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IMPRESSUM

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# Competitive Screening in Insurance Markets with Endogenous Labor Supply

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**Abstract**

We examine equilibria in competitive insurance markets when individuals take unobservable labor supply decisions. Precautionary labor motives introduce countervailing incentives in the insurance market, and equilibria with positive profits can occur even in the standard case in which individuals exogenously differ in risk only. We then extend the model to allow for both privately known risks and labor productivities. This endogenously introduces two-dimensional heterogeneity in the insurance market since precautionary labor effects lead to differences in income and hence risk aversion. Under these circumstances, separating and pooling equilibria exist, which generally differ from those with exogenous two-dimensional heterogeneity considered by the existing literature. Notably, in contrast to standard screening models, profits may be increasing with insurance coverage, and the correlation between risk and coverage can be zero or negative in equilibrium, a phenomenon frequently observed in empirical studies.

*JEL-classification:* D82, G22, J22.

*Keywords:* Insurance Markets, Adverse Selection, Precautionary Labor.

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# 1 Introduction

In the standard screening model going back to Rothschild and Stiglitz (1976), individuals differ only in a single dimension, namely their risk of incurring a loss, and the choice of an insurance contract is their only action explained endogenously. In this simple framework, insurance companies can induce customers to fully reveal their private information by offering contracts that separate the risk types. In particular, equilibrium contracts are such that high risk individuals obtain more insurance coverage than low risks. This positive correlation property has been the basis for much of the empirical research trying to identify adverse selection in specific markets. Yet, several recent studies have found no evidence to support this prediction of the standard screening model.[1] This has been interpreted as indicating that the importance of asymmetric information in these markets is smaller than previously assumed. Subsequent empirical research, however, has shown that the absence of a positive correlation between insurance coverage and risk occurrence does not imply that there is no adverse selection. Finkelstein and Poterba (2004), for instance, find strong evidence for adverse selection along contract features other than coverage in the UK annuity market. In addition, Finkelstein and McGarry (2006) show that preference based selection in the US long-term care insurance market may offset risk based selection so that, in aggregate, those with more insurance are not higher risk. Unfortunately, the basic screening model is not rich enough to account for such phenomena.

Motivated by the empirical findings, one strand of theoretical literature has focused on combining adverse selection and moral hazard in insurance markets. In these models, individuals can reduce their damage probability by an unobserved effort decision, which gives rise to moral hazard. To introduce adverse selection, De Meza and Webb (2001) and Jullien et al. (2006) assume that individuals differ in their privately known risk attitude, which affects their effort decision.[2] These models can indeed generate equilibria where those with more insurance coverage do not have a higher ex post risk. However, they do so by taking a number of

[1] See Cawley and Philipson (1999) for the US life insurance market, Chiappori and Salanié (2000) for the French automobile insurance market, and Cardon and Hendel (2001) for the US health insurance market.

[2] This idea was first put forward informally by Hemenway (1990) and Hemenway (1992). Another approach within this class of models, chosen by Stewart (1994) and Chassagnon and Chiappori (1997), is to assume that agents differ in their effort cost, which is also private information. While these models yield some interesting deviations from the standard Rothschild-Stiglitz model, such as different welfare implications and the coexistence of equilibria, they are unable to explain a zero or negative correlation between ex post risk and insurance coverage.

additional assumptions. In contrast to the standard screening model, Jullien et al. (2006) consider a monopolistic insurer. De Meza and Webb (2001) introduce administrative costs that also drive a wedge between premiums and expected claims. In addition, both models stick to a framework with one-dimensional heterogeneity between agents where ex post risk and risk attitude are perfectly correlated.[3] The question remains whether their results extend to purely competitive settings that allow for a less restrictive structure of heterogeneity.

Indeed, there exist theoretical contributions that extend the basic framework of Rothschild and Stiglitz (1976) to two-dimensional heterogeneity. Such models of adverse selection in competitive insurance markets in which individuals differ in more than one private characteristic have been examined by Smart (2000), Wambach (2000) and Villeneuve (2003). They assume that insurance customers differ in wealth and hence risk aversion in addition to risk, whereby the correlation between risk and risk aversion is not assumed to be perfect. This exogenously introduces a second dimension of heterogeneity. However, in these models, moral hazard is excluded since the individuals' only action is to choose an insurance contract. As standard monotonicity properties hold in each of the two dimensions, countervailing incentives and thus deviations from risk separation only emerge in these models if individuals differ in both characteristics so that the resulting effects work in opposite directions. Also, any equilibrium in such models will exhibit a positive correlation between insurance coverage and risk occurrence.

In this paper, we combine the two approaches outlined above to construct a model that can explain the empirical findings without assuming deviations from perfect competition. In our model, individuals differ in two dimensions *and* take an additional action unobservable to the insurance companies. In contrast to the standard moral hazard approach, however, this action does not affect their damage probability. A natural example of such a situation, which we focus on in this paper, is a setting where individuals differ in both their damage risk and their labor productivity, and choose their labor supply endogenously. We examine possible

[3] De Meza and Webb (2001) assume that some individuals are risk-neutral and hence neither purchase insurance nor take preventive actions. Their expected damage is therefore larger than that of the individuals who purchase partial insurance and take preventive measures due to their higher risk aversion. This generates a negative relationship between individuals' risk and their insurance coverage, even though in a rather special framework. See De Donder and Hindriks (2006) for a more general analysis demonstrating which set of assumptions is needed to generate such an equilibrium. Jullien et al. (2006) also consider a two type model only. They are mainly concerned with the question how risk aversion affects the power of incentives provided by the optimal contract.

equilibria in competitive insurance markets when insurers cannot observe individual risks, productivities, and labor supply. Obviously, this combines the typical informational assumptions underlying standard models of insurance markets and of optimal taxation, which may make our model a useful starting point for analyzing further questions of optimal tax policy in the presence of imperfect insurance markets.

Various interesting economic effects emerge in such a model. First, optimal labor supply reacts to the level of uncertainty and thus depends on the insurance market outcome. On the other hand, the endogeneity of labor supply introduces countervailing incentives in the insurance market as the individuals' marginal willingness to pay for insurance is influenced not only by their risk, but also their labor income. We demonstrate how the resulting interactions between labor and insurance markets affect insurers' ability to screen their customers. It will be shown that the insurance market equilibrium will in general not be fully separating any more when labor supply is endogenous. Also, in contrast to the models with exogenous differences in income, equilibria in which the correlation between risk and insurance coverage is negative can emerge. Thus, our model is able to explain this empirically relevant phenomenon based on purely competitive insurance markets.[4]

The paper is structured as follows. In section 2, fundamental results about labor supply under uncertainty are derived, which will provide the basis for the subsequent analysis of insurance market equilibrium. We demonstrate that, under broad and meaningful assumptions, there is a motive for precautionary labor, i. e. individuals work more in response to increases in risk. The resulting income change in turn affects their marginal willingness to pay for insurance. After having introduced the model of the insurance market in section 3, we first examine the resulting equilibria when there is only one-dimensional heterogeneity and individuals differ only in risk, not in productivity. As will be shown in section 4, the endogeneity of labor supply may make perfect screening impossible even in this simple framework. We then proceed to the two-dimensional case in section 5 to show how the results of Smart (2000), Wambach (2000) and Villeneuve (2003) are altered when endogenous labor is allowed for. The main result of this analysis is the emergence of equilibria where

[4]Chiappori et al. (2006) show that the positive correlation property holds for a large class of models, including competitive models and models with homogeneous risk aversion. This leads Jullien et al. (2006) to conclude that their model with private risk aversion and a monopolistic insurer is the only one that allows for a negative correlation when insurees have private information (p. 17). Our model demonstrates that this is incorrect. In fact, the result by Chiappori et al. (2006) is based on the assumption that, in competitive insurance markets, profits do not increase with coverage in the equilibrium set of contracts. As will turn out below, this property is not necessarily satisfied in our model.

those with more insurance are not higher risks in aggregate. Finally, section 6 concludes. The proofs of sufficient conditions for the existence of the equilibria discussed in the paper are relegated to the appendix.

## 2 Labor Supply under Uncertainty

Models of competitive insurance markets with adverse selection imply that, in general, not all uncertainty can be resolved. In order to introduce endogenous labor supply in the standard adverse selection model by Rothschild and Stiglitz (1976), we therefore need to derive the determinants of labor supply under uncertainty. Notably, we focus on situations in which uncertainty results from an income independent risk to consumption and labor supply is chosen before this risk is realized.[5] This problem was first examined by Netzer and Scheuer (2005) who used the insights of Kimball (1990) to establish a theory of precautionary labor within a model of taxation and social insurance. We briefly discuss their results in this section.

Consider a Bernoulli random variable $\theta(\beta)$ that results from a possible damage $D$ which occurs with probability $p$ and where the parameter $\beta \in [0,1]$ stands for the share of the damage that is insured. It can be used to vary both expected value $\mathrm{E}[\theta(\beta)] = p\,(1-\beta)D$ and variance $\mathrm{Var}[\theta(\beta)] = p\,(1-p)[(1-\beta)D]^2$ of the risk. Preferences are characterized by an additively separable utility function $U(c,L) = u(c)+v(L)$ where $c$ denotes consumption and $L$ denotes labor supply.[6] The standard conditions $u'(c) > 0$, $u''(c) < 0$, $v'(L) < 0$ and $v''(L) < 0$ are assumed. Denote the productivity of an individual by $w$. Firms can observe $w$ and pay wages according to marginal productivity so that earned income is $wL$.

Note that the separability of preferences implies that leisure is a normal good. In addition, let us assume the following:

**Assumption 1.** *Utility $u(c)$ displays non-increasing absolute risk-aversion.*

This common and realistic assumption is needed to obtain precautionary labor effects in the following. The first-order condition for labor supply $L^*$ that maximizes

[5]Eaton and Rosen (1980), Hartwick (2000) and Low (2005) model the case of endogenous labor with wage uncertainty. While this gives rise to different effects, they also find that larger uncertainty should increase labor supply. Parker et al. (2005) find empirical evidence in favor of this prediction.

[6]We need the assumption of separability only to keep the exposition of our labor supply theory concise. As shown by Kimball (1990), the results can be transferred to the case of nonseparable utility. We assume the function $v$ to be at least twice, $u$ at least three times differentiable.

expected utility in the presence of a given consumption risk $\theta(\beta_0)$ is

$$w\,\mathrm{E}\left[u'\left(wL^* - \theta(\beta_0)\right)\right] = -v'(L^*), \tag{1}$$

where E is the expectations operator. (1) is a standard condition stating that labor supply is determined so as to equalize expected marginal utility and disutility from work.[7] To answer the question how risk affects labor supply, we examine the move from $\theta(\beta_0)$ to the risk $\theta(\beta)$, $\beta \neq \beta_0$, which includes a change in variance and a change in expected damage. The latter will have an income effect on labor supply. To focus on the pure variance effect, we assume that the move from $\beta_0$ to $\beta$ is accompanied by a decrease of income by $(\beta - \beta_0)pD$, so that expected income remains constant. This would for example be the case if insurance premiums were adjusted actuarially fairly. We define the corresponding *equivalent precautionary premium* $\Psi(\beta_0, \beta)$ for such a move implicitly as follows:[8]

$$\mathrm{E}\left[u'\left(wL^* - \theta(\beta_0) - \Psi(\beta_0,\beta)\right)\right] = \mathrm{E}\left[u'\left(wL^* - \theta(\beta) - (\beta - \beta_0)pD\right)\right]. \tag{2}$$

Its interpretation is as follows. The expectation-neutral change in risk will have the same effect on the LHS of (1) and therefore on labor supply as a lump-sum reduction of income by $\Psi(\beta_0, \beta)$. Both changes affect the optimality condition in the same way. Therefore, statements about the adjustment of labor supply induced by a change of risk can be restated as income effects triggered by a decrease of income by $\Psi$.

Using the moments of the Bernoulli distribution, we can obtain an explicit expression for $\partial\Psi(\beta_0, \beta)/\partial\beta$ by differentiating (2). Notably, we are interested in the value of this derivative at $\beta = \beta_0$, which gives the income change that would have the same effect on labor supply as a small change in insurance, starting from a situation with insurance $\beta_0$. We obtain after a few rearrangements

$$\left.\frac{\partial\Psi(\beta_0,\beta)}{\partial\beta}\right|_{\beta=\beta_0} = \left(-\frac{\Delta u''(.)/(1-\beta_0)D}{\mathrm{E}[u''(.)]}\right)\left(\frac{1}{2}\frac{\partial \mathrm{Var}}{\partial\beta}\right), \tag{3}$$

where $\Delta u''(.)$ stands for the difference of $u''(.)$ between consumption levels in case of no damage and damage, and $\mathrm{E}[u''(.)]$ is the expected value of $u''(.)$.[9]

The first bracketed term on the RHS of (3) is the *generalized coefficient of ab-*

[7] The sufficient second order condition for a maximum is satisfied.

[8] As shown by Kimball (1990), the discussed premium is simply the equivalent risk premium developed by Pratt (1964), applied to the first derivative of $u$.

[9] The derivation makes use of the fact that $\Psi(\beta_0, \beta_0) = 0$ holds.

*solute prudence* $\eta^G$. As $\beta_0$ converges to 1, i.e. the examined situation converges to a situation without risk, the coefficient $\eta^G$ converges to the prudence $\eta$ as defined by Kimball (1990), which is simply the coefficient of absolute risk aversion for the function $u'(.)$, i.e. $\eta(c) = -u'''(c)/u''(c)$. From (3) follow our implications for labor supply under uncertainty. First, note that a sufficient condition for $\eta^G$ to be positive is that $u'''(.) > 0$. This, in turn, is implied by non-increasing risk aversion and hence by our Assumption 1. An increase in insurance coverage $\beta$ (compensated for its effect on expected damage) will therefore have the same effect on labor supply as an increase in income. Given that leisure is a normal good in our model with separable preferences, this increases the demand for leisure and decreases labor supply.[10] The individual has a motive for *precautionary labor*. The size of the generalized prudence $\eta^G$ indicates how strong this motive is. Lemma 1 summarizes these findings.

**Lemma 1.** *Under Assumption 1, a compensated increase in insurance coverage reduces individuals' labor supply. The strength of this effect increases in the generalized coefficient of prudence* $\eta^G$.

The precautionary labor effect described in Lemma 1 will be the driving force behind our results on possible insurance market equilibria. As individuals' labor income decreases in response to an increase in insurance, their marginal willingness to pay for insurance increases given Assumption 1. This will affect the insurers' ability to screen their customers. Before we demonstrate this, however, we introduce the model in the following section.

# 3 The Model

## 3.1 Preferences for Insurance

Consider a society of individuals characterized by their productivity $w_i$, $i = L, H$, and probability $p_j$, $j = L, H$, of incurring a damage $D$, with the conventions $w_L < w_H$ and $p_L < p_H$. There is a continuum of individuals normalized to unit mass. Let $n_{ij}$ denote the number of individuals with productivity $w_i$ and risk $p_j$. These individuals will be referred to as $ij-$individuals. Let $\bar{p}_i = \sum_j (n_{ij} p_j)/(n_{iL} + n_{iH})$ be the average risk in productivity group $i$ and $\bar{p} = \sum_{i,j} n_{ij} p_j$ the average risk in the entire population.

[10]The income effect on labor supply can easily be derived by introducing a state independent exogenous income $T$ and implicitly differentiating (1) in this slightly changed setup.

Individuals purchase insurance contracts that specify the share $\beta \in \,]0,1]$ of the damage that is covered,[11] and a premium $d \in \mathbb{R}^+$. Given such a contract $C = (\beta^C, d^C)$ from the contract space $\mathcal{C} = \,]0,1] \times \mathbb{R}^+$, optimal labor supply can be determined according to (1). It depends on the insurance contract and is denoted by $L^*_{ij}(\beta^C, d^C)$ or $L^*_{ij}(C)$. Substitution into the expected utility function yields the indirect expected utility function $V_{ij}(\beta^C, d^C)$ or $V_{ij}(C)$, from which indifference curves in the $(\beta, d)$-space can be obtained. Throughout the paper, the notation $A > B$ implies that insurance contract $A$ has a larger coverage and a larger premium than contract $B$.

Lemma 1 implies that, when considering an individual's preferences, we need to account for changes in labor supply and thus consumption levels as we move along an indifference curve in the $(\beta, d)$-space. On the one hand, labor supply is affected by precautionary motives. On the other hand, expected damage and premiums change and cause income effects on labor supply. Altogether, the endogeneity of labor supply may alter the shape and crossing properties of indifference curves compared to the canonical model by Rothschild and Stiglitz (1976).

At a contract $C$, consumption in case of loss is $c^0_{ij} = w_i L^*_{ij} - (1-\beta^C)D - d^C$ and $c^1_{ij} = w_i L^*_{ij} - d^C$ otherwise. Let us consider the slope of an indifference curve of an individual with productivity $w_i$ and risk $p_j$ in this contract

$$\left.\frac{\mathrm{d}d}{\mathrm{d}\beta}\right|_{V_{ij}=\bar{V}} = \mathrm{MRS}_{ij} = \frac{Dp_j u'(c^0_{ij})}{p_j u'(c^0_{ij}) + (1-p_j)u'(c^1_{ij})} > 0, \tag{4}$$

which is positive as in the standard model.[12] Note also that $\mathrm{MRS}_{ij} = p_j D$ at any full coverage contract (where $c^0_{ij} = c^1_{ij}$ holds), an additional result that carries over from the standard model. However, while the curvature of indifference curves in the $(\beta, d)$-space is always concave in the model with exogenous income, this does not necessarily hold when labor supply is endogenous. Notably, if an increase in insurance along an indifference curve leads to a strong reduction in labor supply, consumption may decrease so much that the individual actually has a higher marginal willingness to pay for insurance, given decreasing risk aversion. This would imply that indifference curves are not globally concave, and complicate our equilibrium analysis substantially. In the following lemma we derive a sufficient condition to exclude this

[11] Contracts with zero coverage are not relevant for our analysis. We exclude them to avoid technical problems in the following proofs.

[12] Clearly, indifference curves are still continuous and differentiable since labor supply is a continuous and differentiable function of the insurance contract while utility is continuous and differentiable in labor supply.

problem.

**Lemma 2.** *Indifference curves are concave in the $(\beta, d)$-space if an increase in insurance along an indifference curve leads to (weakly) larger consumption in case of damage.*

*Proof.* In order to examine how the marginal rate of substitution (4) changes as we move up on the indifference curve $d(\beta)$, we need to evaluate the sign of

$$\frac{\partial\,\mathrm{MRS}_{ij}(\beta, d(\beta))}{\partial\beta} = p_j(1-p_j)D\frac{u''(c_{ij}^0)u'(c_{ij}^1)\frac{\partial c_{ij}^0}{\partial\beta} - u''(c_{ij}^1)u'(c_{ij}^0)\frac{\partial c_{ij}^1}{\partial\beta}}{(\partial V_{ij}/\partial d)^2}, \tag{5}$$

where the expression on the RHS follows from differentiating (4), substituting $d(\beta)$ for $d$ and some simplifications. Note that $u''(c_{ij}^0)u'(c_{ij}^1) \leq u''(c_{ij}^1)u'(c_{ij}^0) < 0$ under Assumption 1, since $c_{ij}^0 \leq c_{ij}^1$ if $\beta \leq 1$. It is also clear that $\partial c_{ij}^1/\partial\beta < \partial c_{ij}^0/\partial\beta$ since the higher premium has to be paid in both states of the nature while the larger benefits are only received in case of damage. Hence $\partial c^0 ij/\partial\beta \geq 0$ along the indifference curve is a sufficient condition for (5) to be negative and thus for the indifference curve to be concave. □

Lemma 2 puts an upper bound on the precautionary labor effect that will be assumed to be satisfied for the remainder of this paper.

Apart from the shape of a given individual's indifference curves, the crossing properties of different individuals' indifference curves in a given insurance contract are also crucial for the equilibrium outcomes. Let us first ignore productivity differences and consider individuals that only differ in their risk. In the standard adverse selection model where income is exogenous, it is easy to show that, at any given contract, high risks have a steeper indifference curve than low risks. Put formally, the marginal rate of substitution between coverage and premium given in (4) is increasing in $p_j$. Clearly, the property immediately follows from (4) if $L_{ij}$ is held fixed. By the following definition, we will refer to this as "regular crossing" of indifference curves.

**Definition 1.** *The indifference curves of two individuals that differ only in risk exhibit "regular crossing" at a given contract if the high risk's indifference curve is steeper ($\mathrm{MRS}_{iH} > \mathrm{MRS}_{iL}$). Otherwise, they exhibit "irregular crossing".*

Definition 1 introduces a *local* concept at a given contract. If regular crossing holds in the whole contract space $\mathcal{C}$, as it does in the Rothschild-Stiglitz model, it implies the *global* property of single crossing for indifference curves of two individuals that differ only in risk. As was shown by Netzer and Scheuer (2005), however, regular crossing will not in general hold everywhere in the contract space due to precautionary labor effects. At any given contract with less than full coverage, high risk individuals supply more labor than low risks. If this effect is strong, the resulting higher level of consumption may reduce the high risks' marginal willingness to pay for insurance below that of the low risks due to decreasing risk aversion. The following lemma provides sufficient conditions for regular crossing even if labor supply is endogenous.

**Lemma 3.** *Regular crossing holds at a contract $(\beta, d) \in \mathcal{C}$ if either:*
*(i) the ratio $p_H/p_L$ is sufficiently large,*
*(ii) preferences exhibit CARA or a sufficiently small degree of DARA,*
*(iii) the prudence $\eta^G$ is sufficiently small,*
*(iv) the contract provides full coverage.*

*Proof.* See Netzer and Scheuer (2005), Appendix D. □

If no one of the conditions (i) to (iii) is satisfied, the indifference curve of a low-risk individual can be steeper than that of a high-risk individual in a contract with less than full coverage. On the other hand, regular crossing always holds at full coverage contracts. Together, these results show that the global single crossing property can be violated for indifference curves of individuals that differ in risk only. This possibility is the crucial difference between our model and the existing literature. The existence of precautionary labor effects can introduce countervailing incentives in the insurance market and prevent a simple ordering of the risks with respect to their marginal rate of substitution between coverage and premium.

We next turn to individuals of the same risk but different labor productivities. To get clear-cut results for this case, the following assumption is used.

**Assumption 2.** *Consumption is a normal good.*

**Lemma 4.** *Under Assumption 2 and DARA, the marginal rate of substitution at contract $(\beta, d)$ strictly decreases in productivity if $\beta < 1$. Under CARA or if $\beta = 1$, the marginal rate of substitution is always constant in productivity.*

*Proof.* In order to examine how productivity affects the marginal rate of substitution between coverage and premium given in (4), we need to evaluate the sign of

$$\frac{\mathrm{d}}{\mathrm{d}w_i} \left.\frac{\mathrm{d}d}{\mathrm{d}\beta}\right|_{V_{ij}^*=\bar{V}} = \left(L_{ij}^* + w_i \frac{\partial L_{ij}^*}{\partial w_i}\right) p_j(1-p_j) D \frac{u''(c_{ij}^0)u'(c_{ij}^1) - u''(c_{ij}^1)u'(c_{ij}^0)}{(p_j u'(c_{ij}^0) - (1-p_j)u'(c_{ij}^1))^2}. \tag{6}$$

It is immediate to show that $u''(c_{ij}^0)u'(c_{ij}^1) - u''(c_{ij}^1)u'(c_{ij}^0) = 0$ if absolute risk-aversion is constant or if the insurance contract provides full coverage so that $c_{ij}^0 = c_{ij}^1$. Note furthermore that

$u''(c^0_{ij})u'(c^1_{ij}) - u''(c^1_{ij})u'(c^0_{ij}) < 0$ in the case of decreasing absolute risk-aversion and $\beta < 1$. Then, (6) is negative if and only if $L^*_{ij} + w_i \partial L^*_{ij}/\partial w_i > 0$. By the Slutzky-decomposition, this is equivalent to

$$L^*_{ij} + w_i L^*_{ij}\left(-\frac{\partial L^*_{ij}}{\partial d}\right) + w_i \frac{\partial L^c_{ij}}{\partial w_i} > 0, \tag{7}$$

where $\partial L^c_{ij}/\partial w_i > 0$ denotes the pure substitution effect based on the Hicksian labor supply function $L^c_{ij}$ and $-\partial L^*_{ij}/\partial d < 0$ is the pure income effect. A sufficient condition for (7) to hold is therefore that $1 + w_i(-\partial L^*_{ij}/\partial d) > 0$, which is just saying that consumption is a normal good and hence implied by Assumption 2. □

Hence, under DARA, a low productivity individual's indifference curve will be steeper than the one of a high productivity individual of the same risk type in the interior of the contract space. Clearly, since this *local* property holds everywhere, it implies the *global* property of single crossing for indifference curves of individuals that differ only in productivity.

As we have seen, single crossing may be violated for individuals that differ only in risk. Of course it can also be violated for individuals that differ in *both* dimensions. For example, an $LL$-individual's marginal rate of substitution might well be larger than the one of an $HH$-individual somewhere in the interior of the contract space, while it is flatter at full coverage contracts according to the previous lemmas. This could occur even if labor supply was fixed, i. e. if income was exogenous, simply because productivity and risk affect the willingness to pay for insurance in opposite directions. Hence this violation of single crossing can also occur in the models of two-dimensional adverse selection by Smart (2000), Wambach (2000) and Villeneuve (2003). In our model, however, single crossing can be violated even for $HL$- and $LH$-individuals. This will occur if the reaction of labor supply to risk is sufficiently large, so that it dominates the effect of productivity as discussed in Lemma 4.

With precautionary labor effects, we cannot generally exclude the possibility that any two indifference curves cut more than twice. This, however, would require that utility functions exhibit highly irregular patterns. We shall exclude this with the following assumption, which is a relaxation of the well-known Spence-Mirrlees condition.

**Assumption 3.** *Any two indifference curves of individuals that have different damage probabilities cut at most twice.*

A graphical clarification of this double crossing property is provided by Smart (2000). For any two types that differ in risk (and possibly in productivity) the contract space can be divided in two regions; one in which the high risks have a larger marginal rate of substitution[13] and one in which the opposite holds. The two regions are separated by a line defined by the points of tangency of the two types' indifference curves. Each indifference curve cuts this line at most once.

## 3.2 The Screening Game

The screening game that we consider in the following goes back to Rothschild and Stiglitz (1976). It consists of two stages. There is a large number of risk-neutral firms who first decide whether to enter the market or not. In case they enter, they decide which contract to offer. Each entering firm offers exactly one contract $(\beta, d) \in \mathcal{C}$. The expected profit of such a contract if it is purchased by $a$ low-risk and $b$ high-risk individuals is given by

$$\pi(\beta, d, a, b) = a[d - p_L \beta D] + b[d - p_H \beta D]. \quad (8)$$

Each entering firm pays a fixed entry cost $E > 0$. At the second stage, customers simultaneously choose labor supply and select their preferred contract from the set of offered contracts. In case of indifference between different contracts, they opt for the larger coverage contract.[14] If several firms offer the same contract, customers split equally between them. Finally, the risk is realized, insurance payments are made and consumption takes place.

We are interested in characterizing the set of subgame-perfect Nash equilibria of the described game. Of course, each insurance company must earn nonnegative profits in any such equilibrium. Second, it may not be possible for an inactive firm to earn positive profits by entering the market. This implies that there may be no contract which earns profits larger than $E$ if offered in addition by a new entrant. As it will turn out, the equilibrium set of contracts can contain contracts which earn positive profits. Competition does not eliminate such contracts, because any contract which is slightly more attractive to the consumers would also attract bad risk types and become unprofitable. The existence of fixed entry costs therefore solves the problem of unlimited entry of firms. As more and more firms enter, less customers will purchase from each of them, driving down the firms' profits. Since we are interested in perfectly competitive markets, however, we examine the limit as $E \to 0$.[15]

[13] This region includes all full coverage contracts.

[14] This convention follows the approach of Smart (2000). We will discuss alternative assumptions where appropriate.

[15] This approach is due to Smart (2000). See De Meza and Webb (2001) for an alternative way

Equilibria can be categorized according to properties of the set of contracts which are offered. The following definition gives such a categorization.

**Definition 2.** *An equilibrium is strictly pooling if all individuals purchase the same contract. It is weakly pooling if the $HH$-individuals and/or the $LH$-individuals purchase a contract which is also purchased by low risks. It is separating otherwise.*

First, this definition categorizes equilibria only with respect to which damage risks purchase which contract. This is because the major interest in terms of the insurance market is how different risks select themselves, or are "screened". Second, the focus on high risks for the definition of pooling will prove useful later. Pooling requires all individuals of at least one type $iH$ to be bunched in contracts with low risks. Note finally that in any weakly but not strictly pooling equilibrium at least two different contracts will be offered because not all individuals purchase the same contract. Since, however, at least one high and one low risk type must be bunched in one contract, it can contain at most three different contracts.

# 4 One-Dimensional Heterogeneity

We first assume that individual productivities are publicly observable. In that case, insurance companies offer contracts conditional on productivity, so that an insurance market for each productivity group $w_i$ emerges. Individuals within each of these markets differ only in risk.[16] We consider one such market, in which the concepts of weakly and strictly pooling equilibria coincide. We proceed as follows. First, general properties of equilibria are proven. More specific properties will depend on the exact constellations of marginal rates of substitution in the contract space, and will be described in the following corollary. The results of this section will then be compared to the standard Rothschild-Stiglitz model, where individuals differ only in risk. We point out the differences in the form of testable hypotheses at the end of the section.

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to deal with the problem of positive profits and perfect competition.

[16]The same results obtain if $w_L = w_H$ so that there is no heterogeneity with respect to productivity, or if risk-aversion is constant. In that case, productivity has no influence on indifference curves by Lemma 3.

## 4.1 Separating Equilibria

**Proposition 1.** *In any equilibrium in the $w_i$-market two contracts are offered:*

$$A = (\beta^A, d^A) = (1, p_H D),$$
$$B_i = (\beta^{B_i}, d^{B_i}) = argmax\ V_{iL}(\beta, d) \quad s.t. \quad (i)\ V_{iH}(A) = V_{iH}(\beta, d),$$
$$(ii)\ \pi(\beta, d, n_{iL}, 0) \geq 0.$$

*The low risks purchase $B_i$, the high risks purchase $A$. Equilibrium exists if the average zero profit line of the market does not cut the $iL-$individual's indifference curve through $B_i$.*

*Proof.* First, a pooling equilibrium with a pooling contract $P = (\beta^P, d^P)$ cannot exist. Assume to the contrary that it did, implying $\pi(\beta^P, d^P, n_{iL}, n_{iH}) \geq 0$. For any contract $C = (\beta^C, d^C)$ let $\mathcal{B}_\epsilon(C) = \{(\beta, d) \in \mathcal{C} | (\beta^C - \beta)^2 + (d^C - d)^2 < \epsilon^2\}$, $\epsilon > 0$, be the $\epsilon$-ball around $C$ in $\mathcal{C}$. If $\text{MRS}_{iL} \neq \text{MRS}_{iH}$ in $P$, then for any $\epsilon > 0$, $\exists P' \in \mathcal{B}_\epsilon(P)$ s.t. $V_{iL}(P') > V_{iL}(P)$ and $V_{iH}(P') < V_{iH}(P)$. If offered in addition to $P$, its profits $\pi(\beta^{P'}, d^{P'}, n_{iL}, 0)$ converge to $\pi(\beta^P, d^P, n_{iL}, 0) > 0$ as $\epsilon \to 0$, a contradiction to equilibrium. If $\text{MRS}_{iL} = \text{MRS}_{iH}$ in $P$ then $\beta^P < 1$ due to regular crossing at full coverage. But then for any $\epsilon > 0$, $\exists P' \in \mathcal{B}_\epsilon(P)$ s.t. $P' > P$, $V_{iL}(P') > V_{iL}(P)$, $V_{iH}(P') > V_{iH}(P)$ and $P'$ earns $\pi(\beta^{P'}, d^{P'}, n_{iL}, n_{iH}) > 0$ if offered in addition to $P$, again a contradiction. This last property holds since $\text{MRS}_{iH} > p_H D > \bar{p}_i D$ in the interior of $\mathcal{C}$ and hence in $P$ ($\text{MRS}_{iH} = p_H D$ at full coverage and concavity of indifference curves), so that $P'$ can be chosen above the pool's zero profit line given that $P$ was not below that line.

Hence risks will be separated. Contract $A$ for $iH$-individuals follows since for any $A' \neq A$ satisfying $\pi(\beta^{A'}, d^{A'}, 0, n_{iH}) \geq 0$, $\exists A'' \in \mathcal{B}_\epsilon(A')$ s.t. $V_{iH}(A'') > V_{iH}(A')$ and $\pi(\beta^{A''}, d^{A''}, x, n_{iH}) > 0$ for any $x \geq 0$ (the notation including $x$ captures that $A''$ might also attract low risks). Contract $B_i$ for $iL$-individuals follows since for any $B'_i \neq B_i$ satisfying $\pi(\beta^{B'_i}, d^{B'_i}, n_{iL}, 0) \geq 0$ and incentive compatibility $V_{iH}(A) \geq V_{iH}(\beta^{B'_i}, d^{B'_i})$, $\exists B''_i \in \mathcal{B}_\epsilon(B'_i)$ s.t. $V_{iL}(B''_i) > V_{iL}(B'_i)$, it still satisfies incentive compatibility, and $\pi(\beta^{B''_i}, d^{B''_i}, n_{iL}, 0) > 0$.

The existence condition makes sure that $\nexists Q$ s.t. $V_{iH}(Q) > V_{iH}(A)$, $V_{iL}(Q) > V_{iL}(B_i)$ and $\pi(\beta^Q, d^Q, n_{iL}, n_{iH}) > 0$, as shown by Rothschild and Stiglitz (1976). □

Proposition 1 shows that equilibrium will always by separating. However, more specific results obtain. A crucial distinction arises depending on whether or not the individuals' indifference curves exhibit regular crossing at the contract where the high risks' indifference curve through $A$ intersects the low risks' zero profit line.

**Corollary 1.** *The contract $B_i$ earns positive profits if and only if* $\text{MRS}_{iH} < \text{MRS}_{iL}$ *(irregular crossing) in the contract where the high risks' indifference curve through $A$ intersects the low risks' zero profit line.*

*Proof.* Under regular crossing in the respective contract, the constraint $(ii)$ in the definition of $B_i$ is binding. This holds since the double crossing assumption implies regular crossing in any larger contract $C$ satisfying $V_{iH}(A) = V_{iH}(C)$, so that the corner contract where $(ii)$ is binding maximizes $V_{iL}$. Under irregular crossing, the contract that maximizes $V_{iL}$ is larger than the corner contract. Regular crossing at full coverage together with the double crossing assumption then implies that $B_i$ is the unique point of tangency of the two types' indifference curves, where $(ii)$ is slack. □

The separating equilibria are illustrated in Figure 1. The left panel depicts the situation in which both equilibrium contracts earn zero profits. It corresponds to the standard Rothschild-Stiglitz contract set. The right panel depicts a situation where $B_i$ earns positive profits, which requires irregular crossing and hence cannot occur in the canonical model. As more and more firms enter and offer the profitable $B_i$, each firm obtains a smaller share of the profits until further entry becomes unprofitable. Our results refer to the limit as the entry costs $E$ converge to zero.

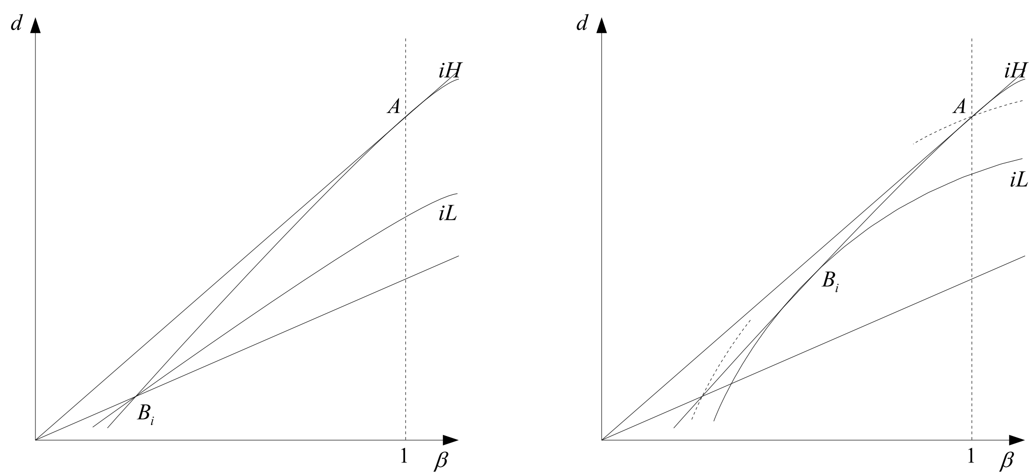


Figure 1: Observable Productivities

Note finally that imperfect separation might occur in the irregular crossing-case whenever the assumption is dropped that individuals who are indifferent pick the larger coverage contract. In the case illustrated in the right panel of Figure 1, a share $\gamma_i \leq \bar{\gamma}_i$ of the $iH$-individuals might instead purchase contract $B_i$, where $\bar{\gamma}_i$ is implicitly defined by $\pi(\beta^{B_i}, d^{B_i}, n_{iL}, \bar{\gamma}_i n_{iH}) = 0$.This consideration carries over to all following cases in which positive profit contracts exist in equilibrium. Then, some share of the indifferent customers bounded above by a zero profit condition might always pick the smaller coverage contract.

In sum, even without assuming two-dimensional heterogeneity, our screening model with endogenous labor supply can explain deviations from the standard Rothschild-Stiglitz model. First, contracts with positive profits are possible in equilibrium. Second, low risks may pay actuarially unfair premiums. For purposes of empirical testing, however, it may be of interest to derive predictions of our model which allow to distinguish it from the standard Rothschild-Stiglitz model even if all equilibrium contracts earn zero profits. In fact, there are such implications of our model. First, by Lemma 4, our model predicts a negative correlation between productivity and the low risks' insurance coverage. This implies that, second, productivity shocks should have a larger effect on low than on high risks' labor supply since the former are in addition affected by the precautionary effect from Lemma 1.

# 5 Two-Dimensional Heterogeneity

In this section, we assume that both individual characteristics, risk and productivity, cannot be observed by the insurance companies.[17] This implies that all four types of individuals act on the same market. We also assume that preferences exhibit DARA, so that differences in productivity are indeed relevant. We proceed as follows. We again prove general properties of possible (separating and pooling) equilibria. More specific equilibrium properties will then again depend on the exact constellations of marginal rates of substitution in the contract space, and will be given in the subsequent corollaries. In particular, we will highlight predictions that distinguish our model from the contributions of Smart (2000), Wambach (2000) and Villeneuve (2003).

## 5.1 Separating Equilibria

**Proposition 2.** *In any separating equilibrium the contracts* $A = (\beta^A, d^A) = (1, p_H D)$ *and* $B_i$, $i = L, H$, *are offered, where*

$$B_i = (\beta^{B_i}, d^{B_i}) = argmax\ V_{iL}(\beta, d) \quad s.t. \quad \begin{array}{l} (i)\ V_{HH}(A) = V_{HH}(\beta, d), \\ (ii)\ \pi(\beta, d, n_{iL}, 0) \geq 0. \end{array}$$

*Low risks with productivity* $w_i$ *purchase* $B_i$. *All high risks purchase* $A$.

*Proof.* The definition of separation requires that no contract is purchased by different risks, i.e. that there exists (at least) a contract only purchased by high risks. Existence of $A$ then follows as in the proof of Proposition 1. Since $A = \text{argmax}\ V_{iH}(\beta, d)$ s.t. $\pi(\beta, d, 0, x) \geq 0$ for any $x > 0$,

[17] This would, for example, be a natural information assumption in a model of optimal taxation in the presence of risk, where the government cannot observe productivities and risk but has to rely on the observation of realized income (see Netzer and Scheuer (2005)). Private insurance markets in such models work as described here.

there can be no other (weakly) profitable contract purchased only by high risks, i.e. both high risk types purchase $A$. Lemma 4 implies that the $HH$-individuals' indifference curve through $A$ is then relevant for incentive compatibility. The contracts $B_i$, $i = L, H$, follow as in the proof of Proposition 1. □

Proposition 2 does not mention existence conditions in the spirit of the condition given in Proposition 1, where existence required that the average zero profit line of the whole market does not cut the low risks' indifference curve through their equilibrium contract. Similar conditions have to be satisfied in the present case as well, but are more complicated. It has to be checked which of the four types would be attracted away from the equilibrium candidate by a new contract. Profitability of such a contract is then calculated by comparing its position relative to the relevant zero profit line. Opposed to the standard case where there is only one zero profit line for the pool, we can have several different pools and corresponding zero profit lines here. Hence, there will be more than one existence condition. It turns out that four such conditions have to be satisfied in our model. They are derived and discussed in the Appendix.

As before, the specific characteristics of the separating equilibrium will depend on the slopes of the low risks' indifference curves at the contract where the $HH$-type's indifference curves through $A$ intersect the low-risks' zero profit line.

**Corollary 2.** *Contract $B_i$ earns positive profits if and only if* $\text{MRS}_{HH} < \text{MRS}_{iL}$ *in the contract where the $HH-$individuals' indifference curve through $A$ intersects the low risks' zero profit line. If $\pi(\beta^{B_L}, d^{B_L}, n_{iL}, 0) > 0$ then $B_L > B_H$. Otherwise $B_L = B_H$.*

*Proof.* The first statement follows as in the proof of Corollary 1. The second statement uses Lemma 4 in addition. If $\pi(\beta^{B_L}, d^{B_L}, n_{iL}, 0) = 0$ and hence $\text{MRS}_{LL} \leq \text{MRS}_{HH}$ in the corner contract $B_L$, it follows that $\text{MRS}_{HL} < \text{MRS}_{LL} \leq \text{MRS}_{HH}$ there by Lemma 4, implying that $B_L = B_H$. If $\pi(\beta^{B_L}, d^{B_L}, n_{iL}, 0) > 0$, i.e. constraint $(ii)$ is slack in the definition of $B_L$, the contract $B_L$ is defined by the point of tangency of the $HH$-individuals' indifference curve through $A$ and an indifference curve of the $LL$-individuals. By Lemma 4, $\text{MRS}_{HL} < \text{MRS}_{LL} = \text{MRS}_{HH}$ in $B_L$. The double crossing assumption then implies that $B_H < B_L$, where $B_H$ can either be a point of tangency ($\pi(\beta^{B_H}, d^{B_H}, n_{HL}, 0) > 0$) or a corner solution ($\pi(\beta^{B_H}, d^{B_H}, n_{HL}, 0) = 0$). □

The separating equilibrium is illustrated in Figure 2, where the three different cases are depicted. The left panel illustrates the case where all contracts earn zero profits, since the low risks' indifference curves are flattest in contract $B_L = B_H$. The middle panel illustrates that contract $B_L$ moves upwards on the $HH$-individuals' indifference curve through $A$ and thus earn positive profits. This requires $\text{MRS}_{LL} >$

$\text{MRS}_{HH}$ in $B_H$, a case that can occur even if labor supply is exogenous as discussed in the introduction. Hence, the cases depicted in the first two panels can already occur in the models of Smart (2000), Wambach (2000) and Villeneuve (2003). The last case, however, requires $\text{MRS}_{HL} > \text{MRS}_{HH}$, i.e. irregular crossing, which is unique to our model. Therefore, the contract $B_H$ may also move up along the $HH$-indifference curve through $A$ and earn profits in equilibrium. Hence, in contrast to the models with exogenous income heterogeneity, our model is able to explain actuarially unfair premiums and a larger coverage even for $HL$-individuals.

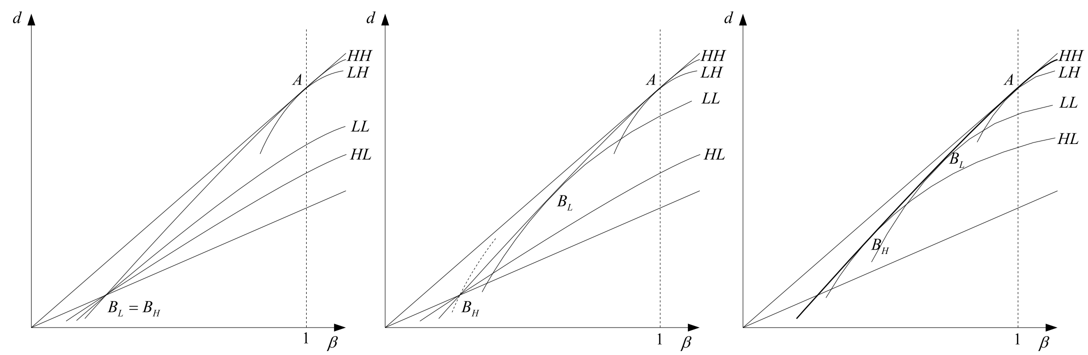


Figure 2: Unobservable Productivities / Separation

A number of observations are worth noting at this point. First, as mentioned in footnote 4, Chiappori et al. (2006) claim that their "non-increasing profits property", which implies that profits do not increase with coverage in the equilibrium set of contracts, is a general property of equilibrium in competitive insurance markets. However, our findings show that this may not be the case. Indeed, the middle panel of figure 3 provides an example of an equilibrium where profits first increase and then decrease with coverage. Despite this deviation form the non-increasing profits property, the separating equilibria will always exhibit a positive correlation between coverage and risk. This result will not, however, carry over to the possible pooling equilibria discussed in the next subsection. Second, for the purpose of empirically distinguishing our setting from that considered by Smart (2000), Wambach (2000) and Villeneuve (2003), it may be useful to note that our model predicts a negative correlation between $w_H$ and the $HL$-types' insurance coverage. Moreover, an increase in $w_H$ should have a larger effect on $HL$- than on $HH$-individuals' labor supply due to the additional precautionary effect.

## 5.2 Pooling Equilibria

**Lemma 5.** *In any pooling equilibrium, the $LH-$individuals will be separated and purchase $A = (\beta^A, d^A) = (1, p_H D)$. A strictly pooling equilibrium therefore does not exist.*

*Proof.* Assume to the contrary that a pooling equilibrium exists in which the $LH$-individuals are bunched in a contract $P$ with low risks, and $P$ earns nonnegative profits. Assume first that the other high risks ($HH$) purchase a different contract $C$. It will then hold that $V_{HH}(C) > V_{HH}(P)$, since $V_{HH}(C) = V_{HH}(P)$ would imply $C > P$ and contradict $V_{LH}(P) > V_{LH}(C)$, due to Lemma 4. Hence any contract $P' \in \mathcal{B}_\epsilon(P)$ will not attract $HH$-individuals for $\epsilon$ small enough. Next, for all low risk types $iL$ that purchase $P$ it has to hold that $\text{MRS}_{iL} = \text{MRS}_{LH}$ in $P$, since otherwise for any $\epsilon > 0$, $\exists P' \in \mathcal{B}_\epsilon(P)$ s.t. $V_{iL}(P') > V_{iL}(P)$ for at least one of those low risk types, $V_{LH}(P') < V_{LH}(P)$ and $\pi(\beta^{P'}, d^{P'}, x, 0) > 0$, for any $x > 0$ (with the reason given in the proof of Proposition 1). Hence positive profits are earned if $P'$ is offered in addition to $P$, a contradiction. Thus $\beta^P < 1$ by Lemmas 3 and 4. Since $\text{MRS}_{HL} < \text{MRS}_{LL}$ at any such contract, only one low risk type $iL$ purchases $P$. But then for any $\epsilon > 0$, $\exists P' \in \mathcal{B}_\epsilon(P)$ s.t. $P' > P$, $V_{iL}(P') > V_{iL}(P)$, $V_{LH}(P') > V_{LH}(P)$ and $\pi(\beta^{P'}, d^{P'}, n_{iL}+x, n_{LH}) > 0$, for any $x \geq 0$, again for the reason described in the proof of Proposition 1.

Assume next that the $HH$-individuals purchase $P$ as well. $\text{MRS}_{HH} \leq \text{MRS}_{LH}$ holds in $P$ due to Lemma 4. For all low risk types $iL$ purchasing $P$, $\text{MRS}_{HH} \leq \text{MRS}_{iL} \leq \text{MRS}_{LH}$ has to be satisfied in $P$, since otherwise for any $\epsilon > 0$ $\exists P' \in \mathcal{B}_\epsilon(P)$ s.t. $V_{iL}(P') > V_{iL}(P)$ for at least one of those low risk types, $V_{kH}(P') < V_{kH}(P)$ for both $k = L, H$, and $\pi(\beta^{P'}, d^{P'}, x, 0) > 0$, for any $x > 0$. Hence $\beta^P < 1$ and for any $\epsilon > 0$, $\exists P' \in \mathcal{B}_\epsilon(P)$ s.t. $P' > P$, $V_{ij}(P') > V_{ij}(P)$ for all types $ij$ that purchase $P$, and $P'$ earns positive profits if offered in addition. This again holds since $\text{MRS}_{HH} > p_H D$ in the interior of $\mathcal{C}$ and hence in $P$ (Lemma 2), so that $P'$ can be chosen above the pool's zero profit line given that $P$ was not below that line. Therefore, $LH$-individuals cannot be pooled with low risks. Existence of contract $A$ follows as in the proof of Proposition 1. □

Given that there cannot be a strictly pooling equilibrium by Lemma 5, three possible candidates remain for a pooling equilibrium. In a "2–contract–equilibrium", the $HH$-individuals purchase the same contract as all the low risks. In addition, there could be two different "3–contract–equilibria" in which one of the two low risk types drops out of the pooling contract. The following three propositions characterize these three types of equilibria. Each is followed by a corollary that summarizes the equilibrium's profit properties depending on the specific relations between the types' marginal rates of substitution between coverage and premium.

**Proposition 3.** *In any weakly pooling "2-contract-equilibrium", contracts $A$ and $P^*$ are offered, where*

$$P^* = (\beta^{P^*}, d^{P^*}) = argmax\, V_{LL}(\beta, d) \quad s.t. \quad (i)\ V_{LH}(A) = V_{LH}(\beta, d),$$
$$(ii)\ \pi(\beta, d, n_{LL} + n_{HL}, n_{HH}) \geq 0.$$

*$LH-$individuals purchase $A$, all others purchase $P^*$. Existence requires $MRS_{HL} \geq MRS_{HH}$ (irregular crossing) in $P^*$.*

*Proof.* The pooling contract $P^*$ in any 2-contract-equilibrium must satisfy constraint (ii). In addition, given that contract $A$ is offered according to Lemma 5, $P^*$ must satisfy $V_{LH}(A) \geq V_{LH}(P^*)$. Assume $V_{LH}(A) > V_{LH}(P^*)$ so that any contract $P' \in \mathcal{B}_\epsilon(P^*)$ will not attract the $LH$-individuals for $\epsilon$ small enough. But since $\text{MRS}_{LL} > \text{MRS}_{HL}$ if $\beta^{P^*} < 1$ and $\text{MRS}_{LL} = \text{MRS}_{HL} < \text{MRS}_{HH}$ if $\beta^{P^*} = 1$, $\text{MRS}_{iL} \neq \text{MRS}_{HH}$ holds in $P^*$ for at least one low risk type $iL$. Therefore, for any $\epsilon > 0$, $\exists P' \in \mathcal{B}_\epsilon(P^*)$ s.t. $V_{iL}(P') > V_{iL}(P^*)$ for at least one low risk type, $V_{HH}(P') < V_{HH}(P^*)$, and $\pi(\beta^{P'}, d^{P'}, x, 0) > 0$ for $x > 0$. Thus $V_{LH}(A) = V_{LH}(P^*)$ must hold, yielding constraint (i). Furthermore, $P^* < A$ and thus $\text{MRS}_{HL} < \text{MRS}_{LL}$, $\text{MRS}_{HH} < \text{MRS}_{LH}$ in $P^*$ according to Lemma 4. It also follows that $\text{MRS}_{LL} \leq \text{MRS}_{LH}$ in $P^*$ since otherwise for any $\epsilon > 0$, $\exists P' \in \mathcal{B}_\epsilon(P^*)$ s.t. $P' > P^*$, $V_{LL}(P') > V_{LL}(P^*)$, $V_{ij}(P') < V_{ij}(P^*)$ for all other types $ij$, and $\pi(\beta^{P'}, d^{P'}, n_{LL}, 0) > 0$.[18] If $\text{MRS}_{LL} < \text{MRS}_{LH}$ in $P^*$, then constraint (ii) must be binding. If not, $\exists P' \in \mathcal{B}_\epsilon(P^*)$ s.t. $P' < P^*$, $V_{LH}(P') < V_{LH}(P^*)$, $V_{ij}(P') > V_{ij}(P^*)$ for all other types $ij$, such that $\pi(\beta^{P'}, d^{P'}, n_{LL} + n_{HL}, n_{HH}) > 0$. Such a contract does not exist if $\text{MRS}_{LL} = \text{MRS}_{LH}$ in $P^*$, since any $P'$ for which $V_{LL}(P') > V_{LL}(P^*)$ also satisfies $V_{LH}(P') > V_{LH}(P^*)$. Constraint (ii) can thus be slack. Altogether, this is equivalent to saying that $P^*$ maximizes $V_{LL}$ subject to (i) and (ii).

The additional condition that $\text{MRS}_{HH} \leq \text{MRS}_{HL}$ in $P^*$ follows since otherwise for any $\epsilon > 0$ $\exists P' \in \mathcal{B}_\epsilon(P^*)$ s.t. $P' < P^*$, $V_{HL}(P') > V_{HL}(P^*)$, $V_{ij}(P') < V_{ij}(P^*)$ for all other types $ij$, and $P'$ earns positive profits if offered in addition. □

Since existence of this type of pooling equilibrium requires irregular crossing, it cannot exist in the models of two-dimensional heterogeneity but exogenous labor supply analyzed by Smart (2000), Wambach (2000) and Villeneuve (2003). As before, additional conditions have to be satisfied for existence. Notably, no contracts may exist that attract away profitable pools from the contract set described in the Proposition. A complete discussion of these conditions is provided in the Appendix. We proceed to show how more specific properties of the equilibrium depend on local crossing-properties of indifference curves.

**Corollary 3.** *Contract $P^*$ earns positive profits if and only if $\text{MRS}_{LH} < \text{MRS}_{LL}$ (irregular crossing) in the contract where the $LH$-individuals' indifference curve through $A$ intersects the zero profit line of the pool of all $LL-$, $HL-$ and $HH-$individuals.*

*Proof.* The proof follows exactly as for Corollary 1. □

The two possible cases described in the corollary are depicted in Figure 3, where the left panel refers to the case in which $P^*$ earns positive profits. In both cases,

[18] Note that similar arguments do not apply to *any* case in which the individuals' marginal rates of substitution differ in $P^*$. A contract $P' > P^*$ that attracts low risks might also attract the $LH$-individuals due to the binding incentive compatibility constraint.

the equilibrium is associated with a positive correlation between coverage and risk since only high risks obtain full insurance. In addition, profits are non-increasing with coverage since the full insurance contract $A$ always makes zero profits.

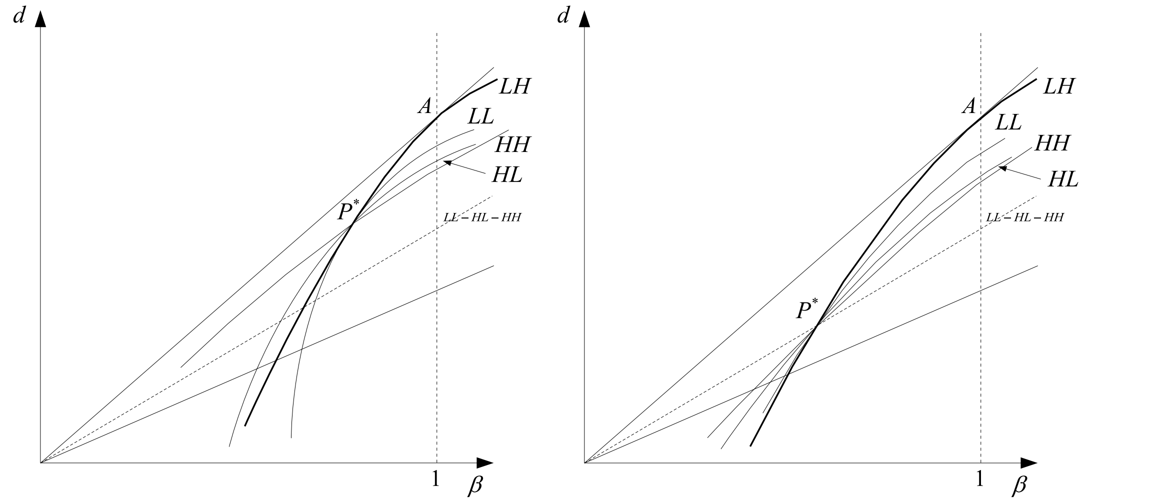


Figure 3: Unobservable Productivities / 2-Contract Pooling

The next proposition characterizes the pooling equilibrium where type $HL$ is separated, called a "type I" equilibrium.

**Proposition 4.** *In any weakly pooling "3-contract-equilibrium of type I", contracts $A$, $P^{**}$ and $C$ are offered, where*

$$P^{**} = (\beta^{P^{**}}, d^{P^{**}}) = argmax\ V_{LL}(\beta, d) \quad s.t. \quad (i)\ V_{LH}(A) = V_{LH}(\beta, d),$$
$$(ii)\ \pi(\beta, d, n_{LL}, n_{HH}) \geq 0.$$

$$C = (\beta^{C}, d^{C}) = argmax\ V_{HL}(\beta, d) \quad s.t. \quad (i)\ V_{HH}(P^{**}) = V_{HH}(\beta, d),$$
$$(ii)\ \pi(\beta, d, n_{HL}, 0) \geq 0.$$

*$LH-$individuals purchase $A$, the $LL-$ and $HH-$individuals purchase $P^{**}$, and the $HL-$individuals purchase $C$. Existence requires* $\text{MRS}_{HL} < \text{MRS}_{HH} \leq \text{MRS}_{LL}$ *in $P^{**}$.*

*Proof.* Obviously, $P^{**}$ has to satisfy (ii) as given in the proposition. Given Lemma 5, $V_{LH}(A) \geq V_{LH}(P^{**})$ also has to be satisfied. Assume $V_{LH}(A) > V_{LH}(P^{**})$, so that any $P' \in \mathcal{B}_\epsilon(P^{**})$ satisfies $V_{LH}(A) > V_{LH}(P')$ for $\epsilon$ small enough, hence does not attract the $LH$-individuals. Then, if $\text{MRS}_{LL} \neq \text{MRS}_{HH}$ in $P^{**}$, $\exists P' \in \mathcal{B}_\epsilon(P^{**})$ s.t. $V_{LL}(P') > V_{LL}(P^{**})$, $V_{HH}(P') < V_{HH}(P^{**})$ and $\pi(\beta^{P'}, d^{P'}, n_{LL} + x, 0) > 0$, for any $x \geq 0$. If $\text{MRS}_{LL} = \text{MRS}_{HH}$ in $P^{**}$ and therefore $\beta^{P^{**}} < 1$, $\exists P' \in \mathcal{B}_\epsilon(P^{**})$ s.t. $P' > P^{**}$, $V_{LL}(P') > V_{LL}(P^{**})$, $V_{HH}(P') > V_{HH}(P^{**})$, and

$\pi(\beta^{P'}, d^{P'}, n_{LL}+x, n_{HH}) > 0$, for any $x \geq 0$. Thus $V_{LH}(A) = V_{LH}(P^{**})$ holds. Also, $P^{**} < A$ and hence $\text{MRS}_{HH} < \text{MRS}_{LH}$, $\text{MRS}_{HL} < \text{MRS}_{LL}$ in $P^{**}$. Next, $\text{MRS}_{LL} \leq \text{MRS}_{LH}$ in $P^{**}$ since otherwise $\exists P' \in \mathcal{B}_\epsilon(P^{**})$ s.t. $P' > P^{**}$, $V_{LL}(P') > V_{LL}(P^{**})$, $V_{ij}(P') < V_{ij}(P^{**})$ for all other types, and $\pi(\beta^{P'}, d^{P'}, n_{LL}, 0) > 0$. If $\text{MRS}_{LL} < \text{MRS}_{LH}$ in $P^{**}$, then constraint (ii) must be binding. If not, $\exists P' \in \mathcal{B}_\epsilon(P^{**})$ s.t. $P' < P^{**}$, $V_{LH}(P') < V_{LH}(P^{**})$, $V_{ij}(P') > V_{ij}(P^{**})$ for all other types $ij$, such that $\pi(\beta^{P'}, d^{P'}, n_{LL}+x, n_{HH}) > 0$ for any $x \geq 0$. Such a contract does not exist if $\text{MRS}_{LL} = \text{MRS}_{LH}$ in $P^{**}$, and constraint (ii) can be slack. Altogether, this is just saying that $P^{**}$ maximizes $V_{LL}$ subject to (i) and (ii). Condition $\text{MRS}_{HH} \leq \text{MRS}_{LL}$ in $P^{**}$ has to be satisfied since otherwise $\exists P' \in \mathcal{B}_\epsilon(P^{**})$ s.t. $P' < P^{**}$, $V_{LL}(P') > V_{LL}(P^{**})$, $V_{kH}(P') < V_{kH}(P^{**}), k = L, H$, and $\pi(\beta^{P'}, d^{P'}, n_{LL}+x, 0) > 0$ for any $x \geq 0$.

By Lemma 4, $\text{MRS}_{HL} < \text{MRS}_{LL}$ in $P^{**}$ and single crossing within the productivity dimension. Therefore, contract $C \neq P^{**}$ for $HL$-individuals, for which $V_{HL}(C) \geq V_{HL}(P^{**})$ and $V_{LL}(C) \leq V_{LL}(P^{**})$ has to hold (incentive compatibility), must satisfy $C < P^{**}$. From $\text{MRS}_{HH} \leq \text{MRS}_{LL} \leq \text{MRS}_{LH}$ in $P^{**}$ and double crossing it follows that $V_{HH}(P^{**}) \geq V_{HH}(C)$ is the relevant incentive compatibility constraint for $C$. The contract $C$ then follows with the argument given for $B_i$ in the proof of Proposition 1. The condition $\text{MRS}_{HL} < \text{MRS}_{HH}$ in $P^{**}$ makes sure that indeed $C \neq P^{**}$. □

Note that the conditions on the marginal rates of substitution given in Proposition 4 do not require irregular crossing. Even with exogenous labor supply and two-dimensional heterogeneity, $\text{MRS}_{HH} \leq \text{MRS}_{LL}$ can occur since the respective individuals differ in both dimensions. The "3-contract equilibrium of type I" therefore exists in the models of Smart (2000), Wambach (2000) and Villeneuve (2003).[19] A discussion of existence conditions in the spirit of Rothschild and Stiglitz can be found in the Appendix. We proceed to illustrate how profits in equilibrium depend on local characteristics of the indifference curves.

**Corollary 4.** *Contract $P^{**}$ earns positive profits if and only if $\text{MRS}_{LL} > \text{MRS}_{LH}$ (irregular crossing) at the contract where the $LH-$individuals' indifference curve through $A$ intersects the zero profit line of the pool of all $LL-$ and $HH-$individuals. Contract $C$ earns positive profits if and only if $\text{MRS}_{HL} > \text{MRS}_{HH}$ (irregular crossing) at the contract where the $HH-$individuals' indifference curve through $P^{**}$ cuts the low risks' zero profit line.*

*Proof.* The proof follows exactly as for Corollary 1. □

While the discussed equilibrium can exist even with exogenous labor supply, positive profits can only occur with irregular crossing, hence only with endogenous labor supply. For simplicity, both panels of Figure 4 depict a situation in which $P^{**}$

[19] However, both Wambach (2000) and Villeneuve (2003) fail to realize this possibility. Wambach (2000) makes a mistake in his argument and therefore erroneously concludes that pooling equilibria do generically not exist. Villeneuve (2003) does not consider this possibility at all.

earns zero profits although this does not need to be the case by Corollary 4. $C$ also earns zero profits in the left panel, but positive profits in the right panel. As becomes clear from Figure 4, the "3-contract-equilibrium of type I" always implies a positive correlation between coverage and risk in aggregate. This is because the contracts $C$, $P^{**}$ and $A$ are ranked with respect to both coverage and average risk of the pool of customers. However, the non-increasing profits property used by Chiappori et al. (2006) to derive this positive correlation result is not necessarily satisfied. For instance, it is possible that $C$, the contract with the lowest coverage, earns zero profits, whereas $P^{**} > C$ earns positive profits.

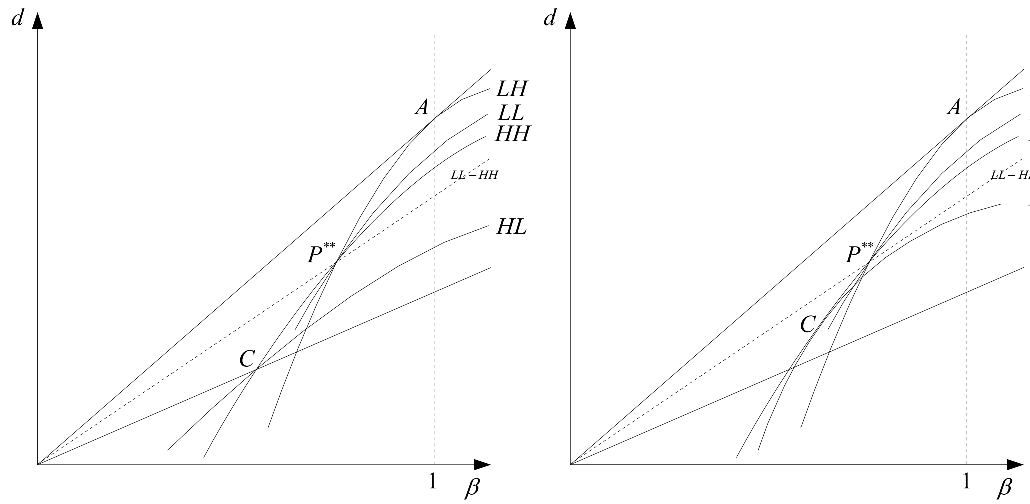


Figure 4: Unobservable Productivities / 3-Contract Pooling I

Finally, the last proposition characterizes the pooling equilibrium where type $LL$ is separated, called a "type II" equilibrium. As will turn out, it is particularly interesting due to the arising correlation between risk and coverage.

**Proposition 5.** *In any weakly pooling "3-contract-equilibrium of type II", contracts $A$, $P^{***}$ and $D$ are offered, where*

$$P^{***} = argmax\ V_{HL}(\beta, d) \quad s.t. \quad (i)\ V_{LH}(A) = V_{LH}(\beta, d), \\ (ii)\ \pi(\beta, d, n_{HL}, n_{HH}) \geq 0.$$

$$D = argmax\ V_{LL}(\beta, d) \quad s.t. \quad (i)\ V_{LH}(A) = V_{LH}(\beta, d).$$

*$LH-$individuals purchase $A$, the $HL-$ and the $HH-$individuals purchase $P^{***}$, and the $LL$-individuals purchase $D$. Existence requires $MRS_{HH} \leq MRS_{HL}$ and* $\mathrm{MRS}_{LH} < MRS_{LL}$ *in $P^{***}$ (both irregular crossing).*

*Proof.* The fact that $P^{***}$ has to satisfy (i) and (ii) as given the the proposition follows exactly as in the previous proof. Therefore $P^{***} < A$ and $\text{MRS}_{HH} < \text{MRS}_{LH}$, $\text{MRS}_{HL} < \text{MRS}_{LL}$ in $P^{***}$. Furthermore, $\text{MRS}_{HL} \leq \text{MRS}_{LH}$ has to hold in $P^{***}$, since otherwise $\exists P' \in \mathcal{B}_\epsilon(P^{***})$ s.t. $P' > P^{***}$, $V_{HL}(P') > V_{HL}(P^{***})$, $V_{kH}(P') < V_{kH}(P^{***}), k = L, H$, and $\pi(\beta^{P'}, d^{P'}, n_{HL} + x, 0) > 0$ for any $x \geq 0$. If $\text{MRS}_{HL} < \text{MRS}_{LH}$ in $P^{***}$, then constraint (ii) must be binding. If not, $\exists P' \in \mathcal{B}_\epsilon(P^{***})$ s.t. $P' < P^{***}$, $V_{LH}(P') < V_{LH}(P^{***})$ and $V_{ij}(P') > V_{ij}(P^*)$ for both $ij = HL, HH$, such that $\pi(\beta^{P'}, d^{P'}, n_{HL} + x, n_{HH}) > 0$ for any $x \geq 0$. Such a contract does not exist if $\text{MRS}_{HL} = \text{MRS}_{LH}$ in $P^{***}$, and constraint (ii) can be slack. Altogether, this is just saying that $P^{***}$ maximizes $V_{HL}$ subject to (i) and (ii). Condition $\text{MRS}_{HH} \leq \text{MRS}_{HL}$ in $P^{***}$ has to be satisfied since otherwise $\exists P' \in \mathcal{B}_\epsilon(P^{***})$ s.t. $P' < P^{***}$, $V_{HL}(P') > V_{HL}(P^{***})$ and $V_{kH}(P') < V_{kH}(P^{***}), k = L, H$, such that $\pi(\beta^{P'}, d^{P'}, n_{HL} + x, 0) > 0$ for any $x \geq 0$.

As in the proof of the previous proposition, $\text{MRS}_{HL} < \text{MRS}_{LL}$ in $P^{***}$ and single crossing in the productivity dimension implies $D > P^{***}$. It immediately follows that $\pi(\beta^D, d^D, n_{LL}, 0) > 0$. From $\text{MRS}_{HH} \leq \text{MRS}_{HL} \leq \text{MRS}_{LH}$ in $P^{***}$ and double crossing, the relevant incentive compatibility constraint for $D$ will be $V_{LH}(A) \geq V_{LH}(D)$. It must be binding and $\text{MRS}_{LL} = \text{MRS}_{LH}$ must hold in $D$, since otherwise $\exists D' \in \mathcal{B}_\epsilon(D)$ s.t. $V_{LL}(D') > V_{LL}(D)$, $V_{ij}(D') < V_{ij}(D)$ for all other types $ij$, and $\pi(\beta^{D'}, d^{D'}, n_{LL}, 0) > 0$. This, however, is just saying that $D$ maximizes $V_{LL}$ subject to the constraint $V_{LH}(A) = V_{LH}(D)$, as given in the proposition. $\text{MRS}_{LH} < \text{MRS}_{LL}$ in $P^{***}$ makes sure that indeed $D \neq P^{***}$. □

The existence of the type II equilibrium is unique to our model, since it requires irregular crossing of indifference curves. Most important is the fact that $D$, purchased by low risks that have a low productivity, has a larger coverage $\beta^D$ and premium $d^D$ than the pooling contract $P^{***}$. Hence, if $n_{LH}$ and $n_{HL}$ are sufficiently small, low risk individuals, which then mainly consist of $LL$-types, purchase on average more insurance (contract $D$) than high risks, which are mainly $HH$-types who purchase $P^{***} < D$. This gives rise to a negative correlation between risk and coverage in equilibrium and might help to explain the empirical puzzle that the positive correlation between risk and coverage predicted by the previous screening models is not observed although adverse selection seems to be a relevant phenomenon in insurance markets. Most interestingly, this negative correlation result is obtained without assuming any deviations from perfect competition and without assuming a one-dimensional structure with only two types as in De Meza and Webb (2001) and Jullien et al. (2006). It is simply based on the possibility of irregular crossing, which in turn naturally results from our setup with two-dimensional heterogeneity and endogenous labor supply.

A discussion of the existence conditions for this type of equilibrium is relegated to the Appendix. We proceed to illustrate how profits in equilibrium depend on local characteristics of the indifference curves.

**Corollary 5.** *Contract $P^{***}$ earns positive profits if and only if* $\mathrm{MRS}_{HL} > \mathrm{MRS}_{LH}$ *(irregular crossing) at the contract where the $LH-$individuals' indifference curve through $A$ intersects the zero profit line of the pool of all $HL-$ and $HH-$individuals. Contract $D$ always earns positive profits.*

*Proof.* The proof follows exactly as for Corollary 1. □

The two possible cases described in the corollary are depicted in Figure 5. As Corollary 5 and the figure make clear, the non-increasing profits assumption used by Chiappori et al. (2006) to derive the positive correlation property is again not satisfied. In fact, $D$ will always make more profits per capita than $P^{***}$ although $D > P^{***}$ since it is only bought by low risks. Thus, the non-increasing profits property cannot be considered as a general characteristic of equilibrium in competitive insurance markets as soon as multidimensional heterogeneity and unobserved actions are accounted for.

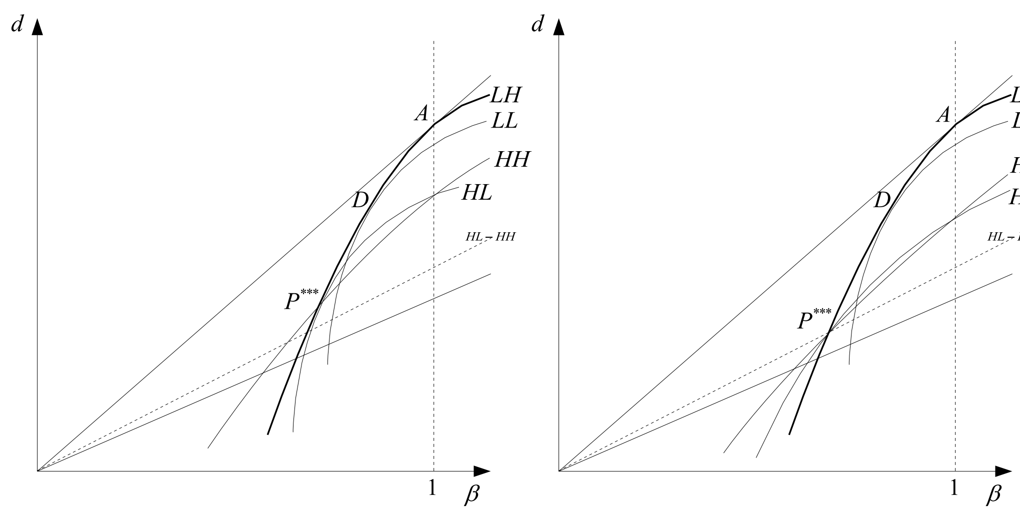


Figure 5: Unobservable Productivities / 3-Contract Pooling II

## 6 Conclusion

Based on recent empirical findings, the theoretical literature on adverse selection has started to realize that screening in most relevant real-world situations is associated with more than one dimension of privately known heterogeneity, and that the resulting countervailing incentives significantly alter the nature of equilibrium compared to the standard model of Rothschild and Stiglitz (1976). These models, however, typically assume that all dimensions of heterogeneity are given exogenously. For instance, the contributions by Smart (2000), Wambach (2000) and Villeneuve (2003) consider competitive insurance markets where individuals differ in both risk and risk preference. Unfortunately, it is not possible to find equilibria in these settings where the correlation between risk occurrence and insurance coverage is zero or negative, a phenomenon that has frequently been observed in empirical studies.

In this paper, we asked the question how insurance market equilibrium may look like if heterogeneity in some dimensions is not given exogenously but arises from the individuals' choices. As a natural example of such a situation, we considered a model where individuals not only differ in risk and select an insurance contract, but also choose their labor supply endogenously, which affects their income and hence risk attitude. While it may not be obvious at first glance why this endogeneity should be relevant, it turned out that the interdependency between insurance market equilibrium and labor supply leads to economic effects that have an impact on possible equilibrium configurations. Notably, it allows for "irregular crossing" in the sense that, among individuals who exogenously *only* differ in risk, high risk individuals have the *lower* marginal willingness to pay for insurance than low risks since they supply more labor and hence are less risk averse.

We show that this possibility will generally lead to equilibria with (i) smaller correlations between risk and coverage than in the standard models, and (ii) positive profit contracts. The latter result might lead to imperfectly separating equilibria even in the simple case of one-dimensional heterogeneity. If individuals differ in both their risk and productivity, equilibria can (i) pool different risk types in one contract, (ii) violate the non-increasing profits property of Chiappori et al. (2006), and therefore (iii) exhibit a zero or negative correlation between risk and coverage. Interestingly, this latter result provides an explanation for the empirical findings without assuming non-competitive insurance markets or special restrictions on the structure of heterogeneity as in De Meza and Webb (2001) and Jullien et al. (2006).

Our model raises a number of issues for further research. First, our informational assumption that risk, productivity and labor supply are privately known by the individuals may make our model a helpful tool for the analysis of policy questions such as taxation under risk and social insurance. Models addressing these issues need to combine multidimensional heterogeneity with the endogenous choice of private insurance and labor supply. We show how this set of assumptions affects the working of insurance markets. Natural questions to ask are about the effects of labor taxes or social insurance in this framework, and, more generally, about the efficiency properties of the equilibria that arise in our model.

Furthermore, as pointed out above, the possibility of irregular crossing is the driving force behind our novel results on competitive screening equilibria. Our model of insurance markets with two-dimensional heterogeneity and endogenous labor supply is just one - though certainly natural - example of a situation where irregular crossing can arise. Our results extend, however, to other settings of competitive screening with irregular crossing preferences. Generally, such preferences can endogenously result from some unobserved decision that does not affect the agent's risk, but only risk aversion. This may not only be a relevant phenomenon to be accounted for in models of insurance, but also of credit markets, portfolio choice, or labor contracts.

# References

Cardon, J., and Hendel, I. (2001), "Asymmetric information in health insurance: Evidence from the national medical expenditure survey," *RAND Journal of Economics,* 32, 408–427.

Cawley, J., and Philipson, T. (1999), "An empirical examination of information barriers to trade in insurance," *American Economic Review,* 89, 827–846.

Chassagnon, A., and Chiappori, P.A. (1997), "Insurance under moral hazard and adverse selection: The case of pure competition," *Discussion Paper, DELTA, Paris.*

Chiappori, P.A., and Salanié, B. (2000), "Testing for asymmetric information in insurance markets," *Journal of Political Economy,* 108, 56–78.

Chiappori, P.A., Jullien, B., Salanié, B., and Salanié, F. (2006), "Asymmetric information in insurance: General testable implications," *RAND Journal of Economics, forthcoming.*

De Donder, P., and Hindriks, J. (2006), "Does propitious selection explain why riskier people buy less insurance?," *Discussion Paper 2006-17, Université catholique de Louvain.*

De Meza, D., and Webb, D.C. (2001), "Advantageous selection in insurance markets," *RAND Journal of Economics,* 32, 249–262.

Eaton, J., and Rosen, H. S. (1980), "Labor supply, uncertainty and efficient taxation," *Journal of Public Economics,* 14, 365–374.

Finkelstein, A., and McGarry, K. (2006), "Multiple dimensions of private information: Evidence from the long-term care insurance market," *American Economic Review, forthcoming.*

Finkelstein, A., and Poterba, J. (2004), "Adverse selection in insurance markets: Policyholder evidence from the uk annuity market," *Journal of Political Economy,* 112, 183–208.

Hartwick, J. M. (2000), "Labor supply under wage uncertainty," *Economics Letters,* 68, 319–325.

Hemenway, D. (1990), "Propitious selection," *Quarterly Journal of Economics,* 105, 1063–1069.

—— (1992), "Propitious selection in insurance markets," *Journal of Risk and Uncertainty,* 5, 247–251.

Jullien, B., Salanié, B., and Salanié, F. (2006), "Screening risk-averse agents under moral hazard: Single-crossing and the CARA case," *Economic Theory, forthcoming.*

Kimball, M. S. (1990), "Precautionary saving in the small and in the large," *Econometrica,* 58, 53–73.

Low, H. (2005), "Self-insurance in a life-cycle model of labor supply and savings," *Review of Economic Dynamics,* 8, 945–975.

Netzer, N., and Scheuer, F. (2005), "Taxation, insurance and precautionary labor," *DIW Discussion Paper No. 516.*

Parker, S. C., Belghitar, Y., and Barmby, T. (2005), "Wage uncertainty and the labor supply of self-employed workers," *Economic Journal,* 115, 190–207.

Pratt, J. W. (1964), "Risk aversion in the small and in the large," *Econometrica,* 32, 122–136.

Rothschild, M., and Stiglitz, J. (1976), "Equilibrium in competitive insurance markets: An essay in the economics of incomplete information," *Quarterly Journal of Economics,* 90, 629–649.

Smart, M. (2000), "Competitive insurance markets with two unobservables," *International Economic Review,* 41, 153–169.

Stewart, J. (1994), "The welfare implications of moral hazard and adverse selection in competitive insurance markets," *Economic Inquiry,* 32, 193–208.

Villeneuve, B. (2003), "Concurrence et antisélection multidimensionelle en assurance," *Annales d'Economie et de Statistique,* 69, 119–142.

Wambach, A. (2000), "Introducing heterogeneity in the rothschild-stiglitz model," *Journal of Risk and Insurance,* 67, 579–592.

# 7 Appendix

In section 5, equilibria were characterized but the question was not addressed whether such equilibria in fact exist. In this appendix, we provide necessary and sufficient conditions for the existence of the equilibria. As in the model by Rothschild and Stiglitz (1976), the fundamental condition for existence is that there is no contract outside the equilibrium set of contracts that attracts a profitable pool of individuals. In looking for such potentially profitable deviations, we can confine ourselves to the area between the zero profit lines of the high and low risks. Clearly, a contract below the low risks' zero profit line could never be profitable. Contracts above the high risks' zero profit line, in turn, would not attract any individual given the equilibria from section 5.

Figure 6 illustrates this area. The thick black lines represent the high and low risks' zero profit lines. We first turn to the separating equilibria defined in Proposition 2. Figure 6 shows the indifference curves of the four types through the contracts $A$ and $B_L = B_H$ of the separating equilibrium for the case that all contracts make zero profits (see Corollary 2). Based on this graphical representation, the necessary and sufficient conditions for the existence of this equilibrium can be stated as follows:

**Corollary 6.** *The separating equilibrium where all contracts make zero profits defined in Corollary 2 exists if and only if the conditions from Corollary 2 are satisfied and there is no contract E*

*(i) in area I in figure 6 such that* $\pi(\beta^E, d^E, n_{LL}, n_{HH}) > 0$*,*

*(ii) in area II such that* $\pi(\beta^E, d^E, n_{HL} + n_{LL}, n_{HH}) > 0$*,*

*(iii) in area III such that* $\pi(\beta^E, d^E, n_{LL}, n_{HH} + n_{LH}) > 0$*, and*

*(iv) in area IV such that* $\pi(\beta^E, d^E, n_{HL} + n_{LL}, n_{HH} + n_{HL}) > 0$*.*

*Proof.* Necessity follows from Corollary 2 and the fact that, if one of the conditions (i) to (iv) is not satisfied, a profitable pooling contract exists that destroys the equilibrium. For sufficiency, note

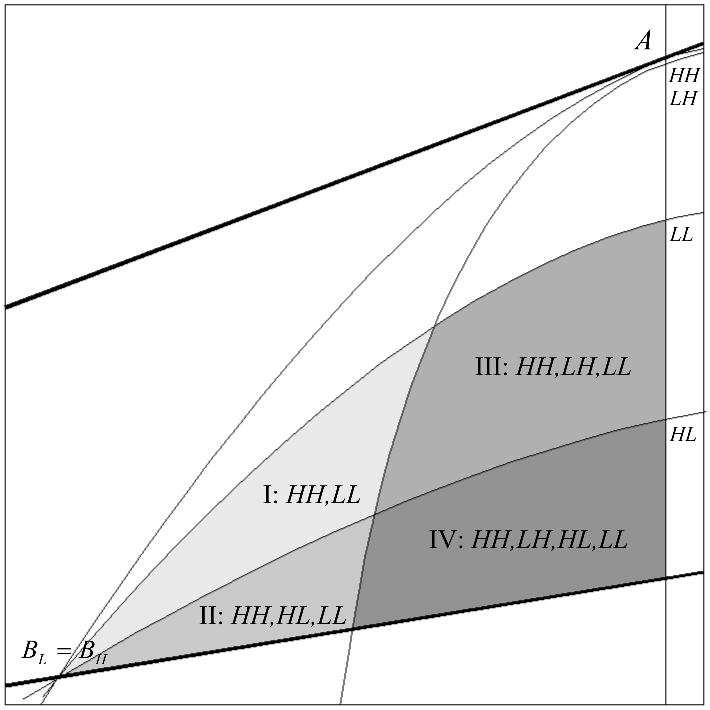


Figure 6: Existence of Separation

first that a contract in any other area between the zero profit lines of the low and high risks either attracts no individual or only high risks. It therefore cannot be a profitable deviation. Moreover, the crossing properties of the indifference curves implied by Lemma 4 and Assumption 3 rule out the emergence of other relevant areas. □

Hence, in contrast to the standard case considered by Rothschild and Stiglitz (1976), four conditions instead of just one need to be satisfied in order to guarantee existence. In the proof of the following Corollary, we show that the existence conditions from Corollary 6 analogously apply to the other separating equilibria defined in Corollary 2.

**Corollary 7.** *Conditions (i) to (iv) from Corollary 6, together with the relevant relations between the marginal rates of substitution, are also sufficient for the existence of the other separating equilibria defined in Corollary 2.*

*Proof.* For the equilibrium where only $B_L$ makes positive profits, note that there exists an additional area to the left of area II and below the $HL$-types' indifference curve through $B_H$ representing contracts that would attract $HH$- and $HL$-types (see the middle graph in figure 2). However, if condition (ii) from Corollary 6 is satisfied, we must have $\pi(\beta^E, d^E, n_{HL}, n_{HH}) < 0$ for all contracts $E$ in this new area. First, the zero profit line of the pool of $HH$- and $HL$-types lies above the zero profit line from condition (ii). Second, the $HL$-types' indifference curve through $B_H$ is concave by Lemma 2. Together, this ensures that, if condition (ii) is satisfied, the zero profit line of the pool of $HH$- and $HL$-types lies above the new area where only these types are attracted.

For the equilibrium where both $B_L$ and $B_H$ make positive profits, the area with contracts attracting only $HH$- and $HL$-types described above also exists but cannot contain profitable contracts if condition (ii) is satisfied by the same argument as above. In addition, in this case, Assumption 3 does not rule out that the indifference curves of the $LH$- and of the $LL$-types and those of the $LH$- and the $HL$-types cross again above the low risks' zero profit line (see right graph in Figure 2). Then, new areas compared to Figure 6 can emerge. However, contracts in these areas would either attract only high risks or a pool of $HL$-, $HH$- and $LH$-types, which cannot be profitable if condition (iv) is satisfied. □

We now turn to the existence of pooling equilibria. First, the 2-contract-equilibrium from Proposition 3 is considered. Figure 7 graphically represents the equilibrium contracts $A$ and $P^*$ together with the relevant indifference curves for the case that $P^*$ makes positive profits (see Corollary 3). The necessary and sufficient conditions for its existence are as follows:

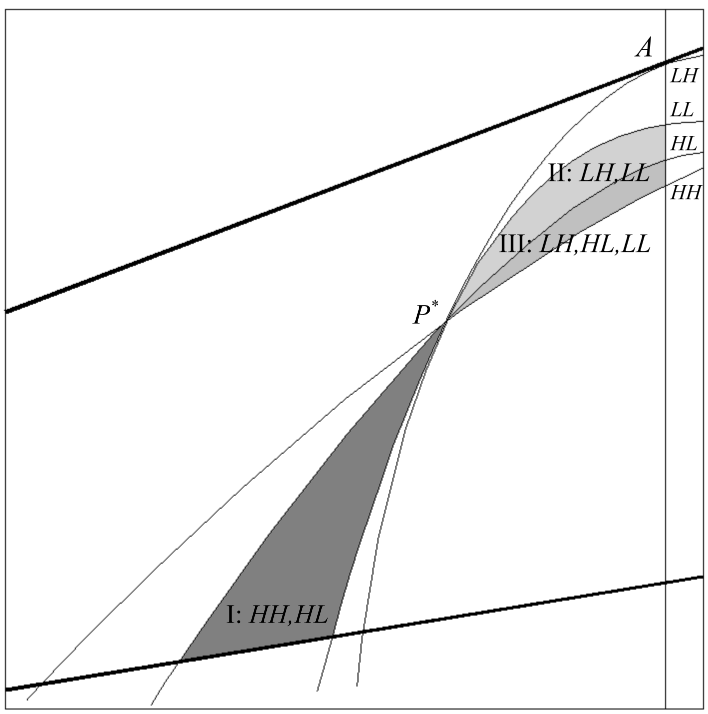


Figure 7: Existence of 2-Contract Pooling

**Corollary 8.** *The weakly pooling 2-contract-equilibrium where $P^*$ makes positive profits defined in Corollary 3 exists if and only if the conditions from Proposition 3* *and Corollary 3 are satisfied and there is no contract E*

*(i) in area I in Figure 7 such that* $\pi(\beta^E, d^E, n_{HL}, n_{HH}) > 0$*,*

*(ii) in area II such that* $\pi(\beta^E, d^E, n_{LL}, n_{LH}) > 0$*, and*

*(iii) in area III such that* $\pi(\beta^E, d^E, n_{LL} + n_{HL}, n_{LH}) > 0$*.*

*Proof.* Necessity is implied by Proposition 3, Corollary 3, and the fact that, if one of the conditions (i) to (iii) is not satisfied, there is a contract outside the equilibrium set that attracts a profitable pool. Sufficiency is established by showing that if conditions (i) to (iii) hold, no other area in Figure 7 can contain profitable deviations. This is obvious for the areas in which contracts would attract no individual or only high risks. Moreover, contracts in the area below area III cannot make positive profits if condition (iii) is satisfied. This is because, first, they are cet. par. associated with a lower premium than those in area III and, second, attract a less favorable pool (all the population rather than all except the $HH$-types). The same holds for contracts in the area to the right of area I in Figure 7. They attract a pool of $LH$-, $HH$- and $HL$-individuals and therefore cannot be profitable given condition (iii) and the concavity of indifference curves. Finally, it can be easily shown that even if the indifference curves of the $HH$- and $HL$-types or of the $HH$- and $LL$-types cross again (be it below or above contract $P^*$), the resulting new areas cannot contain profitable contracts given conditions (i) to (iii). □

Hence, in contrast to the separating equilibria, only three existence conditions are needed for this type of pooling equilibrium. Concerning the existence conditions for the weakly pooling 2-contract-equilibrium where $P^*$ makes zero profits, only two slight modifications are necessary. First, contracts in area I could never be profitable. They attract $HH$- and $HL$-types and hence a less favorable pool than $P^*$, which lies above them and just makes zero profits. Therefore, condition (i) in Corollary 8 is not needed for the existence of this equilibrium. Second, contracts in the area to the right of area I would attract all individuals who purchase $P^*$ in this case, which could not be profitable for the same reason as above. All the other arguments would remain unchanged. Thus, only the two conditions (ii) and (iii) from Corollary 8 would be needed to guarantee the existence of the weakly pooling 2-contract-equilibrium when $P^*$ makes zero profits.

Next, let us consider the weakly pooling 3-contract-equilibrium of type I defined in Proposition and Corollary 4. Figure 8 shows the case where both $P^{**}$ and $C$ make zero profits. Based on this illustration, we can derive sufficient and necessary conditions for the existence of this pooling equilibrium in the following corollary.

**Corollary 9.** *The weakly pooling 3-contract-equilibrium of type I where all contracts make zero profits defined in Corollary 4 exists if and only if the conditions of Proposition 4 and Corollary 4 are satisfied and there is no contract E*

*(i) in area I in Figure 8 such that* $\pi(\beta^E, d^E, n_{HL} + n_{LL}, n_{HH}) > 0$*,*

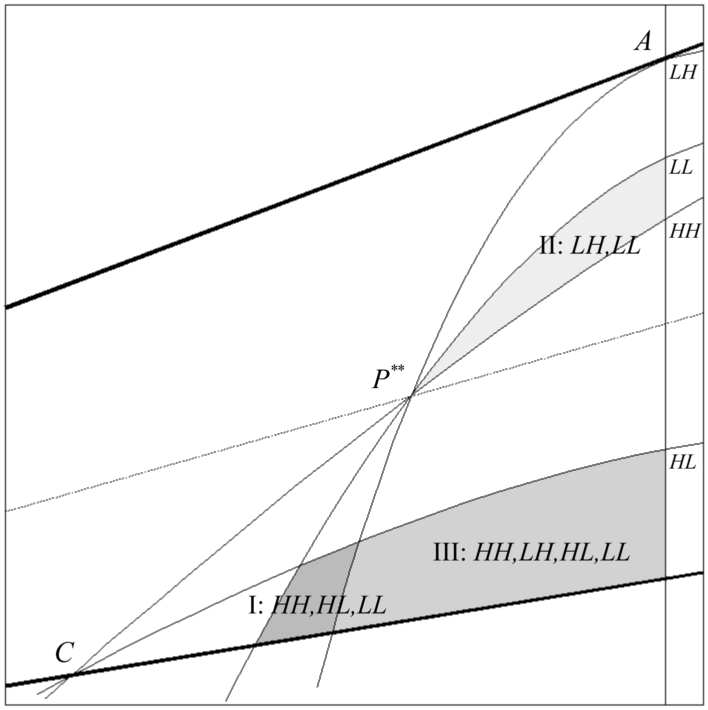


Figure 8: Existence of 3-Contract Pooling I

*(ii) in area II such that* $\pi(\beta^E, d^E, n_{LL}, n_{LH}) > 0$*, and*

*(iii) in area III such that* $\pi(\beta^E, d^E, n_{HL} + n_{LL}, n_{HH} + n_{LH}) > 0$.

*Proof.* Both necessity and sufficiency are established as in the proof of Corollary 8. For sufficiency, note that contracts in the area to the left of area I cannot be profitable if condition (i) is satisfied. This follows from the fact that, in this area, only $HH$- and $HL$-types are attracted and hence the corresponding zero profit line must lie above the one from condition (i). Together with the concavity of the $HL$-types' indifference curve through $C$, this ensures that there cannot be a profitable deviation in this area. By the same argument, contracts in the area between areas II and III in figure 8 are not profitable if condition (ii) is satisfied. Moreover, the contracts represented by the area above area I cannot be profitable as they would attract the $LH$- and $LL$-types only but lie below this pool's zero profit line. Finally, the crossing properties of the indifference curves implied by Lemma 4 and Assumption 3 rule out other relevant areas. □

Again, Corollary 9 and Figure 8 need to be changed only slightly when $P^{**}$ or $C$ make positive profits. First, if $P^{**}$ makes positive profits and thus lies at a point of tangency of the $LH$- and $LL$-types' indifference curves, area I would attract $HL$-, $LH$- and $HH$-individuals. In addition, the area to the left of area I would contain potentially profitable contracts attracting $HL$- and $HH$-individuals. This would need to be ruled out by a fourth condition. Second, if contract $C$ is not on the low risks' zero profit line but at a point of tangency of the $HH$- and $HL$-types indifference curves, another area compared to Figure 8 appears. However, it represents contracts that only attract high risks and are therefore not profitable. Hence, no modification of the existence conditions from Corollary 9 would be necessary.

Finally, Corollary 10 and Figure 9 and give the existence conditions for the pooling 3-contract-equilibrium of type II defined in Proposition and Corollary 5. We focus on the case where $P^{***}$ makes zero profits.

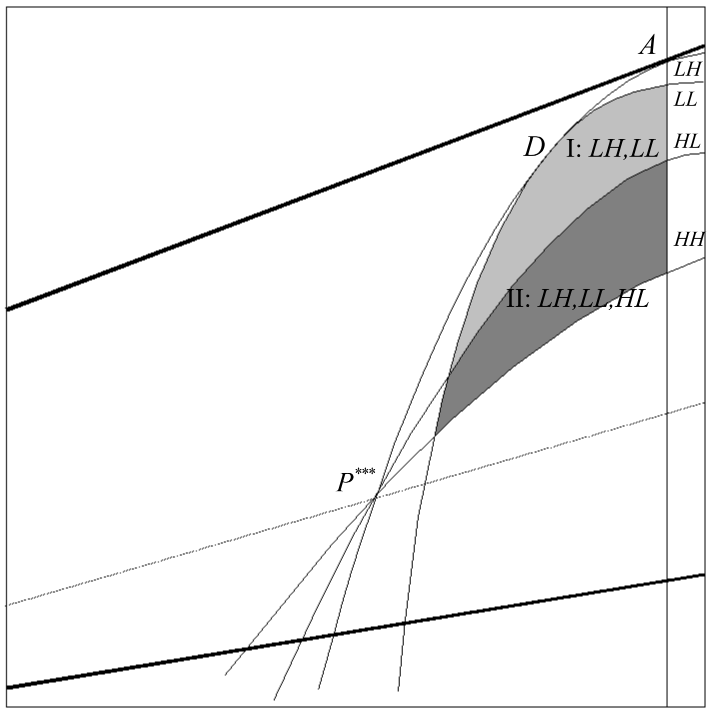


Figure 9: Existence of 3-Contract Pooling II

**Corollary 10.** *The weakly pooling 3-contract-equilibrium of type II where $P^{***}$ makes zero profits defined in Corollary 5 exists if and only if the conditions of Proposition 5 and Corollary 5 are satisfied and there is no contract E*

*(i) in area I in Figure 9 such that $\pi(\beta^E, d^E, n_{LL}, n_{LH}) > 0$, and*

*(ii) in area II such that $\pi(\beta^E, d^E, n_{LL} + n_{HL}, n_{LH}) > 0$.*

*Proof.* Again, the proof is as for Corollary 8. Concerning sufficiency, note that contracts in the area to the left of area II cannot be profitable if condition (ii) is satisfied. This follows from the fact that, in this area, only $LH$- and $HL$-types are attracted and hence the corresponding zero profit line must lie above the one from condition (ii). Together with the concavity of the $HL$-types' indifference curve through $P^{***}$, this ensures that there cannot be a profitable deviation in this area. By an analogous argument, contracts in the regions below this area and below area II are not profitable if condition (ii) is satisfied. Finally, contracts in the area to the Southwest of $P^{***}$ cannot be profitable since they would attract all individuals who purchase $P^{***}$, which lies above them and makes zero profits. □

As can be easily verified, Corollary 10 applies without modifications to the case that $P^{***}$ makes positive profits. Hence, whether the pooling contract makes profits or not, only two existence conditions are needed for the pooling 3-contract equilibrium of type II.