased in the previous period. She or he receives interests and dividends on these assets, may earn capital gains or incur losses on shares, earns labor income, is taxed, and consumes.

Denote the date t price (in units of domestic currency) of a claim to the representative domestic firm i's entire future profits (starting on date $t+1$) by V_t^i. Let x_{t+1}^{vji} be the share of the representative domestic firm i owned by the representative domestic consumer j born in period v at the end of period t. D_t^i denotes the nominal dividends firm i issues on date t. Then, letting A_{t+1}^{vj} (A_{t+1}^{*vj}) be the home consumer's holdings of domestic (foreign) currency denominated bonds entering time $t+1$, the period budget constraint expressed in units of domestic currency is:

$$A_{t+1}^{vj} + \varepsilon_t A_{t+1}^{*vj} + \int_0^a \left(V_t^i x_{t+1}^{vji} - V_{t-1}^i x_t^{vji} \right) di + M_t^{vj}$$

$$= (1+i_t)A_t^{vj} + \varepsilon_t(1+i_t^*)A_t^{*vj} + \int_0^a D_t^i x_t^{vji} di$$

$$+ \int_0^a \left(V_t^i - V_{t-1}^i \right) x_t^{vji} di + M_{t-1}^{vj} + W_t L_t^{vj} - P_t C_t^{vj} - P_t T_t^v, \tag{3}$$

where i_t (i_t^*) is the nominal interest rate on holdings of domestic (foreign) bonds between $t-1$ and t, W_t is the nominal wage, M_{t-1}^{vj} denotes the agent's holdings of nominal money balances entering period t, and T_t^v is a lump-sum net real transfer, which is identical across members of generation v.[13]

The representative domestic consumer born in period v maximizes the intertemporal utility function (1) subject to the constraints (2) and (3). Dropping the j superscript (because symmetric agents make identical choices in equilibrium), optimal labor supply is given by:

$$L_t^v = 1 - LE_t^v = 1 - \frac{1-\rho}{\rho}\frac{C_t^v}{w_t}, \tag{4}$$

which equates the marginal cost of supplying labor with the marginal utility of consumption generated by the corresponding increase in labor income (w_t denotes the real wage, $\frac{W_t}{P_t}$).

Making use of this equation, the first-order condition for optimal holdings of domestic currency bonds yields the Euler equation:

$$C_t^v = \left[\beta(1+i_{t+1})(\frac{P_t}{P_{t+1}}) \right]^{-1} C_{t+1}^v \tag{5}$$

for all $v \leq t$.

Demand for home currency real balances is:

$$\frac{M_t^v}{P_t} = \frac{\chi}{\rho}\frac{1+i_{t+1}}{i_{t+1}}C_t^v. \tag{6}$$

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[13] Given that individuals are born owning no financial wealth, because not linked by altruism to individuals born in previous periods, $A_v^{vj} = A_v^{*vj} = x_v^{vji} = M_{v-1}^{vj} = 0$.

Real domestic currency balances increase with consumption and decrease with the opportunity cost of holding money.

Condition (5) can be combined with the first-order condition for holdings of foreign bonds to yield a no-arbitrage condition between domestic and foreign currency bonds for domestic agents. Absence of unexploited arbitrage opportunities requires:

$$1 + i_{t+1} = \left(1 + i_{t+1}^*\right) \frac{\varepsilon_{t+1}}{\varepsilon_t}. \tag{7}$$

The consumption-based real interest rate between t and $t+1$ is defined by the familiar Fisher parity condition:

$$1 + r_{t+1} = \left(1 + i_{t+1}\right) \frac{P_t}{P_{t+1}} = \frac{1 + i_{t+1}}{1 + \pi_{t+1}^{CPI}}, \tag{8}$$

where π_{t+1}^{CPI} is CPI inflation ($\pi_{t+1}^{CPI} \equiv \frac{P_{t+1}}{P_t} - 1$). PPP ensures that $1 + \pi_t^{CPI} = (1 + e_t)\left(1 + \pi_t^{CPI*}\right)$, where $1 + \pi_t^{CPI*} \equiv \frac{P_t^*}{P_{t-1}^*}$ and $1 + e_t \equiv \frac{\varepsilon_t}{\varepsilon_{t-1}}$. Combining (8) with (7) and making use of PPP shows that $1 + r_{t+1} = 1 + r_{t+1}^* = \left(1 + i_{t+1}^*\right) \frac{P_t^*}{P_{t+1}^*}$: real interest rates are equal across countries in the absence of unexpected shocks that may cause no-arbitrage conditions to fail *ex post*.

Absence of arbitrage opportunities between bonds and shares in the domestic economy requires:

$$1 + i_{t+1} = \frac{D_{t+1}^i + V_{t+1}^i}{V_t^i}. \tag{9}$$

Letting $d_t^i \equiv \frac{D_t^i}{P_t}$ and $v_t^i = \frac{V_t^i}{P_t}$, we can re-write the no-arbitrage condition between bonds and shares as:

$$1 + r_{t+1} = \frac{d_{t+1}^i + v_{t+1}^i}{v_t^i}. \tag{10}$$

As usual, first-order conditions and the period budget constraint must be combined with appropriate transversality conditions to ensure optimality.[14]

2.2   Firms

Output supplied at time t by the representative domestic firm i is a linear function of labor demanded by the firm:

$$Y_t^{\text{Si}} = Z_t L_t^i. \tag{11}$$

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[14] Similar labor-leisure tradeoff, Euler equation, no-arbitrage, and transversality conditions hold for foreign agents.

Z_t is an exogenous economy-wide productivity parameter. Production by the representative foreign firm is a linear function of L_t^{i*}, with productivity parameter Z_t^*.[15]

Output demand comes from several sources: domestic and foreign consumers and domestic and foreign firms. The demand for home good i by the representative home consumer born in period v is:

$$c_t^v(i) = \left(\frac{p_t(i)}{P_{Ht}}\right)^{-\theta} \left(\frac{P_{Ht}}{P_t}\right)^{-\omega} C_t^v$$

obtained by maximizing C^v subject to a spending constraint. *Total demand* for home good i coming from domestic consumers is:

$$
\begin{aligned}
c_t(i) &= a \left[\begin{array}{l} \cdots \frac{n}{(1+n)^{t+1}} c_t^{-t}(i) + \ldots + \frac{n}{(1+n)^2} c_t^{-1}(i) + \frac{n}{1+n} c_t^0(i) \\ + n c_t^1(i) + n(1+n) c_t^2(i) + \cdots + n(1+n)^{t-1} c_t^t(i) \end{array} \right] \\
&= \left(\frac{p_t(i)}{P_{Ht}}\right)^{-\theta} \left(\frac{P_{Ht}}{P_t}\right)^{-\omega} a(1+n)^t c_t,
\end{aligned}
$$

where

$$c_t \equiv \frac{a \left[\begin{array}{l} \cdots \frac{n}{(1+n)^{t+1}} C_t^{-t} + \ldots + \frac{n}{(1+n)^2} C_t^{-1} + \frac{n}{1+n} C_t^0 \\ + n C_t^1 + n(1+n) C_t^2 + \cdots + n(1+n)^{t-1} C_t^t \end{array} \right]}{a(1+n)^t}$$

is aggregate per capita home consumption.[16]

Given identity of intratemporal preferences, the expression for the demand of home good i from foreign consumers born in period v is analogous, and *total* demand for the same good by foreign consumers is

$$c_t^*(i) = \left(\frac{p_t(i)}{P_{Ht}}\right)^{-\theta} \left(\frac{P_{Ht}}{P_t}\right)^{-\omega} (1-a)(1+n)^t c_t^*,$$

where

$$c_t^* \equiv \frac{(1-a) \left[\begin{array}{l} \cdots \frac{n}{(1+n)^{t+1}} C_t^{-t*} + \ldots + \frac{n}{(1+n)^2} C_t^{-1*} + \frac{n}{1+n} C_t^{0*} \\ + n C_t^{1*} + n(1+n) C_t^{2*} + \cdots + n(1+n)^{t-1} C_t^{t*} \end{array} \right]}{(1-a)(1+n)^t}$$

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[15] Because all firms in the world economy are born at $t = -\infty$, after which no new goods appear, it is not necessary to index output and factor demands by the firms' date of birth. As for consumers, we focus on domestic firms below. Foreign firms are symmetric in all respects.

[16] At time 0, home population is equal to a. At time 1, it is $a(1+n)$. Hence, generation 1 consists of an households. Population at time 2 is $a(1+n)^2$. It follows that generation 2 consists of $an(1+n)$ households. Continuing with this reasoning shows that generation t consists of $an(1+n)^{t-1}$ households. Going back in time from $t = 0$, population at time -1 is $\frac{a}{1+n}$. Hence, generation 0 consists of $\frac{an}{1+n}$ households. Population at time -2 is $\frac{a}{(1+n)^2}$. It follows that generation -1 consists of $\frac{an}{(1+n)^2}$ households. Continuing with this reasoning makes it possible to show that generation $-t$ consists of $\frac{an}{(1+n)^{t+1}}$ households.

is aggregate per capita foreign consumption.

Changing the price of its output is costly for the firm, which generates nominal rigidity. Specifically, we assume that the real cost (measured in units of the composite good) of output-price inflation volatility around a steady-state level of inflation equal to 0, is:

$$PAC_t^i = \frac{\kappa}{2}\left(\frac{p_t(i)}{p_{t-1}(i)} - 1\right)^2 \frac{p_t(i)}{P_t}Y_t^i.$$

When the firm changes the price of its output, a set of material goods–e.g., new catalogs, price tags, etc.–need to be purchased. The price adjustment cost (PAC^i) captures the amount of marketing materials that must be purchased to implement a price change. Because the amount of these materials is likely to increase with firm size, PAC^i increases with revenues $((p_t(i)/P_t)Y_t^i)$, which are taken as a proxy for size. The cost is convex in inflation; faster price movements are more costly to the firm.[17] We assume $\kappa \geq 0$. When $\kappa = 0$, prices are flexible.

Total demand for good i produced in the home country is obtained by adding the demands for that good originating in the two countries. Making use of the results above, it is:

$$Y_t^{Di} = \left(\frac{p_t(i)}{P_{Ht}}\right)^{-\theta}\left(\frac{P_{Ht}}{P_t}\right)^{-\omega}Y_t^{DW}. \tag{12}$$

Y_t^{DW} is aggregate world demand of the composite good, defined as $Y_t^{DW} \equiv C_t^W + PAC_t^W$. $C_t^W \equiv (1+n)^t\left[ac_t + (1-a)c_t^*\right]$ and $PAC_t^W \equiv aPAC_t^i + (1-a)PAC_t^{*i}$ denote aggregate world consumption and the world aggregate cost of adjusting prices, respectively.[18]

Given the no-arbitrage condition between bonds and shares (10) and a no-speculative bubble condition, the real price of firm i's shares at time t_0 is given by the present discounted value of the real dividends paid by the firm from $t_0 + 1$ on:

$$v_{t_0}^i = \sum_{s=t_0+1}^{\infty} R_{t_0,s}d_s^i,$$

where

$$R_{t_0,s} \equiv \frac{1}{\prod\limits_{u=t_0+1}^{s}(1 + r_u)}, \quad R_{t_0,t_0} = 1.$$

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[17] The quadratic specification for the cost of adjusting prices, first introduced by Rotemberg (1982), yields dynamics for the aggregate economy that are similar to those resulting from staggered price setting a la Calvo (1983).

[18] The expression for the world aggregate cost of adjusting prices derives from the assumption that the number of firms is constant. In the expression for PAC_t^W, we have already made use of the fact that symmetric firms make identical equilibrium choices. Keeping the i superscript for individual firms' variables allows us to denote several aggregate per capita variables referring to firms by dropping the superscript below.

At time t_0, firm i maximizes:

$$v_{t_0}^i + d_{t_0}^i = \sum_{s=t_0}^{\infty} R_{t_0,s} d_s^i,$$

i.e., the present discounted value of dividends to be paid from t_0 on. At each point in time, dividends are given by net real revenues–$(1 - \overline{\tau}) \frac{p_t(i)}{P_t} Y_t^i$–plus a lump-sum transfer (or tax) from the government–T_t^{fi}–minus costs–$\frac{W_t}{P_t} L_t^i + \frac{\kappa}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 \frac{p_t(i)}{P_t} Y_t^i$. The firm chooses the price of its product and the amount of labor demanded in order to maximize the present discounted value of its current and future profits subject to the constraints (11) and (12), and the market clearing condition $Y_t^i = Y_t^{\text{Si}} = Y_t^{Di}$. Firm i takes the aggregate price indexes, the wage rate, Z_t, world aggregates, and taxes and transfers as given.

Let λ_t^i denote the Lagrange multiplier on the constraint $Y_t^{\text{Si}} = Y_t^{Di}$. Then, λ_t^i is the shadow price of an extra unit of output to be sold in period t, or the marginal cost of time t sales. The first-order condition with respect to $p_t(i)$ yields the pricing equation:

$$p_t(i) = \Psi_t^i P_t \lambda_t^i, \tag{13}$$

which equates the price charged by firm i to the product of the (nominal) shadow value of one extra unit of output–the (nominal) marginal cost $(P_t \lambda_t^i)$–and a markup (Ψ_t^i). The latter depends on output demand as well as on the impact of today's pricing decision on today's and tomorrow's costs of adjusting the output price:

$$\Psi_t^i \equiv \theta Y_t^i \left\{ (\theta - 1) Y_t^i \left[1 - \overline{\tau} - \frac{\kappa}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 \right] + \kappa \Upsilon_t^i \right\}^{-1},$$

where

$$\Upsilon_t^i \equiv Y_t^i \frac{p_t(i)}{p_{t-1}(i)} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 \right) - \frac{Y_{t+1}^i}{1 + r_{t+1}} \frac{P_t}{P_{t+1}} \left(\frac{p_{t+1}(i)}{p_t(i)} \right)^2 \left(\frac{p_{t+1}(i)}{p_t(i)} - 1 \right).$$

If $\kappa = 0$, *i.e.*, if prices are fully flexible, $\Psi_t^i = \theta / [(\theta - 1)(1 - \overline{\tau})]$, the familiar constant-elasticity markup. If $\kappa \neq 0$, price rigidity generates endogenous fluctuations of the markup. Firms react to CPI dynamics in their pricing decisions. Changes in monetary policy generate changes in CPI inflation. Hence, they affect producer prices and the markup. Through this channel, they generate different dynamics of relative prices (the real price of the firm's output is $RP_t^i \equiv \frac{p_t(i)}{P_t}$) and the real economy.

The first-order condition for the optimal choice of L_t^i yields:

$$\frac{W_t}{P_t} = \lambda_t^i Z_t. \tag{14}$$

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Today's real wage must equal the shadow value of an extra unit of labor in production.

Making use of the market clearing conditions $Y_t^{\text{Si}} = Y_t^{Di}$ and $Y_t^{DW} = Y_t^{SW} = Y_t^W$, of the expressions for supply and demand of good i, and recalling that symmetric firms make identical equilibrium choices (so that $p_t(i) = P_{Ht}$) yields:

$$L_t^i = \left(\frac{p_t(i)}{P_t}\right)^{-\omega} \frac{Y_t^W}{Z_t}. \tag{15}$$

This equation can be combined with (13) and (14) to obtain:

$$L_t^i = \left(\Psi_t^i w_t\right)^{-\omega} Y_t^W Z_t^{\omega-1}.$$

Firm i's labor demand is a decreasing function of the markup and of marginal cost. It is an increasing function of world demand of the composite basket and of productivity if $\omega > 1$.

2.3 The Government

We assume that governments in both countries run balanced budgets. The government taxes firm revenues at a rate that compensates for monopoly power in a zero-inflation steady state and removes the markup over marginal cost charged by firms in a flexible-price world. The tax rate is determined by $1-\overline{\tau} = \frac{\theta}{\theta-1}$, which yields $\overline{\tau} = -\frac{1}{\theta-1}$. Because the tax rate is negative, firms receive a subsidy on their revenues and pay lump-sum taxes determined by $T_t^{fi} = \overline{\tau} R P_t^i Y_t^i$. In addition, the government injects money into the economy through lump-sum transfers of seignorage revenues to households: $P_t T_t^v = -\left(M_t^{vj} - M_{t-1}^{vj}\right)$. Similarly for the foreign government.

2.4 Aggregation and Equilibrium

2.4.1 Households

Aggregate per capita consumption and labor supply are obtained by aggregating consumption and labor supply across generations and dividing by total population at each point in time. The aggregate per capita labor-leisure tradeoffs in the two economies are:

$$L_t = 1 - \frac{1-\rho}{\rho}\frac{c_t}{w_t}, \quad L_t^* = 1 - \frac{1-\rho}{\rho}\frac{c_t^*}{w_t^*}.$$

Labor supply rises with the real wage and decreases with consumption.

Consumption Euler equations in aggregate per capita terms contain an adjustment for consumption by the newborn generation at time $t+1$:[19]

$$c_t = \frac{1+n}{\beta(1+r_{t+1})}\left(c_{t+1} - \frac{n}{1+n}C_{t+1}^{t+1}\right), \quad c_t^* = \frac{1+n}{\beta(1+r_{t+1})}\left(c_{t+1}^* - \frac{n}{1+n}C_{t+1}^{t+1*}\right). \tag{16}$$

[19]To understand the presence of C_{t+1}^{t+1} in the aggregate Euler equation, apply the aggregation procedure to both

Newborn households hold no assets, but they own the present discounted value of their labor income. We define human wealth, h_t, as the present discounted value of the household's lifetime endowment of time in terms of the real wage:[20]

$$h_t \equiv \sum_{s=t}^{\infty} R_{t,s} w_s, \quad h_t^* \equiv \sum_{s=t}^{\infty} R_{t,s} w_s^*.$$

The dynamics of h and h^* are described by the following forward-looking difference equations:

$$h_t = \frac{h_{t+1}}{1 + r_{t+1}} + w_t, \quad h_t^* = \frac{h_{t+1}^*}{1 + r_{t+1}} + w_t^*. \tag{17}$$

Using the labor-leisure tradeoff (4), the Euler equation (5), and a newborn household's intertemporal budget constraint, it is possible to show that the household's consumption in the first period of its life is a fraction of the household's human wealth at birth:

$$C_{t+1}^{t+1} = \rho (1 - \beta) h_{t+1}, \quad C_{t+1}^{t+1*} = \rho (1 - \beta) h_{t+1}^*. \tag{18}$$

Aggregate per capita real money demands in the two economies are:

$$m_t \equiv \frac{M_t}{P_t} = \frac{\chi}{\rho} \frac{1 + i_{t+1}}{i_{t+1}} c_t, \quad m_t^* \equiv \frac{M_t^*}{P_t^*} = \frac{\chi}{\rho} \frac{1 + i_{t+1}^*}{i_{t+1}^*} c_t^*. \tag{19}$$

In the absence of arbitrage opportunities between bonds and shares, the aggregate per capita equity values of the home and foreign economies entering period $t + 1$ must evolve according to:

$$v_t = \frac{1 + n}{1 + r_{t+1}} v_{t+1} + \frac{d_{t+1}}{1 + r_{t+1}}, \quad v_t^* = \frac{1 + n}{1 + r_{t+1}} v_{t+1}^* + \frac{d_{t+1}^*}{1 + r_{t+1}}. \tag{20}$$

where $v_t \equiv \frac{a V_t^i}{P_t N_{t+1}}$, $v_t^* \equiv \frac{a V_t^{*i}}{P_t^* N_{t+1}^*}$, and d_t and d_t^* denote aggregate per capita real dividends, equal to $(1 - \overline{\tau}) y_t + T_t^f - w_t L_t - pac_t$ and $(1 - \overline{\tau}^*) y_t^* + T_t^{f*} - w_t^* L_t^* - pac_t^*$, respectively (note that $\overline{\tau} = \overline{\tau}^*$).[21]

sides of the Euler equation $C_t^v = \frac{1}{\beta(1 + r_{t+1})} C_{t+1}^v$. It is:

$$\frac{a \left[\begin{array}{c} \cdots \frac{n}{(1+n)^{t+1}} C_t^{-t} + \ldots + \frac{n}{1+n} C_t^0 \\ + n C_t^1 + \cdots + n(1+n)^{t-1} C_t^t \end{array} \right]}{a(1+n)^t} = \frac{1}{\beta(1 + r_{t+1})} \frac{a \left[\begin{array}{c} \cdots \frac{n}{(1+n)^{t+1}} C_{t+1}^{-t} + \ldots + \frac{n}{1+n} C_{t+1}^0 \\ + n C_{t+1}^1 + \cdots + n(1+n)^{t-1} C_{t+1}^t \end{array} \right]}{a(1+n)^t}.$$

The left-hand side of this equation is equal to c_t. The right-hand side is $\frac{1}{\beta(1 + r_{t+1})} \left[(1 + n) c_{t+1} - n C_{t+1}^{t+1} \right]$.

[20] Blanchard (1985) defines human wealth as the present discounted value of future, exogenous noninterest income. Weil (1989) defines human wealth as the present discounted value of after-tax endowment income. Labor income is endogenous in our model. Our definition of human wealth as the present discounted value of an agent's exogenous endowment of time parallels those of Blanchard and Weil.

[21] pac_t (pac_t^*) is the aggregate per capita cost of nominal rigidity at home (abroad).

The law of motion of aggregate per capita net foreign assets is obtained by aggregating an equilibrium version of the budget constraint (3) across generations alive at each point in time.[22] It is:

$$(1+n) B_{t+1} = (1+r_t) B_t + w_t L_t + d_t - c_t,$$
$$(1+n) B_{t+1}^* = (1+r_t) B_t^* + w_t^* L_t^* + d_t^* - c_t^*. \tag{21}$$

where $B_{t+1} \equiv \frac{A_{t+1} + \varepsilon_t A_{t+1}^*}{P_t}$ and $B_{t+1}^* \equiv \frac{\frac{A_{*t+1}}{\varepsilon_t} + A_{*t+1}^*}{P_t^*}$ (A_* denotes foreign households' holdings of home bonds, A_*^* denotes their holdings of foreign bonds). A country's net foreign assets and net foreign bond holdings coincide in a world in which all shares are held domestically.[23]

Because $d_t = y_t - w_t L_t - pac_t$ and $d_t^* = y_t^* - w_t^* L_t^* - pac_t^*$ in equilibrium, equations (21) become:

$$(1+n) B_{t+1} = (1+r_t) B_t + y_t - c_t - pac_t,$$
$$(1+n) B_{t+1}^* = (1+r_t) B_t^* + y_t^* - c_t^* - pac_t^*. \tag{22}$$

2.4.2 Firms

Aggregate per capita, real GDP in each economy is obtained by expressing production of each differentiated good in units of the composite basket, multiplying by the number of firms, and dividing by population. It is:

$$y_t = RP_t Z_t L_t, \quad y_t^* = RP_t^* Z_t^* L_t^*. \tag{23}$$

For given employment and productivity, real GDP rises with the relative price of the representative good produced, as this is worth more units of the consumption basket.

Aggregate per capita labor demand is:

$$L_t = RP_t^{-\omega} \frac{y_t^W}{Z_t}, \quad L_t^* = RP_t^{*-\omega} \frac{y_t^W}{Z_t^*}, \tag{24}$$

where y_t^W is aggregate per capita world production of the composite good, equal to aggregate per capita world consumption plus the aggregate per capita resource cost of price changes, $c_t^W + pac_t^W$. It is $y_t^W = a y_t + (1-a) y_t^*$, $c_t^W = a c_t + (1-a) c_t^*$, $pac_t^W = a pac_t + (1-a) pac_t^*$. Market clearing requires $y_t^W = c_t^W + pac_t^W$.

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[22] See Ghironi (2000) for details.

[23] Strictly speaking, these equations hold in all periods *after* the initial one. No-arbitrage conditions may be violated between time $t_0 - 1$ and t_0 if an unexpected shock surprises agents at the beginning of period t_0. In Appendix A, we show that using log-linear versions of these equations to determine asset accumulation in the initial period is harmless if one is willing to assume that the steady-state levels of A, A^*, A_*, and A_*^* are all zero. (As we show below, the model pins down the steady-state levels of B and B^* endogenously. Because domestic and foreign bonds are perfect substitutes once no-arbitrage conditions are met, the model does not pin down the levels of A, A^*, A_*, and A_*^*.)

Domestic and foreign relative prices are equal to markups over marginal costs:

$$RP_t = \Psi_t \frac{w_t}{Z_t}, \quad RP_t^* = \Psi_t^* \frac{w_t^*}{Z_t^*} \tag{25}$$

2.4.3 International Equilibrium

For international asset markets to be in equilibrium, aggregate home assets (liabilities) must equal aggregate foreign liabilities (assets), *i.e.*, it must be $\widehat{B}_t + \widehat{B}_t^* = 0$. In terms of aggregates per capita, it must be:

$$aB_t + (1-a) B_t^* = 0. \tag{26}$$

Using (26), the equations in (22) reduce to $y_t^W = c_t^W + pac_t^W$: consistent with Walras' Law, asset market equilibrium implies goods market equilibrium, and vice versa.

2.5 The Steady State

2.5.1 Real Variables

The procedure for finding the steady-state levels of real variables follows the same steps as in Ghironi (2000). As described there, the departure from Ricardian equivalence caused by entry of new households with no assets in each period generates dependence of aggregate per capita consumption growth on the stock of aggregate per capita net foreign assets. This yields determinacy of steady-state real net foreign asset holdings, and thus of the steady-state levels of other real variables in the model.

We denote steady-state levels of variables with overbars. A subscript -1 indicates that the steady state described below is going to be the position of the economy up to and including period $t = -1$ in our exercise.[24] Unexpected shocks can surprise agents at the beginning of period 0, generating the dynamics we describe in the following sections.

Given initial steady-state levels of productivity ($\overline{Z}_{-1} = \overline{Z}_{-1}^* = 1$) and inflation ($\overline{\pi}_{-1}^{PPI} = \overline{\pi}_{-1}^{PPI*} = \overline{\pi}_{-1}^{CPI} = \overline{\pi}_{-1}^{CPI*} = 0$, where $\overline{\pi}_t^{PPI} \equiv \frac{p_t(i)-p_{t-1}(i)}{p_{t-1}(i)}$ and $\overline{\pi}_t^{PPI*}$ is defined similarly), real variables are stationary, in the sense that they return to the initial position determined below following non-permanent shocks to productivity and/or inflation. (The restriction that inflation shocks ought not to be permanent for real variables to return to the steady state described below applies to the general case in which prices are sticky ($\kappa > 0$). If prices are flexible ($\kappa = 0$), real

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[24]There are two reasons for time indexes for steady-state levels of variables. On one side, when we consider non-stationary exogenous shocks, these will cause the economy to settle at a new long-run position. On the other side, we shall see that the levels of nominal variables may exhibit a unit root regardless of stationarity of the exogenous shocks.

variables return to the steady state below also after permanent changes in inflation. When $\kappa > 0$, permanent deviations of domestic or foreign inflation from zero impose permanent resource costs on the economy. These costs generate a different long-run equilibrium for real variables.)

To see the mechanism that determines the steady state at work, consider the home economy, and set aggregate per capita consumption to be constant. It is:

$$\overline{c}_{-1}\left[1 - \frac{\beta\left(1 + \overline{r}_{-1}\right)}{1 + n}\right] = \frac{n}{1 + n}\overline{C}_{\upsilon_{-1}}^{\upsilon}, \tag{27}$$

where $\overline{C}_{\upsilon}^{\upsilon}$ is steady-state consumption by a newborn generation in the first period of its life. We assume $\frac{\beta(1 + \overline{r}_{-1})}{1+n} < 1$ to ensure that steady-state consumption is positive. As we shall see, this assumption is automatically satisfied as long as $n > 0$.

From equation (18) and the definition of h, $\overline{C}_{\upsilon_{-1}}^{\upsilon}$ is:

$$\overline{C}_{\upsilon_{-1}}^{\upsilon} = \rho\left(1 - \beta\right)\frac{1 + \overline{r}_{-1}}{\overline{r}_{-1}}\overline{w}_{-1}. \tag{28}$$

Hence, aggregate per capita consumption as a function of the steady-state real wage is:

$$\overline{c}_{-1} = \frac{n\rho\left(1 - \beta\right)\left(1 + \overline{r}_{-1}\right)}{\overline{r}_{-1}\left[1 + n - \beta\left(1 + \overline{r}_{-1}\right)\right]}\overline{w}_{-1}. \tag{29}$$

Under the assumption that $\overline{Z}_{-1} = 1$, steady-state GDP is:

$$\overline{y}_{-1} = \overline{RP}_{-1}\overline{L}_{-1}. \tag{30}$$

From the pricing equation,

$$\overline{RP}_{-1} = \overline{w}_{-1}, \tag{31}$$

because the monopolistic distortion is removed by the subsidy $\overline{\tau}$. It follows that:

$$\overline{y}_{-1} = \overline{w}_{-1}\overline{L}_{-1}. \tag{32}$$

The labor-leisure trade-off implies:

$$\overline{L}_{-1} = 1 - \frac{1 - \rho}{\rho}\frac{\overline{c}_{-1}}{\overline{w}_{-1}}. \tag{33}$$

Using equations (29), (32), (33), and a steady-state version of the law of motion for home's net foreign assets yields:[25]

$$\overline{B}_0 = \frac{1}{\overline{r}_{-1} - n}\left\{\frac{n\left(1 - \beta\right)\left(1 + \overline{r}_{-1}\right) - \overline{r}_{-1}\left[1 + n - \beta\left(1 + \overline{r}_{-1}\right)\right]}{\overline{r}_{-1}\left[1 + n - \beta\left(1 + \overline{r}_{-1}\right)\right]}\right\}\overline{w}_{-1}. \tag{34}$$

[25]The subscript for initial steady-state asset holdings is 0 rather than −1 because time-0 asset holdings are determined at time −1.

Similarly, foreign steady-state assets are given by

$$\overline{B}_0^* = \frac{1}{\overline{r}_{-1} - n} \left\{ \frac{n(1-\beta)(1+\overline{r}_{-1}) - \overline{r}_{-1}[1+n-\beta(1+\overline{r}_{-1})]}{\overline{r}_{-1}[1+n-\beta(1+\overline{r}_{-1})]} \right\} \overline{w}_{-1}^*. \qquad (35)$$

Substituting equations (34) and (35) in the asset market equilibrium condition, $a\overline{B}_0 + (1-a)\overline{B}_0^* = 0$, yields:

$$\frac{1}{\overline{r}_{-1} - n} \left\{ \frac{n(1-\beta)(1+\overline{r}_{-1}) - \overline{r}_{-1}[1+n-\beta(1+\overline{r}_{-1})]}{\overline{r}_{-1}[1+n-\beta(1+\overline{r}_{-1})]} \right\} [a\overline{w}_{-1} + (1-a)\overline{w}_{-1}^*] = 0.$$

Given non-zero real wages at home and abroad, the only level of the interest rate that satisfies the market clearing condition is such that $\beta(1+\overline{r}_{-1}) = 1$, or

$$\overline{r}_{-1} = \frac{1-\beta}{\beta}. \qquad (36)$$

Substituting this result into equations (34) and (35) yields steady-state levels of domestic and foreign net foreign assets $\overline{B}_0 = \overline{B}_0^* = 0$. Consistent with the fact that the two economies are structurally symmetric in per capita terms, the long-run net foreign asset position is a zero equilibrium. Differently from Obstfeld and Rogoff (1995), this position is pinned down *endogenously* by the model.

Given these results, it is easy to verify that steady-state levels of endogenous variables other than real balances are:[26]

$$\begin{aligned}
\overline{w}_{-1} &= \overline{RP}_{-1} = \overline{w}_{-1}^* = \overline{RP}_{-1}^* = 1, \quad \overline{h}_{-1} = \overline{h}_{-1}^* = \frac{1}{1-\beta}, \\
\overline{y}_{-1} &= \overline{c}_{-1} = \overline{C}_{v-1}^v = \overline{L}_{-1} = \overline{y}_{-1}^* = \overline{c}_{-1}^* = \overline{C}_{v-1}^{v*} = \overline{L}_{-1}^* = \overline{y}_{-1}^W = \overline{c}_{-1}^W = \rho, \\
\overline{\Psi}_{-1} &= \overline{\Psi}_{-1}^* = 1, \quad \overline{pac}_{-1} = \overline{d}_{-1} = \overline{v}_{-1} = \overline{pac}_{-1}^* = \overline{d}_{-1}^* = \overline{v}_{-1}^* = 0.
\end{aligned}$$

2.5.2   Real Money Balances and Nominal Variables

Given steady-state consumption, domestic steady-state real balances are determined by:

$$\overline{m}_{-1} = \chi \frac{1+\overline{i}_{-1}}{\overline{i}_{-1}}.$$

Similarly for foreign.

In a zero-inflation steady state, nominal interest rates at home and abroad are equal to the steady-state real interest rate: $\overline{i}_{-1} = \overline{i}_{-1}^* = \frac{1-\beta}{\beta}$. It follows that real balances are:

$$\overline{m}_{-1} = \overline{m}_{-1}^* = \frac{\chi}{1-\beta}.$$

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[26]See Ghironi (2000) for details.

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Nominal money balances at home and abroad are determined by, respectively:

$$\overline{M}_{-1} = \frac{\chi}{1-\beta}\overline{P}_{-1}, \quad \overline{M}^{*}_{-1} = \frac{\chi}{1-\beta}\overline{P}^{*}_{-1}, \tag{37}$$

Taking the ratio of $\overline{M}_{-1}$ to $\overline{M}^{*}_{-1}$ and using PPP yields:

$$\overline{\varepsilon}_{-1} = \frac{\overline{M}_{-1}}{\overline{M}^{*}_{-1}}. \tag{38}$$

The steady-state exchange rate is determined by the ratio of money supplies. In the analysis below, we assume that monetary policy is conducted by setting the nominal interest rate. In order to pin down the initial steady-state level of the exchange rate, we assume that the initial level of money supplies was set by the domestic and foreign central banks at $\overline{M}_{-1} = \overline{M}^{*}_{-1} = \frac{\chi}{1-\beta}$. Structural symmetry of the two economies implies that the central banks' optimal choice of steady-state money supplies would satisfy $\overline{M}_{-1} = \overline{M}^{*}_{-1}$ if the two authorities had identical objectives. The level $\frac{\chi}{1-\beta}$ conveniently implies $\overline{\varepsilon}_{-1} = \overline{P}_{-1} = \overline{P}^{*}_{-1} = \overline{p}_{-1}(h) = \overline{p}^{*}_{-1}(f) = 1$ ($\overline{p}_{-1}(h)$ and $\overline{p}^{*}_{-1}(f)$ are the steady-state levels of the domestic and foreign PPIs, respectively, which follow from $\overline{RP}_{-1} = \frac{\overline{p}_{-1}(h)}{\overline{P}_{-1}} = \overline{RP}^{*}_{-1} = \frac{\overline{p}^{*}_{-1}(f)}{\overline{P}^{*}_{-1}} = 1$). Because the model does not pin down the steady-state levels of all nominal variables endogenously as functions of the structural parameters only, monetary policy may generate the presence of a unit root in the dynamics of price levels, the exchange rate, and nominal money balances. In this case, steady-state levels of nominal variables will change as a consequence of temporary shocks depending on the nature of monetary policy.

3   The Log-Linear System

The equations that determine domestic and foreign variables can be log-linearized around the steady state. We use sans serif fonts to denote percentage deviations from the steady state.[27] As usual, it is convenient to solve the model for cross-country differences ($x_t^D \equiv x_t - x_t^*$ for any variable x) and world aggregates ($x_t^W \equiv ax_t + (1-a)x_t^*$). The levels of individual country variables can be recovered easily given solutions for differences and world aggregates. Because the focus of this paper is on the relation between the exchange rate and asset accumulation, which are determined by cross-country differences in our setup, we report only the log-linear equations for the cross-country differences between the main variables in this section.

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[27] Percentage deviations of inflation, depreciation, and interest rates from the steady state refer to gross rates. From now on, π denotes the percentage deviation of the corresponding (gross) inflation rate from the steady state.

3.1 No-Arbitrage Conditions

PPP implies that the CPI inflation differential equals exchange rate depreciation:

$$\pi_t^{CPI^D} = \mathsf{e}_t, \tag{39}$$

where $\mathsf{e}_t \equiv \epsilon_t - \epsilon_{t-1}$ and ϵ denotes the percentage deviation of ε from the steady state.

Uncovered interest parity (UIP) implies:

$$\mathsf{i}_{t+1}^D = \epsilon_{t+1} - \epsilon_t. \tag{40}$$

3.2 Households

The relative labor-leisure trade-off is:

$$\mathsf{w}_t^D = \mathsf{c}_t^D + \frac{\rho}{1-\rho}\mathsf{L}_t^D. \tag{41}$$

Log-linear Euler equations imply that the consumption differential obeys:

$$\mathsf{c}_t^D = (1+n)\,\mathsf{c}_{t+1}^D - n\mathsf{C}_{t+1}^{t+1D}. \tag{42}$$

The *ex ante* real interest rate has no effect, because agents in both countries face identical real rates. The random walk result of the standard Obstfeld-Rogoff (1995) model for real variables is transparent here: If $n = 0$, *i.e.*, if no new agents with zero assets enter the economy, the consumption differential between the two countries follows a random walk. Any shock that causes a consumption differential today has permanent consequences on the relative level of consumption. When $n > 0$, the Euler equation is adjusted for consumption of a newborn generation in the first period of its life. It is:

$$\mathsf{C}_t^{tD} = \mathsf{h}_t^D, \tag{43}$$

where h is the deviation from the steady state of the present discounted value of a household's endowment of time over the infinite lifetime in terms of the real wage. The Euler equation for the consumption differential can thus be rewritten as:

$$\mathsf{c}_t^D = (1+n)\,\mathsf{c}_{t+1}^D - n\mathsf{h}_{t+1}^D, \tag{44}$$

where h^D is determined by:

$$\mathsf{h}_t^D = \beta\mathsf{h}_{t+1}^D + (1-\beta)\,\mathsf{w}_t^D. \tag{45}$$

Relative real balances depend on the consumption and nominal interest rate differential:

$$\mathsf{m}_t^D = \mathsf{c}_t^D - \frac{\beta}{1-\beta}\mathsf{i}_{t+1}^D. \tag{46}$$

3.3 Firms

The GDP differential obeys:

$$\mathsf{y}_t^D = \mathsf{RP}_t^D + \mathsf{L}_t^D + \mathsf{Z}_t^D. \tag{47}$$

The relative price differential reflects relative markup and marginal cost dynamics:

$$\mathsf{RP}_t^D = \psi_t^D + \mathsf{w}_t^D - \mathsf{Z}_t^D, \tag{48}$$

where ψ denotes the percentage deviation of the markup (Ψ) from the steady state.[28]

Similarly, the difference between domestic and foreign labor demand depends on the markup differential and on relative marginal cost and productivity:

$$\mathsf{L}_t^D = -\omega \left(\psi_t^D + \mathsf{w}_t^D - \mathsf{Z}_t^D \right) - \mathsf{Z}_t^D. \tag{49}$$

Substituting equations (48) and (49) into (47) yields an expression for the GDP differential as a function of relative markup and cost dynamics:

$$\mathsf{y}_t^D = -\left(\omega - 1 \right) \left(\psi_t^D + \mathsf{w}_t^D - \mathsf{Z}_t^D \right). \tag{50}$$

Combining labor demand (49) with the labor-leisure trade-off (41) yields the equilibrium real wage differential:

$$\mathsf{w}_t^D = \frac{1}{1 + \rho \left(\omega - 1 \right)} \left[\left(1 - \rho \right) \mathsf{c}_t^D - \rho \omega \psi_t^D + \rho \left(\omega - 1 \right) \mathsf{Z}_t^D \right]. \tag{51}$$

From firms' optimal pricing (equation (13) for domestic firms and the analogous equation for foreign), the PPI inflation differential depends positively on the CPI inflation differential and on relative markup and marginal cost growth:

$$\pi_t^{PPI^D} = \pi_t^{CPI^D} + \psi_t^D - \psi_{t-1}^D + \mathsf{w}_t^D - \mathsf{w}_{t-1}^D - \left(\mathsf{Z}_t^D - \mathsf{Z}_{t-1}^D \right). \tag{52}$$

Alternatively, the PPI inflation differential can be written as a function of nominal depreciation and relative real GDP growth, if $\omega \neq 1$:

$$\pi_t^{PPI^D} = \epsilon_t - \epsilon_{t-1} - \frac{1}{\omega - 1} \left(\mathsf{y}_t^D - \mathsf{y}_{t-1}^D \right) \tag{53}$$

Finally, using $1 - \overline{\tau} = 1 - \overline{\tau}^* = \frac{\theta}{\theta - 1}$ and the definitions of domestic and foreign markups, relative markup dynamics depend on current and future pricing decisions:

$$\psi_t^D = -\frac{\kappa}{\theta} \left[\pi_t^{PPI^D} - \beta \left(1 + n \right) \pi_{t+1}^{PPI^D} \right]. \tag{54}$$

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[28] We define the domestic terms of trade following Obstfeld and Rogoff (1995, 1996 Ch. 10) as $\frac{p(h)}{\varepsilon p^*(f)}$, where $p(h)$ ($p^*(f)$) is the producer currency price of the representative home (foreign) good. It is easy to verify that RP^D is the percentage deviation of the terms of trade from the steady state.

3.4 Asset Accumulation

Log-linearizing the laws of motion for the real net foreign bond holdings of domestic and foreign households yields:

$$\mathsf{B}_{t+1} \quad = \quad \frac{1}{1+n}\left(\frac{1}{\beta}\mathsf{B}_t + \mathsf{y}_t - \mathsf{c}_t\right), \tag{55}$$

$$\mathsf{B}_{t+1}^* \quad = \quad \frac{1}{1+n}\left(\frac{1}{\beta}\mathsf{B}_t^* + \mathsf{y}_t^* - \mathsf{c}_t^*\right). \tag{56}$$

Because $\overline{B}_0 = \overline{B}_0^* = 0$, B and B^* are defined as percentage deviations of B and B^* from the steady-state level of domestic and foreign consumption, respectively. As steady-state asset holdings are zero, changes in the real interest rate have no impact on asset accumulation. Bond market equilibrium requires $a\mathsf{B}_t + (1-a)\mathsf{B}_t^* = 0$. Thus, taking the difference of (55) and (56) yields:

$$\mathsf{B}_{t+1} = \frac{1}{1+n}\left[\frac{1}{\beta}\mathsf{B}_t + (1-a)\left(\mathsf{y}_t^D - \mathsf{c}_t^D\right)\right]. \tag{57}$$

Accumulation of aggregate per capita domestic net foreign assets is faster (slower) the larger (smaller) the GDP (consumption) differential.

The equity values of the domestic and foreign economies are such that:

$$\mathsf{v}_t^D = \beta\left(1+n\right)\mathsf{v}_{t+1}^D + \beta\mathsf{d}_{t+1}^D, \tag{58}$$

where d^D is the differential between the percentage deviations of domestic and foreign dividends from steady-state consumption. Similarly, v and v^* are defined as percentage deviations of v and v^*, respectively, from the steady-state level of consumption. (Recall that $\overline{d}_{-1} = \overline{d}_{-1}^* = \overline{v}_{-1} = \overline{v}_{-1}^* = 0$.) The dividend differential is:

$$\mathsf{d}_t^D = \mathsf{y}_t^D - \mathsf{w}_t^D - \mathsf{L}_t^D.$$

Domestic dividends are higher relative to foreign the wider the GDP differential and the smaller the domestic real wage bill relative to foreign. It is easy to verify that $\mathsf{d}_t^D = \psi_t^D$, so that the dynamics of the relative equity value (relative stock market dynamics) reflect the relative behavior of the markup in the two economies:

$$\mathsf{v}_t^D = \beta\left(1+n\right)\mathsf{v}_{t+1}^D + \beta\psi_{t+1}^D. \tag{59}$$

3.5 Monetary Policy

We assume that central banks set interest rates according to simple Taylor-type rules of the form:

$$\mathsf{i}_{t+1} \quad = \quad \alpha_1\mathsf{y}_t + \alpha_2\pi_t^{CPI} + \xi_t, \tag{60}$$

$$\mathsf{i}_{t+1}^* \quad = \quad \alpha_1\mathsf{y}_t^* + \alpha_2\pi_t^{CPI*} + \xi_t^*, \tag{61}$$

21

with $\alpha_1 \geq 0$, $\alpha_2 > 1$. (Recall that i_{t+1} and i_{t+1}^* are set at time t.) The reaction coefficients to GDP and inflation are identical at home and abroad. Because the two economies are identical in all structural features, if central banks with identical objectives independently chose the optimal values of α_1 and α_2, they would choose identical reaction coefficients. ξ and ξ^* are exogenous interest rate shocks. We assume:

$$\xi_t = \mu\xi_{t-1}, \quad \xi_t^* = \mu\xi_{t-1}^*,$$

$\forall t > 0$ ($t = 0$ being the time of an initial, surprise impulse in our exercise), $0 \leq \mu \leq 1$. Hence, $\xi_t^D = \mu\xi_{t-1}^D$.

Equations (60) and (61) yield:

$$i_{t+1}^D = \alpha_1 y_t^D + \alpha_2 \pi_t^{CPI^D} + \xi_t^D.$$

Because PPP implies $\pi_t^{CPI^D} = \epsilon_t - \epsilon_{t-1}$, it is:

$$i_{t+1}^D = \alpha_1 y_t^D + \alpha_2 \left(\epsilon_t - \epsilon_{t-1}\right) + \xi_t^D. \tag{62}$$

Before moving on, we stress that nominal interest rates react to the deviations of GDP from the steady state rather than to the output gap–the deviation of GDP from the flexible price equilibrium– in our benchmark policy specification. This is consistent with Taylor's (1993) original analysis. But a reaction to the output gap is the standard in the recent normative literature on monetary policy. We stick to the Taylor benchmark for essentially two reasons. First, this is a positive, rather than normative, paper. One of its purposes is to try and offer an explanation for dynamics that were observed in the 1990s, a period for which the Taylor-specification fits the U.S. data fairly well.[29] Second, the normative claim that central banks should react to the output gap is borne out of representative agent models subject to rather stringent assumptions. It is not clear that the same result would hold here.

In addition to the assumptions about interest rate setting, we assume that speculative bubbles in prices or the exchange rate are ruled out by the commitment to fractional backing mechanisms as in Obstfeld and Rogoff (1983).

4 Net Foreign Assets and Exchange Rate Dynamics under Flexible Prices

If prices are flexible ($\kappa = 0$), a dichotomy exists between nominal and real variables in the model. Real variables affect nominal ones, but the converse is not true.[30] There is no longer a time-varying, forward-looking markup. Equilibrium profits are zero in all periods, along with the equity

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[29]See Clarida and Gertler (1997) and Clarida, Galí, and Gertler (1998) for evidence from other countries.
[30]Except for real balances, which are a function of the nominal interest rate.

value of both economies. The equations that describe firm behavior in the log-linear system for cross-country differences between real variables simplify as follows.

The difference between domestic and foreign relative prices equals the difference between real wages adjusted by relative productivity:

$$RP_t^D = w_t^D - Z_t^D. \tag{63}$$

Relative employment and GDP are:

$$L_t^D = -\omega w_t^D + (\omega - 1) Z_t^D. \tag{64}$$

$$y_t^D = -(\omega - 1) \left(w_t^D - Z_t^D \right). \tag{65}$$

The solution for real variables other than real balances follows the same steps as in Ghironi (2000). Combining (41) and (64) yields:

$$L_t^D = -\frac{1 - \rho}{1 + \rho (\omega - 1)} \left[\omega c_t^D - (\omega - 1) Z_t^D \right]. \tag{66}$$

From (51),

$$w_t^D = \frac{1 - \rho}{1 + \rho (\omega - 1)} c_t^D + \frac{\rho (\omega - 1)}{1 + \rho (\omega - 1)} Z_t^D. \tag{67}$$

Substituting equations (65) and (67) into (57) with $v_t^D = 0$, we obtain:

$$B_{t+1} = \frac{1}{\beta (1 + n)} B_t - \frac{1 - a}{(1 + n) [1 + \rho (\omega - 1)]} \left[\omega c_t^D - (\omega - 1) Z_t^D \right]. \tag{68}$$

Human wealth can be written as:

$$h_t^D = \beta h_{t+1}^D + \frac{(1 - \beta)(1 - \rho)}{1 + \rho (\omega - 1)} c_t^D + \frac{\rho (1 - \beta)(\omega - 1)}{1 + \rho (\omega - 1)} Z_t^D. \tag{69}$$

Aggregating the consumption functions for individual domestic and foreign households and log-linearizing yields the following expression for the consumption differential:

$$c_t^D = \frac{\rho (1 - \beta)}{\beta (1 - a)} B_t + h_t^D. \tag{70}$$

The consumption differential in each period reflects the net foreign asset position of the two economies and the differential between the expected real wage paths from that period on.

It is easy to show that $c_t^D = w_t^D = L_t^D = y_t^D = 0$ if $\omega = 1$.[31] Unitary intratemporal elasticity of substitution ensures that domestic and foreign consumption, the real wage, employment, and

---

[31]See Ghironi (2000).

GDP are equal regardless of productivity. Hence, to preserve bond market equilibrium, it must be $B_t = B_t^* = 0$ if $\omega = 1$. This is the result first obtained by Corsetti and Pesenti (2001). If the elasticity of substitution between domestic and foreign goods is one, accumulation of net foreign assets plays no role in the transmission of shocks, and current accounts are always zero: $y_t = c_t$ and $y_t^* = c_t^*$. The same result would arise with complete asset markets and $\omega = 1$. Assuming complete markets in one-period, contingent bonds with $\omega \neq 1$ would yield $c_t^D = 0$ through perfect "risk-sharing" between the domestic and the foreign economy. Net foreign assets would respond to relative GDP movements, but they would be determined residually.

Equation (70) can be substituted into equations (68) and (69) to obtain:

$$
\begin{aligned}
B_{t+1} &= \frac{1 + \rho(\omega\beta - 1)}{\beta(1+n)[1 + \rho(\omega - 1)]} B_t - \frac{\omega(1-a)}{(1+n)[1 + \rho(\omega - 1)]} h_t^D \\
&\quad + \frac{(\omega - 1)(1-a)}{(1+n)[1 + \rho(\omega - 1)]} Z_t^D,
\end{aligned}
\tag{71}
$$

$$
\begin{aligned}
h_t^D &= \frac{\beta[1 + \rho(\omega - 1)]}{\rho\omega + \beta(1-\rho)} h_{t+1}^D + \frac{\rho(1-\rho)(1-\beta)^2}{\beta(1-a)[\rho\omega + \beta(1-\rho)]} B_t \\
&\quad + \frac{\rho(1-\beta)(\omega - 1)}{\rho\omega + \beta(1-\rho)} Z_t^D.
\end{aligned}
\tag{72}
$$

Equations (71) and (72) constitute a system of two equations in two unknowns (the endogenous state variable B and the forward-looking variable h^D) plus the exogenous relative productivity term Z^D. We assume:

$$
Z_t = \phi Z_{t-1}, \quad Z_t^* = \phi Z_{t-1}^*,
$$

$\forall t > 0$, $0 \leq \phi \leq 1$. Hence, $Z_t^D = \phi Z_{t-1}^D$. The stock of net foreign assets and the levels of the exogenous productivity parameters describe the state of the (real) economy in each period. We assume that the restrictions on structural parameter values such that the solution of the system (71)-(72) exists and is unique are satisfied. The solution can be written as:

$$
B_{t+1} = \eta_{BB} B_t + \eta_{BZ^D} Z_t^D,
\tag{73}
$$

$$
h_t^D = \eta_{h^D B} B_t + \eta_{h^D Z^D} Z_t^D,
\tag{74}
$$

where η_{BB} is the elasticity of time-$t+1$ assets to their time-t level, η_{BZ^D} is the elasticity of time-$t+1$ assets to the time-t productivity differential between home and foreign (Z_t^D), $\eta_{h^D B}$ is the elasticity of h_t^D to time-t assets, and $\eta_{h^D Z^D}$ is the elasticity of h_t^D to Z_t^D. The values of the elasticities

η as functions of the structural parameters of the model can be obtained with the method of undetermined coefficients as in Campbell (1994).[32]

Given the solutions for real variables, the path of the nominal exchange rate can be determined by using the UIP condition (40) in conjunction with the interest setting rules for the domestic and foreign economy. Combining equation (62) with UIP and rearranging, we obtain:

$$\epsilon_{t+1} - (1 + \alpha_2)\,\epsilon_t + \alpha_2\epsilon_{t-1} = \alpha_1\mathsf{y}_t^D + \xi_t^D. \tag{75}$$

Now, the solution for the GDP differential is:

$$\mathsf{y}_t^D = \eta_{y^D B}\mathsf{B}_t + \eta_{y^D Z^D}\mathsf{Z}_t^D, \tag{76}$$

where $\eta_{y^D B}$ $(\eta_{y^D Z^D})$ is the elasticity of the GDP differential to the net foreign asset position (productivity differential).

Hence, the dynamics of real net foreign assets and the exchange rate are determined by the system:

$$\begin{aligned}
\epsilon_{t+1} - (1 + \alpha_2)\,\epsilon_t + \alpha_2\epsilon_{t-1} &= \alpha_1\mathsf{y}_t^D + \xi_t^D, \\
\mathsf{y}_t^D &= \eta_{y^D B}\mathsf{B}_t + \eta_{y^D Z^D}\mathsf{Z}_t^D, \\
\mathsf{B}_{t+1} &= \eta_{BB}\mathsf{B}_t + \eta_{BZ^D}\mathsf{Z}_t^D, \\
\mathsf{Z}_t^D &= \phi\mathsf{Z}_{t-1}^D, \\
\xi_t^D &= \mu\xi_{t-1}^D,
\end{aligned} \tag{77}$$

which can be easily cast in the notation of Uhlig's (1999) equations (3.19)-(3.21).[33] The roots of the characteristic polynomial for the exchange rate equation are 1 and α_2. The assumption that $\alpha_2 > 1$ is thus sufficient to ensure determinacy. Appendix C shows that the presence of a root on the unit circle does not pose problems for determinacy of the solution given our assumptions on fractional backing.[34] Following Uhlig, we conjecture a solution for the exchange rate of the form:

$$\epsilon_t = \eta_{\epsilon\epsilon}\epsilon_{t-1} + \eta_{\epsilon B}\mathsf{B}_t + \eta_{\epsilon Z^D}\mathsf{Z}_t^D + \eta_{\epsilon\xi^D}\xi_t^D, \tag{78}$$

with elasticities $\eta_{\epsilon\epsilon}$, $\eta_{\epsilon B}$, $\eta_{\epsilon Z^D}$, and $\eta_{\epsilon\xi^D}$.

The conjectured solution and the equations in (77) can be used to obtain expressions for the exchange rate elasticities with the method of undetermined coefficients. The following equation

---

[32]See Appendix B.

[33]The only difference being the absence of the innovations that are featured in a rational expectations setup.

[34]Carlstrom, Fuerst, and Ghironi (2001) explore the consequences of $\alpha_2 \leq 1$ (monetary policies that violate the Taylor principle) along with those of alternative interest setting rules for determinacy of the equilibrium.

must hold:

$$
\begin{aligned}
& \eta_{\varepsilon\varepsilon}\left(\eta_{\varepsilon\varepsilon}\epsilon_{t-1} + \eta_{\varepsilon B}\mathsf{B}_t + \eta_{\varepsilon Z^D}\mathsf{Z}_t^D + \eta_{\varepsilon\xi^D}\xi_t^D\right) \\
& +\eta_{\varepsilon B}\left(\eta_{BB}\mathsf{B}_t + \eta_{BZ^D}\mathsf{Z}_t^D\right) + \eta_{\varepsilon Z^D}\phi\mathsf{Z}_t^D + \eta_{\varepsilon\xi^D}\mu\xi_t^D \\
& -(1+\alpha_2)\left(\eta_{\varepsilon\varepsilon}\epsilon_{t-1} + \eta_{\varepsilon B}\mathsf{B}_t + \eta_{\varepsilon Z^D}\mathsf{Z}_t^D + \eta_{\varepsilon\xi^D}\xi_t^D\right) \\
& +\alpha_2\epsilon_{t-1} \\
= \quad & \alpha_1\left(\eta_{y^D B}\mathsf{B}_t + \eta_{y^D Z^D}\mathsf{Z}_t^D\right) + \xi_t^D.
\end{aligned}
\tag{79}
$$

Equating coefficients on ϵ_{t-1} on the left-hand side and on the right-hand side of (79) yields:

$$
\eta_{\varepsilon\varepsilon}^2 - (1+\alpha_2)\,\eta_{\varepsilon\varepsilon} + \alpha_2 = 0.
$$

This polynomial has roots 1 and α_2. Because $\alpha_2 > 1$, this root would yield unambiguously unstable dynamics for the exchange rate. Hence, we select $\eta_{\varepsilon\varepsilon} = 1$: the exchange rate exhibits a unit root. The intuition is simple: the reaction of interest rates to CPI inflation in an environment in which PPP holds at all points in time (including when an unexpected shock happens) causes today's interest setting to depend also on yesterday's level of the exchange rate. (At time 0, it is $\mathsf{i}_1^D = \alpha_1\mathsf{y}_0^D + \alpha_2\epsilon_0 + \xi_0^D$, because the economy is assumed to be in steady state up to and including $t = -1$.) In turn, this causes today's exchange rate to depend on its past value. Stability imposes that the relevant root be 1.[35]

It is important to note that validity of the Taylor principle ($\alpha_2 > 1$) is not necessary for the exchange rate to exhibit a unit root. When the Taylor principle holds, the solution we are describing is unique. If the Taylor principle does not hold, it is possible to prove that there exists a solution in which the exchange rate does not contain a unit root.[36] However, indeterminacy of the solution when α_2 is smaller than 1 causes existence of sunspot equilibria that may well exhibit a unit root, including the solution described here.

Equating coefficients on B_t in (79) and using $\eta_{\varepsilon\varepsilon} = 1$ yields:

$$
\eta_{\varepsilon B} + \eta_{\varepsilon B}\eta_{BB} - (1+\alpha_2)\,\eta_{\varepsilon B} = \alpha_1\eta_{y^D B},
$$

from which:

$$
\eta_{\varepsilon B} = -\frac{\alpha_1\eta_{y^D B}}{\alpha_2 - \eta_{BB}}.
$$

---

[35]Taylor rules that allow for interest rate smoothing such that $\mathsf{i}_{t+1}^D = \alpha_1\mathsf{y}_t^D + \alpha_2\pi_t^{CPI^D} + \alpha_3\mathsf{i}_t^D + \xi_t^D$ would still induce the presence of a unit root in the level of the exchange rate through the same channel. There would be no unit root in the exchange rate if central banks were setting interest rates to react to the level of the CPI rather than to CPI inflation.

[36]See Carlstrom, Fuerst, and Ghironi (2001).

As $\alpha_2 > 1$ and $\eta_{BB} < 1$ (assets are stationary), $\alpha_2 - \eta_{BB} > 0$. Thus, the sign of $\eta_{\varepsilon B}$–the elasticity of the exchange rate to net foreign assets–is the opposite of that of $\eta_{y^D B}$–the elasticity of the GDP differential to net foreign assets. If this elasticity is negative, accumulation of foreign debt (a capital inflow, $B_t < 0$) results in an appreciation of the exchange rate below its steady-state level. We show that the sign of $\eta_{y^D B}$ is the opposite of the sign of $\eta_{c^D B}$ in Appendix B. For most plausible combinations of values of the structural parameters β, ρ, ω, ϕ, a, and n, it is $\eta_{c^D B} > 0$. Intuitively, accumulation of net foreign assets allows the home economy to sustain a higher consumption path than foreign. It follows that $\eta_{y^D B} < 0$: *ceteris paribus*, accumulation of net foreign assets causes domestic agents to supply less labor than foreign, the domestic real wage and relative price are higher than abroad (*i.e.*, the domestic terms of trade improve), and domestic GDP falls relative to foreign. Hence, $\eta_{\varepsilon B} > 0$.[37]

If central banks do not react to GDP movements ($\alpha_1 = 0$), the exchange rate is not affected by asset accumulation ($\eta_{\varepsilon B} = 0$). The latter matters for the exchange rate because it generates a GDP differential across countries. If this differential has no impact on interest rate setting, it has no effect on the exchange rate either.

Equating coefficients on Z_t^D in (79) and using the previous results yields the elasticity of the exchange rate to productivity:

$$\eta_{\varepsilon Z^D} = -\frac{\alpha_1 \left[(\alpha_2 - \eta_{BB}) \eta_{y^D Z^D} + \eta_{y^D B} \eta_{B Z^D} \right]}{(\alpha_2 - \phi)(\alpha_2 - \eta_{BB})}.$$

Our assumptions ensure that it is $\alpha_2 - \phi > 0$. The sign of $\eta_{\varepsilon Z^D}$ depends on that of $(\alpha_2 - \eta_{BB}) \eta_{y^D Z^D} + \eta_{y^D B} \eta_{B Z^D}$. A favorable shock to relative domestic productivity causes domestic agents to accumulate net foreign assets to smooth consumption dynamics for plausible parameter values if $\phi < 1$ (see below). Hence, $\eta_{B Z^D} > 0$ and $\eta_{y^D B} \eta_{B Z^D} < 0$ if $\phi < 1$. Because $\eta_{y^D Z^D} > 0$ for the same combinations of parameters, a sufficiently aggressive reaction of the central banks to inflation (α_2 large) ensures $\eta_{\varepsilon Z^D} < 0$: *ceteris paribus*, a positive shock to relative domestic productivity generates an appreciation of the exchange rate.[38] If $\alpha_1 = 0$, the exchange rate does not react to relative productivity shocks ($\eta_{\varepsilon Z^D} = 0$). The intuition is similar to that for $\eta_{\varepsilon B}$.

Finally, equating coefficients on ξ_t^D in (79) and solving yields:

$$\eta_{\varepsilon \xi^D} = -\frac{1}{\alpha_2 - \mu}.$$

[37] For example, interpreting periods as quarters, these results hold with $\beta = .99$ (a standard value of the discount factor at quarterly frequency), $\rho = .33$ (in steady state, agents spend one-third of their time working), $\omega = 1.2$ (a conservative choice for this parameter, the results still hold for higher, possibly more realistic values), $\phi = 0$ (no persistence in productivity, the results hold also for ϕ as high as .99), $a = .5$ (the two economies have equal size), and $n = .01$ (population grows by one percent per quarter).

[38] All the results in this paragraph hold for the parameter values mentioned above and with the standard Taylor-reaction of the interest rates to inflation, $\alpha_2 = 1.5$.

Because $\alpha_2 - \mu > 0$ under our assumptions, the elasticity of the exchange rate to the relative interest rate shock is negative: $\eta_{\varepsilon \xi^D} < 0$. An exogenous increase in the domestic interest rate relative to foreign causes the domestic currency to appreciate. The appreciation is larger the smaller $\alpha_2 - \mu$. If central banks react aggressively to inflation (α_2 large), the appreciation triggered by the shock is smaller. To understand the mechanism, suppose $\mu = 0$. In this case, the exchange rate jumps instantly to its new long-run level. The depreciation rate ($\mathsf{e}_t = \epsilon_t - \epsilon_{t-1}$) is zero in all periods after the time of the shock ($t = 0$, during which the depreciation rate equals the initial jump of the exchange rate–$\mathsf{e}_0 = \epsilon_0$). On impact, domestic inflation falls relative to foreign by the extent of the initial appreciation. This causes the interest differential to fall endogenously by α_2 times the initial appreciation. In equilibrium, the interest rate differential must be zero at all points in time, because it must equal expected depreciation in the following period. (At time 0 agents expect no further exchange rate movement in future periods.) Given a 1 percent exogenous impulse to the interest rate differential, it follows that the initial appreciation that is required to keep the interest differential at zero at the time of the shock is smaller the larger α_2. If the interest rate shock is more persistent ($\mu \in (0, 1)$), the exchange rate appreciates by more: a persistent shock generates expectations of continuing appreciation that are incorporated in the initial movement of the exchange rate.

A permanent shock to the interest rate differential ($\mu = 1$) would cause the percentage deviation of the exchange rate from the steady state to increase (in absolute value) by a constant amount in all periods. This implies that the percentage deviation of the exchange rate from the steady state reaches -100 percent in finite time. But a constant deviation of the rate of depreciation from its steady-state level (zero) amounts to a constant, non-zero rate of depreciation (appreciation in this case). An exchange rate that appreciates at a constant rate becomes arbitrarily small, but never actually reaches zero. Thus, the case of a permanent shock raises the issue of the reliability of the log-linear approximation, which becomes less and less informative on the actual path of the exchange rate as its deviation from the steady state becomes larger. The zero-bound on the exchange rate is indeed a non-issue in the case of a constant rate of appreciation. The conclusion of the log-linear model for long-run exchange rate behavior in the case of permanent shocks should be taken with caution.[39]

To summarize, a flexible price world yields the following exchange rate equation:

$$\epsilon_t = \epsilon_{t-1} - \frac{\alpha_1 \eta_{y^D B}}{\alpha_2 - \eta_{BB}} \mathsf{B}_t - \frac{\alpha_1 \left[(\alpha_2 - \eta_{BB}) \eta_{y^D Z^D} + \eta_{y^D B} \eta_{B Z^D} \right]}{(\alpha_2 - \phi)(\alpha_2 - \eta_{BB})} \mathsf{Z}_t^D - \frac{1}{\alpha_2 - \mu} \xi_t^D . \qquad (80)$$

---

[39] Perhaps less justifiably, we do not impose the zero bound on nominal interest rates. See Benhabib, Schmitt-Grohé, and Uribe (2001) and references therein for analyses of the consequences of this zero bound in (closed economy) environments in which monetary policy follows Taylor-type rules.

The nominal exchange rate contains a unit root, but the stock of aggregate per capita real net foreign assets helps predict the exchange rate if central banks react to GDP movements in setting the interest rate. If there is no such reaction (or if there are no productivity shocks that generate movements in real variables), the process for the exchange rate simplifies to:

$$\epsilon_t = \epsilon_{t-1} - \frac{1}{\alpha_2 - \mu}\xi_t^D, \tag{81}$$

which is exactly the random walk result of Meese and Rogoff (1983) if $\mu = 0$.

Equation (81) describes the exchange rate process also if $\omega = 1$. In this case, it is $\eta_{y^D B} = \eta_{y^D Z^D} = \eta_{B Z^D} = 0$, so that $\eta_{\varepsilon B} = \eta_{\varepsilon Z^D} = 0$ (and $\mathsf{B}_t = 0 \; \forall t$). If the intratemporal elasticity of substitution is equal to 1, productivity shocks do not affect the exchange rate regardless of whether or not interest rate setting is reacting to GDP movements. This suggests that models that assume $\omega = 1$ may be poorly suited to analyze the relation between the exchange rate and productivity.

Finally, a unit root in the exchange rate is associated with unit roots in price levels and nominal money balances. Taylor rules of the form (60)-(61) do not generate stationary levels of nominal variables. This is consistent with the empirical evidence in favor of unit roots in these variables.

4.1 Impulse Response Analysis

To evaluate the relevance of asset holdings for exchange rate dynamics under flexible prices, we calculate impulse responses to productivity and interest rate shocks for a plausible parameterization of the model. Periods are interpreted as quarters. We use the parameter values mentioned above: $\beta = .99$, $\rho = .33$, $\omega = 1.2$, $a = .5$, and $n = .01$. Our choice of n is higher than realistic, at least if one has developed economies in mind and n is interpreted strictly as the rate of growth of population.[40] However, we could reproduce the same speed of return to the steady state with slower population growth in a version of the model that incorporates probability of not surviving as in Blanchard (1985). We take $n = .01$ as a proxy for that situation.[41] In contrast, we use a lower than realistic value of ω. Estimates from the trade literature suggest that values significantly above 1 would be reasonable.[42] Our conservative choice of benchmark allows us to show that even small departures from the $\omega = 1$–case generate quite different results. We point out important consequences of higher values of ω below. We assume $\alpha_1 = .5$ and $\alpha_2 = 1.5$, as in the interest rate rule originally advocated by Taylor (1993).

Figures 2-5 show the dynamics of aggregate per capita real net foreign assets, the GDP differential, the exchange rate, and the depreciation rate (the CPI inflation differential) following a

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[40] The average rate of quarterly population growth for the U.S. between 1973:1 and 2000:3 has been .0025.

[41] Alternatively, n could be interpreted as a proxy for the size of deviations from Ricardian equivalence in the economy–which constitute the key mechanism to achieve stationarity of real variables.

[42] See Feenstra (1994), Hummels (1999), and Shiells, Stern, and Deardorff (1986).

1 percent increase in relative domestic productivity. We consider three values of the persistence parameter ϕ in the figures (0, .5, and .75) and omit (but mention) the responses for $\phi = 1$. When $\phi < 1$, the home economy accumulates net foreign assets following the shock (Figure 1). When the shock is temporary ($\phi = 0$), net foreign assets decrease monotonically in the periods after the initial one. A persistent increase in productivity ($0 < \phi < 1$) causes the home economy to continue accumulating assets for several quarters before settling on the downward path to the steady state. The home economy accumulates no assets if the shock is permanent ($\phi = 1$). Domestic GDP and consumption (not shown) rise permanently above foreign exactly by the same amount in the period of the shock. Net foreign asset dynamics triggered by non-permanent shocks are extremely persistent. This is consistent with the evidence in favor of persistence in net foreign assets in Kraay *et al.* (2000), whose regression results support an elasticity of net foreign assets at time $t+1$ to the time-t value that is very close to 1.[43]

The exchange rate appreciates on impact, the more so the more persistent the shock (Figure 3). The initial jump equals the elasticity $\eta_{\varepsilon Z^D}$:

$$\epsilon_0 = -\frac{\alpha_1 \eta_{y^D Z^D}}{\alpha_2 - \phi} - \frac{\alpha_1 \eta_{y^D B} \eta_{B Z^D}}{(\alpha_2 - \phi)(\alpha_2 - \eta_{BB})}. \tag{82}$$

The first part of this expression is negative and originates in the reaction of interest rates to the immediate change in relative GDP caused by the relative productivity shock (in Figure 4, domestic GDP is initially above foreign, more significantly the more persistent the shock). This component of the initial appreciation is larger the smaller α_2 because the reaction of interest rates to inflation causes the exchange rate to move in opposite direction to the reaction to GDP movements (in Figure 5, the inflation differential falls on impact, the more so the more persistent the shock).[44] If $\frac{\eta_{y^D Z^D}}{\alpha_2 - \phi} > -\frac{\partial \eta_{y^D Z^D}}{\partial \phi}$, the first term in (82) is also larger the higher ϕ (because agents anticipate the effects of a persistent shock, which prolongs the time during which domestic GDP is above foreign and the exchange rate appreciates). The second part of (82) is positive, because $\eta_{y^D B} < 0$ and $\eta_{B Z^D} \geq 0$, and reflects the anticipated reaction of future interest rates to the persistent negative GDP differential generated by real asset accumulation (indeed, the GDP differential in Figure 4 becomes slightly negative during the second part of the transition dynamics, as assets slowly return to the steady state). For the same reasons as before, this component of the initial exchange rate jump is larger the less aggressive the reaction of policy to inflation and the more persistent the shock (the latter if $\frac{\eta_{B Z^D}}{\alpha_2 - \phi} > -\frac{\partial \eta_{B Z^D}}{\partial \phi}$).[45] The exchange rate movement caused by anticipated asset

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[43] See also Ghironi (2000).

[44] Figure 5 provides information also on the actual behavior of interest rates following the shock. Because of UIP, the realized interest rate differential at each date equals the rate of depreciation in the following period.

[45] Note that changes in the persistence of shocks have no impact on the elasticity of other endogenous variables to asset holdings.

accumulation is larger the closer η_{BB} to 1. Slow convergence of assets to the steady state ensures a more persistent GDP differential, the effect of which on future interest setting is anticipated by the agents. For realistic parameter values, the effect of the first term in (82) dominates and the exchange rate appreciates on impact.

If $\phi = 0$, the path of the exchange rate is monotonic after the initial downward jump. The exchange rate overshoots its long-run level. Endogeneity of interest rate setting with $\alpha_1 > 0$ and asset dynamics generate overshooting with flexible prices. To understand this, observe that the exchange rate is determined by the following equation in all periods after the initial shock:

$$\epsilon_t = \epsilon_{t-1} - \frac{\alpha_1 \eta_{y^D B}}{\alpha_2 - \eta_{BB}} \mathsf{B}_t.$$

As B becomes positive after the initial period, the exchange rate climbs very slowly towards the new steady-state position (recall that $\eta_{\epsilon B} > 0$). (The depreciation rate in Figure 5 becomes positive–albeit small–as the exchange rate starts moving towards its new steady-state level.) The new steady state is reached when net foreign assets have completed their transition back to zero. The transition is very slow because so is the speed of convergence of net foreign assets (determined by the rate at which new households enter the economy).

If $\phi \in (0, 1)$, delayed overshooting obtains. The initial jump is followed by further appreciation. After the initial period, exchange rate and net assets obey:

$$
\begin{aligned}
\epsilon_t &= \epsilon_{t-1} - \frac{\alpha_1 \eta_{y^D B}}{\alpha_2 - \eta_{BB}} \mathsf{B}_t - \frac{\alpha_1 \left[(\alpha_2 - \eta_{BB}) \eta_{y^D Z^D} + \eta_{y^D B} \eta_{B Z^D} \right]}{(\alpha_2 - \phi)(\alpha_2 - \eta_{BB})} \mathsf{Z}_t^D, \\
\mathsf{B}_{t+1} &= \eta_{BB} \mathsf{B}_t + \eta_{B Z^D} \mathsf{Z}_t^D.
\end{aligned}
$$

A persistent (but not permanent) shock causes the stock of assets to increase until the shock has died out. That puts upward pressure on the exchange rate. However, the shock generates appreciation beyond the initial jump as long as the productivity differential remains positive. As the shock dies out, the dynamics of asset holdings drive the exchange rate to its new long run level, between the initial response and the peak appreciation.

A permanent relative productivity shock ($\phi = 1$) causes no change in net foreign assets. The percentage deviation of the exchange rate from the steady state increases (in absolute value) by the same amount in all periods. The caveat we mentioned above about the reliability of the log-linearization for the path of the exchange rate following permanent shocks apply here.[46]

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[46] We omit impulse responses for variables that are not directly related to the exchange rate. When $\phi = 0$, domestic consumption is slightly above foreign throughout the transition dynamics. The relative price differential falls on impact–which causes stronger output and labor demand at home than abroad. But this movement is reversed in the following periods, in which these differentials are slightly below the steady state. The domestic real wage is higher than abroad throughout the transition. When $\phi \in (0, 1)$, the fall in the domestic real price relative to foreign is more persistent, and so is the initial movement of both employment and real wage differential above the steady state. Domestic consumption rises above foreign by more. Details are available on request.

To further investigate the relation between net foreign assets and the exchange rate, Figure 6 shows their responses to a 1 percent domestic productivity shock with $\phi = 0$ for the benchmark value of n (.01) and for an arbitrary, unrealistically large value (.5), which delivers much faster convergence of real assets to the steady state.[47] The response of the exchange rate settles at its new long-run level much faster when $n = .5$. Put differently, the response of the exchange rate is closer to that of a pure random walk the faster the speed of convergence of net foreign assets to the steady state. Even if the elasticity of the exchange rate to net foreign assets is very small for the parameter values we use ($\eta_{\varepsilon B} = .0014$ [.0005] when $n = .01$ [.5]), near non-stationary net foreign assets generate exchange rate dynamics that can be quite different from those of a random walk.

Figure 7 repeats the exercise for a higher, more realistic value of ω ($\omega = 4$). The range of variation of net foreign assets and the exchange rate is an order of magnitude larger. Cross-country differences caused by asymmetric shocks are bigger if goods are more highly substitutable across countries. Even if $\eta_{\varepsilon B}$ remains small ($\eta_{\varepsilon B} = .0081$ [.0041] when $n = .01$ [.5]), the difference between the $n = .01$ and $n = .5$ cases becomes more pronounced.[48] The extent to which slow convergence to the steady state causes net foreign assets to affect exchange rate dynamics is more relevant the higher the degree of substitutability between domestic and foreign goods in consumption.

Figures 8 and 9 illustrate the reaction to a 1 percent domestic interest rate shock. Equation (81) determines the exchange rate. As expected, the response to the shock is non-stationary (Figure 8) and no overshooting takes place. The cases $\mu = 0$ and $\mu = 1$ have been discussed above. When $\mu \in (0, 1)$, the exchange rate *under*shoots its new long-run level on impact. It continues to appreciate as the shock dies out and eventually settles at its new steady state. The depreciation rate/inflation differential falls and returns to the steady state at a speed that is inversely related to the persistence of the shock.[49]

To summarize our analysis of the flexible price benchmark, the unit root in (80), combined with stationary real net foreign assets and shock processes, unambiguously delivers a non-stationary process for the nominal exchange rate. Because the deviation of net foreign assets from the steady state becomes negligible in finite time following a non-permanent shock, the exchange rate eventually settles on a new long-run position if shocks are not permanent.[50] Notwithstanding the presence

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[47]Note that, if prices were sticky, n would be bounded above by $\bar{r}_{-1}$ for the log-linear system to be stable.

[48]In this case, the exchange rate actually *depreciates* in the long run when $n = .01$.

[49]The value of ω has of course no impact on the effect of interest rate shocks under flexible prices.

[50]It should be noted that $n = 0$, which delivers non-stationary real assets, will not necessarily generate a stationary exchange rate. Keeping the other parameter values at the benchmark, a favorable shock to home productivity with $\phi = 0$ causes domestic net foreign assets to settle at a new (higher) steady-state level by the beginning of the period after the shock. The exchange rate appreciates on impact. But permanently higher assets from $t = 1$ on imply that expected exchange rate depreciation (and the interest rate differential) must be constant in all periods (including that of the shock). (At time 0, expected depreciation between time 0 and time 1 equals $\eta_{\varepsilon B}\bar{B}$, where $\bar{B}$ is the permanent deviation of asset holdings from the steady state.) A constant rate of depreciation in all periods following the initial

of a unit root in the exchange rate, impulse response analysis supports the idea that net foreign asset dynamics help predict the path of the nominal exchange rate to the extent that the elasticity of the latter to net foreign assets is different from zero. The predictive power of net foreign assets is enhanced if their law of motion is near non-stationary and if the elasticity of substitution between domestic and foreign goods is significantly above 1. Finally, the exercise of this section shows that price stickiness is not necessary to obtain exchange rate over- or undershooting following exogenous impulses. Endogenous interest rate setting and asset dynamics are sufficient.

5   Sticky Prices

The exchange rate continues to be determined by equation (75). However, the dynamics of the real GDP differential (and of all other real variables) following productivity and interest rate shocks are now affected by the markup fluctuations generated by nominal rigidity.

It is possible to prove that $\omega = 1$ implies $\mathsf{B}_{t+1} = 0$ $\forall t$ also under sticky prices regardless of other parameter values. Intuitively, equation (50) shows that the GDP differential is always zero regardless of productivity and interest rates if the elasticity of substitution between domestic and foreign goods is one. Because countries are starting off with zero net assets, identical GDP levels imply that the two economies have identical real resources to allocate to consumption in all periods. Thus, the utility maximizing choice entails $\mathsf{c}_t^D = 0$ $\forall t$.

The system on which we focus our attention for the general case $\omega \neq 1$ consists of equations (44), (45), (50), (51), (53), (54), (57), (59), and (75), combined with our assumptions on the shock processes, $\xi_t^D = \mu \xi_{t-1}^D$ and $\mathsf{Z}_t^D = \phi \mathsf{Z}_{t-1}^D$. It is hard to obtain an easily interpretable analytical solution for this system. Thus, we resort to numerical methods. In Uhlig's (1999) notation, the $\underline{\mathsf{x}}_t$ vector of endogenous states is $\left[\mathsf{B}_{t+1}, \epsilon_t, \psi_t^D, \mathsf{w}_t^D, \mathsf{y}_t^D, \mathsf{h}_t^D, \mathsf{v}_t^D\right]'$, the $\underline{\mathsf{y}}_t$ vector of other endogenous variables is $\left[\pi_t^{PPI^D}, \mathsf{c}_t^D\right]'$, and the vector of exogenous driving forces is $\underline{\mathsf{z}}_t = \left[\mathsf{Z}_t^D, \xi_t^D\right]'$. We include h_t^D and v_t^D in the $\underline{\mathsf{x}}_t$ vector to avoid singularity problems in the solution. We use Uhlig's implementation of the method of undetermined coefficients to solve for matrices $\underline{\eta}_{xx}$, $\underline{\eta}_{xz}$, $\underline{\eta}_{yx}$, and $\underline{\eta}_{yz}$ such that: $\underline{\mathsf{x}}_t = \underline{\eta}_{xx}\underline{\mathsf{x}}_{t-1} + \underline{\eta}_{xz}\underline{\mathsf{z}}_t$ and $\underline{\mathsf{y}}_t = \underline{\eta}_{yx}\underline{\mathsf{x}}_{t-1} + \underline{\eta}_{yz}\underline{\mathsf{z}}_t$. The method returns a unique stable solution for the parameter values we consider.[51] We use the baseline parameterization above, which we repeat for convenience: $\beta = .99$, $\rho = .33$, $\omega = 1.2$, $a = .5$, $n = .01$, $\alpha_1 = .5$, and $\alpha_2 = 1.5$. We set κ to 77, the estimate in Ireland (2001), and θ to 6, consistent with Rotemberg and Woodford (1992). These values imply that PPI inflation of 1 percent would generate a resource cost of .385 percent of aggregate per capita real GDP. Our choice of θ implies a steady-state markup (non-adjusted for

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one causes the exchange rate to eventually shoot to infinity.

[51] Speculative bubbles in the exchange rate would violate transversality conditions under sticky prices.

the subsidy $\overline{\tau}$) of 20 percent.

Table 1 shows the solution for $\underline{\eta}_{xx}$, $\underline{\eta}_{xz}$, $\underline{\eta}_{yx}$, and $\underline{\eta}_{yz}$ for different values of the persistence parameters ϕ and μ. A few facts emerge. The exchange rate continues to display a unit root. The intuition is fairly simple. Given the vector $\underline{x}_t$, the conjectured solution for the exchange rate is of the form:

$$\epsilon_t = \eta_{\varepsilon\varepsilon}\epsilon_{t-1} + \eta_{\varepsilon B}\mathsf{B}_t + \eta_{\varepsilon\psi^D}\psi^D_{t-1} + \eta_{\varepsilon w^D}\mathsf{w}^D_{t-1} + \eta_{\varepsilon y^D}\mathsf{y}^D_{t-1}$$
$$+ \eta_{\varepsilon h^D}\mathsf{h}^D_{t-1} + \eta_{\varepsilon v^D}\mathsf{v}^D_{t-1} + \eta_{\varepsilon Z^D}\mathsf{Z}^D_t + \eta_{\varepsilon\xi^D}\xi^D_t.$$

Equation (75) implies that, if the elasticity of y^D_t to ϵ_{t-1} is zero, the equation that determines $\eta_{\varepsilon\varepsilon}$ in the undetermined coefficients method is exactly the same as under flexible prices. This turns out to be the case, and $\eta_{\varepsilon\varepsilon} = 1$ emerges also under sticky prices as a consequence of endogenous interest rate setting with reactions to inflation rates.[52]

Indeed, Table 1 suggests that nominal price rigidity implies that all variables in $\underline{x}_t$ (with the exception of the exchange rate) can be written as functions of B_t, y^D_{t-1}, and the shocks only (i.e., that the values of all other elasticities are zero). (In addition, the exchange rate features the above mentioned unit root.) Put differently, at least for the benchmark parameterization and a number of plausible alternatives, nominal rigidity implies that real variables in the endogenous state vector depend on the current level of the exogenous component of the interest rate differential and on the past level of the GDP differential, in addition to assets accumulated in the previous period and the relative productivity term.

The solution for the exchange rate in Table 1 is of the form:

$$\epsilon_t = \epsilon_{t-1} + \eta_{\varepsilon B}\mathsf{B}_t + \eta_{\varepsilon y^D}\mathsf{y}^D_{t-1} + \eta_{\varepsilon Z^D}\mathsf{Z}^D_t + \eta_{\varepsilon\xi^D}\xi^D_t.$$

On empirical grounds, the sticky-price solution points to the past GDP differential as useful to predict the current exchange rate, along with net foreign assets accumulated in the previous period. As for the flexible-price case, $\eta_{\varepsilon B}$ is positive and small: a capital inflow generates exchange rate appreciation. Accumulation of net foreign debt is associated to higher domestic GDP than foreign because domestic agents supply more labor. The interest rate differential rises in response to a positive GDP differential, which leads to appreciation. The elasticity to the past GDP differential ($\eta_{\varepsilon y^D}$) is negative for similar reasons. Sticky prices introduce persistence in the GDP process beyond its dependence on assets accumulated in the previous period. (Recall that GDP is measured in units of the consumption basket, i.e., by multiplying production of domestic goods by their real price. Stickiness in the latter introduces stickiness in GDP.) As a consequence of GDP persistence,

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[52]This result extends Benigno and Benigno's (2001) finding of a unit root in the exchange rate under sticky prices and Taylor-type monetary policies to an environment in which net foreign assets respond to shocks.

a positive GDP differential yesterday translates into a higher interest rate differential today and, hence, into appreciation. The negative elasticity to productivity ($\eta_{\varepsilon Z^D}$) reflects the fact that higher domestic productivity generates higher domestic output directly, as in the flexible-price world. Again, this leads to a higher domestic interest rate and appreciation.[53] Consistent with uncovered interest parity, an exogenous increase in $\eta_{\varepsilon \xi^D}$ causes the domestic currency to appreciate on impact.

The elasticities in $\underline{\eta}_{xx}$ and $\underline{\eta}_{yx}$ are not affected by changes in the persistence of the productivity and interest rate shocks. As in the flexible-price world, changes in shock persistence affect only the elasticities to the shocks themselves, $\underline{\eta}_{xz}$ and $\underline{\eta}_{yz}$. The elasticity of assets to productivity (η_{BZ^D}) is an increasing function of the persistence of productivity shocks for $\phi < 1$. Intuitively, the more persistent a (non-permanent) favorable productivity shock, the stronger the incentive of households to accumulate assets to smooth consumption ($\eta_{y^D Z^D}$ is unambiguously increasing in ϕ). Interestingly, η_{BZ^D} is negative in the special case $\phi = 1$. The domestic country accumulates debt if there is a permanent favorable shock to relative productivity. The reason is that a permanent favorable productivity shock causes the new long-run level of domestic GDP to be above the initial jump following the shock (GDP stickiness through real price persistence is responsible for this; see also below). As a consequence, optimal consumption smoothing dictates that domestic agents borrow in the anticipation of permanently higher income in the future. The relation between $\eta_{B\xi^D}$ and the persistence of interest rate shocks (μ) is non-monotonic and less easily interpretable. The elasticity of the exchange rate to the productivity differential is larger (in absolute value) the more persistent the latter. As under flexible prices, a more persistent productivity differential generates anticipation of a more persistent interest rate differential, and hence a larger movement of the exchange rate on impact. As expected, the elasticity of the exchange rate to the relative interest rate shock increases with the persistence of the latter.

5.1 Impulse Response Analysis

5.1.1 Productivity Shock

Figure 10 shows impulse responses to a 1 percent favorable shock to relative domestic productivity for the values of ϕ in Table 1.[54] Consider the $\phi = 0$–case. Under flexible prices, the exchange rate overshoots its new long run equilibrium on impact. It then converges monotonically to the new steady state. The home country accumulates assets in the initial period, which it decumulates back to the steady state along the transition dynamics. Sticky prices cause a hump-shaped response of net foreign assets and delayed overshooting, as we observed under flexible prices when the

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[53]Not surprisingly, if $\alpha_1 = 0$ (interest rates do not react to GDP movements), it is $\eta_{\varepsilon B} = \eta_{\varepsilon y^D} = \eta_{\varepsilon Z^D} = 0$ also under sticky prices.

[54]In figures 10 and 11, net foreign assets in each period (B1) are net foreign assets at the end of that period.

productivity shock was persistent. The reason for this difference is exactly that price-stickiness imparts persistence in the dynamics of the GDP differential as described above. Domestic firms initially lower prices more than foreign firms, though the domestic markup rises relative to foreign to preserve profitability. Labor demand falls (not shown), and so does the real wage. However, these movements are quickly reversed. A more persistent favorable GDP differential causes home agents to continue accumulating assets in the first periods after the shock. As under flexible prices, continuing asset accumulation puts upward pressure on the exchange rate. But the added persistence in the GDP differential acts as persistence in productivity under flexible prices, causing further appreciation. When the GDP differential is close to zero, the upward pressure on the exchange rate from asset holdings above the steady state kicks in, and assets and the exchange rate converge slowly to their steady-state levels. Interestingly, the stock market value of the domestic economy relative to foreign is below the steady state throughout the transition, reflecting the fact that the markup is below the steady state for most of the time. As expected, domestic consumption is (slightly) above foreign.

When the productivity shock is more persistent–$\phi \in (0, 1)$–the dynamics of net foreign assets and the exchange rate are similar, with a hump-shaped response of assets and delayed exchange rate overshooting. Persistence in productivity introduces a hump in the response of GDP, consistent with the persistence effect of price rigidity and markup dynamics. (GDP rises with labor demand as the markup falls after the initial increase.) The markup is above the steady state for longer, which causes an initial stock market expansion relative to foreign.

If $\phi = 1$, the domestic country borrows from abroad for the reasons described above. Domestic GDP climbs to a permanently higher level than foreign over time due to price stickiness and markup dynamics. The exchange rate appreciates in all periods (appreciation reaches 20 percent thirty years after the shock, though the caveat mentioned above applies here). The relative markup is permanently above the initial steady state (albeit by very little) and relative PPI inflation is permanently lower. The real wage differential falls initially, as higher productivity and the initial relative markup movement depress domestic labor demand relative to foreign. However, higher long-run GDP at home than abroad results in a positive long-run real wage differential. A permanently higher markup causes the domestic stock market value to rise permanently above foreign. So does domestic consumption.[55]

If one believes that the advent of the "new economy" has shifted U.S. productivity permanently above foreign, our model provides a qualitative account of empirical observations of the past few years. The model also delivers much richer exchange rate dynamics following a productivity shock

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[55]It is easy to solve for the new permanent cross-country differentials using the equations of the model. We leave it to the interested reader as an exercise.

under sticky prices than its antecedent by Obstfeld and Rogoff (1995). The case of a temporary productivity shock is trivial in that model. Supply equations are not binding in the short run, and the shock has no output effect. A permanent shock causes the exchange rate to appreciate permanently in the period of the shock. No further dynamics happen and no overshooting is obtained in either case.

5.1.2 Interest Rate Shock

Figure 11 shows impulse responses to a 1 percent exogenous increase in domestic interest rates for the values of μ in Table 1.

Under flexible prices, the exchange rate jumped immediately to its new long-run equilibrium after an interest rate shock with no persistence ($\mu = 0$). When prices are sticky, the shock affects real variables, which in turn have an impact on exchange rate dynamics. Higher domestic interest rates result in lower PPI inflation at home than abroad, though firms raise the markup component of prices to preserve profitability, and this translates into a higher relative value of home equity. Domestic labor demand falls below foreign (not shown), and so do the real wage, GDP, and (slightly) consumption. The home economy borrows from foreign to mitigate the fall in consumption. The exchange rate appreciates on impact. Its dynamics in the periods after the shock reflect those of GDP differential and asset holdings. A GDP differential below the steady state pushes the exchange rate upwards, consistent with the lowering of the domestic interest rate in response to lower GDP. Net foreign assets below the steady state push the exchange rate downward. Because the elasticity of the exchange rate to the GDP differential ($\eta_{\varepsilon y^D}$) is larger than that to assets ($\eta_{\varepsilon B}$) in absolute value, the former effect prevails, and the exchange rate moves upwards in the first years after the shock. However, the effect of asset dynamics on the exchange rate becomes preponderant once the GDP differential has been (almost entirely) re-absorbed. Approximately five years after the shock, the exchange rate starts moving slowly downward, mirroring the return of assets to the steady state. Eventually, the exchange rate settles on a new long-run position between the impact appreciation in period 0 and the level it had reached due to GDP dynamics. Thus, the exchange rate displays "two-ways" overshooting following a zero-persistence relative interest rate shock: it shoots below the new long-run equilibrium on impact, but it climbs above it when the GDP differential is the main driving force.

If the interest rate shock is persistent–$\mu \in (0, 1)$–the responses of markup, real wage, GDP, and consumption differential become hump-shaped, as the differential in relative prices adjusts gradually. Deviations from the steady state become more persistent. The home economy borrows from abroad to sustain consumption. Consider the case $\mu = .5$. The exchange rate jumps downward and continues to appreciate further until the GDP differential has reached its peak. After that,

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the exchange rate reverses direction along with GDP and climbs slightly. As in the case of a shock with no persistence, once the GDP differential is close to zero, asset dynamics take over, and the exchange rate moves downward to its new steady state. In the case of a persistent shock, the exchange rate undershoots its new long-run equilibrium on impact.[56] When μ rises to .75, the exchange rate converges monotonically to the new steady state. It appreciates further after the shock as the GDP differential continues to climb, and it continues to appreciate also after the latter reverses its movement. In this case, the size of the net debt accumulated by the home economy makes up for the small elasticity of the exchange rate to assets.

A permanent shock ($\mu = 1$) causes domestic PPI inflation to be permanently below foreign. The domestic markup now falls on impact relative to foreign, though it eventually settles on a slightly higher value. This translates into a permanently higher domestic equity value. The initially lower markup generates higher labor demand, which mitigates the negative GDP effect of the shock. The real wage differential is above the steady state for a long time, though its new steady-state level is negative (not shown). The real GDP differential eventually returns to zero from below. Domestic consumption is above foreign for many years, but it eventually settles below, and the domestic economy is permanently in debt. The percentage deviation of the exchange rate from the steady state increases linearly in absolute value. The now usual caveat on the consequences of permanent shock for the path of the exchange rate applies. The interest rate shock has permanent real consequences by generating non-zero steady-state inflation, which imposes a permanent real cost on the economy.

As for the case of a productivity shock, the model delivers richer exchange rate dynamics than the benchmark setup in Obstfeld and Rogoff (1995). It brings a new perspective to bear on Dornbusch's (1976) results on exchange rate overshooting. In particular, our model has the potential for reconciling rational behavior and UIP with the evidence of delayed overshooting in Clarida and Galí (1994) and Eichenbaum and Evans (1995).

6   Conclusions

We presented a theory of exchange rate determination that de-emphasizes the role of exogenous money supply changes and emphasizes the relation between the exchange rate and net foreign assets and the endogeneity of interest rate setting. Our model builds on a stationary version of Obstfeld and Rogoff's (1995) seminal contribution. We relied on the method of undetermined coefficients along the lines of Campbell (1994) and Uhlig (1999) to obtain the solution. This technique is fully consistent with the forward-looking nature of the model. It delivers a process equation for

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[56]Although it is hard to see this from the figure, the new steady-state level is also below the level that the exchange rate reaches with the further appreciation due to the hump-shaped response of GDP in the first years after the shock.

the exchange rate rather than a solution expressed in the form of an infinite summation of future variables. This facilitates interpretation and quantitative work.

We started from a simple model with flexible prices and PPP. Interest rates are set to react to CPI inflation and real GDP movements. The solution for the nominal exchange rate exhibits a unit root, consistent with Meese and Rogoff (1983). However, today's exchange rate depends also on the stock of real net foreign assets accumulated in the previous period. For plausible parameter values, a capital inflow (accumulation of net foreign debt) generates appreciation of the exchange rate. The predictive power of net foreign assets for the latter is stronger the slower their convergence to the steady state following shocks and the higher the degree of substitutability between domestic and foreign goods in consumption.

We introduced price stickiness by assuming that it is costly to change output prices over time. We investigated the relation between asset holdings and the exchange rate quantitatively using a plausible calibration of the model. When prices are sticky, the exchange rate still exhibits a unit root. The current level of the exchange rate depends on the past GDP differential, along with net foreign assets.

The model yields a number of results on exchange rate overshooting. Under flexible prices, the exchange rate overshoots its new long-run level following a temporary (relative) productivity shock. If the shock is persistent, endogenous monetary policy and asset dynamics generate delayed overshooting. Endogenous monetary policy is responsible for exchange rate undershooting after persistent (relative) interest rate shocks. When prices are sticky, temporary shocks to relative productivity result in delayed overshooting. So do persistent shocks. Temporary relative interest rate shocks cause immediate overshooting. No overshooting may happen when interest rate shocks are persistent. The sticky-price dynamics are richer than in the Obstfeld-Rogoff (1995) model. Our model has the potential for reconciling rational behavior and UIP with the empirical results in Clarida and Galí (1994) and Eichenbaum and Evans (1995).

The flexible-price model is capable of delivering exchange rate appreciation after a favorable relative productivity shock under Taylor-type monetary policy. This is one side of the story that one would like to represent formally when trying to explain the recent behavior of the U.S. economy and the dollar exchange rate. However, the flexible price model does not deliver appreciation *cum* accumulation of net foreign debt. Because consumption smoothing is the only motive for asset accumulation, the home economy accumulates assets rather than debt. The sticky-price version of the model generates appreciation, a net foreign debt, and a stock market expansion after a permanent favorable shock to relative productivity. But it remains to be seen whether the advent of the "new economy" has shifted U.S. productivity permanently above foreign. Adding accumulation of physical capital to the scene appears a promising way of generating the dynamics we observe in

the data for non-permanent changes in relative productivity (Ghironi and Lettau, 2001). This is a direction we will pursue in future work, along with rigorous testing of the model's implications and exploring the consequences of deviations from PPP.[57] Another possible extension will be the exploration of the performance of alternative policy rules.

A Aggregate Per Capita Net Foreign Assets in the Initial Period

Aggregating the period budget constraint (3) across generations, dividing by population size, and imposing money market equilibrium and the seignorage rebate yields:

$$(1+n) \frac{A_{t+1} + \varepsilon_t A_{t+1}^*}{P_t} = \frac{1+i_t}{1+\pi_t^{CPI}} \frac{A_t}{P_{t-1}} + \frac{(1+i_t^*)(1+e_t)}{1+\pi_t^{CPI}} \frac{\varepsilon_{t-1} A_t^*}{P_{t-1}}$$
$$+ d_t + w_t L_t - c_t,$$

where it is understood that all variables are in levels rather than deviations from the steady state. Equilibrium aggregate per capita real dividends are $d_t = y_t - w_t L_t - pac_t$. Hence, domestic aggregate per capita net foreign assets obey:

$$(1+n) \frac{A_{t+1} + \varepsilon_t A_{t+1}^*}{P_t} = \frac{1+i_t}{1+\pi_t^{CPI}} \frac{A_t}{P_{t-1}} + \frac{(1+i_t^*)(1+e_t)}{1+\pi_t^{CPI}} \frac{\varepsilon_{t-1} A_t^*}{P_{t-1}}$$
$$+ y_t - c_t - pac_t. \tag{83}$$

Note that we have not used any no-arbitrage condition to obtain this equation. If we impose uncovered interest parity, the Fisher parity relation, and we define $B_{t+1} \equiv \frac{A_{t+1} + \varepsilon_t A_{t+1}^*}{P_t}$, it is immediate to recover:

$$(1+n) B_{t+1} = (1+r_t) B_t + y_t - c_t - pac_t, \tag{84}$$

which we used along with its foreign counterpart to solve for the model's initial steady state in Section 2. Because $pac_t = \frac{\kappa}{2} \pi_t^{PPI^2} y_t$ and PPI inflation is zero in the initial steady state, log-linearization of this equation around the initial steady state yields equation (55). In general, it is not appropriate to use equation (84) (or its log-linear counterpart) to determine asset holdings in the period of an unexpected shock, which generally causes no-arbitrage conditions to be violated *ex post*. Asset dynamics in the initial period must be described by equation (83), which does not incorporate any no-arbitrage condition. However, the initial steady state is such that $\overline{B}_0 = 0$. If we assume that it is also $\frac{\overline{A}_0}{\overline{P}_{-1}} = \frac{\overline{\varepsilon}_{-1} \overline{A}_0^*}{\overline{P}_{-1}} = 0$ (*i.e.*, not only their sum is zero), equation (83) at the

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time of the shock reduces to:

$$(1+n)\frac{A_1 + \varepsilon_0 A_1^*}{P_0} = (1+n)B_1 = y_0 - c_0 - pac_0,$$

with log-linear version:

$$(1+n)\,\mathsf{B}_1 = \mathsf{y}_0 - \mathsf{c}_0,$$

which is exactly what follows from using (55) directly for the initial period. Similarly for the foreign economy.

B   Solution: The Flexible-Price Model, Real Variables

This appendix describes the undetermined coefficients solution of the flexible-price model for real variables other than real money balances. The solution makes it possible to explore the role of changes in relative country size for interdependence across countries.

Equations (71) and (72) can be rewritten as follows:

$$\begin{aligned}
0 &= -(1+n)\left[1+\rho\left(\omega-1\right)\right]\frac{\mathsf{B}_{t+1}}{1-a} + \left[1-\rho+\beta\rho\omega\right]\frac{1}{\beta}\frac{\mathsf{B}_t}{1-a} \\
&\quad -\omega\mathsf{h}_t^D + \left(\omega-1\right)\mathsf{Z}_t^D, \tag{85}\\
0 &= -\left[\rho\omega+\beta\left(1-\rho\right)\right]\mathsf{h}_t^D + \left(1-\beta\right)\left(1-\rho\right)\rho\frac{1-\beta}{\beta}\frac{\mathsf{B}_t}{1-a} \\
&\quad + \left(1-\beta\right)\rho\left(\omega-1\right)\mathsf{Z}_t^D + \beta\left[1+\rho\left(\omega-1\right)\right]\mathsf{h}_{t+1}^D. \tag{86}
\end{aligned}$$

Conjecture the solution:

$$\frac{\mathsf{B}_{t+1}}{1-a} = \widetilde{\eta}_{BB}\frac{\mathsf{B}_t}{1-a} + \widetilde{\eta}_{BZ^D}\mathsf{Z}_t^D, \tag{87}$$

$$\mathsf{h}_t^D = \widetilde{\eta}_{h^D B}\frac{\mathsf{B}_t}{1-a} + \widetilde{\eta}_{h^D Z^D}\mathsf{Z}_t^D. \tag{88}$$

Conjecture (87)-(88) is isomorphic to equations (73)-(74) in the text, with $\eta_{BB} = \widetilde{\eta}_{BB}$, $\eta_{BZ^D} = (1-a)\widetilde{\eta}_{BZ^D}$, $\eta_{h^D B} = \frac{\widetilde{\eta}_{h^D B}}{1-a}$, and $\eta_{h^D Z^D} = \widetilde{\eta}_{h^D Z^D}$. To obtain solutions for the elasticities $\widetilde{\eta}_{BB}$, $\widetilde{\eta}_{BZ^D}$, $\widetilde{\eta}_{h^D B}$, and $\widetilde{\eta}_{h^D Z^D}$, proceed as follows. Substitute the conjecture (87)-(88) into (85) and (86):

$$\begin{aligned}
0 &= -(1+n)\left[1+\rho\left(\omega-1\right)\right]\left(\widetilde{\eta}_{BB}\frac{\mathsf{B}_t}{1-a} + \widetilde{\eta}_{BZ^D}\mathsf{Z}_t^D\right) \\
&\quad + \left[1-\rho+\beta\rho\omega\right]\frac{1}{\beta}\frac{\mathsf{B}_t}{1-a} - \omega\left(\widetilde{\eta}_{h^D B}\frac{\mathsf{B}_t}{1-a} + \widetilde{\eta}_{h^D Z^D}\mathsf{Z}_t^D\right) + \left(\omega-1\right)\mathsf{Z}_t^D, \\
0 &= -\left[\rho\omega+\beta\left(1-\rho\right)\right]\left(\widetilde{\eta}_{h^D B}\frac{\mathsf{B}_t}{1-a} + \widetilde{\eta}_{h^D Z^D}\mathsf{Z}_t^D\right) + \left(1-\beta\right)\left(1-\rho\right)\rho\frac{1-\beta}{\beta}\frac{\mathsf{B}_t}{1-a} \\
&\quad + \left(1-\beta\right)\rho\left(\omega-1\right)\mathsf{Z}_t^D + \beta\left[1+\rho\left(\omega-1\right)\right]\left[\widetilde{\eta}_{h^D B}\left(\widetilde{\eta}_{BB}\frac{\mathsf{B}_t}{1-a} + \widetilde{\eta}_{BZ^D}\mathsf{Z}_t^D\right) + \widetilde{\eta}_{h^D Z^D}\phi\mathsf{Z}_t^D\right],
\end{aligned}$$

41

where we have made use of $Z_{t+1}^D = \phi Z_t^D$ in the second equation.

Now equate the coefficients on $\frac{B_t}{1-a}$:

$$0 = -(1+n)\left[1+\rho(\omega-1)\right]\widetilde{\eta}_{BB} + \left[1-\rho+\beta\rho\omega\right]\frac{1}{\beta} - \omega\widetilde{\eta}_{h^D B},$$

$$0 = -\left[\rho\omega+\beta(1-\rho)\right]\widetilde{\eta}_{h^D B} + (1-\beta)(1-\rho)\rho\frac{1-\beta}{\beta} + \beta\left[1+\rho(\omega-1)\right]\widetilde{\eta}_{h^D B}\widetilde{\eta}_{BB}.$$

The first relation implies:

$$\widetilde{\eta}_{h^D B} = -(1+n)\frac{1+\rho(\omega-1)}{\omega}\widetilde{\eta}_{BB} + \frac{1-\rho+\beta\rho\omega}{\beta\omega}. \qquad (89)$$

Substituting into the second relation, we obtain a second order polynomial in $\widetilde{\eta}_{BB}$:

$$0 = \left\{ \begin{array}{c} \left[\rho\omega+\beta(1-\rho)\right](1+n) \\ +1-\rho+\beta\rho\omega \end{array} \right\} \frac{1+\rho(\omega-1)}{\omega}\widetilde{\eta}_{BB}$$
$$+ (1-\beta)(1-\rho)\rho\frac{1-\beta}{\beta} - \left[\rho\omega+\beta(1-\rho)\right]\frac{1-\rho+\beta\rho\omega}{\beta\omega}$$
$$- \beta\left[1+\rho(\omega-1)\right](1+n)\frac{1+\rho(\omega-1)}{\omega}\widetilde{\eta}_{BB}^2$$
$$\equiv \Gamma(\widetilde{\eta}_{BB})$$

Further manipulation makes it possible to simplify the polynomial as:

$$\Gamma(\widetilde{\eta}_{BB}) = \left\{ \begin{array}{c} (1+\beta+n)\frac{[1+\rho(\omega-1)]^2}{\omega} \\ -n(1-\rho)(1-\beta)\frac{[1+\rho(\omega-1)]}{\omega} \end{array} \right\} \widetilde{\eta}_{BB}$$
$$- \frac{[1+\rho(\omega-1)]^2}{\omega} - \beta(1+n)\frac{[1+\rho(\omega-1)]^2}{\omega}\widetilde{\eta}_{BB}^2.$$

Note that:

$$\Gamma(\pm\infty) = -\infty,$$
$$\Gamma(0) = -\frac{[1+\rho(\omega-1)]^2}{\omega} < 0,$$
$$\Gamma(1) = n(1-\beta)\rho\left[1+\rho(\omega-1)\right] > 0,$$
$$\Gamma\left(\frac{1}{\beta}\right) = -n(1-\rho)\frac{1-\beta}{\beta}\frac{[1+\rho(\omega-1)]}{\omega} < 0.$$

Hence, the polynomial $\Gamma(\widetilde{\eta}_{BB})$ has two roots: one between 0 and 1 and one between 1 and β^{-1} (uninteresting). Once the stable root $\widetilde{\eta}_{BB} \in (0,1)$ has been obtained by solving the quadratic equation $\Gamma(\widetilde{\eta}_{BB}) = 0$, $\widetilde{\eta}_{h^D B}$ follows from (89). For $\widetilde{\eta}_{h^D B}$ to be positive, it must be $\widetilde{\eta}_{BB} < \frac{1-\rho+\beta\rho\omega}{\beta(1+n)[1+\rho(\omega-1)]}$, which is satisfied for plausible parameter values.

Turning to the coefficients on Z_t^D, we have:

$$
\begin{aligned}
0 &= -(1+n)\left[1+\rho\left(\omega-1\right)\right]\widetilde{\eta}_{BZ^D} - \omega\widetilde{\eta}_{h^D Z^D} + \omega - 1, \\
0 &= -\left[\rho\omega + \beta\left(1-\rho\right)\right]\widetilde{\eta}_{h^D Z^D} + \left(1-\beta\right)\rho\left(\omega-1\right) \\
&\quad + \beta\left[1+\rho\left(\omega-1\right)\right]\left(\widetilde{\eta}_{h^D B}\widetilde{\eta}_{BZ^D} + \widetilde{\eta}_{h^D Z^D}\phi\right),
\end{aligned}
$$

From which we obtain:

$$
\begin{aligned}
\widetilde{\eta}_{BZ^D} &= \frac{\beta\left(\omega-1\right)\left(1-\phi\right)}{\left(1+n\right)\left\{\rho\omega + \beta\left(1-\rho\right) - \beta\phi\left[1+\rho\left(\omega-1\right)\right]\right\} + \beta\omega\widetilde{\eta}_{h^D B}}, \\
\widetilde{\eta}_{h^D Z^D} &= -\left(1+n\right)\frac{\left[1+\rho\left(\omega-1\right)\right]}{\omega}\widetilde{\eta}_{BZ^D} + \frac{\omega-1}{\omega}.
\end{aligned}
$$

Both these elasticities are positive for a broad range of sensible parameter values, including those we use.

Inspection of the results obtained thus far shows that the solutions for $\widetilde{\eta}_{BB}$, $\widetilde{\eta}_{BZ^D}$, $\widetilde{\eta}_{h^D B}$, and $\widetilde{\eta}_{h^D Z^D}$ are not affected by changes in the relative size of the two countries. Because η_{BB} and $\eta_{h^D Z^D}$ in equations (73)-(74) are equal to $\widetilde{\eta}_{BB}$ and $\widetilde{\eta}_{h^D Z^D}$, respectively, the elasticity of aggregate per capita net foreign assets to its past level and the elasticity of the permanent income differential to the productivity differential are not affected by country size. This is so because η_{BB} relates a domestic variable to itself and $\eta_{h^D Z^D}$ relates a cross-country differential to another. By definition, relative country size must be irrelevant for these elasticities. Instead, $\eta_{BZ^D} = \left(1-a\right)\widetilde{\eta}_{BZ^D}$ and $\eta_{h^D B} = \frac{\widetilde{\eta}_{h^D B}}{1-a}$ imply that the elasticity of asset accumulation to the productivity differential and of the permanent income differential to net foreign assets change with country size. The larger the domestic economy (the closer a to 1), the smaller the elasticity of net assets to the productivity differential. Intuitively, a productivity differential relative to a foreign economy of negligible size $(a \to 1)$ has no effect on the home economy's incentive to accumulate or decumulate assets. The elasticity of the permanent income differential to net foreign assets is smaller the smaller the home economy (the closer a to 0). If home is a small open economy $(a \to 0)$, a given productivity differential relative to foreign causes a larger movement in domestic aggregate per capita net foreign assets. To keep a given, desired smoothness of the consumption path following a larger reaction of assets to the shock, $\eta_{h^D B}$ must fall.

Given solutions for η_{BB}, η_{BZ^D}, $\eta_{h^D B}$, and $\eta_{h^D Z^D}$, it is easy to recover the other relevant elasticity parameters from the equations in Section 4. In what follows, we assume that parameter values are such that η_{BB}, η_{BZ^D}, $\eta_{h^D B}$, and $\eta_{h^D Z^D}$ are all positive and $\omega > 1$. This is sufficient for us to be able to sign most elasticities below.

The elasticities of the consumption differential to assets and relative productivity are, respec-

tively:

$$\eta_{c^D B} = \frac{\rho(1-\beta)}{\beta(1-a)} + \eta_{h^D B} > 0,$$
$$\eta_{c^D Z^D} = \eta_{h^D Z^D} > 0.$$

As for the real wage and relative price differentials:

$$\eta_{w^D B} = \frac{1-\rho}{1+\rho(\omega-1)}\eta_{c^D B} > 0,$$
$$\eta_{w^D Z^D} = \frac{1-\rho}{1+\rho(\omega-1)}\eta_{c^D Z^D} + \frac{\rho(\omega-1)}{1+\rho(\omega-1)} > 0,$$
$$\eta_{RP^D B} = \eta_{w^D B} > 0, \quad \eta_{RP^D Z^D} = \eta_{w^D Z^D} - 1 < 0.$$

(Plausible parameter values yield $\eta_{w^D Z^D} < 1$.) Finally, the elasticities of the employment and real GDP differentials are:

$$\eta_{L^D B} = -\omega\eta_{w^D B} < 0, \quad \eta_{L^D Z^D} = -\omega\eta_{w^D Z^D} + \omega - 1,$$
$$\eta_{y^D B} = -(\omega-1)\eta_{w^D B} < 0, \quad \eta_{y^D Z^D} = -(\omega-1)(\eta_{w^D Z^D} - 1) > 0.$$

Given $\eta_{w^D Z^D} < 1$, $\eta_{L^D Z^D} > 0 \Leftrightarrow \omega(1-\eta_{w^D Z^D}) > 1$, which holds for plausible parameter values.

C   Determinacy of the Exchange Rate under Flexible Prices

The domestic and foreign interest rate rules can be written in anti-log form as:

$$1 + i_{t+1} = y_t^{\alpha_1}\left(1 + \pi_t^{CPI}\right)^{\alpha_2}\xi_t,$$
$$1 + i_{t+1}^* = y_t^{*\alpha_1}\left(1 + \pi_t^{CPI*}\right)^{\alpha_2}\xi_t^*, \ \xi_t, \xi_t^* > 0.$$

(It is understood than ξ and ξ^* now denote levels rather than percentage deviations from the steady state. Note that, for the interest rate rules to be consistent with the initial steady-state levels of GDP, inflation, and the interest rate in both countries, it must be $\overline{\xi}_{-1} = \overline{\xi}_{-1}^* = \frac{1}{\beta\rho^{\alpha_1}}$, which we assume satisfied.) It follows that the ratio of domestic to foreign interest rate must obey:

$$\frac{1+i_{t+1}}{1+i_{t+1}^*} = \left(\frac{y_t}{y_t^*}\right)^{\alpha_1}\left(\frac{1+\pi_t^{CPI}}{1+\pi_t^{CPI*}}\right)^{\alpha_2}\frac{\xi_t}{\xi_t^*}$$
$$= \left(\frac{y_t}{y_t^*}\right)^{\alpha_1}(1+e_t)^{\alpha_2}\frac{\xi_t}{\xi_t^*}, \tag{90}$$

where the second equality follows from purchasing power parity.

Under flexible prices, the real GDP ratio is exogenous to nominal exchange rate dynamics. Define $u_t^D \equiv \left(\frac{y_t}{y_t^*}\right)^{\alpha_1} \frac{\xi_t}{\xi_t^*} > 0$. Combining (90) with UIP and making use of this definition yields the equation for the rate of depreciation:

$$1 + e_{t+1} = (1 + e_t)^{\alpha_2} u_t^D,$$

or:

$$1 + e_t = (1 + e_{t+1})^{\frac{1}{\alpha_2}} u_t^{D^{-\frac{1}{\alpha_2}}}.$$

Taking logs of both sides, it is:

$$\log (1 + e_t) = \frac{1}{\alpha_2} \log (1 + e_{t+1}) - \frac{1}{\alpha_2} \log u_t^D. \tag{91}$$

We stress that equation (91) holds exactly. No approximation has been taken. Equation (91) is a linear, forward-looking difference equation for the rate of depreciation of the domestic currency. Given the initial level of the exchange rate $\bar{\varepsilon}_{-1}$, uniqueness of the solution for the rate of depreciation at all points in time is sufficient to ensure uniqueness of the solution for the level of the exchange rate. Solving equation (91) forward yields:

$$\log (1 + e_t) = \lim_{T \to \infty} \left(\frac{1}{\alpha_2}\right)^T \log (1 + e_{t+T}) - \sum_{s=0}^{\infty} \left(\frac{1}{\alpha_2}\right)^{s+1} \log u_{t+s}^D. \tag{92}$$

Assuming a well-behaved forcing function $\log u_t^D$, $\alpha_2 > 1$ is sufficient to ensure that the summation term in (92) is well defined. Nevertheless, the condition $\alpha_2 > 1$ alone is not sufficient to ensure that $\lim_{T \to \infty} \left(\frac{1}{\alpha_2}\right)^T \log (1 + e_{t+T})$ is uniquely determined (in particular, that it is equal to zero).

For the limit in (92) to be indeterminate, it must be the case that either $e_{t+T} \to -1$ (the domestic currency is expected to appreciate by 100 percent between $t + T - 1$ and $t + T$) or that $e_{t+T} \to \infty$ (the domestic currency is expected to depreciate at an explosive rate).[58] Would these be rational equilibria under the interest rules we consider? If the domestic currency is expected to appreciate by 100 percent, UIP implies a ratio of domestic to foreign gross interest rates equal to zero. Equation (90) implies that the expectation would be validated by the interest rules, and it would actually be $e_{t+T-1} \to -1$, and so on. Similarly for the case of an explosive depreciation rate. In sum, $\alpha_2 > 1$ alone is not sufficient to rule out self-fulfilling, speculative movements of the exchange rate.

---

[58] From $1 + e_t = \frac{\varepsilon_t}{\varepsilon_{t-1}}$, it follows that $\frac{1}{1+e_t} = \frac{\frac{1}{\varepsilon_t}}{\frac{1}{\varepsilon_{t-1}}}$. Thus, the case of an expected 100 percent appreciation of the home currency corresponds to an explosive depreciation of the foreign currency.

Now recall PPP: $P_t = \varepsilon_t P_t^*$. If the domestic currency depreciates at an explosive rate, it follows that the domestic price level is diverging to infinity at an explosive rate for any foreign price level. We eliminate explosive solutions such as this by imposing that each monetary authority can back its currency with goods, when the currency becomes too devalued. This is the solution proposed by Obstfeld and Rogoff (1983). If the price level shoots to infinity, the real value of the currency $\left(\frac{1}{P}\right)$ falls to zero. But now suppose that the central bank is committed to redeeming money for goods at some strictly positive rate $\left(\frac{1}{P}\right)^{MIN}$. (The implicit assumption is that the resources for this are generated by tax revenues made available to the central bank by the government when need be.) This fractional backing imposes a strictly positive lower bound on the real value of money and causes the explosive equilibrium in which $P \to \infty$ to unravel, thus ruling it out. Similar fractional backing abroad rules out the situation in which the foreign currency is depreciating at an explosive rate.

Thus, the assumption that authorities are committed to fractional backing of their currencies ensures that $\lim_{T \to \infty} \left(\frac{1}{\alpha_2}\right)^T \log\left(1 + e_{t+T}\right) = 0$, and the solution for the rate of depreciation (with no approximation) is uniquely determined by:

$$\log\left(1 + e_t\right) = -\sum_{s=0}^{\infty} \left(\frac{1}{\alpha_2}\right)^{s+1} \log u_{t+s}^D. \tag{93}$$

Because $\overline{e}_{-1} = 0$ and $\overline{u}_{-1}^D = 1$, equation (93) is identical to:

$$\log\left(1 + e_t\right) - \log\left(1 + \overline{e}_{-1}\right) = -\sum_{s=0}^{\infty} \left(\frac{1}{\alpha_2}\right)^{s+1} \left(\log u_{t+s}^D - \log \overline{u}_{-1}^D\right).$$

But $\log\left(1 + e_t\right) - \log\left(1 + \overline{e}_{-1}\right) = d\log\left(1 + e_t\right) = \frac{d(1+e_t)}{1+\overline{e}_{-1}} = de_t = \epsilon_t - \epsilon_{t-1}$ for sufficiently small deviations from the initial steady state. Also, $\log u_{t+s}^D - \log \overline{u}_{-1}^D = d\log u_{t+s}^D = \alpha_1 y_{t+s}^D + \xi_{t+s}^D$ (where ξ^D is now in percentage deviations from the steady state). Thus, given the initial level $\overline{\varepsilon}_{-1}$ the path of the exchange rate is uniquely determined by:

$$\epsilon_t = \epsilon_{t-1} - \sum_{s=0}^{\infty} \left(\frac{1}{\alpha_2}\right)^{s+1} \left(\alpha_1 y_{t+s}^D + \xi_{t+s}^D\right). \tag{94}$$

The root on the unit circle in the characteristic equation for (75) shows up in the presence of a unit root in the level of the exchange rate. We can verify that making use of the assumption $\xi_{t+s}^D = \mu \xi_{t+s-1}^D$, $0 \le \mu \le 1$, and the results for y_{t+s}^D into (94) returns the undetermined coefficients solution of Section 4 for the exchange rate.

Start from considering the term $\sum_{s=0}^{\infty} \left(\frac{1}{\alpha_2}\right)^{s+1} \xi_{t+s}^D$. Using $\xi_{t+s}^D = \mu \xi_{t+s-1}^D$, we obtain:

$$\sum_{s=0}^{\infty} \left(\frac{1}{\alpha_2}\right)^{s+1} \xi_{t+s}^D = \frac{1}{\alpha_2} \xi_t^D \sum_{s=0}^{\infty} \left(\frac{\mu}{\alpha_2}\right)^s = \frac{1}{\alpha_2 - \mu} \xi_t^D. \tag{95}$$

Now consider the term $\sum_{s=0}^{\infty}\left(\frac{1}{\alpha_2}\right)^{s+1}\alpha_1 \mathsf{y}_{t+s}^D$. It is:

$$
\begin{aligned}
\mathsf{y}_{t+s}^D &= \eta_{y^D B}\mathsf{B}_{t+s} + \eta_{y^D Z^D}\mathsf{Z}_{t+s}^D, \\
\mathsf{B}_{t+s} &= \eta_{BB}\mathsf{B}_{t+s-1} + \eta_{BZ^D}\mathsf{Z}_{t+s-1}^D, \\
\mathsf{Z}_{t+s}^D &= \phi\mathsf{Z}_{t+s-1}^D.
\end{aligned}
$$

Given an initial level B_t, the equation for B_{t+s} implies:

$$
\mathsf{B}_{t+s} = \eta_{BB}^s \mathsf{B}_t + \eta_{BZ^D}\sum_{v=0}^{s-1}\eta_{BB}^{s-1-v}\mathsf{Z}_{t+v}^D.
$$

Hence, using $\mathsf{Z}_{t+v}^D = \phi\mathsf{Z}_{t+v-1}^D$ and the result $\sum_{v=0}^{s-1}\left(\frac{\phi}{\eta_{BB}}\right)^v = \frac{1-\left(\frac{\phi}{\eta_{BB}}\right)^s}{1-\frac{\phi}{\eta_{BB}}}$ in the general case in which $\phi \neq \eta_{BB}$, we obtain:

$$
\begin{aligned}
\mathsf{B}_{t+s} &= \eta_{BB}^s\mathsf{B}_t + \eta_{BZ^D}\mathsf{Z}_t^D\sum_{v=0}^{s-1}\eta_{BB}^{s-1-v}\phi^v \\
&= \eta_{BB}^s\mathsf{B}_t + \eta_{BZ^D}\eta_{BB}^{s-1}\mathsf{Z}_t^D\sum_{v=0}^{s-1}\left(\frac{\phi}{\eta_{BB}}\right)^v \\
&= \eta_{BB}^s\mathsf{B}_t + \eta_{BZ^D}\frac{\eta_{BB}^s - \phi^s}{\eta_{BB}-\phi}\mathsf{Z}_t^D.
\end{aligned}
$$

Thus, we can write:

$$
\begin{aligned}
&\sum_{s=0}^{\infty}\left(\frac{1}{\alpha_2}\right)^{s+1}\alpha_1\mathsf{y}_{t+s}^D \\
={}&\frac{\alpha_1}{\alpha_2}\left[\eta_{y^D B}\sum_{s=0}^{\infty}\left(\frac{1}{\alpha_2}\right)^s\mathsf{B}_{t+s} + \eta_{y^D Z^D}\mathsf{Z}_t^D\sum_{s=0}^{\infty}\left(\frac{\phi}{\alpha_2}\right)^s\right] \\
={}&\frac{\alpha_1}{\alpha_2}\left\{
\begin{array}{c}
\eta_{y^D B}\mathsf{B}_t\sum_{s=0}^{\infty}\left(\frac{\eta_{BB}}{\alpha_2}\right)^s \\
+\left[\frac{\eta_{y^D B}\eta_{BZ^D}}{\eta_{BB}-\phi}\sum_{s=0}^{\infty}\left(\frac{1}{\alpha_2}\right)^s(\eta_{BB}^s - \phi^s) + \eta_{y^D Z^D}\sum_{s=0}^{\infty}\left(\frac{\phi}{\alpha_2}\right)^s\right]\mathsf{Z}_t^D
\end{array}
\right\} \\
={}&\frac{\alpha_1\eta_{y^D B}}{\alpha_2-\eta_{BB}}\mathsf{B}_t + \frac{\alpha_1\left[(\alpha_2-\eta_{BB})\eta_{y^D Z^D} + \eta_{y^D B}\eta_{BZ^D}\right]}{(\alpha_2-\phi)(\alpha_2-\eta_{BB})}\mathsf{Z}_t^D. \quad (96)
\end{aligned}
$$

Combining equation (94) with results (95) and (96) returns equation (80).

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Table 1. The sticky-price solution, elasticities

Endogenous states to endogenous states

	B	ϵ	ψ^D	w^D	y^D	h^D	v^D
B	.9944	0	0	0	.3261	0	0
ϵ	.0008	1	0	0	-.4017	0	0
ψ^D	-.0074	0	0	0	-5.3284	0	0
w^D	.0096	0	0	0	1.9855	0	0
y^D	-.0004	0	0	0	.6686	0	0
h^D	.0044	0	0	0	.063	0	0
v^D	-.0312	0	0	0	-10.676	0	0

Endogenous states to shocks

	$\phi=0$ Z^D	$\mu=0$ ξ^D	$\phi=.5$ Z^D	$\mu=.5$ ξ^D	$\phi=1$ Z^D	$\mu=1$ ξ^D
B	.0076	-.0435	.0115	-.0498	-.0602	-.0705
ϵ	-.0093	-.6131	-.0217	-.9067	-.0989	-1.9899
ψ^D	1.3686	.7104	1.3002	.8265	.8804	-.0916
w^D	-.4464	-.2647	-.4206	-.3088	-.1543	.12
y^D	.0156	-.0891	.0241	-.1035	.0548	-.0057
h^D	-.0035	-.0084	-.0053	-.0193	.1607	.0537
v^D	-.2483	1.4235	.5182	4.1245	3.8639	25.0141

Other endogenous variables to endogenous states

	B	ϵ	ψ^D	w^D	y^D	h^D	v^D
π^{PPI^D}	.003	0	0	0	1.2554	0	0
c^D	.011	0	0	0	.0098	0	0

Other endogenous variables to shocks

	$\phi=0$ Z^D	$\mu=0$ ξ^D	$\phi=.5$ Z^D	$\mu=.5$ ξ^D	$\phi=1$ Z^D	$\mu=1$ ξ^D
π^{PPI^D}	-.0871	-.1674	-.1421	-.389	-.3727	-1.9615
c^D	.0002	-.0013	.0008	-.0029	.1764	.1368

Figure 1. The dollar and the U.S. current account
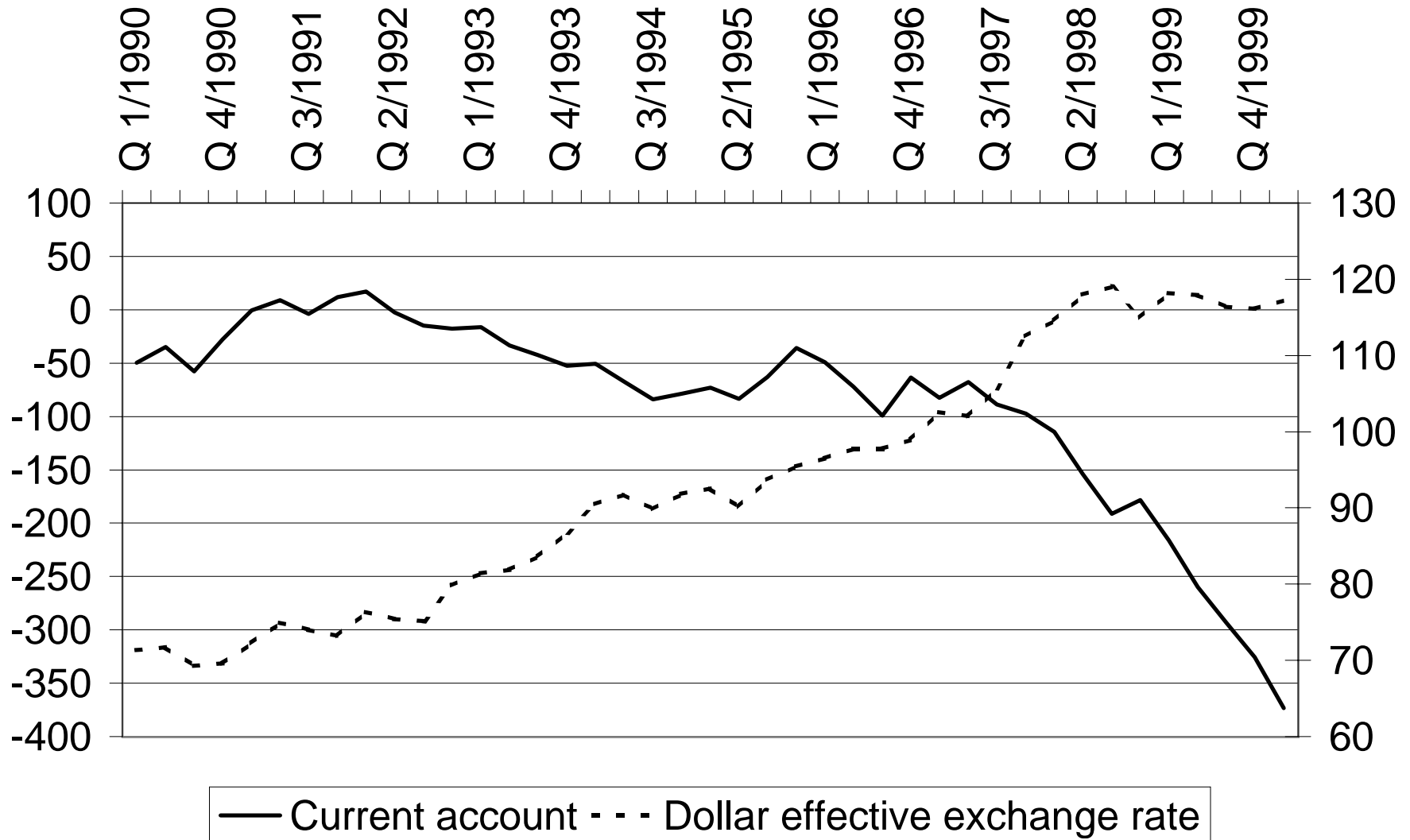
Figure 2. Net foreign assets, productivity shock
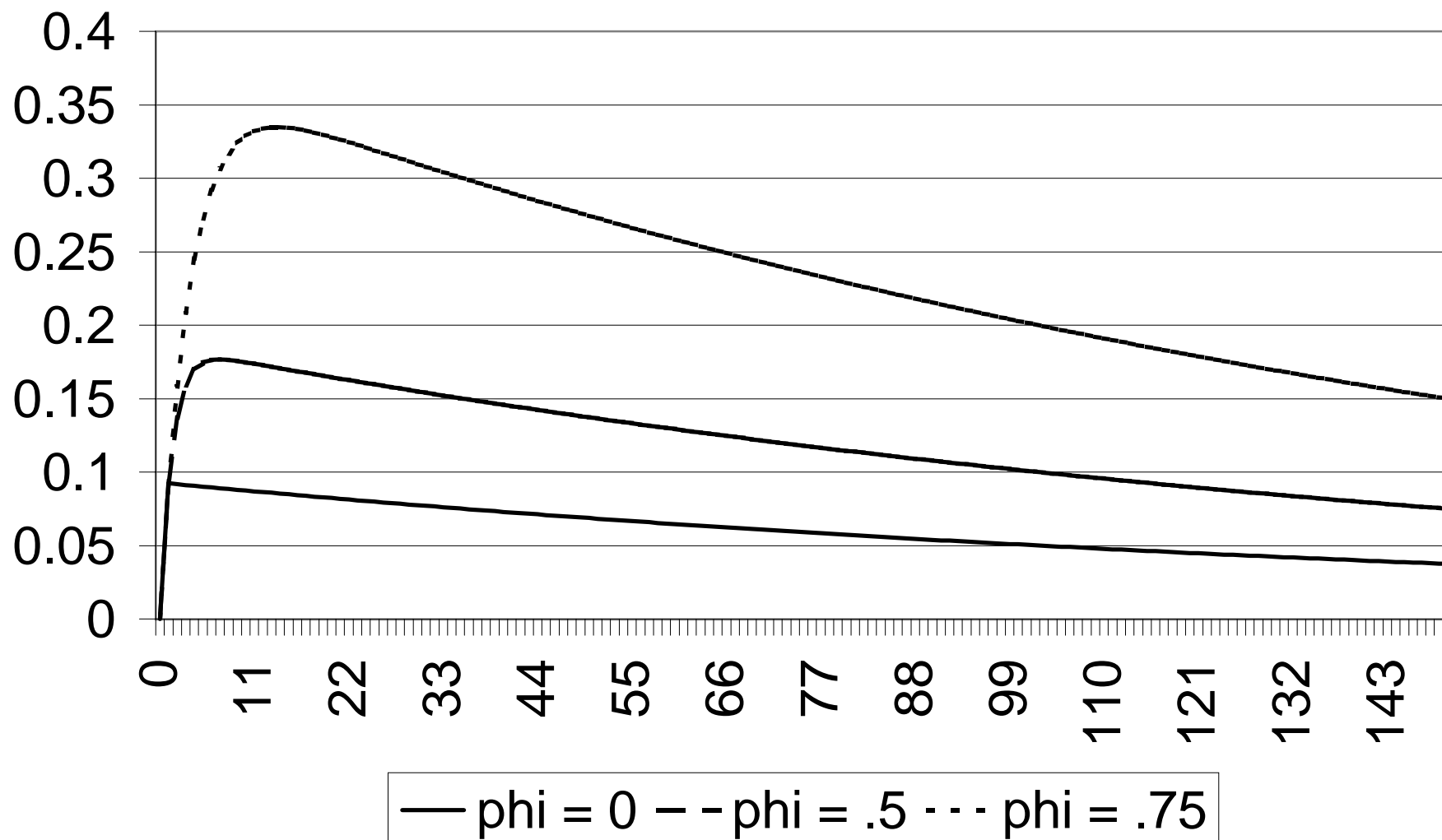
Figure 3. Exchange rate, productivity shock

phi = 0 — — phi = .5 · · · phi = .75
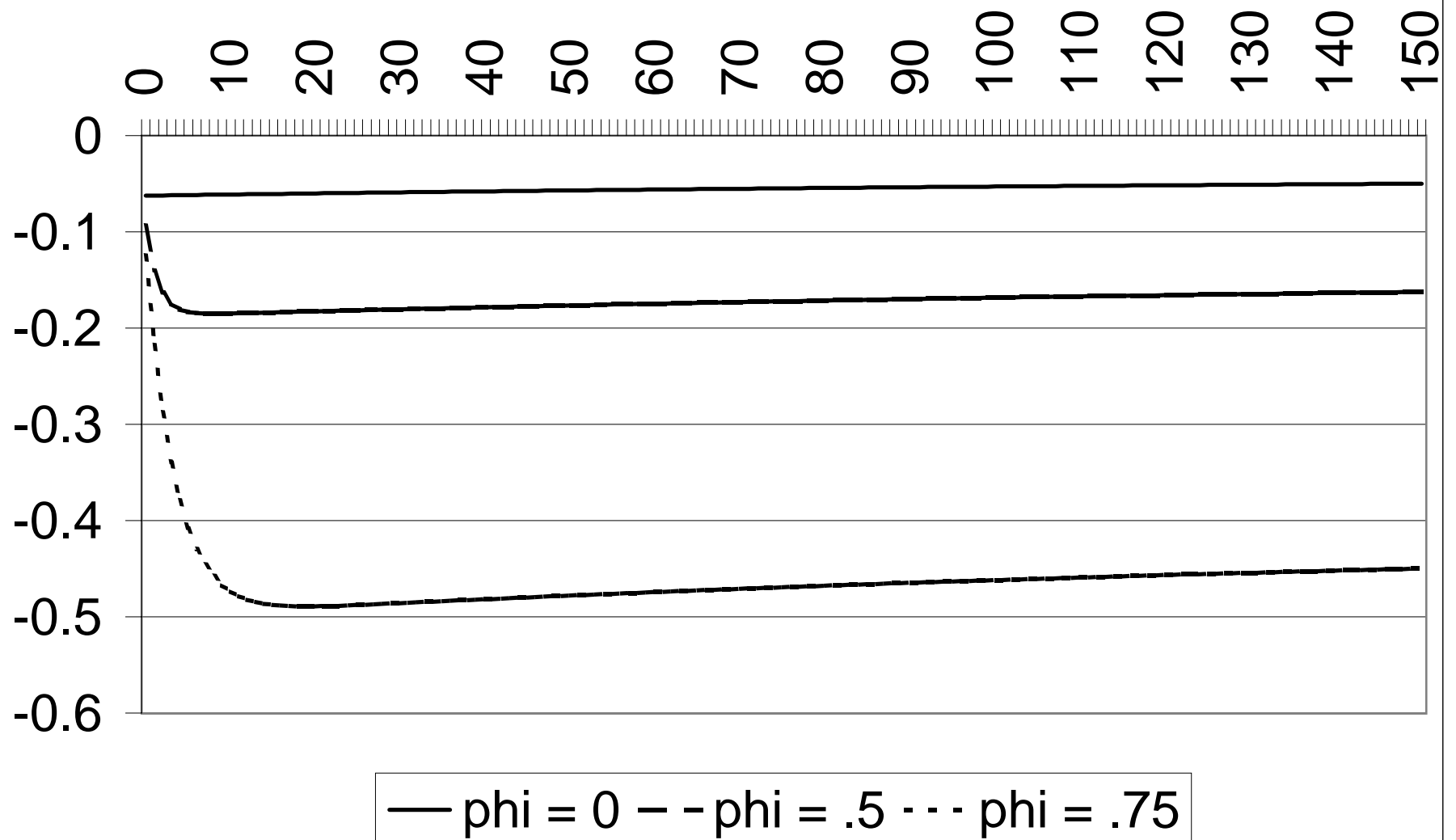
Figure 4. GDP differential, productivity shock
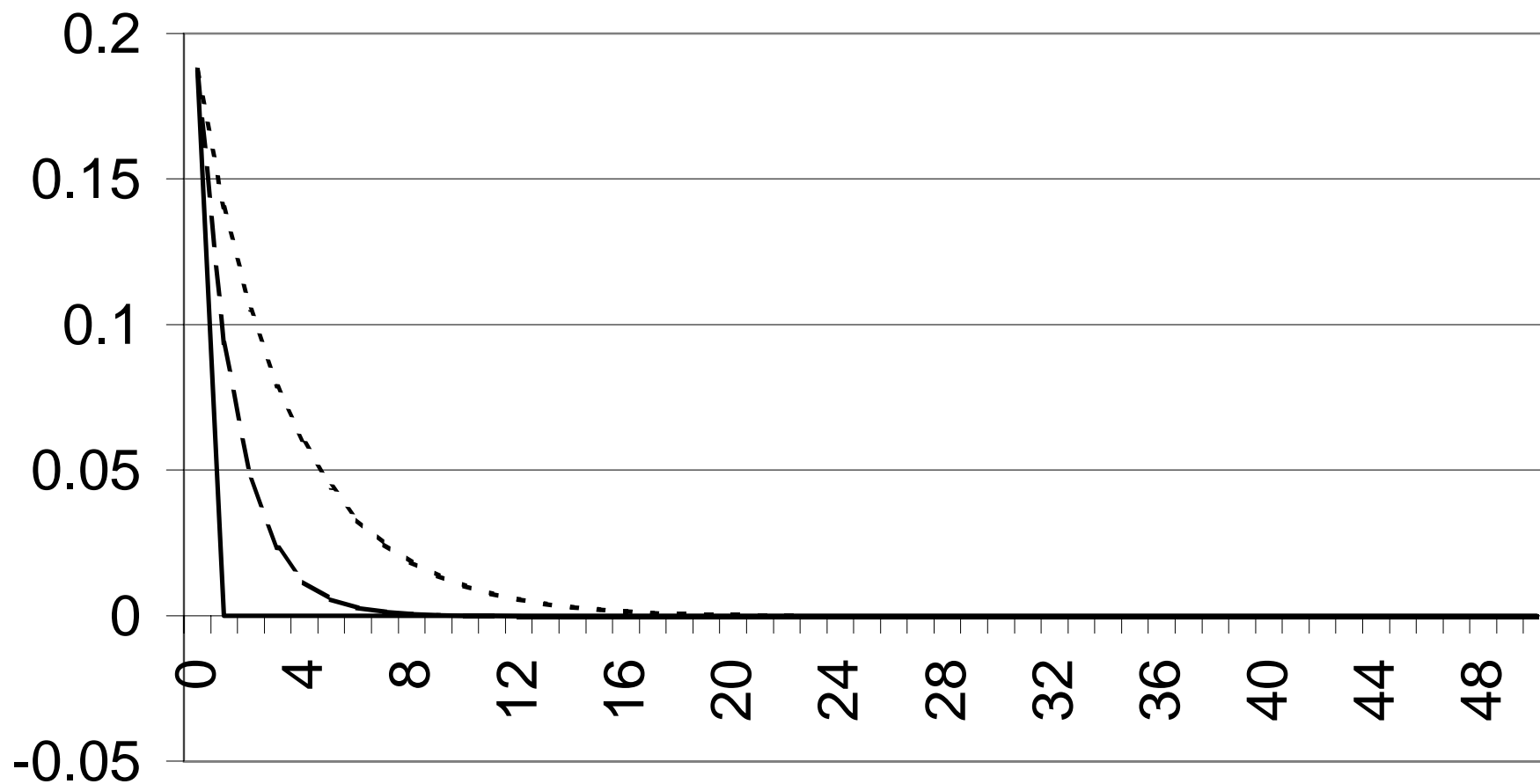
— phi = 0 − − phi = .5 · · · phi = .75

Figure 5. Rate of depreciation, productivity shock

phi = 0 — — phi = .5 · · · phi = .75
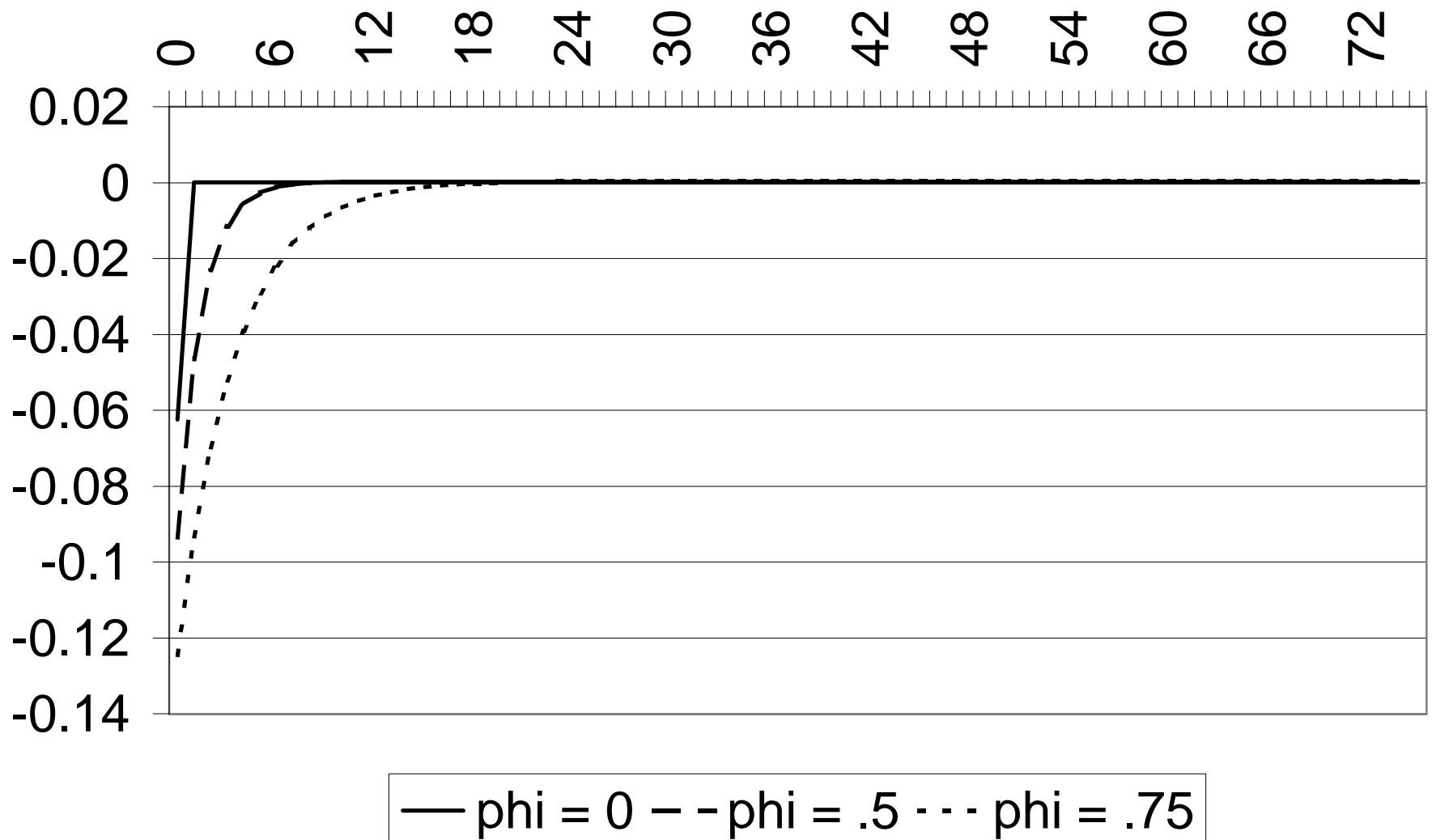
Figure 6. Productivity shock, no persistence
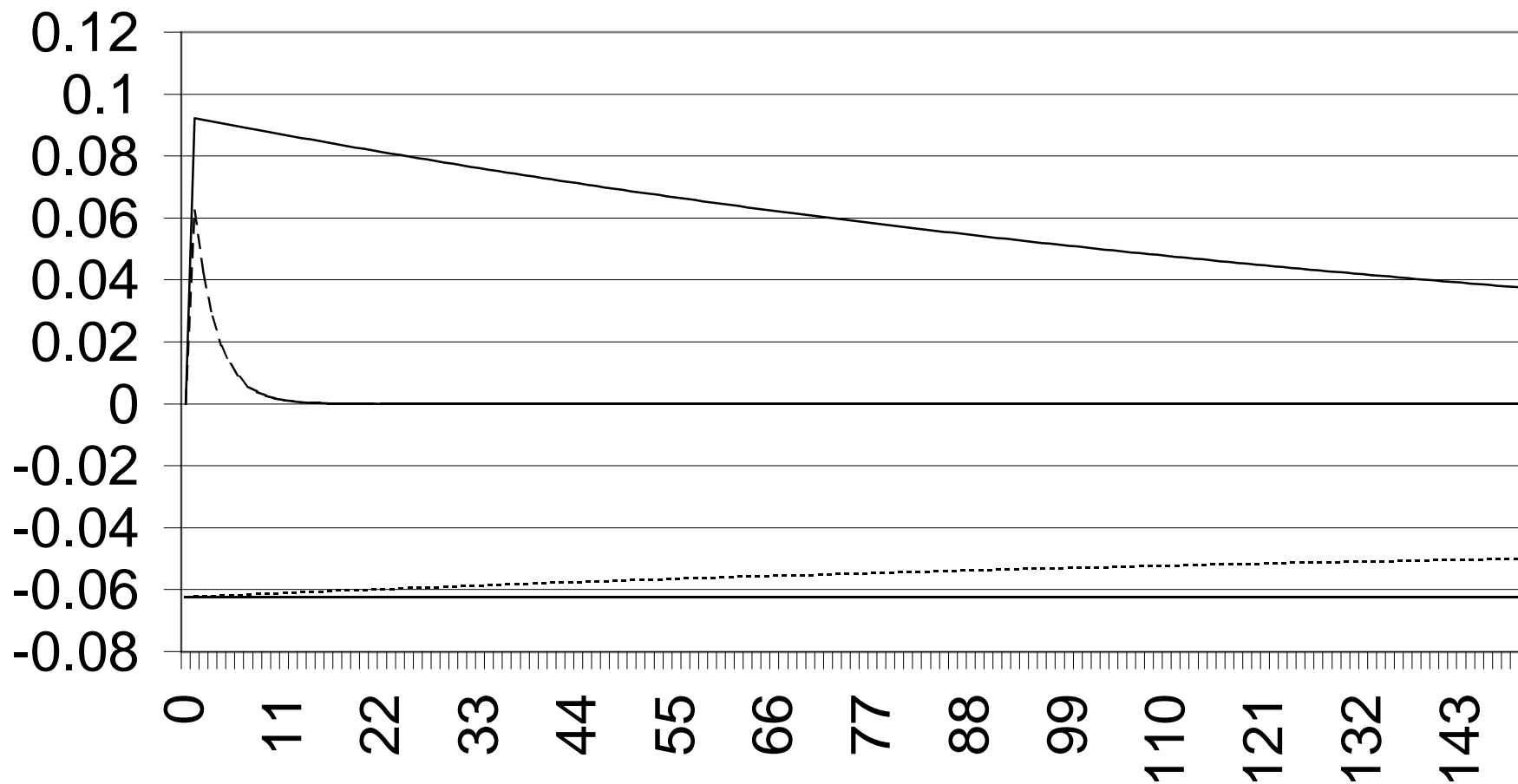
Legend: NFA, n = .01 — NFA, n = .5 --- ER, n = .01 ···· ER, n = .5 -·-·

Figure 7. Productivity shock, no persistence, omega = 4

NFA, n = .01   NFA, n = .5   ER, n = .01   ER, n = .5
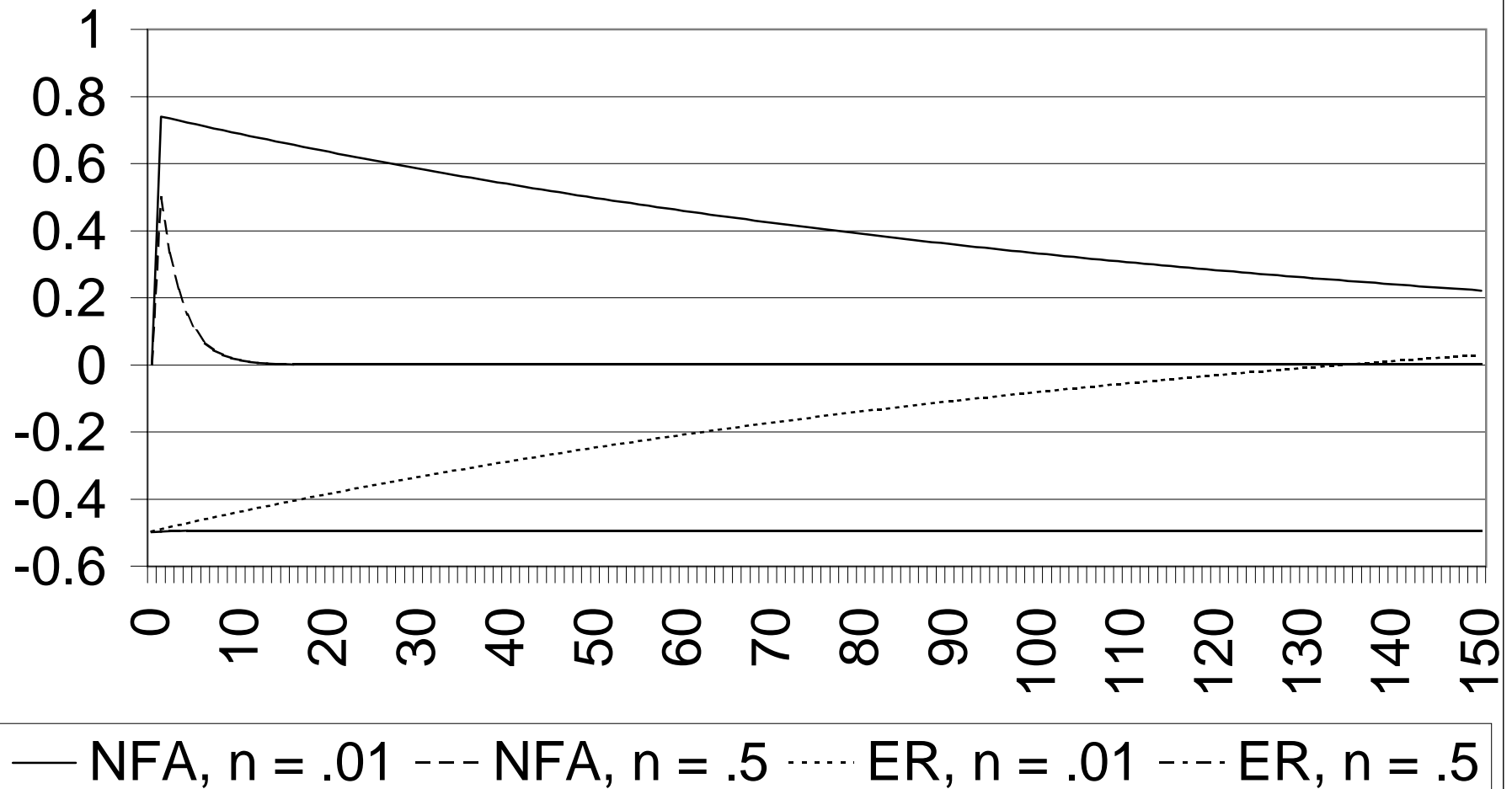
Figure 8. Exchange rate, interest rate shock

Legend: —— mu = 0 – – mu = .5 · · · mu = .75
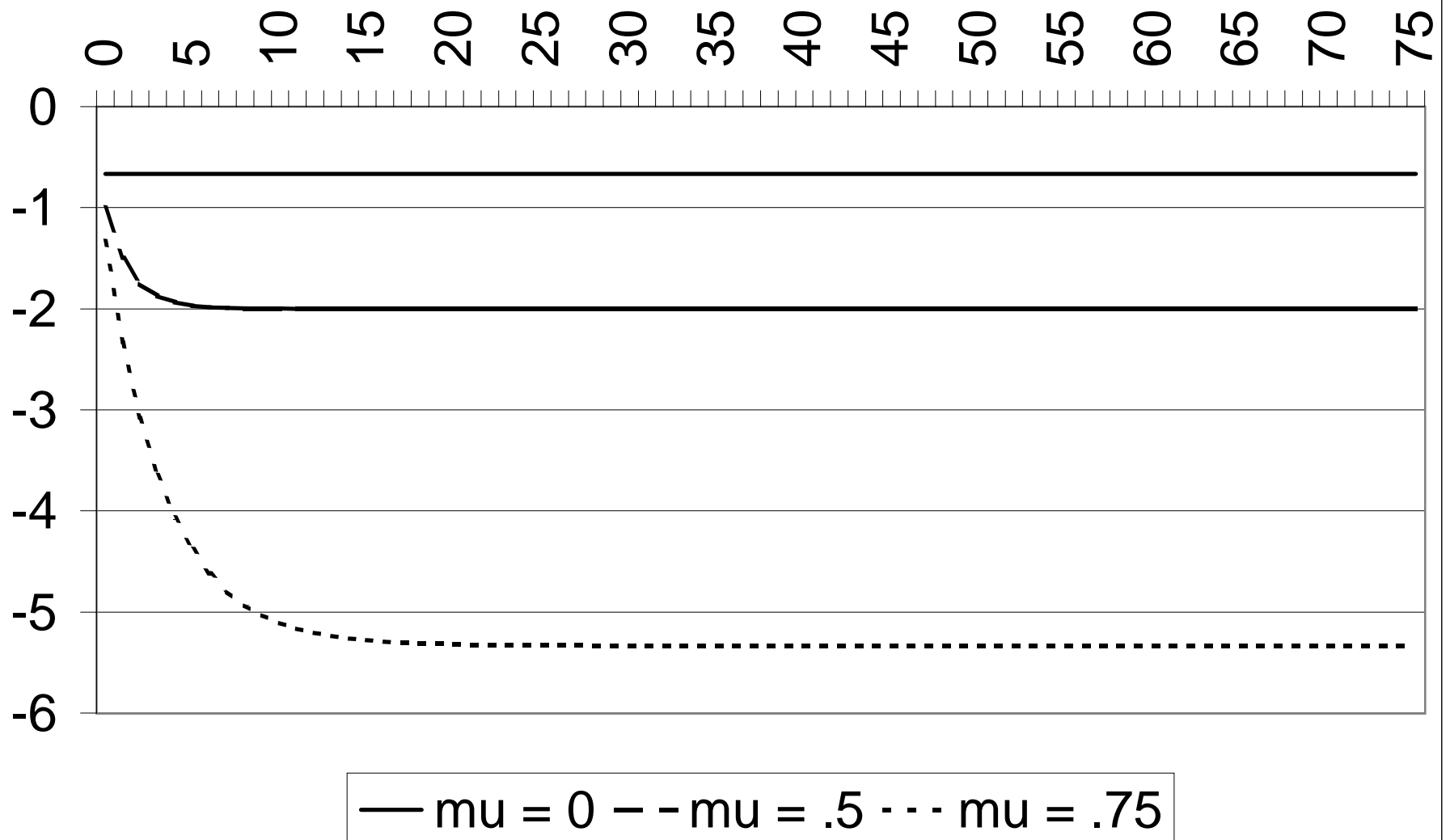
Figure 9. Rate of depreciation, interest rate shock

mu = 0 − − mu = .5 · · · mu = .75
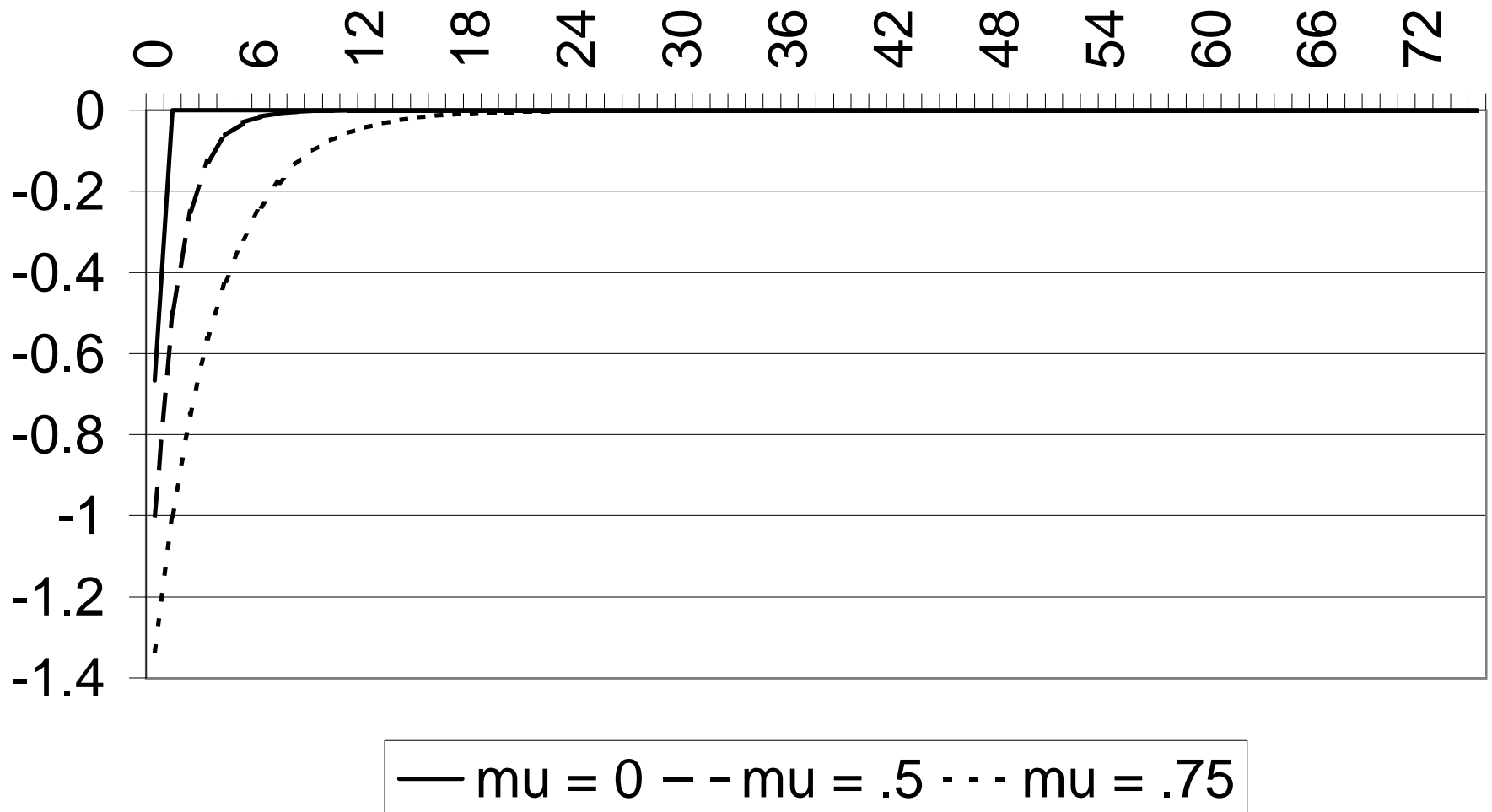
Figure 10. Impulse responses, productivity shock, sticky prices
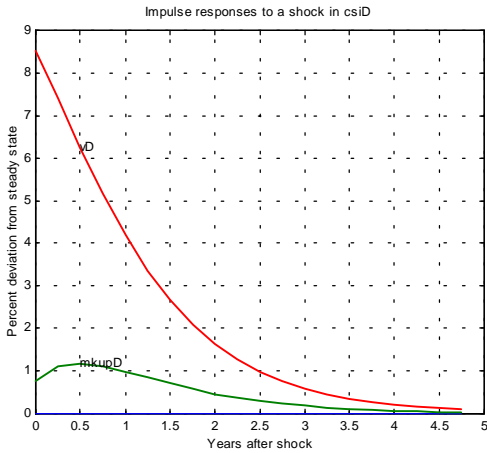
$\phi = 0$
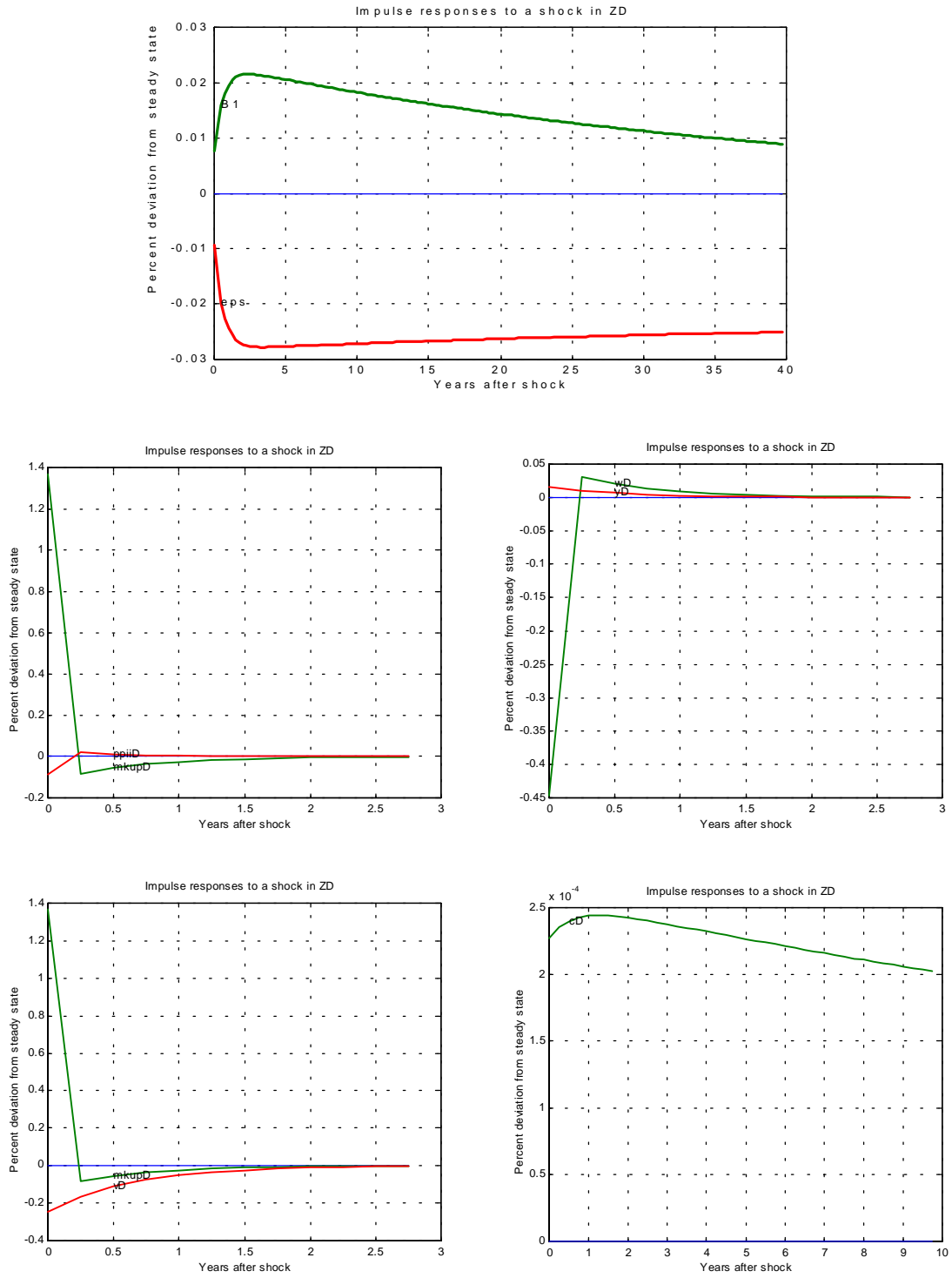
Figure 10, continued

$\phi = .5$
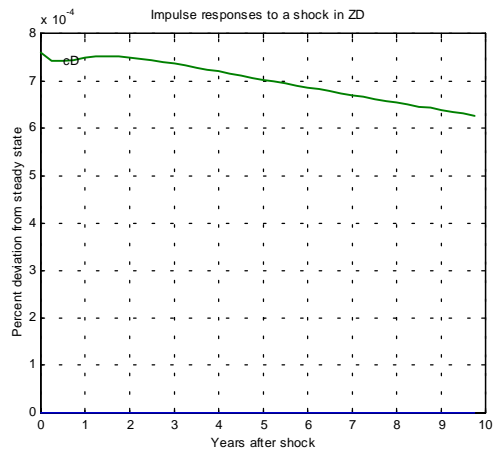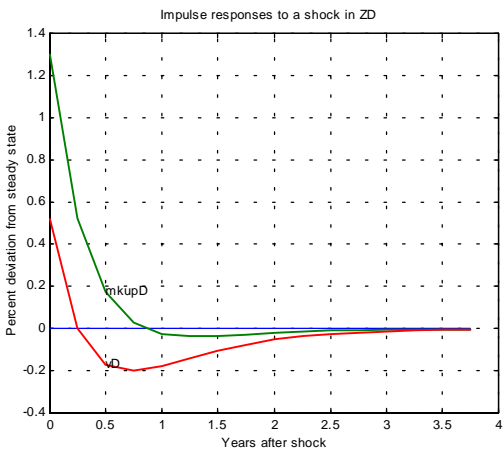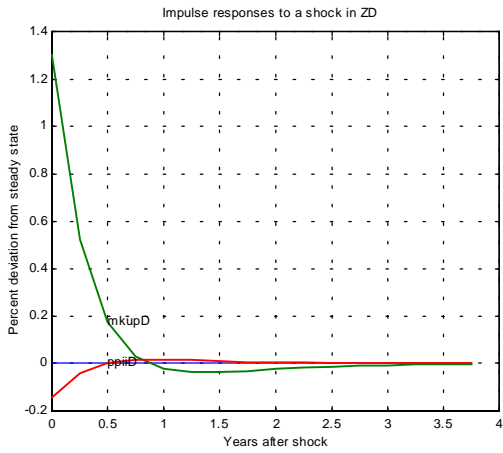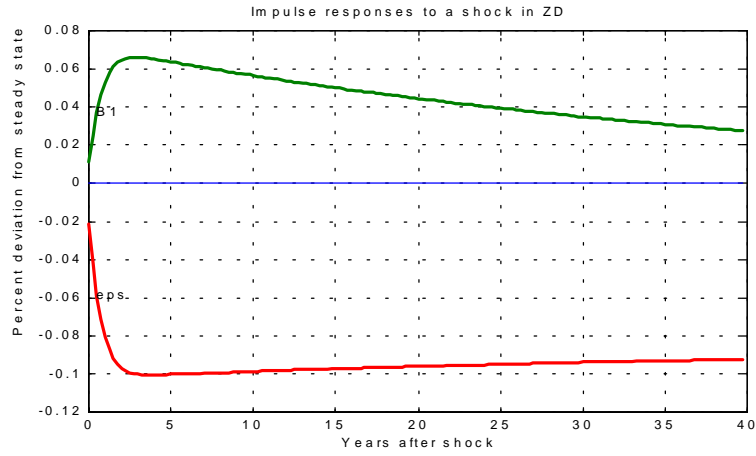
Figure 10, continued

$\phi = .75$
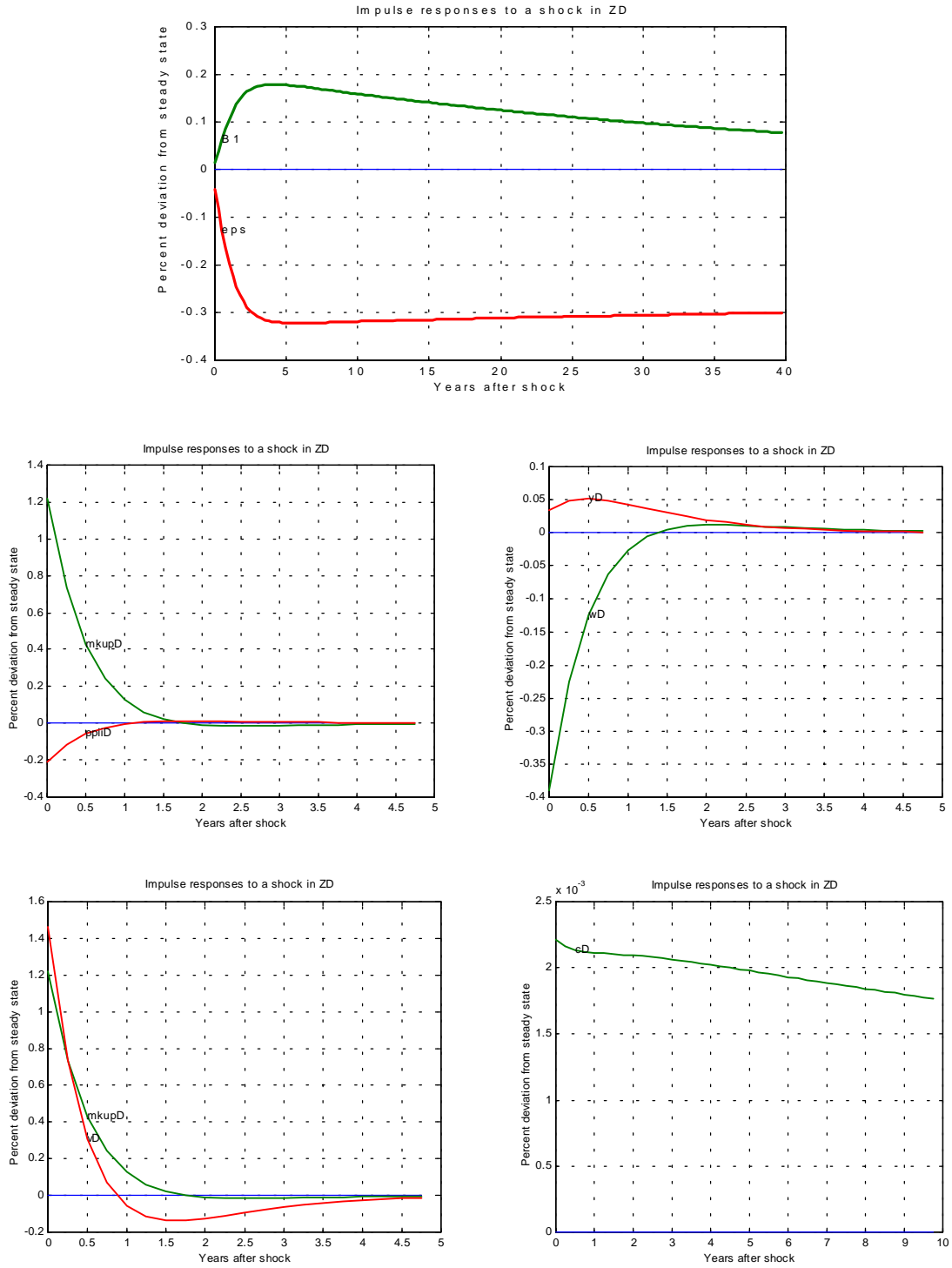
Figure 10, continued

$\phi = 1$
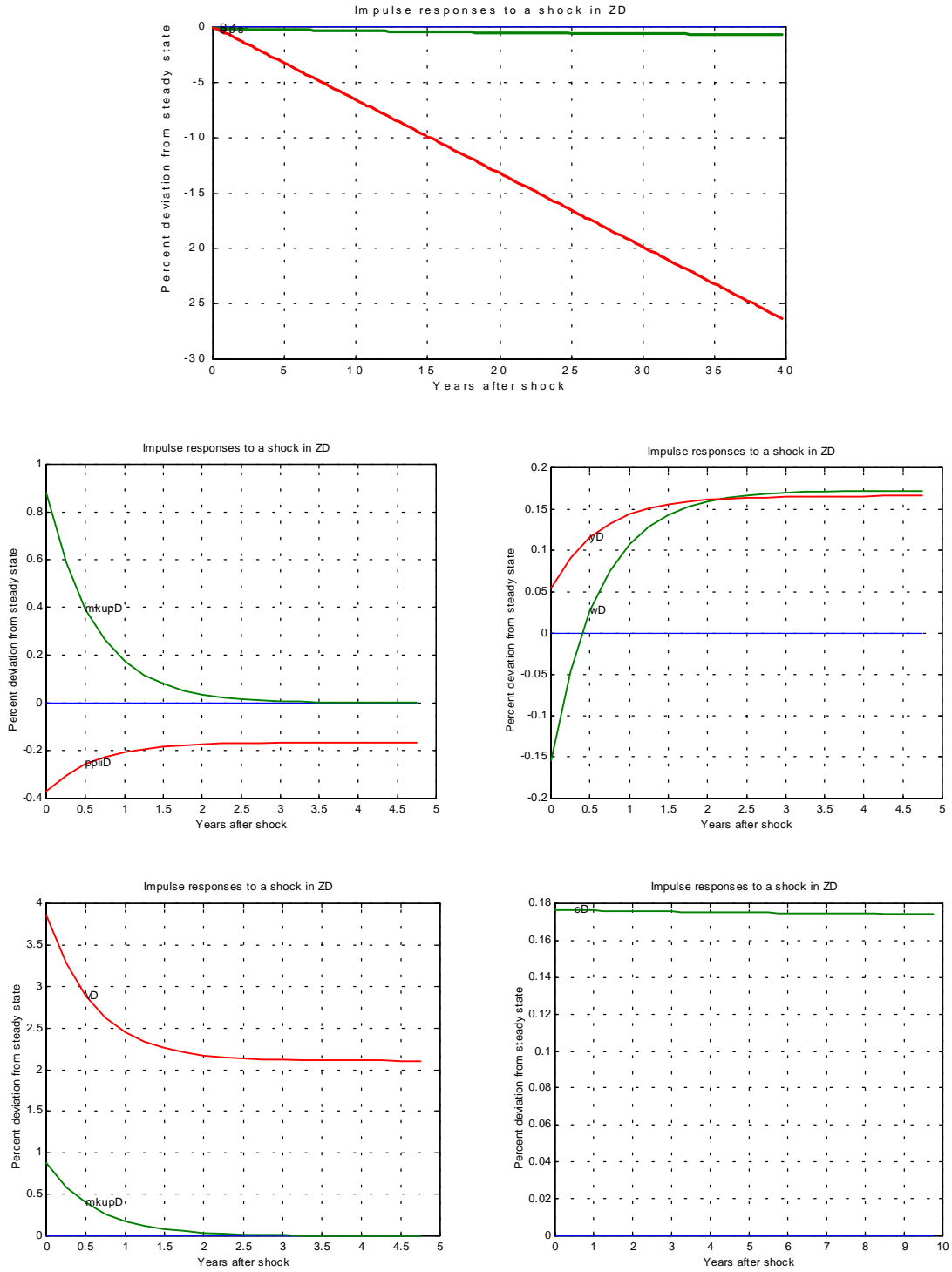
Figure 11. Impulse responses, interest rate shock, sticky prices
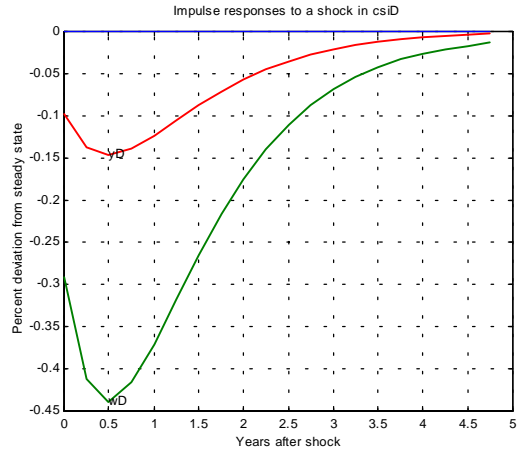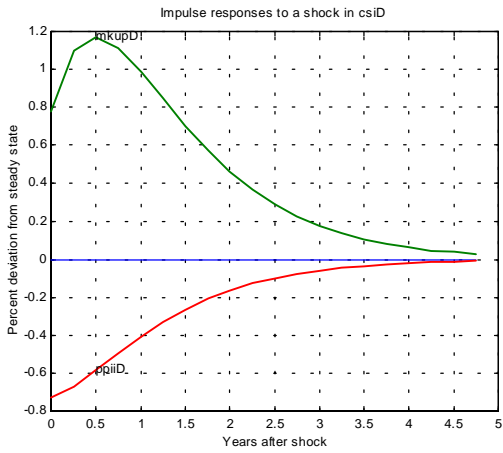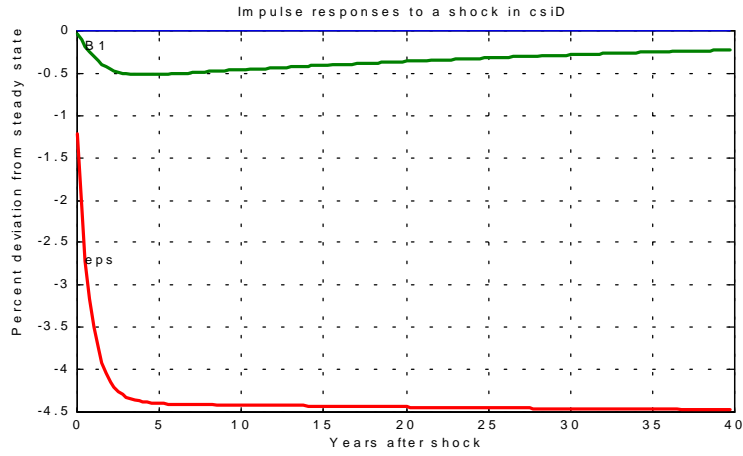
$\mu = 0$
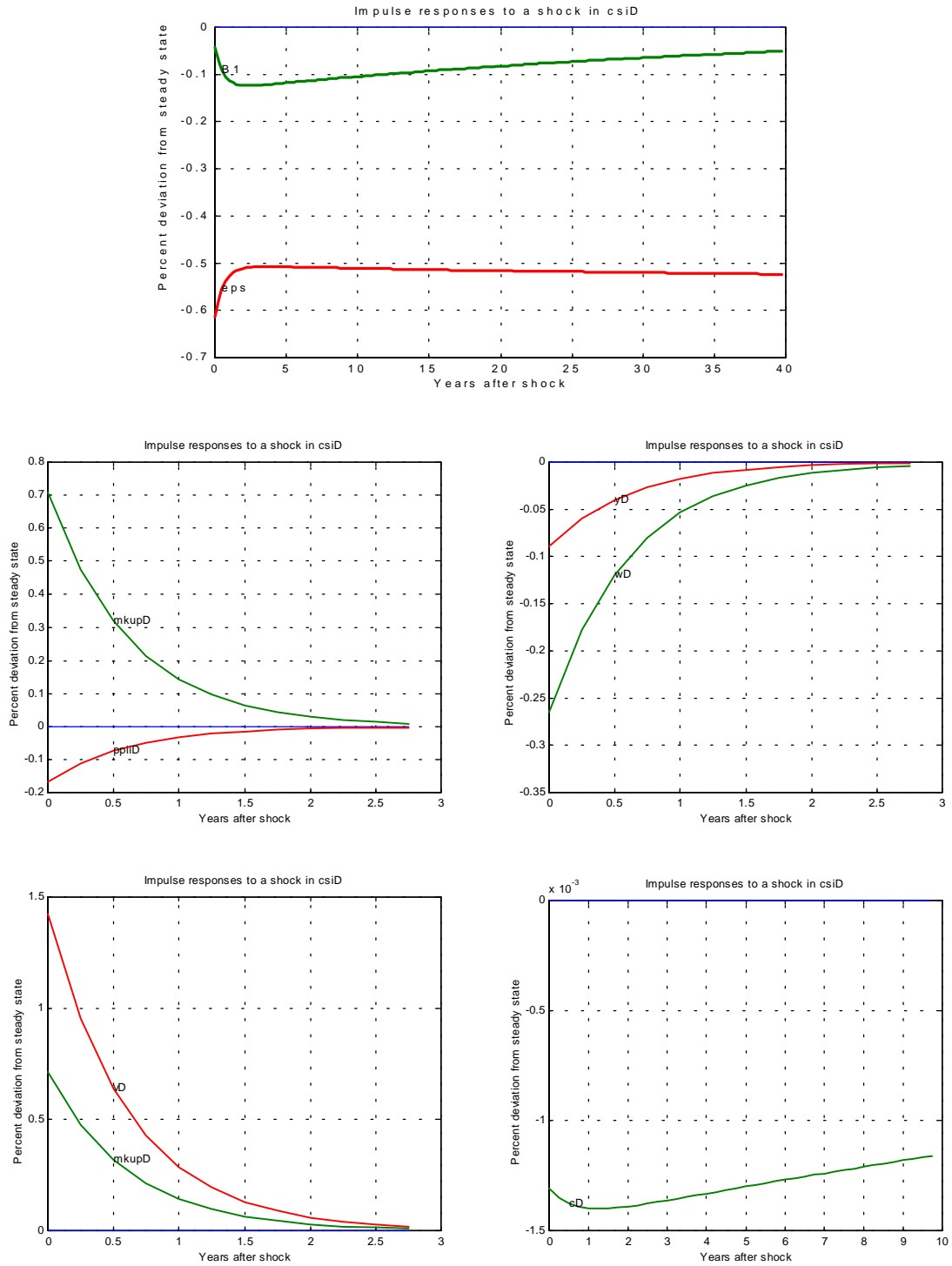
Figure 11, continued

$\mu = .5$
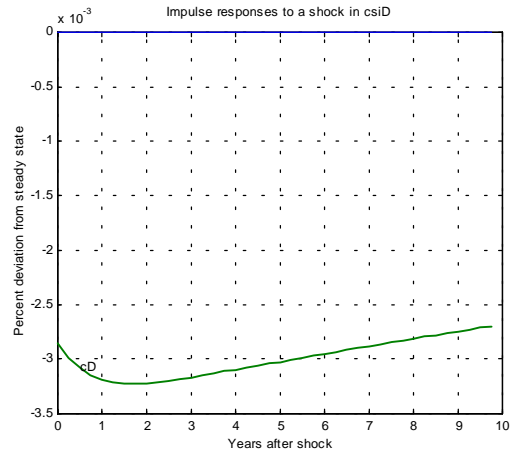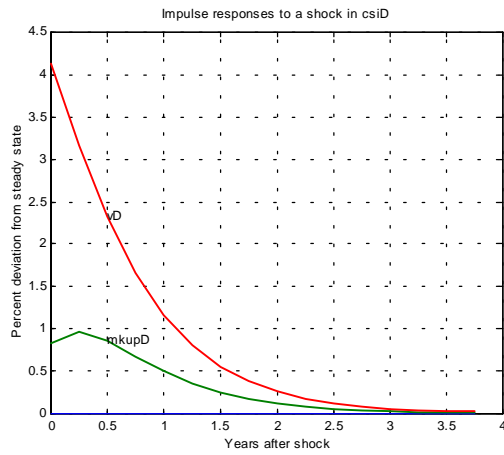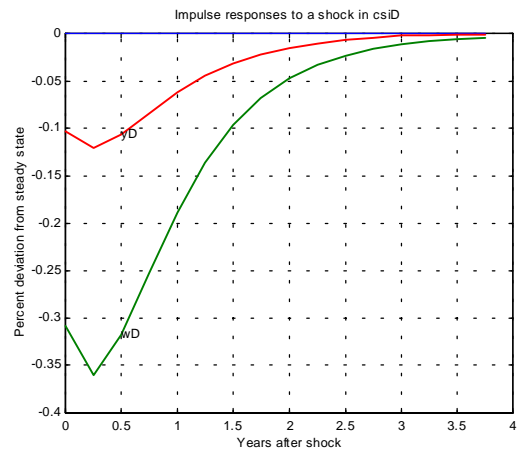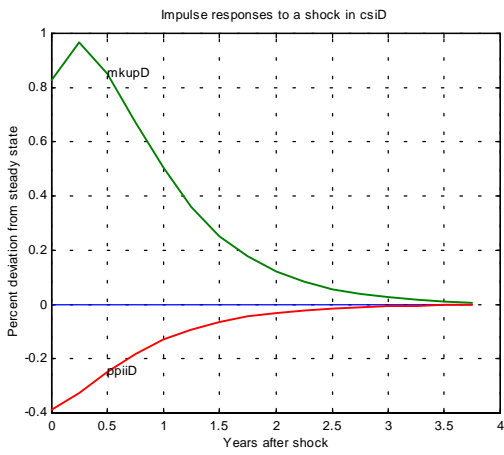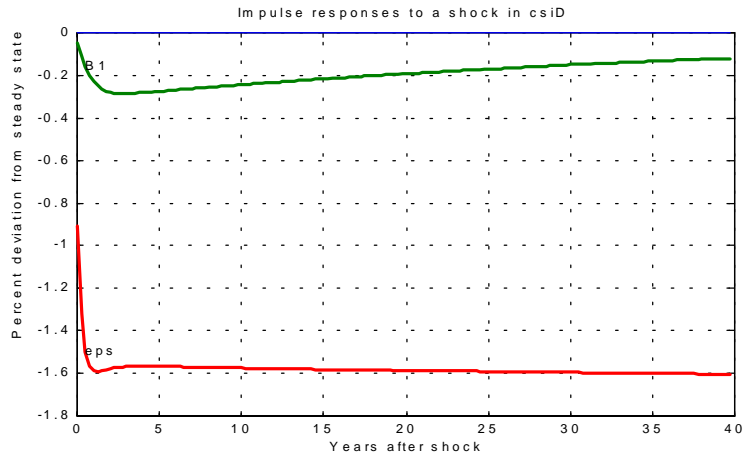
Figure 11, continued

$\mu = .75$

Figure 11, continued

$\mu = 1$

Impulse responses to a shock in csiD

Impulse responses to a shock in csiD

Impulse responses to a shock in csiD

Impulse responses to a shock in csiD