

# **DEPARTEMENT TOEGEPASTE ECONOMISCHE WETENSCHAPPEN**

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## **A mathematical Programming Approach for Supplier Selection using Activity Based Costing**

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SELECTION USING ACTIVITY BASED COSTING**

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**Abstract**

Vendor selection is an important problem in today's competitive environment. Decisions involve the selection of vendors and the determination of order quantities to be placed with the selected vendors. In this research we develop a mathematical programming model for this purpose using an Activity Based Costing approach. The system computes the total cost of ownership, thereby increasing the objectivity in the selection process and giving the opportunity for different kinds of sensitivity analysis. Moreover, it allows the analyst to objectively evaluate alternative purchasing policies due to the underlying analytic and rigorous decision model.

*(Activity Based Costing ; Mixed Integer Linear Programming ; Supplier Selection ; Decision Analysis ; Accounting/Operations Research Interface)*

**1. Introduction**

In this paper we propose a mathematical programming approach using Activity Based Costing information to select several suppliers for several orders over a specific time horizon. This research is motivated by the fact that parts and components are mostly procured outside the company in the open market. In industrialized countries a general shift has been observed away from a vertical integration strategy towards a focused strategy on the core business of the company. In the face of growing global competition and the demands that this places on the management of resources, companies are forced to take advantage of any opportunity to improve their resource utilization.

Supplier selection greatly affects the firm's ability to compete in the market place as purchasing frequently accounts for a large percentage of a product's costs and may involve

long term contracts. The component procurement decision is complicated by the amount and diversity of the potential suppliers. Trying to reap the benefits of a focused strategy such as high efficiency in manufacturing and economies of scale, the problem of vendor selection has intensified enormously and will continue to do so in the near future.

Vendor selection decisions are multi-objective in nature. At least 23 criteria for various selection problems have been identified in the literature (Weber and Current 1991). Traditional approaches were based on selecting least invoice cost suppliers. In modern production environments applying such systems as Just-in-time, Total quality management and Flexible Manufacturing Systems, other criteria become more and more important. Amongst the most significant ones are quality, delivery reliability, electronic data interchange, geographical location and production capacity (Van Holderbeke 1996).

Most of the research dealing with procurement decisions is concerned with selecting one supplier for one order given the multi-objective nature of the problem. Different methods have been suggested in the literature. The simplest one is the “categorical method” (Timmerman 1986) where the relevant criteria for different suppliers are simply categorized in a limited number of classes (e.g. good, satisfactory and unsatisfactory). Many articles discuss the use of “linear weighting models”. The different criteria are weighted and the supplier with the best weighted total score, defined as the weighted sum of the subscores on the different criteria, is selected. Narasimhan (1983) uses the “analytical hierarchy process model” to solve the selection decision. This approach gives the opportunity to determine relative positions of the different suppliers on the basis of pairwise comparison of weights and subscores. Another method used to select a supplier is “data envelopment analysis” (Kleinsorge, Schary and Tanner 1992, Weber 1996). This is a performance evaluation tool based on the linear programming technique in which both quantitative and qualitative elements can be introduced.

Finally, Roodhooft and Konings (1997) use an Activity Based Costing approach for solving the problem. They argue that the above methods rank the criteria ad hoc and that the weights given to different criteria and the ranking of the suppliers on these criteria could reflect subjectivity. They propose to use a net adjusted income cost figure based on activity and cost driver information taking into account supplementary costs caused by the supplier. This corresponds to the total cost of ownership concept that received a lot of attention in recent years (Carr and Ittner 1992, Cavinato 1992, Ellram and Siferd 1993, Ellram 1995).

The literature review above deals with selecting one supplier for one order. However, as Akinc (1993) notes, when the number of parts and components externally bought is large, decisions pertaining to which vendors to keep, what to buy from them and which ones to drop are not as simple as finding the best single source for each part. Mathematical programming models can then be used to solve such problems. The objective is to select the best combination of suppliers for one or more orders on the basis of the criteria defined, given different constraints.

Weber and Current (1993) present a multi-objective approach to generate various options for one given item in a division of a Fortune 500 company. They use price, time and quality data. Akinc (1993) proposes a mathematical programming model with sole sourcing for all parts. Quantity discounts in purchase price make the problem more complicated (Chaudry, Forst and Zydiak 1993, Rosenthal, Zydiak and Chaudry 1995, Sadrian and Yoon 1994). Two major problems associated with these models are the quantification of the criteria and the solution to the trade-off between the criteria.

Vendor selection using an Activity Based Costing system is choosing the combination of suppliers that minimizes the total costs, consisting of price and costs beyond price incurred by the purchasing company, associated with the purchasing decision. As supplier selection and purchasing quantity determination is not a once and for all decision but rather a dynamic

process over time, we will develop a multi-period, multi-item, multi-vendor mathematical optimization program leading to a management decision support system. The system will explicitly take into account constraints on (1) minimum or maximum quantities to buy established by the vendors as well as by the purchasing company, (2) quantity discounts on total quantity possibly over several orders combined, (3) supply lead time and inventory holding costs determining when and how much to order from whom, (4) consolidation of varying numbers of parts to the same supplier, (5) vendor capacities, (6) minimum or maximum number of vendors to employ, (7) geographic preferences and (8) costs associated by using particular suppliers. This will result in a mixed integer linear programming decision model that can be solved with techniques from the field of operations research.

The multidisciplinary approach used in the paper results in a threefold contribution. First, the use of Activity Based Costing tools for supplier selection permits us to determine an objective function that minimizes the total cost of ownership associated with the purchasing decision thereby increasing objectivity in the selection process. Second, we consider the vendor selection problem as a dynamic process so that a multi-period purchasing policy will be developed. Third, the model allows us to evaluate alternative purchasing policies in cost terms by performing different kinds of sensitivity analysis.

The remainder of this paper is organized as follows. In section 2, we consider the use and advantages of Activity Based Costing systems for supplier selection. The mathematical programming model that can be developed to solve the selection problem is introduced in section 3. In section 4, we present a case study to motivate and illustrate the approach. A summary and final conclusions are presented in section 5.

## 2. The Use of ABC Information for Supplier Selection

Management accounting is a system that provides information to decision makers inside the company in order to make better decisions. The analysis of costs throughout the extended value chain of a company is an important topic in today's management accounting literature (Shank and Govindarajan 1992). Activity Based Costing permits us to analyze activities and determine cost drivers for the different activities defined. There exists an important literature dealing with possible applications of the system such as customer profitability analysis, performance management, cost management and pricing decisions.

While suppliers are an important part of the total value chain analysis, the application of Activity Based Costing ideas to the vendor selection problem has received little attention. Roehm, Critchfield and Castellano (1992) discuss the use of the system in a purchasing department. They assign additional purchasing costs to products, but not to suppliers. Ellram (1995) studied the total cost of ownership approach based on 11 case studies. She concludes that Activity Based Costing represents an important opportunity for purchasing. She also states that the supplier selection decision is one of the three major uses of these models.

This paper proposes a multi-vendor, multi-item, multi-period approach for vendor selection based on activity and cost driver information. Ellram and Siferd (1993) and Benett (1996) give an overview of possible activities and cost drivers that can be used to calculate supplementary internal costs caused by the suppliers. In our approach we recognize a hierarchical structure in activities with respect to the purchasing decision : (1) the supplier level, (2) the order level and (3) the unit level activities. Additional internal costs can be defined on these levels.

The use of Activity Based Costing in the mathematical programming decision model has several advantages. First, it is important to note that the quantification of the criteria and



the trade-off between them is no longer a problem because the objective function is defined as the total cost of ownership with respect to the purchasing decision caused by the suppliers. Second, an important advantage of this approach over other methodologies exists in arriving at objective cost measures in a systematic way. Third, the system will enable companies to develop interorganizational activity based management opportunities given the importance of close relationships between the purchaser and a limited number of reliable suppliers. Fourth, the model allows us to answer all sorts of “what if” questions dealing with cost management and strategic decision making such as (1) the cost impact of making different/alternative supplier selections, (2) the consequences of performance improvement by suppliers with respect to the different important criteria and the reduction or elimination by the purchasing company of some of the costs or activities caused by the purchasing decision and (3) the evaluation of alternative company policies with respect to the number of suppliers, order quantities, and minimum and/or maximum quantities to buy.

### **3. The Mathematical Decision Model**

In this section, we present a mathematical programming decision model that can be formulated for supplier selection and order quantity determination. In general, it derives a multi-period purchasing policy minimizing the total cost of ownership taking into account different constraints relevant to the problem. The only assumption used is the fact that the company can place at most one order per time period with each supplier. This assumption is not restrictive, however, as the typical order frequency could determine the length of the time bucket to be a month, a week or even a day.

Before stating the model, we give a summary list of the notation for later reference. The following primitive sets, grouping the key elements of the model, are used :

- $N$  : set of products, index  $i$ ,  
 $M$  : set of time periods, e.g. weekly or monthly time buckets, index  $t$ ,  
 $P$  : set of suppliers, index  $s$ ,  
 $C_s$  : set of discount intervals given by supplier  $s$ ,  $\forall s \in P$ , index  $r$ .

The parameters indicate the data required. We essentially distinguish three hierarchical levels of activities into which the parameters can be subdivided : (1) the supplier level, (2) the order level and (3) the unit level. The first hierarchical level, the supplier level parameters, describe costs incurred and conditions imposed whenever the purchasing company actually uses the supplier over the decision horizon. Some examples are as follows :

- $qc_s$  : quality audit cost incurred by the buyer for the evaluation of supplier  $s$ ,  $\forall s \in P$ ,  
 $mc_s$  : cost of a dedicated purchasing manager for supplier  $s$ ,  $\forall s \in P$ ,  
 $ec_s$  : additional research and development costs due to using supplier  $s$ ,  $\forall s \in P$ ,  
 $slc$  : total supplier level costs,  
 $mins$  : minimum number of suppliers to use,  
 $maxs$  : maximum number of suppliers to use.

The second hierarchical level, the order level parameters, indicate costs incurred and conditions imposed each time an order is placed with a particular supplier. Some important examples of those parameters are as follows :

- $rc_s$  : reception cost per order when purchasing from supplier  $s$ ,  $\forall s \in P$ ,

- $vc_s$  : invoice cost per order placed with supplier  $s$ ,  $\forall s \in P$ ,
- $tc_s$  : transportation cost per order when purchasing from supplier  $s$ ,  $\forall s \in P$ ,
- $oc_s$  : order cost per order when purchasing from supplier  $s$ ,  $\forall s \in P$ ,
- $nc$  : credit notes' cost per credit note,
- $pc_s$  : probability of having a credit note per invoice from supplier  $s$ ,  $\forall s \in P$ ,
- $olc$  : order level costs,
- $minb_s$  : minimum order quantity required by the buyer when purchasing from supplier  $s$ ,  
 $\forall s \in P$ ,
- $maxb_s$  : maximum order quantity required by the buyer when purchasing from supplier  $s$ ,  
 $\forall s \in P$ ,
- $st_s$  : number of periods safety time imposed by the buyer to compensate for delivery uncertainty when purchasing from supplier  $s$ ,  $\forall s \in P$ ,
- $lb_{sr}$  : minimum total quantity to buy in discount interval  $r$  set by supplier  $s$ ,  $\forall s \in P$ ,  
 $\forall r \in C_s$ ,
- $ub_{sr}$  : maximum total quantity to buy in discount interval  $r$  set by supplier  $s$ ,  $\forall s \in P$ ,  
 $\forall r \in C_s$ ,
- $dc_{sr}$  : price discount as a percentage given by supplier  $s$  in discount interval  $r$ ,  $\forall s \in P$ ,  
 $\forall r \in C_s$ .

The third hierarchical level, the unit level parameters, specify costs incurred and conditions imposed related to the units of the products for which a procurement decision has to be made. Some examples are as follows :

- $sc_i$  : setup cost per setup for product  $i$ ,  $\forall i \in N$ ,

- $p_{s_i}$  : probability of defects per unit of product  $i$  bought from supplier  $s$ ,  $\forall s \in P, \forall i \in N$ ,
- $fc_{s_i}$  : the external failure cost of product  $i$  bought from supplier  $s$ ,  $\forall s \in P, \forall i \in N$ ,
- $pf_{s_i}$  : probability of a failure in the field of product  $i$  bought from supplier  $s$ ,  $\forall s \in P$ ,  
 $\forall i \in N$ ,
- $p_{s_i}$  : price of product  $i$  from supplier  $s$ ,  $\forall s \in P, \forall i \in N$ ,
- $h_i$  : inventory holding cost per unit of product  $i$ , per period as a percentage of the  
product's price,  $\forall i \in N$ ,
- $ulc$  : total unit level costs,
- $aulc$  : the additional unit level costs,
- $purc$  : the purchasing cost,
- $invc$  : the inventory holding cost,
- $b_i$  : beginning inventory of product  $i$ ,  $\forall i \in N$ ,
- $d_{it}$  : demand for product  $i$  in period  $t$ ,  $\forall i \in N, \forall t \in M$ .

As for the parameters, the decision variables can also be subdivided into the same three hierarchical levels. The supplier level decision variable models whether or not the supplier will be used by the purchasing company over the planning horizon and is as follows :

$$z_s = 1, \text{ if we buy from supplier } s, 0, \text{ otherwise, } \forall s \in P.$$

The order level decision variables model characteristics of the individual orders placed with the suppliers used. They are defined as follows :

$$y_{st} = 1, \text{ if we buy from supplier } s \text{ in period } t, 0, \text{ otherwise, } \forall s \in P, \forall t \in M,$$

$x_{tst}$  = total amount bought from supplier  $s$  in period  $t$ ,  $\forall s \in P, \forall t \in M$ ,

$v_{srt}$  = 1, if we buy from supplier  $s$  in discount interval  $r$  in period  $t$ , 0, otherwise,  $\forall s \in P$ ,  
 $\forall r \in C_s, \forall t \in M$ ,

$xdt_{srt}$  = total amount bought from supplier  $s$  in discount interval  $r$  in period  $t$ ,  $\forall s \in P$ ,  
 $\forall r \in C_s, \forall t \in M$ .

The unit level decision variables pertain to the units of the products for which a procurement decision has to be made and are defined as follows :

$x_{sit}$  = amount bought of product  $i$  from supplier  $s$  in period  $t$ ,  $\forall s \in P, \forall i \in N, \forall t \in M$ ,

$inv_{it}$  : inventory of product  $i$  at end of period  $t$ ,  $\forall i \in N, \forall t \in M$ ,

$xd_{srti}$  = amount bought of product  $i$  from supplier  $s$  in discount interval  $r$  in period  $t$ ,  
 $\forall s \in P, \forall r \in C_s, \forall t \in M, \forall i \in N$ .

With the notation given above, the mathematical decision model is described below.

Objective : minimize the total cost of ownership ;

$$\text{Min} \quad slc + olc + ulc \quad (1)$$

The objective function (1), which is used to evaluate alternative procurement policies, is a minimization of the total cost of ownership and reflects the cost data in the three hierarchical levels distinguished.

Define the supplier level costs ;

$$slc = \sum_{s \in P} (qc_s + mc_s + ec_s) * z_s \quad (2)$$

The supplier level costs are incurred whenever the purchasing company actually uses supplier  $s$  over the planning horizon, i.e.  $z_s = 1$ . The quality audit cost is not incurred if the supplier  $s$  is not present in the pool of suppliers, i.e.  $z_s = 0$ . This is also the case for the cost of a dedicated purchasing manager who can be put to some alternative use if supplier  $s$  is not chosen. Similarly for the additional research and development costs due to supplier  $s$ . Those costs could potentially be negative if the supplier is actually performing some research and development thereby eliminating this investment for the purchasing company.

Define the order level costs ;

$$olc = \sum_{s \in P} \sum_{t \in M} (rc_s + vc_s + tc_s + oc_s + nc * pc_s) * y_{st} \quad (3)$$

The order level costs are incurred only in those time periods  $t$  when an order is placed with a particular supplier, i.e.  $y_{st} = 1$ . They could consist of the reception cost, the invoice cost, the transportation cost, the ordering cost and the credit notes' cost per order when purchasing from supplier  $s$ .

Define the unit level costs ;

$$ulc = aulc + purc + invc \quad (4)$$

Specifically, the unit level costs consist of the additional unit level costs, the purchasing cost and the inventory holding cost.

Define the additional unit level costs ;

$$aulc = \sum_{i \in N} \sum_{s \in P} \sum_{t \in M} (sc_i * ps_{si} + fc_{si} * pf_{si}) * x_{sit} \quad (5)$$

The additional unit level costs (5) incurred due to ordering from supplier  $s$  in time period  $t$  depend on the total order size,  $x_{sit}$ . There are several components to this cost from which we give two examples. Additional unit level costs are incurred whenever an *additional* setup is required due to problems with the products bought from supplier  $s$ . They are computed as the product of the setup cost per setup for product  $i$ ,  $sc_i$ , with the probability that a defect might happen with a unit of the product during production resulting in extra setups. The probabilities can be computed from historic time series data that are recorded for each supplier  $s$  or from an estimate based on experience. For generality, the setup cost is defined to be product dependent, in case the setup cost is only process dependent we can set  $sc_i$  equal for all products  $i$  that require the particular process. A second example relates to the internal failure costs incurred when a product breaks down during usage in the field and its cause can be attributed to a component bought from supplier  $s$ . Although we realize that some of the cost items might be hard to quantify, we are convinced that in order to be conceptually correct we should take a systems view w.r.t. the costs caused by the supplier. The presence of well developed ABC systems in industry have led us to believe that not only procurement costs should be considered in the sound derivation of a purchasing policy but also additional costs incurred due to problems with the suppliers' products during manufacturing and operation in the field. Moreover, an excellent ABC system will allow analysts to quantify many of the relevant cost elements.

Define the purchasing cost ;

$$purc = \sum_{s \in P} \sum_{r \in C_s} \sum_{t \in M} \sum_{i \in N} p_{si} * (1 - dc_{sr}) * xd_{srti} \quad (6)$$

The purchasing cost depends on the total amount of product  $i$  bought from supplier  $s$  in discount interval  $r$  in each time period  $t$  when an order is placed,  $xd_{srti}$ . This determines the discount rate,  $dc_{sr}$ , that has to be applied to the price,  $p_{si}$ , of the product  $i$ .

Define the inventory holding cost ;

$$invc = \sum_{i \in N} \sum_{t \in M} h_i * \bar{p}_i * inv_{it} \quad (7)$$

The inventory holding cost applies to the total amount of each product  $i$  held in inventory in each time period  $t$ ,  $inv_{it}$ . A supplier selection model should consider inventories explicitly and thus be inherently dynamic, as there is the potential trade-off between ordering more and thus receiving a quantity price discount and the cost of keeping the extra amounts in inventory. To compute the true inventory holding cost and the resulting optimal amount of inventory to hold, the value of the products in inventory should be known. Due to the possible discounts this presents a complex combinatorial problem. Specifically, we do not know the products' prices before the purchasing decision has been made, but we cannot make a purchasing decision before the inventories are known which optimal amounts will result from the products' prices. We could potentially split up the inventory of similar products according to their supplier sources, but then, we would have to know the company's consumption pattern of those inventories to determine the inventory value. This would certainly lead us into unnecessary detailed and complicating issues away from the original problem. Therefore



we take the average price of the product in the market to determine the inventory value of a product,  $\bar{p}_i$ , to which the product's inventory holding cost  $h_i$  has to be applied.

This concludes the derivation of the objective function. Constraints relevant to the procurement problem are as follows.

Satisfy the demand ;

$$b_i + \sum_{s \in P} x_{si(1-st_s)} - inv_{i1} = d_{i1} \quad \forall i \in N \quad (8a)$$

$$inv_{it-1} + \sum_{s \in P} x_{si(t-st_s)} - inv_{it} = d_{it} \quad \forall i \in N, \forall t \in M \setminus \{1\} \quad (8b)$$

The demand for each product in the first period,  $d_{i1}$ , modeled by constraint (8a), can be satisfied either from beginning inventory,  $b_i$ , and/or from purchases from the potential suppliers,  $x_{si(1-st_s)}$ . The amount that remains is end-of-period inventory,  $inv_{i1}$ . To compensate for delivery uncertainty of supplier  $s$ , some companies prefer to implement a safety time offset,  $1-st_s$ , resulting in ordering earlier. However, a late delivery could also result in additional costs, e.g. additional planning and setup costs, additional reception and invoicing costs, which should be taken into account in the objective function either at the order level (3) or at the unit level (5) depending on the relevancy, using a historic probability for late deliveries by supplier  $s$ . Constraints (8b) model the demand for each product  $i$  in later time periods,  $d_{it}$ . This demand is satisfied either from begin-of-period inventory, which equals the ending inventory of the previous period,  $inv_{it-1}$ , and/or from purchases from the potential suppliers,  $x_{sit}$ . Again, the amount that remains is end-of-period inventory,  $inv_{it}$ .

Compute the total amount bought from each supplier ;

$$xt_{st} = \sum_{i \in N} x_{sit} \quad \forall s \in P, \forall t \in M \quad (9)$$

The total amount bought from each supplier  $s$  in each time period  $t$  is the sum of the quantities of each product bought from that supplier in that time period. This total amount has to be known to enforce the following condition.

Impose the minimum and maximum purchasing quantity required by the buyer ;

$$xt_{st} \geq \min b_s * y_{st} \quad \forall s \in P, \forall t \in M \quad (10a)$$

$$xt_{st} \leq \max b_s * y_{st} \quad \forall s \in P, \forall t \in M \quad (10b)$$

$$x_{sit} \leq \left( \sum_{l \in M, l \geq t} d_{il} \right) * y_{st} \quad \forall s \in P, \forall i \in N, \forall t \in M \quad (10c)$$

The conditions above impose that in case an order is placed with supplier  $s$  in period  $t$ ,  $y_{st} = 1$ , then the total amount bought should be at least the minimum (10a),  $\min b_s$ , and at most the maximum (10b),  $\max b_s$ , buying quantity required by the purchasing company. If an order is not placed with supplier  $s$  in period  $t$ ,  $y_{st} = 0$ , condition (10c) will enforce that the amounts of each product that can be bought from the supplier will indeed be zero.

Enforce the bounds on number of suppliers used ;

$$\sum_{s \in P} z_s \geq \min s \quad (11a)$$

$$\sum_{s \in P} z_s \leq \max s \quad (11b)$$

$$z_s \leq \sum_{t \in M} y_{st} \quad \forall s \in P \quad (11c)$$

$$y_{st} \leq z_s \quad \forall s \in P, \forall t \in M \quad (11d)$$

The conditions (11a) and (11b) force the purchasing plan to have at least the minimum number, mins, and at most the maximum number, maxs, of suppliers over the complete time horizon. Using constraint (11c), the decision variable  $z_s$  will be equal to 0, if the model suggests not to buy from the supplier  $s$ , while constraint (11d) forces  $z_s$  to be equal to 1, if during some time period  $t$ , an order has been placed with supplier  $s$ .

Model the discounts ;

$$xd_{srti} \leq \min \left( ub_{sr}, \sum_{l \in M, l \geq t} d_{il} \right) * v_{srt} \quad \forall s \in P, \forall r \in C_s, \forall t \in M, \forall i \in N \quad (12a)$$

$$x_{sit} = \sum_{r \in C_s} xd_{srti} \quad \forall s \in P, \forall i \in N, \forall t \in M \quad (12b)$$

$$xdt_{srt} = \sum_{i \in N} xd_{srti} \quad \forall s \in P, \forall r \in C_s, \forall t \in M \quad (12c)$$

$$xdt_{srt} \geq lb_{sr} * v_{srt} \quad \forall s \in P, \forall r \in C_s, \forall t \in M \quad (12d)$$

$$xdt_{srt} \leq ub_{sr} * v_{srt} \quad \forall s \in P, \forall r \in C_s, \forall t \in M \quad (12e)$$

$$y_{st} = \sum_{r \in C_s} v_{rst} \quad \forall s \in P, \forall t \in M \quad (12f)$$

The constraints (12a) - (12f) model the discounts. In real-life business practice, the discount usually applies to the total quantity bought, irrespective of the product mix (Sadrian and Yoon 1994). Each supplier specifies a number of discount intervals,  $C_s$ , that are bounded by a minimum and maximum total purchasing quantity,  $lb_{sr}$  and  $ub_{sr}$  respectively. The discount intervals are further characterized by a discount rate to be applied to the product's price valid

only within the associated bounds. Consequently, it is not the individual product's amount but rather the sum of the products' amounts that will determine the correct discount interval and its corresponding discount rate. The conceptual modeling approach is as follows. Using decision variables  $xd_{srti}$ , denoting the amount of each product  $i$  bought in each discount interval  $r$  from supplier  $s$  in time period  $t$ , we will force, through the constraints, all but at most one of those decision variables over all discount intervals to be zero. Furthermore,  $xd_{srti}$  will only be nonzero in this discount interval where the total quantity  $xdt_{srt}$  falls. The key decision variable for achieving this result is  $v_{srt}$ , being equal to 1, if the company buys a total quantity from supplier  $s$  in discount interval  $r$  in period  $t$ , and zero otherwise. Constraint (12a) models the fact that in case  $v_{srt}$  equals zero, the amount of the individual product  $i$  should also be zero, otherwise, it is bounded by the minimum of the maximum amount to buy in the particular discount interval  $r$  or the sum of the demands for this product  $i$  from the time period  $t$  on til the end of the planning horizon. A definitional constraint (12b) is needed for the link between the quantity bought in the discount intervals  $xd_{srti}$  and the amount bought,  $x_{sit}$ , used in the demand constraints (8a and 8b). Condition (12c) computes the total quantity bought from supplier  $s$  in discount interval  $r$ ,  $xdt_{srt}$ , as the sum of the individual products' quantities,  $xd_{srti}$ . The lower bound on the total amount to buy in the discount intervals is set by constraint (11d) while constraint (11e) impose the same condition for the upper bound. Finally, condition (12f), takes care of two requirements at the same time : if the company does not order from supplier  $s$  in time period  $t$ ,  $y_{st} = 0$ , then obviously, it cannot buy in any of the discount intervals, and consequently all  $v_{srt} = 0$ . If, however, the company orders from supplier  $s$  in time period  $t$ ,  $y_{st} = 1$ , then the total amount should fall into only one discount interval, only one  $v_{srt} = 1$ . Observe that our treatment of the discounts is a generalization from discounts on amount bought of individual products to discounts on total amount bought. It should be clear from the exposition that the former is still included in our model. Specifically,

if an individual product's amount is within the bounds of a particular discount interval and none other products are bought, then the discount will apply to the price of the individual product. If the discount applies only to individual products we can change our model to accommodate this requirement quite similar to the constraints above.

Integrality and nonnegativity ;

$$z_s \in \{0, 1\} \quad \forall s \in P \quad (13a)$$

$$y_{st} \in \{0, 1\} \quad \forall s \in P, \forall t \in M \quad (13b)$$

$$v_{srt} \in \{0, 1\} \quad \forall s \in P, \forall r \in C_s, \forall t \in M \quad (13c)$$

$$x_{sit} \geq 0, x_{st} \geq 0, x_{d_{srt}} \geq 0, x_{d_{srt}} \geq 0 \quad \forall s \in P, \forall r \in C_s, \forall t \in M, \forall i \in N \quad (13d)$$

To conclude the model specification, constraints (13a) - (13d), impose the proper integrality and nonnegativity conditions that apply to the decision variables. Model (1) - (13d) is a mixed integer linear program that can be solved with specialized optimization software such as LINGO (Cunningham and Schrage 1995) on any IBM compatible 486 or higher PC. Typical computation times are in the order of minutes.

Although we have tried to be quite general by enumerating all major components that should be considered while deriving a sound purchasing policy, the mathematical program (1) - (13d) is very application dependent. It might contain constraints and cost elements that are not always relevant in every purchasing situation as well as new constraints and cost elements that might become relevant depending on the application. For example, it frequently happens that state or federal laws require manufacturing companies to include a specific percentage of local content in their products. Using the following additional notation :

$P_l$  : the set of local suppliers, index  $s$ ,  $P_l \subseteq P$ ,

$f_l$  : fraction of local content required in the total volume purchased,

a possible way of satisfying the local content requirement could be to impose that at least a specified percentage of the total volume purchased over the planning horizon should come from local suppliers :

$$\sum_{s \in P_l} \sum_{t \in M} x_{st}^t \geq f_l * \left( \sum_{s \in P} \sum_{t \in M} x_{st}^t \right) \quad (14)$$

A similar constraint could be derived to model the managerial request from a purchasing company that wants to pursue a high research and development profile, to buy at least a certain percentage of their components from suppliers with important research and development programs.

A particularly relevant concern in the food processing industry is that some goods are often subject to spoilage and thus can only be kept in inventory for a specified amount of time. For example, a constraint could be used to impose that at any point in time, the maximum amount of inventory of a product  $i$  should never exceed 3 periods of demand :

$$inv_{it} \leq \sum_{l=t}^{t+2} d_{il} \quad (15)$$

Within the limits set forth by this and the other constraints, the model is then left to choose the optimal purchasing frequency.

The model presented above will derive a purchasing policy over a specific time horizon indicated by the number of time periods in the set  $M$ . Rather than implementing the policy over the complete time horizon, we suggest to use the model in a dynamic, 'rolling horizon' fashion. In this way, only the purchasing policy resulting for the current period ( $t = 1$ ) should be implemented as this is the decision to be taken right now. The rolling horizon procedure then implies that at the next epoch, i.e. the end of the first time period, which equals the beginning of the second time period, the model should be rerun with all the parameters updated at this time to reflect the changes that have taken place during the first time period, in order to derive a new purchasing policy over the complete time horizon. The policy found at that epoch should then be implemented only for the first time period. As such, as time goes by, period by period, the model is always resolved with updated information about costs and inventories to reflect the present state of the system and always a purchasing policy over the complete time horizon is computed of which only the first time period's policy is implemented.

There are several advantages to working in this a way. First, if the basic time period is one month and the resulting time horizon, say, 6 months, the data for 6 months out into the future will be more unreliable than for the month just ahead. The rolling horizon procedure allows the company to implement the plan only for the upcoming time period while looking well into the future such that end-of-period effects are taken into account. Second, resolving the model period by period while dynamically updating the data to reflect the current situation w.r.t. costs and inventories allows the firm to take performance changes from suppliers as well as those resulting from internal improvements into account. Third, as an additional benefit of the dynamic updating inherent in the rolling horizon procedure, the company will be able to provide accurate feedback to the suppliers about how changes in their performance over time have led to changes in the company's purchasing plan. Moreover, the company will

also be able to ascertain whether observed modifications to the purchasing plan resulted from internal changes or from changes in the performance of its suppliers.

In addition, the model could also be used as a tool to evaluate alternative improvement strategies for the suppliers as well as for the firm. In particular, the model can identify what and to what extent specific changes or improvements suppliers could implement in order for the company to start buying, buy more or buy a different mix of products. In short, our model gives a rigorous basis for answering all sorts of 'what if' questions related to the purchasing function.

#### **4. Case Study**

In order to illustrate the mathematical programming approach for vendor selection based on activity and cost driver information, we consider the case of a medium sized printing company, called Ecoprint, that produces weekly magazines (WM), periodicals (PE) and newspapers (NP) for different publishers. These three product groups require a different quality and size of paper. Paper rolls are bought on a regular basis from three suppliers, Baars, Chan and Debro. Paper procurement costs represent an important percentage of total costs and are comparable to those of the most important competitors.

The company uses an MRP system to determine quantities to be bought for the next six months. The resulting demand in paper rolls for the different products is given in Table 1. There is no beginning inventory for the paper rolls in the first month.



Table 1 : Demand for the Different Products.

Product	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6
WM	60	47	53	78	63	52
PE	40	38	43	47	36	38
NP	25	32	27	19	21	26

Quality and delivery reliability are important criteria in the vendor selection process. Prior experience with suppliers indicates that Baars offers the lowest invoice price with considerable price discounts. However, Baars has the poorest quality reputation in the industry despite regular quality audits done by the purchasing company. A one period delivery safety time is also required for this supplier due to the erratic delivery performance. Furthermore, Baars has no research and development department. Chan is a more expensive supplier with better performance on quality and delivery. However, transportation costs are not included in the net purchasing price and a purchasing manager is part-time responsible for managing the relationship with this company. Debro is the most expensive supplier but is known for excellent quality, high delivery reliability, providing assistance with reception activities at Ecoprint and ordering and invoicing possibilities with Electronic Data Interchange (EDI). There are no quality audits necessary for this supplier.

A few years ago an Activity Based Costing system was introduced. As this system is able to determine activities, cost drivers and cost driver rates associated with the different suppliers, orders and products, it can be used in a mathematical programming model for supplier selection. Table 2 summarizes price schedules and information on supplier level, order level and unit level for the three suppliers. The average prices per roll in the market amount to (in Belgian Francs, BF) BF 3,000 for weekly magazines, BF 4,000 for periodicals and BF 2,000 for newspapers. Inventory holding costs for Ecoprint are about 2% per roll per

month. The results of various alternative purchasing strategies using the information of Table 2 are shown in Table 3.

Table 2 : Information for the Supplier Selection Process.

	Baars	Chan	Debro
Price			
Paper roll WM	2,820	2,900	3,060
Paper roll PE	4,050	4,100	4,200
Paper roll NP	2,000	2,080	2,120
Discount per order	71 to 200 : 3% 201 or more : 6%	101 to 250 : 2% 251 or more : 4%	- -
Supplier level			
Quality Audit	25,000	30,000	0
Purchasing Manager	0	40,000	40,000
R&D	10,000	0	0
Order level			
Transportation	0	5,000	0
Reception	25,000	25,000	15,000
Ordering	18,000	18,000	7,000
Invoicing	12,000	12,000	4,000
Credit note probability	20%	5%	0%
Credit note cost	3,000	3,000	3,000
Safety time (periods)	1	0	0
Min order quantity	5	20	50
Unit level			
Probability of defects per unit	4%	2%	0.5%
Setup cost	10,000	10,000	10,000

At this moment the company uses a purchasing policy where weekly magazines, periodicals and newspapers are bought from Debro, Chan and Baars respectively. This policy

is based on prior experience on quality and delivery reliability. For example, as weekly magazines require the highest quality paper rolls, they are procured from Debro, the supplier with the highest quality performance. Furthermore, the purchasing company tries to combine purchasing orders in order to exploit the quantity discounts. Information on activities, cost drivers and cost driver rates is not taken into account. This implies that supplementary costs associated with the purchasing decision are not yet considered at this point.

The results of this purchasing policy are given in the column labeled 'Model 1' in Table 3. The purchasing company uses all suppliers and minimizes the sum of net purchasing prices and inventory holding costs. Total purchasing costs amount to BF 2,536,406 with discounts from Baars in months 0 and period 3 and from Chan in months 1, 3 and 5. Total cost of ownership amount to BF 3,549,086. Additional unit level costs consist of extra setup costs incurred due to quality defects discovered during production. Order level costs amount to BF 687,300 and are caused by ordering, transportation, reception and invoicing activities for the different orders. A total of 14 orders will be placed to satisfy demand for the next six months. Additional supplier level activities are necessary for the three suppliers selected.

The model developed in the third section of this paper was used to minimize the total cost of ownership to satisfy demand. In addition to net purchasing price and inventory holding costs, it was recognized that additional costs caused by the supplier in the purchasing company (see Table 2) should also be introduced in the supplier selection decision. The results of this analysis are given in the third column of Table 3 labeled 'Model 2'. The optimal purchasing policy, hereafter referred to as the base case, is a sole sourcing decision where Debro, the most expensive supplier with lower additional costs, is preferred. Net purchasing price increases by 4.35% compared to the existing purchasing strategy. Contrary to this purchasing cost increase, cost savings are realized at the different levels defined. At the unit level, due to the improvement in quality of the paper rolls, there are fewer additional

setups. Important cost savings are also realized at the order level. The total number of orders placed to satisfy demand decreases from 14 to 2. Additionally, Debros has an EDI system and offers reception assistance such that supplementary cost savings are realized for the associated activities. Supplier specific costs for Baars and Chan disappear. The combination of these effects results in a considerable decrease in total cost of ownership. Total costs associated with this purchasing policy equal BF 2,829,990. Consequently; the implementation of the mathematical programming model based on activity and cost driver information can achieve cost savings of up to 20% for Ecoprint.

The mathematical programming approach allows for different kinds of sensitivity analysis dealing with cost management and strategic decision making. The effects of performance improvement by the supplier(s) and/or the purchasing company could have an impact on the purchasing policy. For example, suppose that Chan will introduce a quality control system such that the probability of defects per unit, compared to the base case, decreases from 2% to 1%. This improvement clearly affects unit level costs caused by Chan because the number of additional setups will decrease. The quality increase will result in a different optimal procurement policy for the purchasing company as illustrated in the column labeled 'Model 3'. Chan becomes the preferred supplier and Ecoprint realizes supplementary cost savings of BF 57,904 due to better prices and volume discounts. Additional costs increase at all levels when compared to the base case (Model 2).

Cost reduction initiatives in the purchasing company could also influence the importance of the different cost categories considered. The column labeled 'Model 4' in Table 3 shows the results in case the purchasing company, compared to the base case, reduces reception costs by BF 5,000 for every order, setup costs by BF 1,000 for every setup and quality audit costs by BF 5,000 for every quality audit. Debros remains the preferred supplier. Total cost of ownership is reduced by BF 17,600 compared to the optimal policy. Cost

savings are realized at the unit level and the order level. There is no supplier level cost reduction because quality audits are not necessary for Debro and no other supplier enters the solution.

Finally, it is possible to study the consequences on total cost of ownership of alternative supplier selection strategies. Suppose that the purchasing company decides to continue today's policy, as given in Model 1, but takes into account additional costs caused by the suppliers. The column labeled 'Model 5' describes the results for this policy and shows that total cost of ownership amounts to BF 3,078,282. It is useful to compare these results with Model 1. Bundling of orders leads to important order level cost savings. Inventory holding costs increase, but total costs are reduced by 20.3%. When compared to the base case, total costs increase some 8.8% as a consequence of the specific purchasing policy. The column labeled 'Model 6', describes amounts bought and total costs if the purchasing company decides to use at least two suppliers. Baars and Debro are selected by the mathematical program. When compared to the base case, this procurement policy will cause a total cost increase of 2%.

Table 3 : Results of Various Alternative Purchasing Strategies.

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Month 0	Baars : NP 71				Baars : NP 178	Baars : NP 88, WM 193, PE 130
Month 1	Chan : WM 101 Debro : PE 50	Debro : NP 88, WM 193, PE 130	Chan : NP 88, WM 193, PE 130	Debro : NP 61, WM 130, PE 91	Chan : WM 393 Debro : PE 130	
Month 2	Baars : NP 17 Chan : WM 29 Debro : PE 80					
Month 3	Baars : NP 71 Chan : WM 101			Debro : NP 57, WM 130, PE 80		
Month 4	Chan : WM 29 Debro : PE 50	Debro : NP 90, WM 200, PE 124	Chan : NP 90, WM 200, PE 124		Debro : PE 124	Debro : NP 90, WM 200, PE 124
Month 5	Baars : NP 19 Chan : WM 101 Debro : PE 74			Debro : NP 60, WM 133, PE 83		
Month 6	Chan : WM 32					
TOTAL COST	3,549,086	2,829,990	2,772,086	2,812,390	3,078,282	2,887,254
Purchasing cost	2,536,406	2,646,740	2,449,286	2,646,740	2,50,232	2,495,554
Additional unit level costs	162,500	41,250	82,500	37,125	162,500	185,100
Order level costs	687,300	52,000	120,300	63,000	167,750	81,600
Supplier level costs	145,000	40,000	70,000	40,000	145,000	75,000
Inventory holding costs	17,880	50,000	50,000	25,525	96,800	50,000

## 5. Conclusions

Vendor selection greatly affects a firm's competitive position as procurement costs account for a large percentage of total costs. Supplier selection decisions are typically multi-objective in nature as different and sometimes conflicting criteria have to be considered. Our paper introduces a new approach to this problem and defines the objective function as the total cost of ownership associated with the purchasing decision. We introduce a mathematical programming model using Activity Based Costing information to select suppliers over a multi-period horizon. The system increases objectivity in the selection process and allows to perform different kinds of sensitivity analysis.

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