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Multicointegration in US consumption data.

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Abstract

The present paper tests for the existence of multicointegration between real per capita private consumption expenditure and real per capita disposable personal income in the USA. In doing so, we exploit the fact that the flows of disposable income and consumption expenditure on the one hand, and the stock of consumers' wealth, which can be considered as cumulative past discrepancies between the flows of income and expenditure, on the other hand, can be thought of as a stock-flow model, in which multicointegration is likely to occur. We apply recently developed I(2) techniques for testing for multicointegrating relations and find supporting evidence for the existence of multicointegration in US consumption data.

Keywords: Cointegration, multicointegration, I(2) processes, consumption.

JEL code: C12, C13, C22, C32, C51, E21.

1 Introduction.

Modelling consumer's expenditure has been a long-standing occupation of several generations of economists as well as econometricians. Economists have put forward a number of prominent theories that have shaped our views of consumption and saving. These include, amongst others, the formulation of the consumption function suggested by Keynes (1936), the permanent income hypothesis (PIH) of Friedman (1957), and the life-cycle hypothesis of Ando and Modigliani (1963). These contributions have been judged so significant that they have played roles in the awarding of two Nobel prizes in economics.

Applied economists and econometricians for their part, have contributed the error-correction models, that dominates modern time series econometrics, as initially suggested in Davidson, Hendry, Srba, and Yeo (1978). The notion of error-correction mechanisms was first introduced into economics by Phillips (1954) and Phillips (1957) who borrowed the idea from the control engineering literature. These error-correction models were, later, statistically justified by the theory of cointegration (see Engle and Granger, 1987, *inter alia*). In general, error correction models came about as a response to the fact that theoretical economic models often stipulate only the long-run or equilibrium relations between the economic variables. In so doing, they often fail (or are unable) to describe the dynamic adjustment toward these equilibrium relations as well as to take the characteristic features of the actual data into consideration. Applied economists cannot be content with empirical models that yield only long-run or steady-state solutions. Thus, the approach initiated by Davidson et al. (1978), while taking its inspiration from formal theoretical economic models, specifically focused on designing empirical models that explicitly take the salient features of the data into account. The relevance of the empirical models are judged on the basis of several design criteria developed for this purpose. This data-driven approach, which subsequently evolved into what is known as the London School of Economics (LSE) approach, offers a practical way of modelling economic relations in general and modelling consumption functions in particular.

In this paper we model U.S. consumption function following this LSE traditions. The motivation of the paper is as follows. Following Davidson et al. (1978), we assume the existence of a long-run equilibrium relation between consumption expenditure and disposable income, which we assume to be well approximated by I(1) processes in the sequel. In reality, however, this relation need not to hold exactly in every time period. In other words, we assume that consumption expenditure and disposable income are cointegrated. Thus, the resulting savings variable, or cointegration error, defined as the difference between consumption expenditure and disposable income, is stationary. Intuitively, this approach is appealing since generally one cannot spend income without earning it and saving income without spending it also makes little sense.

Furthermore, suppose that in a given period saving is the increment to household wealth such that the savings accumulated over time represent a measure of private wealth. Hence, the three variables: income, expenditure, and the stock of cumulated savings (or wealth), taken together form a stock-flow model, where the difference between income and expenditure is the increment to the stock. Granger and Lee (1989, 1991) were the first to suggest the possible existence of a second cointegrating relation in stock-flow type models, i.e. when the flow variables cointegrate with the stock variable (which itself is created from the past flows). In our case, this corresponds to cointegration between the stock of wealth and the flows of expenditure and income, and thus the income and expenditure variables are multicointegrated in the sense of Granger and Lee (1989, 1991). In a bivariate system, multicointegration means that there exist two cointegrating relations formed by the two original time series and their transformations. This is opposite to the usual cointegration case where only a single cointegrating relation is allowed in a model with only two variables. As we have noted above, the first cointegrating relation arises between the levels of the flow variables, whereas the second cointegrating relation arises from the cumulated equilibrium errors, obtained in the first step, as well as the original variables in levels.

The approximation of the stock of wealth by summation of the past discrepancies between disposable income and consumption expenditure is not new in the econometrics literature. In fact, Stone (1966, 1973) first approximated the stock of wealth held by households by cumulating past savings in his study of the UK consumer expenditures. Clearly, the introduction of wealth effects into the study of consumption behaviour of the economic agents seems not to be unwarranted as some (unobservable to econometrician) wealth stock must undergo some changes when income and expenditure flows fail to match each other [1]. Elaborating on Davidson et al. (1978), Hendry and von Ungern-Sternberg (1981) were the first to incorporate wealth into error-correction framework by stipulating the existence of a long-run relation between the stock of wealth, on the one hand, and the disposable income on the other hand. In other words, using the modern terminology, the consumption function, developed in Hendry and von Ungern-Sternberg (1981), can be considered as a multicointegrating system, which can be statistically tested using already available techniques.

The literature on multicointegration has been rather limited to date [2]. To the best of our knowledge, Lee (1996) is the only published study that tests for multicointegration in US consumption data and finds no evidence for mulicointegration. On the other hand, Granger and Lee (1989, 1991) found support for the presence of multicointegrating relations existing between sectorial production and sales figures across a range of US industries and industrial aggregates. In this case, two cointegrating relations were found amongst the production and sales variables and the stock of inventory defined as the cumulative historic difference between production and sales. In succession, Lee (1992, 1996) and Engsted and Haldrup (1999) detected multicointegration in data for US housing. They found a stationary linear relation amongst the flows of housing units started and completed as well as the stock of housing units under construction; the latter being defined as the cumulated quasi-differences between the number of housing units started and completed in a given period.

While Granger and Lee (1989, 1991) and Lee (1992, 1996) estimated the multicointegrating relations using only the original I(1) variables, Engsted and Haldrup (1999) showed that the statistical inference and estimation of the multicointegrating relations could be carried out in the framework of the Johansen FIML procedure for I(2) variables (Johansen, 1995). In order to apply this procedure,the original I(1) flow variables must be transformed into their cumulated stock variants which then become I(2) series by construction. As advocated by Engsted and Johansen (1999), this transformation of variables is necessary because the I(1) analysis turns out to be invalid in the presence of multicointegration.

In the present paper we address the detection and estimation of a possible multicointegrating relation in the US consumption data on consumer expenditure and disposable income as measured in the National Income and Product Accounts(NIPA) by employing the recent technique developed in Engsted and Haldrup (1999). For this purpose we use the same data set as in Campbell (1987) which spans the years of 1953-1984. We use this limited in time data set for the following reasons. First, it allows us to directly compare our results obtained via use of the I(2) technique with those of Campbell (1987) who, among other things, estimated the marginal propensity to consume (henceforth, MPC) from hypothetical permanent income using only the first cointegration level between the consumers' income and expenditures. In particular, we are interested in determining whether his rejection of the unitary MPC hypothesis could be attributed to omission of the multicointegrating relation in the modelling process. Second, by employing this period we avoid the wealth effects of the stock market boom of the nineties on household consumption decisions. In the presence of a persistently rising stock market (and implied growth in household wealth), the propensity to consume out of disposable income increases, as do borrowing incentives. As the consequence, savings correspondingly decline as the households increasingly rely on the realized and expected positive equity market gains to do their savings for them (Auerback, 2000). This echoed by the fact that the personal saving rate based on the NIPA, estimated around quite a steady eight percent during the sixties and seventies, has collapsed to the levels of around two percent in 1997 and further to the negative area in 1999 for the first time since the Great Depression (Gale and Sabelhaus, 1999). Since in our model the stock of the wealth is approximated only by cumulative past savings, in times of the sky-rocketing stock market of the nineties our measure of wealth diverges from that perceived by consumers by significantly understating the stock of wealth.

Our main findings are following. First, using the Johansen FIML I(2) cointegration procedure for the first time in the cointegration literature we find the supportive evidence for the multicointegration in the US consumption data. The two error correction mechanisms from the corresponding two levels of cointegration have statistically significant adjustment coefficients with anticipated signs when inserted in vector error-correction models suggested in Engsted and Haldrup (1999) for the multicointegrated variables. Second, we cannot reject the null hypothesis that the marginal propensity to consume from the permanent income equals unity. This finding contrasts that obtained in Campbell (1987).

The plan of the paper is as follows. In Sections 2 and 3, we provide the formal definition of multicointegration in the sense of Granger and Lee together with a brief description of the Johansen FIML I(2) estimation technique which we use to make statistical inference as well as for estimation of the multicointegrating relation as it was originally done in Engsted and Haldrup (1999). Next, we present the stock-flow vector error correction models - henceforth VECM - for the multicointegrating variables in Section 4. The data set and the empirical results are described in Section 5. We draw conclusions and discuss possible extensions and limitations of this study in Section 6.

2 The Statistical Model.

We use the consumption-income example presented above for the formal definition of multicointegration. Suppose that income, y_t, and consumption variables, c_t, are integrated of order one. Moreover, assume that the variables in question are cointegrated, i.e. such that there exists some stationary linear combination of these variables, or equivalently, these two variables share a common stochastic trend:

$$s_t = y_t - \frac{1}{\gamma} c_t \ \sim \ I(0). \tag{1}$$

The I(0) variable on the left hand side of (1) , s_t, represents the cointegration error. Multicointegration occurs when the cumulated cointegration error, which is an I(1) stock variable by construction, forms a cointegrating relation CI(1,1) with either one of the original flow variables or both [3]:

$$\sum_{j=1}^{t} \left(y_j - \frac{1}{\gamma} c_j \right) + \phi_1 y_t + \phi_2 c_t \ \sim \ I(0). \tag{2}$$

Notice that (2) represents the stationary linear combination between the flows of consumption and income and the wealth variable which is the cumulated stock of the past discrepancy between income and consumption.

Furthermore, if we adopt the convention that the generated I(2) variables are denoted in capital letters,

$$Y_t = \sum_{j=1}^{t} y_j, \ C_t = \sum_{j=1}^{t} c_j, \ \Delta Y_t = y_t, \ \Delta C_t = c_t,$$

then we can write our multicointegrating relation (2) in the form of a polynomial cointegrating relation

$$Y_t - \frac{1}{\gamma}C_t + \phi_1 \Delta Y_t + \phi_2 \Delta C_t \ \sim \ I(0), \tag{3}$$

which occurs when I(2) variables cointegrate with their first differences, or at least one of them, i.e. we could have that either $\phi_1 = 0$ or $\phi_2 = 0$. For example, in the case when $\phi_2 = 0$ the multicointegrating relation is represented only in terms of the wealth stock and disposable income. Observe that such a relation resembles one that has been suggested in Hendry and von Ungern-Sternberg (1981) in addition to the cointegrating combination (1) which has been suggested earlier in Davidson et al. (1978). Thus, the consumption function of Hendry and von Ungern-Sternberg (1981) with income and wealth effects can be represented as a multicointegrating system.

Note that in the first cointegrating relation (1) we estimate the parameter γ. In Campbell (1987) the γ parameter is defined as the marginal propensity to consume out of the hypothetical permanent income. Campbell (1987) estimates this parameter in equation (1) using both a method of the grid-search and the two-step Engle-Granger procedure, see Engle and Granger (1987). The method applied in this paper can be considered as an alternative estimation method of the parameter of interest. Note that the parameter γ is expected to be either equal to one or be a positive fraction, i.e. $0 < \gamma \leq 1$.

As mentioned by Engsted and Haldrup (1999), the existence of the stationary relation between the stock and flow variables (3) would imply that we can estimate the parameter γ in the first step cointegrating relation (1) at the fast rate of consistency, $O_p(T^{-2})$.

Having defined the statistical and economic models, we consider the estimation and inference procedures as well as the VECM representations for the multicointegrating variables.

3 Estimation and Inference Procedures.

Initially[4], consider the following unrestricted VAR model of order k for the $p \times 1$ vector of variables X_t integrated of order two:

$$X_t = \Pi_1 X_{t-1} + ... + \Pi_k X_{t-k} + \varepsilon_t, \quad t = 1, ..., T, \tag{4}$$

where we assume fixed initial values. The error term is identically, independently distributed N(0,Ω). We also assume here that the roots of the characteristic polynomial of (4) either take value of unity or lie outside the unit circle.

Following Johansen (1995), as an intermediate step we can reparametrize (4) as:

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t, \tag{5}$$

where

$$\Pi = \sum_{i=1}^{k} \Pi_i - I,\ \Gamma_i = -\sum_{j=i+1}^{k} \Pi_j,\ i = 1, ..., k-1.$$

Finally, after one more rearrangement we arrive at

$$\Delta^2 X_t = \Pi X_{t-1} - \Gamma \Delta X_t + \sum_{i=1}^{k-2} \Phi_i \Delta^2 X_{t-i} + \varepsilon_t, \tag{6}$$

where

$$\Gamma = I - \sum_{i=1}^{k-1} \Gamma_i,\ \Phi_i = -\sum_{j=i+1}^{k-1} \Gamma_i,\ i = 1, ..., k-2.$$

The last reformulation (6) is convenient for the subsequent analysis because it displays rather explicitly the reduced rank conditions that characterize the model with I(2) variables. Hence, according to Johansen (1995) the I(2) model nested in the unrestricted VAR involves the following two reduced rank conditions:

$$\underset{p \times p}{\Pi} = \alpha\beta' \quad \underset{(p-r)\times(p-r)}{\alpha'_{\perp}\Gamma\beta_{\perp}} = \xi\eta',$$

where α and β are $p \times r$ matrices, and $\alpha_{\perp}$ and $\beta_{\perp}$ are the respective orthogonal complements of dimension $p \times (p-r)$ with $r < p$, such that by definition we have that $\alpha'_{\perp}\alpha = 0$ and $\beta'_{\perp}\beta = 0$. The matrices ξ and η have the dimensions $(p-r) \times s$ with $(p-r) > s$. Further description of an I(2) model requires more notation. Denote $\overline{\alpha} = \alpha\left(\alpha'\alpha\right)^{-1}$ such that $P_{\alpha} = \overline{\alpha}\alpha'$ is the orthogonal projection matrix onto the vector space spanned by the columns α and correspondingly $\alpha'\overline{\alpha} = \mathbf{I}$ is the identity matrix. Then in addition to already introduced $p \times r$ matrix α we can define the following matrices $\alpha_1 = \overline{\alpha}_{\perp}\xi$ and $\alpha_2 = \alpha_{\perp}\xi_{\perp}$ of the corresponding dimensions of $p \times s$ and $p \times (p-r-s)$ in such a way that α, α_1, and α_2 provide an orthogonal basis for the $p-$dimensional vector space. The same holds for the following matrices β, $\beta_1 = \overline{\beta}_{\perp}\eta$, and $\beta_2 = \beta_{\perp}\eta_{\perp}$ which have dimensions of $p \times r$, $p \times s$, and $p \times (p-r-s)$, respectively.

Using this notation, we can give the condition that rules out the presence of variables which are integrated of order higher than two, i. e. the following matrix needs to be of full rank:

$$\alpha_2'\theta\beta_2 = \alpha_2'\left\{\Gamma\overline{\beta}\overline{\alpha}'\Gamma + \sum_{i=1}^{k-1} i\Gamma_i\right\}\beta_2.$$

In the following we will refer to the numbers r, s, and $p-r-s$ as the *integration indices*. Given the fact that we have p variables in the system (6), these integration indices, respectively, indicate the number of I(0), I(1), and I(2) relations present in the model. Thus, the I(2) model is characterized by the following. There are $p-r-s$ linear combinations that do not cointegrate and represent the common stochastic I(2) trends:

$$p-r-s: \qquad \beta_2' X_t \sim I(2).$$

There are s linear combinations of the X_t variables that cointegrate to the I(1) level referred to as the common stochastic I(1) trends:

$$s: \qquad \beta_1' X_t \sim I(1).$$

The remaining r linear combinations of the X_t variables and often its first differences, ΔX_t, cointegrate to the I(0) level:

$$r: \qquad \beta' X_t - \delta\beta_2'\Delta X_t \sim I(0), \tag{7}$$

where $\delta = \overline{\alpha}'\Gamma\overline{\beta}_2$ is the $r \times (p-r-s)$ matrix. This matrix has an $r \times (r-(p-r-s))$ orthogonal complement $\delta_\perp$, such that $\delta_\perp'\delta = 0$. Thus, in general we have that $r \geq (p-r-s)$. However, as discussed in Engsted and Haldrup (1999) for the multicointegrating system it should be rather common that $r = (p-r-s)$.

It is important to note that for a bivariate I(2) system with $X_t = (Y_t, C_t)'$ this linear combination (7) constitutes the only possible polynomially cointegrating relation defined in (3) with $\beta = (1, -1/\gamma)'$ and $\delta\beta_2' = (\phi_1, \phi_2)$. Therefore this relation is of our primary interest. Hence, we would expect in the multicointegrating system to have one stationary relation[5], $r = 1$, no common I(1) trends, $s = 0$, and one common I(2) trend, $p-r-s = 1$.

As suggested in Johansen (1992) and Kongsted and Nielsen (2002), a representation of multicointegrating relation (7) is not unique in the sense that we can find a coefficient matrix $\widetilde{\delta}$ and the corresponding vector υ with the property $\upsilon'\beta_2 \neq 0$, such that the following relation is I(0) as well:

$$r: \qquad \beta' X_t - \widetilde{\delta}\upsilon'\Delta X_t \sim I(0). \tag{8}$$

By specifying the matrix $\widetilde{\delta} = \delta\beta_2'\beta_2(\upsilon'\beta_2)^{-1}$ and choosing a $p \times 1$ unit vector $\upsilon = (..., 0, 1, 0, ...)'$ with one at the ith position with $i = 1, ..., p$ and zeros at the remaining positions $j \neq i$ with $j = 1, ..., p$, we can select either element of vector ΔX_t to appear in relation (8). Observe that for bivariate system the choice of unit vectors υ is restricted to $\upsilon = (1, 0)'$ or $\upsilon = (0, 1)'$.

Moreover, as noted in Johansen (2002) analysis of I(2) systems is complicated by the fact that rather few hypotheses on the multicointegrating relations allow the usual asymptotic χ^2 inference. As suggested in Kongsted and Nielsen (2002), the solution is to transform the original I(2) system to the I(1) system, for which the theory of inference is well-developed. The transformed system consists of the following I(1) variables $\widetilde{X_t} = ((B'X_t)', (\upsilon'\Delta X_t)')'$, where $B = b_\perp$ is the $p \times (r+s)$ orthogonal complement of a known matrix b for which the orthogonality condition $b'(\beta, \beta_1) = 0$ is satisfied. The advantage of such transformation is that the inference on the parameters of the multicointegrating relations can be achieved using the standard I(1) technique. In particular, the multicointegrating parameter in the transformed system can be expressed as $\widetilde{\delta} = \delta b'b(\upsilon'b)^{-1}$, where δ reflects the normalization rule for β_2 in the original I(2) model and b provides a valid basis for β_2.

In order to address the question of how the models with different integration indices are related we need the following notation. First, consider the restricted I(1) model without any I(2) trends. This corresponds to the case when $p - r = s$, i.e. the matrix $\alpha_\perp'\Gamma\beta_\perp$ has full rank. Thus we have only one reduced rank condition left. Therefore, we denote H_r as a model that has $rank(\Pi) \leq r < p$, whereas H_r^0 denotes the model with the $rank(\Pi) = r$. Therefore H_r^0 is a submodel of H_r or $H_r = \cup_{i=0}^r H_i^0$.

Similarly, we define the more general hierarchical ordering of the models by allowing for the I(2) relations as well. The model with H_{rs} involves two reduced rank conditions: $rank(\Pi) = r < p$ and $rank(\alpha_\perp'\Gamma\beta_\perp) \leq s < (p - r)$. It nests the sub-models H_{rs}^0 with $rank(\alpha_\perp'\Gamma\beta_\perp) = s$ such that the various models are related as follows: $H_{rs} = \cup_{i=0}^s H_{ri}^0$ and $H_{r0} \subset H_{rs} \subset ... \subset H_{rp-r} = H_r^0 \subset H_r \subset H_p$.

The relations amongst the various bivariate models with the different integration indices are viewed best when presented in Table 1, adapted from Johansen (1995).

Insert Table 1 about here.

Recapitulating, the upper-left corner of Table 1 houses the most restricted model H_{00} with $\Pi = \Gamma = 0$ such that we have only the noncointegrating I(2) variables present. This corresponds to the VAR in second differences, see (6). The unrestricted model placed in the lower-right corner is H_p with $p = 2$, where we have only I(0) variables. The remaining models comprise one or another form of cointegration as discussed above. The exception is the model H_0 in the upper-right corner which contains only the noncointegrating I(1) variables such that it corresponds to the VAR model in first differences.

This order of how the various models are nested determines the sequence of the testing procedure for the integration indices in our model. We start testing with the most restrictive model against the unrestricted alternative. In case we reject the hypothesis in question, we proceed to the less restrictive model and so on until the first hypothesis that we cannot reject. This determines the integration indices.

Notice that here we have ignored the deterministic terms for expositional simplicity. However, inclusion of the appropriate deterministic terms seems to be an important issue in the empirical application that follows. In particular, it is desirable to account for the following two features of the data under scrutiny. First, we would like to allow for the presence of the quadratic trend in our model. This is due to the fact that cumulation of the original I(1) variables, which exhibit the trending behavior (see Figure 1), generates the nonlinear deterministic trends. Secondly, we would like to allow for the possibility of having the trend-stationary multicointegrating relation in our model. To see this, note that if the estimated I(0) savings variable in equation (1) has a nonzero mean then by creating the measure of wealth stock as an integral of the savings we create an I(1) variable with a linear trend. As a consequence, this generated wealth stock combined with the original trending I(1) variables in general should form a trend-stationary multicointegrating relation (2) unless these linear trends cancel each other out.

There are two readily developed parametrizations of the deterministic terms in the I(2) models such as Rahbek, Kongsted, and Jørgensen (1999) and Paruolo (1994). The distinguishing feature of the former specification is that it allows for the presence of the trend-stationary multicointegrating relations. However, it does not allow for the existence of the quadratic trends in the model. Hence, we find it inappropriate for the empirical application that follows. On the other hand, the specification of Paruolo (1994) does allow for the quadratic deterministic trends in the model. However, it has a limitation as it does not allow for the trend-stationary multicointegrating relations. Without any other choice left, it is worthwhile taking a closer look at this specification. Then we will discuss how the issue of trend-stationarity can be tested indirectly within this model.[6]

Considered the VAR model (4) with an unrestricted constant, i.e.:

$$X_t = \Pi_1 X_{t-1} + ... + \Pi_k X_{t-k} + \mu + \varepsilon_t, \quad t = 1, ..., T, \tag{9}$$

According to Paruolo (1994), this results in the presence of linear and, most importantly, quadratic trends in the data. This is best seen in the corresponding common stochastic trends representation[7] of the model given in (9):

$$
\begin{aligned}
X_t &= C_2 \sum_{s=1}^{t} \sum_{i=1}^{s} (\varepsilon_i + \mu) + C_1 \sum_{i=1}^{t} (\varepsilon_i + \mu) + C^* (L) \varepsilon_t, \\
X_t &= \tfrac{1}{2} C_2 \mu t^2 + \left(\tfrac{1}{2} C_2 \mu + C_1 \mu\right) t + C_2 \sum_{s=1}^{t} \sum_{i=1}^{s} \varepsilon_i + C_1 \sum_{i=1}^{t} \varepsilon_i + C^* (L) \varepsilon_t,
\end{aligned}
\tag{10}
$$

where

$$
\begin{aligned}
C_2 &= \beta_2 \left(\alpha_2' \theta \beta_2\right)^{-1} \alpha_2' \\
\beta' C_1 &= \overline{\alpha}' \Gamma C_2 \\
\beta_1' C_1 &= \overline{\alpha}_1' \left(I - \theta C_2\right),
\end{aligned}
$$

and the matrix lag polynomial $C^* (L)$ has all the roots strictly outside the unit circle.

The model (10) allows for the I(2) process X_t with quadratic trends given by $\frac{1}{2} C_2 \mu t^2$. Next, by defining the I(1) relations as $\beta_1' X_t$ we annihilate both the stochastic I(2)- and the quadratic deterministic trends such that the resulting I(1) linear combinations have only at most a linear deterministic trend given by $\beta_1' C_1 \mu t$. Finally, due to the equality restriction $\beta' C_1 = \overline{\alpha}' \Gamma C_2$ there are no linear deterministic trends in the multicointegrating relations $\beta' X_t - \delta \beta_2' \Delta X_t$.

Notice that the model (10) does not allow for different stochastic and deterministic orders in either of I(2), I(1), or I(0) directions. In particular, as opposed to the parametrization of Rahbek et al. (1999) it does not allow for the trend-stationary multicointegrating relations. Despite the fact that we do not have a more general model that, by allowing both for quadratic trends and trend-stationary relations, encompasses the specification of Paruolo (1994), we still are able to address the issue of the existence of the linear trend in the multicointegrating relation. This line of argument uses the interesting similarities of the inference procedure for integration indices that exists between these two mentioned model specifications. But before doing this we describe the steps of the inference procedure on the integration indices.

Rahbek et al. (1999) and Paruolo (1994) show that in the presence of the imposed restrictions on the deterministic terms inference on the integration indices is performed in the likelihood-based two-step procedure similar to Johansen (1995). Essentially, at the first step we address the reduced rank of $\Pi = \alpha \beta'$ by testing the restricted model H_r against the unrestricted alternative H_p. For later use in the empirical section, we denote the corresponding test statistic as $S(r)$. Then, by fixing the rank of the matrix Π at each of the following values $r = 0, ..., p-1$ we address the reduced rank of the other matrix $\alpha_\perp' \Gamma \beta_\perp = \xi \eta'$ by testing the restricted model H_{rs} against the alternative $H_{r,p-r}$ model. Finally, because of the fact that in practice the reduced rank of the matrix Π is unknown and since the models are nested as discussed earlier, inference on the integration indices is based on the joint hypothesis of H_{rs} against the unrestricted H_p model. The relevant test statistic is referred to as $S(r,s)$.

As discussed, in the first step both specifications address the number of stationary relations in the model given by the rank of the Π matrix. At this stage, the models of Paruolo (1994) and Rahbek et al. (1999) differ in that the latter allows for the linear trend to be restricted to the cointegration space. Moreover, according to Rahbek et al. (1999) the likelihood-ratio test statistic for the trend exclusion has $\chi^2(r)$ asymptotic distribution for the given rank r of the Π matrix. This result holds regardless of the values of other integration indices, see Corollary 4.1 in Rahbek et al. (1999). Thus, ignoring the I(2) nature of the model we can address the issue of trend-stationarity of the multicointegrating relations using the standard inference procedure.

We have argued above that the bivariate multicointegrating model contains two cointegrating vectors that essentially appear in the form of a single polynomially cointegrating vector. In the next section we demonstrate how these equilibrium relations can be incorporated into VECM representations for multicointegrated variables.

4 The VECM for Multicointegrating Variables.

Engsted and Haldrup (1999) suggest two types of vector error correction models (VECM) for the multicointegrating variables. The first type shows how the flow variables react to deviations from an equilibrium. The second type shows disequilibrium responses in the stock variables.

Engsted and Haldrup (1999) present the VECM for the general case, potentially embracing more than two variables. Since we operate in the bivariate system, we know that multicointegration in such a system implies the following integration indices: $r = 1$, $s = 0$, and $p - r - s = 1$. This knowledge allows us to simplify significantly the presentation of the VECM for the multicointegrating variables which is given below.

Definition 1 *The bivariate <u>flow</u> VECM representation for the multicointegrating variables:*

$$\Delta x_t = \alpha[Q_{t-1} - \delta\beta_2' x_{t-1}] - \zeta_1 \Delta Q_{t-1} + \Phi(L)\Delta x_t + \varepsilon_t, \tag{11}$$

where $x_t = (y_t, c_t)'$, $Q_t = \sum_{j=1}^{t} \beta' x_j$ *represents the stock of cumulative equilibrium errors and* $\Phi(L) = \sum_{i=1}^{k-2} \Phi_i L^i$ *contains the coefficients of the short-run dynamics. The adjustment coefficients* α *and* $\zeta_1 = \Gamma\overline{\beta}$ *have the equal dimensions of* $p \times r$*, with* $\overline{\beta} = \beta(\beta'\beta)^{-1}$*. The matrix* $\delta = \overline{\alpha}'\Gamma\overline{\beta}_2$ *is* $r \times (p - r - s)$*, where* $\overline{\alpha}$ *and* $\overline{\beta}_2$ *are defined similarly to* $\overline{\beta}$*.*

Notice that this VECM incorporates several control mechanisms, as discussed in Hendry and von Ungern-Sternberg (1981), for example. For instance, the integral control mechanism, $[Q_{t-1} -$

$\delta\beta_2' x_{t-1}]$, represents the multicointegrating relation. The proportional control mechanism, $\Delta Q_{t-1} = \beta' x_{t-1}$, represents the first step cointegrating relation between the variables in levels, and lastly, the derivative control mechanism is given by the lagged Δx_t's.

Definition 2 *The bivariate* <u>*stock*</u> *VECM representation for the multicointegrating variables*

$$\begin{aligned} \Delta\widetilde{x}_t &= M\alpha[Q_{t-1} - \delta\beta_2' x_{t-1}] - \widetilde{\zeta}_1 \Delta Q_{t-1} + \\ &\quad + \widetilde{\Phi}(L)\Delta\widetilde{x}_t + M\Phi(L)\overline{\beta}_2 \Delta\beta_2' x_{t-1} + M\varepsilon_t, \end{aligned} \tag{12}$$

where in addition to the variables and the model parameters defined above we have $\Delta\widetilde{x}_t = (\Delta Q_t', \Delta x_t'\beta_2)'$, $M = (\beta, \beta_2)'$, $\widetilde{\zeta}_1 = (\iota_r - M\zeta_1)$ *with* $\iota_r = (1, 0)'$, *and* $\widetilde{\Phi}(L) = M\Phi(L)M^{-1}D_\perp(1)$ *such that* $M^{-1} = (\overline{\beta}', \overline{\beta}_2')$ *and*

$$D(L) = \begin{pmatrix} \Delta & 0 \\ 0 & 1 \end{pmatrix} \quad D_\perp(L) = \begin{pmatrix} 1 & 0 \\ 0 & \Delta \end{pmatrix} \quad D(L)D_\perp(L) = \begin{pmatrix} \Delta & 0 \\ 0 & \Delta \end{pmatrix}.$$

The latter VECM representation is worth commenting further on. First, note that the equilibrium relations are the same as in the former representation. Secondly, the variables that adjust to the previous period disequilibrium state are the stock variable, $Q_t = \sum_{j=1}^{t} \beta' x_j$, as well as the flow variables that appear in the VECM as the first difference of the I(2) trends, $\beta_2' x_t$. Finally, note that the "$\sim$" parameters in (12) retain the same dimension as the parameters without the "$\sim$" sign in (11).

5 The Empirical Application.

In this study we use the same data set as in Campbell (1987)[8]. This data set contains quarterly data for the period of 1953:2 to 1984:4 with 127 observations. The data are the seasonally adjusted time series of real disposable income and real total private consumption expenditure taken from the National Income and Product Accounts with some adjustments made by Blinder and Deaton (1985). The data are in per capita values in units of thousands US$. As noted above, by addressing this sample period we are able to compare our results with those of Campbell.

The upper panel of Figure 1 displays the actual values of the real total disposable income, y_t, and the real total private consumption expenditure, c_t[9]. As seen, both the time series develop very synchronously. Given the results of the ADF test reported in Campbell (1987) and Table 2 that these variables are I(1), this is the first sign that they might be cointegrated. However, we are interested in testing whether the variables in question are multicointegrated.

Insert Table 2 about here.

Insert Table 3 about here.

In order to sort this out, we first transform the variables into their cumulative counterparts, Y_t and C_t, which are shown in the lower panel of Figure 1. Next we use these newly generated I(2) variables to form a parsimonious bivariate VAR(7) model. Table 3 summarizes the results of both the univariate and multivariate diagnostic tests of the estimated residuals. The univariate diagnostic tests comprise: F_{AR8} - test for autocorrelation of most *8*$^{\text{th}}$ order (see Godfrey (1978)); *Normality* - test for the normally distributed residuals (see Doornik and Hansen (1994); F_{HET} - White (1980) test for heteroscedasticity based on the original and squared regressors; F_{ARCH4} - Engle (1982) test for the *4*$^{\text{th}}$ order AutoRegressive Conditional Heteroscedasticity. The multivariate test statistics denoted with the superscript v were derived in Doornik and Hansen (1994) for vector normality, and in Doornik (1995) for vector autocorrelation and vector heteroscedasticity. The graphics, regression output, and residual diagnostic tests were calculated using GiveWin 2.2 and Pc-Give 10.2 (see Doornik and Hendry, 2001a,b).

Taken as a whole, it seems that the model residuals do not display autocorrelation, ARCH effects, and heteroscedasticity when judged on the basis of both from the results of the univariate and the multivariate specification tests. However, their is some deviation from the normality assumption in the equation for C_t. An additional information can be obtained from Figure 2, which provides a graphical analysis of the estimated residuals. It contains the estimated residuals, their correlogram, spectral density, and histogram. As seen, the deviation from normality in the equation for C_t occurs due to a small number of large negative residuals. The most important assumption we require to be fulfilled is the absence of autocorrelation in the VAR residuals, since it introduces nuisance parameters in the limiting distribution of the test statistics. This invalidates the asymptotic critical values that we use in our statistical inference procedure. On the other hand, Gonzalo (1994) showed that the FIML Johansen procedure is rather robust to minor departures from the model assumptions due to non-normality.

The system dynamics is summarized by the eigenvalues of the companion form of (4)

$$(1.006, 0.9839 \pm 0.0733i, 0.867 \pm 0.2793i, 0.6281 \pm 0.4737i, 0.06447 \pm 0.5902i,$$
$$-0.5195 \pm 0.2325i, -0.7123, -0.1898 \pm 0.7429i).$$

A priori, in the bivariate multicointegrating model we would expect two unit roots corresponding to the one common I(2) trend. As seen for the given realization of the stochastic variables in our model we have one explosive eigenvalue, but it needs not be significantly different from unity. Hence, we assume it to be a unit root in the sequel. Furthermore, we have two pairs of comparatively large complex conjugate eigenvalues of moduli 0.986 and 0.910, respectively. The remaining eigenvalues of rather smaller magnitude lie at some other different from the zero frequencies. Thus, the unrestricted VAR model seems to contain at least two unit roots or, possibly, more.

The statistical inference[10] of testing sequentially the hypotheses of the restricted submodel H_{rs} against the unrestricted alternative H_p yields the results displayed in Table 4.

Insert Table 4 about here.

As seen from Table 4 the rank determination is problematic as practically every hypothesis is rejected either at the 10% or 5% significance level. Hence, the results of formal testing suggest that the variables of interest are I(0). This contradicts the decisive evidence from the unit root tests, see Table 2, which suggests that the original variables in levels are I(1). Consequently, the constructed cumulated variables should be I(2) and hence we should have at least one common I(2) trend in the system. On the other hand, given the uniform rejection of the hypothesis $r = 0$, we can conclude that at least one stationary relation exists in our model. Using these considerations, we choose to restrict the values of the integration indices as follows: $r = 1$, $s = 0$, and $p - r - s = 1$. That is we allow for the presence of one common I(2) trend and one stationary multicointegrating relation in the system.

Having chosen the integration indices, we address the issue of possible trend stationarity of the multicointegrating relation as discussed above. The likelihood-ratio test statistics for the exclusion of trend from the multicointegrating relation yields the value of 1.2, which is not significant when compared with the usual critical values of the $\chi^2(1)$ distribution. This is the evidence against the trend-stationarity of the multicointegrating relation. This justifies application of specification of the deterministic terms according to Paruolo (1994).

Insert Table 5 about here.

Table 5 summarizes our estimation and inference results on the parameters of the multicointegrating relation. The unrestricted estimate has the following form:

$$\sum_{j=1}^{t}(y_j - 1.014c_j) - 5.741y_t - 5.664c_t. \tag{13}$$

As pointed out above, it embodies the first level cointegrating relation (1):

$$s_t = y_t - 1.014c_t, \tag{14}$$

where the estimated γ parameter is $\widehat{\gamma} = 1/1.014 = 0.986$. We are interested in testing of the following two hypotheses. First, the hypothesis of the unitary MPC from the permanent income, i.e. $\gamma = 1$. The corresponding likelihood ratio test statistic is 0.064 with the $p-$value 0.800 according to the asymptotic $\chi^2(1)$ distribution. Thus we cannot reject the null hypothesis that the MPC from the permanent income is one, i.e. income and consumption form a cointegrating relation described by $\beta = (1, -1)'$. Second, we test the hypothesis whether the estimate of the MPC reported in Campbell (1987) ($\widehat{\gamma} = 0.941$) is consistent with our results. In other words, we test whether income and consumption are cointegrated with the vector $(1, -1.062)'$ as given in Campbell (1987, see Table I, p. 1260). The likelihood ratio test for imposing this restriction on the obtained vector β yields the value of 7.295, which is significant even at the 1% level according to the asymptotic $\chi^2(1)$ distribution. In summary, our findings sharply contrast with those of Campbell (1987). We cannot reject the null hypothesis of the unitary MPC at the conventional significance levels, whereas we decisively reject the null hypothesis that the MPC takes value reported in Campbell (1987). The difference is attributed to the fact that we employ I(2) analysis and allow for existence of multicointegrating relation in the statistical model.

Furthermore, as suggested in Johansen (1992), after imposing the accepted restriction on the $\beta = (1, -1)'$ vector, we transform the multicointegrating relation (13) into the following one:

$$\sum_{j=1}^{t}(y_j - c_j) - \underset{(0.500)}{14.240}y_t. \tag{15}$$

Following suggestion of Kongsted and Nielsen (2002), the estimate and standard error of the multicointegrating parameter $\widetilde{\delta}$ has been obtained from the standard I(1) analysis on the transformed I(2) system $\widetilde{X_t} = ((B'X_t)', (v'\Delta X_t)')'$ with $X_t = (Y_t, C_t)', B = (1, -1)', b = (1, 1)'$, and $v = (1, 0)'$. The null hypothesis that there are no cointegrating relations in the transformed system is decisively rejected based on the trace test statistic of 21.399[0.005] with $p-$value in the squared parentheses, and the null hypothesis that there is at least one cointegrating vector correspondingly accepted with the trace test statistic of 2.0253[0.155]. This result further reinforces our earlier conclusion on the presence of the multicointegrating relation in the data under scrutiny.

We display the estimated cointegrating and multicointegrating relations in Figure 3.

As the final exercise we place the estimated equilibrium relations in the VECM discussed in Section 4. The flow VECM looks as follows:

$$\begin{pmatrix} \Delta y_t \\ \\ \Delta c_t \end{pmatrix} = \begin{pmatrix} \underset{(0.0027)}{-0.0000} \\ \\ \underset{(0.0026)***}{0.0084} \end{pmatrix} \left(Q_{t-1} - \underset{(0.500)***}{14.240}\ y_{t-1} \right) + \begin{pmatrix} \underset{(0.0888)}{-0.0061} \\ \\ \underset{(0.0859)***}{0.3071} \end{pmatrix} (\Delta Q_{t-1}) + \tag{16}$$

$$+ lags\{\Delta y_t, \Delta c_t\} + constant + error\ term.$$

The stock VECM is

$$\begin{pmatrix} \Delta Q_t \\ \\ \Delta \widehat{\beta}_2' \left(y_t, c_t \right)' \end{pmatrix} = \begin{pmatrix} \underset{(0.0023)***}{-0.0084} \\ \\ \underset{(0.0049)*}{0.0083} \end{pmatrix} \left(Q_{t-1} - \underset{(0.500)***}{14.240}\ y_{t-1} \right) + \begin{pmatrix} \underset{(0.0744)***}{0.6867} \\ \\ \underset{(0.1582)*}{0.3009} \end{pmatrix} (\Delta Q_{t-1}) + \tag{17}$$

$$+ lags\{\Delta y_t, \Delta c_t\} + constant + error\ term.$$

where the stock variable is $Q_t = \sum_{j=1}^{t} (y_j - c_j)$ in both the VECMs. The standard errors are reported below the estimated coefficients in the parentheses. The symbols $***, **, *$ indicate significance at the 1%, 5%, and 10% level, respectively.

As seen, in the flow VECM (16) the income adjustment coefficients that correspond to the integral and proportional mechanisms are found to be insignificant. On the opposite, the consumption adjustment coefficients are significant even at the 1% level and they are correctly signed. Hence, consumption strongly reacts to the past deviations from the found equilibrium relations through both adjustment channels. In the stock model, the first equation is of most interest to us, as it describes the adjustment of the wealth stock to the past disequilibria. Again, the adjustment coefficients are highly significant and correctly signed.

6 Conclusions.

Using the same data set as in Campbell (1987), this study has been first in the literature to detect the presence of multicointegrating relation between the consumption expenditure and disposable income flows that was anticipated by Granger and Lee (1989, 1991). As it was initially suggested by Engsted and Johansen (1999) and implemented in Engsted and Haldrup (1999), we perform statistical inference and estimation of the multicointegrating relation using the I(2) technique based on the Johansen (1995) FIML procedure.

Since we use the same data set as in Campbell (1987), we are able to make comparisons with his parameter estimates. Campbell uses the Engle-Granger two step procedure to obtain the super-consistent estimate of the parameter of his interest. The advantage of using the I(2) technique applied here, is that in the presence of multicointegration we are able to estimate the same parameter at the super-super consistent rate of $O_p(T^{-2})$. Also the two step procedure is generally invalid when series are multicointegrated, see Engsted and Johansen (1999). Our estimation results suggest that the hypothesis that income and consumption variables are cointegrating with the vector $\widehat{\beta} = (1, -1/\widehat{\gamma})' = (1, -1.062)'$ as reported in Campbell (1987) can be rejected at the 1% significance level, where γ denotes the marginal propensity of consumption out of the permanent income. On the other hand, we cannot reject the hypothesis that these variables are cointegrated with the vector $\widehat{\beta} = (1, -1)'$; paradoxically, this hypothesis has been rejected in Campbell (1987). Thus, our results suggest that the marginal propensity of consumption out of the permanent income equals unity, i.e. $\gamma = 1$.

The existence of a multicointegrating relation implies that there are two layers of cointegrating relations in the bivariate model. We incorporated these two estimated equilibrium relations in the error correction models for the multicointegrating variables that were initially proposed by Engsted and Haldrup (1999). The estimated adjustment coefficients that are statistically significant appear to be correctly signed in both VEC models.

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References

Ando, A. and F. Modigliani (1963). The ”life-cycle” hypothesis of saving: Aggregate implications and tests. *American Economic Review 53*(1), 55–84.

Auerback, M. (2000). Q1 US flow funds and the stock market. International Perspective, June 20, 2000: available online: http://www.prudentbear.com.

Banerjee, A., L. Cockerell, and B. Russell (2001). An I(2) analysis of inflation and the markup. *Journal of Applied Econometrics 16*, 221–240.

Blinder, A. S. and A. S. Deaton (1985). The time-series consumption revisited. *Brooking Papers on Economic Activity 1985*(2), 465–511.

Campbell, J. Y. (1987). Does saving anticipate declining labour income? An alternative test of the permanent income hypothesis. *Econometrica 55*, 1249–1273.

Campbell, J. Y. and A. S. Deaton (1989). Why consumption so smooth? *Review of Economic Studies 56*, 357–374.

Davidson, J. E. H., D. F. Hendry, S. Srba, and S. Yeo (1978). Econometric modelling of the aggregate time-series relationship between consumers' expenditure and income in the United Kingdom. *Economic Journal 80*, 661–692.

Doornik, J. (1995). Testing vector autocorrelation and heteroscedasticity in dynamic models. mimeo, Nuffield College, Oxford.

Doornik, J. A. and H. Hansen (1994). A practical test for univariate and multivariate normality. Discussion Paper, Nuffield College, Oxford.

Doornik, J. A. and D. F. Hendry (2001a). *GiveWin: An Interface to Empirical Modelling*. London: Timberlake Consultants Press.

Doornik, J. A. and D. F. Hendry (2001b). *Modelling Dynamic Systems Using PcGive*, Volume II. London: Timberlake Consultants Press.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica 50*, 987–1007.

Engle, R. F. and C. W. J. Granger (1987). Cointegration and error correction: Representation, estimation and testing. *Econometrica 55*, 251–276.

Engsted, T. and N. Haldrup (1999). Multicointegration in stock-flow models. *Oxford Bulletin of Economics and Statistics 61*, 237–254.

Engsted, T. and S. Johansen (1999). Granger's representation theorem and multicointegration. In R. Engle and H. White (Eds.), *Cointegration, Causality and Forecasting, Festschrift in Honour of Clive Granger*, Oxford. Oxford University Press.

Flavin, M. (1993). The excess smoothness of consumption: Identification and interpretation. *The Review of Economic Studies 60*(3), 651–666.

Friedman, M. (1957). *A Theory of the Consumption Function*. Princeton, NJ: Princeton University Press.

Fuller, W. A. (1976). *Introduction to Statistical Time Series*. New York: John Wiley.

Gale, W. G. and J. Sabelhaus (1999). Perspectives on the household saving rate. *Brookings Papers on Economic Activity 1*, 181–224.

Godfrey, L. G. (1978). Testing for higher order serial correlation in regression equations when the regressors include lagged dependent variables. *Econometrica 46*, 1303–1313.

Gonzalo, J. (1994). Five alternative methods of estimating long-run equilibrium relationships. *Journal of Econometrics 60*, 203–233.

Granger, C. W. J. and T. H. Lee (1989). Investigation of production, sales and inventory relations using multicointegration and non-symmetric error correction models. *Journal of Applied Econometrics 4*, S145–S159.

Granger, C. W. J. and T. H. Lee (1991). Multicointegration. In R. F. Engle and C. W. J. Granger (Eds.), *Long-Run Economic Relationships. Reading in Cointegration*, Advanced Texts in Econometrics, Oxford. Oxford University Press.

Haldrup, N. (1998). An econometric analysis of I(2) variables. *Journal of Economic Surveys 12*(5), 595–650.

Hendry, D. F. and T. von Ungern-Sternberg (1981). Liquidity and inflation effects on consumers' expenditure. In A. S. Deaton (Ed.), *Essays in the Theory and Measurement of Consumers' Behaviour: In Honour of Sir Richard Stone*, Cambridge. Cambridge University Press.

Johansen, S. (1992). Testing weak exogeneity and the order of integration in UK money demand data. *Journal of Policy Modeling 14*, 315–335.

Johansen, S. (1995). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Advanced Texts in Econometrics. Oxford: Oxford University Press.

Johansen, S. (2002). The statistical analysis of hypotheses on the cointegrating relations in the I(2) model. mimeo, Department of Statistics and Operations Research, University of Copenhagen.

Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money.* London: Macmillan.

Kongsted, H. C. (2003). I(2) cointegration analysis of small country import price determination. *Econometrics Journal 6*, 53–71.

Kongsted, H. C. and H. B. Nielsen (2002). Analyzing I(2) systems by transformed vector regressions. working paper 2002-20, Institute of Economics, University of Copenhagen.

Lee, T. H. (1992). Stock-flow relationships in US housing construction. *Oxford Bulletin of Economics and Statistics 54*, 419–430.

Lee, T. H. (1996). Stock adjustment for multicointegrated series. *Empirical Economics 21*, 63–639.

Muellbauer, J. and R. Lattimore (1995). The consumption function: A theoretical and empirical overview. In M. H. Pesaran and M. R. Wickens (Eds.), *Handbook of Applied Econometrics*, Oxford and Malden, Mass., pp. 221–311. Blackwell.

Nielsen, H. B. (2002). An I(2) cointegration analysis of price and quantity formation in Danish manufactured exports. *Oxford Bulletin of Economics and Statistics 64*(5), 449–472.

Paruolo, P. (1994). The role of the drift in I(2) systems. *Journal of Italian Statistical Society 3*(1), 93–123.

Phillips, A. W. (1954). Stabilization policy in the closed economy. *The Economic Journal 64*, 290–323.

Phillips, A. W. (1957). Stabilization policy and the time-forms of lagged responses. *The Economic Journal 67*, 265–77.

Rahbek, A., H. . C. Kongsted, and C. Jørgensen (1999). Trend-stationarity in the I(2) cointegration model. *Journal of Econometrics 90*, 265–289.

Stone, R. (1966). Spending and saving in relation to income and wealth. *L'industria 4*, 471–499.

Stone, R. (1973). Personal spending and saving in postwar Britain. In H. C. Bos, H. Linneman, and P. de Wolff (Eds.), *Economic Structure and Development (Essays in Honour of Jan Tinbergen)*, Amsterdam. North-Holland Publishing Co.

Vahid, F. and R. Engle (1993). Common trends and common cycles. *Journal of Applied Econometrics 8*, 341–360.

White, H. (1980). A heteroscedasticity-consistent covariance matrix estimator and a direct test for heteroscedasticity. *Econometrica 48*, 817–838.

Notes

[1]Muellbauer and Lattimore (1995) summarize the economic role of assets on the consumption decisions.

[2]We would like to distinguish between multi- and polynomial cointegration. The former refers to the situation when the focus is on the long-run relations between the original I(1) variables and their I(2) transformation, whereas the latter - on the long-run relations between the original I(2) variables and its first differences or some other I(1) variables. For recent examples of polynomial cointegration analysis see Kongsted (2003), Nielsen (2002), and Banerjee, Cockerell, and Russell (2001).

[3]We assume zero initial values for y_t and c_t. Such scaling has no implications for the further analysis, except that proper allowance for deterministic components in the model will be needed.

[4]The I(2) analysis in VAR models is technically involved. Therefore, in the further discourse we mainly present the skeleton of the inference and estimation procedures we use. For a recent review of the I(2) analysis as well as for the further technical details, see e.g. Haldrup (1998) and the references therein.

[5]Observe that by using the I(2) formulation of the problem the single multicointegrating relation involves the two layers of cointegration that follow from the usual I(1) analysis.

[6]This point has been made by Hans Christian Kongsted.

[7]In the common stochastic trends representation we omit the nuisance parameters introduced by the initial conditions, see Johansen (1995).

[8]This dataset has been used extensively in the literature, for example, in Blinder and Deaton (1985), Campbell and Deaton (1989), Flavin (1993), and Vahid and Engle (1993).

[9]As pointed out by Muellbauer and Lattimore (1995), when modelling the consumption func-

tion one should be concerned with the possibility that the error term grows with the scale of consumption. Consequentially, they suggest using the log transformations of the consumption and explanatory variables in order to remedy this potential problem. Fortunately, this effects are absent in our data, see Figure 1. Therefore in the subsequent analysis we proceed with the variables measured in the natural units. Further advantage of using the data in the present form is that in the framework of the multicointegration analysis it is easier to interpret the measure of wealth in terms of the cumulative savings.

[10]All I(2) analysis has been performed using the I(2) procedure written by Clara M. Jørgensen for the CATS in RATS package(available online http://www.estima.com/procs/i2index.htm).

Table 1: Hierarchy of the various models for $p = 2$.

r	$I(2)$ *model*				$I(1)$ *model*				$I(0)$*model*
0	H_{00}	$\subset$	H_{01}	$\subset$	$H_{02} = H_0^0$	$\subset$	H_0		
							$\cap$		
1			H_{10}	$\subset$	$H_{11} = H_1^0$	$\subset$	H_1	$\subset$	H_2
$p - r - s$	2		1		0				0

Adapted from Johansen (1995).

Table 2: Results of the ADF test.

Variable	Deterministic terms	Augmentation	t-ratio	5% critical value
y_t	Constant, Trend	1,5	-2.027	-3.45[a]
c_t	Constant, Trend	2	-2.224	-3.45

[a]The critical values are reported after Fuller (1976).

Table 3: VAR (7). Residual diagnostic tests.

Univariate analysis			Multivariate analysis	
Y_t :	$F_{AR8}(8, 97)$	$= 1.4002$ [0.2061]	$F^v_{AR8}(32{,}176)$	$= 0.9399$ [0.5646]
C_t :	$F_{AR8}(8, 97)$	$= 1.4423$ [0.1888]		
Y_t :	*Normality* $\chi^2(2)$	$= 2.3321$ [0.31]	*Normality*^{v} $\chi^2(4)$	$= 13.207$ [0.01]
C_t :	*Normality* $\chi^2(2)$	$= 17.043$ [0.00]		
Y_t :	$F_{HET}(28, 76)$	$= 1.0256$ [0.45]	$F^v_{HET}(84{,}222)$	$= 0.8891$ [0.73]
C_t :	$F_{HET}(28, 76)$	$= 1.2864$ [0.19]		
Y_t :	$F_{ARCH4}(4, 97)$	$= 0.2154$ [0.93]		
C_t :	$F_{ARCH4}(4, 97)$	$= 0.2222$ [0.92]		

The corresponding p-values are reported in the square brackets.

Table 4: Test for integration indices.

$p-r$	r	$S(r,s)$		$S(r)$
2	0	30.14*	$21.49**$	20.75**
		30.25	*19.79*	*15.4*
1	1		$7.02**$	$5.33**$
			5.99	*3.8*
$p-r-s$		2	1	0

**,* indicate rejection at the 5% and 10% significance levels, respectively.

The asymptotic 95% quantiles are reported in italics, see Table A1 in Paruolo (1994).

Table 5: Estimation and inference results, $r = 1$, $s = 0$, $p - r - s = 1$.

Unrestricted model		Hypotheses testing	
$\widehat{\beta} = (1, -1.014)'$	$\widehat{\delta} = 5.664$	$H_0 : \gamma = 1,$	
$\beta_1 = 0$		$H_0 : \beta = (1, -1)'$	$\chi^2(1) = 0.064[0.800]$
$\widehat{\beta}_2 = (1.014, 1)'$		$H_0 : \gamma = 0.941,$	
Restricted model		$H_0 : \beta = (1, -1.062)'$	$\chi^2(1) = 7.295[0.007]$
$\widehat{\beta} = (1, -1)'$	$\widehat{\widetilde{\delta}} = \underset{(0.500)}{14.240}$ [1]		
$\beta_1 = 0$			
$\widehat{\beta}_2 = (1, 1)'$			

[1] The parameter $\widetilde{\delta}$ and its standard error estimate are obtained from the standard I(1) cointegration analysis of the transformed I(2) system $\widetilde{X_t} = ((B'X_t)', (v'\Delta X_t)')'$ with $X_t = (Y_t, C_t)', B = (1, -1)', b = (1, 1)', v = (1, 0)'$ as suggested in Kongsted and Nielsen (2002).

Figure 1

Data.

(a) Total real consumption expenditure, c_t, total real disposable income, y_t, per capita values in thousands US$, seasonally adjusted.

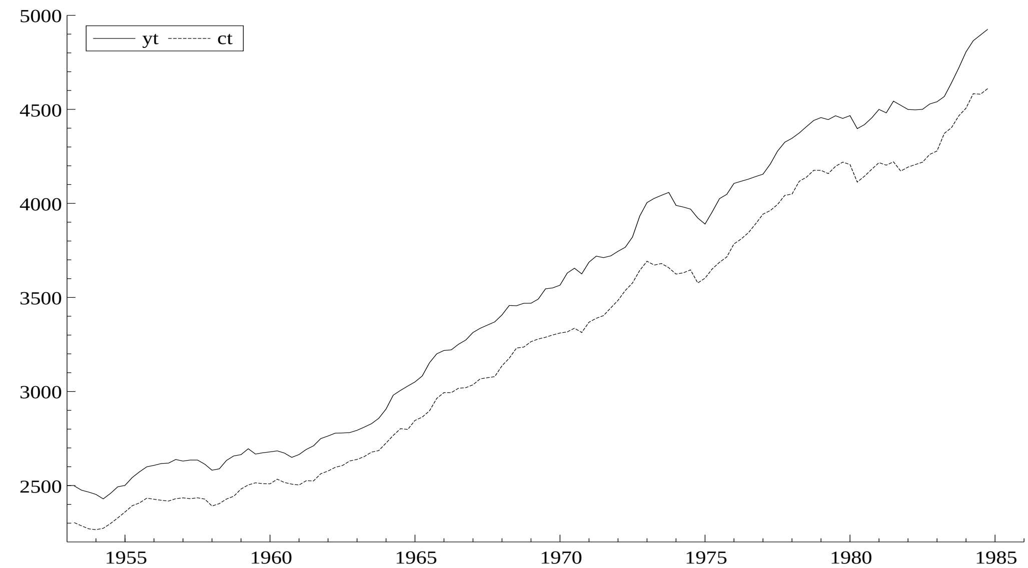
(b) The cumulative series of c_t and y_t, denoted C_t and Y_t, respectively.

Figure 2

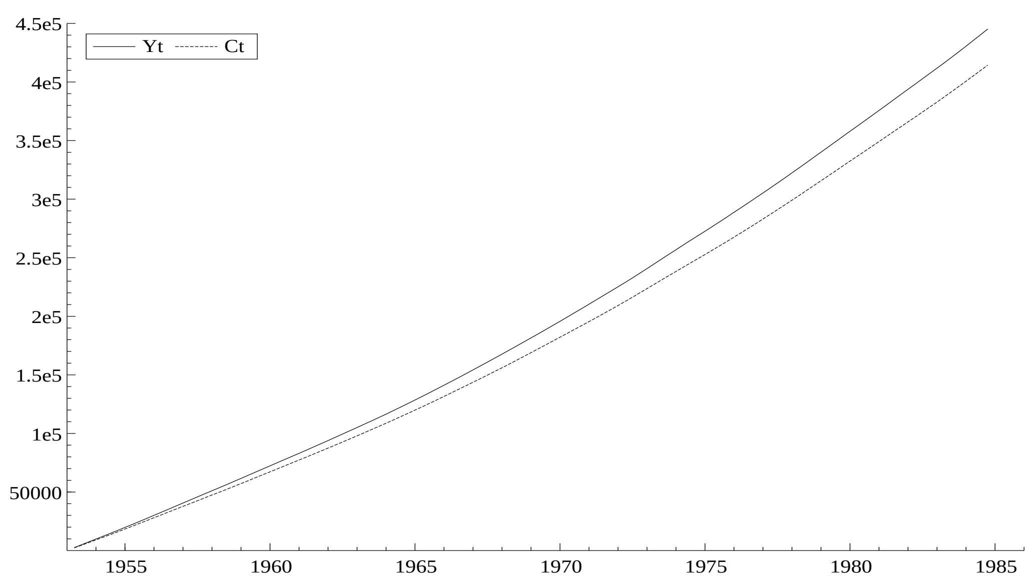
Estimated residuals, their correlogram, spectral density, and histogram.

Figure 3

Paruolo (1994) specification: estimated cointegrating relation (CI): $y_t - 1.014c_t$, restricted cointegrating relation (CIrestr): $y_{t-1} - c_{t-1}$; estimated multicointegrating relation (MCI): $\sum_{j=1}^{t}(y_j - 1.014c_j) - 5.741y_t - 5.664c_t$, and restricted multicointegrating relation (MCIrestr): $\sum_{j=1}^{t}(y_j - c_j) - 14.240y_t$. All in deviations from the respective mean values and adjusted to have the same range.

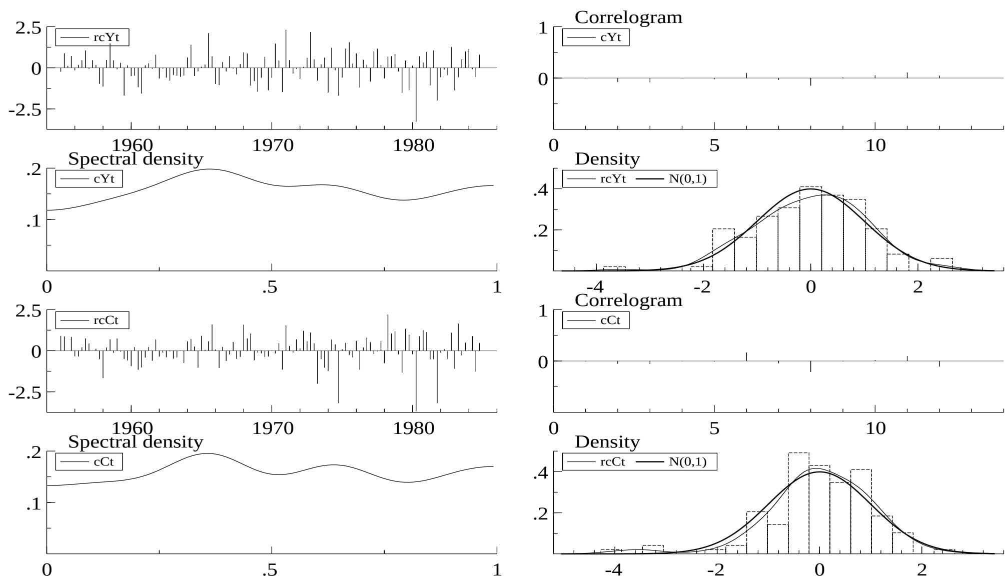

yt
ct
5000
4500
4000
3500
3000
2500
1955
1960
1965
1970
1975
1980
1985

(a)


Yt
Ct
4.5e5
4e5
3.5e5
3e5
2.5e5
2e5
1.5e5
1e5
50000
1955
1960
1965
1970
1975
1980
1985

(b)

Figure 1:



rcYt
2.5
0
-2.5
1960
1970
1980
Correlogram
cYt
1
0
0
5
10
Spectral density
cYt
.2
.1
0
.5
1
Density
rcYt
N(0,1)
.4
.2
-4
-2
0
2
rcCt
Correlogram
cCt
Spectral density
cCt
Density
rcCt
N(0,1)

Figure 2:

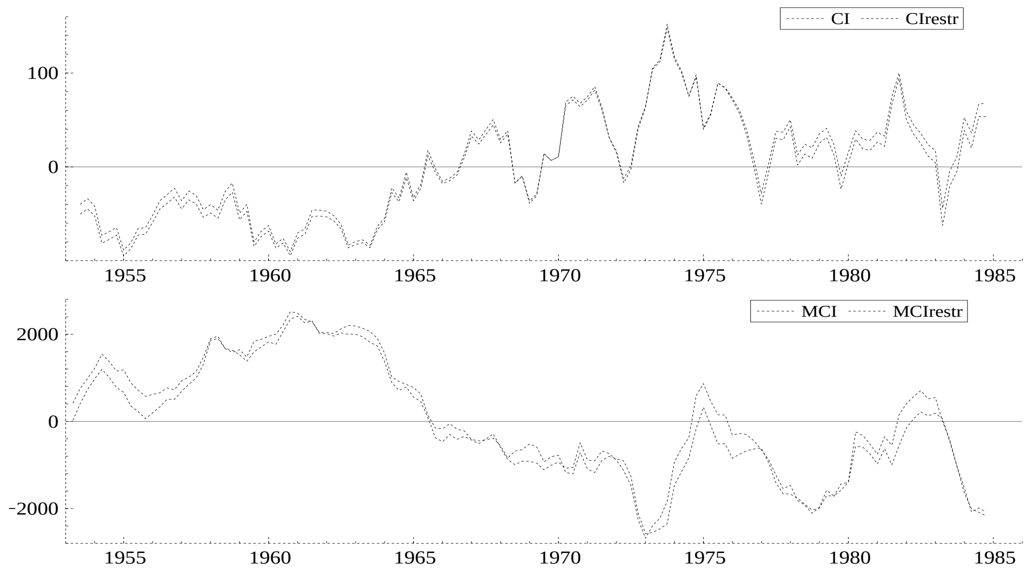

CI
CIrestr
100
0
1955
1960
1965
1970
1975
1980
1985
MCI
MCIrestr
2000
0
-2000
1955
1960
1965
1970
1975
1980
1985

Figure 3: