

DEPARTEMENT TOEGEPASTE ECONOMISCHE WETENSCHAPPEN

ONDERZOEKSRAPPORT NR 9512

Control and Information in a Dynamic Multiple Agent Model

by

Wilfried PAUWELS

Filip ROODHOOFT



Katholieke Universiteit Leuven

Naamsestraat 69, B - 3000 Leuven

ONDERZOEKSRAPPORT NR 9512

**Control and Information in a Dynamic
Multiple Agent Model**

by

Wilfried PAUWELS

Filip ROODHOOFT

CONTROL AND INFORMATION IN A DYNAMIC MULTIPLE AGENT MODEL

Wilfried Pauwels (Professor, Universitaire Faculteiten
Sint-Ignatius Antwerpen)

Filip Roodhooft (Assistant Professor, Katholieke
Universiteit Leuven)¹

The controllability principle says that an individual should be rewarded on the basis of what he controls. This paper studies this principle from a principal agent point of view. A dynamic multiple agent model is constructed and the controllability principle is compared with the agency model's solution. It is shown that from a principal agent's point of view all results that give information about an individual's action should be used to determine his remuneration.

Keywords: *organizational behavior, moral hazard, control, information asymmetry, subgame perfectness*

JEL-classification: *D21, D82, M14*

Introduction

The purpose of this paper is to examine the controllability principle from a principal agent point of view. This management principle says that an individual should be evaluated and rewarded on the basis of what he controls [Choudhury (1986, p.382), Milgrom and Roberts (1992, p.220)]. With respect to the controllability principle three different problems can be defined: the measurement standard problem, the multiple agent problem and the multiple period problem. The measurement standard problem deals with the measurement standard used to reward an individual. This problem has already been studied in the literature before [Antle and Demski (1988), Holmström (1982)].

The multiple agent problem considers the question of relative performance measurement. An individual's evaluation and

¹ Filip Roodhooft, Katholieke Universiteit Leuven, Departement Toegepaste Economische Wetenschappen, Naamsestraat 69, 3000 Leuven, Belgium.

compensation can depend on the results of other individuals (agents). The multiperiod problem deals with the introduction of other period elements in the evaluation and remuneration of an agent. These problems are studied in this paper from a dynamic multiple agent point of view. This combination of multiple agents and multiple periods has not been investigated yet in the principal agent literature.

The basic principal agent model has been extensively discussed in literature [Bamberg and Spremann (1987), Grossman and Hart (1983), Holmström (1979), Kreps (1990), Rasmusen (1989), Ross (1973), Shavell (1979), Strong and Walker (1987)]. The multiple agent extension introduces supplementary agents into the model. A distinction can be made between tournaments [(Green and Stokey (1983), Malcolmson (1986), Mookherjee (1984), Nalebuff and Stiglitz (1983)] and principal agent contracts with moral hazard [Holmström (1982), Mookherjee (1984), Holmström and Milgrom (1990)] or/and adverse selection [Demski and Sappington (1984), Picard (1987) and Zou (1992)]. The multiple period extension deals with a repeated principal agent relationship. The number of periods can be infinite [Malueg (1986), Radner (1985), Rubinstein and Yaari (1983), Spear and Srivastava (1987)] or finite [Fellingham, Newman and Suh (1985), Fudenberg, Holmström and Milgrom (1990), Laffont and Tirole (1988), Lambert (1983, 1984), Malcolmson and Spinnewyn (1988), Radner (1981), Rogerson (1985)].

The first section introduces the two period multiple agent model with moral hazard. It is assumed that effort in one period has no effect on other period results and that agents cannot leave the principal. A second section confronts the controllability principle and the principal agent answer with respect to the evaluation and remuneration of the agents. In the third section an example is presented to illustrate the difference between control and information. A fourth section summarizes the most important conclusions.

I. The dynamic multiple agent model

The model presented consists of one principal and two agents, where principal and agents precommit to a two-period contract. The principal promises to employ the agents for two periods. Both agents agree not to leave the principal after the first period. Each agent chooses an action in every period. These actions are taken out of some finite sets A_t^a , with A_t^a the finite set of actions of agent a in period t . The principal observes a result from a finite set X_t^a for agent a in period t .

Every first period action combination induces a probability distribution on the first period results. We denote by

$\Pi_{ij}(a_1^1, a_1^2)$ the joint probability of result pair (x_{1i}^1, x_{1j}^2) when

action pair (a_1^1, a_1^2) is chosen. Every combination of first period results is possible under every combination of first period actions. The joint probability of result pair (x_{2k}^1, x_{2l}^2) resulting if the second period action combination (a_2^1, a_2^2) is

chosen can be written as $\Pi_{kl}(a_2^1, a_2^2)$. This implies that second

period results cannot be influenced by first period actions.

These results can be used to determine the compensation of both agents in the two periods. First period compensations $s_1^a(i, j)$ can be based on both first period results. Second period results can be used, in combination with first period results, to determine the agents' compensation $s_2^a(i, j, k, l)$ in the second period. An agent's incentive scheme is a scheme where a first period remuneration is determined for every possible first period result combination (x_{1i}^1, x_{1j}^2) and a second period remuneration for every possible combination of first and second period results $(x_{1i}^1, x_{1j}^2, x_{2k}^1, x_{2l}^2)$.

The dynamic multiple agent model considered in this paper consists of three stages. In the first stage the principal

announces the agents' incentive schemes. The agents observe these schemes and use them to choose their strategies. In a second stage both agents simultaneously choose their first period actions out of A_1^a . Nature determines, together with the first period action combination, the probability distribution on first period results. First period results are observed by the principal and both agents. These results, together with the principal's incentive scheme, determine the compensation of both agents in the first period. In a third stage the agents simultaneously choose their second period actions out of A_2^a . Nature determines, together with the second period action combination, the probability distribution on second period results. Second period results are observed and both agents are remunerated on the basis of first and second period results, according to the principal's incentive scheme..

The principal's strategy is the determination of both agents' incentive schemes. Agents choose first and second period actions. A different second period action may be chosen for different first period result combinations (x_{1i}^1, x_{1j}^2) . We denote by $a_2^a(i, j)$ the strategy of agent a in the second period if x_{1i}^1 is the observed result of the first agent and x_{1j}^2 is the result of the second agent in the first period.

The agents' utility functions are permitted to vary with time and are given by:

$$\begin{aligned} U_1^a &= \text{utility function of agent } a \text{ in period 1} \\ &= V_1^a(s_1^a(i, j)) - G_1^a(a_1^a) \end{aligned}$$

$$\begin{aligned} U_2^a &= \text{utility function of agent } a \text{ in period 2} \\ &= V_2^a(s_2^a(i, j, k, l)) - G_2^a(a_2^a(i, j)) \end{aligned}$$

V_t^a are convex and G_t^a are concave functions.

The principal is supposed to be risk neutral. He maximizes his expected benefits minus expected compensations. Expected benefits are given by

$$\begin{aligned} & B(a_1^1, a_1^2, a_2^1(i, j), a_2^2(i, j)) \\ & = \sum_i \sum_j \Pi_{ij}(a_1^1, a_1^2) [F_1(x_{1i}^1, x_{1j}^2) + \sum_k \sum_l \Pi_{kl}(a_2^1(i, j), a_2^2(i, j)) F_2(x_{2k}^1, x_{2l}^2)] \end{aligned} \quad (1)$$

The functions F_1 and F_2 associate with every possible combination of results in the periods considered a gross benefit for the principal. Given strategy combinations $(a_1^1, a_1^2, a_2^1(i, j), a_2^2(i, j))$ and incentive schemes $(s_1^1(i, j), s_1^2(i, j), s_2^1(i, j, k, l), s_2^2(i, j, k, l))$ the expected costs of the principal equal

$$\begin{aligned} & C(a_1^1, a_1^2, a_2^1(i, j), a_2^2(i, j), s_1^1(i, j), s_1^2(i, j), s_2^1(i, j, k, l), s_2^2(i, j, k, l)) \\ & = \sum_i \sum_j \Pi_{ij}(a_1^1, a_1^2) [s_1^1(i, j) + s_1^2(i, j) + \sum_k \sum_l \Pi_{kl}(a_2^1(i, j), a_2^2(i, j)) (s_2^1(i, j, k, l) + s_2^2(i, j, k, l))] \end{aligned} \quad (2)$$

Analogous to the Grossman-Hart analysis of the basic agency problem (1983) the maximization problem of the principal can be decomposed into two stages. In a first step the least costly incentive scheme for every strategy combination of the two agents in both periods is derived. For each possible strategy combinations $(a_1^1, a_1^2, a_2^1(i, j), a_2^2(i, j))$ (2) is minimized with respect to $(s_1^1(i, j), s_1^2(i, j), s_2^1(i, j, k, l), s_2^2(i, j, k, l))$. In a second step the best strategy combination is chosen. This means that $[(1) - (2)]$ is maximized with respect to $(a_1^1, a_1^2, a_2^1(i, j), a_2^2(i, j))$. Properties of the optimal incentive scheme can be deduced from the first stage minimization problem.

New variables can be defined:

$$\begin{aligned} V_1^a(s_1^a(i, j)) &= v_1^a(i, j) \\ V_2^a(s_2^a(i, j, k, l)) &= v_2^a(i, j, k, l). \end{aligned}$$

These variables are the levels of utility the agents enjoy. Let h_1^a and h_2^a be the inverse cost of utility functions of V_1^a and V_2^a . This means that

$$\begin{aligned} h_1^a(v_1^a(i, j)) &= s_1^a(i, j) \\ h_2^a(v_2^a(i, j, k, l)) &= s_2^a(i, j, k, l). \end{aligned}$$

The problem² of minimizing (2) can be written using the new decision variables. Given $(\bar{a}_1^1, \bar{a}_1^2, \bar{a}_2^1(i, j), \bar{a}_2^2(i, j))$, the

following expression (3) is minimized with respect to $(v_1^1(i, j), v_1^2(i, j), v_2^1(i, j, k, l), v_2^2(i, j, k, l))$.

$$\text{Min } \sum_i \sum_j \Pi_{ij}(\bar{a}_1^1, \bar{a}_1^2) [h_1^1(v_1^1(i, j)) + h_1^2(v_1^2(i, j))] + \sum_k \sum_l \Pi_{kl}(\bar{a}_2^1(i, j), \bar{a}_2^2(i, j)) [h_2^1(v_2^1(i, j, k, l)) + h_2^2(v_2^2(i, j, k, l))] \quad (3)$$

$$\text{s. t. 1) } \sum_i \sum_j \Pi_{ij}(\bar{a}_1^1, \bar{a}_1^2) [v_1^1(i, j) + \sum_k \sum_l \Pi_{kl}(\bar{a}_2^1(i, j), \bar{a}_2^2(i, j)) v_2^1(i, j, k, l) - G_2^1(\bar{a}_2^1(i, j))] - G_1^1(\bar{a}_1^1) \geq \theta^1$$

$$\sum_i \sum_j \Pi_{ij}(\bar{a}_1^1, \bar{a}_1^2) [v_1^2(i, j) + \sum_k \sum_l \Pi_{kl}(\bar{a}_2^1(i, j), \bar{a}_2^2(i, j)) v_2^2(i, j, k, l) - G_2^2(\bar{a}_2^2(i, j))] - G_1^2(\bar{a}_1^2) \geq \theta^2$$

$$\text{s. t. 2) } \sum_k \sum_l \Pi_{kl}(\bar{a}_2^1(i, j), \bar{a}_2^2(i, j)) v_2^1(i, j, k, l) - G_2^1(\bar{a}_2^1(i, j)) \geq \sum_k \sum_l \Pi_{kl}(a_2^1(i, j), a_2^2(i, j)) v_2^1(i, j, k, l) - G_2^1(a_2^1(i, j)) \quad \forall(i, j), \forall a_2^1(i, j)$$

$$\sum_k \sum_l \Pi_{kl}(\bar{a}_2^1(i, j), \bar{a}_2^2(i, j)) v_2^2(i, j, k, l) - G_2^2(\bar{a}_2^2(i, j)) \geq \sum_k \sum_l \Pi_{kl}(a_2^1(i, j), a_2^2(i, j)) v_2^2(i, j, k, l) - G_2^2(a_2^2(i, j)) \quad \forall(i, j), \forall a_2^2(i, j)$$

$$\text{s. t. 3) } \sum_i \sum_j \Pi_{ij}(\bar{a}_1^1, \bar{a}_1^2) [v_1^1(i, j) + \sum_k \sum_l \Pi_{kl}(\bar{a}_2^1(i, j), \bar{a}_2^2(i, j)) v_2^1(i, j, k, l) - G_2^1(\bar{a}_2^1(i, j))] - G_1^1(\bar{a}_1^1) \geq$$

$$\sum_i \sum_j \Pi_{ij}(a_1^1, a_1^2) [v_1^1(i, j) + \sum_k \sum_l \Pi_{kl}(\bar{a}_2^1(i, j), \bar{a}_2^2(i, j)) v_2^1(i, j, k, l) - G_2^1(\bar{a}_2^1(i, j))] - G_1^1(a_1^1) \quad \forall a_1^1$$

$$\sum_i \sum_j \Pi_{ij}(\bar{a}_1^1, \bar{a}_1^2) [v_1^2(i, j) + \sum_k \sum_l \Pi_{kl}(\bar{a}_2^1(i, j), \bar{a}_2^2(i, j)) v_2^2(i, j, k, l) - G_2^2(\bar{a}_2^2(i, j))] - G_1^2(\bar{a}_1^2) \geq$$

$$\sum_i \sum_j \Pi_{ij}(a_1^1, a_1^2) [v_1^2(i, j) + \sum_k \sum_l \Pi_{kl}(\bar{a}_2^1(i, j), \bar{a}_2^2(i, j)) v_2^2(i, j, k, l) - G_2^2(\bar{a}_2^2(i, j))] - G_1^2(a_1^2) \quad \forall a_1^2$$

The first constraint is the participation condition for both agents. Both agents require a minimum expected utility θ^a over the two period horizon, when viewed from the beginning of the first period. The second constraint is the incentive compatibility constraint for both agents in the second period.

² We refer to Grossman and Hart (1983) for a discussion of existence problems associated with this minimization problem.

A subgame perfect Nash equilibrium³ is used to determine the agents' strategies. When an agent comes to the second period, he chooses his second period action conditional upon the information he has at that moment. A different second period action may be optimal from the principal's point of view for different combinations of first period results⁴. The third constraint is the incentive compatibility constraint in the first period. Given the optimal second period strategies, each agent chooses a first period action to maximize his two period expected utility.

II. Control and information in the dynamic multiple agent model

An agent's first and second period remuneration can respectively depend on results from the sets X_1^a and X_t^a . The central question of this paper is the presence of a given result in the principal's incentive scheme. The controllability principle and the dynamic multiple agent problem give different answers to this question. The remainder of this section will consider the incentive scheme of the first agent.

Three effects can be defined in a dynamic multiple agent model. The information effect considers the introduction of the second agent's first period result j in $v_1^1(i, j)$ and the second agent's second period result l in $v_2^1(i, j, k, l)$. The time effect deals with the dependency of the first agent's second period remuneration $v_2^1(i, j, k, l)$ on his own first period result i . The third (combined) effect looks at the use of the second agent's first period result j in $v_2^1(i, j, k, l)$.

³ A problem of multiple equilibria can arise in this model. Given the optimal incentive scheme to implement a subgame perfect Nash equilibrium, it is possible that both agents can increase their utilities by choosing another subgame perfect Nash equilibrium. Ma (1988) provides a costless solution for this problem in a one period multiple agent principal agent problem.

⁴ The notion subgame perfect Nash equilibrium is used in a rather unusual way, because subgames begin at decision nodes that begin at combinations of first period results and are no singleton information sets.

We first present a formal definition of the controllability principle. Because it is assumed that effort in one period has no effect on the results in the other period, the controllability principle says that the evaluation and remuneration of an agent in a given period should be independent of other period results. This means that $v_2^1(i, j, k, l)$ is independent of first period results i and j .

The first agent controls the second agent's result if the marginal probability function on that result, given the actions of the second agent, is a nontrivial function of his own actions. Define $\pi_j(a_1^1, a_1^2)$ as the marginal probability of the second agent's first period result given action combination (a_1^1, a_1^2) . The controllability principle tells us that the first period evaluation and remuneration of the first agent should be independent of the first period result of the second agent if

$$\Pi_j(\bar{a}_1^1, a_1^2) = \Pi_j(a_1^1, a_1^2) \quad \forall a_1^1, \forall \bar{a}_1^1, \forall a_1^2, \forall j \quad (3)$$

The first agent does not control the second agent's first period result if, given the second agent's action, he is not able to affect the probability distribution of this result by his own actions. According to the controllability principle, $v_1^1(i, j)$ should then be independent of j .

Define $\pi_1(a_2^1, a_2^2)$ as the marginal probability of the second agent's second period result given action combination (a_2^1, a_2^2) . The second period remuneration of the first agent $v_2^1(i, j, k, l)$ should be independent of the second agent's second period result if

$$\Pi_1(\bar{a}_2^1, a_2^2) = \Pi_1(a_2^1, a_2^2) \quad \forall a_2^1, \forall \bar{a}_2^1, \forall a_2^2, \forall l \quad (4)$$

The controllability principle can be confronted with the principal agent solution. Whereas effort in one period has no effect on the other period results, an agent's second period evaluation and remuneration will depend on his own first period result.

Proposition 1⁵

The second period incentive scheme of an agent will depend on his own first period result.

Proposition 1 considers the time effect and means that $v_2^1(i,j,k,l)$ will always depend on i . Because i gives information about a_1^1 it will also be used in $v_2^1(i,j,k,l)$. This gives a supplementary possibility to use the information content of i .

In order to better understand principal agent conclusions with respect to the introduction of results of other agents in the remuneration of a given agent, we describe the concept of sufficient statistics. Denote by x any data and by

$\Pi_x(\theta_1, \theta_2)$ the probability of the data given parameters θ_1 and

θ_2 . Any function $t(x)$ of the data is called a statistic. From statistical theory it is known that

$\Pi_x(\theta_1, \theta_2) = \Pi_{t(x)}(\theta_1, \theta_2) * \Pi_x(t(x), \theta_1, \theta_2)$ If this can be written as

[Lindley (1965)] $\Pi_x(\theta_1, \theta_2) = \Pi_{t(x)}(\theta_1, \theta_2) * \Pi_x(t(x), \theta_2)$, $t(x)$ is a

sufficient statistic for θ_1 . This means that $\Pi_x(t(x), \theta_1, \theta_2)$ does

not involve θ_1 . This definition is equivalent [Lindley (1965), Theorem 1, p.46] to the definition of De Groot, where a statistic $t(x)$ is defined as a sufficient statistic for θ "if, for any prior distribution of θ_1 , its posterior distribution depends on the observed value of x only through $t(x)$ " [De Groot (1970)] or that given parameter θ_2 an analysis based on $t(x)$, the sufficient statistic, is just as effective as an analysis based on all data x . It is also equivalent [Lindley (1965), Theorem 2, p.47] to Holmström's (1982) sufficient statistic formulation.

⁵ We refer to Appendix I for the proof of Propositions 1 to 4.

Propositions 2 to 4 introduce principal agent conclusions with respect to the use other agents' results in an incentive scheme.

Proposition 2

The first agent's first period optimal remuneration is independent of the second agent's first period result if his own first period result is a sufficient statistic for his first period action choice or⁶

$$\Pi_j(i, a_1^1, \bar{a}_1^2) = \Pi_j(i, \bar{a}_1^2) \quad \forall a_1^1, \forall i, \forall j \quad (5)$$

Proposition 2 considers the first period information effect. The first period remuneration $v_1^1(i, j)$ will not be affected by j if i is a sufficient statistic for a_1^1 .

Proposition 3

The first agent's second period remuneration is independent of the second agent's second period result if his own second period result is a sufficient statistic for his second period action choice or

$$\Pi_1(k, a_2^1(i, j), \bar{a}_2^2(i, j)) = \Pi_1(k, \bar{a}_2^2(i, j)) \quad \forall a_2^1(i, j), \forall k, \forall j \quad (6)$$

Proposition 3 looks at the second period information effect. The first agent's second period remuneration $v_2^1(i, j, k, l)$ will be independent of l if k is a sufficient statistic for a_2^1 .

⁶ Stronger results will be derived in Appendix I. There is a difference between the sufficient statistic concept and the conditions in Appendix I. Whereas the sufficient statistic condition has to hold for all possible actions, the condition in Appendix I is limited to those actions where the Lagrange multiplier is strictly positive.

Proposition 4

The first agent's second period remuneration is independent of the first period result of the second agent if the first period result of the first agent is a sufficient statistic for his first period action choice or⁷

$$\Pi_j(i, a_1^1, \bar{a}_1^2) = \Pi_j(i, \bar{a}_1^2) \quad \forall a_1^1, \forall i, \forall j \quad (5)$$

This last Proposition deals with the combined effect. If j gives supplementary information with respect to a_1^1 (information effect), it will also be used to determine $v_2^1(i, j, k, l)$. This is an extension of the time effect described in Proposition 1.

The controllability principle introduces a result in an agent's incentive scheme if it is controlled by the agent's action choice. The principal agent solution stresses the information content of the results considered. If a result gives supplementary information with respect to an agent's actions, this result should be used for the agent's remuneration.

III. An example

A simple example with two possible actions and two possible results for both agents in every period is presented. The probability distributions on first and second period results are given in Tables 1 and 2.

The first agent controls his own results and the second agent controls the results of both agents in every period. This means that, following the controllability principle, $v_1^1(i, j)$ should be independent of j and $v_2^1(i, j, k, l)$ should not be influenced by j , k and l . For the same reasons $v_1^2(i, j)$ should depend on i and j and $v_2^2(i, j, k, l)$ on k and l .

⁷ This condition reduces to the condition presented in Proposition 2.

	$x_1^1(1) \ x_1^2(1)$	$x_1^1(1) \ x_1^2(2)$	$x_1^1(2) \ x_1^2(1)$	$x_1^1(2) \ x_1^2(2)$
$a_1^1(1) \ a_1^2(1)$	0,48	0,1	0,32	0,1
$a_1^1(2) \ a_1^2(1)$	0,24	0,1	0,56	0,1
$a_1^1(1) \ a_1^2(2)$	0,24	0,3	0,16	0,3
$a_1^1(2) \ a_1^2(2)$	0,12	0,3	0,28	0,3

Table 1. Probability distributions period 1

	$x_2^1(1) \ x_2^2(1)$	$x_2^1(1) \ x_2^2(2)$	$x_2^1(2) \ x_2^2(1)$	$x_2^1(2) \ x_2^2(2)$
$a_2^1(1) \ a_2^2(1)$	0,48	0,11	0,32	0,09
$a_2^1(2) \ a_2^2(1)$	0,24	0,08	0,56	0,12
$a_2^1(1) \ a_2^2(2)$	0,24	0,25	0,16	0,35
$a_2^1(2) \ a_2^2(2)$	0,12	0,3	0,28	0,3

Table 2. Probability distributions period 2

The principal is risk neutral and minimizes his own expected costs. Both agents are risk averse and have utility functions of the form $\sqrt[3]{s}-a$ for both periods with $a = 0$ if the agent chooses action 1 and $a = 0,5$ if the agent chooses action 2. The reservation utilities of the agents are equal to 4. The principal induces action 2 for both agents in period 1 and action 2 for both agents in period 2 for every observed combination of first period results. The agents' levels of utility are presented in Table 3.

Because result j is a sufficient statistic for a_1^2 and i , k and l are no sufficient statistics for respectively a_1^1 , a_2^1 and a_2^2 , the dynamic multiple agent model rewards the agents in the following way. The first agent's remuneration $v_1^1(i,j)$ depends on i and j . Furthermore $v_2^1(i,j,k,l)$ is affected by i,j,k and l . The second agent's remuneration $v_1^2(i,j)$ is independent of i and $v_2^2(i,j,k,l)$ is influenced by j , k and l . This can be easily checked from Table 3.

	AGENT 1	AGENT 2
$v_1^a(1,1)$	0.30	2.17
$v_1^a(1,2)$	2.59	2.78
$v_1^a(2,1)$	3.09	2.17
$v_1^a(2,2)$	2.59	2.78
$v_2^a(1,1,1,1)$	0	1.30
$v_2^a(1,1,1,2)$	2.07	2.61
$v_2^a(1,1,2,1)$	3.30	1.35
$v_2^a(1,1,2,2)$	0	2.54
$v_2^a(1,2,1,1)$	0	1.93
$v_2^a(1,2,1,2)$	3.07	3.25
$v_2^a(1,2,2,1)$	3.71	1.98
$v_2^a(1,2,2,2)$	1.99	3.18
$v_2^a(2,1,1,1)$	0	1.30
$v_2^a(2,1,1,2)$	3.41	2.61
$v_2^a(2,1,2,1)$	3.87	1.35
$v_2^a(2,1,2,2)$	2.72	2.54
$v_2^a(2,2,1,1)$	0	1.93
$v_2^a(2,2,1,2)$	3.07	3.25
$v_2^a(2,2,2,1)$	3.71	1.98
$v_2^a(2,2,2,2)$	1.99	3.18

Table 3. Incentive schemes: principal agent solution

IV. Conclusion

The controllability principle says that an individual should be rewarded on the basis of the results under his own control. An individual controls a given result in a given period if he is able to affect the probability distribution of this result by his action choice.

This principle can be studied from a dynamic multiple agent point of view. Because effort is not observable, all possible results that give new information about the agents' action choices will be introduced in the principal agent solution. Three different inference effects can be defined.

The information effect states that another agent's first (second) period result is introduced in the first (second) period

remuneration of the agent considered if, given his own result and the other agent's first (second) period action, the agent influences the probability distribution of the other agent's result by his own first (second) period action.

The time effect exists when another period result is introduced in an agent's remuneration. With respect to the time effect, it can be shown that the second period evaluation and remuneration of the agent will depend on his own first period result.

The combined effect considers the introduction of another agent's first period remuneration in an incentive scheme. A second period remuneration will be determined by the other agent's first period result if this result is used for the first period remuneration.

Where control plays a central role in the controllability principle, it is information about an agent's actions that is crucial in the principal agent model considered.

References

Antle, R. and J.S. Demski, 1988, The Controllability Principle in Responsibility Accounting, *Accounting Review*, 700-717.

Bamberg, G. and K. Spremann, 1987, *Agency Theory, Information, and Incentives* (Springer-Verlag, Berlin).

Degroot, M.H., 1970, *Optimal Statistical Decisions* (Mc Graw-Hill Book Company).

Demski, J.S. and D.E. Sappington, 1984, Optimal Incentive Contracts with Multiple Agents, *Journal of Economic Theory*, 152-171.

Fellingham, J., D. Newman and Y. Suh, 1985, Contracts without memory in multiperiod agency models, *Journal of Economic Theory*, 340-355.

Fudenberg, D., B. Holmström and P. Milgrom, 1990, Short-Term Contracts and Long-Term Agency Relationships, *Journal of Economic Theory*, 1-31.

Green, J.R. and N.L. Stokey, 1983, A Comparison of Tournaments and Contracts, *Journal of Political Economy*, 349-364.

Grossman, S.J. and O.D. Hart, 1983, An analysis of the principal-agent problem, *Econometrica*, 7-45.

Holmström, B., 1979, Moral Hazard and Observability, *Bell Journal of Economics and Management Science*, 74-91.

Holmström, B., 1982, Moral hazard in teams, *Bell Journal of Economics*, 324-340.

Holmström, B. and P. Milgrom, 1990, Regulating Trade among Agents, *Journal of Institutional and Theoretical Economics*, 85-105.

Horngren, C.T. and G. Foster, 1991, *Cost accounting: a managerial emphasis* (Prentice Hall International, Englewood Cliffs).

Kreps, D.M., 1990, *A course in microeconomic theory* (Harvester Wheatsheaf, New York).

Laffont, J.J. and J. Tirole, 1988, The Dynamics of Incentive Contracts, *Econometrica*, 1153-1175.

Lambert, R., 1983, Long term contracting and moral hazard, *Bell Journal Of Economics*, 441-452.

Lambert, R., 1984, Income Smoothing as Rational Equilibrium Behavior, *Accounting Review*, 604-617.

Lindley, D.V., 1965, *Introduction to Probability and Statistics from a Bayesian Viewpoint: Part 2* (Cambridge University Press, Cambridge).

Ma, C., 1988, Unique Implementation of Incentive Contracts with Many Agents, *Review of Economic Studies*, 555-571.

Malcomson, J., 1986, Rank-Order Contracts for a Principal with Many Agents, *Review of Economic Studies*, 807-817.

Malcomson, J. and F. Spinneweyn, 1988, The multiperiod principal-agent problem, *Review of Economic Studies*, 391-407.

Malueg, D.A., 1986, Efficient Outcomes in a Repeated Agency Model without Discounting, *Journal of Mathematical Economics*, 217-230.

Milgrom, P. and J. Roberts, 1992, *Economics, organization and management* (Prentice Hall International, Englewood Cliffs).

Mookherjee, D., 1984, Optimal Incentive Schemes with Many Agents, *Review of Economic Studies*, 433-446.

Nalebuff, B.J. and J.E. Stiglitz, 1983, Prizes and incentives: towards a general theory of compensation and competition, *Bell Journal of Economics*, 21-43.

Picard, P., 1987, On the Design of Incentive Schemes under Moral Hazard and Adverse Selection, *Journal of Public Economics*, 305-331.

Radner, R., 1981, Monitoring Cooperative Agreements in a Repeated Principal-Agent Relationship, *Econometrica*, 1127-1148.

Radner, R., 1985, Repeated principal-agent problems with discounting, *Econometrica*, 1173-1198.

Rasmusen, E., 1989, *Games and information* (Basil Blackwell, Cambridge).

Rogerson, W., 1985, Repeated Moral Hazard, *Econometrica*, 69-76.

Ross, S.A., 1973, The Economic Theory of Agency: The Principal's Problem, *American Economic Review*, 134-139.

Rubinstein, A. and M. Yaari, 1983, Repeated Insurance Contracts and Moral Hazard, *Journal of Economic Theory*, 74-97.

Shavell, S., 1979, Risk sharing and incentives in the principal and agent relationship, *Bell Journal of Economics and Management Sciences*, 55-73.

Spear, S.E. and S. Srivastava, 1987, On Repeated Moral Hazard with Discounting, *Review of Economic Studies*, 599-617.

Strong, N. and M. Walker, 1987, *Information and Capital Markets* (Basil Blackwell, Cambridge).

Zou, L., 1992, Ownership Structure and Efficiency: An Incentive Mechanism Approach, *Journal of Comparative Economics*, 399-431.

Appendix I

The convexity of the objective function and the linearity of all constraints in (3) imply that the Kuhn-Tucker conditions⁸ are necessary and sufficient for the optimality of the incentive schemes.

The first and second period first order conditions with respect to the first agent's incentive scheme are respectively given by:

$$h_1^{1'}(v_1^1(i, j)) = \lambda_1 + \sum_{a_1^1} \alpha_1(a_1^1) \left[1 - \frac{\Pi_{ij}(a_1^1, \bar{a}_1^2)}{\Pi_{ij}(\bar{a}_1^1, \bar{a}_1^2)} \right] \quad (\text{A.1})$$

and

$$\begin{aligned} h_2^{1'}(v_2^1(i, j, k, l)) &= \lambda_1 \\ &+ \sum_{a_2^1} \frac{\alpha_3(a_2^1, i, j)}{\Pi_{ij}(\bar{a}_1^1, \bar{a}_1^2)} \left[1 - \frac{\Pi_{kl}(a_2^1(i, j), \bar{a}_2^2(i, j))}{\Pi_{kl}(\bar{a}_2^1(i, j), \bar{a}_2^2(i, j))} \right] \\ &+ \sum_{a_1^1} \alpha_1(a_1^1) \left[1 - \frac{\Pi_{ij}(a_1^1, \bar{a}_1^2)}{\Pi_{ij}(\bar{a}_1^1, \bar{a}_1^2)} \right] \end{aligned} \quad (\text{A.2})$$

Characteristics of the second agent's incentive scheme can be derived in an analogous way.

Lemma 1

The participation constraint for any implemented strategy for both agents is binding in the corresponding optimal incentive scheme for each agent.

Proof: Suppose the participation constraint in (3) is not binding for agent 1. Then the agent's expected utility exceeds his reservation utility. If each $v_1^1(i, j)$ is replaced by $[v_1^1(i, j) - \epsilon]$, with $\epsilon > 0$ and sufficiently small, the expected

⁸ The Lagrange multipliers associated with the constraints in (3) are denoted as λ_1 and λ_2 for the participation constraints, $\alpha_1(a_1^1)$ and $\alpha_2(a_1^2)$ for the first period incentive compatibility constraints and $\alpha_3(a_2^1, i, j)$ and $\alpha_4(a_2^2, i, j)$ for the second period incentive compatibility constraints.

costs for the principal can be reduced and all the other constraints of the minimization problem will still be satisfied. The same holds for each $v_2^1(i,j,k,l)$. The participation constraint of agent 2 is binding for the same reasons. Q.E.D.

Lemma 2

The Lagrange multiplier on the second period incentive compatibility constraint is strictly positive for each agent for each combination (i,j) for at least one second period action or

$$\forall(i,j) : \exists a_2^1(i,j) : \alpha_3(a_2^1, i, j) > 0$$

$$\forall(i,j) : \exists a_2^2(i,j) : \alpha_4(a_2^2, i, j) > 0$$

Proof: Suppose $\alpha_3(a_2^1, i, j)$ is equal to 0 for some combination (i,j) and every second period action of agent 1. Then $h_2^{1'}(v_2^1(i,j,k,l))$ is the same for all combinations (k,l) given first period results (i,j) . In this case agent 1 will choose an effort minimizing second period action for any combination (i,j) . The same is true for agent 2. Q.E.D.

Lemma 3

The Lagrange multiplier on the first period incentive compatibility constraint in (1) is strictly positive for each agent for at least one first period action or

$$\exists a_1^1 : \alpha_1(a_1^1) > 0$$

$$\exists a_1^2 : \alpha_2(a_1^2) > 0$$

Proof: Suppose $\alpha_1(a_1^1)$ is equal to 0 for every first period action of agent 1. Then $h_1^{1'}(v_1^1(i,j))$ is the same for all combinations (i,j) . This implies that $v_1^1(i,j)$ is the same for all first period result combinations (i,j) or that the first agent is not able to influence $v_1^1(i,j)$ by his first period action choice. Also he is not able to influence $v_2^1(i,j,k,l)$ by his first period action since $h_2^{1'}(v_2^1(i,j,k,l))$ is not influenced by

different first period action choices of the first agent. In this case agent 1 will choose the effort minimizing first period action. The same is true for agent 2. Q.E.D.

Since h_a^t is a convex function, $h_a^{t'}$ is an increasing function. A larger hand side in the first order conditions (A.1 and A.2) means a larger value of the payment made in utilities. A base payment (Lemma 1) is made to both agents. Lemma 2 and Lemma 3 ensure that agents' incentive schemes depend on the probability distributions on first and second period results.

Proposition 1

The second period incentive scheme of an agent will depend on his own first period result.

Proof: Consider the first order conditions with respect to the first agent's second period incentive scheme (A.2). Because the Lagrange multiplier on the first period incentive compatibility constraint is strictly positive for each agent for at least one first period action (Lemma 3), the first agent's second period incentive scheme will depend on his own first period result. The same holds for the second agent. Q.E.D.

Proposition 2

The first agent's first period optimal evaluation and remuneration is independent of the second agent's first period result if his own first period result is a sufficient statistic for his first period action choice or more specifically

$$\Pi_j(i, a_1^1, \bar{a}_1^2) = \Pi_j(i, \bar{a}_1^2) \quad \forall a_1^1: \alpha(a_1^1) > 0; \forall i; \forall j \quad (\text{A.3})$$

Proof: The first order condition of agent 1 with respect to his first period incentive scheme is given by (A.1). If (A.3) holds, this first order condition can be reduced to

$$h_1^{1'}(v_1^1(i, j)) = \lambda_1 + \sum_{a_1^1} \alpha_1(a_1^1) \left[1 - \frac{\Pi_i(a_1^1, \bar{a}_1^2)}{\Pi_i(\bar{a}_1^1, \bar{a}_1^2)} \right]. \quad \text{The optimal incentive}$$

scheme of agent 1 is independent of the first period result of agent 2. If the optimal incentive scheme of agent 1 is

independent of the second result, $\sum_{a_1^1} \alpha_1(a_1^1) [1 - \frac{\Pi_{ij}(a_1^1, \bar{a}_1^2)}{\Pi_{ij}(\bar{a}_1^1, \bar{a}_1^2)}]$ is

independent of j . This is certainly the case for Lagrange multipliers $\alpha_1(a_1^1)$ equal to 0. If $\alpha_1(a_1^1) > 0$, $\Pi_j(i, \bar{a}_1^2)$ may not

involve a_1^1 unless the Lagrange multiplier vector $\{\alpha_1(a_1^1)\}$ is such that even with every component of the likelihood ratio vector

$\{\Pi_{ij}(a_1^1, \bar{a}_1^2) | \Pi_{ij}(\bar{a}_1^1, \bar{a}_1^2)\}$ varying with j , their inner product is

independent of j . The latter situation arises only by accident (Mookherjee, 1984). Q.E.D.

Proposition 3

The first agent's second period evaluation and remuneration is independent of the second agent's second period result if his own second period result is a sufficient statistic for his second period action choice or more specifically

$$\Pi_1(k, a_2^1(i, j), \bar{a}_2^2(i, j)) = \Pi_1(k, \bar{a}_2^2(i, j)) \quad \forall a_2^1(i, j) : \alpha_3(a_2^1, i, j) > 0; \forall k; \forall j$$

(A.3)

Proof: The first order condition of agent 1 with respect to his second period incentive scheme is given by (A.2). If (A.3) holds, this first order condition can be reduced to

$$\begin{aligned} h_2^{1'}(v_2^1(i, j, k, l)) &= \lambda_1 \\ &+ \sum_{a_2^1} \frac{\alpha_3(a_2^1, i, j)}{\Pi_{ij}(\bar{a}_1^1, \bar{a}_1^2)} [1 - \frac{\Pi_k(a_2^1(i, j), \bar{a}_2^2(i, j))}{\Pi_k(\bar{a}_2^1(i, j), \bar{a}_2^2(i, j))}] \\ &+ \sum_{a_1^1} \alpha_1(a_1^1) [1 - \frac{\Pi_{ij}(a_1^1, \bar{a}_1^2)}{\Pi_{ij}(\bar{a}_1^1, \bar{a}_1^2)}] \end{aligned}$$

The optimal incentive scheme of agent 1 is independent of the second period result of agent 2. The necessity condition is proved in an analogous way as in Proposition 2. Q.E.D.

Proposition 4

The first agent's second period evaluation and remuneration is independent of the first period result of the second agent if the first period result of the first agent is a sufficient statistic for his first period action choice or more specifically

$$\Pi_j(i, a_1^1, \bar{a}_1^2) = \Pi_j(i, \bar{a}_1^2) \quad \forall a_1^1: \alpha(a_1^1) > 0; \forall i; \forall j \quad (\text{A.4})$$

Proof: The first order condition of agent 1 with respect to his second period incentive scheme is given by (A.2). If (A.4) holds, this first order condition can be reduced to

$$\begin{aligned} h_2^{1'}(v_2^1(i, j, k, l)) &= \lambda_1 \\ &+ \sum_{a_2^1} \frac{\alpha_3(a_2^1, i, j)}{\Pi_i(\bar{a}_1^1, \bar{a}_1^2)} \left[1 - \frac{\Pi_{kl}(a_2^1(i, j), \bar{a}_2^2(i, j))}{\Pi_{kl}(\bar{a}_2^1(i, j), \bar{a}_2^2(i, j))} \right] \\ &+ \sum_{a_1^1} \alpha_1(a_1^1) \left[1 - \frac{\Pi_i(a_1^1, \bar{a}_1^2)}{\Pi_i(\bar{a}_1^1, \bar{a}_1^2)} \right] \end{aligned}$$

The optimal incentive scheme of agent 1 is independent of the first period result of agent 2. The necessity condition is proved in an analogous way as in Proposition 2. Q.E.D.

Propositions 2, 3 and 4 can be formulated in an analogous way for the second agent.

