DIW Berlin

German Institute
for Economic Research

Discussion Papers

402

Pio Baake
Kay Mitusch

Competition with Congestible Networks

Berlin, February 2004

Opinions expressed in this paper are those of the author and do not necessarily reflect views of the Institute.

DIW Berlin

German Institute
for Economic Research

Königin-Luise-Str. 5
14195 Berlin,
Germany

Phone +49-30-897 89-0
Fax +49-30-897 89-200

www.diw.de

ISSN 1619-4535

Competition with Congestible Networks

Pio Baake* DIW Berlin

Kay Mitusch† TU Berlin

10th January 2004

Abstract

We analyse competition between two network providers when the quality of each network depends negatively on the number of customers connected to that network. With respect to price competition we provide a sufficient condition for the existence of a unique pure strategy Nash equilibrium. Comparative statics show that as the congestion effect gets stronger quantities will decrease and prices increase, under both Bertrand and Cournot competition. In an example with endogenous capacities it turns out that capacities are strategic substitutes for both modes of ensuing competition. Welfare comparisons between Bertrand and Cournot competition are unambiguous for fixed capacities, but may turn around for endogenous capacities.

JEL– classification numbers: L13, L86.
Keywords: Congestion, networks, Bertrand and Cournot competition

*DIW Berlin, Königin-Luise-Str. 5, 14191 Berlin, Germany, Email: pbaake@diw.de.
Thanks to Rainald Borck and Anette Boom for helpful comments.
†TU Berlin, Sekr. H 33, Straße des 17. Juni 135, 10623 Berlin, Germany, E-mail: mitusch@wip.tu-berlin.de

1 Introduction

Communications networks like the Internet or the telecommunication networks share two main features. First, there are positive network externalities, i.e. consumers' utility from using the network increases with the number of other consumers. But second, there are also negative congestion externalities. Given the transmission capacity of a network infrastructure the number of telephone calls or data packages which can be transferred *at a given speed* is limited. On the other hand, by reducing the speed or introducing short breaks in individual transmissions the amount of connections can be extended even with a given capacity. More generally, while the network capacity does not impose a strict limit on the number of transfers, the *quality* of transfers will decrease when the total number of transfers in the network gets large.

This paper focuses on congestion effects and their implications for competition between network providers. Our leading example are Internet service providers (ISP firms) which offer connection to households and other firms. In view of the huge size of the world wide Internet we abstract from the network externalities. Instead, individual connection decisions are mainly based on the ISPs' connection prices and the qualities of their network services. Measuring quality as the average waiting time for downloads and/or the expected loss rate of data packages, the quality provided by an ISP depends positively on the capacity of its network and negatively on the data flow the ISP has to transfer.[1] Hence we will model congestion effects as smoothly increasing in a firm's demand and decreasing in its capacity.

We will analyze a simple two stage game with two network providers and a continuum of consumers who have to decide with which provider they connect. Consumers differ only in their willingness to pay for a given quality of connection, but quality is affected by congestion effects. Providers face

[1] The network capacity of an ISP is determined by the amount and the quality of its servers and routers as well as its connections to other ISPs and to Internet Exchange Points. The digital nature of the Internet implies that the quality of a connection as perceived by the customers is a smooth and decreasing function of the total data flow the ISP has to handle.

no production cost except capacity cost. In the first stage providers choose their capacities. Competition for consumers takes place in the second stage where we will consider both Bertrand and Cournot equilibria.

A crucial assumption of our model is that congestion effects are 'sufficiently smooth' in the firms' demands. While this assumption rules out kinked-like congestion effects, where network qualities would sharply deteriorate at particular demand levels, it guarantees that in the second stage of the game a unique pure strategy equilibrium exists—under both Bertrand and Cournot competition. While this result is not surprising for Cournot competition, it is in contrast to the so–called 'Edgeworth cycles' which can occur under Bertrand competition with strict capacity constraints. It is also contrary to the existence of multiple pure strategy Bertrand equilibria when firms have smooth and convex cost functions (see Dastidar (1995)).

Comparative statics in the strength of the congestion effect show that the equilibrium quantities are decreasing while the prices are increasing in the strength of congestion. These results hold irrespective of the mode of competition; moreover, it turns out that the differences between the Bertrand and Cournot equilibria are decreasing when the congestion externality gets stronger cet. par.

With respect to the firms' capacity choice in the first stage of the game we investigate an example which shows that capacities are strategic substitutes, regardless of the mode of competition in the subsequent stage. Welfare comparisons show that the two–stage equilibrium with Cournot competition is preferred to the one with Bertrand competition, which is due to the fact that Cournot induces higher investments in capacities (whereas, with given capacities, Bertrand would be preferred). The comparative statics with respect to the strength of congestion effect show that the equilibrium capacities are at first increasing and then decreasing in the strength of congestion. With rather low congestion effects the firms try to restore their network qualities by increasing their capacities. With strong congestion effects this strategy would become too expensive and firms start to reduce their capacities.

Although the sequence of moves and the cost structure assumed in our model are similar to Kreps and Scheinkman (1983), the structure and aim of the model is quite different. While Kreps and Scheinkman assume a strict capacity constraint, our model implies that the initial capacity choice has an indirect and smooth effect on a firm's demand at the competition stage. We do not find that the capacity–Bertrand sequence leads to 'Cournot outcomes' in any sense.

Other related papers are duopoly models with differentiated goods, for example Singh and Vives (1984), Cheng (1985), and Maggi (1995). However, none of these analyze *endogenously* differentiated goods. In our model, vertical product differentiation is an endogenous outcome of firms' capacity choices and consumers' connection decisions.

Endogenous (vertical) product differentiation with respect to the Internet is analyzed for example by Crémer et al. (2000) and Laffont et al. (2001). In contrast to our model, their focus is on product differentiation due to positive network externalities. The papers most closely related to ours are Scotchmer (1985) and, in particular, Engel et al. (1999). These papers also analyze the effects of negative congestion externalities on competition. With respect to road pricing, Engel et al. state conditions for the existence of a Bertrand equilibrium in pure strategies, but their ensuing analysis focuses on welfare comparisons. In particular, they take capacities as given and do not explore the comparative statics in the strength of the congestion effect. Similar holds for Scotchmer who analyzes only symmetric equilibria under the 'everyone buys' assumption and focuses on the relations to club theory and general equilibrium theory.

Section 2 sets out the model. Section 3 analyses Bertrand and Cournot equilibria for fixed capacities, and comparative statics in the congestion effect for symmetric capacities. Section 4 analyses the capacity choices for a specific congestion function, and then takes up the comparative statics in the congestion effect. Section 5 provides some concluding remarks.

2 The Model

There are two firms (Internet service providers, ISPs), $i = 1, 2$, which offer connection to the Internet. On the demand side is a mass one of consumers who are uniformly distributed over the unit interval and indexed by $\theta \in [0, 1]$. Each consumer connects to at most one ISP. If a consumer connects to firm i his utility u depends on his preference parameter θ, on the firm's connection price p_i, and on a negative congestion externality q_i to be explained shortly:

$$u(q_i, p_i, \theta) = 1 - \theta - q_i - p_i \tag{1}$$

The congestion externality q_i increases with the number of consumers connected to firm i and decreases with the firm's capacity k_i. Letting D_i denote the number of firm i's customers, q_i is determined by a differentiable function $q_i(D_i, k_i) \geq 0$ with the following properties (in the relevant ranges):[2]

$$q_i(0, k_i) = 0, \quad q_{iD} > 0, \quad q_{iDD} \geq 0$$
$$q_{ik} < 0, \quad q_{ikk} \geq 0, \quad q_{ikD} \leq 0 \text{ for } D_i > 0.$$

The more customers use a network the more data packages have to be sent on that network and the higher is the expected delay and/or the loss rate. On the other hand, if capacity is expanded the congestion externality decreases gradually. Thus, capacity has a smooth positive impact on the network's quality.

The firms face identical q_i–functions, i.e. if $D_1 = D_2$ and $k_1 = k_2$ then $q_1 = q_2$. Normalizing the connection costs per customer to zero, firms have only capacity cost $c(k_i)$ with $c' > 0$ and $c'' \geq 0$. Firms' profits π_i are given by

$$\pi_i = p_i D_i - c(k_i)$$

We envision the following sequence of moves. In the first stage firms simultaneously choose their capacities k_i. In the second stage they compete for customers by committing simultaneously to either prices (Bertrand) or quantities (Cournot). In this stage capacities are fixed and commonly known to the rivals and the consumers. In the final stage consumers decide on connection. We solve the game by backward induction.

[2]For notational simplicity let $q_{iD} := \partial q_i(D_i, k_i)/\partial D_i$ and so on.

3 Fixed capacities

In the following we will analyze and compare Bertrand and Cournot equilibria when capacities $k_i > 0$ are fixed and given. Concerning Bertrand competition we provide a sufficient condition for the slope of the q_i-function with respect to D_i which guarantees that a unique equilibrium in pure strategies exists.

3.1 Bertrand competition

In the Bertrand game the two firms simultaneously commit to their prices p_1 and p_2 and connect every consumer who buys at that price. Knowing the firms' prices and capacities consumers then decide on whether to connect to one of the firms' networks. Therefore, we will first determine the consumers' decisions and the firms' demand functions. We then proceed by characterizing the firms' pricing decisions.

Concerning consumers' decisions the following is implied by (1): If $D_i > 0$ then there is a critical consumer $\underline{\theta} < 1$ such that $u(q_i, p_i, \theta) > 0 \Leftrightarrow \theta < \underline{\theta}$. It always holds that $D_1 + D_2 = \underline{\theta}$. Moreover, if both firms have positive demand then every consumer must be indifferent between the two firms, i.e. $u(q_1, p_1, \theta) = u(q_2, p_2, \theta)$ for all θ. Similarly, if $D_i > 0 = D_j$ with $i, j \in \{1, 2\}$ and $j \neq i$, then $u(q_i, p_i, \theta) \geq u(q_j, p_j, \theta)$ for all θ. In a demand (Nash) equilibrium these conditions must be satisfied at the corresponding levels of $q_i(D_i, k_i)$. The following lemma establishes that there always exists a unique demand equilibrium and that the firms face ordinary demand functions $D_i(p_i, p_j, k_i, k_j)$. (The exact derivations of the demand functions are given in the proof).

Lemma 1 *For every $k_1, k_2 > 0$ and $p_1, p_2 \geq 0$, the following holds:*
(i) D_1 and D_2 are uniquely defined and continuous functions of p_1, p_2, k_1, k_2.
(ii) If $p_i = 0$ then $D_i > 0$.
(iii) If $D_i > 0$ then D_i is strictly decreasing in p_i and k_j and strictly increasing in p_j and k_i.

Proof See appendix.

Using $D_i(\cdot)$ the firms' profit functions π_i^B are

$$\pi_i^B(p_i, p_j, k_i, k_j) = p_i D_i(\cdot) - c(k_i).$$

Part (ii) of Lemma 1 shows that each firm can earn a strictly positive revenue, part (iii) reveals that each firms' demand is strictly decreasing in its own price if $D_i > 0$. However, the firms' profit functions need not to be strictly concave in their own prices. The following assumption assures that the profit functions are well-behaved and that there exists a unique profit-maximizing price:

Assumption 1 $\quad q_{iDD} \leq q_{iD}(1 + q_{iD})$

In words, convexity of the q–function should not be too strong.[3] While Assumption 1 excludes, for example, a kink–like shaped q–function, we obtain:

Lemma 2 *If Assumption 1 holds, then $\pi_i^B(p_i, p_j, k_i, k_j)$ is strictly quasi–concave in p_i.*

Proof See appendix.

Thus, π_i^B has a unique maximum in p_i. With positive demands for both firms, the firms' reaction functions $p_i^r(p_j, \cdot)$ are implicitly given by:[4]

$$\frac{\partial \pi_i}{\partial p_i} = D_i + p_i D_{ip_i} = D_i - p_i \frac{1 + q_{jD}}{q_{iD}(1 + q_{jD}) + q_{jD}} = 0 \qquad (2)$$

Based on Lemma 2, and once more on Assumption 1, we finally get:

Proposition 1 *If Assumption 1 holds, the Bertrand game has a unique equilibrium in pure strategies $(p_1^B(k_1, k_2), p_2^B(k_2, k_1))$ for any pair of capacities $k_1, k_2 > 0$. It satisfies $p_i^B > 0$ and $D_i^B > 0$ both i.*

Proof See appendix.

Note that, generally, the existence of a unique Bertrand equilibrium in pure strategies cannot to be taken for granted. It is well known that under a usual capacity constraint the 'Edgeworth cycle' phenomenon can arise, i.e. that

[3] Assumption 1 is satisfied by several plausible examples. In particular: $q_i = D_i/k_i$ or $q_i = \exp(D_i/k_i) - 1$, or, for $k_i \geq 1$, $q_i = D_i/(k_i - D_i)$.

[4] Employing (14) in the appendix.

price equilibria exist only in mixed strategies. To assure the existence of a pure strategy equilibrium either capacities must not be too large (namely, not exceeding the Cournot quantities) or the cost functions must be smooth and strictly convex. For the latter case Dastidar (1995) shows that there exists a multiplicity (continuum) of pure strategy Bertrand equilibria. In our model, Assumption 1 suffices to guarantee both the existence and the uniqueness of a pure strategy Bertrand equilibrium.

3.2 Cournot competition

With Cournot competition, the two firms simultaneously commit to sales quantities and adapt prices until that amount is sold. As the demand quantities are effectively chosen by the firms, we will denote them by lower case d_1 and d_2 and the resulting prices as functions $P_i(d_i, d_j, k_i, k_j)$. Since the critical consumer is given by $\underline{\theta} = d_1 + d_2 \leq 1$, (1) implies that

$$P_i(\cdot) = 1 - d_1 - d_2 - q_i(d_i, k_i) \tag{3}$$

In order to avoid a negative price each firm will obey $d_i < 1$. This implies that both firms are able to make positive revenues. Furthermore, it is easy to verify that each firm's profit

$$\pi_i^C(d_i, d_j, k_i, k_j) = P_i(\cdot)d_i - c(k_i)$$

is strictly concave in d_i. Hence, the profit maximizing choice of d_i is uniquely characterized by the first order condition

$$\frac{\partial \pi_i^C}{\partial d_i} = P_i - (1 + q_{iD})d_i = 0. \tag{4}$$

Evaluating the reaction functions $d_i^r(d_j, k_1, k_2)$ as implied by (4) we get

Proposition 2 *The Cournot game has a unique equilibrium in pure strategies $(d_1^C(k_1, k_2), d_2^C(k_1, k_2))$ for any pair of capacities $k_1, k_2 > 0$. It satisfies $d_i^C > 0$ and $P_i^C > 0$ both i.*

Proof See appendix.

3.3 Comparisons and Comparative Statics

In this section we will compare the equilibria under Bertrand and Cournot competition with respect to quantities, prices and social welfare. We then discuss how the equilibria change if the strength of the congestion effect increases.

3.3.1 Bertrand, Cournot, and Welfare

Using (2) and (4) we obtain the standard result that Bertrand competition is more competitive than Cournot competition:

Proposition 3 *Bertrand competition leads to higher quantities and lower prices than Cournot competition, i.e.* $D_i^B > d_i^C$ *and* $p_i^B < P_i^C$ *both* i*, for all* $k_1, k_2 > 0$.

Proof Evaluating (2) at the price which is implied by the Cournot reaction function, i.e. at $p_i = P_i(d_i^r(d_j, \cdot), \cdot)$ given by (4), yields

$$\frac{\partial \pi_i^B}{\partial p_i} = D_i + (1 + q_{iD}) D_i D_{ip_i} = D_i \left[1 - \frac{(1 + q_{iD})(1 + q_{jD})}{q_{iD}(1 + q_{jD}) + q_{jD}} \right] < 0$$

From this the claim follows. ■

While prices are lower under Bertrand competition, they are still higher than the welfare-maximizing prices:

Proposition 4 *Bertrand competition leads to lower quantities than the first best, for all* $k_1, k_2 > 0$.

Proof In our context aggregate welfare is simply given by (using $D_1 + D_2 = \underline{\theta}$):

$$\begin{aligned} W &= \frac{D_1}{\underline{\theta}} \int_0^{\underline{\theta}} [1 - \theta - q_1(D_1, k_1)]\, d\theta + \frac{D_2}{\underline{\theta}} \int_0^{\underline{\theta}} [1 - \theta - q_2(D_2, k_2)]\, d\theta \\ &= \underline{\theta} - \frac{1}{2}\underline{\theta}^2 - D_1 q_1 - D_2 q_2 \end{aligned}$$

Differentiating with respect to D_i yields the Pareto–optimality conditions:

$$1 - \underline{\theta} - q_i - D_i q_{iD} = 0 \quad \text{both } i. \tag{5}$$

However, in the Bertrand equilibrium the first order condition is given by (2) which, together with the price equation $p_i = 1 - \underline{\theta} - q_i$ (from (1)) yields

$$1 - \underline{\theta} - q_i - \left(1 + \frac{q_{jD}}{q_{iD}(1 + q_{jD})}\right) D_i q_{iD} = 0 \quad \text{both } i.$$

Comparison with (5) shows that the Bertrand quantities are below the first–best quantities. ■

3.3.2 Comparative statics in the strength of congestion

The comparative statics with respect to the congestion effect will be discussed for the symmetric case, i.e., the case in which the firms have the same capacities $k := k_1 = k_2 > 0$. This also implies that the Bertrand as well as the Cournot equilibria are symmetric. We introduce a shift parameter ϕ such that the firms' congestion externalities are given by $q(\overline{D}, k)$ with $\overline{D} := \phi D$. If ϕ is increased the congestion effect gets ceteris paribus stronger, both absolutely and marginally. Furthermore, note that $\phi = 0 \Rightarrow q = 0$ leads to the standard Bertrand and Cournot equilibria with linear demand and zero production costs, i.e. to $(p^B, D^B) = (0, 1/2)$ and $(P^C, d^C) = (1/3, 1/3)$, respectively.

Summarizing the comparative statics with respect to the congestion effect we obtain:

Proposition 5 *The equilibrium quantities D^B and d^C are strictly decreasing while the prices P^B and p^C are strictly increasing in ϕ. Furthermore, with $q_{\overline{DDD}} \geq 0$, the differences in quantities and prices between the Bertrand and Cournot equilibria, i.e. $D^B - d^C$ and $p^C - P^B$, are strictly decreasing in ϕ. For the limit $\phi \to \infty$ it holds that $D^B \to 0$ and $d^C \to 0$ and $(P^C - p^B) \to 0$.*

Proof See appendix.

That the equilibrium quantities are decreasing is intuitive since the market gets tighter when congestion gets stronger. On the other hand, the increase of the prices is less obvious since the good gets less attractive if the congestion externality gets stronger. Two effects are at work here. First, by increasing its price each firm can restore the quality of its good. Second, an increase in the congestion effect leads to a decrease in the marginal product substitutability of the firms' products. That is, the stronger the congestion effect the more will any demand shift affect the qualities of the firms' products. Therefore, competition is softened and the firms will increase their prices. For the same reason the difference between the Cournot and Bertrand equilibria is decreasing in the congestion effect. Note that these results are in line with similar results in the literature on exogenous product differentiation which shows that a decrease in the degree of product substitutability implies an increase of Bertrand and Cournot prices, a decrease of quantities, and a decrease of the price/quantity differences between Bertrand and Cournot equilibria.[5]

For a numerical example, the left hand diagram of Figure 1 shows the equilibrium prices p^B and P^C as functions of $\phi \in [0, 10]$, and right hand diagram the equilibrium quantities D^B and d^C.[6]

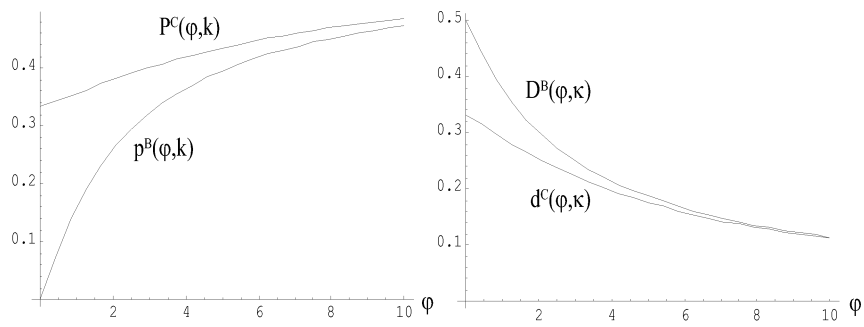

Figure 1: Prices and quantities in ϕ

Note that in this example the limiting price which will be approached for very large ϕ is strictly below one, namely $\lim_{\phi\to\infty} p^B = \lim_{\phi\to\infty} P^C = 2/3$. This also implies that the limiting value of congestion externality is positive, $\lim_{\phi\to\infty} q = 1/3$.

[5]A concise statement of these properties can be found in Singh and Vives (1984) for linear demand. See also the literature cited there or in Cheng (1985).

[6]The figure is based on $q_i = \phi D_i/(k_i - \phi D_i)$ with $k_i = 5$.

4 Endogenous Capacities

We now turn to the first stage of the game in which firms simultaneously choose their capacities. Since this stage is difficult to solve for arbitrary q_i-functions we first characterize the firms' incentives to invest and then use a particular example which allows us to analyze the firms' equilibrium capacities.

4.1 Optimal Capacities for Bertrand and Cournot

Let $\pi_i^B(p_i^B(\cdot), p_j^B(\cdot), k_i, k_j)$ denote the reduced profit functions under Bertrand competition. Proposition 1 implies that π_i^B is differentiable in k_i. Since π_i^B is bounded and capacity costs are weakly convex, π_i^B must attain a maximum at some $k_i \geq 0$. Applying the envelope theorem and using $D_{ik_i} = q_{ik} D_{ip_i}$ and $D_{ip_j} = -D_{ip_i}/(1+q_{jk})$, [7] the firms' first order conditions with respect to their capacities are given by:

$$\frac{\partial \pi_i^B(\cdot)}{\partial k_i} = D_i^B(\cdot)\left[-q_{ik} + \frac{1}{1+q_{jD}} \frac{\partial p_j^B(\cdot)}{\partial k_i}\right] - c'(k_i) \leq 0 \text{ and } \frac{\partial \pi_i^B(\cdot)}{\partial k_i} k_i = 0. \tag{6}$$

The term $-q_{ik}$ in (6) is the 'direct effect' of an increase in k_i on the firm's profit. Since $q_{ik} < 0$ and $q_{ikD} \leq 0$, the direct effect is positive and increasing with D_i^B. The other term in the brackets reflects the 'strategic effect' which is due to the influence of k_i on the equilibrium price p_j^B. Using (2) in order to determine the sign of $\partial p_j^B(\cdot) \big/ \partial k_i$ we obtain that $\partial p_j^B(\cdot) \big/ \partial k_i < 0$ holds for all capacities $k_i, k_j > 0$ (see appendix). Hence, by increasing its own capacity firm i will cause a decrease in the other firm's price p_j^B. This effect reduces the firm's incentive to invest.

Turning to Cournot competition we denote the reduced profit functions by $\pi_i^C(d_i^C(\cdot), d_j^C(\cdot), k_i, k_j)$. Again, since $\pi_i^C(\cdot)$ is bounded there must exist a maximum in k_i. Applying the envelope theorem and using $\partial P_i/\partial k_i = -q_{ik}$ and $\partial P_i/\partial d_j = -1$ (see (3)), we can write the firms' first order conditions as

$$\frac{\partial \pi_i^C(\cdot)}{\partial k_i} = d_i^C(\cdot)\left[-q_{ik} - \frac{\partial d_j^C(\cdot)}{\partial k_i}\right] - c'(k_i) \leq 0 \text{ and } \frac{\partial \pi_i^C(\cdot)}{\partial k_i} k_i = 0. \tag{7}$$

[7]For these expressions see the appendix, proof of Lemma 1.

Again, the direct effect $-q_{ik}$ is positive and increasing in d_i^C. However, comparative statics with respect to k_i yields that $\partial d_j^C(\cdot) \big/ \partial k_i < 0$ holds for all capacities $k_i, k_j > 0$ (see appendix). By increasing its own capacity firm i will cause a decrease in the other firm's quantity d_j^C. Therefore, also the strategic effect (the second term in the brackets) is positive.

Comparing the investment incentives under Bertrand and Cournot competition the above results indicate that an unambiguous statement about the firms' capacity decisions is not possible without further assumptions on the q_i–functions. With given capacities Bertrand competition leads to higher demand than Cournot competition which implies (together with $q_{ikD} \leq 0$) that the direct effect, which is always positive, is higher under Bertrand competition. On the other hand, the indirect effect is negative under Bertrand competition but positive under Cournot. Thus, Cournot may lead to higher capacities than Bertrand competition. This is indeed the case in the following example.

4.2 An Example

In order to get some more insights into the nature of the two–stage equilibria we now analyze an example. The q_i–function is given by

$$q_i(\phi D_i, k_i) = \begin{cases} \frac{\phi D_i}{k_i - \phi D_i} \text{ for } D_i < \frac{k_i}{2\phi} \\ 1 \text{ otherwise.} \end{cases}$$

We assume linear and low capacity costs, $c(k_i) = k_i / 100$, which ensure that both firms will invest. Solving at first the competition stage with given capacities, for Bertrand and Cournot, and then using (6) resp. (7) we obtain the reaction functions $k_i^B(k_j, \phi)$ and $k_i^C(k_j, \phi)$, shown in Figure 2 for $\phi = 1$.

Since the reaction functions in the capacity game are decreasing, capacities are strategic substitutes under both Bertrand and Cournot competition. Finally, solving the system of reaction functions we obtain a unique and symmetric equilibrium in capacities under Bertrand (k^B) as well as under Cournot (k^C). It turns out that investment incentives are higher under Cournot competition, i.e. $k^C > k^B$.

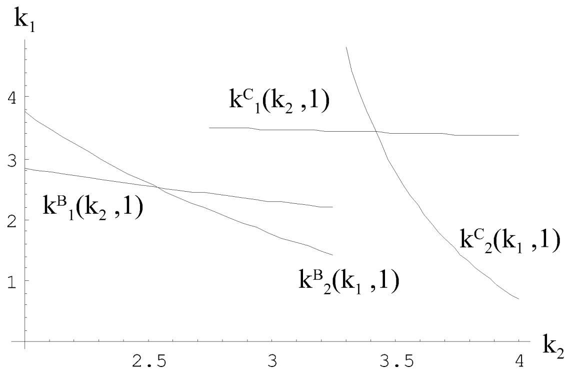

Figure 2: Capacity–reaction functions for Bertrand and Cournot

Turning to comparative statics, Figure 3 shows $k^B(\phi)$ and $k^C(\phi)$ for $\phi \in [1, 10]$. The equilibrium capacities are non–monotonic in the strength of the congestion effect, ϕ. At first they are increasing in order to compensate for the congestion effect. However, since profits are bounded capacities will not be increased without limits. Thus, when ϕ gets very large, compensating for the congestion effect will become unprofitable. As profits are then squeezed, the equilibrium capacities will eventually decrease in ϕ.

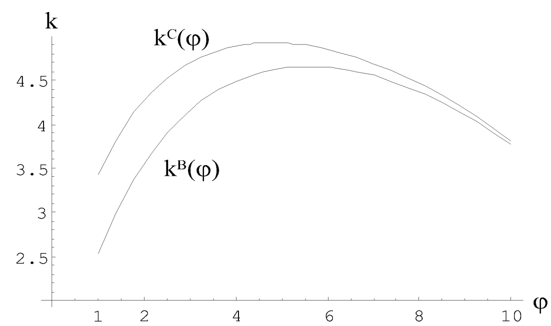

Figure 3: Equilibrium capacities with Bertrand and Cournot competition

Moreover, considering the equilibrium prices and quantities we obtain the same qualitative pattern as in the case of fixed capacities (compare Figure 1). Under both Bertrand and Cournot competition prices are monotonically increasing and quantities are monotonically decreasing in the congestion externality ϕ. While firms may increase their capacities if congestion gets stronger, the additional investments do not suffice to keep prices and quantities constant. Furthermore, Cournot prices are always higher than Bertrand prices, while the reverse is true for quantities.[8] Since we also have $k^C(\phi) > k^B(\phi)$, there is less congestion in the Cournot equilibrium, i.e. $q(\phi d^C, k^C) < q(\phi D^B, k^B)$.

This observation leads us to compare social welfare under the two modes of competition.[9] While Proposition 5 shows that with *given* capacities Bertrand competition is welfare superior to Cournot competition, using $k^B(\phi)$, $k^C(\phi)$ and calculating the corresponding welfare levels shows that Cournot competition leads to higher welfare as compared to Bertrand competition. With endogenous capacities the higher investment incentives under Cournot more than compensate higher prices.

5 Concluding Remarks

A pervasive problem of communications networks is that the speed and quality of transfers often deteriorates during times of intensive use. This suggests that firms which provide Internet services face a trade-off between the quantity and the quality of service they can offer. In this paper we explored the implications of congestion effects in a simple duopoly model. We discussed both Bertrand and Cournot competition under congestion. The analysis of Bertrand competition is more involved in this setting since consumers will not only respond to prices but also to the endogenous qualities. Under a fairly mild assumption the existence of a unique equilibrium in pure strategies can be assured for Bertrand (and of course for Cournot). This provides the basis for comparative statics analysis and, based on a particular example, for the analysis of endogenously determined capacities.

Generally, we showed that an increase in the strength of congestion leads to higher equilibrium prices and lower equilibrium demand. This holds irrespective of the mode of competition and whether capacities are given or

[8]The differences between Bertrand and Cournot equilibria in prices, quantities, and capacities are decreasing in ϕ, and vanishing for $\phi \to \infty$.

[9]Welfare equals W, as given in the proof of Proposition 4, minus capacity cost.

endogenous. With endogenous capacities, our example showed that capacities are strategic substitutes, for both modes of ensuing competition. An increase in the strength of congestion leads at first to an increase and then to a decrease of the equilibrium capacities.

The differences between Bertrand and Cournot equilibria concerning prices, quantities, and capacities (if endogenous) are decreasing in the strength of congestion. This is in line with similar results in the literature on exogenous product differentiation. Also in line with standard results, we found that Cournot competition leads to higher prices, lower quantities, and higher capacities (if endogenous) than Bertrand competition.

Welfare comparisons have to take account of the congestion effects. Since Cournot leads to higher prices and lower quantities than Bertrand, it follows that for fixed capacities the relation of network use to network capacity is lower under Cournot, so that the quality provided in Cournot equilibrium is higher. Still, for *given* capacities we could show that Bertrand competition leads to a more efficient combination of quantities and qualities of connections (although it is no longer the first best allocation when there is a congestion effect). However, with endogenous capacities our example showed that the ranking of the two modes of competition might turn around, since Cournot competition induces higher investments in capacity due to higher profit expectations. This effect may lead to a more efficient combination of capacities, quantities and qualities of connections under Cournot competition compared to Bertrand competition.

Appendix

Proof of Lemma 1

Proof for part (i). For brevity we write the q_i–function as $q_i(D_i)$. Recall that $q_i(0) = 0$.

If $p_i \geq 1$ then $D_i = 0$, see (1). If $p_j \geq 1 > p_i$ then $D_j = 0$ and $D_i = \underline{\theta}$ which is determined by $1 - \underline{\theta} - p_i - q_i(\underline{\theta}) = 0$, see (1). Since q_i is strictly increasing in D_i, $\underline{\theta}$ is uniquely defined, and $\underline{\theta} \in (0, 1)$. Hence, if at least one firm sets $p_i \geq 1$ then D_1 and D_2 are uniquely defined.

Now assume $p_i < 1$ for both i. Let $\alpha \in [0,1]$ be firm 1's market share, i.e.

$$D_1 = \alpha\underline{\theta} \quad \text{and} \quad D_2 = (1-\alpha)\underline{\theta} \tag{8}$$

For any $\alpha \in [0,1]$ the following equations uniquely define two numbers $\underline{\theta}^i(\alpha) \in (0,1)$:

$$1 - \underline{\theta}^1 - q_1(\alpha\underline{\theta}^1) - p_1 = 0 \tag{9}$$

$$1 - \underline{\theta}^2 - q_2((1-\alpha)\underline{\theta}^2) - p_2 = 0 \tag{10}$$

They have the properties

$$\underline{\theta}^1_\alpha = -\frac{q_{1D}\underline{\theta}^1}{1 + q_{1D}\alpha} < 0 \quad \text{and} \quad \underline{\theta}^2_\alpha = \frac{q_{2D}\underline{\theta}^2}{1 + q_{2D}(1-\alpha)} > 0 \tag{11}$$

With the help of these variables we can show that the demand system $(\alpha, \underline{\theta})$ is in equilibrium when one of the following cases holds:

If $\alpha \in (0,1)$ then (1) requires

$$1 - \underline{\theta} - q_1(\alpha\underline{\theta}) - p_1 = 1 - \underline{\theta} - q_2((1-\alpha)\underline{\theta}) - p_2 = 0 \tag{12}$$

By (9) and (10) this is equivalent to $\underline{\theta} = \underline{\theta}^1(\alpha) = \underline{\theta}^2(\alpha)$, and (11) implies that α and $\underline{\theta}$ are uniquely determined.

If $\alpha = 1$ then (1) requires $1 - \underline{\theta} - p_2 \leq 1 - \underline{\theta} - q_1(\underline{\theta}) - p_1 = 0$. By (9) this implies $\underline{\theta} = \underline{\theta}^1(1)$.

If $\alpha = 0$ then (1) requires $1 - \underline{\theta} - p_1 \leq 1 - \underline{\theta} - q_2(\underline{\theta}) - p_2 = 0$. By (10) this implies $\underline{\theta} = \underline{\theta}^2(0)$.

One finally observes the uniqueness and continuity of the demand system over all the different cases.

Proof for part (ii). None of the cases discussed above is consistent with $p_i = D_i = 0$.

Proof for part (iii). With $D_i > 0$ and $D_j = 0$, D_i is implicitly given by

$$1 - D_i - q_1(D_i) - p_i = 0. \tag{13}$$

Differentiating (13) with respect to p_i reveals $D_{ip_i} < 0$. If $D_iD_j > 0$, we are in the case described by (12). To determine the derivations D_{iv} for

$v \in \{p_i, p_j, k_i, k_j\}$ we differentiate the four equations in (12) and (8). Solving the resulting system of equations reveals:

$$D_{ip_i} = \frac{-(1+q_{jD})}{q_{iD}(1+q_{jD})+q_{jD}} < 0, \; D_{ip_j} = \frac{1}{q_{iD}(1+q_{jD})+q_{jD}} > 0 \quad (14)$$

$$D_{ik_i} = \frac{-q_{ik}(1+q_{jD})}{q_{iD}(1+q_{jD})+q_{jD}} > 0, \; D_{ik_j} = \frac{q_{jk}}{q_{iD}(1+q_{jD})+q_{jD}} < 0. \quad (15)$$

Proof of Lemma 2

If p_i is such that $D_j = 0$ then $D_i = 1 - q_i - p_i$. Hence, $D_{ip_i} = -1/(1+q_{iD})$ and:

$$\frac{\partial \pi_i}{\partial p_i} = D_i + p_i D_{ip_i} = D_i - p_i \frac{1}{1+q_{iD}} \quad (16)$$

In this region π_i is strictly concave since

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = 2D_{ip_i} + p_i D_{ip_i p_i} = D_{ip_i}\left[2 + p_i \frac{q_{iDD}}{(1+q_{iD})^2}\right] < 0$$

When D_j becomes positive, we have

$$\frac{\partial \pi_i}{\partial p_i} = D_i + p_i D_{ip_i} = D_i - p_i \frac{1+q_{jD}}{q_{iD}(1+q_{jD})+q_{jD}} = 0. \quad (17)$$

Comparing (16) and (17) reveals that π_i has a kink, its slope making a downward shift as is implied by

$$\frac{1}{1+q_{iD}} < \frac{1+q_{jD}}{q_{iD}(1+q_{jD})+q_{jD}}$$

After the kink the derivative of π_i is given by (17), and π_i will generally not be concave. However, in the following we show that if π_i is weakly convex in p_i then it is strictly decreasing. This implies that π_i is strictly quasi–concave in that region, and hence overall.

To show that $\partial^2 \pi_i / \partial p_i^2 \geq 0 \Rightarrow \partial \pi_i / \partial p_i < 0$ consider first $\partial^2 \pi_i / \partial p_i^2$:

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = 2D_{ip_i} + p_i D_{ip_i p_i} = D_{ip_i}\left[2 + p_i (D_{ip_i})^2 \left[q_{iDD} - \frac{q_{jDD}}{(1+q_{jD})^3}\right]\right] \quad (18)$$

where $D_{ip_i p_i}$ has been evaluated on the basis of (14). Since $D_{ip_i} < 0$ it follows:

$$\frac{\partial^2 \pi_i}{\partial p_i^2} \geq 0 \iff 2 \leq p_i (D_{ip_i})^2 \left[\frac{q_{jDD}}{(1+q_{jD})^3} - q_{iDD}\right]$$

which also implies that

$$\frac{q_{jDD}}{(1+q_{jD})^3} - q_{iDD} > 0, \quad \text{i.e.} \quad q_{jDD} - q_{iDD}(1+q_{jD})^3 > 0$$

These two inequalities together with $D_{ip_i} < 0$ imply that $\partial^2\pi_i / \partial p_i^2 \geq 0$ iff

$$p_i D_{ip_i} \leq \frac{2(1+q_{jD})^3}{D_{ip_i}\left[q_{jDD} - q_{iDD}(1+q_{jD})^3\right]} = \frac{2(1+q_{jD})^2(q_{iD}(1+q_{jD}) + q_{jD})}{q_{iDD}(1+q_{jD})^3 - q_{jDD}}$$

where (14) has been used. This gives us an upper bound for $\partial\pi_i / \partial p_i$:

$$\frac{\partial \pi_i}{\partial p_i} = D_i + p_i D_{ip_i} \leq D_i + \frac{2(1+q_{jD})^2(q_{iD}(1+q_{jD}) + q_{jD})}{q_{iDD}(1+q_{jD})^3 - q_{jDD}} \tag{19}$$

Since $D_i \leq 1$ and the RHS of (19) is increasing in q_{iD} and q_{iDD} we set $D_i = 1$ and $q_{iD} = q_{iDD} = 0$ in (19) to obtain

$$\frac{\partial \pi_i}{\partial p_i} \leq 1 - \frac{2(1+q_{jD})^2 q_{jD}}{q_{jDD}} \tag{20}$$

Assumption 1 implies that the right–hand side of (20) is negative. Thus, $\partial\pi_i / \partial p_i < 0$ if $\partial^2\pi_i / \partial p_i^2 \geq 0$.

Proof of Proposition 1

Lemma 2 implies that the reaction functions $p_i^r(p_j, \cdot)$ are continuous for all $p_j \in [0, 1]$. Since $0 < p_i^r(p_j, \cdot) < 1$, there exists an equilibrium in pure strategies. To prove uniqueness, we show that $0 \leq \partial p_i^r(p_j, \cdot) / \partial p_j < 1$ for all p_i, p_j such that $D_i D_j > 0$. Start by '≥ 0' and note that

$$\frac{\partial p_i^r}{\partial p_j} = -\frac{\partial^2 \pi_i}{\partial p_i \partial p_j} \Big/ \frac{\partial^2 \pi_i}{\partial p_i^2} \Rightarrow \text{ sign } \frac{\partial p_i^r}{\partial p_j} = \text{ sign } \frac{\partial^2 \pi_i}{\partial p_i \partial p_j}$$

The expressions in the proof of Lemma 1 yield:

$$D_{ip_j} = -\frac{D_{ip_i}}{1+q_{jD}} \quad \text{and} \quad D_{ip_ip_j} = (D_{ip_i})^3 \left[q_{jDD}\frac{1+q_{iD}}{(1+q_{jD})^3} - \frac{q_{iDD}}{1+q_{jD}}\right]$$

From this we get

$$\frac{\partial^2 \pi_i}{\partial p_i \partial p_j} = D_{ip_j} + p_i D_{ip_ip_j} = \frac{D_{ip_i}}{1+q_{jD}}\left[-1 + p_i (D_{ip_i})^2 \left[q_{jDD}\frac{1+q_{iD}}{(1+q_{jD})^2} - q_{iDD}\right]\right] \tag{21}$$

Since $D_{ip_i} < 0$, (21) shows that $\partial p_i^r / \partial p_j > 0$ iff

$$p_i (D_{ip_i})^2 \left[q_{jDD}\frac{1+q_{iD}}{(1+q_{jD})^2} - q_{iDD}\right] < 1 \tag{22}$$

Using

$$p_i = D_i \frac{q_{iD}(1+q_{jD})+q_{jD}}{1+q_{jD}}$$

in (22), $\partial p_i^r / \partial p_j > 0$ is equivalent to

$$q_{jDD} < \frac{(1+q_{jD})(D_i\, q_{iDD}(1+q_{jD})+q_{iD}(1+q_{jD})+q_{jD})}{D_i(1+q_{iD})} \tag{23}$$

Since the right–hand side of (23) is decreasing in D_i and increasing in q_{iD} and q_{iDD} we can set $D_i = 1$ and $q_{iDD} = q_{iD} = 0$ to obtain

$$q_{jDD} \le q_{jD}(1+q_{jD})$$

as a sufficient condition for $\partial p_i^r / \partial p_j > 0$. This condition is satisfied by Assumption 1.

To show $\partial p_i^r / \partial p_j < 1$ it suffices to show

$$\frac{\partial^2 \pi_i}{\partial p_i^2} - \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} > 0$$

Substituting (18) and (21) we get

$$\frac{\partial^2 \pi_i}{\partial p_i^2} - \frac{\partial^2 \pi_i}{\partial p_i \partial p_j} = -D_i D_{ip_i}\, q_{jDD}\, q_{iD} - (1+q_{jD})^2 (D_i D_{ip_i}\, q_{iDD}\, q_{jD} - 2q_{jD} - 1) > 0.$$

Proof of Proposition 2

The reactions functions $d_i^r(d_j,\cdot)$ satisfy $0 < d_i^r(d_j,\cdot) < 1$ for all $d_j \in [0,1]$. Hence, there exists a Nash equilibrium in pure strategies. Using (4) and noting that

$$\frac{\partial^2 \pi_i}{\partial d_i^2} = -2(1+q_{iD}) - q_{iDD} d_i < -2, \; \frac{\partial^2 \pi_i}{\partial d_i \partial d_j} = p_{iD_j} = -1$$

we get

$$\frac{\partial d_i^r(d_j,\cdot)}{\partial d_j} = -\left. \frac{\partial^2 \pi_i}{\partial d_i \partial d_j} \right/ \frac{\partial^2 \pi_i}{\partial d_i^2} \Rightarrow -1/2 < \frac{\partial d_i^r(d_j,\cdot)}{\partial d_j} < 0,$$

which implies that the equilibrium is unique.

Proof of Proposition 5

In order to simplify notation we define $Q(\phi, D, k) := q(\phi D, k)$ (and will make use of $\phi Q_\phi = D Q_D$ and $\phi Q_{D\phi} = D Q_{DD} + q_D$). We can combine the equilibrium conditions for Bertrand and Cournot by introducing an indicator variable $v \in [0,1]$ such that $v = 0$ refers to Cournot competition and $v = 1$ to Bertrand competition. Let $D^*(\phi, v, k)$ denote the equilibrium quantities, i.e., $D^B = D^*(\phi, 1, k)$ and $d^C = D^*(\phi, 0, k)$. Based on symmetry and (2) resp. (4), these are implicitly defined by the solutions of

$$1 - 2D - Q(\phi, v, k) = D(1 + Q_D) - v\frac{D}{1 + Q_D}$$

which is equivalent to

$$D^*(\phi, v, k) = \frac{(1-Q)(1+Q_D)}{3 - v + 4Q_D + Q_D^2} \tag{24}$$

and the equilibrium prices are given by

$$P(\phi, v, k) := 1 - 2D^* - Q(\phi, D^*, k). \tag{25}$$

Using (24) and (25) we can prove the proposition by showing

$$\frac{\partial D^*}{\partial \phi} < 0 \text{ and } \frac{\partial P^*}{\partial \phi} < 0 \forall\, v \in [0,1] \tag{26}$$

$$\frac{\partial^2 D^*}{\partial \phi \partial v} < 0 \text{ and } \frac{\partial^2 P^*}{\partial \phi \partial v} > 0 \forall\, v \in [0,1] \tag{27}$$

where (27) together with (26) and $D^B - d^C > 0$ and $P^C - p^B > 0$ implies that the differences between the Bertrand and Cournot equilibria are decreasing in ϕ. Comparative statics of $D^*(\phi, v, k)$ (see (24)) with respect to ϕ leads to

$$\frac{\partial D^*}{\partial \phi} = -\frac{D^*}{\phi}\frac{Q_D(2 + v + 4Q_D + 2Q_D^2) + D^* Q_{DD}(1 + v + 2Q_D + Q_D^2)}{(1 + Q_D)(3 - v + 5Q_D + 2Q_D) + D^* Q_{DD}(1 + v + 2Q_D + Q_D^2)} \tag{28}$$

which reveals that $\partial D^* / \partial \phi < 0$ for all $v \in [0,1]$. Using this in the derivation of (25) yields $\partial P^* / \partial \phi > 0\ \forall\ v \in [0,1]$. Turning to (27), we want to show negativity of

$$\begin{aligned}\frac{\partial^2 D^*}{\partial \phi \partial v} &= \frac{\partial}{\partial v}\left[\frac{\partial D^*}{\partial \phi}\right] + \\ &\left(\frac{\partial}{\partial D^*}\left[\frac{\partial D^*}{\partial \phi}\right] + \frac{\partial}{\partial Q_D}\left[\frac{\partial D^*}{\partial \phi}\right] Q_{DD} + \frac{\partial}{\partial Q_{DD}}\left[\frac{\partial D^*}{\partial \phi}\right] Q_{DDD}\right)\frac{\partial D^*}{\partial v}\end{aligned}$$

From (24) one shows that $\partial D^* / \partial v > 0$. Next, we have $q_{\overline{DDD}} \geq 0 \Leftrightarrow Q_{DDD} \geq 0$. Partial differentiations of the RHS of (28) with respect to v, D^*, Q_D and Q_{DD} show that

$$\frac{\partial}{\partial v}\left[\frac{\partial D^*}{\partial \phi}\right] < 0, \frac{\partial}{\partial Q_{DD}}\left[\frac{\partial D^*}{\partial \phi}\right] < 0 \text{ and } \frac{\partial}{\partial D^*}\left[\frac{\partial D^*}{\partial \phi}\right] + \frac{\partial}{\partial Q_D}\left[\frac{\partial D^*}{\partial \phi}\right] Q_{DD} < 0$$

where the last inequality is implied by Assumption 1 and $D^{*2} < D^*$. Hence it follows that $(\partial^2 D^*)/(\partial\phi\partial v) < 0$. Concerning prices, differentiating (25) and using the above properties of D^* shows that $(\partial^2 P^*)/(\partial\phi\partial v) > 0$.

Finally turn to the limiting properties for $\phi \to \infty$, which implies $Q_D = \phi q_{\overline{D}} \to \infty$ for any given $D > 0$. To show $D^* \to 0$ suppose to the contrary that D^* approaches some strictly positive limit for $\phi \to \infty$. But then the RHS of (24) goes to zero, contradicting the assumption. Hence it follows $D^* \to 0$. Therefore the difference $D^B - d^C$ vanishes, and therefore the price difference must also vanish.

Proof of $\partial p_j^B(\cdot)\big/\partial k_i < 0$

Differentiating (2) with respect to k_i and using $0 \leq \partial p_i^r(p_j,\cdot)/\partial p_j < 1$ leads to

$$\text{sign}\ \frac{\partial p_j^B(\cdot)}{\partial k_i} = \text{sign}\ \left[\frac{\partial^2\pi_i}{\partial p_i\partial k_i}\frac{\partial^2\pi_j}{\partial p_j\partial p_i} - \frac{\partial^2\pi_j}{\partial p_j\partial k_i}\frac{\partial^2\pi_j}{\partial p_j\partial p_j}\right]$$

Substituting $D_{ik_i} = q_{ik}D_{ip_i}$ and $D_{jk_i} = q_{ik}\ D_{ip_i}/(1+q_{jD})$ and rearranging terms shows that $\partial p_j^B(\cdot)\big/\partial k_i < 0$ holds if

$$q_{iD}D_{ip_i}D_j + 1 > 0 \tag{29}$$

holds. But since $p_j^B(\cdot) = -\ D_j/D_{jp_j} < 1$ and $p_j^B(\cdot) < 1 - D_j$ we also have

$$D_j < \frac{1+q_{iD}}{q_{iD}\left(2+q_{jD}\right)+q_{jD}+1}.$$

Hence, (29) is satisfied in the price equilibrium.

Proof of $\partial d_j^C(\cdot)\big/\partial k_i < 0$

Differentiating (4) with respect to k_i and using $-0.5 < \partial d_i^r(\cdot)/\partial d_j < 0$ yields

$$\text{sign}\ \frac{\partial d_j^C(\cdot)}{\partial k_i} = -\ \text{sign}\ \frac{\partial^2\pi_i}{\partial d_i\partial k_i}.$$

Since $\partial^2\pi_i/\partial d_i\partial k_i = (-q_{iDk})d_i^C(\cdot) - q_{ik} > 0$, $\partial d_j^C(\cdot)\big/\partial k_i$ is negative.

References

[1] Cheng, L. (1985): Comparing Bertrand and Cournot equilibria: a geometric approach, *RAND Journal of Economics*, 16(1), 146-52.

[2] Crémer, J., Rey, P. and Tirole, J. (2000): Connectivity in the Commercial Internet, *Journal of Industrial Economics*, 48, 433 - 472.

[3] Dastidar, K. (1995): On the existence of pure strategy Bertrand equilibrium, *Economic Theory*, 5, 19-32.

[4] Engel, E., Fischer, R. and Galetovic, A. (1999): Toll competition among congested roads, *Serie Economica No. 54,* Universidad de Chile.

[5] Kreps, D. and Scheinkman, J. (1983): Quantity precommitment and Bertrand competition yield Cournot outcomes, *Bell Journal of Economics,* 14, 326-337.

[6] Laffont, J., Marcus, S., Rey, P. and Tirole, J. (2001): Internet Interconnection and the Off-Net-Cost Principle, IDEI, mimeo

[7] Maggi, G. (1996): Strategic trade policies with endogenous mode of competition, *American Economic Review*, 86(1), 237-58.

[8] Scotchmer, S. (1985): Profit–Maximizing Clubs, *Journal of Political Economy*, 27, 25-45.

[9] Singh, N. and Vives, X. (1984): Price and quantity competition in a differentiated duopoly, *RAND Journal of Economics*, 15(4), 546–54.