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Sven Husmann
Andreas Stephan

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DIW Berlin
German Institute for Economic Research
Königin-Luise-Str. 5
14195 Berlin
Tel. +49 (30) 897 89-0
Fax +49 (30) 897 89-200
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On Estimating an Asset's Implicit Beta*

Sven Husmann (European University Viadrina)

Andreas Stephan (European University Viadrina and DIW Berlin)

Abstract

Siegel (1995) has developed a technique with which the systematic risk of a security (beta) can be estimated without recourse to historical capital market data. Instead, beta is estimated implicitly from the current market prices of exchange options that enable the exchange of a security against shares on the market index. Because this type of exchange options is not currently traded on the capital markets, Siegel's technique cannot yet be used in practice. This article will show that beta can also be estimated implicitly from the current market prices of plain vanilla options, based on the Capital Asset Pricing Model. We provide empirical evidence on implicit betas using prices of exchange options from the EUREX over years 2000 to 2004.

JEL-Classification: G12.

Keywords: Capital Asset Pricing Model; Beta; Option Pricing.

*Corresponding author: Prof. Dr. Sven Husmann, Assistant Professor for International Accounting, Department of Business Administration and Economics, Europa-Universität Viadrina Frankfurt (Oder), Große Scharrnstraße 59, 15230 Frankfurt (Oder), Germany. E-mail: husmann@euv-frankfurt-o.de.

1 Introduction

The Capital Asset Pricing Model (CAPM) of *Sharpe* (1964) and *Lintner* (1965) continues to be of central importance to the valuation of risk-bearing securities, in theory as well as in practice. The CAPM is still widely used in estimating the cost of capital of firms and evaluating their performance. Unfortunately, empirical findings of the CAPM are poor. Empirical problems may be caused by theoretical problems, such as simplifying assumptions, or by difficulties in implementing tests of the CAPM.

According to the CAPM, the expected rate of return on a security depends primarily on its systematic risk (beta), which is normally estimated by means of a regressive analysis of historical capital market data. Of all of the numerous empirical tests of the CAPM, the study by *Fama and French* (1992) in particular generated much attention. According to this study, beta has hardly any explanatory power for the expected rate of return on a security. In fact, the expected rate of return depends much more on the size of a company and the book-to-market ratio. *Berk* (1995) showed nonetheless that these effects can also be traced back to a flawed measurement of beta.[1] Statistical errors can be caused in particular by the fact that beta changes through time.[2] In order to avoid this problem, *Siegel* (1995) proposes a method with which beta can be estimated from current options prices, without recourse to historical capital market data. However, practical application of this method requires that exchange options be traded that entitle the exchange of securities for shares on the market index. Presently, such options are not traded on the capital market. The purpose of this paper is to universalize Siegel's method so that beta can also be estimated from plain vanilla options.

The *Siegel* (1995) method is based on estimation of implicit volatility according to *Latané and Rendleman* (1976), whose technique is considered the standard in option pricing today.[3] *Siegel* (1995) ties this technique together with the valuation of exchange options according to *Margrabe* (1978) in order to estimate implicit beta. *Siegel* (1997), *Campa and Chang* (1998), and *Walter and Lopez* (2000) use similar approaches to obtain implied correlation of currencies from currency options. Recently, *Skintzi and Refenes* (2005) propose a method in forecasting future index

[1] *Fama and French* (2004) discuss the empirical problems that may be caused by difficulties in implementing valid tests of the CAPM.

[2] *Skintzi and Refenes* (2005) and *Longin and Solnik* (2001), for example, observed that correlations of stocks returns increase in highly volatile or bear markets.

[3] See *Blair et al.* (2001) for recent studies on the predictive ability of implied volatility.

correlation called *implied correlation index* that is also based on current option prices.

In this article the *Siegel* (1995) method will be universalized in that the implicit density function of an underlying asset is estimated implicitly from the theoretical CAPM prices of plain vanilla options.[4] The beta of an underlying asset results from the moments of this density function. The theoretical basis for calculation of implicit, risk-neutral density functions originates from *Ross* (1976) and *Breeden and Litzenberger* (1978) and has been used in numerous works to this day: *Rubinstein* (1994), *Darman and Kani* (1994), *Jackwerth and Rubinstein* (1996), and *Brown and Toft* (1999) estimate implicit risk-neutral density functions with the help of a modified binomial model (Implied Binomial Trees). *Shimko* (1993), *Jarrow and Rudd* (1982), and *Longstaff* (1995) estimate the price functions of options directly from their observed market prices, in dependence on the exercise price, and from there derive risk-neutral density functions. *Aït-Sahalia and Lo* (2000) and *Jackwerth* (2000) determine a clear difference between risk-neutral and subjective expectations and attempt to draw conclusions from this regarding the risk aversion of market participants. *Jackwerth* (2000) arrives furthermore at the result that the historical capital market rates of return are approximately lognormally distributed.

The notation and model assumptions are explained in section 2. In section 3, a model is presented with which calls can be evaluated based on the CAPM when rates of return are distributed lognormally. On this basis, it is possible to estimate beta implicitly from the prices of ordinary calls in section 4. In section 5 we apply this approach to estimating betas from call options traded at the EUREX. Section 6 summarizes the results.

2 Assumptions and Notation

The valuation of options in section 3 is based on the assumptions of the one-period CAPM:

1. Risk-averse investors maximize the μ-σ-utility of their end-of-period wealth.

2. Investors have homogeneous expectations about assets returns; the instantaneous rate of return on any asset and the market portfolio have a joint normal distribution.[5] Investors may borrow or lend unlimited amounts at the risk-free rate.

[4] *Dennis and Mayhew* (2002) investigated the relative importance of beta in explaining the prices of stock options traded on the Chicago Board Options Exchange.

[5] For a definition of bivariate normal distribution, see Appendix A.

3. Markets are frictionless. Information is costless and simultaneously available to all investors. There are no market imperfections such as transaction costs, taxes, or restrictions on short selling.

The following notation is used throughout the paper:

K	Exercise price on an option
$p(\tilde{X}_c)$	Price of a call on an asset S with cashflow $\tilde{X}_c$
$p(\tilde{X}_{cm})$	Price of a call on the market index $\tilde{X}_m$ with cashflow $\tilde{X}_{cm}$
$p(\tilde{X}_{ce})$	Price of an exchange option call with cashflow $\tilde{X}_{ce}$
$p(\tilde{X}_s)$	Price of an underlying asset S with cashflow $\tilde{X}_s$
$p(\tilde{X}_m)$	Current Market index
n_s	Number of shares of asset S to be exchanged under the exchange option
n_m	Number of shares of the market index under the exchange option
$\tilde{R}_s$	Standardized cashflow of an underlying asset, $\tilde{R}_s = \tilde{X}_s \,/\, p(\tilde{X}_s)$
$\tilde{R}_m$	Standardized cashflow of the market portfolio, $\tilde{R}_m = \tilde{X}_m \,/\, p(\tilde{X}_m)$
β_s	Beta of an underlying asset S with respect to the market index
r_f	Instantaneous risk-free rate of interest
$\tilde{r}_s$	Instantaneous rate of return on asset S
$\tilde{r}_m$	Instantaneous rate of return on the market index
μ_s	Expected instantaneous rate of return on asset S
μ_m	Expected instantaneous rate of return on the market index
σ_S	Instantaneous variance of the rate of return on asset S
σ_m	Instantaneous variance of the rate of return on the market index
ρ	Instantaneous correlation between the rates of return on asset S and on the market index

In the case of the given parameters for bivariate normal distribution of rates of return, the following applies for the expected values, variances and covariances of the securities' cash flow and market portfolio's standardized cash flow[6]

$$E\,[\tilde{X}_s] = p(\tilde{X}_s)\, e^{\mu_s + \frac{1}{2}\sigma_s^2}, \tag{1}$$

$$E\,[\tilde{R}_m] = e^{\mu_m + \frac{1}{2}\sigma_m^2}, \tag{2}$$

$$Var\,[\tilde{R}_m] = e^{2\mu_m + \sigma_m^2}\left(e^{\sigma_m^2} - 1\right), \tag{3}$$

$$Cov\,[\tilde{X}_s, \tilde{R}_m] = p(\tilde{X}_s)\, e^{\mu_m + \frac{1}{2}\sigma_m^2 + \mu_s + \frac{1}{2}\sigma_s^2}\left(e^{\rho\,\sigma_m\sigma_s} - 1\right). \tag{4}$$

[6]The moments of lognormal distribution can be calculated with the help of the integrals (38), (39) and (40) indicated in Appendix A.

For the standard definition of beta, the following results in the case of bivariate normal distribution[7]

$$\beta_s = \frac{Cov\,[\tilde{R}_s, \tilde{R}_m]}{Var\,[\tilde{R}_m]} = \frac{e^{\mu_s + \frac{1}{2}\sigma_s^2} \cdot (e^{\rho\,\sigma_s \sigma_m} - 1)}{e^{\mu_m + \frac{1}{2}\sigma_m^2} \cdot (e^{\sigma_m^2} - 1)}\,. \tag{5}$$

To simplify matters the time-to-maturity of an option is set equal to one throughout the paper.[8]

3 The Model

3.1 Option Pricing in an Incomplete Lognormal Market

In an incomplete lognormal market the CAPM may be used for option pricing.[9] The well-known *certainty equivalent valuation formula* of the single-period CAPM is[10]

$$p(\tilde{X}_c) = \frac{E\,[\tilde{X}_c] - \lambda \cdot Cov\,[\tilde{X}_c, \tilde{R}_m]}{1 + r_f^*} \qquad \text{where} \quad \lambda = \frac{E\,[\tilde{R}_m] - (1 + r_f^*)}{Var\,[\tilde{R}_m]}\,. \tag{6}$$

In order to be able to apply this equation to the valuation of a call, the expected cash flow of the call and the covariance between the cash flow of the call and the rates of return on the market portfolio must first be determined. Under the assumption of lognormally distributed rates of return, we derive[11]

$$E\,[\tilde{X}_c] = p(\tilde{X}_s) \cdot e^{\mu_s + \frac{1}{2}\sigma_s^2} \cdot \Phi\,(d_1) - K \cdot \Phi\,(d_2)\,, \tag{7}$$

$$\begin{aligned} Cov\,[\tilde{X}_c, \tilde{R}_m] = {} & p(\tilde{X}_s) \cdot e^{\mu_s + \frac{1}{2}\sigma_s^2 + \mu_m + \frac{1}{2}\sigma_m^2} \cdot (e^{\rho\,\sigma_s\sigma_m} \cdot \Phi\,(d_3) - \Phi\,(d_1)) \\ & - K \cdot e^{\mu_m + \frac{1}{2}\sigma_m^2} \cdot (\Phi\,(d_4) - \Phi\,(d_2))\,, \end{aligned} \tag{8}$$

$$d_1 = (\ln(p(\tilde{X}_s)/K) + \mu_s)/(\sigma_s) + \sigma_s, \tag{9}$$

$$d_2 = (\ln(p(\tilde{X}_s)/K) + \mu_s)/(\sigma_s), \tag{10}$$

$$d_3 = (\ln(p(\tilde{X}_s)/K) + \mu_s)/(\sigma_s) + \sigma_s + \rho\,\sigma_m, \tag{11}$$

$$d_4 = (\ln(p(\tilde{X}_s)/K) + \mu_s)/(\sigma_s) + \rho\,\sigma_m. \tag{12}$$

If we insert (7) and (8) in (6), after further conversion we get a representation that allows a comparison with the valuation equation according to *Black and Scholes*

[7]For a general definition of beta, see *Copeland et al.* (2005), p. 152.

[8]However, one can easily adjust the model to any time-to-maturity t different from one year using the following transformations: $\mu = t \cdot \mu_{p.a.}$, $\sigma^2 = t \cdot \sigma_{p.a.}^2$ and $r_f = t \cdot r_{f,p.a.}$.

[9]Options are redundant securities in a complete market. However, the empirical results of *Vanden* (2004) indicate that options are nonredundant for explaining the returns on risky assets.

[10]See *Copeland et al.* (2005), p. 157.

[11]See Appendix B. Put prices follow from put-call parity.

(1973)[12]

$$p(\tilde{X}_c) = p(\tilde{X}_s)\,\theta_1 - K\,e^{-r_f}\,\theta_2 \tag{13}$$

where
$$\theta_1 = e^{\mu_s+\frac{1}{2}\sigma_s^2-r_f}\left(\Phi\left(d_1\right) - \lambda\, e^{\mu_m+\frac{1}{2}\sigma_m^2}\left(e^{\rho\sigma_s\sigma_m}\Phi\left(d_3\right) - \Phi\left(d_1\right)\right)\right) \tag{14}$$
$$\theta_2 = \Phi\left(d_2\right) - \lambda\, e^{\mu_m+\frac{1}{2}\sigma_m^2}\left(\Phi\left(d_4\right) - \Phi\left(d_2\right)\right). \tag{15}$$

This model can be applied to the special case of complete markets. On complete markets, a risk-neutral valuation always leads to the correct valuation result.[13] In a risk-neutral world, the rate of return of the expected cash flow of a given risk-bearing financial title and that of the market portfolio equal the risk-free interest rate[14]

$$\mu_s + \sigma_s^2/2 = r_f\ , \tag{16}$$
$$\mu_m + \sigma_m^2/2 = r_f\ . \tag{17}$$

This correlation can also be intuitively justified. Market participants may only expect a risk premium for their risk-bearing financial title if they cannot nullify the risk through diversification of their portfolio. Because systematic risk can be nullified through diversification in complete markets, the market price of the risk is zero. From (16) and (17) follows $\lambda = 0$, $\theta_1 = \Phi\left(d_1\right)$ and $\theta_2 = \Phi\left(d_2\right)$. The valuation equation (13) is reduced accordingly with risk-neutral valuation to

$$p(\tilde{X}_c \,|\, \lambda = 0) = p(\tilde{X}_s)\,\Phi\left(\tfrac{\ln(p(\tilde{X}_s)/K)+r_f+\sigma_s^2/2}{\sigma_s}\right) - K e^{-r_f}\,\Phi\left(\tfrac{\ln(p(\tilde{X}_s)/K)+r_f-\sigma_s^2/2}{\sigma_s}\right)\ , \tag{18}$$

which equals the valuation equation of *Black and Scholes* (1973).

3.2 Pricing Options on the Market Index

Jarrow and Madan (1997) developed a valuation model for calls on the market index that is also based on the assumptions of the CAPM and lognormally distributed rates of return. If we use the notation established above, then the value of a call on the

[12] *Ritchken* (1985a) developed a similar valuation equation for options based on the CAPM. This model is not consistent with the *Black and Scholes* (1973) model in the case of risk-neutral valuation, however.

[13] See *Cox and Ross* (1976).

[14] If the expected instantaneous rate of return on a security equals μ_s and the rate of return is lognormally distributed, then the rate of return on the expected cash flow equals $\mu_s + \sigma_s^2/2$.

market index equals[15]

$$p(\tilde{X}_{cm}) = p(\tilde{X}_m)\,\theta_{m1} - K\,e^{-r_f}\,\theta_{m2} \tag{20}$$

$$\text{where}\quad \theta_{m1} = e^{\mu_m+\frac{1}{2}\sigma_m^2-r_f}\left(\Phi\left(dm_1\right) - \lambda\, e^{\mu_m+\frac{1}{2}\sigma_m^2}\left(e^{\sigma_m^2}\Phi\left(dm_3\right) - \Phi\left(dm_1\right)\right)\right), \tag{21}$$

$$\theta_{m2} = \Phi\left(dm_2\right) - \lambda\, e^{\mu_m+\frac{1}{2}\sigma_m^2}\left(\Phi\left(dm_1\right) - \Phi\left(dm_2\right)\right), \tag{22}$$

$$dm_1 = \left(\ln(p(\tilde{X}_m)/K) + \mu_m + \sigma_m^2\right)/\left(\sigma_m\right), \tag{23}$$

$$dm_2 = \left(\ln(p(\tilde{X}_m)/K) + \mu_m\right)/\left(\sigma_m\right), \tag{24}$$

$$dm_3 = \left(\ln(p(\tilde{X}_m)/K) + \mu_m + 2\sigma_m^2\right)/\left(\sigma_m\right). \tag{25}$$

This valuation equation is solely a special case of (13); for calls on the market index, the following apply: $\rho = 1$, $\mu_s = \mu_m$ and $\sigma_s = \sigma_m$.

4 Implicit Beta

4.1 Estimating Beta Using Exchange Options

Siegel (1995) assumes that continuous security trading on perfect capital markets is possible.[16] This standard assumption of options price theory enables a risk-neutral valuation of options and is equivalent to the assumption of complete capital markets.[17] Because the theoretical option prices in the case of risk-neutral valuation are independent of the correlation of cash flow of the underlying asset with that of the market portfolio, beta cannot be implicitly estimated from simple options.

[15] *Jarrow and Madan* (1997) define the parameter μ_m as the rate of return of the expected value, while we use it to identify the expected rate of return. In order to establish comparability with our results, the parameter μ must be replaced with $\mu + \sigma^2/2$ in the work by *Jarrow and Madan* (1997),

$$p(\tilde{X}_{cm}) = (a + b\,K)\,p(\tilde{X}_m)\,e^{\mu_m+\frac{1}{2}\sigma_m^2}\,\Phi\left(dm_1\right) - a\,K\Phi\left(dm_2\right) - b\,p(\tilde{X}_m)^2\,e^{2(\mu_m+\sigma_m^2)}\,\left(dm_3\right) \tag{19}$$

$$\text{where}\quad a = \left(e^{\sigma_m^2-r_f} - e^{-(\mu_m+\frac{1}{2}\sigma_m^2)}\right)/\left(e^{\sigma_m^2}-1\right)$$
$$b = \left(e^{\mu_m+\frac{1}{2}\sigma_m^2-r_f}-1\right)/\left(p(\tilde{X}_m)\,e^{2(\mu_m+\sigma_m^2)}\,\left(e^{\sigma_m^2}-1\right)\right).$$

If we furthermore assume that the investor's planning horizon and the time to maturity of the option are identical, following elementary conversions, the valuation equation (20) results from the valuation equation (19). However, for the special case of calls on the market index, the *Ritchken* (1985a) model is not identical with the *Jarrow and Madan* (1997) model.

[16] See Assumption 1 in *Siegel* (1995).

[17] See *Cox et al.* (1979).

Siegel (1995) therefore recourses to exchange options, which securitize the right for exchange of a financial title for shares on the market portfolio. The theoretical price of an exchange option in terms of risk-neutral valuation depends on the correlation of the cash flow of a financial title with the rates of return of the market portfolio and is therefore generally suitable for determining implicit beta factors. The risk-neutral valuation of exchange options is based on *Margrabe* (1978),

$$p(\tilde{X}_{ce}) = n_s\, p(\tilde{X}_s)\, \Phi\left(\frac{\ln\left(\frac{n_s\, p(\tilde{X}_s)}{n_m\, p(\tilde{X}_m)}\right) + \sigma_e^2/2}{\sigma_e}\right) - n_m\, p(\tilde{X}_m)\, \Phi\left(\frac{\ln\left(\frac{n_s\, p(\tilde{X}_s)}{n_m\, p(\tilde{X}_m)}\right) - \sigma_e^2/2}{\sigma_e}\right), \quad (26)$$

whereby the volatility σ_e depends on the volatilities of the underlying assets and the correlation of their rates of return,

$$\sigma_e^2 = Var\,[\tilde{r}_s - \tilde{r}_m] \; = \; \sigma_s^2 + \sigma_m^2 - 2\,\rho_{sm}\,\sigma_s\,\sigma_m\,. \quad (27)$$

Siegel (1995) assumes that three types of options are traded on the capital market: options on a common asset, options on the market index, and options that entitle the exchange of securities for shares on the market index. His idea for determination of implicit beta factors consists of first estimating the volatilities of the two underlying assets and the volatility σ_e of the exchange option implicitly from traded options. The correlation coefficient is then derived from correlation (27),

$$\rho_{sm} = (\sigma_s^2 + \sigma_m^2 - \sigma_e^2)\,/\,(2\,\sigma_s\,\sigma_m). \quad (28)$$

According to *Siegel* (1995), this results in the beta factor of the asset,

$$\beta_s^{Siegel} := \rho_{sm}\,\sigma_s\,/\,\sigma_m \; = \; (\sigma_s^2 + \sigma_m^2 - \sigma_e^2)\,/\,(2\,\sigma_m^2). \quad (29)$$

Leland (1999) describes definition (29) as modified beta. Even in risk-neutral valuation, this definition does not equal the standard definition of beta[18]

$$\beta_s = \frac{Cov\,[\tilde{R}_s, \tilde{R}_m]}{Var\,[\tilde{R}_m]} \; = \; \frac{e^{\rho\,\sigma_s\sigma_m} - 1}{e^{\sigma_m^2} - 1}\,. \quad (30)$$

Regardless of this, from a practical view there is the problem - as *Siegel* (1995) himself notes - that exchange options are not currently traded on the capital markets.

4.2 Estimating Beta Using Plain Vanilla Options

On incomplete markets, beta can be estimated implicitly with the valuation equations (13) and (20). As a result of the state of data typically given on the capital

[18] Inserting (16) and (17) in (5) results in (30).

market, a two-stage process for estimating implicit beta is advisable. In a first step, expectations of the market participant with regard to the market index are estimated. Based on the valuation equation (20) for options on the market index, the sum of the squared relative differences between the empirical options prices $p\big(\tilde{X}_{cm}\big)^*$ and theoretical options prices (20) is minimized through selection of the parameter $\hat{\mu}_m$ and $\hat{\sigma}_m$,[19]

$$\min_{\mu_m,\sigma_m} \quad \sum_{i=1}^{I} \left(\frac{p\big(\tilde{X}_{cm}\big)_i^* - p\big(\tilde{X}_{cm}\big)_i}{p\big(\tilde{X}_{cm}\big)_i} \right)^2 . \tag{31}$$

Based on the parameters $\hat{\mu}_m$ and $\hat{\sigma}_m$, estimated in the first step, the parameters $\hat{\mu}_s$ and $\hat{\sigma}_s$ can be determined with the same method for any asset S ,

$$\min_{\mu_s,\sigma_s} \quad \sum_{j=1}^{J} \left(\frac{p\big(\tilde{X}_c\big)_j^* - p\big(\tilde{X}_c\big)_j}{p\big(\tilde{X}_c\big)_j} \right)^2 \tag{32}$$

Through the application of relative instead of absolute differences, it is avoided that in-the-money options influence estimations of the parameters much stronger than out-of-the-money options.

In the minimization, the correlation coefficient $\hat{\rho}$ cannot be estimated independently of the parameters $\hat{\mu}_s$ and $\hat{\sigma}_s$, as the CAPM equilibrium condition must be considered as an additional condition for the underlying asset,

$$p(\tilde{X}_s) = \frac{E\,[\tilde{X}_s] - \lambda \cdot Cov\,[\tilde{X}_s, \tilde{R}_m]}{1 + r_f^*} \qquad \text{where} \quad \lambda = \frac{E\,[\tilde{R}_m] - (1 + r_f^*)}{Var\,[\tilde{R}_m]} . \tag{33}$$

Following several conversions, inserting (1), (2), (3) and (4) in (33) results in

$$e^{\mu_s + \sigma_s^2/2} = \frac{e^{r_f}\left(e^{\sigma_m^2} - 1\right)}{\left(e^{\sigma_m^2} - 1\right) + \left(e^{-(\mu_m + \frac{1}{2}\sigma_m^2 - r_f)} - 1\right)\left(e^{\rho\,\sigma_m \sigma_s} - 1\right)} . \tag{34}$$

Implicit beta (5) of an asset S can be calculated with the estimated parameters. In order to calculate beta, (34) must be resolved accordingly,

$$\rho\;\sigma_m\,\sigma_s = \ln\left(1 + \frac{e^{2\mu_m + \sigma_m^2}\left(e^{\sigma_m^2} - 1\right)}{e^{\mu_m + \frac{1}{2}\sigma_m^2} - e^{r_f}} \cdot \frac{e^{\mu_s + \frac{1}{2}\sigma_s^2} - e^{r_f}}{e^{\mu_m + \frac{1}{2}\sigma_m^2 + \mu_s + \frac{1}{2}\sigma_s^2}} \right) , \tag{35}$$

and inserted in (5).

[19] This technique is also applied by *Rubinstein* (1994) for the estimation of implicit risk-neutral density functions.

5 Empirical Illustration

We apply the new technique to call options traded at the Eurex, the European electronic exchange (futures exchange) based in Frankfurt, Germany. It is one of the world's largest derivatives exchanges. Implicit betas are estimated for those nine call options on stocks with the highest trading volumes at the Eurex.[20] The following data screening procedures were applied: options with no transactions or missing volume were removed. Furthermore, options with expiration dates smaller than 30 days or larger than two years were deleted.[21] In order to keep the empirical application as simple as possible, we also disregard from dividend payments.

Table 1 displays the names of the underlying assets as well as the frequencies of trade (number of transactions), volume of transactions in units and volume of transactions in € (turnover) for each of the nine options over the years 2000 to 2004.[22] It is worth noting that the call options, e.g. on Allianz AG, EON AG, Muenchener Rueckversicherung AG or SAP AG show an increase both in terms of transactions as well as in volumes over this period, demonstrating the growing importance of the option trade in general. The highest volumes can be observed for call options on the market index; the DJ EuroStoXX 50 had about a 33 billion € turnover in 2004. On the other hand, there are examples of call options, e.g on Deutsche Telekom AG where trade volume in € decreased during the observation period.

For each trading day of the year, the model is estimated using non-linear least squares.[23] In order to reduce the impact of influential observations, we assign weights less than one to out-of-money calls.[24] In the estimations, we employ the restriction that $\mu_m + 0.5\sigma_m^2 > r_f$ to ensure that the instantaneous rate of return of the market index is always greater than the instantaneous risk-free rate of interest, r_f.

Figure 1 shows the estimates of μ_m and σ_m for each day obtained in the first step

[20] We thank the Deutsche Boerse AG for kindly providing the data.

[21] Call options with expiration dates less than 30 days yielded implausible estimates or aggravated convergence problems in the model estimations.

[22] The descriptive statistics of Table 1 are based on the call options traded at the Eurex; this explains the difference in comparison to the market statistics for all options reported by the EUREX available at http://www.eurexchange.com/index.html.

[23] The estimation was carried out using the proc model procedure in SAS 9.1. We also tried estimations on a weekly basis. Overall, the results on a weekly basis are very similar to those obtained from the day-to-day estimations.

[24] The following definitions were used for the weights. Let $ratio = strike\ price/current\ stock\ price$. If $ratio > 1$ then the weight in the estimation is given as $2 \cdot (1 - \Phi((ratio - 1)/0.3))$. For example, if $ratio = 1.3$ then $weight = 0.317$, if $ratio = 1.6$ then $weight = 0.045$.

of the analysis (refer to eq. 31). These estimations are based on the observed transactions of call options on the DJ Eurostoxx 50 index. It can be stated that the overall pattern of μ_m and σ_m over time appears to be quite plausible. We observe an increase of μ_m and σ_m in the second half of 2002, but the estimates gradually decline afterwards. We also find a strong correlation between μ_m and σ_m^2.

Table 2 contains the location and dispersion statistics for the estimated implicit betas using the obtained μ_s and σ_s for each call from the second step estimation. Again, the estimation is carried out for each day with observation weights for out-of-money calls as described above and the restriction that $\mu_m + 0.5\sigma_m^2 > r_f$ is used. Generally, there are 253 to 255 trading days per year. Table 2 also displays the number of days of the year for which the parameter estimates are obtained in the second step. Missing estimates of μ_s and σ_s for some days are mainly the result of non-convergence in the model estimations, and sometimes due to an insufficient number of observations. Furthermore, estimates of μ_s and σ_s that give $|\rho| > 1$ (refer to eq. 35) are set to missing.

Table 2 shows that the yearly averages of estimated betas for the underlying assets are in plausible ranges. The computed 95% confidence intervals (C.I.) for expected values of betas show that in all cases expected values of betas are significantly different from zero. The implicit beta estimates for the technology-company Nokia corporation are higher than the implicit beta estimates for E.ON AG, a big utility company, again confirming the plausibility of the results. Another interesting result is that implicit betas show some considerable variation over time. For instance, Deutsche Telekom AG had implicit betas larger than one over the years 2000 to 2002, but afterwards had lower implicit betas, a development which can also be observed for SAP. Accordingly, an analysis based on the assumption of time-invariance on betas might provide misleading evidence.

6 Summary and Conclusions

This article presents a technique with which beta can be estimated implicitly from the prices of plain vanilla options, without recourse to historical capital market data. The fundamental idea resembles that of *Latané and Rendleman* (1976) in the estimation of implicit volatilities from options prices: beta is estimated implicitly from options traded on the capital market, under the assumption of normally distributed rates of return based on the CAPM. To illustrate the applicability of this new approach, we provide evidence on implicit beta estimates using data on call options from the EUREX. We find that most of implicit betas are in a plausible range, and the dispersion of betas within years appears to be reasonable. The estimation results highlight that beta values change over the years, which implies that the results from the conventional regressive analysis using historical data to obtain betas might be misleading if time-invariance of beta is assumed. This issue will be an interesting avenue for future research.

Appendix A: The Lognormal Distribution

The definition of density of normal distribution is

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \tag{36}$$

$\Phi(\cdot)$ is the standard normal distribution ($\mu = 0$ and $\sigma = 1$). A variate is lognormally distributed if its natural logarithm is normally distributed. The definition of bivariate normal distribution is

$$f(x,y) = \frac{1}{2\pi\sqrt{\sigma_x^2\sigma_y^2(1-\rho^2)}} \cdot e^{-\frac{1}{2(1-\rho^2)}\left(\frac{(x-\mu_x)^2}{\sigma_x^2} - 2\rho\frac{(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} + \frac{(y-\mu_y)^2}{\sigma_y^2}\right)}. \tag{37}$$

Two variates are bivariate lognormally distributed if their natural logarithms are bivariate normally distributed. In order to be able to calculate the moments of lognormally distributed variates (1), (2), (3) and (4), the simplifications of the following special integrals are required:

$$\int_a^\infty e^{cx} f(x)\,dx = e^{c\mu+\frac{1}{2}(c\sigma)^2} \cdot \Phi\left(\frac{-a+\mu+c\sigma^2}{\sigma}\right) \tag{38}$$

$$\int_{-\infty}^\infty \int_a^\infty e^{cx_2} f(x_1,x_2)\,dx_1\,dx_2 = e^{c\mu_2+\frac{1}{2}(c\sigma_2)^2} \cdot \Phi\left(\frac{-a+\mu_1+c\rho\sigma_1\sigma_2}{\sigma_1}\right) \tag{39}$$

$$\int_{-\infty}^\infty \int_a^\infty e^{x_1}e^{cx_2} f(x_1,x_2)\,dx_1\,dx_2 = e^{\mu_1+\frac{1}{2}\sigma_1^2+c\mu_2+\frac{1}{2}(c\sigma_2)^2+c\rho\sigma_1\sigma_2} \cdot \Phi\left(\frac{-a+\mu_1+\sigma_1^2+c\rho\sigma_1\sigma_2}{\sigma_1}\right) \tag{40}$$

In order to keep the proofs of (38), (39) and (40) concise in the following, it is convenient to use the conditional density. The definition of the conditional density is

$$f(x_2|x_1) = \frac{f(x_1,x_2)}{f(x_1)}.$$

If we apply this definition to the bivariate normal distribution, we get

$$f(x_2|x_1) = \frac{1}{\sqrt{2\pi\sigma_2^2(1-\rho^2)}} \cdot e^{-\frac{\left(x_2-\left(\mu_2+\rho\frac{\sigma_2}{\sigma_1}(x_1-\mu_1)\right)\right)^2}{2\sigma_2^2(1-\rho^2)}}. \tag{41}$$

Note that the conditional density of the bivariate normal distribution equals the density of the normal distribution with the parameters

$$\mu_{x_2|x_1} = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x_1-\mu_1) \text{ und} \tag{42}$$

$$\sigma_{x_2|x_1}^2 = \sigma_2^2(1-\rho^2) \tag{43}$$

We next prove equation (38).

$$\int_a^\infty e^{cx}\cdot\frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx = \int_a^\infty \frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{x^2-2x\mu-2c\sigma^2x+\mu^2}{2\sigma^2}}dx$$
$$= \int_a^\infty \frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{x^2-2x(\mu+c\sigma^2)+\mu^2}{2\sigma^2}}dx$$
$$= \int_a^\infty \frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{x^2-2x(\mu+c\sigma^2)+(\mu+c\sigma^2)^2-(\mu+c\sigma^2)^2+\mu^2}{2\sigma^2}}dx$$
$$= \int_a^\infty \frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{(x-(\mu+c\sigma^2))^2}{2\sigma^2}}\cdot e^{-\frac{-(\mu+c\sigma^2)^2+\mu^2}{2\sigma^2}}dx$$
$$= e^{c\mu+\frac{1}{2}(c\sigma)^2}\int_a^\infty \frac{1}{\sigma\sqrt{2\pi}}\cdot e^{-\frac{(x-(\mu+c\sigma^2))^2}{2\sigma^2}}dx$$

Equation (38) follows with $1-\Phi\left(\frac{a-(\mu+c\sigma^2)}{\sigma}\right) = \Phi\left(\frac{-a+\mu+c\sigma^2}{\sigma}\right)$. The proof for equation (39) is given under consideration of the conditional density indicated above,

$$\int_{-\infty}^\infty\int_a^\infty e^{cx_2}\,f(x_1,x_2)\,dx_1\,dx_2 = \int_a^\infty\left[\int_{-\infty}^\infty e^{cx_2}\,f(x_2|x_1)\,dx_2\right]f(x_1)\,dx_1\ .$$

The integral in brackets can be interpreted in that the expected value and the variance according to (42) and (43) are transformed and the equation (38) is subsequently used,

$$\int_a^\infty\left[\int_{-\infty}^\infty e^{cx_2}\,f(x_2|x_1)\,dx_2\right]f(x_1)\,dx_1 = \int_a^\infty\left[e^{c\left(\mu_2+\rho\frac{\sigma_2}{\sigma_1}(x_1-\mu_1)\right)+\frac{1}{2}c^2\left(\sigma_2^2(1-\rho^2)\right)}\right]f(x_1)\,dx_1$$
$$= e^{c\mu_2-c\rho\frac{\sigma_2}{\sigma_1}\mu_1+\frac{1}{2}c^2\sigma_2^2-\frac{1}{2}c^2\sigma_2^2\rho^2}\int_a^\infty e^{c\rho\frac{\sigma_2}{\sigma_1}x_1}f(x_1)\,dx_1\ .$$

If we define the helping variable $c^* := c\rho\frac{\sigma_2}{\sigma_1}$, we arrive at the equation (39) after application of (38) and shortening of the terms in exponents. The proof for equation (40) can be shown analogously,

$$\int_{-\infty}^\infty\int_a^\infty e^{cx_2}e^{x_1}\,f(x_1,x_2)\,dx_1\,dx_2 = \int_a^\infty\left[\int_{-\infty}^\infty e^{cx_2}\,f(x_2|x_1)\,dx_2\right]e^{x_1}f(x_1)\,dx_1$$
$$= e^{c\mu_2-c\rho\frac{\sigma_2}{\sigma_1}\mu_1+\frac{1}{2}c^2\sigma_2^2-\frac{1}{2}c^2\sigma_2^2\rho^2}\int_a^\infty e^{c\rho\frac{\sigma_2}{\sigma_1}x_1+x_1}f(x_1)\,dx_1\ .$$

If we define the helping variable $c^{**} := c\rho\frac{\sigma_2}{\sigma_1}+1$, we arrive at the desired result (40) after repeated application of equation (38) and shortening of the terms in exponents.

Appendix B: Option Pricing Using the CAPM

In order to calculate the expected value of a call (7), we use equation (38),

$$
\begin{aligned}
E\,[\tilde{X}_c] &= \int_{-\infty}^{\infty} \max\Big(p(\tilde{X}_s)\, e^{r_s} - K, 0\Big)\; f(r_s)\, dr_s \\
&= p(\tilde{X}_s) \int_{\ln\left(K/P(\tilde{X}_s)\right)}^{\infty} e^{r_s}\, f(r_s)\, dr_s \;-\; K \int_{\ln\left(K/P(\tilde{X}_s)\right)}^{\infty} f(r_s)\, dr_s \\
&= p(\tilde{X}_s) e^{\mu_s + \frac{1}{2}\sigma_s^2} \cdot \Phi\left(\frac{\ln\left(p(\tilde{X}_s)/K\right) + \mu_s + \sigma_s^2}{\sigma_s}\right) - K\Phi\left(\frac{\ln(p(\tilde{X}_s)/K) + \mu_s}{\sigma_s}\right). \qquad (44)
\end{aligned}
$$

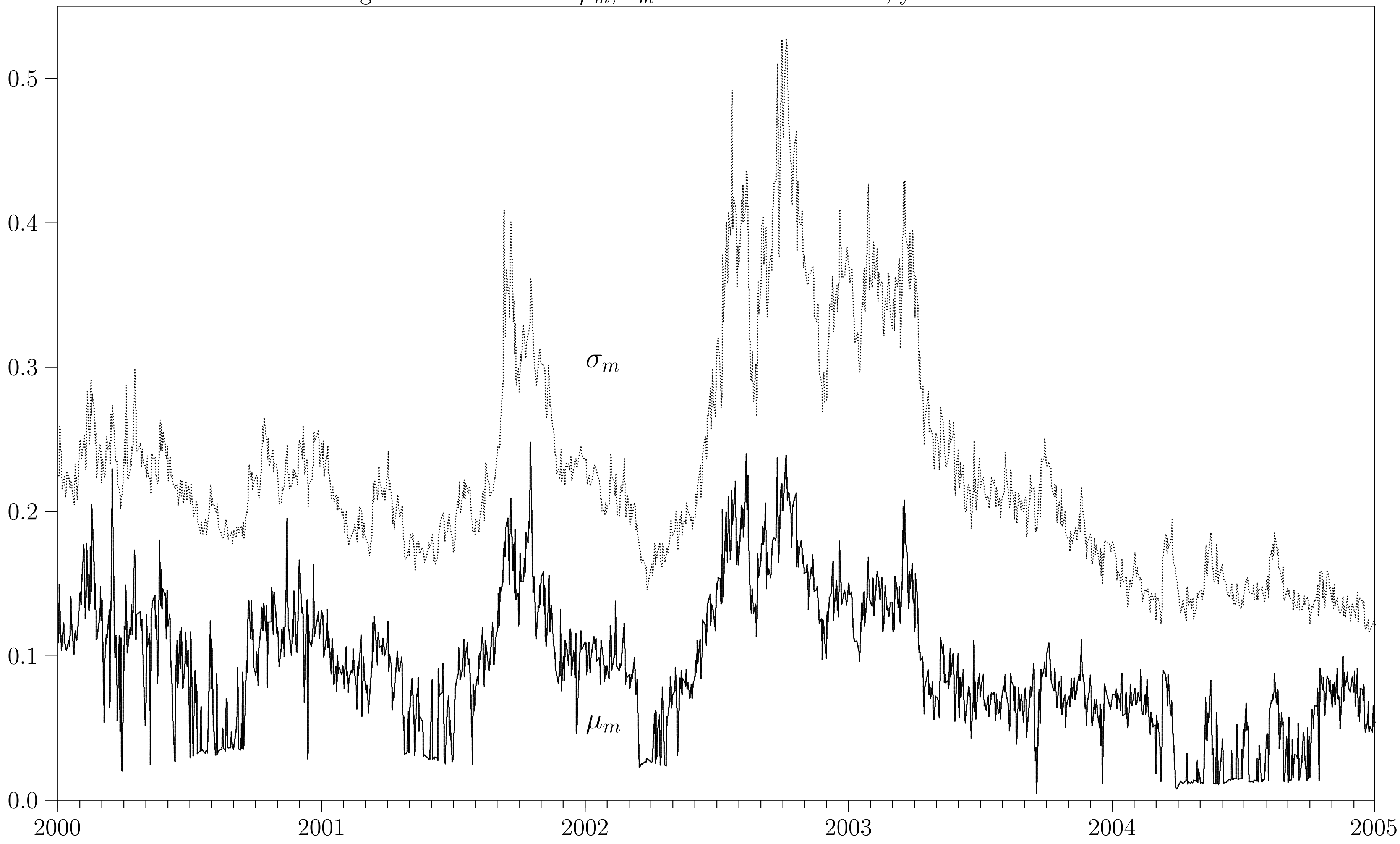
We can simplify the calculation of the covariance through application of the decomposition theorem. From equations (39) and (40) result

$$
\begin{aligned}
E\,[\tilde{X}_c \tilde{R}_m] &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \max(p(\tilde{X}_s)\, e^{r_s} - K, 0) \cdot\, e^{r_m}\, f(r_s, r_m)\, dr_m\, dr_s \\
&= \int_{-\infty}^{\infty}\int_{\ln\left(K/P(\tilde{X}_s)\right)}^{\infty} (p(\tilde{X}_s)\, e^{r_s} - K) \cdot\, e^{r_m}\, f(r_s, r_m)\, dr_m\, dr_s \\
&= p(\tilde{X}_s) \cdot \int_{-\infty}^{\infty}\int_{\ln\left(K/P(\tilde{X}_s)\right)}^{\infty} e^{r_s} e^{r_m}\, f(r_s, r_m)\, dr_m\, dr_s - K \cdot \int_{-\infty}^{\infty}\int_{\ln\left(K/P(\tilde{X}_s)\right)}^{\infty} e^{r_m}\, f(r_s, r_m)\, dr_m\, dr_s \\
&= p(\tilde{X}_s) \cdot\, e^{\mu_s + \frac{1}{2}\sigma_s^2 + \mu_m + \frac{1}{2}\sigma_m^2 + \rho\,\sigma_s\sigma_m} \cdot \Phi\left(\frac{-\ln\left(K/p(\tilde{X}_s)\right) + \mu_s + \sigma_s^2 + \rho\,\sigma_s\sigma_m}{\sigma_s}\right) \\
&\quad - K \cdot\, e^{\mu_m + \frac{1}{2}\sigma_m^2} \cdot \Phi\left(\frac{-\ln\left(K/p(\tilde{X}_s)\right) + \mu_s + \rho\,\sigma_s\sigma_m}{\sigma_s}\right)\;. \qquad (45)
\end{aligned}
$$

Following the decomposition theorem, we arrive at the covariance (8) with (44) und (45), after elementary conversions.

Appendix C: Tables and Figures

Figure 1: Estimates of μ_m, σ_m for the EuroStoXX50, years 2000-2004



(For data description and sources, see Section 5)

Table 1: Description of Call Options and the Names of Underlying Assets

Call Option On	SECU		2000	2001	2002	2003	2004
DJ EURO	OES	♯ transactions	8069	13214	15810	17976	14578
STOXX 50 INDEX		♯ contracts [million]	4.14	9.99	20.19	29.84	31.21
		volu in € [billion]	7.82	16.1	29.85	35.67	33.09
ALLIANZ AG	ALV	♯ transactions	4712	6766	10425	12129	10108
		♯ contracts [million]	0.5	0.99	3.32	13.97	16.04
		volu in € [billion]	0.12	0.13	0.25	0.68	0.73
DEUTSCHE	DBK	♯ transactions	7636	9518	10434	7963	7562
BANK AG		♯ contracts [million]	3.49	5.38	5.91	5.63	5.85
		volu in € [billion]	3.05	2.92	2.27	2.32	2.7
DAIMLER	DCX	♯ transactions	8603	9392	10567	7989	7125
CHRYSLER AG		♯ contracts [million]	3.35	5.75	5.64	4.2	4.57
		volu in € [billion]	0.92	1.6	1.61	0.79	1.22
DEUTSCHE	DTE	♯ transactions	11919	11118	10886	8310	7932
TELEKOM AG		♯ contracts [million]	6.27	11.89	9.46	8.54	8.03
		volu in € [billion]	3.19	2.33	0.87	0.78	0.61
E.ON AG	EOA	♯ transactions	1876	3622	4116	4387	4076
		♯ contracts [million]	0.29	1.17	0.64	1.2	1.68
		volu in € [billion]	0.09	0.17	0.13	0.2	0.65
MUENCHNER	MUV	♯ transactions	2342	2511	4898	7890	7711
RUECKVERS AG		♯ contracts [million]	0.23	0.3	0.96	7.24	10.28
		volu in € [billion]	0.04	0.03	0.09	0.43	0.29
NOKIA CORP.	NOA	♯ transactions	6061	11284	11557	8575	8502
		♯ contracts [million]	2.27	8.37	11.41	7.28	9.57
		volu in € [billion]	1.09	2.19	1.84	0.69	0.64
SAP AG	SAP	♯ transactions	—	4894	11387	10107	7924
		♯ contracts [million]	—	0.61	3.81	9.42	7.85
		volu in € [billion]	—	0.65	2.49	7.59	5.25
SIEMENS AG	SIE	♯ transactions	10657	13713	13013	9798	7639
		♯ contracts [million]	2.3	4.44	4.63	4.3	3.86
		volu in € [billion]	3.13	2.39	1.83	1.34	1.87

Table 2: Estimation Results for Implicit Betas

SECU		2000	2001	2002	2003	2004
ALV	mean Beta	0.464	0.585	0.952	1.480	1.007
	95% C.I.	[0.38,0.54]	[0.50,0.67]	[0.87,1.03]	[1.40,1.56]	[0.91,1.10]
	♯ days	211	214	229	207	202
DBK	mean Beta	0.811	0.966	1.047	1.002	0.916
	95% C.I.	[0.72,0.90]	[0.87,1.06]	[1.00,1.10]	[0.93,1.08]	[0.83,1.01]
	♯ days	226	227	236	223	199
DCX	mean Beta	1.163	1.051	1.297	1.183	1.254
	95% C.I.	[1.10,1.23]	[0.96,1.14]	[1.26,1.33]	[1.13,1.23]	[1.20,1.31]
	♯ days	206	180	214	212	152
DTE	mean Beta	2.120	1.767	1.468	0.561	1.124
	95% C.I.	[2.04,2.20]	[1.63,1.90]	[1.39,1.55]	[0.49,0.64]	[1.05,1.20]
	♯ days	215	187	218	251	174
EOA	mean Beta	0.875	0.594	0.712	0.725	0.685
	95% C.I.	[0.78,0.97]	[0.49,0.69]	[0.66,0.76]	[0.66,0.79]	[0.59,0.78]
	♯ days	107	154	194	181	122
MUV	mean Beta	0.945	0.511	1.002	1.476	0.890
	95% C.I.	[0.79,1.10]	[0.40,0.62]	[0.91,1.09]	[1.39,1.56]	[0.79,0.99]
	♯ days	129	147	210	200	202
NOA	mean Beta	1.924	2.373	1.723	1.014	1.641
	95% C.I.	[1.79,2.06]	[2.21,2.54]	[1.62,1.83]	[0.93,1.10]	[1.58,1.70]
	♯ days	201	197	219	237	160
SAP	mean Beta	—	1.311	1.106	0.787	0.489
	95% C.I.	—	[1.13,1.49]	[0.99,1.22]	[0.69,0.88]	[0.40,0.58]
	♯ days	—	122	226	239	210
SIE	mean Beta	1.334	1.439	1.233	0.938	0.915
	95% C.I.	[1.23,1.44]	[1.34,1.54]	[1.17,1.30]	[0.87,1.01]	[0.84,0.99]
	♯ days	229	211	237	238	196

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