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TELECOMMUNICATION CARRIER SELECTION UNDER VOLUME DISCOUNTS: A CASE STUDY

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Telecommunication Carrier Selection under Volume Discounts: a Case Study

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ABSTRACT

During 2001 many of the European mobile phone markets have reached saturation, and hence mobile phone operators have shifted their attention from growth and market share to cutting costs. One way of doing so is to reduce spending on international calls which are routed via network operating companies (carriers). These carriers charge per call-minute for each destination and may use a joint business volume discount to price their services. We developed a software system that supports the decision of allocating destinations to carriers. The core of this system is a min-cost flow routine that is embedded in a branch-and-bound framework. Our system not only solves the operational problem to optimality, it is also capable of performing what-if analyses and sensitivity analysis. It has been implemented at a major telecommunication services provider. The main benefits realized are twofold:
the business process of allocating carriers to destinations has been structured and the costs arising from routing international calls have significantly decreased.

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Background

The telecommunications industry has changed dramatically over the last decade. At least two different, parallel developments caused this change. First, within the European Union, the telecommunication markets have been privatised. Consequently, a market that was previously monopolized has now become a competitive one. The impact of this development on the industry has been enormous. Second, on a global level, the advances in digital technology, in particular the introduction of mobile phones, have rendered a large increase in the variety of services, as well as in turn over. The challenge of handling this increased complexity is omnipresent in the market since the late 90s.

In the resulting turbulence, new telecommunication service providers (**telcos**), with different business concepts have come to existence, and many have tried to gain market share with aggressive marketing techniques The competition has been and still is fierce; for instance, in the Netherlands, the number of mobile operators (telecommunications service providers offering mobile phone based services) has increased from one, the previously state owned KPN, to six in 2001. For some of their services, mobile operators make use of networks that are either operated (and owned) by the mobile operator itself or are operated by another telco. More specifically, each mobile operator owns a radio network of radio frequency antenna's that communicate with the mobile phones and are connected to the wired net. All mobile calls are routed to their destinations via the wired net. Since most mobile operators do not own a wired net, they rely on wired networks of other telcos. In the remainder of this paper we refer to telcos offering wired network capacity to mobile operators as **carriers**.

Since roughly 2001, many telcos have good reasons to take their financial position into serious consideration. For some, this is due to the enormous investments made, for instance to acquire UMTS frequencies. Many companies therefore started to consider being cash flow positive, or even a positive operational result as an important performance indicator. Reducing cost is an obvious way to improve these indicators. This attention for cost reduction as it entered the market in 2000 is for example is illustrated by the following quotes from annual reports (2001) of major telecommunication service providers:

- "In order to optimize management and take full advantage of economies of scale, the company will develop policies designed to improve efficiency .... These actions will be co-ordinated through a policy based on the containment of costs and the proper materialization of synergies" (from the 2001 annual report of Telefonica (see http://www.telefonica.es)).
- "In addition, the E-3 program was introduced to generate savings and greater efficiency; the aim is to achieve sustained improvements of around EUR 1.5 billion" (see the website of Deutsche Telekom: http://www.telekom.de).
- "...One of the main objectives is COST-REDUCTION" (from the chairman of the board of Belgacom, at the presentation of the 2001 annual report (see http://www.belgacom.be)).

One way of reducing costs is to decrease the spending on calls routed via the wired networks of carriers.

This paper deals with a carrier selection problem encountered by a major telecommunication services provider. We will refer to this provider as our client. In particular, we deal with the carrier selection problem for **international calls**.
An international call is any call that generates traffic on carrier networks across the border.

In subsequent years, the market hasn't been less turbulent than in 2001. Some telcos and carriers have clearly been more successful then others, and as the market reaches a more mature phase, segmentation has also become an issue. In several segments price is not the only important order winner, but quality of service is of at least equal importance. Hence, many telcos have shifted or are shifting their attention towards quality. Such developments will no doubt continue, and we point out that the case described in this paper can serve as a starting point for developments in which quality of service plays a prominent role. We are confident that the procurement problems and models, as well as the solution strategy discussed in this paper will continue to have practical applications, both in the telecom industry, as well as in others.

Introduction to the Problem

Our client operates a country-wide mobile radio frequency network, which is connected to a wired network. From this wired network the calls are routed to their destinations. A **destination** is a geographical entity such as a country or a city; it is characterized by a unique number sequence. The final goal of a call may be a phone that is connected to the wired network, or in case of a mobile destination, the radio frequency network of our client or some other mobile operator. International calls are all routed to a central hub where the wired national network is connected to the international telecommunication networks of the carriers. Examples of such carriers are Teleglobe, COCT, Versatel and France Telecom.
Carriers price their services by means of a price per call-minute for each of the destinations in their network. To ensure that the customers of our client have proper connections, our client has to select a carrier for each destination. As an example, our client can select COCT as the current carrier for the calls with destination Mexico. Bills from the carriers are on an aggregated basis. At the end of the billing period, say each month, our client pays to each of the carriers for each of the destinations, the price times the number of call-minutes sent via this carrier.

Now, the cost oriented procurement problem that our client faces is to route call-minutes to carriers so as to minimize the sum of the amounts of the end of year bills of the carriers. The following two complicating factors must however be taken into account.

Carriers have invested enormous amounts in the development of the wired networks and strive to utilize their capacity fully. To do so they need estimates and insights concerning the amount of usage expected. In fact, our client, as well as other telecommunication companies, is expected to make statements and enter into agreements about how much network capacity it is going to use in the near future. This results in lower- and upper bounds on the available capacity, or may even result in lower and upper bounds on how much capacity to use. Thus, a contract with a carrier may contain agreements about lower bounds and upper bounds on the number of call-minutes allocated to that carrier. Obviously, these agreements should not rely on prices alone, but also on forecasts on the number of call-minutes.

Some carriers offer so-called volume discounts. These carriers distinguish several volume intervals, which are specified by giving lower- and upper thresholds in number of call-minutes, and set different prices for each of the intervals (an example will be given in Section 2). By the end of the year, the carrier determines the total number of call-minutes received over all destinations and determines the appropriate interval. Subsequently the billing is based on the prices according to that appropriate interval. This implies that, in order to minimize total costs, our client has to take into account the appropriate end of year intervals. Of course, during the year it is not yet known in which interval our client ends up in, since this depends on an unknown number of future calls, and the carriers that are selected to route these calls. However, as we will see later, these decisions have a non-negligible impact on the profitability.

This paper deals with the development of optimization software that helps our client to solve the problem described above which we name 'carrier selection problem under volume discounts' (CSPV). In the next section we define the problem exactly and we discuss the application-specific characteristics. Then we review related literature, and formulate an integer program for the CSPV. We propose an algorithm for solving the problem, and we describe a software system that is implemented in our client's organization that supports the decision-making process in this area. Finally, we discuss the benefits that have resulted from using the software system.

Before we proceed with the problem definition, let us spend a few words on the timing, the relevance and the dimensions of the project.

- Timing. At present, so-called carrier switching software systems have become available. These systems are able to digitally assign a given carrier to the appropriate destination without manual intervention, and our client has implemented such a system. Thus, it has now become very easy to route the calls to the right carriers once the optimal selections are known. The availability and implementation of this carrier switching software implied the need for a way to find the right carriers. The software system presented in this paper determines the optimal selections and generates input for the carrier switching software to set the selections accordingly. Thus, not only the current market circumstances incite our client to solve the CSPV, the organization itself has the IT tools installed to utilize a software system that solves CSPV.
- Relevance. There is a significant amount of money involved. Indeed, yearly expenses of our client to carriers exceed ten million Euros. Moreover, it has become clear that the policy of assigning destinations to carriers that offer

the lowest price per call-minute (assuming that the yearly total amount ends up in the highest interval of that carrier) may yield a non-optimal solution, if at all feasible. Thus, solving the CSVP, and solving it to optimality, contributes to profitability.

- Dimensions. The number of destinations distinguished by the carriers has risen to well over 10.000 and carriers may use up to. Therefore (optimal) manual carrier selection at a given time instant is an enormous if not unmanageable task. Even more, there is reason to reconsider the current selection whenever there are changes in the forecast, the prices or other data. Having a software system available facilitates the business process of revising the carrier selection whenever there are changes in the relevant data.

These arguments motivated our client to develop and implement a software system for CSVP.

Problem Definition

A deeper understanding of the problem is probably best provided by studying a simple example as given in Figure 1.

	Carrier A price		Carrier B price	Forecast
	Interval: [0-1.500.000)	Interval: [1.500.000-∞]		
Destination 1	0.40	0.20	0.25	1.000.000
Destination 2	0.60	0.40	0.35	1.000.000

Figure 1: Example instance of CSPV

The instance described in Figure 1 has two destinations and two carriers. Carrier B has a price per call-minute of 0.25 (0.35) for Destination 1 (2), whereas for Carrier A the price depends on the total amount of calls that it handles: if this amount is less than 1.5 million call-minutes the prices are respectively 0.40 and 0.60, otherwise the prices are 0.20 and 0.40 for Destinations 1 and 2 respectively. The forecast for both destinations is 1 million call-minutes.

One solution of the instance in Figure 1 is to select Carrier B for both destinations, thereby routing 2.000.000 call-minutes via Carrier B. This results in a total cost of 600.000. Alternatively, one may select Carrier A for both destinations, which results in a equally high total cost of 600.000 since the combined forecasts of 2.000.000 exceed the threshold of 1.500.000 of the second price interval of Carrier A. A more subtle solution can be found when the call-minutes for Destination 2 are equally divided between the two carriers, and Carrier A is selected for all call-minutes to Destination 1. This results in routing 1.500.000 call-minutes via Carrier A, yielding prices from the second interval, and routing 500.00 call-minutes via Carrier B. The total cost of this solution amounts to 575.000.

Presumably, part of the complexity of the example stems from the fact that in an optimal solution the forecast for a single destination had to be divided over several carriers. On the other hand, it is also possible to construct hard problem instances whose optimal solutions do not feature such a property. More formally, we have shown that the optimization problem CSVP is NP-Hard (see Goossens, Maas, van de Klundert and Spieksma (2003)).

CSPV considers at a given moment in time (referred to as the run date) the following inputs:

1. Actuals: for each destination and for each carrier, the amount of call-minutes that the carrier has received for the destination since the start of the year.
2. Forecast: for each destination and for each month between the run date and the end of the year, the expected number of call-minutes,
3. Prices: for each destination and for each interval of every carrier, the current price for one call-minute of capacity of the carrier for that destination.
4. Intervals: for each interval of each carrier, the lower- and upper thresholds specifying the interval for which these prices are valid. For each carrier, these intervals apply to the total number of call-minutes over the year.
5. Monthly lower- and upper bounds per carrier. Regardless of the interval structure, carriers often impose lower and upper bounds on the total capacity available for each month.

The CSVP is now to select one or more carriers for each of the destinations, and to decide how many call-minutes to send via each of the selected carriers, while respecting the lower bounds and the upper bounds, so as to minimize the yearly procurement costs.

Discussion

Before and during the development of the software application solving the CSVP, a number of issues came up.

- Frequency. A key question that came up in our conversations with the management of our client's organization concerns the frequency with which to solve the CSPV. An important topic in addressing this question is the extent to which uncertainty is present in the input parameters. For instance, if the (perceived) quality of the forecasts is high, CSPV need not be solved as frequently as in a setting where the forecasts are not so good. More generally, it is obvious that for some of the input parameters described above, the realization can be different from the given input. Uncertainties are present in:

1. The monthly forecasts. Of course, as time proceeds, the actually routed number of call-minutes in a particular month may differ from the forecasted number of call-minutes for a specific destination. In addition, the forecast itself can change over the year. For example, the forecast for the number of call-minutes to some destination for December made in January typically differs from the forecast made in November. Currently, in our client's organization, a monthly forecast for each of the destinations is generated.
2. Prices may change over the year. Most carriers announce their prices for a three-month period in which the prices remain fixed. Although there are usually no extreme changes in these numbers, it is very hard to predict what the exact prices will be after such a period. We have chosen to estimate future prices by assuming that the current price will be valid till the end of the year.

It is clear that all forecast and price data required for an optimal solution are not available until the year is over. However, as mentioned before, the decisions have to be made during the year, i.e., the CSPV is a real-time problem. We concluded this discussion by stipulating that the software system should satisfy the following requirements: I) it should be able to solve CSPV at any given moment in time optimally under the assumption that forecasts are perfect and prices remain fixed, II) Whenever there are updates for the actuals, the forecasts, or the prices, the CSPV problem is solved again, and the new solution will be implemented. This means that the problem will be solved at least on a monthly basis.

- Operational/Tactical use versus strategic/tactical use. Right from the beginning, the solution software for the CSPV model has been intended to serve two purposes. First of all, the software has been designed to solve the operational/tactical procurement problem CSVP. Secondly, the software has been designed to support the strategic/tactical task of contract negotiations. The software enables the user to easily perform several types of scenario studies. Forecasts, interval lower- and upper thresholds, lower and upper bounds, and prices can all be easily adapted, and then the problem can be solved again, without affecting the actual carrier selection.

- Quality of carriers. Our client measures the quality of connections of the carriers with respect to each of the destinations. From a customer service viewpoint, a certain carrier may be regarded undesirable for a certain destination because of the quality of the connection that it provides for that destination. Prior to our involvement, these measurements were used to exclude certain carriers for certain destinations. In the development of our application we added the possibility to turn the quality measurements into a price (called a penalty) that then could be added to the price given by the carrier for that destination. In doing so, our client is able to manage the trade off between quality and price of carriers and their corresponding destinations.

- Brokers. Brokers are parties that offer short-term capacity for certain destinations at low rates. In the near future, brokers are expected to play a more prominent role in the international telecommunication network industry. However, our client has not yet decided to purchase capacity from these brokers. Partly this is due to the fact that quality of connections offered by brokers is unreliable. (Brokers do not own their networks, and at the time of an offer, the network from which a broker is going to procure capacity might still be undecided.) Moreover, the impact of accepting broker offers on expected numbers of routed call-minutes, and therefore on the end of year discount rates are uncertain at the time of acceptance. Therefore, it is hard to determine the financial consequences from accepting broker deals. Consequently, while scoping the project, it was decided not to take brokers into consideration.

Related Literature and Modelling

The CSPV is connected to the field of procurement optimization. Such a setting arises when we let the carriers take the role of suppliers (or vendors), and view the call-minutes for the different destinations as the products that are to be procured. Related literature can therefore be found in this area that is gaining importance. We therefore borrow from procurement terminology in the discussion below. The CSVP can be characterized by the fact that the suppliers use a so-called joint business volume discount. A special feature of our setting is further that inventory costs or ordering costs do not play a role since call-minutes are not a physical item and we do not need to order them or store them.

Since Harris in 1913 derived the well-known EOQ formula, work on the EOQ has been extended to incorporate different quantity discount regimes. Rosenthal and Munson [1998] describe different forms of quantity discounts and in particular make the distinction between all-unit discounts and incremental discounts. An example of incremental discounts arises when the first 1.000 items go for 10 dollar per piece, while all units purchased above 1.000 go for 5 dollars per piece. Thus, under incremental discounts, not all units go for the same price. In case of all-unit discounts, the price according to the interval in which the total number of purchased items lies, is applied to all items. In the previous example this would imply that it costs 10.000 dollars to purchase 1.000 items, but only 5.005 to procure 1.001 items. This phenomenon has become known as the more for less paradox. The CSPV has all-unit discounts, and therefore the more for less paradox applies to it. (See for instance Xu, Lu and Glover (2000) for recent work on a lot sizing problem involving all-unit discounts.)

Apart from being the originator of the discount structures, EOQ problems are of limited interest for CSVP. Indeed, EOQ problems consider the operational problem of how much to order, whereas CSVP also deals with the more tactical issue of from whom to order. Such vendor selection problems have been studied by various authors.

Katz, Sadrian and Tendick [1994] (see also [Sadrian & Yoon 1994]) discuss a procurement problem that is related to the CSVP. The main distinction is the fact that in their case, which arises in the context of purchasing telecommunication network hardware, the discount intervals are stated in terms of business volume, i.e. dollar value. Thus, per vendor, the discount depends on the total dollar value of purchases from that vendor, whereas in our case it depends on the total number of units purchased from that vendor (carrier). Their problem also differs from CSVP in another respect, namely they distinguish between purchases on a commitment bases, and purchases on an as-ordered bases. They stress the importance of sourcing flexibility and thereby wish to explicitly model the fact that they do not want to purchase all future items via committed contracts. Likewise they explicitly consider the number of vendors for each item, and consider constraints on these numbers of these vendors, and the percentages of supply given to each of the vendors.

Degraeve and Roodhooft (2000) view vendor selection problems from a total cost of ownership perspective. In this approach, accounting techniques are used to identify all relevant costs associated with a vendor. Based on this, a mathematical program is developed to decide upon which suppliers to order from, when and how much (see also Degraeve, Labro and Roodhooft (1997)).

Rosenthal, Zydiak and Chaudry [1994] consider the problem of vendor selection with bundling. Bundling refers to the idea of selling a bundle of items, where the price of (types of) items depends on other (types) of items in the bundle. Their MILPs that are solved using LINDO are simplified and improved upon by Sarkis and Semple [1999].

Crama, Pascual and Torres [2001] consider a problem in which the discount structure is exactly the same as in CSVP (they call it total quantity discount). Their problem is motivated by an application in which a company is buying raw materials for an industrial process. However, in their model, they deal with the additional complexity that forecasts are given for the outputs, whereas the prices refer to the inputs, and there are several ways to transform inputs into outputs. Thus, not only do they have a procurement problem, they also have a production problem, which yields their problem to be significantly more complex than CSVP.

All of the references cited above solve small to medium sized problems, where there are around 10 suppliers, and the number of products is well below hundred, or in the case of Katz, Sadrian and Tendick [1994], 500. In our case, our client stated explicit performance measurements regarding to problem size that exceeded these numbers significantly. On a routine basis, they wanted to solve instances with 10.000 destinations (products). Hence the project started with a feasibility study, in order to determine the problem sizes that can be realistically solved in an acceptable amount of time, and the corresponding technological requirements in terms of hardware and software. Our success in solving the problem fast enough to meet the requirements is largely due to the combinatorial structure of the mathematical programming formulation for CSVP. Therefore we now proceed by presenting this mathematical programming formulation.

We define continuous decision variables as follows:

x_{ijkt} the number of call-minutes sent via carrier *i* to destination *j* in month *t* for the price in interval *k*.

Further we define the binary decision variable y_{ik} to be:

1 if the total number of call-minutes routed via carrier *i* falls in interval *k*,

y_{ik} =

0 otherwise.

Further, we define the following parameters:

p_{ijkt} = price per call-minute for destination *j* in interval *k* of carrier *i* in month *t*,

d_{jt} = forecast for destination *j* in month *t,*

LT_{ik} = lower threshold of interval *k* of carrier *i*,

UT_{ik} = upper threshold of interval *k* of carrier *i*,

LB_{it} = lower bound for the number of call-minutes routed via for carrier *i* in month *t*,

UB_{it} = upper bound for the number of call-minutes routed via for carrier *i* in month *t*.

Now, the model can be defined as:

$$Min \quad \Sigma_{ijkt} \; p_{ijkt} \, x_{ijkt} \qquad (1)$$

$$\text{s.t.} \quad \Sigma_{ik} \, x_{ijkt} = d_{jt} \qquad \forall j,t \qquad (2)$$

$$\Sigma_{k} \, y_{ik} = 1 \qquad \forall i \qquad (3)$$

$$\Sigma_{jt} \, x_{ijkt} \geq LT_{ik} \; y_{ik} \qquad \forall i,k \qquad (4)$$

$$\Sigma_{jt} \, x_{ijkt} \leq UT_{ik} \, y_{ik} \qquad \forall i,k \qquad (5)$$

$$\Sigma_{jk} \, x_{ijkt} \geq LB_{it} \qquad \forall i,t \qquad (6)$$

$$\Sigma_{jk} \, x_{ijkt} \leq UB_{it} \qquad \forall i,t \qquad (7)$$

$$x_{ijkt} \geq 0 \qquad \forall i,j,t \qquad (8)$$

$$y_{ik} \in \{ 0,1 \} \qquad \forall i,k \qquad (9)$$

Constraints (2) ensure that all forecasted call-minutes are routed via some carrier. Constraints (3) and (9) ensure that exactly one interval is selected for each carrier, while constraints (4) and (5) ensure that the prices corresponding with the selected interval are the prices paid. Finally, constraints (6) and (7) model the requirement that in each month and for each carrier the total number of call-minutes is above the given lower bound and less than the given upper bound. Constraints (8) are non-negativity constraints, and constraints (9) are the integrality constraints. The correctness of the model is not hard to verify.

Notice that our formulation uses a price for each destination for each interval of a carrier. Literature usually assumes that there is a percentage involved, i.e., the quotient of the price in the *k*-th interval and the price in the first interval is identical for all products. In the instances that we encountered, this was not the case. Thus, instead of having a set of prices as input for each carrier and a percentage for each interval, we have a set of prices for each interval of each carrier. Notice further, that model (1)-(9) does not explicitly take the actuals into account. However, one easily argues that by modifying the thresholds LT_{ik} and UT_{ik} ($\forall\, i,k$), one can account for the call-minutes that have already been routed.

A typical Mixed Integer Linear Programming approach to solve model (1)-(9) would be to solve the linear relaxation that arises when relaxing the integrality constraints (9). A branch and bound procedure then allows solving the problem to optimality. Our solution approach is based on a different approach. We explicitly enumerate all feasible combinations of the y_{ik} values. That is, for each possible carrier interval selection we solve the remaining problem.

This remaining problem is formulated in appendix A ((10)-(16)), where it is also shown that this problem is a min-cost flow problem. For min-cost flow problems, there exist specialised algorithms from network flow theory that allow one to solve such problems much faster than a standard linear programming solver does. On the other hand, explicitly enumerating all feasible combinations of y_{ik} may appear to be more time-consuming than the branching process that results from solving (1)-(9) by branch and bound.

In a preliminary computational study we solved instances of the min-cost flow problem (10)-(16) by state-of-the-art LP solvers as well as by dedicated min-cost flow routines. The test instances were real life instances of moderate size; more specifically, these had 5 carriers, 10 months and about 5000 destinations. Apart from some initial problems we encountered with numerical stability (which resulted from scaling), dedicated min-cost flow routines turned out to be faster on our test instances than state-of-the-art LP solvers. The latter typically took several minutes to solve the problem instances, whereas the min-cost flow routines took several seconds on a 800 Mhz, 128 Ram, Pentium.

Based on the outcome of this feasibility study we decided to use IGSystems' CS2 code for solving the min-cost flow problem. CS2 is a so-called double scaling algorithm (see Ahuja, Magnanti and Orlin (1993)) available at http://www.cs2.com. The computational results were good enough to develop software for actual carrier selection optimisation as well as for decision support purposes. In particular, the CS2 code solves the problem quickly enough to perform scenario studies on-line (i.e., while waiting for the results).

The basic algorithm that we used for solving model (1)-(9) can now be described as follows:

Step 1. Enumerate all possible ways of selecting a single interval of each carrier,
Step 2. For each of these ways, solve a min-cost flow problem, and store the solution found,
Step 3. Output the best solution.

Software Application

The software system has as its core the algorithm described above. It is embedded in a user-friendly multi-threaded windows application that is able to communicate its input and output to other software. This application is named BeCR (Best Cost Routing). A screen shot is given in Figure 2. Due to the fact that the application is mainly used by people who are familiar with spreadsheet-based data processing, BeCR extracts its input from Excel files. Some of these Excel files regarding actuals, forecasts, et cetera are generated by other software applications, other Excel files are maintained and kept up to date by the person responsible for BeCR. The output of BeCR can be viewed within BeCR, but can also be exported to Excel. The communication with the carrier selection software is based on XML and technically more involved than just encoding the solution of the problem. The application integration is file based, but doesn't require other human activity than basic file management. It runs on a normal PC with acceptable response times, so that no investments in hardware are required.

Benefits

A first achievement of the project was the recognition that allocating carriers to destinations (the CSVP) is a nontrivial problem where optimal decision-making requires mathematical effort. Moreover, considering the annual business volume involved, it was easily accepted that there was value in optimal decision-making. Elimination of sub-optimalities in the current settings and processes was an important issue, and benefits were expected to be substantial in the future due to changing market conditions (carriers using more intervals, increased importance of negotiating based on upper and lower bounds). BeCR has realized these benefits. Due to different market circumstances and due to the current usage of penalties, it is hard to make a precise statement concerning the amount of money saved. However, a conservative estimate is that BeCR saves at least 1% of the yearly expenses related to carriers.

A second and substantial benefit arises from enabling our client to analyse consequences of changes in input parameters like monthly lower and upper bounds. Clearly, this option supports the negotiating process of our client with the carriers. Investigating the flexibility of the carriers, and using this flexibility, also contributes to a more efficient process.

Two other benefits are more operational. Firstly, the process of carrier selection which used to be extremely time consuming, is now much more efficient. Secondly, by modelling and formalizing the process, our client has been able to improve its data integrity, and thereby the process is also much more reliable and controlled.

Our client is now ready to easily reap the benefits of financial procurement optimization and hence has more time to pick the higher hanging fruits of contract negotiation and accurate forecasting. In addition, the focus of many players in the turbulent mobile operator market has again shifted. In our client's case, the quality of the services offered to its clients, and therefore the quality of the services purchased from the carriers has become a major issue as well. The software system described in this paper offers possibilities to deal with these quality issues. Moreover, it has served as a starting point for more advanced models in which quality issues are dealt with in a way that fits to the corporate strategy focus on quality.

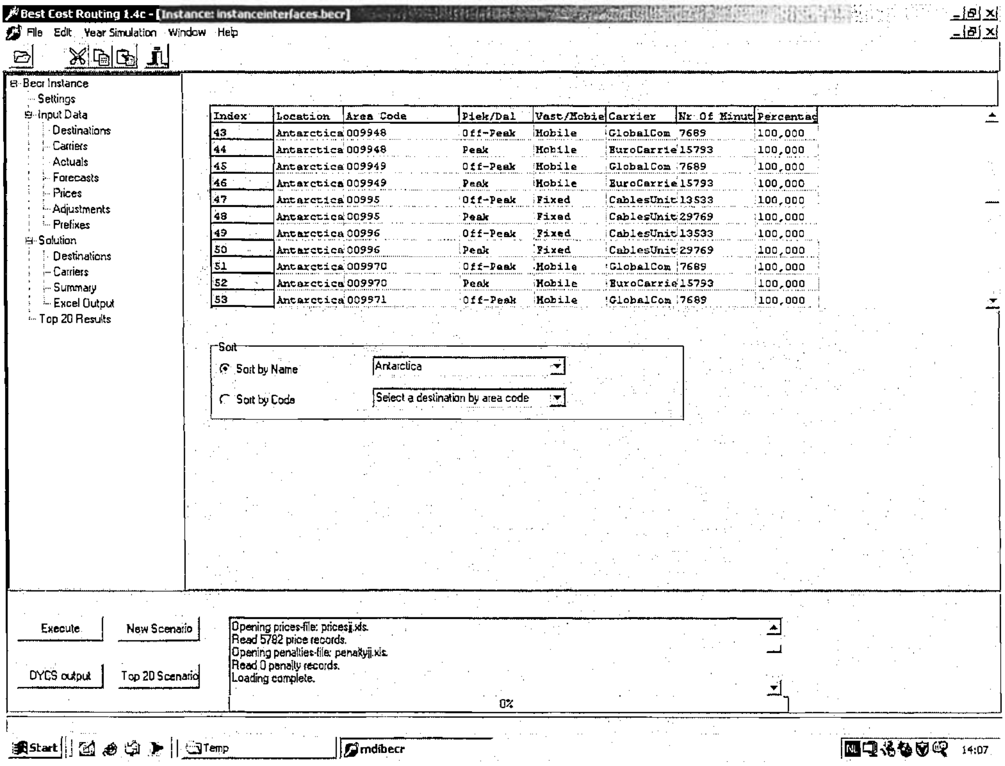

Figure 2

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Appendix A

In this appendix we show that the CSVP when **given** an interval for each carrier becomes a min-cost flow problem. Thus, consider model (1)-(9), and assume that values for the y_{ik} variables are given. Then we can derive the following formulation (notice that the index *k* has disappeared since the intervals have been specified):

Min	$\Sigma_{ijt}\ p_{ijt}\ x_{ijt}$		*(10)*
s.t.	$\Sigma_i\ x_{ijt} = d_{jt}$	$\forall j,t$	*(11)*
	$\Sigma_{jt}\ x_{ijt} \geq LT_i$	$\forall i$	*(12)*
	$\Sigma_{jt}\ x_{ijt} \leq UT_i$	$\forall i$	*(13)*
	$\Sigma_j\ x_{ijt} \geq LB_{it}$	$\forall i,t$	*(14)*
	$\Sigma_j\ x_{ijt} \leq UB_{it}$	$\forall i,t$	*(15)*
	$x_{ijt} \geq 0$	$\forall i,j,t$	*(16)*

Let us now argue how an instance of (10)-(16) gives rise to a min-cost flow instance. To do so, we construct a graph. This graph has four sets of nodes : there is a node for each pair consisting of a destination *j* and a month *t* (called dm-nodes), there is a node for each pair consisting of a carrier *i* and a month *t* (called month-nodes), there is a node for each carrier *i* (called carrier-nodes), and finally there is a (single) source node *s.* The demand of the source node *s* equals $-\Sigma_{jt}\ d_{jt}$, the demand of a dm-node equals $d_{jt}\ \forall j,t,$ and all the other demands are 0. There is an arc from *s* to each carrier-node *i*. With such an arc we associate a lower bound LT_i and upper bound UT_i; these arcs have 0 cost. Further, there is an arc from each carrier-node *i* to each carrier-month node *(i,t)*. These arcs have lower bound LB_{it} and upper bound UB_{it}; these arcs have also 0 cost. Finally, there is an arc from each carrier-month node *(i,t)* to each dm-node (*j,t*) that corresponds to the same month. These arcs have a lower bound of 0 and infinite upper bounds; the cost of the corresponding arc equals p_{ijt}. See Figure 3 for a picture of the resulting graph. We leave it to the reader to verify that a feasible flow corresponds to a feasible carrier selection and vice versa.

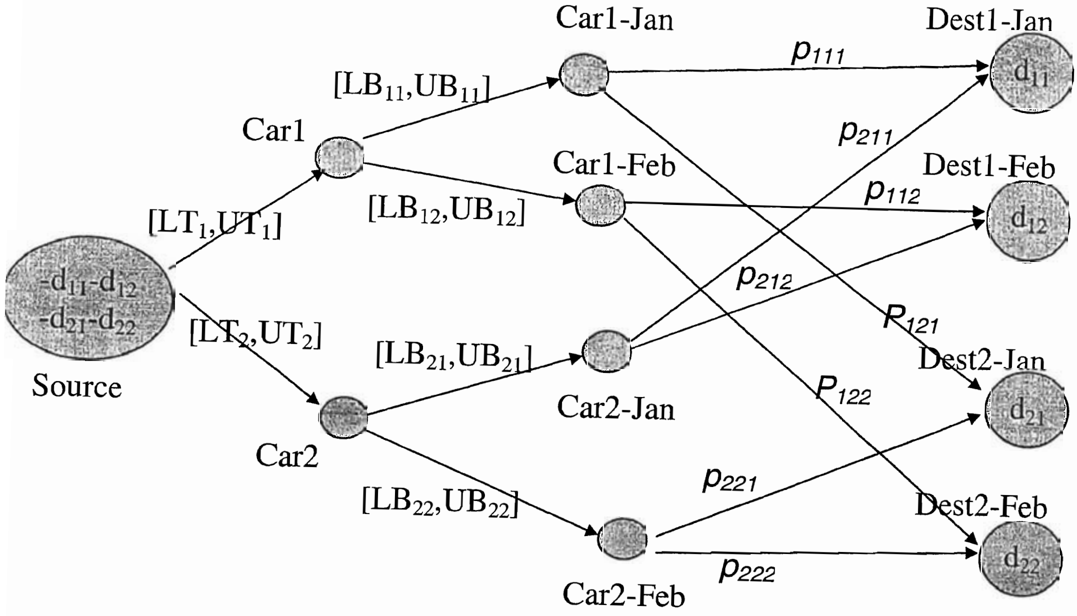
Car1-Jan
Dest1-Jan
p111
d11
[LB11,UB11]
Car1
p211
Car1-Feb
Dest1-Feb
p112
[LT1,UT1]
[LB12,UB12]
d12
-d11-d12
-d21-d22
p212
[LT2,UT2]
P121
[LB21,UB21]
Dest2-Jan
Source
P122
Car2-Jan
d21
Car2
p221
[LB22,UB22]
Dest2-Feb
d22
p222
Car2-Feb

Figure 3