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493

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Berlin, Juni 2005

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IMPRESSUM

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ISSN print edition 1433-0210
ISSN electronic edition 1619-4535

Scale-Free Networks in International Trade

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12 May 2005

Draft - Work in Progress

Abstract

The paper analyzes the network structure of international trade. Adapting a network approach developed in the physical sciences, we propose that international trade functions like a scale-free network. For each commodity group we calculate a characteristic parameter which reflects the structure of its trading network. We then insert this variable into an expanded gravity model to explore the effect of the network structure on the value of bilateral trade. The estimation suggests that, inter alia, globalization has reduced the *value* of trade per product group.

Keywords: networks, international trade, gravity model
JEL Codes: F10, F15, L14

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1 Introduction

Many relationships in nature, society and the economy are structured as networks. Economic theory, however, has been slow to integrate networks into its analysis due to two methodological challenges. First, social networks require rather sophisticated mathematical tools. Secondly, networks that were traditionally analyzed in mathematics were inappropriate for describing economic networks. However, recent developments in network theory derived from the physical sciences can be adapted, thus advancing the economic analysis of networks.

The aim of this paper is to demonstrate the importance of network effects in international trade using an extension of the gravity model. Our approach was inspired by [9] who was the first to consider a network/search view of trade. Rauch's analysis incorporates the fact that differentiated products are, contrary to theory, not traded on markets but in networks into the gravity model by classifying products according to their differentiability into three distinct classes, and estimating the model for each class separately. Broadly speaking, Rauch uses the term network as a proxy for the costliness of the matching process, where differentiated products are traded in networks and homogenous goods on organized markets. We apply a different notion of the term network that takes into account the fact that every good, irrespective of its characteristics can only be traded in an international network.

In this paper, we analyze the *structure* of the network of international trade for various product groups. Our approach entails three innovations. First, from a theoretical point of view, we offer an adaption of the physical sciences approach to network theory in the field of economics. Second, from a methodological point of view, we expand the gravity model of international trade by adding, for each product group, an explicit network term into the model. Third, from an empirical point of view, we demonstrate that the network structure of international trade has a significant effect on the volume of its bilateral trade.

This paper is organized as follows: section 2 provides an overview of network theory and discusses networks in international trade, section 3 calculates a single parameter that characterizes the connectivity of a network and estimates the extended gravity model, and section 4 concludes.

2 Network Theory

2.1 Small-world networks

By the term network we understand a set of elements, called vertices, which are connected to each other through interactions, called edges [5]. Adapting this definition to international trade, each country is a node and the trading links are edges. While there are many more examples of economic systems as complex networks, economists have only recently begun to focus on networks in the economy (two notable examples are [4] and [12]). One limiting factor for analyzing networks in economics has been the complicated mathematics [5]. Network analysis is complicated by interactions that posses a intricate topology, by diversified nodes (e. g. more or less wealthy agents) and edges (e. g. the volume of a transaction), and by dynamically evolving networks [10].

Due to these complexities, one of two simplifying assumptions can be adapted to proceed in the analysis. Either a simplistic topology of the network is assumed in order to analyze its interactions. Or the interactions are assumed to be *binary* interactions (i. e. of relevance is only whether a connection between nodes exist or not). In the following we adapt the second approach. We will therefore focus on whether one country exports to another country at all, neglecting the volume of these exports.

Economists also often neglect networks because for years mathematicians have analyzed classes of networks which seemed to be of little relevance in economics. Originally, the most important distinction between different types of networks has been whether the network was structured or random. That is, if a network did exhibit some kind of regularity it was called a structured network, if it had been modelled to describe a structure that had evolved through a process of uncoordinated actions by agents it was called a random network.

Structured networks are highly clustered, i. e. two neighboring vertices of a vertex tend to be closely connected with each other, too. On the other hand they have rather large average path length, i. e. two arbitrarily chosen vertices must use in general a large number of intermediary vertices to connect to each other.

Random networks, on the other hand, have short average path lengths and small clustering coefficients.

A third class of networks, so called small world networks, can be understood as a combination of random and structured network. They posses two important features [2]:

1. preservation of the local neighborhood (clustering)

2. the average shortest distance between all possible pairs of vertices increases logarithmically with the number of vertices n.

Small-world networks can be further distinguished according to the probability distribution of the number of connections each node has, i. e. its *degree distribution*. There are then at least two representations of small-world networks: (a) single-scale (or exponential) networks, characterized by a connectivity distribution with an exponentially decaying tail and (b) scale-free networks, characterized by a connectivity distribution that follows a power law [2].

Most research in the field of small-world networks has been traditionally devoted to single-scale networks because researches believed that the overwhelming majority of networks displayed their properties. Indeed, empirical research has found relevant examples of such networks, including electric power grid systems and the number of flight connections of an airport [2].

In a seminal article, Barabási and Albert [5] introduce the concept of scale-free networks. Real-world examples are the actor collaboration network and the World Wide Web. The connectivity distribution of scale-free networks follows a power law, that is, $P(x) \sim ax^{-\gamma}$, with x denoting the number of connections per node. This implies that even though vertices with a large number of connections are rare, they are statistically significant.

For example, most of the web sites on the internet have only a few outgoing and incoming links. However, a small number of sites, such as Yahoo, act as hubs and tend to be extremely well connected. A similar pattern can be observed with the pool of all actors. In many movies a large number of unknown actors are cast with a few famous actors. These few famous actors usually are well connected. The supporting actors, despite forming the overwhelming bulk of the actors' pool, tend to be rather poorly connected. Clearly, modelling the process that leads to the emergence of a scale-free network has to differ in some important aspects from that of an exponential network. Barabási and Albert [1] identified two important mechanisms that are responsible for the emergence of power law scaling: preferential attachment and a growing network. Whereas the exponential models assume that the number N of vertices remains fixed, scale-free networks require that their number must be growing by adding new nodes. This implies that scale-free networks can be modelled by starting with a small number m_0 of nodes and then adding at each period a new node with $m \leq m_0$ edges. These new edges have to be connected to the network, but whereas the connection procedure for exponential networks is characterized by choosing a vertex with uniform probability, scale-free networks require that the probability Π that a node i will receive an edge of a new node is proportional to its connectivity x_i, that is, $\Pi(x_i) = \frac{x_i}{\sum_j x_j}$. Clearly, after t periods this network has $N = t + m_0$ nodes and mt edges and for $t \to \infty$ the degree distribution follows a power law, i. e. $P(x) \sim 2m^2 x^{-\gamma}$, with an exponent $\gamma = 3$ which is independent of time [1].

2.2 Scale-free networks

Given our prior knowledge of the international trading system, we hypothesize that international trade takes place within a scale-free network, i. e. that it posses for every good (a) a high clustering coefficient, (b) a short average path length, and (c) a degree distribution that follows a power law. We propose that these properties are unifying principles. This section will motivate our hypothesis from a theoretical point of view. In particular, we discuss the implicit assumption that the distribution of trading links follows a power law and we consider ways in which the model of scale-free networks and international trade may diverge.

We have not calculated the average path lengths and clustering coefficients explicitly. Nevertheless, it is rather obvious with regard to (a) that if two countries, say France and Italy, trade with another country, say Germany, they tend to trade with each other too, and with regard to (b) that countries that are far apart have to use only a small number of intermediaries to connect to each other.

Condition (c) is also intuitively plausible, especially when considering inter-industry trade. Countries export those goods in which they are specialized. This implies that for a given good only a few countries export heavily whereas the majority is rather poorly connected. Consider for example the product group crude oil. There are only a handful of countries that produce oil. However, these few countries supply oil to almost every other country. Therefore, if we plot the number of trade links per exporting country against the number of countries which have that many links, we expect to discover a distribution which has very high values in the beginning but radically decaying values to the right on the x-axis.

On the other hand, a large amount of international trade is intra-industry trade. A good example is butter, which is produced all over the world. Still a significant amount of trading is taking place because, for instance, the French buy butter from Ireland and the Irish buy French butter. Therefore, a more complex but also more even pattern of trade might be observed in the case of butter. There are more countries than in the case of crude oil exporting to a significant number of other countries and there are fewer countries with a negligible number of export links.

Our hypothesis that the trading network can be modelled as a scale-free network is further supported by the fact that it evolves through a process of preferential attachment. That is, countries tend to import a certain good from countries that are already established exporters of that good.

There are some potential divergences between the characteristics of scale-free networks and of the world trading system. First, a scale-free network must be growing while in the world economy the number of countries remains largely fixed. However, this is, in our view, not a hinderance for our empirical analysis because the aim of this requirement is to ensure that not every node is connected to the whole network after a couple of time periods. In our case we may consider the time periods to be quite large, new links are added in decades rather than in days.[1]

[1] Furthermore, during the time span that is covered by our empirical analysis (1976-2002) the number of countries has indeed grown significantly with the disintegration of the USSR and the Federal Republic of Yugoslavia, the partition of Czechoslovakia, the independence of Eritrea and East Timor etc.

Second, a somehow more intriguing feature of international trade that is not matched by the model of [1] is that the number of trading links a country has is decisive in increasing its probability of receiving additional links and is reducing the probability that another country receives an additional link. Once an importing country is obtaining a certain good from one country its need to connect to a second exporting country will shrink. This implies that the number of edges a new vertex can have, too, should be subject to some kind of randomization. A third dissimilarity would be the fact that links are added not only by nodes that are new to the system but between nodes that already exist.

Third, discussing small-world properties in the context of trade in disaggregated goods might be inappropriate. A valid critique would be that bilateral trade largely takes place without intermediating countries. However, this is not always true. For instance, there are notable exceptions such as the trade in diamonds which revolves around the Netherlands, and countries like Singapore and Hongkong which act as trading hubs. Furthermore, even though there are no intermediating countries, there are indeed many intermediating distributors, traders etc., that is *nodes* which have to be passed before a good reaches its destination.

These are certainly exciting topics that should be explored further in future research. In this paper, however, we focus on condition (c), namely on the connectivity distribution of a country with respect to a particular good. In this context, our example of Irish butter indicates the the issue of the distribution of connectivity is closely intervened with that of classification and aggregation. If we only considered the product Irish Butter, we would find a far more uneven distribution of export links than for oil.

These discrepancies between the world trading network and the model of scale-free networks concern the *formation* of networks. However, exploring this issue in greater detail is beyond the scope of this paper as we are only interested in the *structure* of the trading network, that is, whether it connectivity distribution displays scale-free properties.

2.3 Characterizing scale-free networks with power law functions

With the analysis of scale-free networks power laws have gained prominence in network theory. If the degree distribution of a good k follows a power law, its functional form can be given by $P(x_k) \sim a_k x^{-\gamma_k}$, with x_k denoting the number of links, γ_k a constant exponential parameter and a_k representing a multiplicative factor. However, recognizing power laws in real-world networks is challenging as for certain ranges power law and exponential (or respectively logarithmic) distributions display similar shapes. Fortunately, a rather applicable method for recognizing power law distributions emerges by examining the log-log plot of the data. The reason is straightforward: taking logs on both sides results in $\log y_k = \log a_k x^{-\gamma_k} = \log a_k - \gamma_k \log x$, that is, if the data follows a power law in the original scale, its log-log plot should be a linear function with a slope of $-\gamma_k$.

An interesting property of power law distributions is that they are invariant of scale, and it is exactly this property which gives scale-free networks their name.[2] The term scale-free derives from the fact that the shape of a power-law function does not change if the scale of measurement is changed. More formally, a power law distribution $p(\cdot)$ satisfies $p(bx) = g(b)p(x)$ [8]. This implies that for two values l and m the ratio of $p(l)$ and $p(m)$ remains unchanged irrespective of scale. This property distinguishes power laws from exponential functions which are sensitive to changes in scale (hence exponential networks are also sometimes called single-scale networks).

One characteristic of power law functions that contrasts with our aim to capture the network characteristics of trade in a single value is the fact that the power law function which we use possesses two parameters, a_k and γ_k. However, it is in order to consider only the exponential parameter. Because we are considering density functions, we have to normalize them according to:

$$1 = \int_{x_{min}}^{\infty} p(x)dx = a_k \int_{x_{min}}^{\infty} x^{-\gamma_k} dx = \frac{a_k}{1-\gamma_k} \left[x^{-\gamma_k+1} \right]_{x_{min}}^{\infty}. \qquad (1)$$

If $\gamma_k > 1$, it follows that $a_k = (\gamma_k - 1)x_{min}^{\gamma-1}$, which implies for the normalized expression of the power law function:

$$p(x) = \frac{\gamma_k - 1}{x_{min}} \left(\frac{x}{x_{min}} \right)^{-\gamma_k}. \qquad (2)$$

[2] It is regularly asserted that scale-free networks derive their name from the fact that power laws do not have a "mean" connectivity. However, the last explanation is somehow imprecise. Under certain conditions it is possible that an average amount of connections can be established (even though interpreting this moment might not be very meaningful). The mean of a random variable x, which is distributed according to a power law, is given by $E(x) = \int_{x_{min}}^{\infty} xp(x)dx = a_k \int_{x_{min}}^{\infty} x^{-\gamma+1} dx = \frac{a_k}{2-a} \left[x^{-\gamma+2} \right]_{x_{min}}^{\infty}$. A mean exists for $\gamma > 2$; and this is a requirement that is fulfilled for most real world networks [5]. What is actually meant by stating that these distributions do not have a mean connectivity is that there is no value where the distribution has a peak, that is, they do not have a characteristic value around which the distribution is centered.

That is, once we are dealing with normalized power law functions, the exponential parameter fully characterizes its scaling properties, and it is sufficient to estimate only that parameter.

Another obstacle in modelling real world networks using power law functions is that one has to choose a minimum threshold $x_{min} > 0$. A minimum threshold is needed because the power law functions we are interested in have a negative exponent and have, therefore, a singularity at $x = 0$. However, it should be noted that we can expect to find $x = 0$ in our trade data rather frequently.

3 Data and Estimation

3.1 The network variable

In addition to the well known economic determinants, social, political and institutional factors are crucial albeit unobservable determinants of international trade. The γ variable however can be understood as a pooled measure for the various unobservable product and production characteristics that determine the structure of the trading network. For example, if some good is produced by a large number of countries, then the γ variable will tend to be small. If some good is so differentiated that the trade of this good requires a large amount of social interaction, and only a small number of countries have such well established ties, then the γ variable will tend to be large. Different goods will tend to absorb these countervailing effects differently, and thereby produce unique trading structures.

After expanding the gravity model with the γ-variable, we will calculate a distinct coefficient for the relationship of a given network structure and the amount of trade.

One major advantage of this approach is that, unlike [9], we do not have to classify goods first with regard to their differentiability into discrete classes in order to deduce their trading structure, but are able to provide a continuous measure of the trading structure. By doing so, we can capture implicitly the degree of homogeneity of a good. We are then able to distinguish between determinants of trade that should be attributed to the respective countries (e. g. GDPs, Distance etc.) and determinants that can be attributed to the specific good that is traded.

A simple reason why we could expect that an uneven trading structure tends to *increase* the *value* of trade is that the exporting countries can restrict the supply deliberately and act as if they had a mono- or oligopoly (which would result precisely in a smaller volume, but in a larger value of exports). On the other hand, we may argue no less convincingly that a small number of well connected countries tends to reduce supply in a manner that the lower volume outweighs a possibly higher price, and thereby reduces the value of trade.

An important feature of the trading network is that the number of vertices and, therefore, the number of edges that can be found is rather small.[3] This implies that the statistical noise may be quite large and fitting the data to a power law function might be difficult. Using data from the Direction of Trade Statistics Database (DOTS) of the IMF, we plot in figure 1 the fre-

[3]There are $\frac{n(n-1)}{2}$ possible edges with n denoting the number of countries, as one country can have at most $n-1$ links. The exact value of n, however, is ambiguous and may range from 192 (member states of the UN) to more than 220.

quency distribution of the number of trading partners per country.[4] Even

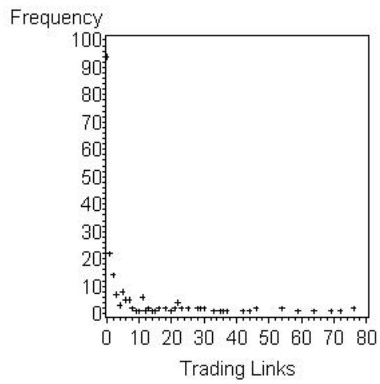

Figure 1: Frequency distribution of the number of trade partners per country.

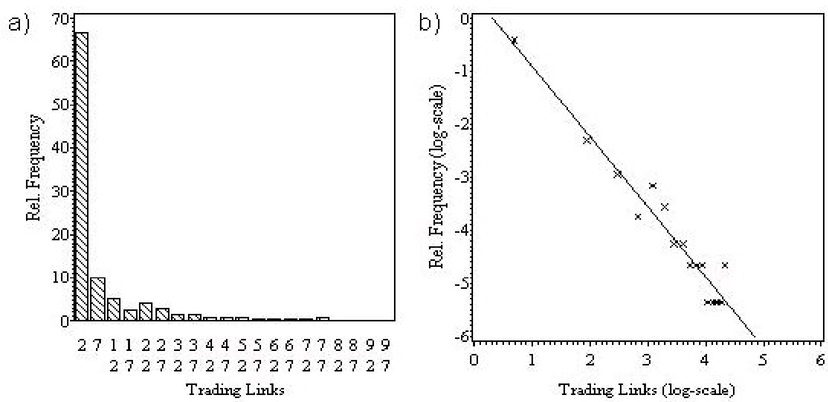

Figure 2: The distribution of (a) the number of trade partners and (b) the log-log plot with a superimposed best line fit. Bins of the width 5 have been applied.

though the distribution is right-skewed, it is erratic and for most of the range the data might be fitted equally well by a uniform distribution. This

[4]In this case countries are thought to have a trading link if they trade at least one good with each other, further below, however, we will disaggregate into different product groups and use, therefore, other and more disaggregated data.

is due to the fact that we have a small number of observations. As argued above, the number of trade links can be expected to follow a power law, as it is probable that countries are asymmetrically connected to the rest of the world.

We can solve this problem either by increasing the amount of observations (i. e. for example by considering trade between regions) or by binning the observations. We adopt the second approach as the first requires data that is not readily available. In the histogram of figure 2a, we can recognize a smooth power law relationship, a presumption which is reinforced by inspecting the log-log plot. (figure 2b)

Another problem, already mentioned above, stems from the fact that we have a large amount of zero observations in our data which we cannot approximate with a power law function because of its singularity at $x = 0$. We resolve this issue by using the midpoint of the lowest bin as x_{min}.

We now turn to our calculation of the γ_k. The data has been taken from the Trade and Production Database of the CEPII[5]. The dataset contains, inter alia, the bilateral trade flows for 27 product groups, classified according to the International Standard Industrial Classification (ISIC) at the 3-digit level, for a large number of countries (approximately 210).[6] The database

[5]http://http://www.cepii.com/anglaisgraph/bdd/TradeProd.htm

[6]The product groups and their respective codes are: food products (311), beverages (313), tobacco (314), textiles (321), wearing apparel, except footwear (322), leather products (323), footwear, except rubber or plastic (324), Wood products, except furniture (331), furniture, except metal (332), paper and products (341), printing and publishing (342), industrial chemicals (351), other chemicals (352), petroleum refineries (353), rubber products (355), plastic products (356), pottery, china, earthenware (361), glass and products (362), other non-metallic mineral products (369), iron and steel (371), non-ferrous metals (372), fabricated metal products (381), machinery, except electrical (382) , machinery, electric (383), transport equipment (384), professional and scientific equipment (385), other manufactured products (390).

is based on data from the World Bank, which in turn originated from the UN COMTRADE Database for trade and the UNIDO industrial statistics for production. The extension of the World Bank database with regard to trade was done by using data from the CEPII's BACI database.

Given that we only consider the existence of bilateral trading links but not the volume of bilateral trade, describing the trading network is straightforward: We start with a matrix in which for every product group the exports F_{ij} of country i to county j are denoted in the respective cells. Then we apply the following algorithm: A new matrix is created with the same number of rows and columns. If the volume of trade surpasses a certain percentage p of international trade the value in the original matrix is replaced by 1 in the new matrix, otherwise by 0. This is repeated for all cells. The sum of each row then denominates the number of significant trading links per country. Then bins of the wide z are created and the number of trading links in each bin is calculated. Finally, the distribution is fitted to a power law function by nonlinear regression in order to estimate the exponential parameter, that is the network parameter γ_k for each product category.

3.2 Summary statistics

Our results for the $k = 27$ product groups are given in the form of summary statistics in table 1 for $p = 0.001$ and $z = 3$ for the years 1976, 1980, 1990, 2000, and 2002. As it is evident from figure 3 the distribution of γ_k is fitted rather well by a log-normal distribution for any given year.[7] For instance,

[7] We apply the following specification for the density:
$f(y) = \frac{1}{y-\theta}\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\left(\frac{\log(y-\theta)-\zeta}{\sigma}\right)^2\right)$, for $y > \theta$.

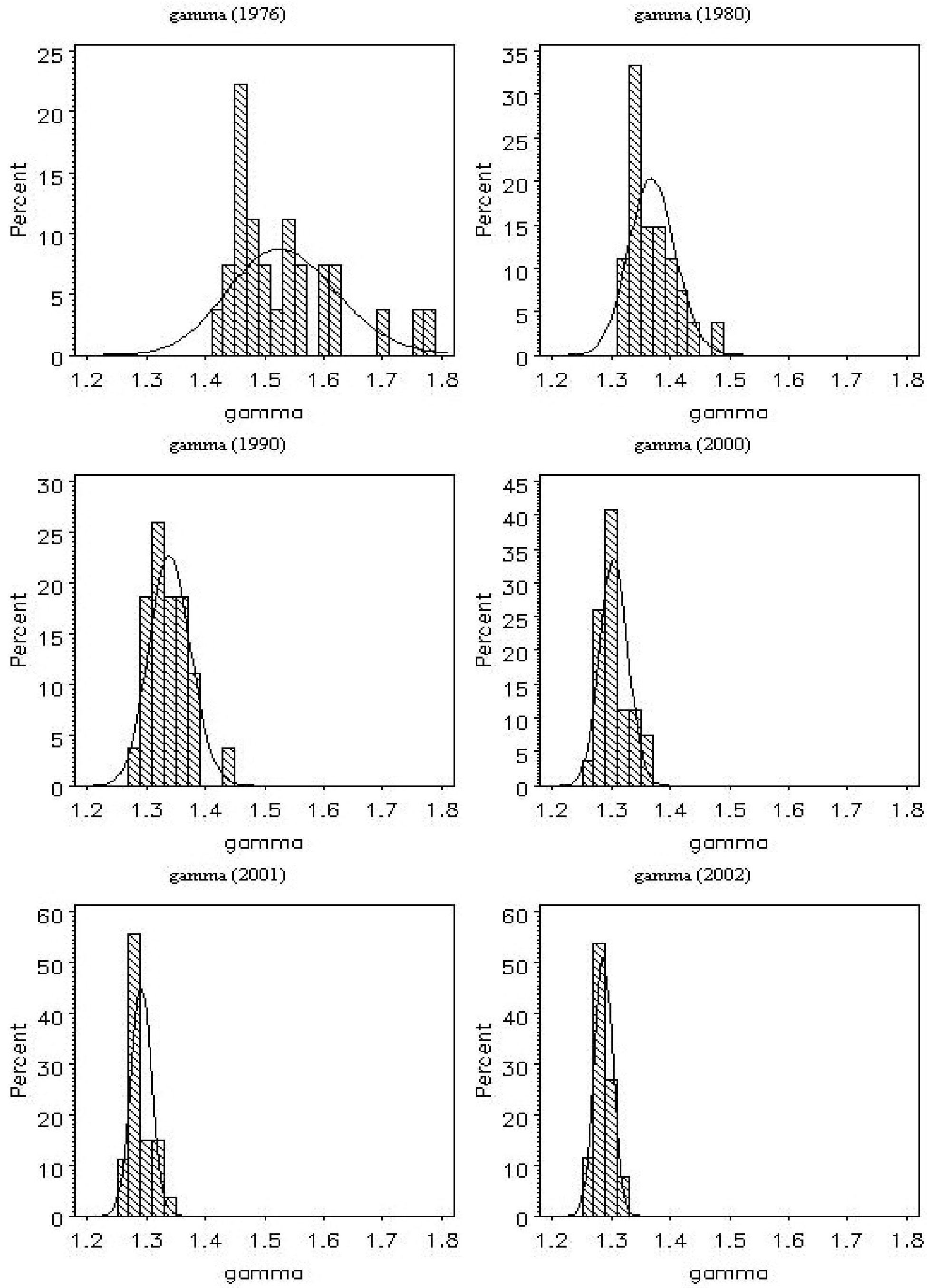

Figure 3: The distribution of the γs, fitted by a log-normal distribution.

Year	$\overline{\gamma}$	$\gamma_{0.5}$	γ_{min}	γ_{max}	$\sqrt{s^2}$
1976	1.534	1.499	1.422	1.782	0.095
1980	1.369	1.361	1.311	1.483	0.039
1990	1.339	1.331	1.286	1.449	0.036
2000	1.304	1.297	1.269	1.361	0.024
2002	1.286	1.280	1.259	1.324	0.016

Table 1: Summary statistics for γ of all product groups.

for 2000 we estimated $\zeta = 0.2654$, $\sigma = 0.0183$ assuming a threshold parameter $\theta = 0$. The distribution is clearly right-skewed and eventually a cutoff emerges.

A comparison of the γ_k over time reveals that significant changes have taken place in the trading network. The most remarkable feature is the continuous decline of the values of γ_k. The mean value for γ in 1976 was 1.534, whereas in 2002 its value was only 1.286, a drop of 16 percent. The maximal and minimal values display a similar pattern. This feature implies that the distribution of connectivity has become *less* uneven as more and more countries are becoming better connected for most of the goods. Therefore, the γs reflect the increased integration of countries in international trade.

From that point of view, γ could be interpreted as an indicator of globalization. Further insights into the trading network can be gained by comparing γ accros to product groups k. in figure 4 we have plotted the development of γ for three different product groups: food products, petroleum refineries and scientific equipment. We find that γ is always the largest for petroleum refineries, whereas it is more or less the same for food products and scientific equipments. This does not suggest a clear causal relationship between characteristics of goods and the value of γ. Scientific equipment, for instance,

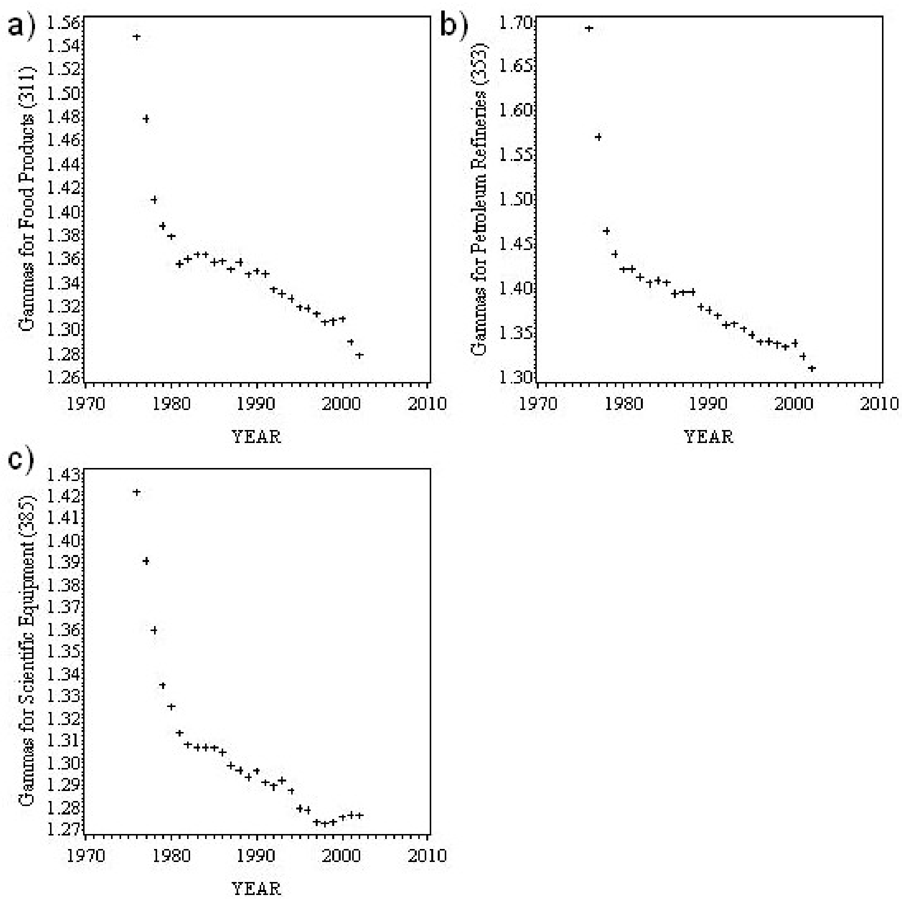

Figure 4: Development of γ for different product gropus

requires a sophisticated production process, strongly implying that only few countries should be able to supply this product group to the world market. Food products (as an aggregated product group), on the other hand, requires little specialization . However, both product groups display largely the same trading structure, indicating that many other effects have a role to play in determining the trading structure. In the case of food, for instance, protectionist policies might prohibit many countries from exploiting their comparative advantage.

3.3 Estimation of the gravity model

The gravity model of international trade predicts at its core a relationship for trade flows based on Newton's famous Law of Universal Gravitation. There are several theoretical models that are capable of producing "gravity" between countries, including [3], the monopolistic competition approach and the Heckscher-Ohlin model. Empirically, the gravity model approach is well established through numerous studies.

We estimate an expanded gravity model using data from the CEPII's Trade and Production database. Data on GDP and GDP per capita was retrieved from the World Bank's country fact book.

We apply the following general (4-way) specification with all continuous variables in logs:

$$
\begin{aligned}
F_{ijkt} = & \alpha_i + \beta_j + \lambda_t + \nu_k + \omega(GDP_{it}GDP_{jt}) + \sigma(PGDP_{it}PGDP_{jt}) \\
& + \delta DISTANCE_{ij} + \kappa NET_{tk} + \epsilon ADJACENT_{ij} + \psi COLONY_{ij} \\
& + \xi COMLANG_{ij} + \eta EEC_{ij} + \theta EFTA_{ij} + u_{ijkt},
\end{aligned}
\tag{3}
$$

with F_{ijkt} denoting the bilateral exports from country i to country j of good k in period t. On the right hand side of equation 3 we specify typical gravity model predictors, that is the product of the GDPs ($GDP_{it}GDP_{jt}$) and GDPs per capita ($PGDP_{it}PGDP_{jt}$) of both countries, their great circle distance ($DISTANCE_{ij}$) and several dummy variables indicating the *closeness* of two countries, that is whether they share a common border ($ADJACENT$), speak a common language ($COMLANG$), whether colonial ties are present ($COLONY$) or whether they are members of the same trading bloc (EU and $EFTA$). We have also included time-constant country-specific effects ($\alpha_i,\ \beta_j$), a time-effect that is constant across cross-sectional units (λ_t) and an intercept for the various goods (ν_k).

We then extend these classic gravity model predictors by the variable NET. This variable is the same as the γ in equation 2 (we have decided to rename this variable because in equation 3 Greek letters are used as coefficients). It is used as a proxy for the network structure of the trading network in which a particular good is traded. By including this variable we differentiate between determinants that can be attributed to particular goods, thereby providing an extension to [9].

There are nevertheless major differences between both approaches. Whereas [9] was concerned about the homogeneity of goods, we focus on the homogeneity of a good's trading structure. Even though it can be argued that both approaches converge with the level of disaggregation, the data we apply requires that we assume that a competitive market persists when a good's trading structure is even. An uneven trading structure is understood as a proxy for costly matching processes as there will be fewer suppliers inclined to provide many variations.

Following [7] we can denote equation 3 in compact matrix notation.

$$X = D_i\alpha + D_j\beta + D_k\nu + D_t\lambda + Z\rho + \mu, \tag{4}$$

with the $(N\text{x}N\text{x}K\text{x}T)\text{x}1$ matrix X denoting a vector of bilateral exports of good k at time t, D_i and D_j denoting dummy-variable matrices, capturing respectively the origin and target country effects and D_t representing a dummy-variable matrix capturing the time effects. Their magnitudes can be retrieved from [7]. The matrix Z is the non-dummy subset of the design matrix with magnitude $(N\text{x}N\text{x}K\text{x}T)\text{x}M$.

Following [6] and [7], we employ a static fixed effects approach, which provides consistent estimators under moderate assumptions. Furthermore, it is highly possible that the individual effects are correlated with the explanatory variables. We will therefore focus on static fixed effects estimation, even though estimating fixed-effect models has the severe drawback that the time constant effects are wiped out.

Simple (pooled) OLS was rejected and our focus on more complex models vindicated by the F-test for heterogenous country specific effects. We applied the Breusch-Pagan Lagrangian multiplier test and reject the null of an homoscedastic error-structure ($\chi_{(1)} = 1 \exp 6$, $p < 0.0001$). We have also applied the Wooldridge-test for autocorrelation and found the null of no-autocorrelation is rejected ($F_{(1,139409)} = 7794, 767$, $p < 0.0001$).

We apply a robust estimation of a fixed effects model by using Stata's *xtgee* command with first-differenced data. As a robustness check, we first estimate as a base model equation 3 without the NET variable (table 2). Then we add the net variable in an extended model (also table 2). The traditional gravity model predictors behave as expected. The estimated coefficient of GDP is about unity and highly significant. The estimated coefficient of GDP per capita is slightly negative, but not significant. Both results are plausible and in accordance with the literature [9].

We find that the network coefficient is positive and highly significant, implying that the value of bilateral trade increases when a good's trading structure is inhomogeneous, that is when there are only few suppliers. This somewhat surprising result. implies that specialization leads, ceteris paribus, to an increase in the value of bilateral trade. In other words, the entry of new suppliers into the trading network in the last three decades (i. e. a reduction of γ) has lead to a decrease in the value in bilateral trade. This further implies that the price-effects must have been stronger than the quantity-effect.

Table 2: Fixed-effects with dependent variable bilateral trade

Variable	**Coefficient** (Std. Err.)	**Coefficients with γ** (Std. Err.)
GDP	1.318***	1.316***
	(0.082)	(0.082)
GDP per capita	-0.013	-0.012
	(0.075)	(0.075)
NET	–	0.912***
	–	(0.314)
1998	-0.042***	-0.040***
	0.004	(0.004)
1999	-0.076***	-0.075***
	(0.004)	(0.004)
2000	0.020***	0.018***
	(0.005)	(0.005)
2001	-0.003	0.006
	(0.004)	(0.005)
2002	-0.025***	-0.022***
	(0.004)	(0.004)
N	605981	605981
Log-likelihood	.	.
$\chi^2_{(7)}$	3705.891	3716.803

Significance levels : $*$: 10% $**$: 5% $***$: 1%

4 Conclusions

In this paper we argue that the economy can be viewed as a network of exchange relations. We demonstrated our notion with the example of international trade, drawing on network theory first developed in the physical sciences. We conclude that scale-free networks display striking similarities to the way countries export goods.

We then analyzed the trading network of different product groups using the framework of scale-free networks. In particular, we analyzed the connectivity distribution for various product groups and calculated a parameter that can be used to describe its scaling properties. One important result demonstrates that the world economy has become more integrated in the last three decades, i. e. the number of countries which act as exporters has increased considerably for most goods.

We also inquired whether this trend has lead to an increase in the value of trade by estimating a gravity-model. We found that the effect of this trend was negative, indicating that importers must have become better off due to declining prices.

We demonstrated that applying a network perspective on trade is not only of theoretical interest but it also leads does to new empirical applications. By extending the traditional gravity model with a parameter that is capable of measuring the structure of trade we provided one such application. Further work on this approach might involve using more detailed.

Several other applications of network theory in the field of international economics appear promising. These include the clustering and small world properties of the trading network, and the analysis of clustering coefficients and average path lengths for various goods.

Acknowledgments

We would like to thank Holger Gerhardt for invaluable research assistance and Philip Schröder and Dieter Schumacher for helpful comments, and CEPII for providing access to the trade data. The usual disclaimer applies.

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