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Persistence in Foreign Exchange Rates : Derivation of the Multivariate Persistence Estimates and Their Associated Standard Errors

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Abstract

The paper provides a technical discussion on how the multivariate persistence measures described in Van de Gucht, Kwok and Dekimpe (1993) can be estimated both when cointegration is present and absent. Procedures to derive the associated asymptotic standard errors are discussed.

The (non-normalized) multivariate persistence matrix used in Van de Gucht et al. (1993) is given by the following expression:

$$VC(\tau_t) = \tilde{A}(1)\ \Sigma\ \tilde{A}(1)' \ , \tag{1}$$

where $\tilde{A}(1)$ contains the sum of the moving-average coefficients in a VMA model and where Σ is the corresponding residual variance-covariance matrix. To derive estimates of the respective elements of $VC(\tau_t)$, consistent estimates of $\tilde{A}(1)$ and Σ are needed. In the absence of a cointegrating relationship, one can estimate a VAR model

$$\tilde{\Pi}(L)\ \Delta\vec{S}_t = \vec{u}_t \ , \tag{2}$$

where $\Delta\vec{S}_t$ denotes a Mx1 vector of exchange growth rates $\Delta S_{i,t}$, and $\vec{u}_t$ is an Mx1 vector of white noise innovations with mean zero and variance-covariance matrix Σ. The autoregressive matrix polynomial $\tilde{\Pi}(L)$ is equal to $(I_M - \tilde{\Pi}_1 L - \ldots - \tilde{\Pi}_p L^p)$, where I_M is an identity matrix of order M and $\tilde{\Pi}_i$ is an (MxM) matrix of unknown coefficients. An estimate of $\tilde{A}(1)$ can be obtained by inverting $\tilde{\Pi}(1) = (I_M - \tilde{\Pi}_1 - \ldots - \tilde{\Pi}_p)$, and $\hat{\Sigma}$ is obtained as the estimated residual variance-covariance matrix.

When the exchange-rate series are cointegrated, one should use an error-correction model rather than a model in the first differences. As such, a lagged term in the levels is added to equation (2), which becomes

$$\Delta\vec{S}_t = \tilde{\Pi}_1\ \Delta\vec{S}_{t-1} + \ldots + \tilde{\Pi}_p\ \Delta\vec{S}_{t-p} + \Gamma\ \vec{S}_{t-p-1} + \vec{u}_t \ . \tag{3}$$

The number of cointegrating vectors is indicated by the rank of Γ. If Γ has rank r ($0 \le r < M$), it can be written as the product

(4) $$\Gamma = \alpha\ \beta' ,$$

where α and β are full-rank (Mxr) matrices. The columns of β indicate the cointegrating vectors, and the elements of α give the error-correction weights. A FIML approach to test for and estimate these cointegrating vectors has been developed in Johansen (1988) and Johansen and Juselius (1990), and we refer to these studies for further details.

If cointegration has been established, one can estimate $\tilde{A}(1)$ as (see e.g. Johansen, 1991; Juselius, 1991a,b; Juselius and Hargreaves, 1992):

(5) $$\tilde{A}(1) = \beta_{\perp}\ [\alpha'_{\perp}\ (I_M - \tilde{\Pi}_1 - \ldots - \tilde{\Pi}_p)\ \beta_{\perp}]^{-1}\ \alpha'_{\perp} ,$$

where $\alpha_{\perp}$ and $\beta_{\perp}$ are Mx$(M$-$r)$ matrices who are orthogonal to α and β, respectively.

Next, we present the derivation of an expression for the asymptotic variance of the multivariate persistence estimates when no cointegration is present. The results are presented for an unrestricted VAR model. Similar expressions can be derived for a restricted model (where we drop e.g. all regressors whose coefficients have a t-ratio less than in absolute value), and can be obtained from the authors upon request. Because of computational and distributional complexities, we advocate the use of a bootstrapping procedure to derive the standard errors of the multivariate persistence estimates when the exchange rates are cointegrated.

To estimate $\tilde{A}(1)$ in the absence of cointegration, the VAR model given in equation (2) is used. First, we derive the asymptotic variance of the elements of $VC(\tau_t)$. For ease of notation, we denote $VC(\tau_t)$ by Q:

(6) $$Q = \tilde{A}(1)\ \Sigma\ \tilde{A}(1)' ,$$

where $\tilde{A}(1)=[\tilde{\Pi}(1)]^{-1}$. Note that the elements of Q (i.e. Q_{ij} ; i,j = 1, ..., M) correspond to the unscaled persistence estimates. As indicated in Van de Gucht et al. (1993), the scaled or normalized persistence estimates (indicated by P_{ij}) are obtained by dividing the elements in the jth column of Q by $\mathrm{Var}(\Delta S_{j,t})$. The standard errors for P_{ij} are therefore given by the ratio of the standard error of Q_{ij} to the variance of $\Delta S_{j,t}$. Stacking all observations, we can rewrite (2) as

(7) $$\tilde{\Pi}(L)\ \Delta SS' = U' ,$$

where $\Delta SS'$ is equal to the $(M\mathrm{x}T)$ matrix $[\Delta\vec{S}_1, ..., \Delta\vec{S}_T]$, and U' is equal to the $(M\mathrm{x}T)$ matrix $[\vec{u}_1, ..., \vec{u}_T]$. After writing equation (7) in vector form, we obtain

(8) $$Vec(\Delta SS) = (I_M \otimes \Delta SS_0)\ \vec{\Theta} + Vec(U) ,$$

where ΔSS_0 is equal to the $(T\mathrm{x}MP)$ matrix $[\Delta SS_{-1}, ..., \Delta SS_{-P}]$, and where $\vec{\Theta}$ is equal to $\mathrm{Vec}([\tilde{\Pi}_1, ...,\tilde{\Pi}_p]')$. For an unrestricted VAR model, $\vec{\Theta}$ is of order PM^2. All elements of Q are scalar functions of $\vec{\Theta}$, and we denote this functional relationship by $Q_{ij}(\vec{\Theta})$. Hence, if the variance-covariance matrix of $\vec{\Theta}$ is known, one can apply the delta method to derive the asymptotic variance of $Q_{ij}(\vec{\Theta})$. In matrix notation:

(9) $$Avar\{Vec[Q(\vec{\Theta})]\} = \frac{\partial Vec[Q(\vec{\Theta})]}{\partial\vec{\Theta}'}\ Avar(\vec{\Theta})\ [\frac{\partial Vec[Q(\vec{\Theta})]}{\partial\vec{\Theta}'}]' .$$

The variances of interest (i.e. $\mathrm{Var}[Q_{ij}(\vec{\Theta})]$) are given by the corresponding diagonal elements of the $(M^2\mathrm{x}M^2)$ matrix $\mathrm{Avar}\{\mathrm{Vec}[Q_{ij}(\vec{\Theta})]\}$. After some algebra, it can be shown that[1,2]

(10) $$Avar(\vec{\Theta}) = S \otimes [(\Delta SS_0)'(\Delta SS_0)]^{-1} ,$$

and

$$
\begin{aligned}
\frac{\partial Vec[Q(\vec{\Theta})]}{\partial \vec{\Theta}'} = & - [\{\tilde{\Pi}(1)\}^{-1} \otimes \{\tilde{\Pi}(1)\}^{-1}] * [2/(T-MP)] * [I_M \otimes U'(\Delta S S_0)] \\
(11) \qquad & + [I_M \otimes \{\tilde{\Pi}(1)\}^{-1}\Sigma] * [\{\tilde{\Pi}(1)\}^{-1} \otimes \{\tilde{\Pi}(1)'\}^{-1}] * [I_M \otimes S_P] \\
& - [\{\tilde{\Pi}(1)\}^{-1}\Sigma \otimes I_M] * [\{\tilde{\Pi}(1)^{-1}\}' \otimes \{\tilde{\Pi}(1)\}^{-1}] * [\frac{\partial Vec\tilde{\Pi}(1)}{\partial \vec{\Theta}'}] \, ,
\end{aligned}
$$

$$
(12) \qquad where \quad S = \frac{U'U}{T-MP} \, , \quad and \quad S_P = [I_M, I_M, \ldots, I_M] \, .
$$

Footnotes

1. Useful suggestions from Kevin Lee and Richard Pierse are greatly appreciated.

2. These expressions differ from the formulas given in Pesaran et al. (1993), which only consider the variance of the diagonal elements of the persistence matrix, and which use a different normalization factor. Indeed, our normalization factor (i.e. Var($\Delta S_{j,t}$)) does not depend on $\vec{\Theta}$. Pesaran et al., on the other hand, use the conditional variance as normalization factor, which depends on $\vec{\Theta}$ and which therefore affects the partial derivatives. As indicated in Pesaran et al., both normalization factors can be used, but our normalization has the appealing properties that (1) it results in a straightforward generalization of the univariate measure proposed by Cochrane (1988), and (2) that the derivation of the associated standard errors is computationally more tractable.

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