# User Costs versus Waiting Services and Depreciation in a Model of Production

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## Abstract

The paper develops an extension of a one period model of production involving beginning and end of the period capital stocks along with output and input flows that is due to Hicks and Edwards and Bell. This generalized Austrian model of production takes into account that end of the period capital stocks result from: (i) purchases of new investment goods; (ii) internal construction of firm capital stock components and (iii) holdings of (depreciated) capital goods that were held by the firm at the beginning of the period. These different methods of creating end of period holdings of capital stocks generally have different resource requirements and hence the one period production possibilities set is more complex than the usual one. This general model of production is used to justify the decomposition of the Jorgensonian user cost of capital into separate waiting services and depreciation components.

## Keywords

Production theory, user cost of capital, waiting services, depreciation, Austrian models of production, net versus gross investment.

## Journal of Economic Literature Classification Codes

D24, D92.

## 1. Introduction

Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006) adapted Diewert and Morrison's (1986) model of production in order to look at the determinants of the growth in the real income generated by the production sector of an economy. These determinants turned out to be: (i) productivity growth; (ii) changes in real output prices and (iii) changes in the use of capital and labour by the market production sector. These recent income generating growth accounting studies were somewhat unconventional in that they did not use the traditional user cost of capital in order to value the contribution of capital services to production[2]; instead these studies took depreciation out of the user cost of capital and treated it as an offset to gross investment. When depreciation is taken out of the traditional user cost of capital, the resulting price of capital services is known as the price of *waiting services*.[3] The question we want to address in this note is whether it is legitimate (from the viewpoint of basic production theory) to take depreciation out of the user cost of capital and to then treat depreciation as separate decision variable which can then be an offset to gross investment.

It turns out that it is not straightforward to answer this legitimacy question. In section 2, we explain the starting point of our analysis: the Neo-Austrian model of production that was proposed by Hicks (1961) and Edwards and Bell (1961). However, this model is not sufficiently general to deal with depreciation in a realistic manner so the model is generalized in section 3 where we show that this Generalized Austrian model provides a justification for removing depreciation from the usual user cost formula and for treating depreciation as a separate decision variable. Section 4 concludes.

## 2. The Neo-Austrian Model of Production

In this section, we will review a generalization of the one period Austrian model of production that dates back to Böhm-Bawerk (1891).[4] This Neo-Austrian model of production is based on a well established model of production that is used both by economists and thoughtful accountants as the following two quotations will show:

"We must look at the production process during a period of time, with a beginning and an end. It starts, at the commencement of the Period, with an Initial Capital Stock; to this there is applied a Flow Input of labour, and from it there emerges a Flow Output called Consumption; then there is a Closing Stock of Capital left over at the end. If Inputs are the things that are put in, the Outputs are the things that are got out, and the production of the Period is considered in isolation, then the Initial Capital Stock is an Input. A Stock Input to the Flow Input of labour; and further (what is less well recognized in the tradition, but is equally clear when we are strict with translation), the Closing Capital Stock is an Output, a Stock Output to match the Flow Output of Consumption Goods. Both input and output have stock and flow components; capital appears both as input and as output." John R. Hicks (1961; 23).

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[2] The traditional user cost of capital was developed by Jorgenson and his coworkers; see Jorgenson (1963) (1989), Jorgenson and Griliches (1967) and Christensen and Jorgenson (1969).
[3] See Rymes (1968) (1983) on the concept of waiting services.
[4] Further contributions to this model were made by von Neumann (1937), Hicks (1946; 230), Malinvaud (1953) and Diewert (1977; 108-111) (1980; 472-474).

"The business firm can be viewed as a receptacle into which factors of production, or inputs, flow and out of which outputs flow...The total of the inputs with which the firm can work within the time period specified includes those inherited from the previous period and those acquired during the current period. The total of the outputs of the business firm in the same period includes the amounts of outputs currently sold and the amounts of inputs which are bequeathed to the firm in its succeeding period of activity." Edgar O. Edwards and Philip W. Bell (1961; 71-72).

Hicks and Edwards and Bell obviously had the same model of production in mind: in each accounting period, the business unit combines the capital stocks and goods in process that it has inherited from the previous period with "flow" inputs purchased in the current period (such as labour, materials, services and additional durable inputs) to produce current period "flow" outputs as well as end of the period depreciated capital stock components which are regarded as outputs from the perspective of the current period (but will be regarded as inputs from the perspective of the next period). Their model can be viewed as an Austrian model of production in honour of the Austrian economist Böhm-Bawerk (1891) who viewed production as an activity which used raw materials and labour to further process partly finished goods into finally demanded goods.

We will formalize this Neo-Austrian model of production as follows. Let Y denote a vector of outputs produced by the production unit under consideration over a period of time and let $P_Y$ denote the corresponding vector of prices. Let X denote a vector of flow inputs (labour services, materials, energy inputs) used by the production unit during the period and let $P_X$ be the corresponding input price vector. In addition to these flow inputs, there are N capital assets (including inventories) that contribute to production. These assets (transportation equipment, machinery, structures, inventories, goods in process, etc.) are broken down into equivalence classes by their age, the amount that they have been utilized and their physical characteristics. Thus N will generally be very large. At the beginning of the period, the production unit has available the vector $K_B$ of beginning of the period capital stocks at its disposal[5] and the vector prevailing market prices for these assets is $P_{KB}$.[6] At the end of the period, the production unit produces the vector $K_E$ of capital assets and the vector of anticipated end of period market prices for these assets is $P_{KE}$. To clarify the fact that the dimensionality N of these capital quantity and price vectors will generally be very large, suppose that one unit of the first type of capital is purchased at the beginning of the beginning of the production period and is utilized at normal rates during the period. Suppose that at normal rates of utilization, this type of asset contributes capital services for M periods. Let the first M components of $K_B$ and $K_E$ correspond to this type of capital with new units recorded in component 1, one period old components recorded in component 2, ..., and M−1 period old components recorded in component M. The beginning of

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[5] In the algebra below, we assume that the production unit purchases these beginning of the period capital stocks at the prevailing prices for these assets. In practical applications of the theory, the unit will simply be purchasing these capital stock components from itself. The important point is that there are beginning of the period opportunity costs for selling these inherited stocks that the firm faces.

[6] It is important to realize that this price vector does not consist only of the prices prevailing for new assets at the beginning of the period. For the used components of the beginning of the period capital stocks, the prices are the opportunity costs that are either reflected in second hand asset markets at the beginning of the period or estimated discounted cash flows that the asset is expected to generate over its expected life. Thus for second hand assets that are held over from the previous period, the corresponding prices in the vector of beginning of the period asset prices $P_{KB}$ will generally be imputed prices instead of market transaction prices.

the period purchase of one unit of a new asset of this type would be recorded by adding 1 to the first component of $K_B$ and the corresponding end of period output of this asset would be recorded by adding 1 to the second component of $K_E$. We assume that the production unit faces the one period nominal opportunity cost of capital or interest rate r. We assume that all revenues received during the period are simply cumulated without any reinvestment of funds and payments for flow inputs are settled up at the end of the period. The production unit's production possibilities set is a set of feasible inputs and outputs, $(Y,X,K_B,K_E)$, that is denoted by the set S. Thus the production unit's *one period intertemporal profit maximization problem* is the following one:

$$(1) \quad \max_{Y,X,K_B,K_E} \{(1+r)^{-1}(P_Y \cdot Y - P_X \cdot X + P_{KE} \cdot K_E) - P_{KB} \cdot K_B : (Y,X,K_B,K_E) \in S\}$$

where $P_Y \cdot Y$ denotes the inner product of the vectors $P_Y$ and $Y$, etc. Note that we have treated the prices $P_Y$ and $P_X$ of the period sales and flow input purchases as *end of the period prices* and hence the corresponding value flows are discounted to their beginning of the period equivalents using the beginning of the period nominal interest rate r. From a practical measurement perspective, it is more useful to multiply the objective function in (1) through by (1+r) and after performing this multiplication, we obtain the following equivalent *profit maximization problem* where prices are "antidiscounted" or "appreciated" to the end of the period rather than to the beginning of the period:[7]

$$(2) \quad \max_{Y,X,K_B,K_E} \{P_Y \cdot Y - P_X \cdot X + P_{KE} \cdot K_E - (1+r)P_{KB} \cdot K_B : (Y,X,K_B,K_E) \in S\}.$$

The one period profit maximization problem defined by (2) is Diewert's (1977; 108-111) (1980; 472-474) formalization of the Hicks (1961) and Edwards and Bell (1961) accounting framework. There are two specializations of the model that could be considered for national income accounting purposes:

- An *ex post version* that uses the *actual* end of period t price as the price $P_{KE}$ in (2) or
- An *ex ante version* that uses an *anticipated* end of period t price as the price $P_{KE}$ in (2).

Diewert (1980; 476), Hill and Hill (2003) and Schreyer (2009) endorsed the ex ante version for most purposes, since it will tend to be smoother than the ex post version and it will generally lead to user costs that are closer to rental or leasing prices for the assets.

While the above neo-Austrian model of production may be satisfactory for some purposes such as for production function studies, it is not completely satisfactory for national income accounting purposes, since the role of depreciation is only implicit in the above model. Hence, in the following section, we will generalize the Neo-Austrian model of production in order to make the role of *depreciation* (or more accurately, *deterioration*) explicit.

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[7] This is consistent with accounting treatments of assets at the beginning and end of the accounting period and cash flows that occur during the period. "Here $A_{t-1}$ is discounted as a flow dated t−1 and $C_t + A_t$ as a flow at t. This accords with the assumption conventional in discrete compounding that flows occur at the end of each period." K.V. Peasnell (1981; 56). $A_{t-1}$ and $A_t$ are Peasnell's counterparts to our $P_{KB}K_B$ and $P_{KE}K_E$ and $C_t$ is Peasnell's cash flow counterpart to our $P_Y \cdot Y - P_X \cdot X - r\, P_{KB}K_B$. These timing conventions are discussed in more detail in Diewert (2005b; 8) and they are consistent with the use of end of period user costs as discussed in Diewert (2005a).

## 3. A Generalized Austrian Model of Production and Depreciation

A problem with the above Neo-Austrian model of production is that depreciation seems to be missing from the model. More fundamentally, the problem with the model is that the end of the period capital stocks, $K_E$, can be generated in three distinct ways:

- As depreciated beginning of the period capital stocks;
- By purchasing new units of the various capital stocks from external production units and
- By building internally new units of the various capital stocks.

If a new asset is *purchased externally*, then this asset purchase will appear in one of the components of $P_X \cdot X$, the value of flow input purchases by the production unit during the period. If a new asset is *constructed* by the production unit during the period, then this newly constructed asset will show up as a component of the end of the period capital stock vector, $K_E$, and the materials and labour that went into building this new asset will also show up in various components of the input vector X. Finally, if an asset is merely held over the period so that it was present as a component in the beginning of the period vector of capital stocks, $K_B$, then it will also show up as a component in the end of the period vector of capital stocks, $K_E$, unless the asset was sold or retired during the period. Thus there are three distinct ways for "producing" an end of period capital stock component and *their resource requirements can be quite different*. Suppose that no components of the initial capital stock vector $K_B$ are sold during the period. Denote the depreciated end of period vector of capital stocks that correspond to $K_B$ by $K_D$, the end of period *depreciated vector of initially held capital stocks*. Denote the vector of *newly purchased capital equipment components* by $I_1$[8] and the vector of *newly internally produced capital equipment components* by $I_2$. In general, the resource requirements and production outcomes for producing the depreciated vector of capital stocks $K_D$ and the vectors of new additions to capital stocks, $I_1$ and $I_2$, are not equivalent. Thus the production possibilities set is not a set of $(Y,X,K_B,K_E)$; rather it is a set of $(Y,X,K_B,K_D,I_1,I_2)$. Since our focus here will be on describing depreciation, we will simplify the model and combine $I_1$ and $I_2$ into I, a *new investment aggregate of both purchased and internally produced investment goods*. Thus our new production possibilities set is a set of quantity vectors of the form $(Y,X,K_B,K_D,I)$. Let S denote this new feasible set of outputs and inputs for the production unit. Thus our end of period vector of capital stocks, $K_E$, is equal to the sum of the depreciated initial capital stocks held by the production unit, $K_D$, plus *new gross investment* (both purchased and created over the period), I:

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[8] There is a complication associated with purchases of investment goods which is neglected here: if the investment good is purchased during the beginning of the period, it may give a stream of capital services during the period under consideration and at the end of the period, it will be a (partially depreciated) addition to our list of end of period capital stocks whereas if the investment good is purchased at the end of the production period, it will be a brand new addition to end of period capital stocks. This difficulty can be overcome conceptually by distinguishing a finer classification of capital goods according to their degree of depreciation during the period of purchase. Other (more sensible) solutions to this problem have been proposed by Balk (2008) and Schreyer (2009). Typically, applied economists and accountants neglect this problem and simply regard purchases of investment goods as if they yield no capital services during the period of purchase and are new additions to capital stocks at the end of the accounting period. This somewhat inaccurate treatment of new investment purchases will be approximately correct if the accounting period is fairly short.

(3) $K_E \equiv K_D + I$

Now substitute (3) into the objective function for (2), and we obtain the following *Generalized Austrian one period profit maximization problem*:

(4) $\max_{Y,X,K_B,K_D,I} \{P_Y \cdot Y - P_X \cdot X - (1+r)P_{KB} \cdot K_B + P_{KE} \cdot (K_D + I) : (Y,X,K_B,K_D,I) \in S\}$.

It may be somewhat surprising that both the end of period inherited capital stocks, $K_D$, and the newly purchased or newly produced capital goods, I, are priced at the same end of period prices, $P_{KE}$. The reason for this is that we distinguished capital not only by its physical type (e.g., types of machine or types of structure) but also by its state, which includes the age of the asset. Thus newly produced or purchased capital goods will necessarily fall into components of $K_E$ which are different from the components corresponding to $K_D$, since these depreciated assets will be at least one period old and hence cannot correspond to the components of I. Thus in fact, the nonzero components of $K_D$ and I will generally have different prices. Note that the objective function in the profit maximization problem defined by (4) can be rewritten as follows:[9]

(5) $P_Y \cdot Y - P_X \cdot X - (1+r)P_{KB} \cdot K_B + P_{KE} \cdot (K_D + I) = P_Y \cdot Y + P_{KE} \cdot I - P_X \cdot X - (1+r)P_{KB} \cdot K_B + P_{KE} \cdot K_D$
$$= P_Y \cdot Y + P_{KE} \cdot I - P_X \cdot X - GVK$$

where the *gross value of capital services*, GVK, for the production unit is defined as follows:

(6) $GVK \equiv (1+r)P_{KB} \cdot K_B - P_{KE} \cdot K_D$.

The above expression for the gross value of capital services does not look very familiar so it is useful to consider the following special case of the general framework. Suppose that $K_B$ is a scalar so that there is only one capital stock and suppose that depreciation is geometric at the constant depreciation rate $\delta$ where $0 < \delta < 1$. Then the end of the period depreciated capital stock, $K_D$, is equal to $(1 - \delta)$ times the beginning of the period capital stock; i.e., we have:

(7) $K_D = (1 - \delta)K_B$.

Substituting (7) into (6) leads to the following expression for the gross value of capital services:

(8) $GVK \equiv (1+r)P_{KB}K_B - P_{KE}(1 - \delta)K_B = [rP_{KB} + \delta P_{KE} - (P_{KE} - P_{KB})]K_B$

where the expression in square brackets is a familiar discretization of Jorgenson's (1963) continuous time *user cost of capital*, $P_U$:[10]

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[9] Note that the vector of new investment goods either internally produced by the production unit or purchased externally is valued at the prevailing end of the period prices for components of the capital stock, $P_{KE}$. As noted above, if the investment vector I contains units of externally purchased capital goods, then the purchase prices for these capital goods will be embedded in the $P_X$ vector and corresponding quantities purchased will be embedded in the X vector.

[10] We are neglecting tax considerations here. Formula (9) was derived by Christensen and Jorgenson (1969; 302) as an approximation to a continuous time formula. The above simple discrete time derivation of (9) was used by

(9) $P_U \equiv rP_{KB} + \delta P_{KE} - (P_{KE} - P_{KB})$.

Thus it can be seen that definition (6) for the value of capital services is a generalization of the familiar Jorgensonian expression for the gross value of capital services. Note that in the geometric model of depreciation, the technology restricts $K_D$ so that it is no longer an independent variable so that in this case, the profit maximization problem (4) collapses to the following simpler problem, which drops $K_D$ as a decision variable:

(10) $\max_{Y,X,K_B,I} \{ P_Y \cdot Y + P_{KE} \cdot I - P_X \cdot X - P_U K_B : (Y,X,K_B,(1-\delta)K_B,I) \in S \}$.

However, the geometric model of depreciation can only be a rough approximation to the "truth": increased maintenance and renovation activity will generally lead to end of the period capital stocks which are not as run down as they would be had these restoration activities not taken place and similarly, increased utilization of capital equipment will generally lead to end of the period assets which have shorter expected remaining asset lives. Thus the one period intertemporal production model defined by (4) is a more realistic depiction of reality than the traditional one period model defined by (10), which is based on rigid geometric depreciation.

We now need to face up to a problem with the more general model (4). The problem is that the expression for the gross value of capital services does not seem to have depreciation or physical deterioration in it; i.e., price effects seem to be very much intertwined with depreciation effects in our formula (6) for the gross value of capital services in the general case. However, this difficulty can be remedied if we define the *physical depreciation vector*, or perhaps more accurately, the *deterioration vector*, D, as the difference between the vector of beginning of the period stocks, $K_B$, and the vector of depreciated end of period stocks, $K_D$:[11]

(11) $D \equiv K_B - K_D$.

The above definition of deterioration is the main new idea in this paper. Note that it is a purely "physical" definition in that it does not make use of any prices, whereas depreciation is generally understood as a loss in value of the asset (either at a point in time or over time) due to the effects of aging (at normal rates of utilization).[12] Thus the traditional definition of depreciation makes use of prices whereas our model of deterioration is defined in purely physical terms. Note also that our definition of the vector D is somewhat similar to the definition of inventory change in that the components of this vector can be either positive or negative.[13]

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Diewert (1974; 504), (1980; 472-473), (1992; 194) and by Hulten (1996; 155). For further discussions of user costs and the associated accounting problems, see Jorgenson (1989), Hulten (1990), Balk (2008) and Schreyer (2009).

[11] Note that the definition of D applies only to assets which were held by the production unit through the entire period so that we are assuming that no beginning of the period assets held by the firm were sold during the period. The case where beginning of the period assets are sold during the period under consideration can be dealt with at the cost of additional algebraic complexity; Diewert and Smith (1994) deal with these complexities for the case of inventory stocks.

[12] See Triplett (1996), Jorgenson (1996) and Schreyer (2009).

[13] However, inventory assets will have associated deterioration components in the vector D equal to 0 (unless there is spoilage or theft).

It may be useful to give a specific example of the physical depreciation vector. Suppose that there is an asset that is subject to one hoss shay depreciation; i.e., it gives a steady flow of capital services for say 3 periods and then it becomes worthless. Suppose that a production unit purchases one new unit of the asset at the beginning of the period and that no other capital stock components are necessary for its operations. Then the dimensionality of the beginning and end of period capital stock vectors is 3; i.e., the production unit could use various combinations of brand new assets, one period old assets or two period old assets, so that $N = 3$. In our example, the beginning of the period vector of capital stock inputs is $K_B = [1,0,0]$ and the corresponding end of period vector of depreciated or deteriorated capital stock outputs is $K_D = [0,1,0]$. Thus D defined by (11) in this case is $D = K_B - K_D = [1,-1,0]$.

Now use (11) to solve for $K_D$ which turns out to be equal to $K_B - D$ and replace $K_D$ in (4) by this expression. Making this substitution, (4) becomes the following equivalent one period intertemporal profit maximization problem:

(12) $\max_{Y,X,K_B,D,I} \{P_Y \cdot Y - P_X \cdot X - (1+r)P_{KB} \cdot K_B + P_{KE} \cdot (K_B - D + I) : (Y,X,K_B,K_B-D,I) \in S\}$.

The objective function in the profit maximization problem defined by (12)[14] can be rewritten as follows:[15]

(13) $P_Y \cdot Y - P_X \cdot X - (1+r)P_{KB} \cdot K_B + P_{KE} \cdot (K_B - D + I)$
$\quad = P_Y \cdot Y + P_{KE} \cdot I - P_X \cdot X - (1+r)P_{KB} \cdot K_B + P_{KE} \cdot (K_B - D)$
$\quad = P_Y \cdot Y + P_{KE} \cdot I - P_X \cdot X - [rP_{KB} - (P_{KE} - P_{KB})] \cdot K_B - P_{KE} \cdot D$
$\quad = P_Y \cdot Y + P_{KE} \cdot I - P_X \cdot X - NVK - VD$

where the *net value of capital services*, NVK, and the *value of physical depreciation* or deterioration over the period (at end of period prices), VD, for the production unit are defined as follows:

(14) $NVK \equiv [rP_{KB} - (P_{KE} - P_{KB})] \cdot K_B$ [16];
(15) $\quad VD \equiv P_{KE} \cdot D$.

For the example involving one hoss shay depreciation introduced below definition (11), assuming that the end of period price of a new asset, $P_{KE1}$, is greater than the end of period price of a one period old asset, $P_{KE2}$, we find that the value of physical depreciation in this example using (15) is $VD = P_{KE} \cdot D = [P_{KE1},P_{KE2},P_{KE3}] \cdot [1,-1,0] = P_{KE1} - P_{KE2} > 0$.

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[14] Similar decompositions of pure profits equal to the right hand side of (13) can be found in Diewert (2006) and Balk (2008) (2010).
[15] Note that the vector of new investment goods I which is either internally produced by the production unit or purchased externally is valued at the prevailing end of the period prices for components of the capital stock, $P_{KE}$.
[16] If there are N types of capital, the end of period anticipated price of the nth type of capital, $P_{KEn}$, can be set equal to $(1+i_n)P_{KBn}$, where $P_{KBn}$ is the beginning of the period price of the nth type of capital and $i_n$ is the anticipated asset inflation rate for asset n. Using this notation, (14) becomes $NVK = \sum_{N=1}^{N} [r - i_n]P_{KBn}K_{Bn}$ where $r - i_n$ can be interpreted as an asset specific real interest rate for asset n.

Definition (15) seems to be a satisfactory definition of the value of physical depreciation in the Generalized Austrian model of production (even though it will be very difficult to implement this definition in practice). Note that the net value of capital services, NVK, is equal to the gross value of capital services, GVK, less the value of depreciation, VD.

At this stage, it is convenient to shift from the production possibilities set S that was defined at the beginning of this section, which it will be recalled, involved feasible combinations of outputs Y, inputs X, beginning of the period capital stocks $K_B$, the depreciated end of period counterparts $K_D$ to the beginning of the period capital stocks and I, the vector of externally purchased and internally constructed new assets. We will now use S and definition (11) in order to define a new production possibilities set $S^*$ that involves combinations of Y, X, $K_B$, I and D so that D replaces $K_E$ as a decision variable:

(16) $(Y,X,K_B,D,I) \in S^*$ if and only if $(Y,X,K_B,K_B-D,I) \in S$.

Using (13) and (16), our Generalized Austrian production profit maximization problem can be written as follows:

(17) $\max_{Y,X,K_B,D,I} \{ P_Y \cdot Y + P_{KE} \cdot I - P_X \cdot X - [rP_{KB} - (P_{KE} - P_{KB})] \cdot K_B - P_{KE} \cdot D : (Y,X,K_B,D,I) \in S^* \}$

$= \max_{Y,X,K_B,D,I} \{ P_Y \cdot Y + P_{KE} \cdot I - P_{KE} \cdot D - P_X \cdot X - [rP_{KB} - (P_{KE} - P_{KB})] \cdot K_B : (Y,X,K_B,D,I) \in S^* \}$.

In the first line of (17), we interpret the value of depreciation, $P_{KE} \cdot D$, as part of primary inputs whereas in the second line of (17), we interpret it as an offset to the value of gross investment, $P_{KE} \cdot I$. Either interpretation appears to be consistent with the Generalized Austrian production theoretic framework.

Note that the difference between the earlier profit maximization problem (4) and the new one (17) is this: in (4), the vector of depreciated end of period capital stocks $K_D$ (that corresponded to the beginning of the period vector of capital stocks held by the production unit $K_B$) appeared as a decision variable whereas in (17), $K_D$ has been replaced by D as a decision variable. Is this replacement justified? The intuitive justification is this: the end of period stocks are affected by how intensively machines are used during the period; more intensive utilization rates will lead to depreciated equipment that fall into different equivalence classes of used equipment, which are usually less valuable the more intensively the equipment is used. Similarly structures and engineering structures will fall into different equivalence classes of structures depending on maintenance policies. Thus the replacement of $K_D$ by D can be justified on intuitive grounds as well as on algebraic grounds.

It seems worthwhile at this point to pause and note that the vector of depreciation variables D is not a completely independent vector of "inputs" like labour or capital in a traditional one output, two input production function model, $Y = f(L,K)$ where L and K are only restricted to be nonnegative scalars and Y is output with f being the production function. In our present generalized Austrian model, D is indeed a vector of decision variables but the feasible choices of D are restricted by the technology defined by the production possibility set $S^*$ *and* the choices made by the production manager for the other inputs and outputs in the one period production

plan, $Y,X,K_B$ and I.[17]  To see that depreciation is a decision variable, note that depreciation of structures can be retarded by proper maintenance so that even in the case of geometric depreciation, there will be an interval where depreciation is a variable.  For machines, the interval will generally be greater. If one simply mothballs a new machine, its characteristics will be  totally unchanged so a lower bound to depreciation in this case is 0.  The  upper bound is equal to complete destruction (due to improper operations or maintenance) or wearing out of the machine by intensive use during the production period.  In either case, the depreciation variable will only be a  bounded interval (in the case of geometric depreciation) which is determined by the environment and technology whereas in traditional production theory, an input x is only bounded from below by 0 and is unbounded from above.  But even in this  traditional case, the upper bound is not really plus infinity; it is some  possibly large number, which is determined by congestion costs.  But in traditional production theory, we generally do not take the upper bound on the use of an input into explicit account because we know a rational production manager will never hire so much x that he or she is approaching the upper bound.  The situation is more complex with respect to outputs.  Consider the case of an oil refinery, which uses crude oil, labor and capital as inputs.  It can produce a variety of refined products, say 5 of them but there will only be a  relatively narrow range of variability with respect to the  product mix, which is determined by the technology and the amount of crude oil input  that is used during the production period.  In fact, there may be rigidities built into some refineries; i.e., for a given amount of crude oil, the outputs of the 5 types of refined products vary in exact proportion to the amount of crude input, given the beginning of the period capital stock and the minimal amount of labour required to run the refinery. Thus the technology restricts the set of feasible output vectors in multi-output, multi-input production functions.  This is  exactly analogous to our treatment of depreciation as an input vector; the vector of  depreciation "inputs" is not an arbitrary nonnegative vector; the D vector is  constrained by the initial vector of beginning of the period stocks and the maintenance and utilization technology of the production unit. In some cases, with "normal" maintenance policies in place, the D vector will be exactly determined by the $K_B$ vector.  Thus in this case, the technology limits the set of feasible D vectors just as the refining technology can restrict the output vector in some cases.  However, in both cases, we treat both the output vector Y and the input vector D as a vector of  decision variables.

To show that our "new" accounting scheme is consistent with traditional accounting practices, we define the *net investment vector* $I_N$ as the difference between the vector of end of period and beginning of period capital stocks, $K_E$ and $K_B$ respectively, and show that net investment is equal to the familiar gross investment vector I less the depreciation vector D:

(18) $I_N \equiv K_E - K_B$
$= (K_D + I) - (D + K_D)$ using (3) and (8)
$= I - D.$

Roughly speaking, Diewert and Morrison (1986) used the first line of (17) in their two stage maximization procedure.  In the first stage, they maximized the value added generated by the

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[17] Similarly, the vector of end of period depreciated capital stocks $K_D$ that correspond to the starting vector of capital stocks $K_B$ are also decision variables but the feasible choices for $K_D$ are again restricted by the technology defined by the production possibility set S *and* the choices made by the production manager for the other inputs and outputs in the one period production plan, $Y,X,K_B$ and I.

private production sector of an economy subject to the constraints of technology; i.e., their first stage maximization problem was the following one, which defines the economy's *market sector gross domestic product function*, π:

(19) $\pi(P_Y,P_{KE};X,K_B,D) \equiv \max_{Y,I} \{P_Y \cdot Y + P_{KE} \cdot I : (Y,X,K_B,D,I) \in S^*\}$.

The second stage DM maximization problem is the following one which involves the gross value of capital services:

(20) $\max_{X,K_B,D} \{\pi(P_Y,P_{KE};X,K_B,D) - P_X \cdot X - [rP_{KB} - (P_{KE} - P_{KB})] \cdot K_B - P_{KE} \cdot D\}$.

On the other hand, Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006) maximized the net value added generated by the private production sector of an economy subject to the constraints of technology; i.e., their first stage maximization problem was the following one, which defines the *economy's market sector net domestic product function*, $\pi^*$:

(21) $\pi^*(P_Y,P_{KE},P_{KE};X,K_B) \equiv \max_{Y,I,D} \{P_Y \cdot Y + P_{KE} \cdot I - P_{KE} \cdot D : (Y,X,K_B,D,I) \in S^*\}$.

The second stage DMN maximization problem is the following one which involves the net value of capital services:

(22) $\max_{X,K_B} \{\pi^*(P_Y,P_{KE},P_{KE};X,K_B) - P_X \cdot X - [rP_{KB} - (P_{KE} - P_{KB})] \cdot K_B\}$.

Thus it appears that both the gross capital services model of production and the net capital services model have an equally valid justification from the viewpoint of the Generalized Austrian model of production. The choice of which model to use depends on the particular problem at hand.

## 4. Conclusion

The question that this note attempts to answer is: can depreciation be taken out of the usual user cost of capital and be treated as a separate decision variable? Using a generalization of the Hicks (1961) and Bell and Edwards (1961) Neo-Austrian model of production, the answer appears to be yes. Thus there does not appear to be any logical difficulty in treating depreciation as a offset to gross investment in a model of production as has been done recently by Diewert, Mizobuchi and Nomura (2005), Diewert and Lawrence (2006), Lawrence, Diewert and Fox (2006) and Balk (2008) (2010).

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