DIW Berlin
German Institute
for Economic Research

Discussion Papers

514

Chetan Ghate

Transitional Dynamics in a Growth Model with Distributive Politics

Berlin, September 2005

IMPRESSUM

DIW Berlin
Deutsches Institut für Wirtschaftsforschung
Königin-Luise-Str. 5
14195 Berlin
Tel. +49 (30) 897 89-0
Fax +49 (30) 897 89-200
www.diw.de

ISSN print edition 1433-0210
ISSN electronic edition 1619-4535

.

Transitional Dynamics in a Growth Model with Distributive Politics*

Chetan Ghate

Indian Statistical Institute, New Delhi

September 23, 2005

*I thank Debajyoti Chakrabarty, Reinaldo Garcia, and Claudia Trentini for helpful comments. This paper was written while the author was visiting the department of economics at the University of Sydney. Their hospitality is gratefully acknowledged.
Correspondence to: Chetan Ghate, Assistant Professor, Planning Unit, Indian Statistical Institute - Delhi Center, 7 S.J.S Sansanwal Marg, New Delhi - 110016, India. Tel: 91-11-5149-3938. Fax: 91-11-5149-3981. E-mail: cghate@isid.ac.

Abstract

This paper constructs a heterogenous agent model of endogenous distribution and growth. When the labor leisure choice of agents is exogenous, the factor holding ratios of households converges to a mass point that is independent of the initial distribution of capital in the steady state. There is complete equality and every household's preferred tax rate equals the growth maximizing tax rate. There is no distributive conflict in the long run. When the labor leisure choice of households is endogenous, there is *also* complete convergence in the factor holding ratios of agents in the steady state. However, the tax rate under majority voting is less than the growth maximizing tax rate which leads to distributive conflict in the long run. These results extend the model of endogenous distribution and growth in Das and Ghate (2004) in two ways. First, we assess the impact of redistributive politics on growth by looking at the effect of income inequality on the tax rate and labor supply. Second, the model is solved using a more empirically plausible specification of the government budget constraint in which households vote over the tax rate on capital income instead of a tax on wealth. The general insight gained from the analysis is that characterizing the transitional dynamics in a model of redistributive politics and growth is not an intractable proposition.

KEYWORDS: Distributive Conflict, Endogenous Distribution, Median Voter Theorem, Endogenous Growth

JOURNAL OF ECONOMIC LITERATURE Classification Number: **P16:** Political Economy of Capitalism; **E62:** Fiscal Policy; **O40** Economic Growth.

1 Introduction

Following the seminal work of Alesina and Rodrik (1994) - henceforth AR - a large body of theoretical work has addressed the impact of income distribution on economic growth via the implied pressure for redistribution.[1] However, in summarizing the recent literature on income distribution and growth, Drazen (2000, p. 473), observes that several growth and distribution models (where inequality is defined in terms of the functional distribution of income), "lack transitional dynamics", a feature "dictated ... by the difficulty in solving for a simultaneous political and economic equilibrium." Das and Ghate (2004) - henceforth DG - to the best of our knowledge, are the first authors to add transitional dynamics to the growth and distribution framework of AR. DG show that the steady state factor holding ratios across agents converges to a mass point that is independent of the initial distribution of capital. Because there is convergence in factor holding ratios in DG, every household's preferred tax rate is the same, and equal to the growth maximizing tax rate in the long run. There is no distributive conflict. This contrasts with AR, in which the steady state factor holding ratios of agents is pinned down by the initial distribution of capital across agents. This perpetuates distributive conflict.

[1] See Aghion, Caroli, and Garcia-Penalosa (1999) for an extensive review.

This paper constructs a model of redistributive politics and growth with transitional dynamics along the lines of DG with two differences. First, we endogenize the household's labor-leisure choice. This makes the growth rate, the distribution of wealth, labor supply, and the tax rate are simultaneously endogenous.[2] Such a framework allows us to study the impact of income distribution on growth via the impact that redistributive politics has on both the tax rate as well as distortions to labor supply. Second, we consider a more empirically plausible specification of the government budget constraint in which public infrastructure - the source of labor augmentation in the model - is financed by a tax on *capital income* as opposed to a tax on the *capital stock.* Like DG, the equilibrium tax rate is determined by majority voting.

The model leads to several interesting results. When labor is exogenous, we show that the factor holding ratios of agents converges to a mass point that is independent of the initial distribution of capital, like Stiglitz (1969). This implies that there is perfect convergence of interest across individuals about the tax rate, or unanimity. There is no distributive conflict in the long run. These results are consistent with DG. When

[2]This contrasts with both AR and DG. In AR, the growth rate and the tax rate are endogenous, while distribution is exogenous. In DG, the growth rate, the tax rate, and distribution are endogenous, but labor is exogenous.

we endogenize leisure, there is still perfect convergence in the factor holding ratio's of agents, i.e., the median and average household's factor holding ratios coincide in the steady state. However, since households value leisure, their preferred tax rate is lower than the growth maximizing tax rate as households work less and choose to tax themselves less. This leads to lower growth. This creates distributive conflict.

In terms of the impact of inequality on growth, our results suggest that more inequality leads to lower growth, as in DG and AR. This is because a capital poor median voter prefers a high tax on capital income, which is larger than the growth maximizing tax rate. However, in the long run, because the median agent 'catches up' to the average agent, his preferred tax on capital income falls, and growth rises. However, in the steady state, because agents value leisure, they work less. Hence, they choose to tax themselves less. This leads to lower growth, as factor holding ratio convergence occurs on the left hand side of the growth maximizing tax rate, even though there is more equality in the long run. This implies that more equality leads to lower growth; although more equality first leads to higher growth and then lower growth in the steady state.[3]

[3] The non-linear effect of growth and inequality is consistent with recent empirical evidence that suggests that while too much equality can be harmful for growth, too much inequality can also be harmful for growth (Banerjee and Duflo, 2003).

The paper is structured as follows. Section 2 outlines the model. Section 3 characterizes the optimal tax rate of households under endogenous and exogenous leisure. Section 4 discusses the implication for optimal tax rates on growth and inequality in the long run. Section 5 concludes.

2 The Model

We first solve the household's problem with labor is supplied endogenously. The population, or number of households, N, is given. Each household is differentiated on the basis of its capital holdings, K_h, whose distribution is assumed to be continuous on a finite support, R_+. We assume that the distribution of K_h is skewed to the right, which implies that the median household's capital holdings is less than the mean household's.[4] The aggregate stock of capital is given by $K = \sum_1^N K_h$. Capital is the only accumulable factor in the model.

A single good is produced in the economy according to a Cobb-Douglas production technology, given by

$$Y_t = K_t^a (G_t H_t)^{1-a}, \tag{1}$$

where Y_t is aggregate output at time period t, K_t denotes the aggregate capital

[4] As will be seen later, this construct permits us to use the capital-labor holding of the median voter relative to the mean voter as an index of wealth inequality in the model.

stock in the economy, H_t is the aggregate labor supply in each period, and G_t is a public infrastructure input which is the source of labor augmentation. Following the endogenous growth literature, we interpret K as both physical as well as human capital. Hence $a \in [0, 1]$ is the private return to physical capital as well as human capital. We require the regularity condition,

$$a > \frac{1}{2}, \tag{2}$$

to ensure that the return to capital is positive in equilibrium.[5]

We assume that the public infrastructure input, G, is financed by a specific tax, $\tau \in [0, 1]$, on capital income in each period. This specification is more empirically plausible, and departs from both AR and DG, who assume that infrastructure is financed by a tax on the capital stock, or wealth. The government budget constraint is balanced in each period, and given by

$$G_t = \tau_t r_t K_t, \tag{3}$$

where r_t is the competitive rate of return to capital. Given (1), the rental rate to

[5]With a narrower interpretation of K as physical capital, it would be empirically implausible to assume that $a > \frac{1}{2}$, but it is not so when capital is interpreted more broadly as we do here. Further, according to Barro and Sala-i-Martin (1995, p. 38), even a value of $\alpha = .75$ is quite reasonable.

capital, r_t, and the wage rate, w_t, are given by,

$$r_t = \phi(\tau_t) H_t^{\frac{1-a}{a}}, \tag{4}$$

and

$$w_t = \xi(\tau_t) H_t^{\frac{1-2a}{a}} K_t, \tag{5}$$

respectively, where, $\phi(\tau_t) = a^{\frac{1}{a}} \tau_t^{\frac{1-a}{a}}$, and $\xi(\tau_t) = (1-a) a^{\frac{1-a}{a}} \tau_t^{\frac{1-a}{a}}$.[6] This allows us to write the after tax rental - wage ratio as

$$\frac{r_t(1-\tau)}{w_t} = \frac{aH_t(1-\tau)}{(1-a)K_t}. \tag{6}$$

Without any loss of generality, we assume that capital depreciates fully in each period.

Following Aghion and Bolton (1997), agents are assumed to live for a single period. In each period, household's are endowed with a single unit of time which they allocate optimally between labor and leisure. At the end of the period, a replica of each agent is born, for which agents leave a bequest. Hence, at time t, the h^{th} household derives utility over consumption, C_{ht}, a bequest K_{ht+1}, and leisure, $1 - H_{ht}$, where H_{ht} is the amount of labor supplied by the h^{th} household in time period t. The utility function $U : \mathbb{R}^3_+ \rightarrow R_+$ satisfies the standard properties, and is assumed to be Cobb-Douglas. The timing of events is as follows. Production occurs at the beginning of

[6]Note that both the return to capital and the wage rate are in increasing in the tax rate.

each period. Once production occurs, households make their consumption, bequest, and labor supply decisions, and then die. We assume that the tax rate is known before households make their consumption, bequest, and labor supply decisions.

The household's problem is to maximize

$$Max_{C_{ht},K_{ht+1},H_{ht}} C_{ht}^{\alpha} K_{ht+1}^{\beta} (1 - H_{ht})^{1-\alpha-\beta} \tag{7}$$

subject to

$$C_{ht} + K_{ht+1} \leq w_t H_{ht} + r_t(1 - \tau_t) K_{ht}, \tag{8}$$

where $\alpha \in (0, 1)$, and $\beta \in (0, 1)$. The optimization exercise implies the following household decision rules,

$$C_{ht} = \frac{\alpha}{\beta} K_{ht+1}, \tag{9}$$

$$K_{ht+1} = \frac{\beta}{\alpha + \beta} \{w_t H_{ht} + r_t(1 - \tau_t) K_{ht}\}, \tag{10}$$

and

$$H_{ht} = (\alpha + \beta) - (1 - \alpha - \beta)[\frac{r_t(1 - \tau_t)}{w_t}] K_{ht}. \tag{11}$$

Equations (9), (10), and (11), summarize the individual household equations in the model. Equation (9) is the Euler equation for consumption. Equation (10) is the household capital accumulation equation: it says that next period's capital is proportional to current disposable income (i.e., wage income plus after-tax capital income).

Equation (11) is the household labor supply equation and is increasing in the tax rate on capital income: i.e., $\frac{\partial H_{ht}}{\partial \tau_t} > 0$.

To obtain the aggregate labor supply and capital accumulation equations, we aggregate across households. Noting that $\sum_1^N H_{ht} = H_t$, using (6), and re-arranging equation (11) leads to an expression for aggregate labor supply determined endogenously as a function of the tax rate,

$$H_t = H(\tau_t) = \frac{N(\alpha+\beta)(1-a)}{(1-a)+a(1-\alpha-\beta)(1-\tau_t)}. \tag{12}$$

Let $\delta(\tau_t) = (1-a)+a(1-\alpha-\beta)(1-\tau_t)$.[7] Intuitively, a rise in the tax rate reduces a household's return to capital, and therefore its capital income. This leads households to supply more labor. When $\alpha+\beta = 1$, households do not value leisure, and supply their entire time endowment as labor exogenously. This implies that $H_t = N$, or that the aggregate labor supply is simply the household per-period time endowment (1) multiplied by the number of households (N). Having determined H, equation (10) implies

$$K_{t+1} = \frac{\beta}{\alpha+\beta}\{\xi(\tau_t)H_t^{\frac{1-a}{a}} + \phi(\tau_t)(1-\tau_t)H_t^{\frac{1-a}{a}}\}K_t. \tag{13}$$

Equations (12) and (13) denote the aggregate decision rules for labor and capital,

[7]When $\tau = 1$, $H = N(\alpha+\beta)$, and when $\tau = 0$, $H = \frac{N(\alpha+\beta)(1-a)}{(1-a)+(1-\alpha-\beta)} < N(\alpha+\beta)$. It is easy to verify that $H^{'}(\tau) > 0 \quad \forall \tau \in [0,1]$.

respectively.

We now obtain the growth maximizing tax rate. Define the economy growth rate as $g_{t+1} = \frac{K_{t+1}}{K_t}$. To obtain an expression for g_{t+1}, we begin with the household capital accumulation equation, (10). Substituting out the expression for H_{ht} (using (11) in (10)), aggregating across households, and simplifying implies

$$K_{t+1} = \beta\{Nw_t + r_t(1-\tau_t)K_t\}. \tag{14}$$

From equation (6), the wage rate can be expressed as $w_t = \frac{(1-a)K_t r_t}{aH_t}$. Using this expression for w_t, the expression for the rental rate in (4), and substituting the expression for H_t from (12) into equation (14) implies that

$$g_{t+1} = \text{constant} \cdot \{(1-a) + a(1-\tau_t)\}(\tau_t H_t)^{\frac{1-a}{a}}, \tag{15}$$

where the constant term is given by, constant $= \frac{(\alpha+\beta)}{\beta} a^{\frac{1-a}{a}}$. Equation (15) in conjunction with equation (12) determines the long run endogenous growth rate of the economy. Note that the growth-tax curve takes the inverse U-shape form (like Barro (1990)), which leads to a unique growth maximizing tax rate. This is because the tax rate enters both positively (both directly as well as through aggregate labor supply) as well as negatively in expression (15). The positive effect of a higher tax rate comes from the growth enhancing effect of more infrastructure (G), as well as the positive effect of higher labor supply from (12). However, when the tax rate on capital income becomes sufficiently high, this reduces the net return to capital, which reduces investment and growth. Hence, there exists a unique growth maximizing tax rate, which we denote as τ_g^e.

2.1 Exogenous Labor-Leisure

We now solve the problem when agents supply their entire time endowment inelastically in each period, i.e., $\alpha + \beta = 1$. When labor is exogenous, the wage rate is given by $w_t = \xi(\bar{\tau_t})K_t$, where $\xi(\bar{\tau_t}) = (1-a)a^{\frac{1-a}{a}}\tau_t^{\frac{1-a}{a}}$, while the return to capital is given by, $r_t = \phi(\bar{\tau_t}) = \{a\tau_t^{1-a}\}^{\frac{1}{a}}$, where we have normalized $N = 1$. Deriving the household decision rules like before, and aggregating the household capital accumulation equations leads to the aggregate capital accumulation equation,

$$K_{t+1} = \frac{\beta}{\alpha+\beta}[w_t + r_t(1-\tau_t)K_t] = \frac{\beta}{\alpha+\beta}[(\tau_t r_t)^{1-a} - \tau_t r_t]K_t. \tag{16}$$

Define the growth rate $g_{t+1} = \frac{K_{t+1}}{K_t}$. It is straightforward to verify from (16) that the growth maximizing tax rate is[8]

$$\tau_x^g = \frac{1-a}{a}, \tag{17}$$

[8]To obtain an expression for the growth maximizing tax rate, note that by Euler's theorem, $Y_t = \frac{\partial Y}{\partial K}K_t + \frac{\partial Y}{\partial H}H_t = r_t K_t + w_t$ where we normalize H to 1. This implies $w_t + r_t(1-\tau_t)K_t = w_t + r_t K_t - \tau_t r_t K_t = Y_t - r_t K_t$. Substituting out for Y_t and differentiating with respect to the tax rate yields the desired result.

where τ_x^g denotes the growth maximizing tax rate when $\alpha+\beta=1$.

We now provide a sufficient condition for the existence of a unique growth maximizing tax rate under endogenous labor-leisure ($\alpha+\beta<1$) and compare it with the growth maximizing tax rate under exogenous labor-leisure ($\alpha+\beta=1$).

Proposition 1 *Suppose $\alpha+\beta=1$. Then, the unique growth maximizing tax rate is given by*

$$\tau_x^g = \frac{1-a}{a}. \tag{18}$$

Suppose $\alpha+\beta<1$. If $2a-1>a(1-\alpha-\beta)(1-a)$, then there exists a unique growth maximizing tax rate under endogenous labor leisure which exceeds the growth maximizing rate under exogenous labor leisure,i.e.,

$$\tau_e^g > \tau_x^g = \frac{1-a}{a}. \tag{19}$$

Proof. See Appendix. ∎.

As shown in the appendix, the growth maximizing tax rate is obtained from differentiating the expression for the growth rate with respect to τ_t in (15). After some manipulation of the first order equation, this leads to the expression,

$$\underbrace{\frac{a\delta(\tau_t)}{(1-a\tau_t)}}_{MC} = \underbrace{\frac{(1-a)[(1-a)+a(1-\alpha-\beta]}{a\tau_t}}_{MB}. \tag{20}$$

Note when $\alpha+\beta=1$, or labor is supplied exogenously by households, then the above expression becomes,

$$\underbrace{\frac{(1-a)}{a\tau_t}}_{MB} = \underbrace{\frac{a}{(1-a\tau_t)}}_{MC}, \tag{21}$$

which leads to the growth maximizing tax rate when $\alpha+\beta=1$: $\tau_x^g = \frac{1-a}{a}$.

It is more convenient to re-write equation (20) as[9]

$$\underbrace{(1-a)\{a\tau_t-(1-a)\}}_{MC} = \underbrace{(1-\alpha-\beta)a[(1-a)(1-a\tau)-a(1-\tau)a\tau]}_{MB}. \tag{22}$$

Equation (22) can be used to plot the marginal cost and benefit schedules corresponding to the growth maximizing tax rate under $\alpha+\beta<1$ and $\alpha+\beta=1$. This is illustrated in Figure (1). The growth maximizing tax rate is obtained when the marginal benefit of an increase in the tax rate on capital income is exactly equal to the marginal cost of higher taxes. However, as (22) shows, changes in $\alpha+\beta$ only affect the marginal benefit schedule, and not the marginal cost schedule. In particular, as $\alpha+\beta\to 1$, the marginal benefit of higher taxes falls for each value of the tax rate. This leads to a reduction in the growth maximizing tax rate. When $\alpha+\beta=1$,

[9]See the appendix for details.

Figure 1: The Growth Maximizing Tax Rate under Exogenous and Endogenous Labor-Leisure

the marginal benefit schedule intersects the marginal cost schedule at $\tau_x^g = \frac{1-a}{a}$: in this case, the marginal benefit is a horizontal line and equal to zero for all feasible values of the tax rate. Intuitively, when labor is endogenous, the tax rate maximizes the net return to capital as well as aggregate labor supply. Under exogenous labor supply, aggregate labor is invariant with respect to the tax rate. Hence, the growth maximizing tax rate is greater when labor-leisure is endogenous.[10]

[10] However, by endogenizing leisure, the growth tax curves are no longer identical. To see this, from (12), when $\alpha + \beta = 1$, $H = N$. When $\alpha + \beta < 1$, $H_t = \frac{N(1-a)(1-\alpha-\beta)}{\delta(\tau_t)} < N, \forall \tau \in [0,1]$ as $\frac{(1-a)(1-\alpha-\beta)}{\delta(\tau_t)} < 1$. This implies that the growth-tax curve under endogenous leisure lies everywhere below the growth-tax curve under exogenous leisure.

3 Optimal Tax Rate under Majority Voting when $\alpha + \beta < 1$.

We would like to verify whether the equilibrium tax rate under majority voting yields the growth maximizing tax rate when $\alpha + \beta < 1$, and $\alpha + \beta = 1$, as derived in Proposition (1). We first consider the case where $\alpha + \beta < 1$, and derive the transitional dynamics governing the law of motion of household capital holdings. We then characterize the optimal tax rate.

Like DG, for any household, h, let $\eta_{ht} = \frac{K_{ht}}{K_t}$, $\eta_h \in [0, 1]$, denote the relative capital holdings of the h^{th} household relative to the aggregate capital stock.[11] The dynamic law of motion of household specific capital holdings is given by [12]

$$\eta_{ht+1} = \eta_{ht}\{1 + \frac{\xi(\tau_t)[\frac{\frac{H_{ht}}{H_t}}{\eta_{ht}} - 1]}{\xi(\tau_t) + \phi(\tau_t)(1 - \tau_t)}\}. \tag{23}$$

Equation (23) is the index of inequality in the model and governs the transitional dynamics of relative capital holdings of the h^{th} household.[13]

Proposition 2 *In the steady state, the factor holding ratios of agents converge to a*

[11]When $\eta_h = 1$, then the h^{th} household owns the entire capital stock.

[12]We divide (10) by (13) and simplify to get (23).

[13]It is easy to verify from equation (23) that the transition to the steady state is monotonic and there is a unique stable steady state.

mass point that is independent of the initial distribution of capital, i.e.,

$$\frac{H_h}{H} = \eta_h = \frac{1}{N} \;\; \forall h. \tag{24}$$

This holds for all feasible values of the tax rate.

Proof. See Appendix ■

Proposition (2) shows that irrespective of the initial distribution of capital, all agents become identical in the steady state. In the long run, every agent is a 'representative' agent, and identical with respect to their relative capital holdings. This implies complete equality as in the steady state every household will be endowed with the same share of the capital stock, $\frac{1}{N}$, and labor hours as the average household.[14] This is also true for the median household.

To derive an expression for the equilibrium tax rate under median voting, we first obtain the indirect utility function of households. We first manipulate the utility function to write it as $logU_{ht} = V_{ht} = \text{constant} + log[\frac{K_{ht+1}}{w_t}] + (\alpha+\beta)log(w_t)$. Note that $\frac{K_{ht+1}}{w_t} = H_{ht} + \frac{r_t(1-\tau_t)}{w_t}K_{ht}$, and $H_{ht} + \frac{a}{1-a}\frac{H_t}{K_t}K_{ht}(1-\tau_t) = (\alpha+\beta)[1+\frac{a}{1-a}\frac{H_t}{K_t}K_{ht}(1-\tau_t)]$.

[14] These results are similar to the results of Saint-Paul and Verdier (1993) who also obtain complete equality in the steady state. In their model - like here - income distribution evolves endogenously. As income distribution becomes more equal, tax rates decline. Further, increased inequality may be good for growth, provided that this implies more support for public education.

Substituting these into the $V_{ht} = log(U_{ht})$ yields,

$$V_{ht} = \text{constant} + log\{1 + \frac{a}{1-a}\frac{H_t}{K_t}K_{ht}(1-\tau_t)\} + (\alpha+\beta)log(w_t). \qquad (25)$$

Substituting for H_t in (12) into the above expression and simplifying yields

$$V_{ht} = \text{constant} + log\{1 + aN(\alpha+\beta)\frac{(1-\tau_t)}{\delta(\tau_t)}\eta_{ht}\} + (\alpha+\beta)log(w_t). \qquad (26)$$

We assume that individual's care not only about how their optimal choices affect individual labor supply, both aggregate H as well. It is sufficient to note that for any given values of K_t and K_{ht} the indirect utility function of single peaked with respect to τ_t. By the median voter theorem, this implies that the median household's ideal tax rate is the equilibrium tax rate in the economy.[15] This corresponds to the political tax determined under majority voting.

Taking η_{ht} as given, the optimal tax rate of household's is obtained from the household's first order condition with respect to (26). The next proposition summarizes the optimal tax rate of households.

Proposition 3 *The optimal tax rate for the h^{th} household, τ_{ht}, is determined by the*

[15] As is well known, this is a sufficient condition for the median voter theorem to hold.

first order condition,

$$\frac{\delta(\tau_{ht})(1-a)}{a\tau_{ht}} = \frac{aN(1-a)\eta_{ht}}{\{(1-a)+a[(1-\alpha-\beta)+(\alpha+\beta)N\eta_{ht}](1-\tau_{ht})\}} + (2a-1)(1-\alpha-\beta) = g(\eta_{ht}). \tag{27}$$

The optimal tax rate is decreasing in the relative capital holdings of the h^{th} household.

PROOF. See Appendix. ∎

It will be easier to characterize the optimal tax of households relative to the growth maximizing tax rate if we substitute $\delta(\tau_t)$ into (27) and re-write it as,

$$\underbrace{\frac{(1-a)[(1-a)+a(1-\alpha-\beta)]}{a\tau_{ht}}}_{MB} = \underbrace{\frac{aN(1-a)\eta_{ht}}{\{(1-a)+a[(1-\alpha-\beta)+(\alpha+\beta)N\eta_{ht}](1-\tau_{ht})\}} + a(1-\alpha-\beta)}_{MC}. \tag{28}$$

Equation (28) characterizes the optimal tax rate of households. First, from (28) it is easily verified that as η_h increases the optimal tax rate of households falls. Intuitively, the right hand side of equation (27) corresponds to the marginal cost schedule of a rise in the tax rate facing households. The first term on the right hand side of equation (27) in increasing in η_h. Hence, a higher η_h pushes the marginal cost up for each tax rate. This reduces the household's preferred tax rate.[16] This is intuitive: the more

[16]Hence, $\tau_{ht} = \underbrace{g(\eta_{ht})}_{-}$, where g_{ht} is defined in (27).

capital rich households are, the less their preferred tax on capital.

Second, equation (28) allows us to rank households in terms of their capital holdings and preferred tax rates. For capital-rich households (relative to the mean), $\eta_h > \frac{1}{N}$. This implies their preferred tax on capital will be less than a capital poor household whose capital holdings are less than the average, $\eta_h < \frac{1}{N}$. This is because the marginal cost for an increase in the tax rate is higher for the capital rich households. Hence, their preferred tax on capital is less compared to a capital poor household.

From Proposition 2 however, the households' factor holding ratios converge to the steady state where $\eta_h = \frac{1}{N}$. Hence, the median household's preferred tax rate is identical to the mean households preferred tax rate in the steady state. Substituting $\eta_h = \frac{1}{N}$ into (27), the preferred tax rate of all households in the steady state is given by

$$\underbrace{\frac{(1-a)[(1-a)+a(1-\alpha-\beta)]}{a\tau_h}}_{MB} = \underbrace{\frac{a(1-a)}{(1-a\tau_h)} + a(1-\alpha-\beta)}_{MC}, \quad h = \frac{1}{N}. \tag{29}$$

This is also the median household's preferred tax rate since all households are identical in the steady state. Setting $h = m$ in (29) yields the political tax rate, i.e., the optimal tax rate of the median voter in the steady state. Like DG and AR, we define distributive conflict as the difference between the median household's preferred tax rate and the growth maximizing tax rate. Note that the median voter's preferred tax rate under majority voting in the steady state is determined by (29), while the growth maximizing tax rate is determined by equation (15). We re-write (15) as

$$\underbrace{\frac{(1-a)[(1-a)+a(1-\alpha-\beta)]}{a\tau_t}}_{MB} = \underbrace{\frac{a(1-a)}{1-a\tau_t} + \frac{a^2(1-\alpha-\beta)(1-\tau)}{1-a\tau_t}}_{MC}. \tag{30}$$

The left hand side of both (30) and (29) denote the marginal benefit schedule from higher taxes. Note that the marginal benefit schedule for changes in the tax rate in the steady state for households is identical to the marginal benefit schedule from the growth maximizing tax rate. The difference lies in the marginal cost schedules of the growth maximizing tax rates and the marginal cost schedule for households in the steady state. In particular, since $\frac{a(1-\tau)}{(1-a\tau)} < 1$, for all $\tau \in [0,1]$, the marginal cost a rise in the tax rate is higher for households in the steady state *for each level of the tax rate*, with the difference between the two marginal cost functions an increasing function of the tax rate. Thus, for higher values of the tax rate, the optimal tax of households in the steady state - as well as the median household's preferred tax rate - *is less than* the growth maximizing tax rate.

Proposition 4 *Let $\alpha + \beta < 1$. In the steady state, the preferred tax rates of all*

Figure 2: The Steady State Tax Rate versus the Growth Maximizing Tax Rate

households - including the median - converges to the 'average' household's preferred tax rate. However, this tax rate is strictly less than the growth maximizing tax rate, τ_e^g.

Figure (2) illustrates the dynamics behind Proposition (4). Intuitively, since households value leisure, they work less. Hence, they chose to tax themselves less. We start with the marginal cost schedule of a household who owns very little but positive amounts of capital.[17] As households become more capital rich, the marginal cost of higher taxes rise, and so the preferred tax on capital falls until it equals the

[17]When the h^{th} owns no capital, i.e., $\eta_h = 0$, his marginal cost curve is flat. In this case, his preferred tax on capital approaches 1.

growth maximizing tax rate. This is where the marginal cost schedules of both the growth maximizing tax rate and the h^{th} household's tax rate coincide. However in the steady state because households value leisure they work less. Their preferred tax on capital is less than the growth maximizing tax rate. There is distributive conflict, which reflects the household's preference for leisure.

3.1 Optimal Tax rate under Majority Voting when $\alpha + \beta = 1$.

Following the same steps as before, the relative capital holdings of households evolves according to

$$\eta_{ht+1} = \eta_{ht}\{1 + \frac{\xi(\bar{\tau_t})[\frac{1}{\eta_{ht}} - 1]}{\xi(\bar{\tau_t}) + \phi(\bar{\tau_t})(1 - \tau_t)}\}. \tag{31}$$

This implies that $\eta_{ht} = 1, \forall h$ in the steady state. There is complete equality in the steady state. To determine the optimal tax rate of households, we obtain the first order conditions of households with respect to their preferred tax rates. The indirect utility function of households is given by,

$$V_{ht} = \text{constant} + log\{1 + \frac{a}{1-a}(1 - \tau_t)H_t\eta_{ht}\} + (\alpha + \beta)log(w_t). \tag{32}$$

Since agents take H as given, the first order condition is given by

$$\frac{\frac{a}{1-a}\eta_{ht}H_t}{1 + \frac{a}{1-a}\eta_{ht}H_t(1 - \tau_t)} = (\alpha + \beta)\frac{1-a}{a}\tau_t. \tag{33}$$

Setting $\alpha + \beta = 1$ implies that the optimal tax of the h^{th} household is given by,

$$\tau_{ht} = (1-a)\{1 + \frac{1-a}{a\eta_{ht}}\} = \underbrace{f(\eta_{ht})}_{-}, \tag{34}$$

which shows that the optimal tax rate is decreasing in the relative capital holdings of the h^{th} household. Setting $h = m$ and $\eta_{mt} = 1$ into this expression implies that $\tau_x^m = \frac{1-a}{a}$, which is the median household's preferred tax rate. Note that this is identical to the growth maximizing tax rate, (18), in the steady state .

Proposition 5 *When $\alpha + \beta = 1$, the growth maximizing tax rate is identical to the equilibrium tax rate under majority voting in the steady state. There is no distributive conflict in the long run.*

Proposition (5) suggests that distributive conflict vanishes in the long run when $\alpha+\beta = 1$, which is consistent with DG. Interestingly, both the growth maximizing tax rate as well as the optimal tax rate for households in the steady state are independent of β when $\alpha + \beta = 1$. This is not the case when household value leisure: as can be seen from (29), the optimal tax of households depends on β.

4 Inequality and Growth

The above results allow us to characterize impact of inequality on growth. We have shown that when the median voter is capital poor relative to the average household, his preferred tax rate on capital will exceed the growth maximizing tax rate. Hence, more inequality leads to lower growth. However, over time, because redistribution through the tax rate equalizes the factor holding ratios of agents, the median agent becomes more capital rich, his preferred tax rate on capital falls, and growth rises. Hence, if the initial distribution of capital holdings is highly skewed wit K_m less than $K_{\frac{1}{N}}$, there is higher inequality and lower growth. In the long run, the factor holding ratio's of all agents converge to a mass point. The median household becomes the average household in the steady state. This implies their preferred tax rates are identical. However, since households also value leisure, they chooses to work less and, hence, choose a lower tax in the steady state. This reduces the tax rate under majority voting relative to the growth maximizing tax rate. This leads to lesser growth. The non-linear effect of growth and inequality is consistent with recent empirical evidence that suggests that the while too much inequality is harmful for growth, too much equality can also be harmful for growth (Banerjee and Duflo, 2003).

Proposition 6 *Suppose the initial distribution of capital is such that the median household is capital poor, i.e.,* $\eta_m < \frac{1}{N}$. *Then higher inequality leads to lower growth. In the long run, while there is more equality, the tax rate determined by majority voting is less than the growth maximizing tax rate. There is higher equality but lower growth in the steady state.*

Note that a social planner would not want to raise taxes to achieve the growth maximizing tax rate, since raising taxes would not be optimal for households.

5 Conclusion

This paper constructs a general model of distributive conflict and economic growth along the lines of AR and DG. The novel feature of this paper is that the growth rate, the tax rate, labor supply, and distribution, are all endogenous. Several interesting results emerge. First, unanimity, or the convergence of household specific factor holding ratios continues to hold. This implies greater political consensus over policy choices in the long run. However, the equilibrium tax rate is less than the growth maximizing tax rates when agents value leisure which leads to distributive conflict in the steady state. The results of this paper extend the work of Das and Ghate (2004) in which the labor-leisure choice is exogenous. We also use a more plausible specification of the government budget constraint where we tax capital income instead of wealth (the capital stock). The model suggests that if the median voter is sufficiently poor, higher inequality will lead to lower growth. However, because distribution is endogenous, in the steady state the political tax rate preferred by households falls over time. This leads to more equality but lower growth in the steady state. Our results show that characterizing the transitional dynamics in a model of growth and endogenous distribution is not an intractable proposition.

A Proofs

Proof. Proposition (1). Log-differentiating (15) with respect to τ_t, and re-arranging, yields the following first order condition for the unique growth maximizing tax rate,

$$\frac{a}{(1-a)+a(1-\tau_t)} = \frac{1-a}{a\tau_t} + \frac{(1-a)(1-\alpha-\beta)}{\delta(\tau_t)} \tag{35}$$

Multiplying through both terms in (35) by $\delta(\tau_t)$ and simplifying implies

$$\underbrace{\frac{a\delta(\tau_t)}{(1-a\tau_t)}}_{MC} = \underbrace{\frac{(1-a)[(1-a)+a(1-\alpha-\beta)]}{a\tau_t}}_{MB}. \tag{36}$$

Substituting for $\delta(\tau_t) = (1-a) + a(1-\alpha-\beta)(1-\tau_t)$ above, equation (20) can be simplified to

$$\underbrace{(1-a)\{a\tau_t - (1-a)\}}_{MC} = \underbrace{(1-\alpha-\beta)a[(1-a)(1-a\tau) - a(1-\tau)a\tau]}_{MB}. \tag{37}$$

Notice that changes in α and β only lead to changes in the marginal benefit schedule. Let $\alpha + \beta < 1$. To obtain Figure (1), , evaluating the left hand side of (37) when $\tau = \{0, 1\}$ implies $LHS(0) = -(1-a)^2$ and $LHS(1) = (2a-1)(1-a)$, with the marginal cost schedule increasing linearly in τ and intersecting the x-axis at $\tau = \frac{1-a}{a}$. Evaluating the right hand side of (37) when $\tau = \{0, 1\}$ implies $RHS(0) = (1 - \alpha - \beta)a(1-a)$ and $RHS(1) = (1-\alpha-\beta)a(1-a)^2$, with the marginal benefit schedule decreasing in $\tau, \forall\tau \in [0, 1]$. Notice that when $\tau = \frac{1-a}{a}$, the marginal benefit term is positive. Hence, $\tau = \frac{1-a}{a}$ cannot be the growth maximizing tax rate. Since, the marginal benefit is falling, when $\alpha + \beta < 1$, the growth maximizing tax under endogenous labor leisure exceeds the growth maximizing tax rate when labor-leisure is exogenous. ∎

Proof. Proposition (2). Setting $\eta_{ht+1} = \eta_{ht} = \eta_h$ in (23) implies that

$$\frac{H_{ht}}{H_t} = \eta_h \ \ \forall h. \tag{38}$$

Dividing equation (11) by the expression for H_t in (12) and simplifying yields,

$$\frac{H_{ht}}{H_t} = \frac{\delta(\tau_t)}{N(1-a)} - \frac{a(1-\alpha-\beta)}{(1-a)}\frac{K_{ht}}{K_t}(1-\tau_t). \tag{39}$$

Since equation (23) implies that

$$\frac{\frac{H_{ht}}{H_t}}{\eta_{ht}} = 1 \tag{40}$$

in the steady state, dividing both sides of (39) by $\frac{K_{ht}}{K_t}$, setting $\frac{\frac{H_{ht}}{H_t}}{\eta_{ht}} = 1$, and simplifying yields the result. ∎

Proof. Proposition (3). The h^{th} agent's indirect utility function is given by,

$$V_{ht} = \text{constant} + \underbrace{log\{1 + aN(\alpha+\beta)\frac{(1-\tau_{ht})}{\delta(\tau_t)}\eta_{ht}\}}_{TermI} + \underbrace{(\alpha+\beta)log(w_t)}_{TermII}. \tag{41}$$

Evaluating the first term (I) and simplifying yields

$$\frac{\partial TermI}{\partial \tau_{ht}} = \frac{-aN(\alpha+\beta)(1-a)\eta_{ht}}{\{a[(1-\alpha-\beta)+(\alpha+\beta)N\eta_{ht}](1-\tau_{ht})\}}\frac{1}{\delta(\tau_{ht})}. \tag{42}$$

Evaluating the first term (II) and simplifying yields

$$\frac{\partial TermII}{\partial \tau_{ht}} = \frac{\xi^{'}(\tau_{ht})}{\xi(\tau_{ht})} + \frac{(1-2a)}{a}\frac{H^{'}(\tau_{ht})}{H(\tau_{ht})}. \tag{43}$$

Note that $\frac{\xi^{'}(\tau_{ht})}{\xi(\tau_{ht})} = \frac{(1-a)}{a\tau_{ht}}$, while $\frac{H^{'}(\tau_{ht})}{H(\tau_{ht})} = \frac{a(1-\alpha-\beta)}{\delta(\tau_{ht})}$. Substituting these expressions back into (43), noting (42), and re-arranging terms yields (27). ∎

B References

Aghion, Phillip and Bolton, P., 1997. A Theory of Trickle-Down Growth and Development, *Review of Economic Studies*, 64: 151-172.

Aghion, Philippe, Caroli, Eve, and Garcia-Penalosa, Cecilia, 1999, Inequality and Economic Growth: The Perspective of the New Growth Theories, *Journal of Economic Literature*, 37(4), ppm. 1615-60

Alesina, Alberto and D. Rodrik, 1994. Distributive Conflict and Economic Growth, *Quarterly Journal of Economics*, 109: 465-490.

Barro, Robert J., 1990, Government Spending in a Simple Model of Endogenous Growth, *Journal of Political Economy*, 98:103-25.

Barro, Robert J. and Xavier Sala-i-Martin, 1995, *Economic Growth*, Cambridge, Mass: MIT Press.

Banerjee, Abhijit, and Esther Duflo, 2003, Inequality and Growth: What can the Data Say ? *Journal of Economic Growth*, Vol.8, 267-299.

Das, Satya, and C. Ghate, 2004, Endogenous Distribution, Politics, and the Growth-Equity Tradeoff. *Contribution to Macroeconomics*, Berkeley Electronic Press, Vol. 4(1), Article 6.

Drazen, Allan, 2000, *Political Economy in Macroeconomics*, Princeton, New Jersey: Princeton University Press.

Saint-Paul, Gilles and T. Verdier, 1993, Education, Democracy, and Growth, *Journal of Development Economics*, 42: 399-407.

Stiglitz, Joseph, 1969, Distribution of Income and Wealth Among Individuals, *Econometrica*, 37: 382-397.

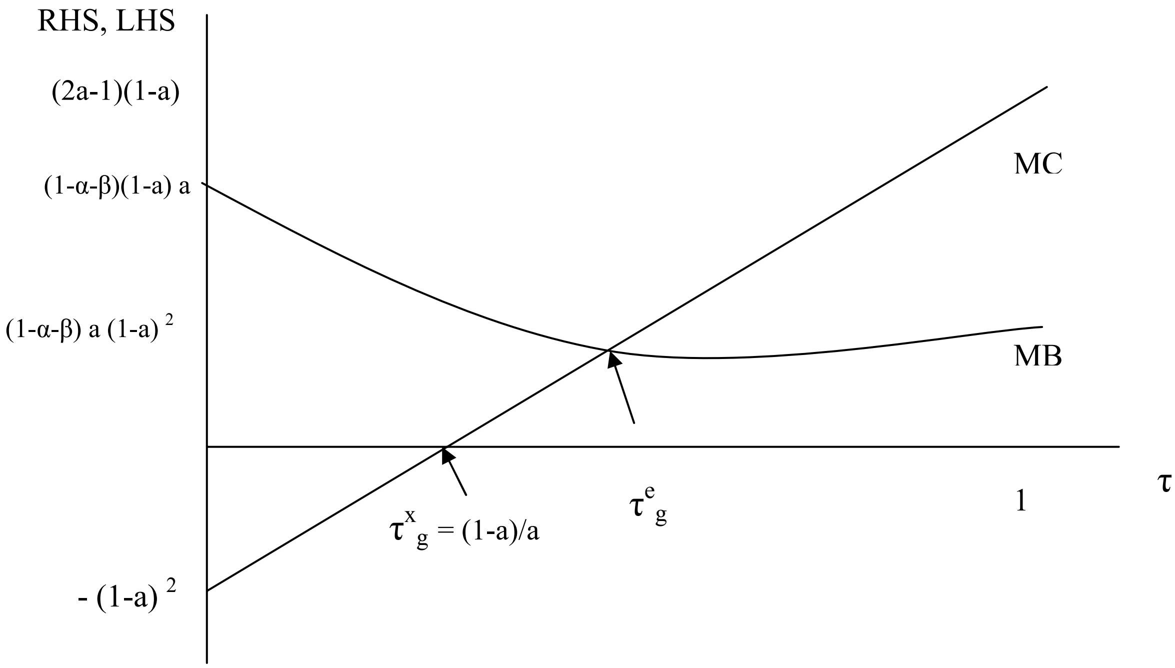

FIGURE 1

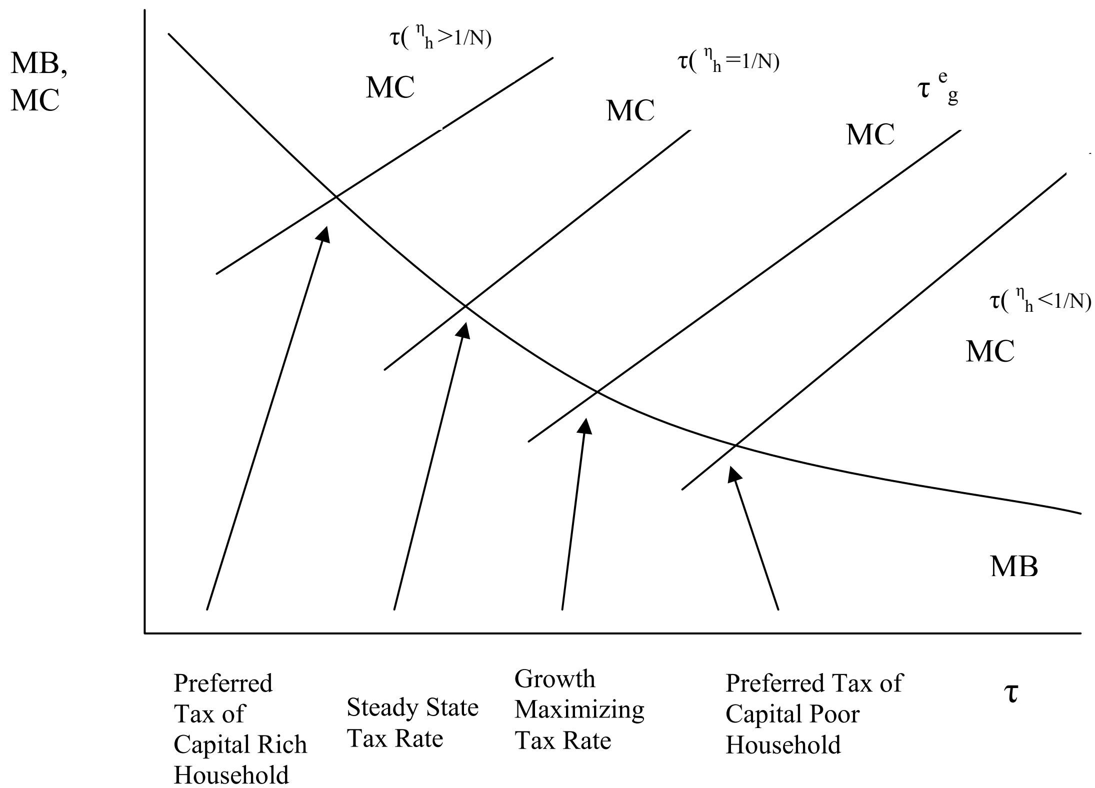
MB,
MC
τ(η h >1/N)
MC
τ(η h =1/N)
MC
τ e g
MC
τ(η h <1/N)
MC
MB
Preferred Tax of Capital Rich Household
Steady State Tax Rate
Growth Maximizing Tax Rate
Preferred Tax of Capital Poor Household
τ

FIGURE 2