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STRATEGIC INVESTMENT AND PRICING DECISIONS IN A CONGESTED TRANSPORT CORRIDOR

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Abstract

This paper studies pricing and investment decisions on a congested transport corridor where the elements of the corridor are controlled by different governments. A corridor can be an interstate highway or railway line, or an inter-modal connection. We model the simplest corridor: two transport links in series, where each of the links is controlled by a different government. Each link is used by transit as well as by local traffic; both links are subject to congestion. We consider a two stage non-cooperative game where both governments strategically set capacity in the first stage and play a pricing game in the second stage. Three pricing regimes are distinguished: (i) differentiated tolls between local and transit transport, (ii) one uniform toll on local and transit traffic, and (iii) only the local users can be tolled. Numerical analysis illustrates all theoretical insights. A number of interesting results are obtained. First, transit tolls on the network will be inefficiently high. If only local traffic can be tolled, however, the Nash equilibrium tolls are inefficiently low. Second, raising the toll on transit through a given country by one euro raises the toll on the whole trajectory by less than one euro. Third, higher capacity investment in a given region not only reduces optimal tolls in this region under all pricing regimes but it also increases the transit tolls on the other link of the corridor. Fourth, capacities in the different regions are strategic complements: when one country on the corridor increases transport capacity, it forces the other country to do the same. Fifth, we find interesting interactions between optimal capacities and the set of pricing instruments used: capacity with differentiated tolls is substantially higher than in the case of uniform tolls but overall welfare is lower. Finally, if transit is sufficiently important, it may be welfare improving not to allow any tolling at all, or to only allow the tolling of locals.

Keywords: congestion pricing, transport investment, transit traffic
JEL: H23, H71, R41, R48

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0. Introduction

The purpose of this paper is to study pricing and investment decisions on a congested transport corridor of which each of the links are under the jurisdiction of a different government. Fiscal and expenditure externalities give rise to strategic pricing and investment behaviour by the various governments involved. Potential applicability of the analysis includes investment and pricing on Trans European Networks ("TEN's" basically a border-crossing highway, rail or multimodal system) in Europe and the interstate highway system in the US. Moreover, it is equally relevant for pricing and investment decisions for inter-modal trips where the transfer facility (ports, airports, freight terminal) and the upstream or downstream infrastructure is controlled by different governments or by different private monopolists. The paper yields new theoretical insights, and it illustrates the results using numerical simulation analysis.

Interstate highways in the US and the TEN's in Europe have raised many policy questions. The two most prominent ones are on tolling and on investment. Allowing tolling by different governments will help to control congestion and generates resources for investments, but there is a fear of too high taxes on transit. When it comes to investment, the general idea is that, without federal help, investments in corridors that are used intensively by transit would be too low. Obviously both questions are linked: allowing tolling may help to overcome insufficient investment, but the net efficiency gain is not clear.

The approach we take focuses on models of interregional competition and considers various tolling or user charge possibilities for the governments involved. Specifically, the model set up contains two serial links where each of the links is controlled by one government[1]. The two links together form a corridor for transit traffic but each of the links is also used by local traffic. Both links are subject to congestion. We consider a two stage game where both governments set capacity in the first stage and play a Nash pricing game in the second stage. We follow De Borger,

[1] Government stands here for a public body that represents faithfully the interests of the local voters. This can be a country government, a state government in a federation, or the officials of a smaller constituency like a city controlling a port.

Proost, Van Dender (2005) in explicitly distinguishing 3 pricing regimes: (i) tolls can be differentiated between transit and local users; (ii) only one uniform toll can be charged to local and transit traffic; and (iii) only local users can be charged.

This paper builds in a natural way on several strands of literature. First, a number of papers have considered pricing decisions for congested facilities, assuming a simple parallel network setting and excluding tax competition. In an early contribution, Braid (1986) studied Cournot and Bertrand pricing rules for congested facilities in a symmetric private duopoly setting. This work was extended in various directions. For example, Verhoef, Nijkamp, Rietveld (1996) considered competition between a private road and a free-access road, and compared the second-best optimal tolls with those obtained when both roads are privately owned. De Palma and Lindsey (2000) use a bottleneck model of congestion and compare three types of ownership structure: a private road competing with a free access road, two competing private roads, and competition between a private and a public operator. More recently, Van Dender (2005) highlighted the important distinction between facility-specific traffic (e.g., traffic to access a port or airport) and other traffic on the network (e.g., local traffic not using port or airport facilities). All these papers implicitly consider a parallel network structure, they do not deal with tax competition, and they ignore capacity competition.[2] Second, De Borger et al. (2005) studied tax competition for transit transport in a simple network setting, assuming welfare-maximizing governments. However, unlike the current paper they focus on parallel networks and ignore the possibility of capacity investment as a strategic variable. Third, recent work looks specifically at tax exporting in the transport sector within a serial network setting. For example, Levinson (2001) analyses US States' choice of instruments for financing transportation infrastructure. He shows that jurisdictions are more likely to opt for toll-financing instead of fuel taxes, for example, when the share of non-residential users is large. His model does not include capacity decisions, however. Both De Palma and Leruth (1989) and De Borger and Van Dender (2005) do study two stage games in capacities and prices for parallel congested facilities. However, they do not look at issues of tax and capacity competition on a serial transport corridor. Moreover, they do not consider the range of pricing instruments nor the

[2] Acemoglu and Ozdazgar (2005) recently provide a detailed theoretical analysis of competition and efficiency on parallel network markets. They show that more competition among oligopolists can reduce efficiency on congested markets. Moreover, pure strategy equilibria may not exist, especially when congestion functions are highly nonlinear. However, they do not consider capacity competition.

interaction between capacity choice and pricing regimes studied in the current paper. Finally, our model setting can be compared to the problem of airline alliances studied by Brueckner (2001). In an airline alliance, airlines cooperate in the international interhub (cross atlantic) markets, but the hubs are feeded and connected by local lines that are operated by each of the carriers. The alliance reduces competition on the interhub market but avoids the double margins on the feeding or connecting local lines. Compared to Brueckner's paper, we concentrate on the serial network (forgetting the parallel interhub part) and we have congested infrastructure so that capacity decisions matter. Finally, we have local governments as decision makers rather than profit maximising airline operators.

A number of interesting results are obtained. First, if transit can be tolled we find that all tolls are inefficiently high. However, if only local traffic can be tolled, Nash equilibrium tolls are inefficiently low: tolls are shown to be smaller than the marginal external congestion cost imposed on local traffic. The reason is that higher local tolls would attract too much transit traffic and hence reduce welfare. Second, the pricing behavior for transit transport boils down to a variant of the double marginalization problem for successive monopolies in the industrial organization literature (see, e.g., Tirole (1993)). It is shown that reaction functions in transit tolls are negatively sloped, so that increasing the transit toll in one region by one euro raises the total toll on transit users for the whole trajectory by less than a euro. Third, at the capacity stage of the game, we show that capacity reaction functions are plausibly upward sloping: capacities are strategic complements. Fourth, we find that capacity changes strongly affect optimal tolling behavior. Higher capacity investment in a region not only lowers optimal tolls in this region under all pricing regimes, but it also increases tolls on transit in the other region. Moreover, there are interesting interactions between optimal capacities and the pricing instruments used: optimal capacity with differentiated tolls is higher than in the case of uniform tolls but welfare is lowest; the largest optimal capacity results when only local tolls are used. Fifth , if transit is sufficiently important, it may be welfare improving not to allow any tolling at all, or to only allow the tolling of locals. Sixth, it is well known that, in a tax competition setting (see Kanbur and Keen (1993)), the smaller country has an interest to go after the revenue objective. We find similar behaviour in the case of uniform tolls.

The paper concentrates on cases where there is always some local and some transit traffic, but some extreme cases are interesting too. First, if there is no transit, there is no strategic interaction and the first best solution can be achieved if all traffic can be tolled. Second, when transit is tolled but local demand is negligible, the two-stage game reduces to a pure standard duopoly problem in which the optimal tolls on transit are both independent of the level of capacity and of the slope of the congestion function.

The structure of the paper is as follows. In a first section we describe the setup of the model. In Section 2 we look in detail at the pricing stage of the game. We study, for different tolling regimes, the countries' optimal choice of transport tolls, conditional on given capacities and the tolls imposed on the other network link. We explicitly analyze the characteristics of the toll reaction functions and the resulting Nash equilibrium for the simplified case of linear demand and cost functions. Section 3 deals with the first stage of the game, where regions decide on capacity, given the pricing behaviour at the second stage. In Section 4, we present some numerical results of the tax-capacity game to illustrate the main theoretical insights. We identify the welfare losses due to the lack of coordination between governments and we analyze the importance of three parameters: the share of transit, the slope of the congestion function and the relative size of the two countries. Finally, Section 5 concludes with some generalisations and caveats.

1. Model structure

The simple setting we consider consists of two serial links; it is assumed that pricing of each link is the responsibility of a different government. We assume each link carries local traffic and transit traffic. Local traffic uses only the local link. Transit traffic, by definition, passes through the two links. Link capacities can be augmented through investments; however, once capacity is chosen, both links are potentially congestible. The distinction between the parallel network, analyzed in De Borger et al. (2005), and the serial setting considered in the current paper, is illustrated on Figure 1.

Both governments are assumed to maximise a local welfare function that reflects two concerns, viz. (i) the travel conditions of its local users and the associated welfare, and (ii) total tax revenues on the link it controls. Transit traffic is supposed to have its origin and destination outside the two-link network, so that the two governments are not interested in the transport costs and the welfare of transit traffic[3]. Finally, we assume that all traffic flows are uniformly distributed over time and are equal in both directions, allowing us to focus on one representative unit period and one direction.

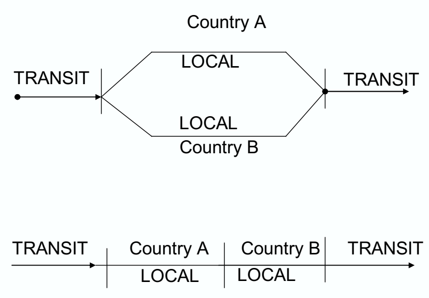

Figure 1: Parallel versus serial competition

[3] We could, for example, add local traffic originating in one of the countries that contributes to transit through the other country. This would imply a third category of traffic that reacts to the sum of the local toll and the transit toll abroad. This complicates matters but does not yield additional insights.

Turning to the specification of the model, demand for local transport in regions A and B is represented by the strictly downward sloping and twice differentiable inverse demand functions $P_A^Y(Y_A)$ and $P_B^Y(Y_B)$, respectively, where Y_A and Y_B are the local flows on both links. As is common in the transport literature, prices $P_i^j(.)$ are generalised prices including resource costs, time costs and tax payments or user charges. Similarly, overall demand for transit traffic is described by the strictly downward sloping inverse demand function $P^X(X)$, where X is the transit traffic flow that passes through both regions A and B.

Turning to the cost side, the generalised user cost for transit, denoted as g^X, equals the sum of the time and resource costs of travel plus the transit tolls in both A and B:

$$g^X = C_A(V_A R_A) + \tau_A + C_B(V_B R_B) + \tau_B$$
$$\text{with } V_i = X_i + Y_i$$

In this expression, the $C_i(.)$ are the time plus resource costs on link i, and R_i is the inverse of capacity[4]. The user cost function is twice differentiable and strictly increasing in $V_i R_i$, the total traffic volume relative to capacity. Making time costs a function of volume-capacity ratio is a common practice in transport economics[5]. The transit tolls are denoted τ_i. Similarly, the generalised user cost functions for local use of links A and B are given by, respectively:

$$g_A^Y = C_A(V_A R_A) + t_A .$$

$$g_B^Y = C_B(V_B R_B) + t_B .$$

The t_i are the tolls on local transport.

Transport equilibrium for transit and local traffic implies

$$P^X(X) = g^X = C_A(V_A R_A) + \tau_A + C_B(V_B R_B) + \tau_B \qquad (1)$$

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[4] A trick we borrowed from de Palma and Leruth (1989).
[5] See Small (1992) for a discussion of the congestion functions for different modes. In the industrial organisation literature an ┘ shaped congestion function is used by Kreps and Scheinkman (1983) to show that a 2 stage capacity and price game gives the same results as a one stage Cournot game in quantities. Our model does not fit into this category because of the different shape of the congestion function and because of the difference in objective functions of the agents.

$$P_A^Y(Y_A) = g_A^Y = C_A(V_A, R_A) + t_A \qquad (2)$$

$$P_B^Y(Y_B) = g_B^Y = C_B(V_B, R_B) + t_B \qquad (3)$$

2. Strategic transport pricing in a serial corridor

In this section, we study the second stage of the tax-capacity game and focus on strategic pricing behaviour of the two governments, conditional on capacity levels. Here the methodology closely follows De Borger et al. (2005). Three different assumptions are made on the tolling instruments available: we consider the case where governments have access to differentiated tolls on local and transit traffic, we look at uniform tolls and, finally, we study the case where only local traffic can be tolled. In each case we first discuss the reduced-form demand system that expresses all demand functions as functions of the policy variables only. Next we derive the optimal tax rules for a given region. As the reaction functions and the resulting Nash equilibrium in taxes are rather cumbersome in general, we finally study strategic behaviour using linear demand and user cost functions. Throughout this section we only report the main insights; technical details are provided in appendices.

2.1 The case of differentiated tolls

2.1.1. The reduced-form demand system

We start from the equilibrium conditions (1), (2) and (3) given above. This system can easily be solved for the three transport volumes demanded as functions of the four tax rates and the two capacity levels:

$$X = X^r(\tau_A, \tau_B, t_A, t_B, R_A, R_B)$$

$$Y_A = Y_A^r(\tau_A, \tau_B, t_A, t_B, R_A, R_B)$$

$$Y_B = Y_B^r(\tau_A, \tau_B, t_A, t_B, R_A, R_B)$$

These reduced-form demand functions are an interesting short-cut because they already incorporate feedback effects of congestion on demand. This is the reason why any tax change of one of the governments affects all the transport flows, including local traffic flows abroad.

Unless otherwise noted we limit our analysis to the domain where all flows are strictly positive[6]. In Appendix 1 we show that the demand function for transit transport has the following properties:

$$\frac{\partial X^r}{\partial \tau_A} < 0,\ \frac{\partial X^r}{\partial \tau_B} < 0,\ \frac{\partial X^r}{\partial t_A} > 0,\ \frac{\partial X^r}{\partial t_B} > 0$$
$$\frac{\partial X^r}{\partial R_A} > 0,\ \frac{\partial X^r}{\partial R_B} > 0 \tag{4}$$

Expression (4) implies that higher transit taxes in an arbitrary region reduce overall transit demand and, as higher local taxes in any given region reduce congestion, they raise transit demand. Higher investment in capacity in either country raises transit demand.

Similarly, the reduced-form demand for local transport in region A has the following characteristics (again, see Appendix 1):

$$\frac{\partial Y_A^r}{\partial \tau_A} > 0,\ \frac{\partial Y_A^r}{\partial \tau_B} > 0,\ \frac{\partial Y_A^r}{\partial t_A} < 0,\ \frac{\partial Y_A^r}{\partial t_B} < 0$$
$$\frac{\partial Y_A^r}{\partial R_A} > 0,\ \frac{\partial Y_A^r}{\partial R_B} < 0 \tag{5}$$

This shows that transit taxes reduce transit demand and hence congestion, raising local demands, whereas local taxes reduce local demand. Moreover, raising capacity abroad in B attracts more transit, increases congestion and, as a consequence, decreases local traffic in A. Finally, a local capacity increase in A raises local traffic demand in this region.

2.1.2. Optimal toll rules

We focus on region A. Consider the problem of determining the tolls on local and transit traffic that maximizes local welfare, conditional on the existing capacities in both regions and taking tax levels in B as given. Region A solves:

$$\underset{t_A,\tau_A}{Max} \quad W_A = \int_0^{Y_A} (P_A^Y(y))dy - g_A^Y Y_A + t_A Y_A + \tau_A X - K_A \frac{1}{R_A}. \tag{6}$$

[6] This assumption is necessary to guarantee differentiable demand functions.

where K_A is the constant unit rental cost of capacity[7], and the demand functions are the reduced-form demands just described. We show in Appendix 1 that the first-order conditions with respect to t_A and τ_A imply the following tax rules:

$$t_A = (Y_A + X)C'_A R_A \quad LMEC_A \quad XC'_A R_A \tag{7}$$

$$\tau_A = LMEC_A \quad X \left[\frac{\frac{\partial Y_A^r}{\partial t_A}}{\frac{\partial z_A}{\partial t_A} \frac{X^r}{\tau_A}} \right] \tag{8}$$

where $LMEC_A = Y_A C'_A R_A$ is the local marginal external cost and $z_A(X, R_A, t_A)$ stands for the (non reduced) demand function for local traffic. The local marginal external cost is the extra congestion cost imposed on local road users by one extra car on the link. These tax rules have the same structure as in the parallel network case (De Borger et al. (2005)) although, see below, they imply very different strategic tolling behaviour. The reduced-form demand derivatives given in (7)-(8) imply that both taxes exceed local marginal external cost. Moreover, in Appendix 1 we show that the transit tax exceeds the local tax. These results imply tax exporting (taxing transit at a higher rate than local demand) and tax competition (taxing a common tax base without any regard to the effects on the other region's revenue). The latter yields tolls on local traffic above marginal cost to reduce congestion and attract more transit.

2.1.3. Tax reaction functions for linear demand and cost functions

Note that (7)-(8) implicitly describe region A's reaction functions: they give optimal taxes for given tax rates in B, at given capacity levels in both regions. To study some of the properties of these reaction functions and to get insight into regions' strategic behaviour, we simplify the analysis by assuming linear demand and cost functions. Let transit and local demands be given by, respectively:

$$P^X(X) = a \quad bX$$
$$P_A^Y(Y_A) = c_A \quad d_A Y_A$$
$$P_B^Y(Y_B) = c_B \quad d_B Y_B$$
$$with\ a, b, c_A, d_A, c_B, d_B > 0$$

[7] We assume constant returns to scale in capacity, an assumption more justified for road than for rail. We return to this assumption in our concluding section.

Cost functions for transport time (and resources) are specified as:

$$C_A(X+Y_A) = \alpha_A + \beta^*_A (X+Y_A)$$
$$C_B(X+Y_B) = \alpha_B + \beta^*_B (X+Y_B)$$
$$where\ \beta^*_A = \beta_A R_A,\ \beta^*_B = \beta_B R_B$$
$$and\ \alpha, \beta > 0$$

Note that demands and costs are linear in generalized prices and transport volumes, respectively. The cost function assumes that congestion is determined by the ratio of the traffic flow relative to capacity (remember that the R_k $(k = A, B)$ are inverse capacities). The formulation in terms of the β* is convenient because at the pricing stage of the game we hold capacities constant.

In Appendix 1 we show that the toll reaction functions for region A that follows from these specifications can be written as:

$$\tau_A = c_A^\tau - (\frac{1}{2})\tau_B - (\frac{1}{2} z_1^B) t_B$$

$$t_A = c_A^t + (\frac{1}{2} L^A)\tau_B + (\frac{1}{2} z_1^B L^A) t_B$$

where the parameters c_A^τ, c_A^t, z_1^B and L^A are all rather complex functions of demand and cost parameters. Note that z_1^B (which is negative, see Appendix 1) gives the effect of an exogenous increase in transit transport in region B on the demand for local transport in that region. Moreover, we have $-1 < L^A < 0$.

Interpretation of the signs of taxes in region B on optimal taxes in A is then clear. We find that an increase in the transit tax in B induces region A to optimally reduce both its transit tax and the tax on local traffic. The higher tax on transit in B reduces transit demand and hence reduces congestion in A. The optimal response in A is therefore to reduce both taxes. Similarly, a higher local tax in B induces region A to optimally raise transit as well as local taxes in A. The higher local tax in B reduces congestion in B, and attracts more transit. This also raises congestion in A. Therefore, country A raises its tax rates on all traffic on its territory**.**

Note that, despite the very different setting, the structure of the reaction functions bears some close resemblance to well-known results in industrial organisation. For example, it implies that an increase in the transit toll in one region is partially compensated by a reduction in the transit toll imposed by the other region. More specifically, a one euro toll increase on transit in B induces region A to reduce its toll by 0.5 euro, so that the overall transit toll for the whole trajectory rises by 0.5 euro only. This phenomenon is reminiscent of the pricing behaviour of successive monopolies, where in the case of linear demands and costs a cost increase by one unit raises the final price by exactly 0.5 euro (see Bresnahan and Reiss (1985), Tirole (1993)).

Together with the equivalent expressions for B, we have four reaction functions that can be solved for the Nash equilibrium in taxes. We denote this solution, which depends on the capacity levels, as $\tau_k^{NE}(R_A, R_B), t_k^{NE}(R_A, R_B), \quad k = A, B$. Unfortunately, despite the simplicity of the model (linear demands and costs), the expression that describes the partial effects of capacities on Nash equilibrium taxes are cumbersome, and even the signs of these derivatives are hard to determine analytically. Intuitively, one expects a capacity increases in A to reduce Nash equilibrium taxes in A, because of lower congestion (although it also implies extra revenue-raising capacity). A capacity increase in B raises congestion in A and is therefore likely to raise taxes in A. We expect, therefore:

$$\frac{\partial \tau_A^{NE}}{\partial R_A} < 0, \frac{\partial t_A^{NE}}{\partial R_A} < 0, \frac{\partial \tau_A^{NE}}{\partial R_B} > 0, \frac{\partial t_A^{NE}}{\partial R_B} > 0$$

Numerical analysis, see Section 4, confirms these signs.

2.2. The case of uniform tolls

The procedure to derive the reduced-form demand system is entirely analogous to the differentiated tolling case; the only difference is that we set the local toll $t_k\,(k = A, B)$ and the transit toll $\tau_k\,(k = A, B)$ equal. We denote the uniform tolls by $\theta_k\,(k = A, B)$. Derivations are summarized in Appendix 2. There we determine the following signs for the partial effects of the uniform taxes and capacity changes on transit demand $X = X^r(\theta_A, \theta_B, R_A, R_B)$:

$$\frac{\partial X^r}{\partial \theta_A} < 0, \; \frac{\partial X^r}{\partial \theta_B} < 0$$

$$\frac{\partial X^r}{\partial R_A} < 0, \; \frac{\partial X^r}{\partial R_B} < 0$$

Both tax rates reduce transit demand. Capacity increases raise demand for transit. Similarly, we have the following partial effects for the local demand function $Y_A = Y_A^r(\theta_A, \theta_B, R_A, R_B)$:

$$\frac{\partial Y_A^r}{\partial \theta_A} < 0, \frac{\partial Y_A^r}{\partial \theta_B} > 0$$

$$\frac{\partial Y_A^r}{\partial R_A} > 0, \frac{\partial Y_A^r}{\partial R_B} < 0$$

A higher tax rate in A reduces local demand in A; an increase in the tax rate abroad reduces transit demand and, hence, raises local demand in A. Capacity increases in A (B) raise (reduce) local demand in A.

The optimal uniform tax can be written as, see Appendix 2.

$$\theta_A = LMEC_A - \frac{X}{\frac{\partial Y_A^r}{\partial \theta_A} + \frac{\partial X^r}{\partial \theta_A}}$$

Noting the signs derived before, it follows that the tax rate exceeds the local marginal external cost. The difference positively depends on the importance of transit.

Finally, in the linear demand and cost case, the reaction function for region A can be written as (see Appendix 2):

$$\theta_A = c_A^\theta + m^A \theta_B$$

where m^A <0. This shows that the reaction function is downward sloping. A higher tax in B reduces transit demand through both regions, reducing congestion in A. This induces this region to reduce its uniform tax.

Nash equilibrium taxes are denoted by $\theta_k^{NE}(R_A, R_B)$, $k = A, B$. Again, the derivatives of these tax expressions with respect to capacities are not easily determined, even for the linear case. However, one expects:

$$\frac{\partial \theta_A^{NE}}{\partial R_A} < 0, \frac{\partial \theta_A^{NE}}{\partial R_B} > 0$$

Higher capacity in A reduces congestion and hence one expects lower taxes. More capacity in B attracts more traffic to A and suggests higher taxes. Numerical analysis confirms this intuition; see Section 4.

2.3. The case of local tolls only

The case of local tolls only is analyzed in detail in Appendix 3. The derivatives of the reduced-form demand functions with respect to the local tolls are identical to those for the differentiated tolling case. Indeed, the only difference is that the transit toll is set to zero. The optimal local toll is shown to be given by:

$$t_A = LMEC_A \left[1 \quad \frac{\frac{\partial X^r}{\partial t_A}}{\frac{\partial Y_A^r}{\partial t_A}} \right]$$

where, importantly, the term between square brackets is between zero and one. This implies that the optimal tax is positive but smaller than the local marginal external cost. Finally, in the linear demand and cost cases, reaction functions are found to be linear and downward sloping: a higher local tax in B reduces local demand but attracts more transit, which passes through both A and B. Hence, congestion in A rises, reducing local traffic demand in A. This reduces the local marginal external cost so that country A reduces its tax rate on local demand. The intuition is that by doing so, country A raises local demand and thus congestion, which is the only way to reduce transit through its territory.

Note that in this case expectations on the partial effects of capacity changes on tolling behaviour are not obvious. More capacity in A attracts more transit, which remains un-tolled. To the extent that the capacity increase yields less congestion one expects the region to set a lower tax; this is confirmed in our numerical illustrations in section 4.

3. Strategic capacity choices: the first stage of the game

In this section we study the capacity competition game, taking into account the implications of capacity choices for pricing behaviour derived in the previous section. Given the complexity of strategic capacity choices and its interaction with pricing at the second stage, the capacity game of this general case does not yield transparent theoretical results. To get some preliminary insights on the nature of capacity interaction between the two links we therefore start out by briefly considering some special cases, viz. capacity competition in the absence of local traffic (subsection 3.1) and the case where tolls are not used at all (subsection 3.2). This second case allows us to exclusively focus on strategic capacity choices. Moreover, it is not uninteresting in its own right, because in some European countries within the EU, congestion tolls are indeed not used at all. The general case where both tolls and capacities are strategically used is considered in subsection 3.3. Throughout we focus on linear demands and costs.

3.1. Capacity competition without local traffic

Consider first the special case of zero local demand. Since transit welfare does not enter the local welfare function there is no congestion externality, and the objective function of each region simply consists in maximizing the transit tax revenues minus capacity costs. It is easily shown that the pricing solution then boils down to the standard private duopoly result (Tirole (1993), Gibbons (1992)). We find that the only Nash equilibrium in tolls: (i) is symmetric, even if the free-flow cost parameters differ; (ii) is independent of capacities, and (iii) is independent of the slope of the congestion function. Moreover, the capacity reaction functions are unambiguously positively sloped.

3.2. Capacity competition when congestion tolls are not used

If tolls are not used at all, the optimal capacity choice problem of the country government reduces to:

$$\underset{R_A}{Max} \quad W_A = \int_0^{Y_A} (P_A^Y(y))dy \quad g_A^Y Y_A \quad K_A \frac{1}{R_A} \tag{9}$$

The first-order condition is given by:

$$Y_A \frac{\partial g_A^Y}{\partial R_A} = \frac{K_A}{R_A^2} \qquad (10)$$

Using the definition of the generalized cost for the linear case, it can be written in implicit form as:

$$\psi(R_A, R_B) = \beta_A V_A Y_A + \tau_A R_A Y_A \left(\frac{\partial Y_A^r}{\partial R_A} + \frac{\partial X^r}{R_A}\right) - \frac{K_A}{R_A^2} = 0 \qquad (11)$$

Expression (11) implicitly describes the reaction function in capacity for region A. In Appendix 4 we show that its slope is highly plausibly positive so that, when tolls are not used at all, capacities in the two regions will be strategic complements The intuition is clear. Suppose region B raises capacity. This attracts more transit through both A and B so that, in order to dampen the negative welfare effect on local demand, country A reacts by also raising capacity. Of course, when tolls can be used as well, the increase in capacity also affects tolling behavior at the second stage, and our conjecture of positively sloped reaction functions may have to be amended.

3.3. Tax-capacity competition: the general case

The general case is complex due to the nonlinearities in the capacity reaction functions and the interaction with the pricing game. As an example, take the case of differentiated tolling. Optimal capacity at the first stage of the game is defined implicitly by the first order condition for a maximum of the local welfare function defined in (6), taking into account the dependency of optimal Nash equilibrium taxes at the second stage of the game on capacity.

The first-order condition of maximizing (6) with respect to inverse capacity in A can, using the equality between generalized price and cost, be written as:

$$Y_A \frac{dg_A^Y}{dR_A} - t_A \frac{dY_A^r}{dR_A} = \tau_A \frac{dX^r}{dR_A} + Y_A \frac{\partial t_A^{NE}}{\partial R_A} + X \frac{\tau_A^{NE}}{R_A} - \frac{K_A}{R_A^2} \qquad (12)$$

Interpretation of (12) is conceptually simple. The right hand side reflects the capacity cost savings realised by a decrease in capacity. The left hand side gives the increase in user cost for local traffic (first term on the left hand side) and the change in tax revenues (other left hand side terms) caused by the decrease in capacity. Note that in

(12) all taxes are evaluated at their Nash equilibrium values, and the total derivatives capture direct capacity effects and indirect effects via tax adjustments:

$$\frac{dg_A^Y}{dR_A} = \beta_A \left[V_A \quad R_A\left(\frac{dY_A^r}{dR_A} \quad \frac{dX^r}{dR_A}\right) \right] \frac{\partial t_A^{NE}}{\partial R_A}$$

$$\frac{dY_A^r}{dR_A} = \frac{\partial Y_A^r}{\partial R_A} + \sum_{k=A,B} \frac{\partial Y_A^r}{\partial t_k} \frac{\partial t_k^{NE}}{\partial R_A} + \sum_{k=A,B} \frac{\partial Y_A^r}{\partial \tau_k} \frac{\partial \tau_k^{NE}}{\partial R_A}$$

$$\frac{dX^r}{dR_A} = \frac{\partial X^r}{\partial R_A} + \sum_{k=A,B} \frac{\partial X^r}{\partial t_k} \frac{\partial t_k^{NE}}{\partial R_A} + \sum_{k=A,B} \frac{\partial X^r}{\partial \tau_k} \frac{\partial \tau_k^{NE}}{\partial R_A}$$

The first-order condition (12) can be simplified by substituting the above total derivatives and using the first-order conditions for optimal taxation in region A. These could be written (see Appendix 1)

$$(t_A - \beta_A \tau_A V_A R_A)\frac{\partial Y_A^r}{\partial t_A} + (\tau_A - \beta_A \tau_A Y_A R_A)\frac{\partial X^r}{\partial t_A} = 0$$

$$(t_A - \beta_A \tau_A V_A R_A)\frac{\partial Y_A^r}{\partial \tau_A} + (\tau_A - \beta_A \tau_A Y_A R_A)\frac{\partial X^r}{\partial \tau_A} + X = 0$$

Using these results in (12), and noting the definition of the local marginal external cost, we find that the optimal capacity rule in region A can be written as:

$$\begin{aligned} &\beta_A \tau_A V_A Y_A - (\tau_A + LMEC_A)\left[\frac{\partial X^r}{\partial R_A} + \frac{\partial X^r}{\partial t_B}\frac{\partial t_B^{NE}}{\partial R_A} + \frac{\partial X^r}{\partial \tau_B}\frac{\partial \tau_B^{NE}}{\partial R_A}\right] \\ &- (t_A - LMEC_A)\left[\frac{\partial Y_A^r}{\partial R_A} + \frac{\partial Y_A^r}{\partial t_B}\frac{\partial t_B^{NE}}{\partial R_A} + \frac{\partial Y_A^r}{\partial \tau_B}\frac{\partial \tau_B^{NE}}{\partial R_A}\right] = \frac{K_A}{R_A^2} \end{aligned} \qquad (13)$$

The left hand side captures all welfare effects of demand changes induced by capacity: the first term is the direct user cost increase of capacity changes at constant demand, the second and third terms represent the net welfare effects via induced demand changes. These terms are the product of the total demand change (direct and indirect via tax adjustments in the other region) and the deviation of taxes and local marginal external costs. Observe that tax adjustments in A do not appear in this expression, because taxes in A have been determined optimally.

A special case is when no transit traffic exists. In that case the optimal local toll equals the local marginal congestion costs, and the left hand side of (13) reduces to the first term only: the direct costs of a reduction of capacity. We then obtain a first best result because there is no strategic interaction between both governments.

Expression (13) implicitly describes the reaction function in capacities for the first stage of the game. Unfortunately, determining the sign of the slope of this reaction function is difficult because demands and Nash equilibrium taxes are all highly nonlinear in capacity. We did not attempt to do so, but will rely on numerical analysis below. The numerical results do confirm that capacity reaction functions are plausibly upward sloping in the general case as well.

For completeness sake, consider the other pricing regimes. In the case of uniform taxes the first-order condition can be rewritten as (see Appendix 5):

$$\beta_A V_A Y_A - (\theta_A + \theta_B - LMEC_A)\left[\frac{\partial Y_A^r}{\partial R_A} + \frac{\partial X_A^r}{\partial R_A} + \left(\frac{\partial Y_A^r}{\partial \theta_B} + \frac{\partial X_A^r}{\partial \theta_B}\right)\frac{\partial \theta_B^{NE}}{\partial R_A}\right] = \frac{K_A}{R_A^2} \qquad (14)$$

Interpretation is as before. The first term represents the direct effect of increasing user costs for local traffic. The second term represents the induced losses of local and transit traffic multiplied by the net tax margin (Tax minus local marginal external costs).

Finally, for local tolls only we can derive (see Appendix 5 for derivation) the following optimal capacity condition:

$$\beta_A V_A Y_A - (t_A - LMEC_A)\left[\frac{\partial Y_A^r}{\partial R_A} + \frac{\partial Y_A^r}{\partial t_B}\frac{\partial t_B^{NE}}{\partial R_A}\right] - LMEC_A\left[\frac{\partial X^r}{\partial R_A} + \frac{\partial X^r}{\partial t_B}\frac{\partial t_B^{NE}}{\partial R_A}\right] = \frac{K_A}{R_A^2} \qquad (15)$$

We again see a first term that represents direct user cost losses of a reduction of capacity. The second term represents the change in welfare of induced local traffic that does not pay its external congestion cost. The third term represents the gain in welfare when under-priced transit traffic decreases as a result of a decrease in capacity.

4. A numerical illustration

This section illustrates the results using numerical simulation analysis. We first explain the calibration of the no toll reference equilibrium for the symmetric case with identical regions (subsection 4.1). Next we discuss the outcomes of the first stage of the game, i.e., the Nash equilibrium tolls for given capacities under the three different pricing regimes (subsection 4.2). Then the results of the complete two-stage pricing and capacity game are considered (subsection 4.3). Moreover, the importance of transit and of the slope of the congestion function for the results is highlighted. Finally, we conclude this section by discussing the role of the relative size of the two countries (subsection 4.4).

4.1 Calibration of the reference case

Assume initially that the two regions are ex ante symmetric. Moreover, the no toll reference case is constructed such that local and transit demand each account for 50% of total traffic in a given region. Local and transit demand each amount to 1300 trips per time unit. Capacity was set at 2000 in each region; which implies the reference value for inverse capacities $R_A = R_B = 0.0005$[8].

The zero toll equilibrium was used as the reference situation to calibrate the parameters of the model. In other words, all parameters (demand function parameters, slope of the congestion function, the capacity cost, etc.) were calibrated so that the parameters reproduced the zero toll Nash equilibrium consistent with the transport volumes and capacities assumed. The set of parameters that resulted from the calibration procedure is reproduced in Appendix 6.

4.2. Optimal pricing at given capacities: the pricing stage of the game

In Table 1 we report results for the pricing game at fixed capacities. We consecutively report 5 different equilibria: the no toll situation to which the parameters were calibrated, three Nash equilibrium outcomes (differentiated tolls,

[8] Note that these capacity levels are chosen so that, in the absence of tolling, they reflect the optimal values for each of the countries, given the cost of capacity. This will become important when we endogenize the choice of capacity.

uniform tolls, local toll only) and the centralized solution. The centralized solution reflects the optimal policies when the two-link serial transport corridor would be operated by one welfare maximizing government.

Results indicate the following. First, the equilibrium with tax differentiation yields very high taxes on transit; this follows from the fact that transit yields no benefits except tax revenues to the individual regions. The consequence is a drastic reduction in transit transport. Welfare of the individual countries rises substantially: despite the toll on local transport the generalized cost of local transport only increases slightly; this is due to the lower time cost associated with much lower transit demand. Of course, the welfare increase in the two countries is due to the fact that the welfare of transit traffic is not incorporated into the countries' individual welfare functions. Incorporating the reduction in welfare for transit, which is almost wiped out, implies that total welfare declines compared to no tolls at all. Second, in the case of uniform taxes we observe taxes substantially exceeding local marginal external costs, yielding a reduction in countries' individual welfare compared to the tax differentiation case. Third, if only local tolls are optimized we see, consistent with the prediction from the theoretical sections, taxes that are (slightly) below local marginal external cost. Interestingly, in this case not only is the countries' individual welfare higher than in the reference case, but even accounting for transit welfare this solution improves overall welfare. Transit is obviously better off because it is not tolled and local transport is, reducing congestion. Finally, the centralized optimal solution yields a uniform toll of about 30% the level at the uniform toll Nash equilibrium; it leads to higher regional welfare as well as overall welfare.

Of course, the results are highly sensitive to the importance of transit in the initial equilibrium. Calibrating the model for a 10% share of transit in the reference situation, we find that all symmetric Nash equilibria, except the differentiated tolling case, improve overall welfare. Moreover, uniform tolls are much lower than before.

Variable	Unit	No tolls	Nash Equilibrium differentiation	Nash Equilibrium uniform	Nash Equilibrium local tolls only	Centralised - differentiation
Local demand	**Trips**	1300	1249	758	1221	1156
Transit demand	**Trips**	1300	407	758	1305	1156
Trip volume, country level	**Trips**	2600	1657	1516	2526	2312
Generalised price, local	**Euro/Trip**	65.4	74.0	156.3	78.7	89.6
Generalised price, transit	**Euro/Trip**	130.9	430.4	312.7	129.1	179.3
Local Toll	**Euro/Trip**	**0.0**	**19.8**	**103.9**	**14.1**	**27.7**
Transit Toll	**Euro/Trip**	**0.0**	**161.1**	**103.9**	**0.0**	**27.7**
Local MEC	**Euro/Trip**	15.6	14.9	9.1	14.6	13.8
Global MEC	**Euro/Trip**	31.1	19.8	18.1	30.2	27.7
Local CS	**1000 Euro**	142	131	48	125	112
Tax revenue, country level	**1000 Euro**	0	90	158	17	64
Welfare, country level	**1000 Euro**	142	221	206	142	176
Transit welfare (CS)	**1000 Euro**	284	28	96	286	224
Overall welfare	**1000 Euro**	567	470	508	571	576
Change compared to Non toll	**%**	0	-17.07	-10.45	0.59	1.57

Table 1: Results of symmetric pricing game for fixed capacity of 2000 calibrated with 50% transit traffic in no toll equilibrium (welfare figures do not include capacity costs; they are constant for all scenarios)

4.3. Strategic capacity choices and pricing

In this subsection we turn to the complete capacity-pricing game. First we illustrate the impact of capacity changes on optimal tolling behaviour; next we discuss the results of the full solution to the two-stage game.

4.3.1. Exogenous capacity adjustments and optimal tolling behaviour

Note that in the theoretical sections it proved difficult to produce clear-cut analytical results for the signs of the partial derivatives of taxes with respect to capacities:

$$\frac{\partial \tau_k^{NE}}{\partial R_A}, \frac{t_k^{NE}}{R_A} \quad (k = A, B), \quad \frac{\partial \theta_k^{NE}}{\partial R_A} \quad (k = A, B), \quad \frac{\partial t_k^{NE}}{\partial R_A} \quad (k = A, B)$$

We therefore first illustrate the effect of increasing capacity on Nash equilibrium taxes. Given fixed inverse capacities, R_A and R_B, and assuming linear demand and cost curves, we can solve the pricing game analytically to determine the variables $X, Y_A, Y_B, t_A, t_B, \tau_A$ and τ_B as functions of the inverse capacities. This has been done for the three tax regimes: differentiated tolls, uniform tolls and local tolls only. We can then evaluate these expressions for various values of capacity in a given region, say A, holding all other variables and parameters constant. In all other respects countries A and B are treated as identical.

We summarize the results in Figure 2, where we concentrate on two cases: uniform tolls and local tolls only. Consider the uniform tolling case. The figure shows the effect of capacity changes in country A on the optimal uniform taxes of both regions. In the initial situation where the capacities in A and B are identical we have a Nash equilibrium with uniform tolls equal to 104 in both regions. Halving capacity in A increases the uniform toll in A but forces B to reduce its toll slightly. The elasticity of the Nash tolls with respect to a capacity change in A is in absolute terms twice as large for the toll in A than for the toll in B. This reaction is confirmed if we cut capacity by half once more and arrive at tolls of 124 in A and 91 in B. The exogenous lowering of capacity in A increases the local marginal external congestion cost and this is an important ingredient of the optimal uniform tax in A. Country B now faces a transit demand with a lower residual willingness to pay and is forced to cut its uniform tolls in order to protect its revenues. With the exogenous decrease in transport capacity in A, transit demand will decrease, local demand in country A will contract strongly while local demand in country B will expand as a result of less transit. The same profile of reactions can be found in the differentiated tolling case (not shown on Figure 2): an exogenous reduction of capacity in A will increase the local tax and transit tax at home and will decrease the local and transit tax in B.

When only local demand can be tolled, an exogenous reduction of capacity in A will reduce the local toll in A (see Figure 2, lower left corner). An important difference with the uniform case is that the tax on local traffic in B now stays approximately constant. This can be explained by the fact that the risk of increasing congestion caused by higher transit traffic, due to capacity reductions and increased tolls in A, is much less severe than in the uniform toll or differentiated toll case.

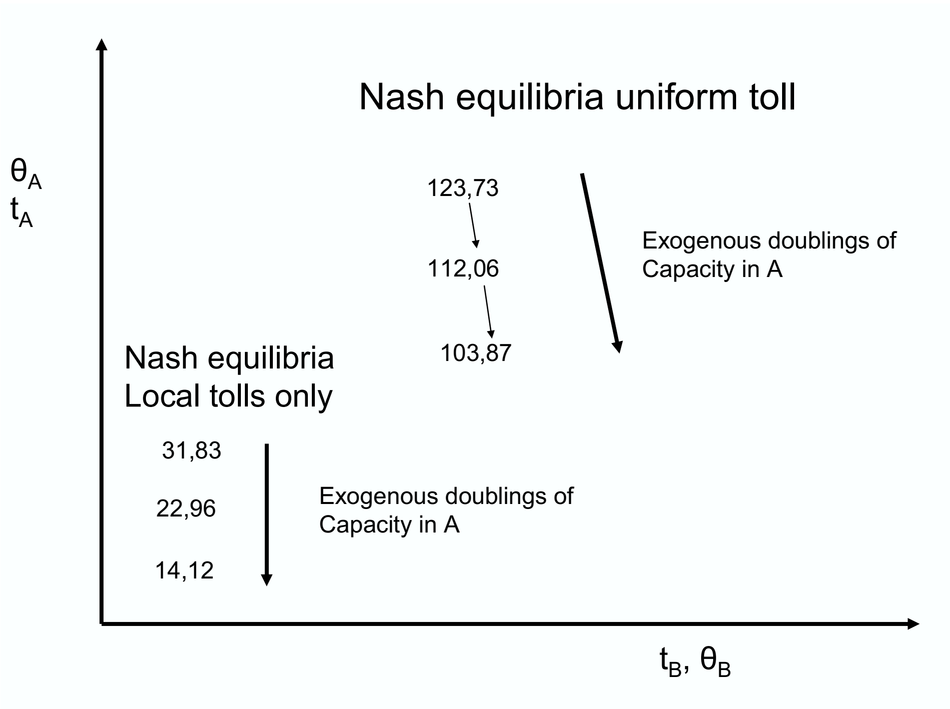

Figure 2: Shifts in Nash equilibria for uniform tolls and differentiated tolls as a result of exogenous changes in the transport capacity in A.

4.3.2. Strategic tolling and capacity choices on a transport corridor

Some results for the complete two-stage game are summarized in Table 2. First, consider the implications of the different pricing regimes for the optimal capacity choice in equilibrium. We observe that, compared to the no-toll capacity of 2000, optimal capacities decline for all three tolling regimes. This is not surprising. The no-toll equilibrium was calibrated such that the observed capacity was optimal in the Nash equilibrium, and the use of tolls reduces overall transport demand in each country. One therefore expects that capacity is lower than when no tolls are charged. Note that the capacity reduction strongly differs with the tolling regime in place. Optimal capacity is smallest when regions compete on the basis of uniform tolls; in that case high uniform tolls to tackle congestion lead to strong reductions in demand and, hence, low capacities. The optimal capacity is much larger when regions only use local tolls. In that case high transit demand implies a relatively small demand reduction compared to the no-toll equilibrium; capacity is much higher than with tolls on transit as a consequence. The case of differentiated tolls takes up an intermediate position: capacity is somewhat higher than in the uniform toll case. Transit demand is low, but local demand is much higher than with uniform tolls.

Second, note that the optimal tolls on local transport are, for all tolling regimes, higher than in the case capacity could not be optimally chosen. This is not surprising, because we showed in the previous subsection that capacity reductions at the first stage of the game induce regions to raise tolls on local transport at the second stage. Capacity reductions raise congestion, giving rise to higher taxes on local demand. The optimal toll on transit slightly declines in comparison with the case of fixed capacity. It is the joint effect of capacity expansions in the own region and in the competing region.

In Table 1, we have seen that, when transit is sufficiently important, allowing individual countries to toll actually reduces overall welfare, except for the case where countries can only toll local traffic. The intuition for this result is clear: a country will only toll its home traffic if its local consumer surplus plus tax revenue on locals increases, and any tax increase on local traffic benefits transit through lower congestion. So for the case of local tolls only, toll incentives are compatible with overall welfare. Interestingly, in Table 2 we see that this result still holds when we include capacity choice. In all cases, when tolling is introduced, it becomes interesting to reduce the overall capacity level, which will affect transit welfare negatively. However, for the case of local tolls only, this does not outweigh the benefit received from tolling local traffic.

Finally, note that we have only considered the case where transit and local demand make up 50% each of total transport in a given region. It is clear that tolling behaviour and capacity choices may be substantially affected by assuming different shares of transit. A share of only 10% transit in the no toll Nash equilibrium is analysed in Table 3; the two regions are still assumed to be symmetric. This case has the same total demand function, the same congestion function and the same unit capacity cost as the previous 50%-50% case discussed in Table 2.

Results from comparing Tables 2 and 3 are as follows. The optimal uniform tolls in the 10% transit case are substantially lower than in the 50%-50% case, despite the increase in local marginal external cost. This follows from the decreased tendency for tax exporting behaviour. In the case of local tolls only, the Nash equilibrium local toll in Table 3 is higher than in Table 2, and it hardly differs from the (higher) local marginal external cost; this again reflects the lower importance of transit. Relative capacity reductions in the differentiated and uniform tolling cases are also slightly smaller in the case with low transit shares. This is due to the fact that the benefits of capacity provision are enjoyed to a larger extent by local traffic. Finally, note that the centralized solution in the 10% transit case yields the same optimal tolls and capacity levels as in the 50%-50% case considered in Table 2; this follows from our assumption of identical total demand and congestion functions.

We further also checked the sensitivity of the symmetric results by increasing the slope of the congestion function (β) by 20%. We found (these results are not shown) that this implies higher optimal capacities, higher tolls and lower volumes; moreover, and that it does not alter the welfare ranking of the five capacity-price equilibria we study.

Variable	Unit	No tolls	Nash Equilibrium differentiation	Nash Equilibrium uniform	Nash Equilibrium local tolls only	Centralised-differentiation
Local demand	Trips	1300	1219	732	1215	1233
Transit demand	Trips	1300	396	732	1301	1233
Trip volume, country level	Trips	2600	1616	1465	2516	2467
Generalised price, local	Euro/Trip	65.4	79.0	160.7	79.7	76.6
Generalised price, transit	Euro/Trip	130.9	434.1	321.3	130.6	153.3
Time cost	Euro/Trip	32.7	23.9	23.3	32.6	22.8
inverse capacity	1/trips	0.00050	0.00058	0.00062	0.00051	0.00036
Capacity	Trips	2000	1732	1618	1945	2791
capacity relative to reference		1.00	0.87	0.81	0.97	1.40
volume/capacity ratio		1.30	0.93	0.91	1.29	0.88
Local Toll	Euro/Trip	0.0	22.3	104.7	14.4	21.1
Transit Toll	Euro/Trip	0.0	160.4	104.7	0.0	21.1
Local MEC	Euro/Trip	15.6	16.8	10.8	14.9	10.6
Global MEC	Euro/Trip	31.1	22.3	21.7	31.0	21.1
Local CS	Euro	141779	124726	45011	123815	127602
Tax revenue, country level	Euro	0	90793	153333	17536	52160
Cost of capacity	Euro?	37383	32371	30251	36346	52160
Welfare, country level	Euro	104395	183147	168093	105005	127602
Transit welfare (CS)	Euro	283557	26363	90023	283945	255204
Overall welfare	Euro	492348	392658	426209	493956	510409
Share country A		20,22	21.20	46.64	39.44	21.26
Change compared to Non toll	%	0	-20.25	-13.43	0.33	3.67
Change compared to Centralized-different.	%	-3.54	-23.07	-16.50	-3.22	0.00

Table 2: Results of symmetric two-stage capacity and pricing game with 50% transit in no toll equilibrium

Variable	Unit	No tolls	Nash Equilibrium differentiation	Nash Equilibrium uniform	Nash Equilibrium local tolls only	Centralised-differentiation
Local demand	Trips	2334	2216	2008	2219	2220
Transit demand	Trips	259	82	223	270	247
Trip volume, country level	Trips	2593	2297	2232	2489	2467
Generalised price, local	Euro/Trip	66.0	77.0	96.3	76.7	76.6
Generalised price, transit	Euro/Trip	132.0	429.9	192.7	113.6	153.3
Time cost	Euro/Trip	32.7	22.4	22.2	23.5	22.2
inverse capacity	1/trips	0.00051	0.00039	0.00040	0.00038	0.00036
Capacity	Trips	1958	2575	2524	2653	2791
capacity relative to reference		1.00	1.32	1.29	1.35	1.43
volume/capacity ratio		1.32	0.89	0.88	0.94	0.88
Local Toll	Euro/Trip	0.0	21.35	40.9	19.9	21.1
Transit Toll	Euro/Trip	0.0	159.26	40.9	0.0	21.1
Local MEC	Euro/Trip	28.5	20.6	19.0	20.0	19.0
Global MEC	Euro/Trip	31.7	21.3	21.2	22.5	21.1
Local CS	Euro	253831	228794	187994	229530	229684
Tax revenue, country level	Euro	0	60323	91185	44182	52160
Cost of capacity	Euro?	36593	48126	47180	49580	52160
Welfare, country level	Euro	217238	240991	231999	224132	229684
Transit welfare (CS)	Euro	56407	5611	41776	61297	51041
Overall welfare	Euro	490883	487594	505774	509561	510409
Share country A		44.25	44.25	49.42	45.87	43.99
Change compared to Non toll	%	0	-0.67	3.03	3.80	3.98
Change compared to Centralized-different.	%	-3.83	-4.47	-0.91	-0.17	0.00

Table 3: Results of symmetric two-stage capacity and pricing game with 10% transit in no toll equilibrium

4.4 Role of relative country size

In this subsection we illustrate the role of asymmetries in country size. It is well known in the tax competition literature (see Kanbur and Keen (1993)) that, in a cross border context, the small country has a larger incentive to undercut the tax rate of its neighbour. This typically holds for cigarettes or gasoline. The intuition is that the big country loses more revenue by lowering taxes because it can never gain abroad what it loses at home. Our context is different in two respects. Firstly, the country governments we analyze are by assumption maximizers of local welfare rather than pure revenue maximizers. Secondly, the transit tax base is the same for both countries and does not shift from one country to another like the demand for sigarettes would do.

In Tables 4 and 5 we report the results of the capacity and price competition for the case where a corridor runs through two different countries. The first one is a "small" country where, in the absence of tolling, transit is as important as local traffic (similar to our 50% transit case of Table 2); the second country is a "large" country where, in the absence of tolling, transit is only 10% of total traffic (similar to the 90%, 10% case of Table 3). Because the transit traffic is the same in the small and the large country, the large country has a higher total demand than the small country. As the cost of capacity is kept constant, the slope of the congestion function in the large country has been increased to have the same generalised cost in the no toll equilibrium.

The behaviour of both countries depends now strongly on the type of tolling instrument that is available. We see that in the Nash differentiated toll case, both countries charge high tolls on transit and we have the same double marginalisation problem as in the symmetric case. When only uniform tolls are available, the behaviour of the small and large country are very different. The small country A now has an interest to favour the revenue motive and accept inefficient pricing for its local users, while the large country gives much higher weight to the local users. In the local tolls only case, we have again fairly similar results for both countries. In terms of overall welfare (small + large country), the ranking of the solutions is similar to the symmetric case. Worst is the tax discrimination case, followed by the uniform case, the no toll case and the local tolls only case. It is interesting to note that the small country can spoil the welfare gains the large country could generate with a uniform tax. In the symmetric 10% transit case, shown in Table 3, the uniform solution was better than the no tolling case. This is no longer true here because the small country has in a serial network a strong incentive to abuse its monopoly power.

Variable	Unit	No tolls	Nash Equilibrium differentiation	Nash Equilibrium uniform	Nash Equilibrium local tolls only	Centralised-differentiation
Local demand	Trips	1300	1209	654	1207	1231
Transit demand	Trips	1300	383	849	1297	1182
Trip volume, country level	Trips	2600	1592	1504	2504	2413
Generalised price, local	Euro/Trip	65.4	80.7	173.7	80.9	77.0
Generalised price, transit	Euro/Trip	130.9	438.4	282.0	131.9	170.4
Time cost	Euro/Trip	32.8	24.8	23.7	33.5	22.9
inverse capacity		0.00046	0.00056	0.00057	0.00049	0.00034
Capacity	Trips	2161	1774	1758	2032	2926
capacity relative to reference		1.00	0.82	0.81	0.94	1.35
volume capacity ratio		1.20	0.90	0.86	1.23	0.82
Local Toll	Euro/Trip	0.0	23.2	117.2	14.8	21.4
Transit Toll	Euro/Trip	0.0	152.6	117.2	0.0	21.4
Local MEC	Euro/Trip	15.5	17.6	9.6	15.3	10.9
Global MEC	Euro/Trip	31.1	23.2	22.1	31.8	21.3
Local CS	Euro	141676	122563	35907	122239	126967
Tax revenue, country level	Euro	0	86422	176290	17846	51680
Cost of capacity	Euro?	40384	33168	32859	37988	54684
Welfare, country level	Euro	101292	175817	179338	102097	123963
Transit welfare (CS)	Euro	283353	24593	120923	281987	234275
Overall welfare	Euro	1296275	1221876	1276962	1317004	1328312
Share country A		7.81	14.39	14.04	7.75	9.33
Share country B		70.33	83.60	76.49	70.84	73.03
Change compared to Non toll	%	0	-5.74	-1.49	1.60	2.47
Change compared to Centralized-different.	%	-2.41	-8.01	-3.87	-0.85	0.00

Table 4: Results of asymmetric two-stage capacity and pricing game for (small) country A and overall results for A and B

Variable	Unit	No tolls	Nash Equilibrium differentiation	Nash Equilibrium uniform	Nash Equilibrium local tolls only	Centralised-differentiation
Local demand	Trips	11700	10125	9395	10197	10200
Transit demand	Trips	1300	383	849	1297	1182
Trip volume, country level	Trips	13000	10508	10245	11494	11382
Generalised price, local	Euro/Trip	65.4	94.8	108.3	93.4	93.3
Generalised price, transit	Euro/Trip	130.9	438.4	282.0	131.9	170.4
Time cost	Euro/Trip	32.8	31.9	31.1	33.1	31.2
inverse capacity	1/trips	0.00005	0.00006	0.00006	0.00006	0.00006
Capacity	Trips	19445	16163	16141	17003	17915
Capacity relative to reference		1.00	0.83	0.83	0.87	0.92
volume capacity ratio		0.67	0.65	0.63	0.68	0.64
Local Toll	Euro/Trip	0.0	30.2	44.5	27.6	29.5
Transit Toll	Euro/Trip	0.0	163.8	44.5	0.0	29.5
Local MEC	Euro/Trip	28.0	29.1	27.0	27.9	26.5
Global MEC	Euro/Trip	31.1	30.2	29.5	31.4	29.5
Local CS	Euro	1275088	955015	822393	968783	969334
Tax revenue, country level	Euro	0	368564	456009	281956	335609
Cost of capacity	Euro?	363458	302114	301701	317819	334869
Welfare, country level	Euro	911630	1021466	976701	932920	970074

Table 5: Results of asymmetric two-stage capacity and pricing game for (large) country B

5. Summary and caveats

In this paper we have analyzed the strategic behaviour of country governments that each operate one individual link of a congested transport corridor. We studied a two stage game in capacities and prices under three different pricing regimes.

The conclusions are easily summarized. With respect to optimal tolling behavior we showed that, when transit can be tolled, strategic behavior implies that a unit increase in the transit toll in one region raises the total toll on transit users for the whole trajectory by less than one unit. Moreover, transit tolls are inefficiently high. However, if only local traffic can be tolled, Nash equilibrium tolls are inefficiently low: tolls are shown to be smaller than the marginal external congestion cost imposed on local traffic. The reason is that higher local tolls would attract too much transit traffic and hence reduce welfare.

At the capacity stage of the game, we showed that capacities in the two regions are likely to be strategic complements: reaction functions are plausibly upward sloping so that higher capacity on one link of the corridor induces the operator of the other link to invest in capacity as well. Moreover, we find that capacity changes strongly affect optimal tolling behavior. Higher capacity investment in a region not only lowers optimal tolls in this region under all pricing regimes, but it also affects tolls on transit in the other region. We further find interesting interactions between optimal capacities and the pricing instruments used: optimal capacity with differentiated tolls is higher than in the case of uniform tolls; the largest optimal capacity results when only local tolls are used. Finally, we emphasized the role of the share of transit and of asymmetric country sizes for the results.

Although the analysis was based on a very simple model, it may have potential applicability in a number of cases where capacity and pricing decisions in regions are strategically chosen. This includes investment and pricing on Trans European Networks (basically a border-crossing highway system) in Europe and the interstate highway system in the US. Moreover, it is equally relevant for pricing and investment decisions for long distance rail trips, and with minor adaptation the analysis also applies to inter-modal freight trips where the transfer facility (ports, airports, freight terminal) and the upstream or downstream infrastructure is controlled by different governments.

Several avenues for extension could be considered. One is to generalise the results for transport corridors through n (rather than just 2) countries. Another is to explore the implications of relaxing the assumption of constant returns in capacity expansion. A third extension could be to integrate the results for serial networks with those obtained for parallel networks. A fourth extension would be to pay specific attention to the timing of the game with, say, one country leading in the capacity extension. A further extension would be to examine a cooperative process. Finally, one could examine other assumptions on the behaviour of the countries' decision makers and explicitly develop some of the political economy aspects of tax and capacity decisions.

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Appendix 1: The case of differentiated tolls

In this appendix we derive the most relevant results for the differentiated tolling case. The procedure is the same as in De Borger et al (2005) but requires some adjustment for the particular serial setting considered here.

Characteristics of the reduced-form demands

We start from the equilibrium conditions:

$$P^X(X) = C_A(V_A R_A) + \tau_A + C_B(V_B R_B) + \tau_B \tag{A1.1}$$

$$P_A^Y(Y_A) = C_A(V_A R_A) + t_A \tag{A1.2}$$

$$P_B^Y(Y_B) = C_B(V_B R_B) + t_B \tag{A1.3}$$

Noting that $V_k = Y_A + X$, first solve the two last equations for local transport as a function of transit demand, the local tax rate and capacity in a given region:

$$Y_A = z_A(X, R_A, t_A) \tag{A1.4}$$

$$Y_B = z_B(X, R_B, t_B) \tag{A1.5}$$

Application of the implicit function theorem to (A1.2) implies:

$$\frac{\partial z_A}{\partial X} = \frac{C_A^{'} R_A}{\frac{\partial P_A^Y}{\partial Y_A} - R_A C_A^{'}} < 0 \tag{A1.6}$$

$$\frac{\partial z_A}{\partial t_A} = \frac{1}{\frac{\partial P_A^Y}{\partial Y_A} - R_A C_A^{'}} < 0 \tag{A1.7}$$

$$\frac{\partial z_A}{\partial R_A} = \frac{V_A C_A^{'}}{\frac{\partial P_A^Y}{\partial Y_A} - R_A C_A^{'}} < 0 \tag{A1.8}$$

where $C_A^{'} = \frac{\partial C_A(V_A R_A)}{\partial (V_A R_A)}$.

An analogous result is derived for B. Interpretation is simple: an exogenous increase in transit reduces the demand for local transport, as it raises local congestion and hence generalised user cost. Raising the local tax, at a given transit level, reduces local demand for transport. Finally, increasing capacity (decreasing R), increases the local transport flow.

Note that (A1.6) implies that, for k=A,B:

$$\left(1+\frac{\partial z_k}{\partial X}\right) = \frac{\dfrac{\partial P_k^Y}{\partial Y_k}}{\dfrac{\partial P_k^Y}{\partial Y_k} - C_k' R_k} > 0 \qquad \text{(A1.9)}$$

Moreover, substituting (A1.4)-(A1.5) into (A1.1) yields:

$$P^X(X) = C_A\left[(X + z_A(X,t_A)), R_A\right] + \tau_A + C_B\left[(X + z_B(X,t_B)), R_B\right] + \tau_B$$

Solution of this expression for transit demand X yields the reduced form demand for transit as a function of all four tax rates. Using the implicit function theorem we derive the following results for A (analogous results hold for B):

$$\frac{dX}{dt_A} = -\frac{1}{\Delta}\left(C_A' R_A \frac{\partial z_A}{\partial t_A}\right) \qquad \text{(A1.10)}$$

$$\frac{dX}{d\tau_A} = -\frac{1}{\Delta} \qquad \text{(A1.11)}$$

$$\frac{dX}{dt_B} = -\frac{1}{\Delta}\left(C_B' R_B \frac{\partial z_B}{\partial t_B}\right) \qquad \text{(A1.12)}$$

$$\frac{dX}{d\tau_B} = -\frac{1}{\Delta} \qquad \text{(A1.13)}$$

$$\frac{dX}{dR_A} = +\frac{1}{\Delta}\left(C_A' V_A + C_A' R_A \frac{\partial z_A}{\partial R_A}\right) \qquad \text{(A1.14)}$$

$$\frac{dX}{dR_B} = +\frac{1}{\Delta}\left(C_B' V_B + C_B' R_B \frac{\partial z_B}{\partial R_B}\right) \qquad \text{(A1.15)}$$

where

$$\Delta = -\left[\frac{\partial P^X}{\partial X}\right] + C_A' R_A\left(1 + \frac{\partial z_A}{\partial X}\right) + C_B' R_B\left(1 + \frac{\partial z_B}{\partial X}\right)$$

Using (A1.9) immediately yields $\Delta > 0$. Since simple algebra also implies:

$$V_k + R_k \frac{\partial z_k}{\partial R_k} = \frac{V_k \dfrac{\partial P_k^Y}{\partial Y_k}}{\dfrac{\partial P_k^Y}{\partial Y_k} - R_k C_k'} > 0$$

we then also easily show that the effects of taxes and capacity on reduced form demand are:

$$\frac{\partial X^r}{\partial \tau_A}<0,\ \frac{\partial X^r}{\partial \tau_B}<0,\ \frac{\partial X^r}{\partial t_A}>0,\ \frac{\partial X^r}{\partial t_B}>0$$
$$\frac{\partial X^r}{\partial R_A}<0,\ \frac{\partial X^r}{\partial R_B}<0 \qquad \text{(A1.16)}$$

where reduced form demand is denoted by the superscript r. Moreover, we have $\left|\frac{\partial X^r}{\partial \tau_A}\right| > \left|\frac{\partial X^r}{\partial t_A}\right|$.

Finally, to determine the impact of taxes and capacities on local demands, we have (see A1.4-A1.5):

$$\frac{\partial Y_A^r}{\partial t_A}=\frac{\partial z_A}{\partial X}\frac{\partial X^r}{\partial t_A}+\frac{\partial z_A}{\partial t_A}$$
$$\frac{\partial Y_A^r}{\partial \tau_A}=\frac{z_A}{X}\frac{\partial X^r}{\partial \tau_A}$$
$$\frac{\partial Y_A^r}{\partial t_B}=\frac{z_A}{X}\frac{\partial X^r}{\partial t_B}$$
$$\frac{\partial Y_A^r}{\partial \tau_B}=\frac{z_A}{X}\frac{\partial X^r}{\partial \tau_B}$$
$$\frac{\partial Y_A^r}{\partial R_A}=\frac{\partial z_A}{\partial X}\frac{\partial X^r}{\partial R_A}+\frac{\partial z_A}{\partial R_A}$$
$$\frac{\partial Y_A^r}{\partial R_B}=\frac{z_A}{X}\frac{\partial X^r}{\partial R_B}$$

Substituting previous results, it follows after simple algebra:

$$\frac{\partial Y_A^r}{\partial \tau_A}>0,\ \frac{\partial Y_A^r}{\partial \tau_B}>0,\ \frac{\partial Y_A^r}{\partial t_A}<0,\ \frac{\partial Y_A^r}{\partial t_B}<0$$
$$\frac{\partial Y_A^r}{\partial R_A}<0,\ \frac{\partial Y_A^r}{\partial R_B}>0 \qquad \text{(A1.17)}$$

Optimal taxes

We focus on region A. Consider the problem of determining the welfare optimal taxes on local and transit transport, conditional on the existing capacities in both regions and taking tax levels in B as given. Region A solves:

$$\underset{t_A,\tau_A}{Max} \quad W_A = \int_0^{Y_A} P_A^Y(y)\,dy - g_A^Y Y_A + t_A Y_A + \tau_A X - K_A \frac{1}{R_A}.$$

where K_A is the unit rental cost of capacity, and the demand functions are the reduced form demands just described. The first-order condition with respect to the local tax rate can be written as:

$$P_A^Y \frac{\partial Y_A^r}{\partial t_A} - g_A^Y \frac{\partial Y_A^r}{\partial t_A} - Y_A \frac{\partial g_A^Y}{\partial t_A} + t_A \frac{\partial Y_A^r}{\partial t_A} + Y_A + \tau_A \frac{\partial X^r}{\partial t_A} = 0$$

Differentiating

$$P_A^Y(Y_A) = g_A^Y = C_A(V_A R_A) + t_A$$

and using equality of generalized price and generalized cost in equilibrium allows us to rewrite this expression, after simple manipulation, as follows:

$$\left(t_A - C_A' Y_A R_A\right)\frac{\partial Y_A^r}{\partial t_A} + \left(\tau_A - C_A' Y_A R_A\right)\frac{\partial X^r}{\partial t_A} = 0$$

A similar procedure is used to show that the first-order condition with respect to τ_A can be written as:

$$\left(t_A - C_A' Y_A R_A\right)\frac{\partial Y_A^r}{\partial \tau_A} + \left(\tau_A - C_A' Y_A R_A\right)\frac{\partial X^r}{\partial \tau_A} + X = 0$$

To determine the optimal taxes, we write the system in matrix notation and solve by Cramers' rule. We find, using similar manipulations as described in De Borger et al. (2005), the following tax rule for local traffic and transit, respectively:

$$t_A = (Y_A + X)C_A' R_A = LMEC_A + X C_A' R_A$$

$$\tau_A = LMEC_A + X\left[\frac{\frac{\partial Y_A^r}{\partial t_A}}{\frac{\partial Y_A^r}{\partial t_A}\frac{\partial X^r}{\partial \tau_A}}\right]$$

where $LMEC_A = Y_A C_A' R_A$ is the local marginal external cost.

To show that the transit tax exceeds the local tax subtract the two taxes, use previous results and explicitly substitute the definition of Δ. We find:

$$\tau_A - t_A = -X\left\{\frac{\partial P^X}{\partial X} - C_B' R_B\left[1 - \frac{\partial z_B}{X_B}\right]\right\} > 0$$

Tax reaction functions: The case of linear demands and costs

We report results for country A; all results for B are derived analogously. Using linear specifications , the demand for local transport in A conditional on transit demand and the local tax is given by:

$$Y_A = z_0^A - z_1^A X - z_2^A t_A$$

where

$$z_0^A = \frac{c_A - \alpha_A}{d_A + \beta^*_A}, z_1^A = \frac{\beta^*_A}{d_A + \beta^*_A}, z_2^A = \frac{1}{d_A + \beta^*_A}$$

Substituting these functions in the equilibrium condition for transit yields:

$$X^r = \gamma_0 + \gamma_1(\tau_A + \tau_B) - (\gamma_1 z_1^A)(t_A) - (\gamma_1 z_1^B)(t_B)$$

where:

$$\gamma_0 = \frac{a - \alpha_A - \alpha_B - \beta^*_A z_0^A - \beta^*_B z_0^B}{\Delta}$$

$$\gamma_1 = -\frac{1}{\Delta} < 0$$

and $\Delta = b + \beta^*_A(1 - z_1^A) + \beta^*_B(1 - z_1^B) > 0$.

The first order conditions for optimal local and transit taxes for country A can be written as:

$$t_A = (Y_A + X)C'_A R_A - \beta^*_A(Y_A - X)$$

$$\tau_A = LMEC_A - X\left[\frac{\frac{\partial Y_A^r}{\partial t_A}}{\frac{\partial \tau_A}{\partial t_A}\frac{X^r}{\tau_A}}\right] - \beta^*_A Y_A - X\left[\frac{z_2^A - (z_1^A)^2\gamma_1}{z_2^A\gamma_1}\right]$$

Substituting for transit demand X, using the definitions of the various parameters and making use of the specification of Δ we find, after simple algebra, the reaction functions:

$$\tau_A = c_A^\tau - (\frac{1}{2})\tau_B - (\frac{1}{2}z_1^B)t_B$$

$$t_A = c_A^t - (\frac{1}{2}L^A)\tau_B - (\frac{1}{2}z_1^B L^A)t_B$$

where $$c_A^{\tau} = \frac{1}{2}\Delta\left[\beta_A^* z_0^A \quad \gamma_0\right],$$

$$c_A^{t} = \left[\beta_A^* \gamma_0 \quad \frac{1}{2} T^A (\ _0 \quad {}_A^* z_0^A \ _1)\right] \frac{L^A}{T^A \gamma_1},$$

and $$L^A = \frac{T^A \gamma_1}{1 - z_1^A (1 \quad \gamma_1 T^A)}; \; T^A = \beta_A^* (1 \quad z_1^A)$$

Note that simple algebra shows that $-1 < L^A \quad 0$.

Existence of equilibrium in prices follows as in De Borger et al (2005).

Appendix 2: The uniform tolling case

Reduced-form demands

Going through exactly the same derivations as before we easily derive:

$$P^X(X) = C_A\left[X \quad z_A(X, \theta_A, R_A)\right] R_A \quad \theta \quad C_B\left(X \quad z_B(X, \ _B, R_B)\right) R_B \quad _B$$

Using the implicit function theorem, we obtain the partial effects of the uniform taxes on transit demand (the definition of $\Delta > 0$ is the same as before):

$$\frac{\partial X^r}{\partial \theta_A} = \frac{1}{\Delta}\left(1 \quad C_A' R_A \frac{\partial z_A}{\ _A}\right) \quad 0$$

$$\frac{\partial X^r}{\partial \theta_B} = \frac{1}{\Delta}\left(1 \quad C_B' R_B \frac{\partial z_B}{\ _B}\right) \quad 0$$

$$\frac{\partial X^r}{\partial R_A} = \frac{1}{\Delta}\left[C_A'\left(Y_A \quad R_A \frac{\partial z_A}{R_A}\right)\right] \quad 0$$

$$\frac{\partial X^r}{\partial R_B} = \frac{1}{\Delta}\left[C_B'\left(Y_B \quad R_B \frac{\partial z_B}{R_B}\right)\right] \quad 0$$

$$\frac{\partial Y_A^r}{\partial \theta_A} < 0, \frac{Y_A^r}{\ _B} > 0$$

$$\frac{\partial Y_A^r}{\partial R_A} < 0, \frac{Y_A^r}{R_B} > 0$$

Optimal tax rule

The first-order condition to the problem

$$\underset{\theta_A}{Max} \quad W_A = \int_0^{Y_A} (P_A^Y(y))dy \quad g_A^Y Y_A \quad \theta_A(Y_A \quad X) \quad \frac{K_A}{R_A},$$

can be written as:

$$P_A^Y \frac{\partial Y_A^r}{\partial \theta_A} - g_A^Y \frac{\partial Y_A^r}{\partial \theta_A} \quad Y_A \frac{\partial g_A^Y}{\partial \theta_A} \quad \theta_A(\frac{\partial Y_A^r}{\partial \theta_A} \quad \frac{\partial Y_A^r}{\partial \theta_A}) \quad (Y_A^r \quad X^r) \quad 0$$

To simplify, use:

$$P_A^Y(Y_A) = g_A^Y + C_A\left[(X \quad Y_A)R_A\right] \quad \theta_A,$$

differentiate with respect to θ_A and substitute to obtain:

$$-Y_A\left[C_A' R_A\right]\left(\frac{\partial Y_A^r}{\partial \theta_A} \quad \frac{\partial X^r}{\partial \theta_A} \quad 1\right) \quad \theta_A(\frac{\partial Y_A^r}{\partial \theta_A} \quad \frac{\partial Y_A^r}{\partial \theta_A}) \quad (Y_A^r \quad X^r) \quad 0$$

Solving for the tax yields:

$$\theta_A = LMEC_A \quad \frac{X}{\frac{\partial Y_A^r}{\partial \theta_A} + \frac{\partial X^r}{\partial \theta_A}}$$

Noting the signs derived before, it follows that the tax rate exceeds the local marginal external cost.

Tax reaction function: linear demand and cost

In the case of uniform tolls, the reduced form demand for transit can be written as:

$$X^r = \gamma_0 \quad \gamma_A \theta_A \quad \gamma_B \theta_B$$

where

$$\gamma_0 = \frac{a - \alpha_A \beta_A^* z_0^A \quad \alpha_B \beta_B^* z_0^B}{\Delta}$$

$$\gamma_A = -\frac{1 + z_1^A}{\Delta} < 0$$

$$\gamma_B = -\frac{1 + z_1^B}{\Delta} < 0$$

Uniform tax increases reduce transit demand.

The optimal tax rule for A is given by:

$$\theta_A = \beta_A^* \eta_A LMEC_A \frac{X}{\frac{\partial Y_A^r}{\partial \theta_A} + \frac{X^r}{\theta_A}} \quad {}^*_A Y_A \quad {}_A X$$

$$\text{where } \eta_A = \frac{1}{\gamma_A(1+z_1^A) \quad z_2^A} < 0 .$$

Reaction functions can be written as:

$$\theta_A = c_A^\theta \quad m^A \theta_B$$
$$\theta_B = c_B^\theta \quad m^B \theta_A$$

where $c_A^\theta = \frac{\beta_A^* z_0^A + (\beta_A^* \eta_A z_1^A \quad _A)\ _0}{1-\beta_A^* z_2^A \quad (\beta_A^* \eta_A z_1^A \quad _A)\ _A}$, $m^A = \frac{(\beta_A^* \eta_A z_1^A - \ _A)\ _B}{1-\beta_A^* z_2^A \quad (\beta_A^* \eta_A z_1^A \quad _A)\ _A}$; coefficients for B are defined analogously. The sign of the slopes of the reaction functions can be shown to be negative. Indeed, using the definitions of η_A and γ_A, and substituting for Δ in the resulting expressions, the numerator of m^A can be shown to be negative, the denominator positive. This shows that the reaction function is downward sloping.

<u>Appendix 3: The case of local tolls only</u>

<u>Optimal tax rule</u>

The first-order condition to the problem

$$\underset{t_A}{Max} \quad W_A = \int_0^{Y_A} (P_A^Y(y))dy \quad g_A^Y Y_A \quad t_A Y_A \quad \frac{K_A}{R_A}$$

yields, using the same simple manipulations as in previous cases:

$$t_A \frac{\partial Y_A}{\partial t_A} - \left(Y_A C_A' R_A\right) \left(\frac{\partial Y_A^r}{\partial t_A} + \frac{\partial X^r}{\partial t_A} \right) = 0$$

Solving for the optimal local toll leads to:

$$t_A = LMEC_A \left[1 \quad \frac{\frac{\partial X^r}{\partial t_A}}{\frac{\partial Y_A^r}{\partial t_A}} \right]$$

Importantly, the term between square brackets can be shown to be between zero and one. That it is smaller than one is obvious, since $\frac{\partial Y_A^r}{\partial t_A} < 0, \frac{\partial X^r}{\partial t_A} > 0$. To see that the bracketed term is positive it suffices to show that $\frac{\partial Y_A^r}{\partial t_A} + \frac{X^r}{t_A} < 0$. To do so, elaborate as follows:

$$\frac{\partial Y_A^r}{\partial t_A} + \frac{X^r}{t_A} = \frac{\partial X^r}{\partial t_A} - \frac{z_A}{X}\frac{X^r}{t_A} + \frac{X^r}{t_A} - \frac{z_A}{t_A} = \frac{\partial X^r}{\partial t_A} + \left(1 - \frac{z_A}{X}\right)\frac{X^r}{t_A}$$

Then substitute for $\frac{\partial X^r}{\partial t_A}$ and use Δ to find:

$$\frac{\partial Y_A^r}{\partial t_A} + \frac{X^r}{t_A} = -\frac{1}{\Delta}\left\{\frac{z_A}{t_A}\right\} C_A' R_A \left(1 - \frac{z_A}{X}\right) < 0$$

The implication is economically important. It implies that the optimal tax is positive but smaller than the local marginal external cost.

<u>Tax reaction functions: linear demands and costs</u>

Finally, in the case of local taxes only, we have that the demand for transit only depends on the two local taxes; the coefficients are the same as those defined for the tax differentiation case. The optimal tax rule can be written as:

$$t_A = -Y_A C_A' R_A \left[1 - \frac{\frac{\partial X^r}{\partial t_A}}{\frac{\partial Y_A^r}{\partial t_A}}\right] = -\beta_A^* Y_A s_A, \quad s_A = \left(1 - \frac{z_1^A \gamma_1}{z_2^A + (z_1^A)^2 \gamma_1}\right)$$

Note that $0 < s_A \le 1$. Substituting the reduced form demand for local transport and working out leads to the following reaction function for country A's optimal local tax as a function of the local tax in B:

$$t_A = \tilde{c}_A^t - r_A t_B$$

where $\tilde{c}_A^t = \frac{s_A \beta_A^* \gamma_A (z_0^A + z_1^A \gamma_0)}{1 - s_A \beta_A^* \gamma_A ((z_1^A)^2 \gamma_1 - z_2^A)}$, $r_A = \frac{s_A \beta_A^* \gamma_A z_1^A z_1^B \gamma_1}{1 - s_A \beta_A^* \gamma_A ((z_1^A)^2 \gamma_1 - z_2^A)}$.

Since the numerator of the slope coefficient is negative and the denominator positive, it follows that the slope of the reaction function is negative.

Appendix 4. Capacity competition in the absence of tolling

For the simplified case of zero taxes, the optimal capacity choice problem reduces to:

$$\underset{R_A}{Max} \quad W_A = \int_0^{Y_A} (P_A^Y(y))dy \quad g_A^Y Y_A \quad K_A \frac{1}{R_A}$$

The first-order condition is given by:

$$Y_A \frac{\partial g_A^Y}{\partial R_A} = \frac{K_A}{R_A^2}$$

Alternatively, it reads:

$$Y_A \frac{\partial\left[C_A(V_A R_A)\right]}{\partial R_A} = \frac{K_A}{R_A^2}, \quad V_A \quad Y_A^r(X, R_A) \quad X^r(R_A, R_B) \qquad (A4.1)$$

We are interested in the reaction function in capacities, i.e., the optimal capacity in A, conditional on capacity in B. Expression (A4.1) implicitly defines this reaction function, which we denote $R_A^R(R_B)$. Writing it in implicit form yields:

$$\psi(R_A, R_B) = Y_A \frac{\partial\left[C_A(V_A R_A)\right]}{\partial R_A} - \frac{K_A}{R_A^2} \quad 0 \qquad (A4.2)$$

In the remainder of this appendix we focus on the linear demand and cost case. Then we have:

$$\frac{\partial\left[C_A(V_A R_A)\right]}{\partial R_A} = \frac{\left[\alpha \quad \beta_A R_A (X \quad Y_A)\right]}{R_A} \quad \beta_A (X \quad Y_A) \quad \beta_A R_A \left(\frac{\partial X^r}{R_A} \quad \frac{\partial Y_A^r}{R_A}\right)$$

The final term between brackets is given by:

$$\frac{\partial X^r}{\partial R_A} \quad \frac{\partial Y_A^r}{\partial R_A} \quad \frac{X^r}{R_A} \quad \frac{z_A}{R_A} \quad z_1^A \frac{X^r}{R_A} \quad \frac{X^r}{R_A}(1 \quad z_1^A) \quad \frac{V_A}{R_A} z_1^A$$

so that simple algebra yields:

$$\frac{\partial\left[C_A(V_A R_A)\right]}{\partial R_A} = \beta_A (1 \quad z_1^A)\left[V_A \quad R_A \frac{\partial X^r}{R_A}\right] \qquad (A4.3)$$

Note that this expression, by the first order condition (A4.2), must be positive: a capacity reduction in A raises travel cost. Substituting (A4.3) in (A4.2) leads to:

$$\psi(R_A, R_B) = Y_A \left\{ \beta_A (1 \quad z_1^A)\left[V_A \quad R_A \frac{\partial X^r}{\partial R_A}\right] \right\} \quad \frac{K_A}{R_A^2} \quad 0 \qquad (A4.4)$$

To determine the slope of the reaction function we use the implicit function theorem:

$$\frac{\partial R_A^R(R_B)}{\partial R_B} = -\frac{\dfrac{\partial\psi}{\partial R_B}}{\dfrac{\partial\psi}{\partial R_A}} \tag{A4.5}$$

where the denominator is negative by the second order condition of the optimal capacity choice problem. The sign of (A4.5) therefore depends on the numerator only. To determine its sign, differentiate (A4.4) with respect to inverse capacity in B:

$$\frac{\partial\psi}{\partial R_B} = Y_A^r \beta_A(1-z_1^A)\left[\frac{\partial V_A}{\partial R_B} - R_A\frac{\partial^2 X^r}{\partial R_A\,\partial R_B}\right] + \beta_A(1-z_1^A)\left(V_A - R_A\frac{\partial X^r}{\partial R_A}\right)\frac{\partial Y_A^r}{\partial R_B} \tag{A4.6}$$

This can be simplified as follows. First, noting that in the linear case A1.14) reduces to:

$$\frac{\partial X^r}{\partial R_A} = \frac{\beta_A V_A}{\Delta}(1-z_1^A) \tag{A4.7}$$

we have

$$\frac{\partial^2 X}{\partial R_A\,\partial R_B} = \left\{\beta_A(1-z_1^A)\left[\frac{\Delta\dfrac{\partial V_A}{\partial R_B} - V_A\dfrac{\partial\Delta}{\partial R_B}}{\Delta^2}\right]\right\}.$$

Substituting this result in (A4.6) and slightly rewriting yields:

$$\begin{aligned}\frac{\partial\psi}{\partial R_B} = {} & Y_A^r\beta_A(1-z_1^A)\left[1-\frac{\beta_A(1+z_1^A)}{\Delta}R_A\right]\frac{\partial V_A}{\partial R_B} \\ & + Y_A^r\beta_A(1-z_1^A)\left[\frac{\beta_A(1+z_1^A)}{\Delta^2}V_A R_A\right]\frac{\partial\Delta}{\partial R_B} \\ & + \beta_A(1-z_1^A)\left[V_A - R_A\frac{\partial X^r}{\partial R_A}\right]\frac{\partial Y_A^r}{\partial R_B}\end{aligned} \tag{A4.8}$$

Second, using the definition of Δ, we have

$$1-\frac{\beta_A(1+z_1^A)}{\Delta}R_A = \frac{1}{\Delta}\left[\;\right]\beta_A R_A(1-z_1^A) > 0$$

Third, we further easily show, see (7):

$$V_A - R_A\frac{\partial X^r}{\partial R_A} = \frac{V_A}{\Delta}\left[\;\;\beta_A R_A(1-z_1^A)\right] > 0 \tag{A4.9}$$

Fourth, again using the definition of Δ, we have by differentiation:

$$\frac{\partial\Delta}{\partial R_B} = -\left[\beta_B(1+z_1^B) + \beta_B R_B \frac{\partial z_1^B}{\partial R_B}\right] \quad \text{(A4.10)}$$

where $\frac{\partial z_1^B}{\partial R_B} = -\frac{d_B}{d_B+\beta_B R_B} < 0$. Working out (A4.10) then shows that:

$$\frac{\partial\Delta}{\partial R_B} = -\left[-\frac{\beta_B (d_B)^2}{d_B+\beta_B R_B}\right] > 0 \quad \text{(A4.11)}$$

Finally, note that

$$\frac{\partial Y_A^r}{\partial R_B} = z_1^A \frac{\partial X^r}{\partial R_B} > 0 \quad \text{(A4.12)}$$

$$\frac{\partial V_A}{\partial R_B} = \frac{\partial X^r}{\partial R_B} + \frac{\partial Y_A^r}{\partial R_B} = \frac{\partial X^r}{\partial R_B}(1+z_1^A) < 0 \quad \text{(A4.13)}$$

Substituting (A4.9), (A4.11), (A4.12) and (A4.13) in (A4.8), and using the definition of total demand, $V_A = Y_A + X$, then gives after simple manipulation:

$$\frac{\partial\psi}{\partial R_B} = -\beta_A(1+z_1^A)\frac{1}{\Delta}\left[\beta_A R_A(1+z_1^A) + Y_A^r(1+2z_1^A) + X z_1^A\right]\frac{\partial X^r}{\partial R_B}$$
$$+ Y_A^r \beta_A(1+z_1^A)\left[\frac{\beta_A(1+z_1^A)}{\Delta^2} V_A R_A \frac{\beta_B (d_B)^2}{d_B+\beta_B R_B}\right] \quad \text{(A4.14)}$$

This expression consists of two terms. The second is positive, the first is ambiguous. Since the term

$$\frac{\partial X^r}{\partial R_B} < 0$$

the first term will also be positive provided

$$Y_A^r(1+2z_1^A) + X z_1^A < 0. \quad \text{(A4.15)}$$

This will be the case if transit is relatively important and the impact of transit on local demand ($z_1^A < 0$) is sufficiently large in absolute value.

Note that it is quite plausible that (A4.15) is satisfied; a sufficient condition is that ($\left|z_1^A\right| > 0.5$, or alternatively, that $d_A < \beta_A R_A$. The left hand side is the slope of the local inverse demand function, the right hand side is the slope of the congestion function at given capacity. A sufficient (but by no means necessary) condition is, therefore, a sufficiently sloped congestion function.

The implication is that capacities in the two regions will be strategic complements (i.e., capacity reaction functions are upward sloping) if transit is sufficiently important, that is, if transit demand is non-negligible and if more transit appreciably reduces local demand through congestion effects. The intuition is clear. Suppose region B raises capacity. This attracts more transit through both A and B so that, in order to dampen the negative welfare effect on local demand, country A reacts by also raising capacity.

Note, however, that when transit is rather unimportant and if it does not much affect local demand then the reaction functions may in principle be negatively sloped. In that case, higher capacity in B induces A to reduce capacity: more transit is attracted through both regions by the capacity increase in B, but this hardly affects local demand in A so that, given the cost of capacity expansion, it is not worthwhile to expand capacity. In fact, capacity is reduced if the welfare loss due to slightly more congestion is more than compensated by the marginal capacity cost savings.

Appendix 5: The optimal capacity choice rule for the cases of uniform taxes and local taxes only

In the case of uniform taxes the first-order condition can be rewritten as:

$$Y_A \frac{dg_A^Y}{dR_A} - \theta_A \left(\frac{dY_A^r}{dR_A} + \frac{dX_A^r}{dR_A} \right) = (Y_A + X) \frac{\partial \theta_A^{NE}}{\partial R_A} + \frac{K_A}{R_A^2}$$

with:

$$\frac{dg_A^Y}{dR_A} = -\beta_A + \left[Y_A + R_A \left(\frac{dY_A^r}{dR_A} + \frac{dX^r}{dR_A} \right) \right] \frac{\partial \theta_A^{NE}}{\partial R_A}$$

$$\frac{dY_A^r}{dR_A} = \frac{\partial Y_A^r}{\partial R_A} + \sum_{k=A,B} \frac{Y_A^r}{\theta_k} \frac{\partial \theta_k^{NE}}{R_A}$$

$$\frac{dX^r}{dR_A} = \frac{\partial X^r}{\partial R_A} + \sum_{k=A,B} \frac{X^r}{\theta_k} \frac{\partial \theta_k^{NE}}{R_A}$$

Interpretation is as before. The first order condition equates marginal costs and benefits of capacity expansion, where the indirect effects via tax adjustment are taken into account. Substituting the total derivatives and using the first-order condition for optimal tax setting by region A, the optimal capacity choice rule can be reformulated as :

$$\beta_A \theta_A V_A Y_A - (t_A + LMEC_A)\left[\frac{\partial X_A^r}{\partial R_A} + \left(\frac{\partial Y_A^r}{\partial \theta_B} + \frac{\partial X_A^r}{\partial \theta_B}\right)\frac{\partial \theta_B^{NE}}{\partial R_A}\right] = \frac{K_A}{R_A^2}$$

Finally, for the case of local tolls only we have:

$$Y_A \frac{dg_A^Y}{dR_A} - t_A \frac{dY_A^r}{dR_A} = Y_A \frac{\partial t_A^{NE}}{\partial R_A} + \frac{K_A}{R_A^2}$$

where

$$\frac{dg_A^Y}{dR_A} = -\beta_A\left[V_A + R_A\left(\frac{dY_A^r}{dR_A} + \frac{dX^r}{dR_A}\right)\right] + \frac{\partial t_A^{NE}}{\partial R_A}$$

$$\frac{dY_A^r}{dR_A} = \frac{\partial Y_A^r}{\partial R_A} + \sum_{k=A,B} \frac{\partial Y_A^r}{\partial t_k} \frac{\partial t_k^{NE}}{\partial R_A}$$

$$\frac{dX^r}{dR_A} = \frac{\partial X^r}{\partial R_A} + \sum_{k=A,B} \frac{\partial X^r}{\partial t_k} \frac{\partial t_k^{NE}}{\partial R_A}$$

This can be rewritten as, using the optimal tax condition for region A, as:

$$\beta_A V_A Y_A - (t_A + LMEC_A)\left[\frac{\partial Y_A^r}{\partial t_B}\frac{\partial t_B^{NE}}{\partial R_A}\right] - LMEC_A\left[\frac{\partial X^r}{\partial R_A} + \frac{\partial X^r}{\partial t_B}\frac{\partial t_B^{NE}}{\partial R_A}\right] = \frac{K_A}{R_A^2}$$

Appendix 6: Calibration of parameters for numerical example

Remember that the parameters a, b, c, d describe the demand for local and transit transport, and α, β determine the congestion function. Moreover, $\beta^* = \beta / R = 0.0005 *$, and K is the cost of capacity. All parameters are the same in A and B, reflecting symmetry.

a	567.11
b	0.34
c	283.56
d	0.17
α	34.34
β*	0.01
β	23.92
K	18.69

Table A6.1. Calibration constants (identical for regions A and B)

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