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New Networks, Competition and Regulation

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Abstract

We consider a model with two firms operating their individual networks. Each firm can choose its price as well as its investment to build up its network. Assuming a skewed distribution of consumers, our model leads to an asymmetric market structure with one firm choosing higher investments. While access regulation imposed on the dominant firm leads to lower prices, positive welfare effects are diminished by strategic investment decisions of the firms. Within a dynamic game with indirect network effects leading to potentially increased demand, regulation can substantially lower aggregate social welfare. Conditional access holidays can alleviate regulatory failure.

JEL-Classification: L51, L13, D43

Keywords: Regulation, network effects, natural monopoly

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1 Introduction

The development of broadband networks such as VDSL and 3thrd generation mobile networks promises a range of new services like triple play (telephone, TV and internet services being offered over a single network), video telephony and other real time services requiring high bandwidth and fast data transmission. However, to realize these services existing bandwidth has to be increased via fiber to home, the buildup of new mobile infrastructures or investments in cable networks to allow for data streams in both directions. While these investments are substantial their economic value is uncertain. Higher bandwidth and faster data transmission is not valuable per se. Customers' willingness to pay for connections to new networks depends on the services offered and thus on complementary innovations in downstream markets. This interdependence between innovations in vertically related markets points to potentially higher risks associated with investments in new communication infrastructures. Additionally, the sunk nature of infrastructure investments and network effects can lead to market failures as new monopolies may emerge.

There is no widely held opinion on how regulation should react to the emergence of new broadband infrastructures. The FCC turned a 180° after a heated debate so much so regulation is now on the retreat in the USA. The European regulation authorities interpret the deployment of new infrastructures as a particularly complicated scenario which calls for equally sophisticated regulation in order to guarantee that markets do not return to the stable monopolies of former times.

The political debate is accompanied by a scientific discussion in which both sides find strong support. On the one hand, there are authors who emphasize the importance of competition generated by unbundling and access regulation. Cave [4] and Cave and Vogelsang [5] propose an investment ladder and interpret telecommunication networks as a collection of potentially unbundled services which should be offered separately. Entrants should be allowed to buy regulated access at any point in the network. Some of these firms may climb higher on the ladder by investing in own infrastructures while others may be satisfied with pure resale of unbundled services.

This idea of establishing different types of competition, on the other hand, is challenged by Bourreau and Dogan [2], [3]. They point out that there is a conflict between facility based and service based competition because access regulation diminishes the value of the facilities owned by the incumbent as well as those owned by his competitors. Hazlet [10] demonstrates that access regulation will typically hurt both, incumbents and potential entrants. A simulation study by Zarakas et al. [15] indicates that investment would have been higher and more into facility and less into service based competition under higher access rates.

Additional concerns are raised by authors who emphasize the importance of investment incentives. They conclude that regulation authorities can or do not take sufficient care of these incentives. Gans [7], for instance, points out that access regulation may lead to an unnecessary delay of investment. Gans and King [8] and Pindyck [14] show that ex ante uncertainty has to be taken into account when ex post regulation imposes restrictions on prices. They suggest either a markup on prices motivated by the option pricing principle or regulation holidays during which the regulation authority is committed not to interfere with the market.

In this paper we focus on regulation in dynamic markets which exhibit network effects. We consider a simple two period model with two firms operating their own networks, sunk investments in infrastructure and uncertainty about future demand. In each period firms first decide on their investment, then they compete amongst each other to connect consumers to their networks. Uncertainty about future demand is incorporated by assuming indirect network effects such that the probability of higher demand in the second period depends positively on the number of connections in teh 1st period. Furthermore, assuming a skewed distribution of consumers our model leads to an asymmetric market structure with one firm operating the greater network. With respect to regulation we restrict the analysis to access regulation imposed on the dominant firm, i.e., on the firm operating the greater network, but we consider different regulatory regimes.

Our starting point is the observation that regulation faces a trade-off between flexibility and commitment: the government can either impose short run case by case decisions which react flexibly to the actual state of nature or it can fix its regulation decision in the long run. While the latter suffers from an obvious inflexibility the case by case approach may reduce the ex ante commitment power of the regulator considerably.

We assume that the regulation authority must refrain from conditioning its policy on future events. We distinguish the following regimes: Long run regulation fixes the access price for both periods in advance and does not allow the regulator to react on the realized demand or the firms' investment decisions. Medium run regulation maximizes social welfare in each period without conditioning on the firms' investment decisions. Short run regulation is based on both the realized demand and the investment. Since the lack of commitment in medium and short run regulation will be shown to handicap the regulator seriously, we will also introduce several commitment rules. More specifically, we explore the performance of regulation constrained by a zero-profit condition, access holidays conditioned on market development and a protected monopoly.

While comparing these different regulatory regimes, several effects have to be taken into account. Though regulated prices increase static efficiency they also alter the investment decisions of the firms. With respect to the smaller firm, two effects have to be taken into account. First, lower prices generally lead to lower investment. Second, regulation alters the pricing game such that the smaller firm's investment incentives are increased ceteris paribus. The welfare effect of this second effect can be negative as an unregulated market may attract too much entry. Considering the dominant firm regulation lowers investment unambiguously. Furthermore, lack of regulatory commitment induces the firms to reduce investments further. With indirect network effects the probability of higher demand in the second period is decreased which lowers expected welfare. Our example shows that there is not much to gain but much to lose from regulation. With rather high investment costs firms refrain from investing under medium regulation. Long run regulation leads to lower expected welfare when compared to an unregulated market. Regulation constrained by a zero-profit condition and access holidays can improve welfare but can not avoid a breakdown of investment incentives completely.

The paper is organized as follows. In section 2 we present the static and the dynamic allocation problem. In section 3 we discuss the interplay between profit maximizing firms and a welfare maximizing regulator in a static set-up. In section 4 we introduce uncertainty about future demand and the network effect and demonstrate that the regulator has severe problems in dealing with the implied consequences. Section 5 considers medium run regulation amended by a zero-profit restriction, access holidays as well as a protected but otherwise not regulated monopoly. In section 6 we will shortly summarize our main findings.

2 The Model

2.1 The Allocation Problem

We consider a model with two firms M and N each operating a network of size m and n, respectively. Consumers are distributed in the interval $[0, 1]$ according to a distribution function $F(\mu)$ with

$$F'(\mu) = f(\mu) \geq 0, f'(\mu) < 0 \text{ for all } \mu < 1 \text{ and } f(1) = 0. \tag{1}$$

The mass of consumers is normalized to one. Consumers located at μ can connect to network $j = m, n$ if $\mu \leq j$. We assume that consumers have a quasi-linear utility function with constant marginal utility of income normalized to 1. Thus, demand for connections to either network m or n depends on the network prices p and q respectively, and on a shift parameter Δ. Let $X(p,q,\Delta)$ and $Y(p,q,\Delta)$ denote demand for connections to m and n. We assume that X and Y have the following properties[1]

$$\begin{aligned} X_p &< 0 \text{ and } X_{pp} \leq 0 \ \forall\ p \text{ with } X > 0, && (2)\\ Y_q &< 0 \text{ and } Y_{qq} \leq 0 \ \forall\ q \text{ with } Y > 0, && (3)\\ 0 &< X_q, Y_p < |X_p|\,;\ 0 < Y_p, X_q < |Y_q|\,;\ 0 \leq X_{pq}, Y_{qp} \ \forall\ p,q \text{ with } X,Y > 0 && (4) \end{aligned}$$

Firms can decide on their prices p and q and the size of their networks m and n. Throughout the paper we assume that investments in m and n are irreversible, that is, firms can not decrease the size of their networks, and that investment decisions are made prior to their pricing decisions Firms bear constant marginal costs $c > 0$ for connecting consumers. Fixed costs are linearly increasing with the size of the firms' networks. Assuming $m \geq n$ and defining $\tilde{X}(p,\Delta) := X(p,\infty,\Delta)$, profits Π^m and Π^n are given by

$$\begin{aligned} \Pi^m(p,q,\Delta,m,n) &= (F(m) - F(n))\,(p-c)\,\tilde{X} + F(n)(p-c)X - rm && (5)\\ \Pi^n(p,q,\Delta,n,z) &= F(n)(q-c)Y - rn && (6) \end{aligned}$$

where $r > 0$ measures the marginal (annualized) network costs. To simplify the analysis we impose the following restriction for the relation between costs and demand

$$(p-c)X(p,0,\Delta) < \frac{r}{f(0)} \text{ and } (q-c)Y(0,q,\Delta) < \frac{r}{f(0)} \ \forall\ p,q \geq 0. \quad (7)$$

Denoting consumers' indirect utility functions by[2]

$$\begin{aligned} \tilde{V}(p,\Delta) &= \int_p^{\infty} \tilde{X}\,d\hat{p}\\ V(p,q,\Delta) &= \int_{(p,q)}^{(\infty,\infty)} (X\,d\hat{p} + Y\,d\hat{q}) \end{aligned}$$

and defining $G(m,n) := F(m) - F(n)$ and $H(n) = F(n)$ total welfare is given by

$$W = G\tilde{V} + HV + \Pi^m + \Pi^n.$$

Within this framework we analyze the impact of access obligations imposed on the network operator M who is assumed to install the greater network. We assume that access

[1] Subscripts denote partial derivatives. To simplify the notation we will omit the arguments of the functions where this does not lead to any confusion. In our numerical example we will use a linear specification with zero cross derivatives.

[2] We can use the line integral for defining $V(\cdot)$ since demand does not depend on income. Hence the order of integration is not relevant.

leads to perfect competition for the consumers connecting to M. Given $m \geq n$ and that M has to provide access for an access charge a, the price p can not exceed $a + c$. Therefore, access regulation is equivalent to direct price regulation of the dominant network provider. To simplify the analysis further, we also assume that the regulator has complete information.

2.2 Dynamics

In order to generate a dynamic investment problem we assume that this market is repeated twice with the shift parameter Δ_1 in the first period and an uncertain shift parameter in the second period. We assume that demand is potentially increasing and that the probability of an increase depends on the total quantity of connections in the first period. Using subscripts to denote network sizes and number of connections in period 1 and 2, we assume that the second period shift parameter is

$$\widetilde{\Delta} = \begin{cases} \Delta_2 & \text{with probability} \quad \sigma(Q(\cdot)) \\ \Delta_1 & \text{with probability} \quad 1 - \sigma(Q(\cdot)) \end{cases} \tag{8}$$

$$\text{with} \quad : \quad Q(\cdot) := G\widetilde{X}_1 + H(X_1 + Y_1) \tag{9}$$

with $\Delta_2 > \Delta_1$ and $\sigma' > 0 > \sigma''$. Note that σ depends on the aggregate amount of connections in period 1, i.e., we model industry specific network effects rather than firm specific network effects. Moreover, with this specification we capture indirect network effects which could result from the observation that the incentives of service providers to develop new applications are positively correlated with the size of their markets, i.e., the number of consumers connected to the networks.

2.3 Regulation

Second Best Regulation As we will not allow side payments to the firms throughout the following the second best optimal solution of this allocation problem is a natural reference point. With second best regulation, the regulator is able to choose ex ante pricing rules which depend on the entire observable history h up to the moment when the prices have to be selected. In particular, the regulator can select the following almost perfect incentive mechanism

$$P^*(h) = \begin{cases} p^* & \text{if } h = h^* \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

which enforces the history h^* of his choice with the threat of a zero price punishment for deviators.

Third Best Regulation Constraints It is the central goal to introduce the trade-offs between flexibility and commitment of third best regulation into this model. For this purpose we restrict the regulation policy in a simple manner. We assume that the regulator fixes prices and that he can only condition his pricing policy on observable past history. In order to gain full flexibility he has to wait until the shift of demand is realized and until the investment decisions are implemented. To have full commitment the regulator has to fix his price in advance without being able to even react to a demand shift. This way we distinguish three third best regulation regimes: The long run regulator (LR) fixes a price in stage 0.5 before the market starts, the medium run regulator (MR) selects a pricing policy p^m after the demand shift is observable, and the short run regulator (SR) picks a fully flexible pricing policy p^s conditional also on the firms' investment decisions. The resulting decision structure is shown in table 2 with the understanding that only one type of regulator is active.

	Agent	Period 1		Period 2
0.5	LR	p	$=$	p
1	Nat	level Δ_1		level Δ_1 or Δ_2
1.5	MR	p_1^m		p_2^m
2	M/N	(m_1, n_1)		(m_2, n_2)
2.5	SR	p_1^s		p_2^s
3	N	q_1		q_2

Timing of Decisions

We will demonstrate each of these regimes to be flawed. In the dynamic game we will therefore also discuss, to what extent it is possible to overcome regulatory failure by partial commitment devices. We will introduce zero profit constraints for the MR as well as access holidays h under which the regulator does not become active unless he observes either Δ_1 or a persistent monopoly in period 2. We also compare the outcomes with a protected and unregulated monopoly pm where the regulator does not allow competition.

To disentangle the quite complex consequences of these regimes we will first analyze a simple one period model. We then turn to the more complicated two period model.

3 One Period Regulation

In the following we will first characterize the equilibrium conditions for second best regulation and for an unregulated market outcome. We then derive the optimal regulatory decisions under MR and SR. To illustrate the results and to analyze the impact of regulation on welfare in more detail we use a numerical example.

3.1 Second Best Solution

Consider first the benchmark solution in which we determine the potential welfare gain by assuming that the pricing rule p^* is only restricted by the firms' participation constraint condition. The firms decide on their investments and their prices such that p satisfies $p \leq p^*$. Focusing on the mechanism specified in (10), assume that both firms adhere to m^* and n^*, respectively. Then, the regulated equilibrium prices are given p^* and $q^* = q^r(p^*)$ implicitly defined by

$$\Pi_q^n = H\left[(q-c)Y_q + Y\right] = 0 \tag{11}$$

for all $n^* > 0$. Additionally, we have $q^{*\prime} > 0$ which is due to $Y_p, Y_{qp} \geq 0$.
Considering the firms' investment decisions, (10) implies that deviation from m^* or n^* can not be optimal as long as

$$\Pi^m(p^*, q^*, \Delta, m^*, n^*) \quad \geq \quad 0 \text{ and} \tag{12}$$

$$\Pi^n(p^*, q^*, \Delta, m^*, n^*) \quad \geq \quad \Pi^n(p^*, q^*(0,n), \Delta, m^*, n) \;\forall\; n \neq n^*, n < m^*. \tag{13}$$

While (12) establishes the usual zero profit constraint for the dominant firm, (13) together with (7) implies an equivalent positive profit constraint for the smaller firm. Therefore, the optimal choice of p^*, m^* and n^* can be derived from maximizing

$$L^*(\cdot) = G\tilde{V} + HV + (1-\lambda)\Pi^m + (1-\nu)\Pi^n \tag{14}$$

with respect to p^*, m^*, n^* (λ and ν with $\lambda, \nu \leq 0$ denote the multipliers for (12) and (13)). Using (11) the first order conditions for L^* are given by[3]

$$L_p^* \quad = \quad G(-\tilde{X}) + H(-X - Yq^{*\prime}) + (1-\lambda)\left[\Pi_p^m + \Pi_q^m q^{*\prime}\right] + (1-\nu)\Pi_p^n \tag{15}$$

$$L_m^* \quad = \quad f(m^*)\tilde{V} + (1-\lambda)\Pi_m^m = 0 \tag{16}$$

$$\Rightarrow \quad \Pi_m^m = -\frac{1}{1-\lambda} f(m^*)\tilde{V} < 0 \tag{17}$$

$$L_n^* \quad = \quad -f(n)(\tilde{V} - V) + (1-\lambda)\Pi_n^m + (1-\nu)\Pi_n^n \leq 0; L_n^* \; n = 0 \tag{18}$$

$$\Rightarrow \quad \Pi_n^n = \frac{1}{(1-\nu)}\left[f(n^*)(\tilde{V} - V) - (1-\lambda)\Pi_n^m\right] \text{ for } n > 0 \tag{19}$$

$$\text{with} \quad : \quad f(n^*)(\tilde{V} - V) < 0 \text{ and } \Pi_n^m = -f(n^*)(p^* - c)(\tilde{X} - X) < 0$$

[3] In order to simplify the notation we ommit the indices of all variables indicating partial derivatives.

Considering λ and ν first, $\lambda = 0$ can be excluded by inspection of (16) and $f(1) = 0$. On the other hand, $\Pi^n(q^*, p^*, m^*, n^*) \geq 0$ may not be binding. Inspection of (19) shows that regulation leads to $\Pi^n_n > 0$ and thus to $\Pi^n \geq 0$ as long as $\tilde{V} - V + (1-\lambda)(p^* - c)(\tilde{X} - X) > 0$ holds. With the two networks being rather close substitutes from the consumers' perspective, $\left|\tilde{V} - V\right|$ is small while $(p^* - c)(\tilde{X} - X)$ is strictly bounded away from zero. Therefore, high substitutability implies $\Pi^n > 0$ and (15) reduces to

$$\Pi^m_p + \Pi^m_q q^{*\prime} = \frac{1}{1-\lambda}\left[G\tilde{X} + H(X + Yq^{*\prime}) - \Pi^n_p\right]. \tag{20}$$

which mirrors the generalized Ramsey price.

3.2 Market outcome without Regulation

Turning to the case without regulation we assume $m > n$ w.l.o.g. The firms' first order conditions for their prices are given by

$$\Pi^m_p = G\left[(p-c)\tilde{X}_p + \tilde{X}\right] + H\left[(p-c)X_p + X\right] = 0 \tag{21}$$

$$\Pi^n_q = H\left[(q-c)Y_q + Y\right] = 0 \tag{22}$$

Note that $X_{pq}, Y_{qp} > 0$ and $X_q, Y_p > 0$ imply that prices are strategic complements and that $\tilde{X}_{pq} \geq 0$ leads to

$$(p^c - c)\tilde{X}_p + \tilde{X} > 0 > (p^c - c)X_p + X. \tag{23}$$

(23) indicates that choosing its price the dominant provider compromises between monopoly profits in the region $G(m,n)$ and competition in region $H(n)$. Letting $p^c(m,n)$ and $q^c(n,m)$ denote the solutions of (21) and (22), simple comparative statics reveals

$$sign\ p^c_m = -sign\ \Pi^m_{pm}\Pi^n_{qq} > 0 \text{ since } \Pi^m_{pm} = f(m)\left[(p^c - c)\tilde{X}_p + \tilde{X}\right] > 0 \tag{24}$$

$$sign\ q^c_m = sign\ \Pi^m_{pm}\Pi^n_{qp} > 0 \text{ since } \Pi^n_{qp} = H\left[(q^c - c)Y_{qp} + Y_p\right] > 0. \tag{25}$$

and

$$sign\ p^c_n = -sign\ \Pi^m_{pn}\Pi^k_{qq} < 0 \text{ since } \Pi^m_{pn} = f(n)\left[(p^c - c)X_p + X\right]\left[\frac{H}{G} + 1\right] < 0 \tag{26}$$

$$sign\ q^c_n = sign\ \Pi^m_{pn}\Pi^k_{qp} < 0 \text{ since } \Pi^n_{qp} = H\left[(q^c - c)Y_{qp} + Y_p\right] < 0 \tag{27}$$

Turning to the firms' investment decisions, using (25) and (26) and employing the envelope theorem we get

$$\Pi^m_m = f(m)(p^c - c)\tilde{X} + H(p^c - c)X_q q^c_m - r = 0 \quad (28)$$

$$\Rightarrow f(m)(p^c - c)\tilde{X} - r = -H(p^c - c)X_q q^c_m < 0 \quad (29)$$

$$\Pi^n_n = f(n)(q^c - c)Y + H(q^c - c)Y_p p^c_n - r \leq 0;\ \Pi^n_n\ n = 0 \quad (30)$$

$$\Rightarrow f(n)(p^c - c)Y - r = -H(q^c - c)Y_p p^c_n > 0 \text{ for } n > 0 \quad (31)$$

Inspection of (29) and (31) shows that the investment decisions of both M and N are influenced by the effects which m and n have on price competition. While M has an incentive to increase its network in order to soften price competition, N has a strategic incentive to decrease n. Note that these strategic effects are in line with second best regulation as long as the networks are rather close substitutes (see (17) and (19), respectively).

3.3 Medium Run Regulation

In contrast to second best regulation medium run regulation restricts the regulator to choose an ex ante price and not a pricing rule. Therefore, medium run regulation does not only limit the strategy space of the regulator, it also leads to different strategic effects with respect to the firms' investment decisions. More precisely, the investment incentives of N are ceteris paribus higher as compared to second best regulation and to competition.

Using the timing specified in section 1.3 we start with the firms' pricing decisions. Assuming that regulation is binding, the optimal price q^m of firm N is again given by $q^r(p^m)$ (see (11)). Employing the envelope theorem the firms' investment decisions are characterized by

$$\Pi^m_m = f(m)(p^m - c)\tilde{X} - r = 0 \quad (32)$$

$$\Pi^n_n = f(n)(q^m - c)Y - r \leq 0,\ \Pi^n_n n = 0 \quad (33)$$

Inspection of (32) and (33) shows that the firms' network sizes only depend on p^m and that there are no strategic effects the firms take into account. Let $m^m(p^m)$ and $n^m(p^m)$ denote the solutions of (32) and (33), respectively, and note that simple comparative statics leads to

$$sign\ m^{m\prime} = sign\ f(m)\left[\tilde{X} + (p^m - c)\tilde{X}_p\right] > 0. \quad (34)$$

$$sign\ n^{m\prime} = sign\ f(n)\left[(q^m - c)Y_p\right] > 0. \quad (35)$$

Turning to the optimal price p^m and taking into account that the firms are free to choose their investment levels, the regulator's maximization problem can be written as

$$\max_{p^m} W^m = G\tilde{V} + HV + \Pi^m + \Pi^n \tag{36}$$

Employing the envelope theorem leads to

$$\begin{aligned} W_p^m &= G(-\tilde{X}) + H(-X - Yq^{m\prime}) + \Pi_p^m + \Pi_q^m q^{m\prime} + \Pi_p^n \\ &\quad + m^{m\prime} f(m)\tilde{V} - n^{m\prime} f(n^m)\left[(\tilde{V} - V) + (p^m - c)(\tilde{X} - X)\right] \end{aligned} \tag{37}$$

Comparing (37) with the respective first order condition under second best regulation, i.e., (15), shows that the pricing problem is complicated by the impact which p^m has on the firms' investment decisions. While $m^{m\prime} f(m)\tilde{V}$ and $n^{m\prime} f(n^m)\tilde{V}$ are strictly positive (see (34) and (35)), the sign of the last term on the RHS of (37) depends on the degree of substitution between the firms' networks. With networks being close substitutes we again have $(\tilde{V} - V) + (p^m - c)(\tilde{X} - X) > 0$ and a decrease in n has a positive effect on welfare. Note further, that this allows for solutions p^m such[4]

$$f(0)(q^m - c)Y - r = 0 \text{ and } m^{m\prime} f(m)\tilde{V} > -F(m^m)(p^m - c)\tilde{X}_p. \tag{38}$$

This kind of solution underlines two properties of medium run regulation. First, regulation can serve as a commitment device to keep prices low and to deter entry which may also increase the profits of the regulated firm. Second, amending medium run regulation with the option to forbid entry can increase welfare. Forbidding market entry and choosing p^m such that

$$m^{m\prime} f(m)\tilde{V} = -F(m^m)(p^m - c)\tilde{X}_p$$

would clearly raise social welfare.

3.4 Short Run Regulation

The third regulatory regime we consider is the case in which the regulator chooses the access price p^s after the firms have made their investments but before the smaller network selects its price q^s.

Since we again have $q^s = q^r(p^s)$, the regulator chooses p^s according to

$$W_p^s = G(-\tilde{X}) + H(-X - Yq^{s\prime}) + \Pi_p^m + \Pi_q^m q^{s\prime} + \Pi_p^n = 0 \tag{39}$$

[4] Simple calculations show that $\frac{d}{dp^p}\left[f(n^p(p^p))(q^p - c)Y - r\right] < 0$ as $n^p(p^p)$ tends to 0.

Substituting the respective derivatives of the profit functions and rearranging (39) leads to

$$(p^s - c)\left[G\tilde{X}_p + H(X_p + q^{s\prime}X_q)\right] = -(q^s - c)H\left[Y_p + q^{s\prime}Y_q\right]. \tag{40}$$

Using $X_q < |X_p|$, $q^{s\prime} = -\left[(q^s - c)Y_{qp} + Y_p\right] / \left[(q^s - c)Y_{qq} + 2Y_q\right] < 1$ and $q^r(p^s)$ reveals

$$sign\ (p^s - c) = sign\ \left[Y_p + q^{s\prime}Y_q\right]. \tag{41}$$

While it is difficult to determine the sign of the RHS in (41) under general demand functions, linear demand functions with $Y_{qq} = Y_{qp} = 0$ lead to

$$sign\ \left[Y_p + q^{s\prime}Y_q\right] > 0 \tag{42}$$

and therefore to ex post regulated prices above marginal costs. Since $(p^s - c) > 0$ is a necessary condition for any investment to take place, we continue by assuming that (42) holds. Furthermore, letting $p^s(m, n)$ denote the solution of (40) simple comparative statics with respect to m and n shows

$$sign\ p^s_m = sign\left[f(m)(p^s - c)\widetilde{X}_p\right] < 0 \tag{43}$$

$$sign\ p^s_n = sign[f(n)(q^s - c)(Y_p + q^{s\prime}Y_q) + (p^s - c)(-\widetilde{X}_p + X_p + q^{s\prime}X_q)] > 0 \tag{44}$$

Finally, the first order conditions for the firms' investment decisions are given by

$$\Pi^m_m = \left[\begin{array}{c} f(m)(p^s - c)\tilde{X} + p^s_m G\left[(p^s - c)\tilde{X}_p + \tilde{X}\right] \\ +p^s_m H\left[(p^s - c)(X_p + X_q q^{s\prime}) + X\right] \end{array}\right] - r = 0 \tag{45}$$

$$\Pi^n_n = f(n)(q^s - c)Y + p^s_n H(q^s - c)Y_p - r \leq 0;\ \Pi^n_n\ n = 0 \tag{46}$$

Hence, while firm M's investment is determined by p^s the strategic effect with respect to anticipated regulation tends to decrease m further (see (43)). On the other hand, (44) reveals that firm N can increase p^s by increasing its own network. Comparing these observations with the results under second best regulation and under an unregulated market outcome shows that short run regulation turns the strategic behavior of the firms upside down.

Finally, note that once networks are given (40) does not ensure positive profits. In fact, the two period model shows that negative second period profits due to (40) can lead to a complete breakdown of first period investments.

3.5 Example

In order to illustrate the above results and to compare the welfare implications of the different regimes we now analyze a numerical example. The distribution function is given by

$$F(\mu) = \mu(2-\mu)$$

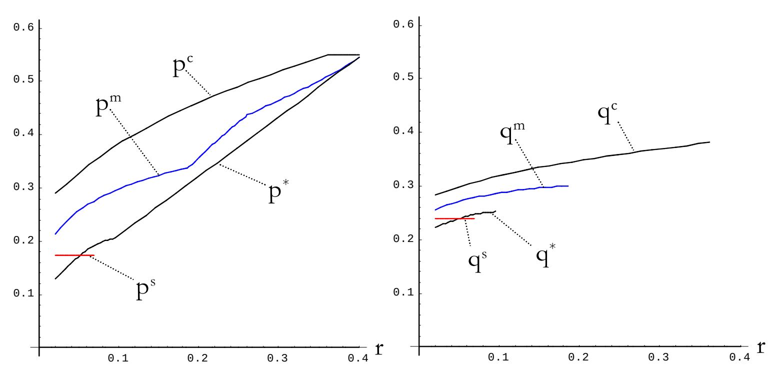
The demand functions $\widetilde{X}$, X and Y are[5]

$$\begin{aligned}
\widetilde{X}(p,\Delta) &= \Delta - p \\
X(p,q,\Delta) &= \frac{\Delta}{1+\alpha} - \frac{p}{(1-\alpha)(1+\alpha)} + \frac{\alpha q}{(1-\alpha)(1+\alpha)} \\
Y(q,p,\Delta) &= \frac{\Delta}{1+\alpha} - \frac{q}{(1-\alpha)(1+\alpha)} + \frac{\alpha p}{(1-\alpha)(1+\alpha)} \\
\text{with} &: \alpha = 0.75 \text{ and } \Delta = 1, 2
\end{aligned}$$

For the firms costs we assume $c = 0$ and $r \in [0.02, 0.55]^6$.

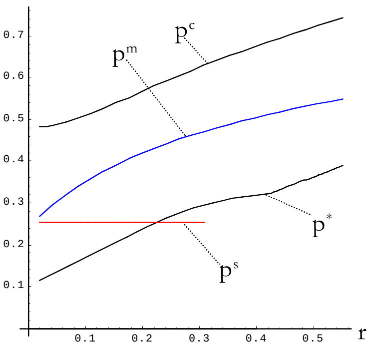
The following graphs compare the firms' prices, network sizes, profits, and the aggregate welfare for the second best solution (*), the market outcome without regulation (^{c}), and for short (^{s}) and medium (^{m}) run regulation.

Prices without regulation are obviously too high. No matter which regime we compare with the unregulated outcome, prices are always significantly lower with regulation.

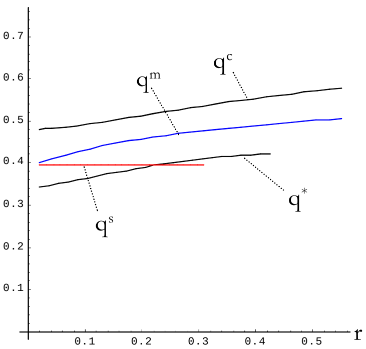


Prices p with $\Delta = 1$ Prices q with $\Delta = 1$

[5] These functions can be derived from a simple Dixit utiliy function.
[6] The upper limit for r is due to the analysis of the dynamic model in the next section.

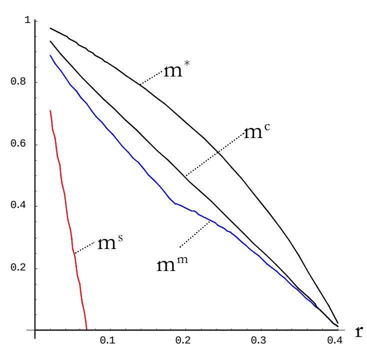


Prices p with $\Delta = 2$

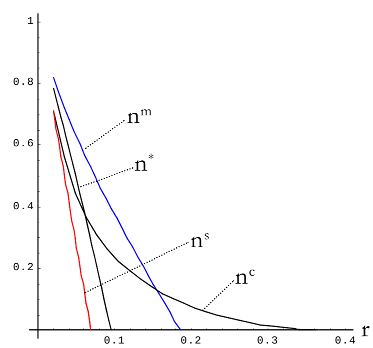


Prices q with $\Delta = 2$

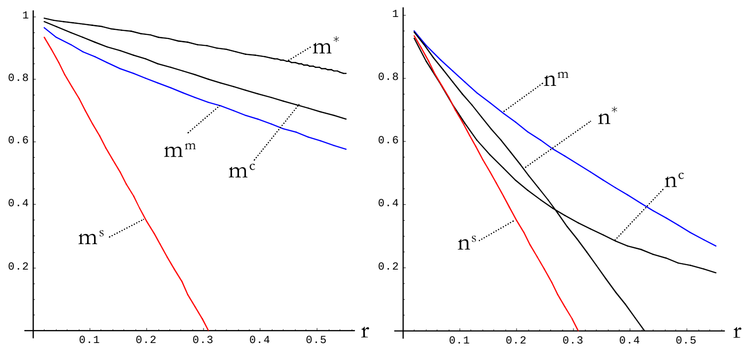
However, the network sizes are not uniformly smaller under regulation. With the second best solution, the reason for a larger network m is easily explained. Since network sizes are restricted by the participation constraints rather than by incentive compatibility, m can be chosen such that the marginal social value of network expansion equals its shadow costs. On the other hand, the second best solution does not call for a uniformly larger network n. The interdependence of the two profit functions implies that an increase of n must be accompanied by an increase of p. Therefore we have $n^c > n^*$ for medium (for $\Delta = 1$) and large (for $\Delta = 2$) values of r.



Network m with $\Delta = 1$



Network n with $\Delta = 1$

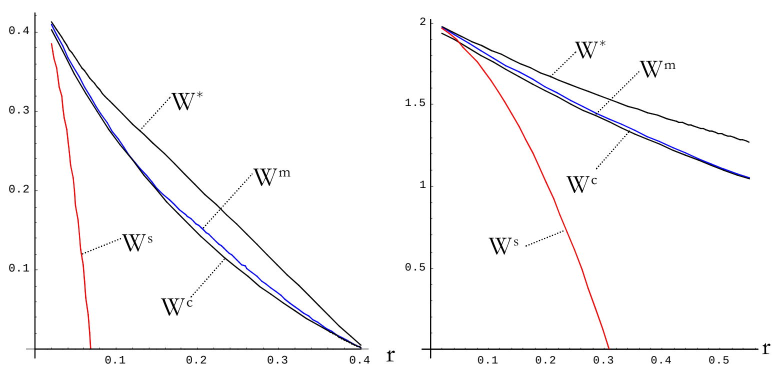


Network m with $\Delta = 2$ Network n with $\Delta = 2$

Under medium run regulation the lower prices generate the expected change of m. Given $p^m < p^c$, M reduces its investments by choosing a smaller network than it would under unregulated competition. On the other hand, increasing n does not affect p^m. Thus, the investment incentives of N are higher and we get $n^m > n^c$ for small r or high demand, i.e., $\Delta = 2$.

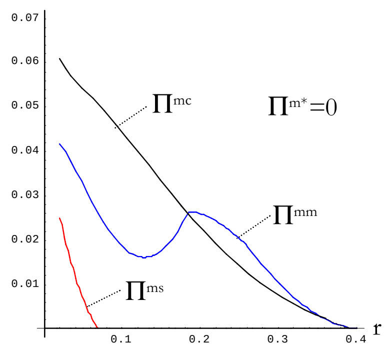
Under short run regulation the incentive problem is drastic. Prices are set after the network sizes have been chosen so that the regulator does not care for disincentive effects. While the regulator fixes p^s above short run marginal costs the low profit margin can not avoid a market breakdown as we have $m = n = 0$ for medium and large r.

Comparing welfare in the four allocations shows that while the second best solution clearly dominates neither of the two third best regulation regimes is able to realize a significant part of this potential surplus. While short run regulation fails miserably, the allocation under medium run regulation is only slightly better than the unregulated outcome.

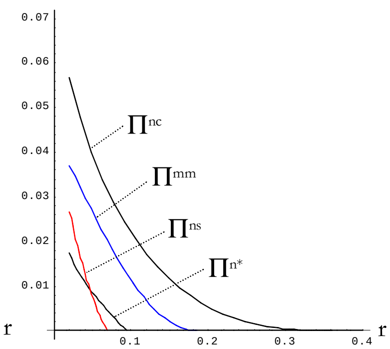


Welfare with $\Delta = 1$ Welfare with $\Delta = 2$

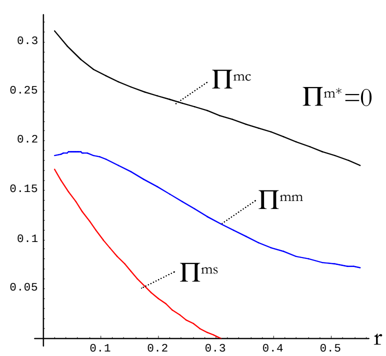
A short look at the firms' profits reveals that similar welfare does not mean similar allocation. Every regulation regime leads to a redistribution of rents from the firms to consumers (as long as there is no market breakdown). For $\Delta = 1$ and medium network costs, the redistribution under medium run regulation is at the expense of N while M profits from regulation. This case reflects the above mentioned scenario in which regulation aims to deter entry in order to increase the investment incentives of the regulated firm.



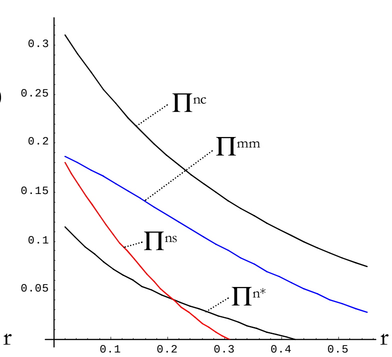
Profits Π^m with $\Delta = 1$



Profits Π^n with $\Delta = 1$



Profits Π^m with $\Delta = 2$



Profits Π^n with $\Delta = 2$

4 The Dynamic Model

We now turn to the analysis of the two period model in which demand in the second period can be either the same or higher as compared to the first period. Due to the assumed network effect (see (8)) the probability of high demand in period 2 increases with the number of connections supplied in period 1. Hence, both firms as well as the regulator have an incentive to increase their networks and to lower prices in period 1. Taking into account that investments are irreversible first period investments have to balance the positive effects from increasing the probability for higher demand and the potential second period losses if demand does not increase. With regulation this consideration is complicated by the fact that the regulator can base its second period decisions on existing networks. Our example shows that the implied ratchet effect under medium run regulation leads to significant welfare losses for medium and high investment costs.

4.1 Second Best Regulation

Starting with second best regulation note first that the regulator can condition its second period decisions on all past decisions. That is, the regulator can choose a second period pricing rule which depends on first and second period investments as well as on first period prices. This allows the regulator to control all investment decisions as well as first period prices.

Let $\phi := (m_1^*, n_1^*, q_1^*, p_1^*, m_{21}^*, n_{21}^*, m_{22}^*, n_{22}^*, p_{21}^*, p_{22}^*)$ denote the vector of the regulator's decision variables (in the following the first subscript refers to the period, the second to the realized demand in period 2, i.e., to either $\Delta_1 = 1$ or $\Delta_2 = 2$). Taking into account the firms' zero profit conditions and using $q_{2i}^* = q^r(p_{2i}^*)$ the regulator's maximization problem can be formulated as ($0 < \rho \leq 1$ denotes the discount factor):

$$\begin{aligned}
\max_{\phi} W &= G_1\tilde{V}_1 + H_1 V_1 + \Pi_1^m + \Pi_1^n && (47)\\
&\quad +\rho \left[\begin{array}{c} \sigma(G_{22}\tilde{V}_{22} + H_{22}V_{22} + \Pi_{22}^m + \Pi_{22}^n) \\ +(1-\sigma)(G_{21}\tilde{V}_{21} + H_{21}V_{21} + \Pi_{21}^m + \Pi_{21}^n) \end{array} \right] \\
\text{s.t.} \quad &: \; E\Pi^m := \Pi_1^m + \rho\left[\sigma\Pi_{22}^m + (1-\sigma)\Pi_{21}^m\right] \geq 0 && (48)\\
&: \; E\Pi^n := \Pi_1^n + \rho\left[\sigma\Pi_{22}^n + (1-\sigma)\Pi_{21}^n\right] \geq 0 && (49)\\
&: \; E\Pi^n \geq E\Pi^n\big|_{q_1,\widetilde{\phi}} \;\; \forall\; q_1 \neq q_1^* && (50)\\
&: \; m_{22}^*, m_{21}^* \geq m_1^*;\;\; m_{22}^*, n_{21}^* \geq n_1^* && (51)
\end{aligned}$$

where $\widetilde{\phi}$ refers to the optimal *ex ante* punishment if N deviates by choosing $(n_1, q_1) \neq (n_1^*, q_1^*)$ in the first period. In the following we assume that negative second period profits implied $\widetilde{p}_{2i} = 0$ with $i = 1, 2$ ensure that (50) holds.

Comparing (47) with the static case and focusing on second period decisions, $m_1^* > 0$ or $n_1^* > 0$ allow for corner solution such that no further investment takes place in the second period. With indirect network effects this will be the case whenever demand remains low in the second period.

Turning to the first period variables let $\lambda, \nu \leq 0$ and $\psi_i^m, \psi_i^n \leq 0$ with $i = 1, 2$ denote the Lagrange multipliers for (48), (49) and (51). Defining $W_{2i}^* := G_{2i}\tilde{V}_{2i} + H_{2i}V_{2i} + \Pi_{2i}^m + \Pi_{2i}^n$ with $i = 1, 2$ and $Q^* := G_1\tilde{X}_1 + H_1(X_1 + Y_1)$ evaluated at p_1^*,q_1^*, m_1^* and n_1^* we get

$$\begin{aligned}
\frac{\partial L}{\partial p_1^*} &\leq 0; \ \frac{\partial L}{\partial p_1^*} p_1^* = 0 \qquad (52)\\
\text{with} &: \ \frac{\partial L}{\partial p_1^*} = \left[\begin{array}{c} G_1(-\tilde{X}_1) + H_1(-X_1) + (1-\lambda)\Pi_{1p}^m + (1-\nu)\Pi_{1p}^n \\ +\rho\sigma' Q_p^*(W_{22}^* - W_{21}^*) \end{array} \right] \\
\frac{\partial L}{\partial q_1^*} &\leq 0; \ \frac{\partial L}{\partial q_1^*} q_1^* = 0 \qquad (53)\\
\text{with} &: \ \frac{\partial L}{\partial q_1^*} = \left[\begin{array}{c} H_1(-Y_1) + (1-\lambda)\Pi_{1q}^m + (1-\nu)\Pi_{1q}^n \\ +\rho\sigma' Q_q^*(W_{22}^* - W_{21}^*) \end{array} \right].
\end{aligned}$$

with $Q_p^*, Q_q^* < 0$. Note that the second row of $\partial L/\partial p_1^*$ and $\partial L/\partial q_1^*$, respectively, is due to the assumed network effect. Since welfare is strictly increasing in demand we must have $W_{22} > W_{21}$ and p_1^* as well as q_1^* tend to be lower when compared to the static case. A corresponding result can be obtained for m_1^* and n_1^*:

$$\begin{aligned}
\frac{\partial L}{\partial m_1^*} &\leq 0; \ \frac{\partial L}{\partial m_1^*} m_1^* = 0 \qquad (54)\\
\text{with} &: \ \frac{\partial L}{\partial m_1^*} = \left[\begin{array}{c} f(m_1^*)\tilde{V}_1 + (1-\lambda)\Pi_{1m}^m \\ +\rho\sigma' Q_m^*(W_{22} - W_{21}) + \psi_2^m + \psi_1^m \end{array} \right] \\
\frac{\partial L}{\partial n_1^*} &\leq 0; \ \frac{\partial L}{\partial n_1^*} n_1^* = 0 \qquad (55)\\
\text{with} &: \ \frac{\partial L}{\partial n_1^*} = \left[\begin{array}{c} -f(n_1^*)(\tilde{V}_1 - V_1) + (1-\lambda)\Pi_{1n}^m + (1-\nu)\Pi_{1n}^n \\ +\rho\sigma' Q_n^*(W_{22} - W_{21}) + \psi_2^n + \psi_1^n \end{array} \right]
\end{aligned}$$

The network effect implies an increase of m_1^* and n_1^*. This is simply due to $Q_m^* = f(m_1^*)\widetilde{X}_1 > 0$ and $Q_n^* = f(n_1^*)(-\widetilde{X}_1 + X_1 + Y_1) > 0$. Furthermore, considering the influence of $\psi_2^m, \psi_2^n \leq 0$ (54) and (55) indicate that irreversibility reduces the optimal network investments in the first period.

4.2 Market outcome without Regulation

Analyzing the market game without regulation the second period can be easily characterized. While prices $p_{2i}^c(m_{2i}, n_{2i})$ and $q_{2i}^c(n_{2i}, m_{2i})$ are given by the solutions of (21) and (22), investment decisions depend on m_1 and n_1. Using the envelope theorem and $m_{2i} \geq m_1$ and $n_{2i} \geq n_1$ we get the following first order conditions:

$$\frac{\partial \Pi_{2i}^{mc}}{\partial m_{2i}} \leq 0,\ \frac{\partial \Pi_{2i}^{mc}}{\partial m_{2i}}(m_{2i}-m_1)=0 \text{ and } \frac{\partial \Pi_{2i}^{nc}}{\partial n_{2i}} \leq 0,\ \frac{\partial \Pi_{2i}^{nc}}{\partial n_{2i}}(n_{2i}-n_1)=0. \tag{56}$$

Let $m_{2i}^c(m_1,n_1)$ and $n_{2i}^c(m_1,n_1)$ denote the solutions of (56). Expected profits in the first period are given by

$$E\Pi^{mc} \ :\ = \Pi_1^m + \rho\left[\sigma \Pi_{22}^{mc} + (1-\sigma)\Pi_{21}^{mc}\right] \geq 0 \tag{57}$$

$$E\Pi^{nc} \ :\ = \Pi_1^n + \rho\left[\sigma \Pi_{22}^{nc} + (1-\sigma)\Pi_{21}^{nc}\right] \geq 0 \tag{58}$$

Maximizing expected profits with respect to p_1 and q_1 leads to (using $Q^c := G_1\tilde{X}_1 + H_1(X_1+Y_1)$ evaluated at p_1, q_1, m_1 and n_1)

$$\frac{\partial E\Pi^{mc}}{\partial p_1} = \Pi_{1p}^m + \rho\sigma' Q_p^c(\Pi_{22}^{mc} - \Pi_{21}^{mc}) = 0 \tag{59}$$

$$\frac{\partial E\Pi^{nc}}{\partial q_1} = \Pi_{1q}^n + \rho\sigma' Q_q^c(\Pi_{22}^{nc} - \Pi_{21}^{nc}) = 0. \tag{60}$$

The network effect gives both firms an incentive to lower their prices in order to increase the probability of high demand in the second period. Note further, that this incentive tends to be higher for firm M since a decrease of p_1 increases the quantities on both market segments G_1 and H_1. Furthermore, the strategic complementarity between p_1 and q_1 does not change since we have

$$\sigma'' < 0 \text{ and } Q_{pq}^c = H_1(X_{1pq} + Y_{1qp}) > 0.$$

Turning to the optimal investment in period 1, let $p_1^c(m_1,n_1)$ and $p_1^c(m_1,n_1)$ denote the solutions of (59) and (60). The first order conditions for m_1 and n_1 are given by[7]

$$\frac{\partial E\Pi^{mc}}{\partial m_1} \leq 0;\ \frac{\partial E\Pi^{mc}}{\partial m_1} m_1 = 0 \tag{61}$$

$$\text{with} \ :\ \frac{\partial E\Pi^{mc}}{\partial m_1} = \begin{bmatrix} \Pi_{1m}^m + \rho\sigma' Q_m^c(\Pi_{22}^{mc} - \Pi_{21}^{mc}) \\ +q_{1m}^c\left[\Pi_{1q}^m + \rho\sigma' Q_q^c(\Pi_{22}^{mc} - \Pi_{21}^{mc})\right] \\ +\rho\left[\frac{d\Pi_{22}^{mc}}{dm_1} + (1-\sigma)\frac{d\Pi_{21}^{mc}}{dm_1}\right] \end{bmatrix} \tag{62}$$

$$\frac{\partial E\Pi^{nc}}{\partial n_1} \leq 0;\ \frac{\partial E\Pi^{nc}}{\partial n_1} n_1 = 0 \tag{63}$$

$$\text{with} \ :\ \frac{\partial E\Pi^{nc}}{\partial n_1} = \begin{bmatrix} \Pi_{1n}^n + \rho\sigma' Q_n^c(\Pi_{2i}^{nc} - \Pi_{2i}^{nc}) \\ +p_{1n}^c\left[\Pi_{1p}^n + \rho\sigma' Q_p^c(\Pi_{22}^{nc} - \Pi_{21}^{nc})\right] \\ +\rho\left[\sigma\frac{d\Pi_{22}^{mc}}{dn_1} + (1-\sigma)\frac{d\Pi_{21}^{mc}}{dn_1}\right] \end{bmatrix} \tag{64}$$

[7] Since m_{2i}^c and n_{2i}^c are kinked functions, we use partial derivatives defined as $\lim_{\widetilde{m}_1 \searrow m_i} \partial m_{2i}^c / \partial \widetilde{m}_1$ and $\lim_{\widetilde{n}_1 \searrow n_i} \partial n_{2i}^c / \partial \widetilde{n}_1$.

The direct impact of m_1 and n_1 on σ again leads to higher investment incentives when compared to the static case. Considering the investment incentives of firm M note that while we have $d\Pi_{21}^{mc}/\, dm_1 < 0 \Leftrightarrow m_{21}^c = m_1$, the potential losses due to high investments in the first period are alleviated by a corresponding increase in p_{21}^c. In contrast, with $n_{21}^c = n_1$ a further increase in n_1 would also intensify second period price competition. Therefore, the difference between the firms' incentives to invest in the first period tends to be higher as compared to the one period game.

4.3 Medium Run Regulation

Starting with second period decisions and taking into account $m_{2i} \geq m_1$ and $n_{2i} \geq n_1$ let $m_{2i}^m(m_1, n_1, p_{2i}^m)$ and $n_{2i}^m(n_1, m_1, p_{2i}^m)$ denote the solutions of (see (32) and (33))

$$\frac{\partial \Pi_{2i}^{mm}}{\partial m_{2i}} \leq 0,\ \Pi_{2im}^{mm}(m_{2i} - m_1) = 0 \text{ and } \frac{\partial \Pi_{2i}^{nm}}{\partial n_{2i}} \leq 0,\ \Pi_{2in}^{nm}(n_{2i} - n_1) = 0. \tag{65}$$

Since firms are free to choose their investments in both period the regulator's decision in period 2 is characterized by

$$\begin{aligned} \frac{dW_{2i}^m}{dp_{2i}^m} &= G(-\tilde{X}_{2i}) + H(-X_{2i} - Y_{2i}q_{2i}^{m\prime}) + \Pi_{2ip}^m + \Pi_{2iq}^m q_{2i}^{m\prime} + \Pi_{2ip}^n \\ &\quad + \tilde{V}_{2i} f(m_{2i}) \frac{\partial m_{2i}^m}{\partial p_{2i}^m} - f(n_{2i}^m)\left[(\tilde{V}_{2i} - V_{2i}) + (p_{2i}^m - c)(\tilde{X}_{2i} - X_{2i})\right] \frac{\partial n_{2i}^m}{\partial p_{2i}^m} \end{aligned} \tag{66}$$

Comparing (66) with (37) shows that the regulator has essentially two options. Either he economizes on the given network sizes and chooses p_{2i}^m in accordance with short run regulation (see (39)) or he selects p_{2i}^m in order to induce further investments. While the firms' profits are strictly positive in the latter case, prices according to short run regulation can obviously lead to negative profits. Note further, that short run regulation is more likely to be optimal the greater the investments in the first period.

Turning to first period decisions, let $p_{2i}^m(m_1, n_1)$ denote the solution of (66) and let $\Pi_{2i}^{mm}(m_1, n_1)$ and $\Pi_{2i}^{nm}(n_1, m_1)$ denote the firms' reduced profit functions in period 2. Given m_1 and n_1 as well as p_1^m, the optimal price q_1^m can be simply characterized by

$$\frac{\partial E\Pi^{nm}}{\partial q_1} = \Pi_{1q}^n + \rho\sigma' Q_q^m(\Pi_{22}^{nm} - \Pi_{21}^{nm}) = 0. \tag{67}$$

With $q_1^m(m_1, n_1, p_1^m)$ as the solution of (67) the firms' optimal investments decisions are given by

$$\begin{aligned}
\frac{\partial E\Pi^{mm}}{\partial m_1} &\leq 0; \frac{\partial E\Pi^{mm}}{\partial m_1} m_1 = 0 && (68)\\
\text{with} &: \frac{\partial E\Pi^{mm}}{\partial m_1} = \begin{bmatrix} \Pi_{1m}^{mm} + \Pi_{1q}^{mm} q_{1m}^m \\ +\rho\sigma'(Q_m^m + Q_q^m q_{1m}^m)(\Pi_{22}^{mm} - \Pi_{21}^{mm}) \\ +\rho\left[\sigma \frac{d\Pi_{22}^{mm}}{dm_1} + (1-\sigma)\frac{d\Pi_{21}^{mm}}{dm_1}\right] \end{bmatrix}\\
\frac{\partial E\Pi^{nm}}{\partial n_1} &\leq 0; \frac{\partial E\Pi^{nm}}{\partial n_1} n_1 = 0 && (69)\\
\text{with} &: \frac{\partial E\Pi^{nm}}{\partial n_1} = \begin{bmatrix} \Pi_{1n}^{nm} + \rho\sigma' Q_n^m(\Pi_{22}^{nm} - \Pi_{21}^{nm}) \\ +\rho\left[\sigma \frac{d\Pi_{22}^{nm}}{dn_1} + (1-\sigma)\frac{d\Pi_{21}^{nm}}{dn_1}\right] \end{bmatrix}
\end{aligned}$$

While the direct impact of m_1 as well as n_1 on σ points to higher investments incentives, the potential negative effect of $d\Pi_{21}^{mc}/\, dm_1$ is rather high. With $m_{21} = m_1$ and $n_{21} = n_1$ an increase of m_1 increases the expected losses due to regulation if the regulator adheres to short run regulation in the second period. Our example shows that this ratchet effect may well imply strong underinvestment in period 1, that is, it can lead to a situation in which firms do not invest in the first period.

Finally, let $m_1^m(p_1)$ and $n_1^m(p_1)$ denote the solutions of (68) and (69). and consider the optimal choice of p_1^m. Using W_{2i}^m to denote realized welfare in period 2, the resultant first order condition can be written as

$$\begin{aligned}
\frac{dEW^m}{dp_1^m} &= \begin{bmatrix} G_1(-\tilde{X}_1) + H_1(-X_1 - Y_1 \frac{dq_1^m}{dp_1^m}) + \frac{d\Pi_1^{mm}}{dp_1^m} + \frac{d\Pi_1^{nm}}{dp_1^m} \\ +f(m_1^m)\widetilde{V}_1^m \frac{dm_1^m}{dp_1^m} + f(n_1^m)(-\widetilde{V}_1^m + V_1^m)\frac{dn_1^m}{dp_1^m} \\ +\rho\left[\sigma' \frac{dQ^m}{dp_1^m}(W_{22}^m - W_{21}^m) + \sigma \frac{dW_{22}^m}{dp_1^m} + (1-\sigma)\frac{dW_{21}^m}{dp_1^m})\right] \end{bmatrix} && (70)\\
\text{with} &: \frac{dQ^m}{dp_1^m} = G_1 \tilde{X}_1' + H_1(X_{1p} + Y_{1p}) + H_1(X_{1q} + Y_{1q})\frac{dq_1^m}{dp_1^m}\\
&\quad +f(m_1^m)\widetilde{X}_1 m_1^{m\prime} + f(n_1^m)(-\widetilde{X}_1 + X_1 + Y_1)n_1^{m\prime}
\end{aligned}$$

where $dW_{2i}^m/\, dp_1^m$ indicates the impact which p_1^m has on m_1^m and n_1^m and thus on welfare in the second period if it is optimal to economize on existing networks instead of inducing further investments. The network effect is again taken into account by the third row in (70). While the first term is clearly negative, the last two terms show that the regulator has an incentive to compensate the potential ratchet effect by increasing p_1^m. As long as $m_{2i}^m = m_1^m$ and $n_{2i}^m = n_1^m$ holds, we also have $dW_{21}^m/\, dp_1^m = \partial W_{21}^m/\, \partial m_1^m \; dm_1^m/\, dp_1^m + \partial W_{21}^m/\, \partial n_1^m \; dn_1^m/\, dp_1^m > 0$ for $dm_1^m/\, dp_1^m$, $dn_1^m/\, dp_1^m > 0$.

4.4 Long Run Regulation

Compared to medium run regulation, long run regulation requires the regulator to choose a price which holds in both periods. While this restricts the regulator's flexibility it is a simple mechanism to overcome the ratchet effect associated with medium run regulation.

The firms' decisions concerning prices and network investments in the second period are analogous to medium run regulation. Thus, let p^l denote the regulated price fixed in period 1 and let $m_{2i}^l(m_1, n_1, p^l)$ and $n_{2i}^m(n_1, m_1, p^l)$ denote the solutions of

$$\Pi_{2im}^{ml} \leq 0,\ \Pi_{2im}^{ml}(m_{2i} - m_1) = 0 \text{ and } \Pi_{2in}^{nl} \leq 0,\ \Pi_{2in}^{nl}(n_{2i} - n_1) = 0 \tag{71}$$

with $q_{2i}^l = q^r(p^l)$. Turning to the first period, firm's N optimal price q_1^l is implicitly given by

$$\frac{\partial E\Pi^{nl}}{\partial q_1} = \Pi_{1q}^{nl} + \rho\sigma' Q_q^l(\Pi_{22}^{nl} - \Pi_{21}^{nl}) = 0. \tag{72}$$

Letting $q_1^l(m_1, n_1, p^l)$ denote the solution of (72) and considering the firms' investments decisions we get

$$\begin{aligned} \frac{\partial E\Pi^{ml}}{\partial m_1} &\leq 0; \frac{\partial E\Pi^{ml}}{\partial m_1} m_1 = 0 & (73) \\ \text{with} &: \frac{\partial E\Pi^{ml}}{\partial m_1} = \begin{bmatrix} \Pi_{1m}^{ml} + \Pi_{1q}^{ml} q_{1m}^l \\ +\rho\sigma'(Q_m^l + Q_q^l q_{1m}^l)(\Pi_{22}^{ml} - \Pi_{21}^{ml}) \\ +\rho\left[\sigma \frac{d\Pi_{22}^{ml}}{dm_1} + (1-\sigma)\frac{d\Pi_{21}^{ml}}{dm_1}\right] \end{bmatrix} \\ \frac{\partial E\Pi^{nl}}{\partial n_1} &\leq 0; \frac{\partial E\Pi^{nl}}{\partial n_1} n_1 = 0 & (74) \\ \text{with} &: \frac{\partial E\Pi^{nl}}{\partial n_1} = \begin{bmatrix} \Pi_{1n}^{nl} + \rho\sigma' Q_n^l(\Pi_{22}^{nl} - \Pi_{21}^{nl}) \\ +\rho\left[\sigma \frac{d\Pi_{22}^{nl}}{dn_1} + (1-\sigma)\frac{d\Pi_{21}^{nl}}{dn_1}\right] \end{bmatrix} \end{aligned}$$

Comparing (73) and (74) with (68) and (69) reveals that the firms' strategic considerations are quite similar. However, the potential negative impact of $d\Pi_{2i}^{ml}/\,dm_1$ and $d\Pi_{2i}^{mm}/\,dn_1$ is equal to the losses from overinvestment in the one period model. Therefore, the ratchet effect present under medium run regulation vanishes.

Finally, let $m_1^l(p^l)$ and $n_1^l(p^l)$ denote the solutions of (73) and (74) and let W_{2i}^l denote realized welfare in period 2. The first order condition for p^l is given by

$$\frac{dEW^m}{dp^l} = \left[\begin{array}{c} G_1(-\tilde{X}_1) + H_1(-X_1 - Y_1 \frac{dq_1^l}{dp^l}) + \frac{d\Pi_1^{ml}}{dp^l} + \frac{d\Pi_1^{nl}}{dp^l} \\ +f(m_1^l)\widetilde{V}_1^l \frac{dm_1^l}{dp^l} + f(n_1^m)(-\widetilde{V}_1^l + V_1^l)\frac{dn_1^l}{dp^l} \\ +\rho \left[\sigma' \frac{dQ^l}{dp_1^l}(W_{22}^l - W_{21}^l) + \sigma \frac{dW_{22}^l}{dp^l} + (1-\sigma)\frac{dW_{21}^l}{dp^l}) \right] \end{array} \right] \quad (75)$$

$$\text{with} \quad : \quad \frac{dQ^l}{dp^l} = G_1 \tilde{X}_1' + H_1(X_{1p} + Y_{1p}) + H_1(X_{1q} + Y_{1q})\frac{dq_1^l}{dp^l}$$

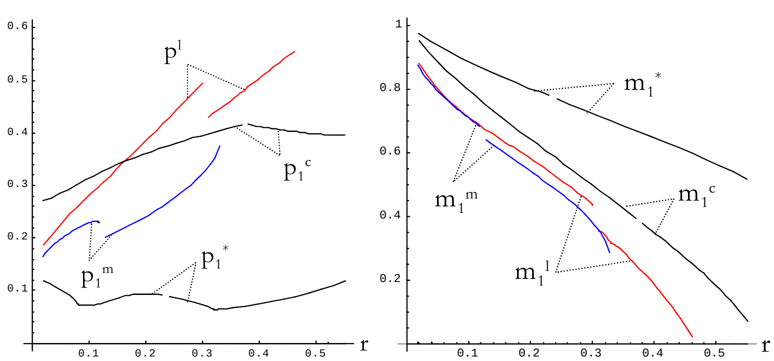
$$+f(m_1^l)\widetilde{X}_1 m_1^{l\prime} + f(n_1^l)(-\widetilde{X}_1 + X_1 + Y_1)n_1^{l\prime}.$$

$$\text{and} \quad : \quad \frac{dW_{2i}^l}{dp^l} = \frac{\partial W_{2i}^l}{\partial p^l} + \frac{\partial W_{2i}^l}{\partial m_{2i}^l}\frac{dm_{2i}^l}{dp^l} + \frac{\partial W_{2i}^l}{\partial n_{2i}^l}\frac{dn_{2i}^l}{dp^l}$$

As expected, the optimal price under long run regulation has to balance the direct welfare effects in period 1, the positive effects due to the network effect and the impact which regulation has on welfare in the second period. Compared to medium run regulation a relatively high price p^l not only stimulates investment in the first period, it also induces the prospect of higher profits if demand shifts upwards in the second period. Therefore, the consideration between exploiting the network effect and providing investment incentives points to $p^l > p_1^m$.

4.5 Example

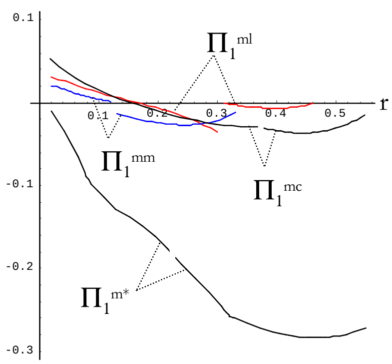
Starting with second best regulation, indirect network effects lead to lower prices and higher investments of M when compared to the one period model.[8] With medium run regulation and unregulated competition, the relationship between first period prices p_1 and network sizes m_1 is similar to the relationship in the one period model provided network costs are low.



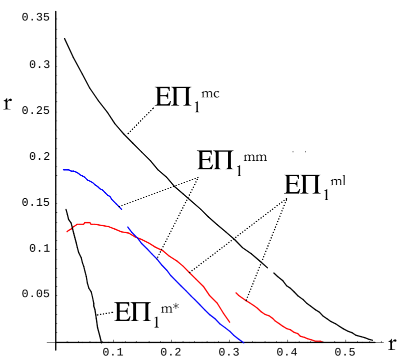
First period prices of M First period networks of M

[8] We left out the prices and the net sizes of N. The discontinuities indicate, where N leaves the market under the different regimes.

With higher investment costs the above mentioned ratchet effect under medium run regulation becomes crucial. Negative first period payoffs and second period losses due to low demand and short run regulation are no longer covered by positive profits if second period demand is high. Therefore, the regulator has to increase the price in the first period to sustain investment incentives. Higher first period prices, however, diminish first period demand and hence erode the network effect. Overall it comes to a sudden market breakdown at $r \approx 0.32$.



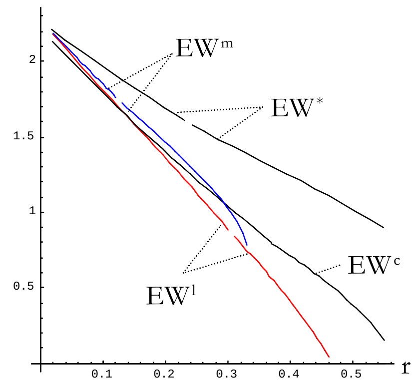
First period profits of M



Expected profits of M

Such a sudden market breakdown does not exist for long run regulation. While high investment costs can be compensated by higher first period prices, the natural limit for this mechanism is given by first period demand and again the network effect. The higher the investment costs, the lower firm's M investment and the lower the probability for higher demand in the second period.

Considering expected welfare, medium run regulation dominates long run regulation and unregulated competition as long as investment costs are relatively low. With high investment costs unregulated competition leads to higher expected welfare when compared to medium and long run regulation. While this is due to the ratchet effect under medium run regulation, long run regulation serves as a simple commitment device but suffers from the induced inability to react to the actually realized demand in the second period.



Expected Welfare

Finally, note that with relatively low investment costs the potential welfare gains due to regulation are rather low. On the other hand, high investment costs imply substantial welfare losses due to medium or long run regulation.

5 Zero Profit Constraints and Access Holidays

Given the results for medium run regulation and the huge welfare loss implied by market breakdown due the ratchet effect, it is natural to ask, whether it is possible to improve regulation by simple commitment devices which may help to overcome some of the incentive problems analyzed above. First, we check whether the market breakdown problem under medium run regulation can be alleviated by a simple zero-profit constraint in the second period. Then we consider access holidays which allow regulation only after an unsuccessful market development, that is only, if the demand in the second period is the same as in the first period. In order to complete the comparison between possible regulatory regimes we also consider the case in which regulation simply protects one firm's monopoly but does not intervene in the monopoly pricing and investment decisions.

5.1 Zero profit constraints

Amending medium run regulation with a zero-profit restriction (m_0) can alter the regulator's decision only if optimal prices would not lead to any further investment. That is, (66) has to be modified such that

$$\frac{dW_{2i}^{m_0}}{dp_{2i}^{m_0}} \leq 0 \text{ and } \Pi_{2i}^{m_0}(m_{2i}, n_{2i}) \geq 0 \qquad (76)$$

holds. While one would expect, that (76) tends to increase the investment incentives of firm M in the first period and would therefore allow the regulator to choose a lower price in the first period, inspection of (67), (68) and (69) indicates that some countervailing effects should be taken into account as well. Most importantly, both firms' investment incentives depend positively on the difference between second period profits with high and low demand as indicated by the terms $\rho\sigma'(Q_m^m + Q_q^m q_{1m}^m)(\Pi_{22}^{mm} - \Pi_{21}^{mm})$ and $\rho\sigma' Q_n^m(\Pi_{22}^{nm} - \Pi_{21}^{nm})$ in (68) and (69), respectively. Therefore, ensuring higher profits in the second period also has a negative impact on first period investments. In fact, our example shows that this countervailing effect can induce the regulator to choose a higher price in the first period as compared to unrestricted medium run regulation.

5.2 Access Holidays

Similar to a zero-profit restriction access holidays (h) can be interpreted as an ex ante commitment which narrows the regulator's strategy space in the second period. While providing access holidays may be an unconditional ex ante commitment we consider access holidays conditioned on the development of the market. More specifically, we analyze a situation in which the regulator commits itself not to intervene if demand realized in the second period is high.

Analyzing this kind of regulation formally, we can build on our analysis for an unregulated market outcome. Assuming $m_{21}^h = m_1^h$ and $n_{21}^h = n_1^h$ and substituting Π_{21}^{mc} and Π_{21}^{nc} by Π_{21}^{ms} and Π_{21}^{ns} in (59)—(64), respectively, shows that access holidays tend to increase the firms' investment incentives by increasing the relative profits which can be earned if demand shifts upwards. Note, however, that this positive effect is limited since the regulator can still intervene in the second period if demand is low. Hence, the firms are facing the potential losses due to short run regulation once again. While access holidays may alleviate the problem of regulation induced underinvestment in the first period, access holidays can not solve the problem of market breakdown completely.

5.3 Protected Monopoly

Finally, the problem of potential market and regulation induced distortions with respect to price competition and the investment behavior of the unregulated firm may also call for a protected monopoly (pm) which is well-known from the patent literature, but which is seldom mentioned when discussing investment in telecommunication networks.

In our model a protected monopoly leads to simple monopoly prices and investments m_{2i}^{pm} in the second period:

$$\Pi_{2ip}^{pm} = F(m_{2i})((p-c)\tilde{X}_{2i} + \tilde{X}_{2i}) = 0 \tag{77}$$

$$\Pi_{2im}^{pm} = f(m_{2i})(p-c)\tilde{X}_{2i} - r \leq 0,\ \Pi_{2im}^{pm}(m_{2i} - m_1) = 0 \tag{78}$$

Let p_{2i}^{pm} and m_{2i}^{pm} denote the solutions of (77) and (78) and let Π_{2i}^{pm} denote the respective profits. First period decisions can be characterized by (with $Q^{pm} = F(m_1)\tilde{X}_1$)

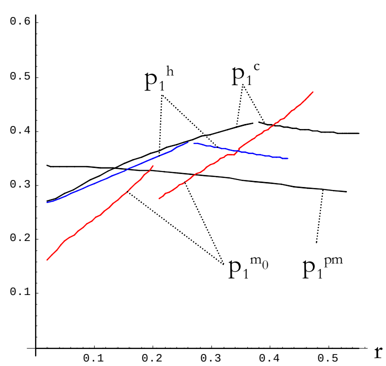
$$\frac{\partial E\Pi^{pm}}{\partial p_1} = \Pi_{1p}^{pm} + \rho\sigma' Q_p^{pm}(\Pi_{22}^{pm} - \Pi_{21}^{pm}) = 0 \tag{79}$$

$$\frac{\partial E\Pi^{pm}}{\partial m_1} \leq 0;\ \frac{\partial E\Pi^{pm}}{\partial m_1} m_1 = 0 \tag{80}$$

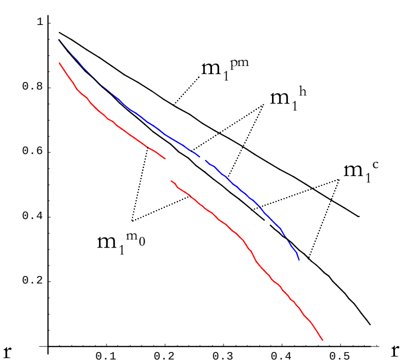
$$\text{with} : \frac{\partial E\Pi^{pm}}{\partial m_1} = \begin{bmatrix} \Pi_{1m}^{pm} + \rho\sigma' Q_m^{pm}(\Pi_{22}^{pm} - \Pi_{21}^{pm}) \\ +\rho\left[\sigma\frac{d\Pi_{22}^{pm}}{dm_1} + (1-\sigma)\frac{d\Pi_{21}^{pm}}{dm_1}\right] \end{bmatrix}$$

5.4 Example

As expected the market breakdown is alleviated by the zero profit constraint, but the implied negative incentive effects in period 1 also lead to relatively higher prices as compared to unconstrained medium run regulation. Overall, zero profit medium run regulation is worse than the unregulated solution for medium and large network costs $r \geq 0.2$.

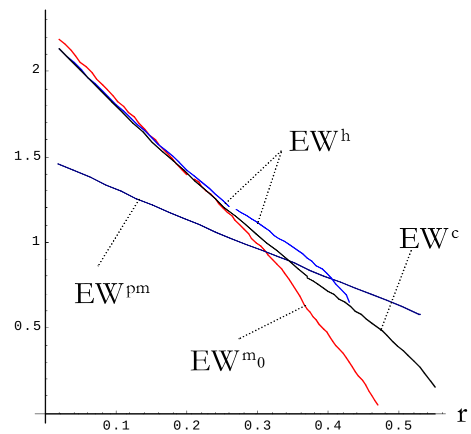


First period prices of M



First period networks of M

Access holidays with regulatory intervention only in period 2 and only if demand is low allow larger profits when demand is high. Hence, the market breakdown is postponed to $r \approx 0.48$. Furthermore, investment incentives are increased such that—for a large range of network costs—prices are lower and network sizes are higher when compared to the unregulated market outcome. Therefore, this regulation scheme performs very well for a large range of medium and large network costs.



Expected Welfare EW

Our model combines elements of a natural monopoly (duplication of fixed costs) with product differentiation gains. It is therefore not surprising that natural monopoly considerations eventually dominate so that the solution in which N is not allowed to enter the market is the best outcome for very large $r \geq 0.41$.

6 Summary

It is well-known that telecommunication networks generate allocation problems which cannot be solved perfectly by a competitive mechanism. Innovation, natural monopolies and network effects generate externalities which lead to market failures, and which are therefore widely accepted as challenging tasks for ex ante regulation. In this paper we demonstrate that this conclusion is very optimistic. Before an active regulation policy is justified, it has to be shown that a regulator has a realistic chance to improve the market outcome if generally accepted restrictions are imposed on potential regulatory measures. Instead of allowing every mechanism that satisfies the firms' participation constraints we restricted our attention to a "third best" world in which the regulator can readjust his decision until history determines the condition on which the policy is conditioned. This way we introduced a natural trade-off between flexibility and commitment of the regulation policy.

Our example shows that several commonly held beliefs on the welfare potential of active regulation are premature. Even in the one period model, third best regulation allows to realize only a small part of the potential welfare gains under second best regulation. Total surplus under third best regulation was almost indistinguishable from social welfare under unregulated competition. Instead, we observed significant redistribution from the producers to the consumers.

While this redistribution is a matter of social preferences, it generates a serious market breakdown in the dynamic model. With medium run regulation the lack of commitment makes it impossible to guarantee profits if network costs are high. With network effects the implied market breakdown is even worse because a lack of investment in the first period does not only destroy the actual consumer welfare, it also eliminates the chance for further growth.

Network effects are also decisive for the relatively bad performance of long run regulation. In the two period model with steady demand, long run regulation allows the realization of the same allocation as medium run regulation in the one period model. Network effects, however, call for relatively lower prices and higher investments in the first period, i.e., for flexible pricing policies. Solving the commitment problem at the costs of flexibility reduces the welfare gains of regulation considerably.

Finally, our analysis of limited commitment devices demonstrates that regulation can be improved by restricting the regulator's possibilities to intervene in the market. While access holidays conditional on the market development can increase social welfare they do not resolve the breakdown problem for high investment costs. The good performance of the protected monopoly shows that it is a mistake to ignore fundamental economic insights gained in the patent literature.

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