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Endogenous Distribution, Politics, and Growth*

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Endogenous Distribution, Politics, and Growth

Abstract

This paper generalizes the analysis of distributive conflict, politics, and growth developed by by Alesina-Rodrik (1994). We construct a heterogenous-agent framework in which *both* growth and the distribution of wealth are endogenous. Due to adjustments in the distribution of wealth, the composition of factor ownership across households equalizes in the long run. This implies that the optimal tax rate is the same for all households and equals the growth maximizing tax rate. Hence, there is *no* distributive conflict in the long run. When the model is augmented with a non-political redistributive policy, the model predicts that long run growth exhibits a negative monotonic relationship with respect to this policy, i.e., a redistributive policy that leads to a more equitable wealth distribution unambiguously reduces growth in the long run.

Keywords: Median Voter, Endogenous Growth, Wealth Distribution, Distributive Conflict, Redistributive Policy.

Journal of Economic Literature Classification Number: **D31** Personal Income and Wealth Distribution; **E62:** Fiscal Policy; **O40** Economic Growth; **P16:** Political Economy of Capitalism.

1 Introduction

A burgeoning literature now documents the impact of wealth distribution on economic growth.[1] In one class of models, the exogenous initial distribution of wealth engenders a balance of power in which distributive conflict influences optimal policy choices in equilibrium (Bertola, 1993; Perotti, 1993; Persson & Tabellini, 1994; Alesina & Rodrik, 1994; Aghion & Howitt, 1998). In these models, a greater level of inequality leads to voted policies which reflect a greater demand for redistribution. In Alesina-Rodrik model for example, since the transfer is funded by a tax on capital, a redistributive tax decreases the after-tax return to capital, discourages investment, resulting in a lower equilibrium growth rate.

An alternative class of models links wealth distribution to economic growth when capital markets are imperfect (Loury, 1981; Galor & Zeira, 1993; Banerjee & Newman, 1993; Benabou, 1995; Aghion & Bolton, 1997; Aghion & Howitt, 1998). In these models, redistributive policies that reduce investment inequality foster aggregate production by relaxing the credit constraints imposed by imperfect capital markets. This raises growth in the long run.

The present paper relates to the former class of models just described. A common feature of these models is that the distribution of wealth or factor ownership is exogenous or pre-specified. For instance, the Alesina-Rodrik model starts with a given initial distribution of labor and capital holding across households (agents) who are infinitely lived. While there is capital accumulation by all households however, there is no transitional dynamics and the ratio of capital holding between any two households remains the same for all time periods and equal to the intially specified ratio – i.e. the "distribution of factor ownership is time-invariant" (page 473). In this sense, wealth distribution is exogenous. Similarly, in Persson & Tabellini (1994),

[1] See Benabou (1995) and Aghion et. al. (1999) for an exhaustive survey of this literature.

the young are born with a draw from a distribution of skills. However, income inequality across the young – who constitute the voting population – remains the same over time, and equals the exogenously specified distribution of skills.

This paper generalizes the pioneering papers by Alesina-Rodrik and Persson-Tabelini, especially the former, by developing a model of *endogenous* wealth and income inequality among the voting population. Persson & Tabellini (1994, page 618) write: "... how income and distribution and economic growth are jointly determined in political equilibrium is not very well understood." Our model can be viewed as an attempt to seek some understanding of this complex issue. Instead of infinite life time as in Alesina-Rodrik, it assumes finite life time of agents and this gives rise to transitional dynamics.[2] The equilibrium tax rate, the economy's growth rate, as well as distribution of factor ownership all evolve endogenously, and, in the steady state, they are independent of the initial configurations. Endogenous distribution opens up a new array of related questions. For instance, we ask whether, starting from a given distribution of factor ownership composition, to what value the composition converges to? Answering this would indicate how far –if at all– political conflict over redistribution "conflicts" with overall economic growth.

Our analysis yields results which, we believe, are interesting and seem to offer many general insights. We begin by considering a straightforward generalization of the A-R model that has a single politically determined policy: namely, a capital income tax. A striking conclusion emerges: starting from any given distribution of factor composition, the distribution becomes degenerate in the long run.[3] In other words, the distribution of factor composition converges perfectly across households in the long run, i.e., capital holding bears the same proportion to

[2] As Drazen (2000, page 473) writes, "One criticism of all these models is the lack of transitional dynamics. This is dictated ... by the difficulty in solving for a simultaneous economic and political equilibrium...".

[3] The distribution of wealth or income does not become degenerate.

skills for all households. This implies that the optimal tax rates of all households (including the median household) are the same and equal to the growth maximizing tax rate i.e. unanimity holds. In this sense, there is no conflict between politics and growth. This is important because it enables us to identify economies, in general, where redistributive politics may hamper economic growth.

We then consider a non-political redistributive transfer policy in which a given proportion of tax proceeds are transferred back to the household sector in a uniform lump-sum fashion. We assume that the distribution of skills is skewed to the right, so that, compared to the mean household, the median household is poorer and receives more of the transfer (proportionally). Such a transfer enables the median household to accumulate more capital than otherwise. In the long run, the median household's capital-labor (or capital-skill) ratio becomes greater than the mean. As a result, its most preferred capital tax rate is less than the growth-maximizing tax rate. This implies that, unlike when there is no nonpolitical redistributive policy, complete convergence does not hold, and distributive conflict reappears. But a more redistributive policy in the form of a higher proportion of tax proceeds being transferred back to the household sector implies, on one hand, a more equitable distribution (as one would expect), and, on the other hand, a lower capital income tax and lower growth rate. Thus the standard positive association between equity and growth is reversed. This, in a sense, resurrects the age-old trade-off between growth and equity.

Finally, our model also predicts that a positive technology shock leads to higher growth *as well as* a higher capital tax in the long run. This means that within a country – having a given technology parameter – the growth rate and the capital tax rate may be negatively or positively related (of which only one combination is the equilibrium one), but, across countries differentiated on the basis of technology, the (cross-country) correlation between the equilibrium growth rate and tax rate is positive.

What are the critical differences between Alesina & Rodrik (1994), Persson & Tabellini (1994), and our model? First, because distribution is endogenous in our model, there is no causal link from distribution to growth. This implies that one can only speak of an "association" between the two. Given this, the relevant questions become: (a) how distribution and growth are correlated along the transition path; and (b), how long-run growth and long-run inequality respond to shocks in the basic parameters of an economy such as a positive shock to technology. Second, in both the Alesina-Rodrik (A-R henceforth) and Persson-Tabellini (P-T henceforth) models, there is only one redistributive policy, which is political. However, the enactment of a policy based on the(ir) result that lesser inequality causes higher growth implies the existence of a redistributive policy that is *non*-political. In our model, we explicitly consider two redistributive policies, one political and the other non-political. We then pose the question as to whether a more redistributive policy, that is not politically manipulable, leads to higher long-run growth via adjustments in the policy that is politically manipulable. Finally, while differences in skills is the primitive source of heterogeneity as in Persson & Tabellini, in our model, the voting population holds labor as well as capital, and capital holdings grow.

Our model also contrasts with the existing literature relating the distribution of wealth to long run growth that does not consider political economy. For instance, in Galor-Zaira (1993), the initial distribution of wealth has an impact on long-run growth of output – leading to multiple equilibria. In contrast, in our model, the long run growth of output and the long-run distribution of wealth are independent of the initial distribution of wealth; instead, they depend on basic parameters of the economy such as the distribution of innate skill across the population, technology, as well as direct re-distributive policies. Hence long-run distribution is endogenous, somewhat similar to Matsuyama (2000). However, the difference is that while in Matsuyama (2000) there is no source of heterogeneity across households except for wealth and thus complete equality of wealth is a possible long-run outcome, in our model the heterogeneity in the distribution of innate skill implies that perfect equality cannot obtain in the long run.

In summary, compared to the existing literature, the novel features of our analysis are the following:

1. Our model incorporates transtional dynamics. This permits short-run as well as long-run characterization of inequality and growth. In particular, the distribution of factor ownership composition changes from its initial configuration and evolves endogenously.

2. Distribution, political equilibrium and growth are *all* simultaenously determined.

The paper proceeds as follows. Section 2 outlines a basic model of endogenous distribution, growth, and distributive conflict. Section 3 introduces a nonpolitical redistributive policy and derives the aggregate implications from the introduction of the policy. Section 4 concludes.

2 The Basic Model

The population or the number of households in the economy is given. Each household is endowed with one unit of labor which is inelastically supplied to the market. Households are differentiated on the basis of a basic skill level, L_h, whose distribution is assumed to be continuous on a finite support in R_+. This distribution is pre-determined and constitutes the source of basic heterogeneity in the model.[4] No further assumption such as skewness is necessary for the analysis of this section. For simplicity however, we assume that the

[4]Alternatively, we can interpret L_h as just labor time supplied by household h, its distribution being based on how 'lazy' households are vis-a-vis one another.

distribution of L_h is skewed to the right. This implies that, L_m, the median skill level, is less than $\bar{L}$, the mean skill level. It permits us, as will be seen later, to use the capital holding of the median household relative to that of the mean household as a simple index of wealth inequality.

Let $\int_{h \in H} L_h dh \equiv L$, where H is the total number of households and L is the total endowment of skill. For notational convenience, we normalize $H = 1$. Thus $L = \bar{L}$.

2.1 Production

A single good is produced in the economy. The production function follows Barro (1990) and A-R:

$$Q_t = A\bar{K}_t^{\alpha} G_t^{1-\alpha} \bar{L}^{1-\alpha}, \tag{1}$$

where Q_t is aggregate output at time t, $\bar{K}_t$ denotes the mean/aggregate capital, G_t is a public-infrastructure input and $A > 0$ is an index of technology. Following the endogenous growth literature, we interpret K as physical as well as human capital. Hence, α is the private return to physical and human capital. We require a regularity condition:

$$\alpha > \frac{1}{2}, \tag{R1}$$

which, as will be seen later, ensures that the net return to capital in equilibrium is positive.[5]

The input G_t is financed by a (specific) tax on capital income.[6] As in A-R, such capital-

[5]With a narrower interpretation of K as physical capital, it would be empirically implausible to assume $\alpha > 1/2$, but it is not so when capital is interpreted more broadly as we do here. Further, according to Barro and Salai-Martin (1995, page 38), even a value of alpha equal to 0.75 is quite reasonable.

[6]This is equivalent to a wealth tax.

income taxation should be viewed broadly as a redistributive policy that, on one hand, reduces the incentive to accumulate, and at the same time act as a transfer income to relatively unskilled labor – in terms of improving the marginal product of labor through an increase in G_t (see A-R, page 466).[7]

The government budget constraint is satisfied in all time periods, i.e.,

$$G_t = \tau_t \bar{K}_t. \tag{2}$$

The competitive factor rewards are:

$$r_t = \tilde{r}(\tau_t) \equiv \alpha A \tau_t^{1-\alpha} \bar{L}^{1-\alpha}, \tag{3}$$

$$w_t = \phi(\tau_t)\bar{K}_t, \text{ where } \phi(\cdot) \equiv (1-\alpha)A\tau_t^{1-\alpha}\bar{L}^{-\alpha}, \tag{4}$$

where r_t is the rent earned by capital and w_t denotes the wage rate. Note that an increase in τ enhances the marginal product of both factors. This constitutes the source of gain from the tax to household income. Finally, as in A-R, without loss of generality, we let the rate of capital depreciation be zero.

2.2 The Household's Problem

Following Aghion and Bolton (1997), Picketty (1997), and Das (2000, 2001), we assume that agents live for a single period.[8] At the end of the period, a replica is born to each agent, and agents pass on a bequest to their children. Households derive utility from consumption, C_{ht}, and the amount of the good bequested (at time t) to time $t+1$, K_{ht+1}. Production occurs in

[7] On page 471-472, they also provide examples of redistributive policies that act as a direct tax on capital.

[8] Since every one lives for one period, there is no time-inconsistency problem.

the beginning of each period. Once production occurs, agents make consumption and bequest decisions. Hence, the bequest can be interpreted as inherited capital, with capital defined before.

The utility function, $U : \Re^2_+ \rightarrow \Re_+$, satisfies the standard properties, and, for the sake of tractability, is assumed to be Cobb-Douglas: $U_{ht} = C_{ht}^{1-\beta} K_{ht+1}^{\beta}, \quad 0 < \beta < 1$. The budget constraint facing an agent h is given by

$$C_{ht} + K_{ht+1} \leq \phi(\tau_t)\bar{K}_t L_h + [1 + \tilde{r}(\tau_t) - \tau_t]K_{ht}. \tag{5}$$

We assume that the skill level of a household does not change over time or generations, i.e., one can think of skill – or habit – as being 100% genetic. Thus, each household is identified by a given L_h. There is no dynamic stochastic process governing the evolution of L_h. The benefit of this assumption is that it offers considerable analytical tractability. The cost is that it does not permit to say anything about social mobility. However, social mobility, although an important problem in its own right, is not our focus.

The tax rate is already known when households make their consumption and bequest decisions. Their optimization exercise implies the following Euler equation for consumption:

$$C_{ht} = \frac{1-\beta}{\beta} K_{ht+1}. \tag{6}$$

It also leads to the following asset accumulation equation for household h and the corresponding aggregate accumulation equation:

$$K_{ht+1} = \beta \left\{ \phi(\tau_t) L_h \bar{K}_t + [1 + \tilde{r}(\tau_t) - \tau_t] K_{ht} \right\}, \tag{7}$$

$$\bar{K}_{t+1} = \beta \left\{ \phi(\tau_t) \bar{L} + [1 + \tilde{r}(\tau_t) - \tau_t] \right\} \bar{K}_t. \tag{8}$$

Substituting the Euler equation for consumption into the utility function yields

$$U_{ht} = \text{Constant} \cdot K_{ht+1}. \tag{9}$$

Finally, substituting equation (7) into equation (9) yields the household's indirect utility function:

$$V_{ht} = \text{Constant} \cdot \{\phi(\tau_t) L_h \bar{K}_t + [1 + \tilde{r}(\tau_t) - \tau_t] K_{ht}\}. \tag{10}$$

It is sufficient to note that for any value of $\bar{K}_t$ and K_{ht}, the indirect utility is single peaked with respect to τ_t. This implies that the median household's optimal tax rate is also the equilibrium tax rate under majority voting. However, before characterizing the prefered tax rate of the median voter, we first prove that the median household is unique – which is an implication of our assumption that for any given h, L_h does not vary over time. The invariance of the median voter's identity over time ensures analytical tractability of our model of endogenous distribution and politics.

2.3 Household Ranking and Uniqueness of the Median Household

Define $n_{ht} \equiv K_{ht}/\bar{K}_t$. Dividing equation (7) by equation (8), we get

$$n_{ht+1} = n_{ht}\left[1 + \frac{\phi(\cdot)(L_h/n_{ht} - \bar{L})}{\phi(\cdot)\bar{L} + 1 + \tilde{r}(\cdot) - \tau_t)}\right]. \tag{11}$$

We start by tracking the economy from an initial period in which the tax rate is exogenous and not politically determined. Note then that the dynamic process (11) leads to a steady state where

$$\begin{aligned} & n_h = \frac{L_h}{\bar{L}} \\ & \Rightarrow \phi(\tau)\bar{K} + [1 + \tilde{r}(\tau) - \tau]\frac{K_h}{L_h} = \bar{K}\left[\phi(\tau) + \frac{1 + \tilde{r}(\tau) - \tau}{\bar{L}}\right]. \end{aligned} \tag{12}$$

Here and onwards, we follow the convention of letting variables without the time subscript denote their steady state values. The relation (12) implies that K_h/L_h is same for all h, i.e. the ranking of households in terms of capital held and disposable income is the same in terms of L_h. This 'alignment' of K_h with L_h in terms of ranking implies that the median household is identified by the ranking of L_h only, i.e. by $L_h = L_m$.

Now suppose that the tax rate 'becomes' political and is determined by majority voting. The economy goes off the steady state. However, irrespective of what the tax rate is, (7) implies that the next period's capital stock holding of household h also has the same ranking as L_h. Further, this remains true for all successive time periods, off and on the steady state, as long as the households do not face asymmetric skill or preference shocks so as to change the initial ranking of households on the L_h scale. We assume away such shocks, which implies that the median household's identity is unchanged even though τ may change over time.

2.4 Analysis of Optimal Tax

How does the optimal tax rate compare across the households? Given (10), since the indirect utility of any particular household is single-peaked with respect to τ_t, the optimal tax for household h is given by the first order condition:[9]

$$\frac{\phi'(\tau_t)L_h}{n_{ht}} + [\tilde{r}'(\tau_t) - 1] = 0. \tag{13}$$

From this equation, we can regard the marginal cost (MC) of a tax increase on disposable income as equal to 1, while the marginal benefit (MB) of a tax increase on disposable income (actually the MB/MC ratio) equal to $\phi'(\tau_t)L_h/n_{ht} + \tilde{r}'(\tau_t)$. These are illustrated in Figure 1. Consider two households: one is labor-rich and capital-poor and the other is labor-poor and capital-rich, i.e., the ratio L_h/n_{ht} is more for the former. Notice that the MB of a tax increase on disposable income is greater for the former. As a result, the optimal tax for the former household is higher as shown $(\tau_1 > \tau_2)$. Intuitively, a labor-rich-capital-poor household cares less about net capital income than a labor-poor-capital-rich household. Hence, the optimal tax rate is higher for the former.

Using the definitions of $\phi(\cdot)$ and $\tilde{r}(\cdot)$ function, the first-order condition (13) yields the following closed-form expression for the optimal tax rate of the h^{th} household at time period t, τ_{ht}:

$$\tau_{ht} = \left\{ A(1-\alpha)\bar{L}^{1-\alpha} \left[\frac{(1-\alpha)L_h}{n_{ht}\bar{L}} + \alpha \right] \right\}^{\frac{1}{\alpha}}. \tag{14}$$

Equation (14) implies that the most preferred tax rate for a particular household depends

[9] The second-order condition can be easily verified.

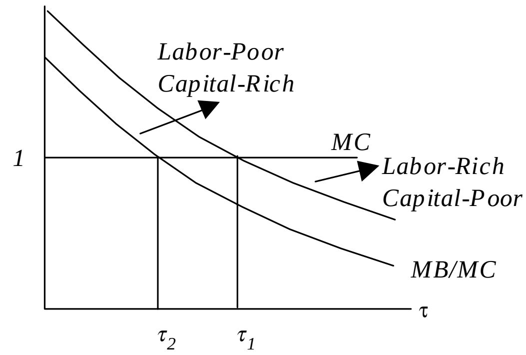

Figure 1: Optimal Tax for Households with Different Factor Holding Compositions

on the ratio of two ratios, namely, $n_{ht}/(L_h/\bar{L})$. From now on, unless specified otherwise, let "relative" mean relative to the mean household. Thus τ_{ht} is negatively related to the ratio of its relative capital holding to its relative skill. The economy's aggregate/mean capital stock and skill matter, because they determine the (economy-wide) marginal products of capital and labor. The individual capital holding and skill matter because they determine the individual disposable income.

It follows that for any given household, τ_{ht} is negatively related to n_{ht}. Also, note that the optimal tax rate for any household is always positive. Moreover, this is bounded from below by the tax rate which will be chosen if a household's labor income were zero.[10]

In particular, the equilibrium tax rate at any t is given by the optimal tax of the median household, i.e.,

$$\tau_{mt} = \left\{ A(1-\alpha)\bar{L}^{1-\alpha} \left[\frac{(1-\alpha)L_m}{n_{mt}\bar{L}} + \alpha \right] \right\}^{\frac{1}{\alpha}}, \tag{15}$$

where n_{mt} is the relative capital holding of this household.

[10] This is equal to the tax rate which maximizes the after-tax return to capital, $\tilde{r}(\tau_t) - \tau_t$.

2.5 Steady State

The dynamics of the economy is fundamentally described by (11) with $h = m$ and τ_t equal to τ_{mt} given by (15). The former governs the dynamics of the relative capital holding of the median household. Substituting $n_{mt+1} = n_{mt} = n_m$ in (11), it follows that along the steady state,

$$n_m = \frac{L_m}{\bar{L}} \Leftrightarrow \frac{K_m}{L_m} = \frac{\bar{K}}{\bar{L}}. \tag{16}$$

That is, the median household's composition of factor holdings is equal to that of the mean household. Indeed, (11) holds for all households, i.e.,

$$n_h = \frac{L_h}{\bar{L}} \Leftrightarrow \frac{K_h}{L_h} = \frac{\bar{K}}{\bar{L}} \quad \forall h. \tag{17}$$

In other words, compared to any given household, a more skilled household accumulates more capital in the long run and there is complete convergence of capital-labor ratio holdings across households in the steady state. This implies that every household's preferred tax rate is the same, i.e., there is unanimity in the long run. Moreover, this tax rate is equal to

$$\tau = \left[A(1-\alpha)\bar{L}^{1-\alpha}\right]^{\frac{1}{\alpha}}. \tag{18}$$

To see this intuitively, note that in terms of the MB/MC and MC curves depicted in Figure 1, each household's MB/MC curve collapses to that of the mean household and its intersection with the MC=1 line gives τ in equation (18). This does not happen in the A-R model because factor ownership compositions are time-invariant and exogenously given.

We note a technical point here. Using (18), the net return to capital, $\kappa \equiv r - \tau$, is equal to $(2\alpha - 1)\tau/(1-\alpha)$, which may not positive for all $\alpha < 1$. This is where our regularity condition (R1), i.e. $\alpha > 1/2$, comes in; it assures that $\kappa > 0$.

Turning to the economy's growth rate, define $g_t \equiv K_t/K_{t-1}$, the growth rate at time t. From (8), this implies that at any time t,

$$g_{t+1} = \beta[\phi(\tau_t)\bar{L} + 1 + \tilde{r}(\tau_t) - \tau_t] = \beta(1 + A\bar{L}^{1-\alpha}\tau_t^{1-\alpha} - \tau_t). \tag{19}$$

This shows a non-monotonic relationship between growth and the tax rate. On one hand, an increase in τ increases the marginal products of labor and capital and thus tends to increase disposable income. On the other hand, it lowers after-tax income. Hence, there is a trade-off in the net effect on disposable income, savings, and subsequently, capital accumulation, from a rise in τ. Moreover, similar to the A-R model, from (19) we see that there is a unique growth-maximizing tax rate equal to

$$t_g = \left[A(1-\alpha)\bar{L}^{1-\alpha}\right]^{\frac{1}{\alpha}}. \tag{20}$$

However, unlike in A-R, this is *same* as the equilibrium tax rate in the steady state given by equation (18). Hence, long-run growth is maximized at the political equilibrium. It follows from the convergence of capital-labor ratio holdings across households. This, we believe, is a very interesting departure from the exogenous-distribution framework of A-R and P-T. We see this as a useful bench-mark case where political equilibrium coincides with maximization of growth. The benefit of identifying this economic environment is that the inefficiency resulting from politics in a more realistic economy can be seen insightfully in terms of a deviation from such an environment. Indeed in the next section, we examine such a deviation.

In terms of comparative statics, we note from (18) that $d\tau/dA > 0$. This is because a positive technology shock enhances the marginal product of both labor and capital and thus raises the marginal gain from a tax increase. Hence, everyone's preferred tax rate is higher. However, the tax rate is independent of the preference parameter β. On the other hand, from (19), the long-run growth rate is an increasing function of both A and β. An important corollary derived from the effects of a technology shock is that the cross-country correlation between the tax rate and the growth rate will be positive when countries are ranked in terms of their levels of technology. This contrasts with an *intra-country* relationship between the tax and growth rates, which may be negative or positive depending on the range of the tax rate.

We have not mentioned anything about inequality yet. Our assumption that $L_m < \bar{L}$ implies that in the steady state, $n_m = K_m/\bar{K} < 1$. Hence, we can take n_m, the median-mean wealth ratio, as the indicator of inequality, and, a higher n_m implies a more *equal* distribution of wealth. Note also that, along the steady state, a household's disposable income and indirect utility are both proportional to a household's holding of capital. Hence, the magnitude of n_m would also indicate inequality in terms of income and utility. In other words, inequality in terms of wealth, income and utility are synonymous in our model.

Further, we observe directly from (16) that the long run wealth inequality is the same as the inequality in skill, i.e., more generally, the distribution of long-run wealth is the same as that of the skill. Within the purview of the model this is only "natural."[11] Since the innate skill distribution is exogenous, unlike the tax rate or the growth rate, the level of inequality is not affected, for example, by a technology shock.[12]

[11]If skill can be enhanced by education and there are capital market imperfections, then the distribution of long-run wealth or income inequality will not be equal to that of the innate skill distribution.

[12]However, a uniform additive skill shock to all households would increase n_m and lower inequality.

More generally, we can define inequality in terms of coefficient of variation.[13] Note from (17) that the standard deviation of K_h equals $c_L \bar{K}_t$, where c_L is the coefficient of variation of L_h. Hence the coefficient of variation of wealth is equal to c_L, which is also invariant with respect to a technology shock.

2.6 Comparing With the A-R Model

Figure 2 illustrates the comparison and reconciliation with the A-R model in a simple manner. The non-monotonic relationship between the growth rate and the tax rate – given by equation (19) – is depicted in the top panel. The tax rate that maximizes the aggregate/average welfare is also the one that maximizes the growth rate. (This holds in the A-R model as well as in Barro (1990).)

The bottom panel graphs equation (14): the optimal tax as a negative function of the ratio of relative capital holding to relative skill. In the A-R model the median voter's relative capital holding is assumed to be less than its relative labor endowment, or, the relative skill in terms of our model. Hence, $\rho_m \equiv n_m/(L_m/\bar{L}) < 1$. Accordingly, the economy operates effectively in the right-hand side of the growth-maximizing tax rate. Suppose that initially $\rho_m = \rho_m^0$. The tax rate is read off the horizontal axis and the economy's growth rate is g^0. Now, if ρ_m increases to ρ_m^1, i.e., the distribution becomes more equitable, we see that the tax rate falls and the economy's growth rate jumps up to g^1. This is the central proposition of the A-R model. In contrast, in our model, the distribution is endogenous and every household's relative capital holding adjusts and converges in the steady state to its relative skill. That is, $\rho_h = 1$, for all h, including the median household. There is unanimity in voting for the tax rate. Political equilibrium implies the growth-maximizing tax rate, τ_g. This is an interesting

[13] Caselli and Ventura (1999) and Das (2000, 2001) also use this as the measure of inequality.

bench-mark situation where there is no conflict between politics and efficiency.

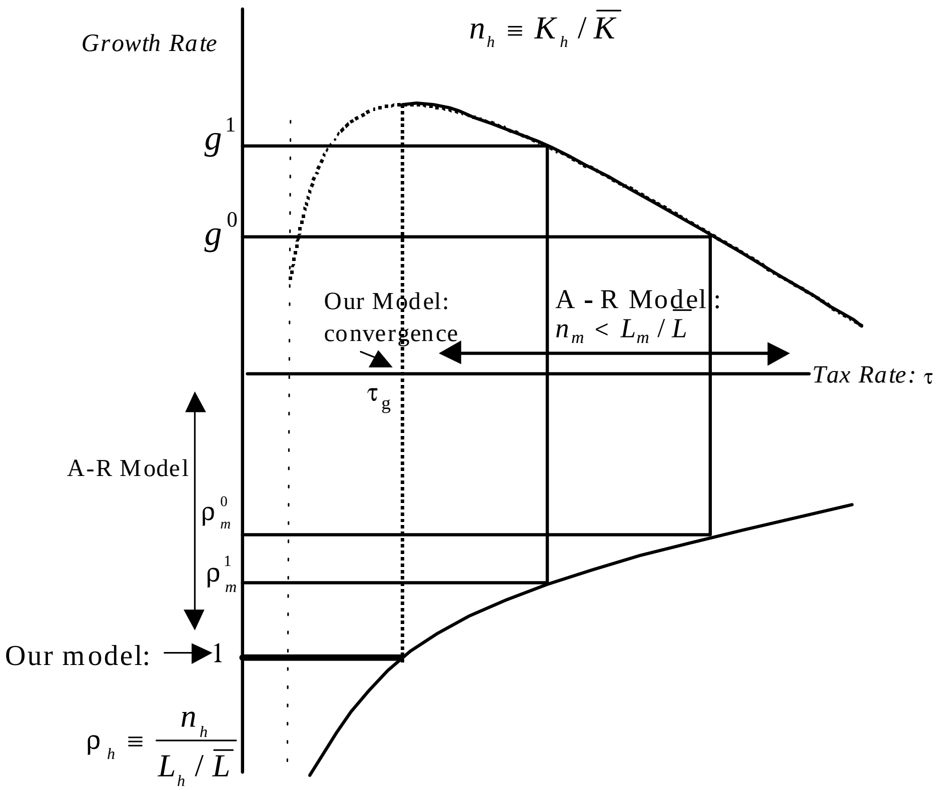

Figure 2: Comparison between Alesina-Rodrik Model and Ours

2.7 Transitional Dynamics

Suppose there are skill shocks to households (without changing their ranking in terms of L_h) such that initially the median voter's relative capital holding is not equal to its steady state value. How does the economy adjust over time? The transitional dynamics are governed by equations (11) and (15). To address this issue, for simplicity, we confine ourselves to dynamics around the neighborhood of the steady state.

Totally differentiating equation (11) and evaluating the derivative by using the steady state condition $L_m/n_{mt} = \bar{L}$, we get

$$0 < \left.\frac{dn_{mt+1}}{dn_{mt}}\right|_{n_{mt}\to L_m/\bar{L}} = \frac{1+\tilde{r}(\tau)-\tau}{\phi(\tau)\bar{L}+1+\tilde{r}(\tau)-\tau} < 1. \tag{21}$$

This implies that the transition path of inequality is monotonic and stable. Thus, starting from $n_{m0} \neq L_m/\bar{L}$, the economy converges monotonically to the long run level of inequality defined by the basic source of heterogeneity in the model, L_h. Given the dynamics of n_{mt}, the dynamics of the tax rate are evident from (15). The optimal tax along the transition path decreases or increases over time as $n_{m0} \lesseqgtr L_m/\bar{L} \Leftrightarrow \rho_m \lesseqgtr 1$.

How does the growth rate change during the transition periods? Interestingly, from Figure 2, we can readily infer that it increases over time – and tends to converge to the maximized growth rate – irrespective of whether $\rho_{m0} \lesseqgtr L_m/\bar{L}$ initially.

In summary, the endogeneity of wealth distribution implies a configuration of the long-run growth rate, tax rate, and the degree of inequality, which is quite different from the case where the distribution of wealth is exogenous. Our model constitutes a bench-mark example in which convergence in terms of composition of factor holdings occurs. Growth-maximizing tax rate and hence maximized growth prevail at the political equilibrium. As comparative statics, a (neutral) technology improvement leads to an increase in both the long-run tax rate and the long-run growth rate, but does not affect inequality. Furthermore, over the transition period, the growth rate and inequality are positively or negatively related depending on whether the initial level of inequality falls short of or exceeds its steady state level.

In what follows, we consider an important extension to our bench-mark model.

3 Redistributive Policy and Redistributive Politics

The implicit but central message of the A-R model and the P-T model is that policies that induce lesser inequality raise the long run growth rate of the economy as well. However, their models do not contain any independent, exogenous policy instrument that can influence distribution. In order to ascertain the impact of redistributive policies on growth, one must then introduce a policy instrument which is not politically manipulable.

The simplest way to capture this within our framework is to assume that a fraction θ, $0 < \theta < 1$, of tax revenues is disbursed uniformly across households. Specifically, while the revenues generated are equal to $\tau_t \bar{K}_t$, a portion of it, $\theta \tau_t \bar{K}_t = T_t$, is transferred back to the households, and, $(1-\theta)\tau_t \bar{K}_t = G_t$ is used in production. The policy parameter θ is exogenous, whereas τ_t is politically determined as before. While the former corresponds to a redistributive policy, redistributive politics enters through the latter. We assume that $L_m < \bar{L}$, so that $n_{mt} < 1$ and an increase in n_{mt} would mean a more equitable distribution. We now ask how an increase in θ affects long-run growth. [14]

Our main result here is that, along the steady state, while an increase in θ increases n_m and thus reduces inequality, it unambiguously *reduces* growth. Hence, the standard positive link between equality and growth is completely reversed, and we are 'back' in the realm of an equity-growth tradeoff. We see this by observing that starting from the steady state in which there is no non-political redistributive policy, and the relative capital endowment of the median household is equal to its relative skill, a redistributive program makes the median household relatively capital-rich. Hence, this household chooses a capital-income tax rate that is less

[14] It can be argued of course that in a real economy θ is political while τ is not. However, the political-economy literature on taxation does not tell us which policy instruments are political and which are not. Assuming θ to be non-political enables us to analyze the impact of a directly redistributive program on growth and distribution.

than the growth-maximizing tax rate. Indeed, more of such redistribution makes the median household more capital-rich (in relative terms) and induces it to lower its preferred tax rate. This causes the growth rate to fall. In terms of Figure 2, the economy in the steady state operates solely in the region that is on the left-hand side of τ_g.

To see this formally, let the aggregate production function be the same as before. Given that $G_t = (1-\theta)\tau_t K_t$, the competitive factor rewards are:

$$r_t = \tilde{r}(\tau_t)(1-\theta)^{1-\alpha}; \quad w_t = \phi(\tau_t)(1-\theta)^{1-\alpha}\bar{K}_t, \tag{22}$$

where $\tilde{r}$ and ϕ are as defined in the basic model. The household problem is modified to include the transfer in the budget constraint:

$$\underset{C_{ht},\, K_{ht+1}}{\text{Maximize}}\; U_{ht} = C_{ht}^{1-\beta} K_{ht+1}^{\beta}, \quad \text{subject to} \quad C_{ht} + K_{ht+1} \leq w_t L_h + (1 + r_t - \tau_t)K_{ht} + T_t.$$

However, the same first-order conditions obtain. Accordingly, the household-level and aggregate capital accumulation equations are:

$$K_{ht+1} = \beta\left\{\phi(\tau_t)(1-\theta)^{1-\alpha}L_h\bar{K}_t + [1+\tilde{r}(\tau_t)(1-\theta)^{1-\alpha} - \tau_t]K_{ht} + \tau_t\theta\bar{K}_t\right\} \tag{23}$$

$$\bar{K}_{t+1} = \beta\left\{\phi(\tau_t)(1-\theta)^{1-\alpha}\bar{L} + [1+\tilde{r}(\tau_t)(1-\theta)^{1-\alpha} - \tau_t]\bar{K}_t + \tau_t\theta\bar{K}_t\right\}, \tag{24}$$

respectively. Finally, the following indirect utility function obtains:

$$V_{ht} = \text{Constant}\cdot\left\{\phi(\tau_t)(1-\theta)^{1-\alpha}L_h\bar{K}_t + [1+\tilde{r}(\tau_t)(1-\theta)^{1-\alpha} - \tau_t]K_{ht} + T_t\right\}. \tag{25}$$

We assume that individuals – in their calculation of the marginal benefits from and the marginal cost of τ_t – disregard the effect of an increase in τ_t on total tax proceeds. Hence, treating the term, $T_t = \tau_t \theta K_t$, as given, the maximization of indirect utility with respect to τ_t for a given $\bar{K}_t$ and K_{ht} gives the most preferred tax rate of household h. We obtain a generalization of (14):

$$\tau_{ht} = \left\{ A(1-\alpha)\bar{L}^{1-\alpha}(1-\theta)^{1-\alpha} \left[\frac{(1-\alpha)L_h}{n_{ht}\bar{L}} + \alpha \right] \right\}^{\frac{1}{\alpha}}. \tag{26}$$

As expected, the optimal tax for any household is a decreasing function of θ.

Finally, dividing (23) by (24) gives the dynamics for the household accumulation of relative capital holdings:

$$n_{ht+1} = n_{ht} \left[1 + \frac{\phi(\tau_t)(1-\theta)^{1-\alpha}(L_h/n_{ht} - \bar{L}) + \theta\tau_t(1/n_{ht} - 1)}{\phi(\tau_t)(1-\theta)^{1-\alpha}\bar{L} + 1 + \tilde{r}(\tau_t)(1-\theta)^{1-\alpha} - \tau_t + \theta\tau_t} \right]. \tag{27}$$

3.1 Steady State

Since a uniform transfer maintains the ranking of households in terms of disposable income, the median household's identity remains unchanged as in the basic model. Substituting $h = m$, from (26) and (27), the steady state conditions are:

$$\tau = \left\{ A(1-\alpha)\bar{L}^{1-\alpha}(1-\theta)^{1-\alpha}[\frac{(1-\alpha)L_m}{n_m\bar{L}} + \alpha] \right\}^{\frac{1}{\alpha}} \equiv \varphi(\underset{-}{n_m}, \underset{-}{\theta}) \tag{28}$$

$$(1-\alpha)A\bar{L}^{1-\alpha}(1-\theta)^{1-\alpha}\left(n_m - \frac{L_m}{\bar{L}} \right) = \theta\tau^{\alpha}(1-n_m). \tag{29}$$

These two equations determine the equilibrium τ and n_m. From the first equation, an increase in n_m reduces τ, as in the basic model. Additionally, for given n_m, an increase in θ increases the median household's disposable income, which reduces its demand for τ. From the second equation, given $L_m < \bar{L}$, it is easy to verify that if $n_m < L_m/\bar{L}$, then the l.h.s. of (29) is negative and the r.h.s. is positive. The opposite holds if $n_m > 1$. This implies

$$\frac{L_m}{\bar{L}} < n_m < 1. \tag{30}$$

Thus, in contrast to the basic model, n_m exceeds $L_m/\bar{L}$, i.e., the redistributive program implies that long-run inequality is less. The difference between n_m and $L_m/\bar{L}$ can be interpreted as *excess equality*, arising from the presence of the redistributive program. Formally, let Δ denote this excess equality, where

$$\Delta = 1 - \frac{L_m/\bar{L}}{n_m}. \tag{31}$$

It is important to note that inequality (30) implies that the median household holds a *higher* capital/skill ratio than the mean household. This occurs, while, in terms of basic endowments, the median household is relatively skill-poor and capital-poor. Because the proportion of transfers received relative to pre-transfer income is *higher* for the median than the mean household, the median household's relative capital accumulation is higher than in the absence of the transfer program.

Also note that factor-composition convergence does not hold. This is because transfers are uniformly disbursed, not proportional to pre-transfer income. However, there is an *interval*-convergence, not point-convergence, in the sense that the standard deviation of the difference between relative capital holding and relative skill across households is dictated by θ. To see this formally, note that, in general, for any household h in the steady state, its relative capital holding is determined by an equation analogous to (29), which is:

$$(1-\alpha)A\bar{L}^{1-\alpha}(1-\theta)^{1-\alpha}\left[n_h - \frac{L_h}{\bar{L}}\right] = \theta\tau^{\alpha}(1-n_h). \tag{32}$$

Substituting (28) above, eliminating τ, using (31), and rearranging give

$$n_h - \frac{L_h}{\bar{L}} = \frac{\theta[1-\Delta(1-\alpha)](1-L_h/\bar{L})}{1+\theta[1-\Delta(1-\alpha)]}. \tag{33}$$

It is easy to see that the mean of $n_h - \frac{L_h}{\bar{L}}$ is zero. Its standard deviation is equal to

$$\sigma(n_h - L_h/\bar{L}) = \frac{\theta[1-\Delta(1-\alpha)]}{1+\theta[1-\Delta(1-\alpha)]}c_L, \tag{34}$$

where, as before, c_L denotes the coefficient of variation of skill distribution. In the special case of $\theta = 0$, $\sigma(\cdot)$ reduces to zero. This means that, initially, off the steady state, the difference $n_h - L_h/\bar{L}$ may be very widely dispersed, but as the economy moves to the steady state, the standard deviation of this difference converges to a given value which is a function of the magnitude of θ.

Since absolute convergence breaks down, the growth-maximizing rate does not coincide with the median voter's preferred tax rate in the long run. This implies that there is distributive conflict in the presence of a nonpolitical redistributive policy.

3.2 Comparative Statics

Return to equations (28) and (29). If we substitute the former into the latter and eliminate τ, we have one equation in one variable, namely, n_m:

$$\theta = \frac{n_m - L_m/\bar{L}}{\alpha(1-n_m) + (1-\alpha)(1/n_m - 1)L_m/\bar{L}}. \tag{35}$$

Note that the r.h.s. is increasing in n_m. Therefore, an increase in redistribution leads to a decrease in long run inequality. However, agents being heterogenous with respect to the basic skill parameter, a rise in θ does not completely eliminate inequality. Indeed, in the limit, when $\theta = 1$, n_m still remains bounded away from unity.[15]

How does an increase in θ affect the long run tax rate? It falls for two reasons. Directly, a rise in θ bestows a positive wealth effect on every household. Thus, it reduces the median household's 'demand for τ'. Indirectly, the median household's relative wealth, n_m, increases. This also reduces its demand for τ. The latter effect arises because wealth distribution is endogenous.

We now define the *effective tax rate* as $\Theta_t = (1-\theta)\tau_t$, which will be useful in understanding the effect of redistributive policy on growth. Note that an increase in θ leads to a decrease in Θ via a decrease in τ for the two reasons described above. In addition, by definition, Θ decreases as θ increases at any given τ. Thus $d\Theta/d\theta < 0$ unambiguously. Turn now to equation (24),

[15] This result contrasts, for example, with the central implication of Saint-Paul & Verdier (1993), in which although distribution is endogenous, income dispersion shrinks over time and the economy converges to full equality.

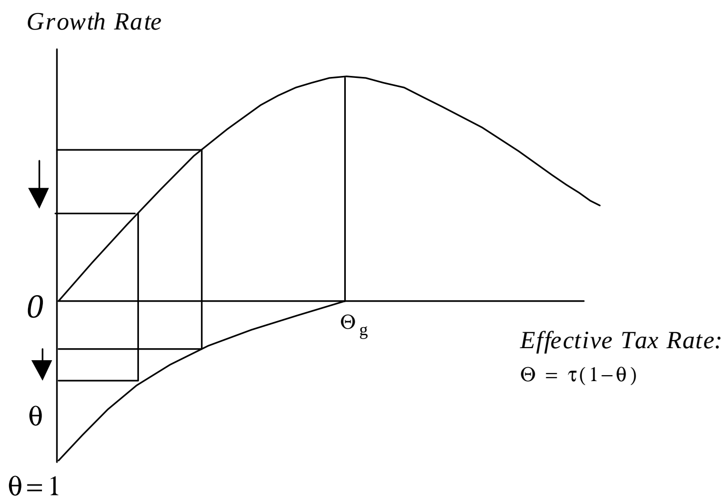

Figure 3: EFFECTIVE TAX RATE AND THE LONG-RUN GROWTH RATE

which implies

$$\begin{aligned} g_{t+1} &= \beta[\phi(\tau_t)(1-\theta)^{1-\alpha}\bar{L} + 1 + \tilde{r}(\tau_t)(1-\theta)^{1-\alpha} - \tau_t + \theta\tau_t] \\ &= \beta\{A\bar{L}^{1-\alpha}[\tau_t(1-\theta)]^{1-\alpha} + 1 - \tau_t(1-\theta)\}, \quad \text{using the definitions of } \phi(\cdot) \text{ and } \tilde{r}(\cdot) \\ &= \beta[1 + A\bar{L}^{1-\alpha}\Theta_t^{1-\alpha} - \Theta_t]. \end{aligned} \tag{19'}$$

Observe that the same non-monotonic relationship holds between the growth rate and the tax rate as in the basic model holds, except that it is the effective tax rate here. This is exhibited in the top panel of Figure 3. (Compared to Figure 2, only the scales are drawn differently.) The bottom panel depicts the negative relationship between the effective tax rate and the redistributive policy parameter θ.

The effect of an increase in θ on the long-run growth rate is seen immediately now. Note that when $\theta = 0$, $\tau = \Theta_g$, the growth-maximizing tax rate. Hence for any $\theta \in [0, 1]$, $\Theta \leq \Theta_g$, and, thus the economy operates in the *left*-hand side of the "growth-tax" relationship. As θ

increases, Θ falls and from the top panel we see that the growth rate decreases. Hence more redistribution leads to a lower growth rate – which is in sharp contrast to A-R and P-T.[16]

3.3 Transitional Dynamics

As in the basic model, the dynamic adjustment path is stable and monotonic. This is proven in Appendix 1. Thus, similar to the basic model, depending on whether $n_{m0} \gtrless n_m$, inequality and the tax rate increase or decrease over time. However, since in the steady state the economy is at a point along the left-hand arm of the growth-tax curve, the dynamics of the growth rate are different. Whereas in the basic model the growth rate would increase over time as long as $n_{m0} \neq n_m$, here, the growth rate increases or decreases depending on whether $n_{m0} \gtrless n_m$.

In what follows we examine the effect of an increase in θ on the dynamics of inequality, tax rate and growth rate. Assume that, initially, at $t = 0$, the economy is in the steady state (i.e. $n_m = n_{m0}$), and, an unanticipated permanent increase in the policy parameter θ occurs. Totally differentiating (27) for $h = m$, using the fact that initially, $\phi(\tau_0)(1-\theta)^{1-\alpha}(L_h/n_{m0} - \bar{L}) + \theta\tau_0(1/n_{m0} - 1) = 0$, and substituting the expression of τ in (26), we get the expression of the change in inequality in period 1:

$$\frac{dn_{m1}}{d\theta} = \frac{x_0(1-n_{m0})}{(1+\kappa_0)x_0/\tau_0 + 1 + \theta x_0} > 0, \quad \text{where } x_0 \equiv \frac{(1-\alpha)L_m}{n_{m0}\bar{L}} + \alpha. \tag{36}$$

Recall that κ is the net return to capital. Thus, in the short run also, an increase in θ reduces inequality. Furthermore, it is proven in Appendix 2 that the magnitude of change is less than

[16] The implication is that an increase in θ reduces the long-run growth rate. Were θ introduced in the A-R model with exogenous n_m ($< L_m/\bar{L}$), a part of the right-hand side of the growth-tax relationship would have been relevant. Hence, starting from $\theta = 0$, an increase in θ would have initially increased the growth rate and then reduced the growth rate beyond a critical point. However, distribution being endogenous in our model, a higher θ lowers the growth rate *unambiguously.*

that of the long-run effect, as one would expect. However, unlike the long-run effect, the magnitude of the short run effect is affected by the technology parameter A. To see this, note that from (36), the magnitude of $dn_{m1}/d\theta$ is an increasing function of the original tax rate, which, in turn, is an increasing function of the parameter A. Hence, the higher value of A, the greater is the short-run effect of an increase in θ on inequality.

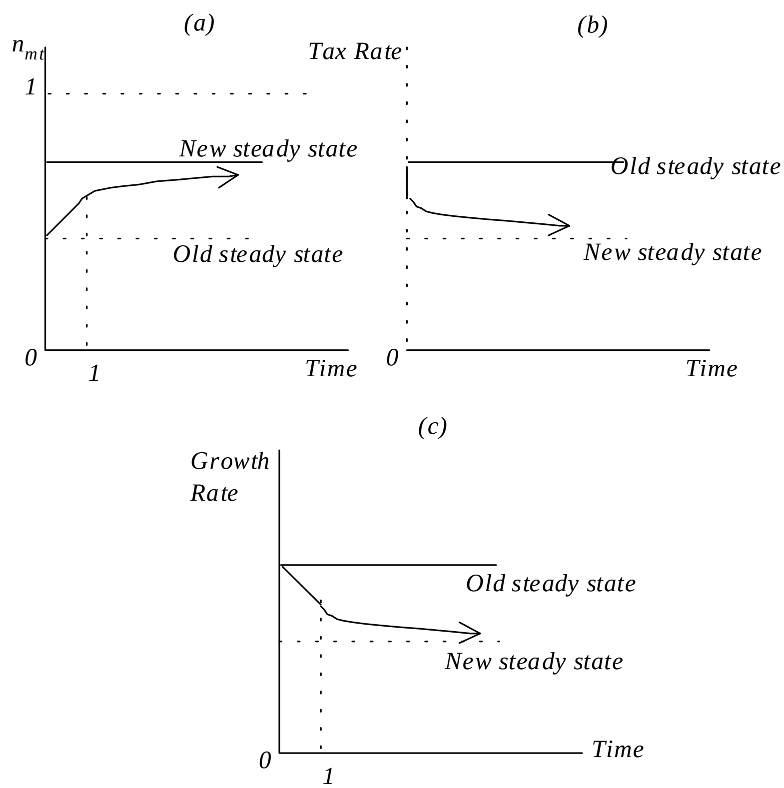

Figure 4: Dynamics of Inequality, Tax Rate & Growth

Figure 4, panel (a), depicts how an unanticipated permanent rise in θ affects the dynamics of inequality: there is initially a discrete downward jump in inequality (n_{m1} increases), followed by a gradual convergence to the new steady state. The tax rate falls in period 0 itself as $\partial\tau_t/\partial\theta < 0$. With n_{mt} increasing over time from period 1, the tax rate continues to fall over time. The dynamics of the growth rate follows directly from that of the tax rate: it decreases over time. These are depicted in panels (b) and (c) of Figure 4.

4 Summary

This paper has attempted to formulate a framework which results in the joint determination of inequality and growth in a political equilibrium. There is a unique non-degenerate distribution of wealth and income in the steady state, which follows from the difference in the innate ability of individuals. In this respect, our model is similar to Caselli and Ventura (1999). In terms of the political equilibrium, our model follows the median-voter approach of Alesina-Rodrik and Persson-Tabellini. More specifically, the model is closer to the A-R model in that the politics is confined to tax on capital income.

We find that the endogeneity of distribution offers novel insights. In the absence of any exogenous redistributive program, the factor-holding ratios across households converge in the long run. Every agent is a representative agent in the political arena. Unanimity holds and the growth-maximizing tax rate is chosen. Thus, there is no distributive conflict.

In contrast, with an inequality-reducing nonpolitical transfer policy in place, the composition of factor holdings does not converge. However, there is interval-convergence in the sense that the dispersion of factor-holding composition is "bounded" by the magnitude of the transfer policy. Political equilibrium does not lead to the choice of a growth-maximizing effective tax rate. Moreover, in contrast to the A-R model, it is less than the growth-maximizing tax rate, since the median household's capital-skill ratio exceeds that of the mean household. This implies, in stark contrast to A-R, that a more equitable transfer policy unambiguously lowers long-run growth.

Although the model of the paper is specific, an 'example' so-to-speak, it seems to offer a few general insights. First, it suggests that long-run dynamic adjustments of wealth and income distribution engender ‘greater’ political consensus over policy choices. Second, we find that a non-political tax-transfer policy that leads to a more equitable distribution hurts long-run growth. Most of all, the model hopes to prove a general point that the joint analytical determination of inequality, growth, and a political equilibrium is not an intractable proposition. The specific model achieves tractability by assuming limited life time and an economy in which the identity of the median household does not change over time. Hopefully, for future research, this approach would suggest other ways to ensure tractability in similar models and at the same time offer more generality. For example, what happens when individuals vote on a tax schedule rather than a tax rate. Also, there are several sources of individual heterogeneity. We have considered innate-skill heterogeneity, so as to illustrate the contrast with the existing literature as sharply as possible. Other sources of heteroneity such as various types individual preference shocks should be considered and their implications toward long-run distribution and growth be systematically studied.

Appendix 1

We prove that the dynamic adjustment path of n_{mt}, as given in (27) for $h = m$, is stable and monotonic.

We begin with some preliminaries. From the definition of $\phi(\cdot)$ and using (28), we obtain the steady state relationship:

$$\tau = \phi(\tau)(1-\theta)^{1-\alpha}\bar{L}\left[\frac{(1-\alpha)L_m}{n_m\bar{L}} + \alpha\right]. \tag{A1}$$

Also, equation (35) implies

$$\frac{(1-\alpha)L_m}{n_m\bar{L}} + \alpha = \frac{n_m - L_m/\bar{L}}{1-n_m}. \tag{A2}$$

The last two relations imply

$$\begin{aligned} &\theta\tau = \phi(\cdot)(1-\theta)^{1-\alpha}\bar{L}\frac{n_m - L_m}{1-n_m} \\ &\Leftrightarrow \phi(\tau)(1-\theta)^{1\alpha}\frac{L_m}{n_m} + \frac{\theta\tau}{n_m} = \phi(\tau)(1-\theta)^{1-\alpha}\bar{L} + \theta\tau. \end{aligned} \tag{A3}$$

Next, by differentiating (28),

$$n_m\frac{d\tau}{dn_m} = -\frac{\tau\mu}{\alpha}, \quad \text{where } \mu \equiv \frac{(1-\alpha)L_m}{(1-\alpha)L_m + \alpha n_m\bar{L}} < 1-\alpha. \tag{A4}$$

We now return to the dynamic equation (27) for $h = m$. Differentiating it with respect to n_{mt} and evaluating the derivative by using the steady state conditions (28), (29) and (35), we obtain

$$\begin{aligned}\frac{dn_{mt+1}}{dn_{mt}} &= 1 - \frac{\phi(\tau)(1-\theta)^{1-\alpha}L_m/n_m + \theta\tau/n_m}{\phi(\tau)(1-\theta)^{1-\alpha}\bar{L} + 1 + \kappa + \theta\tau} \\ &\quad + \frac{\phi'(\tau)(1-\theta)^{1-\alpha}(L_m/n_m - \bar{L}) + \theta(1/n_m - 1)}{\phi(\tau)(1-\theta)^{1-\alpha}\bar{L} + 1 + \kappa + \theta\tau} n_m \frac{d\tau}{dn_m} \\ &= \frac{1 + \kappa - \phi(\tau)(1-\theta)^{1-\alpha}\mu(\bar{L} - L_m/n_m)}{\phi(\tau)(1-\theta)^{1-\alpha}\bar{L} + 1 + \kappa + \theta\tau}, \qquad \text{(A5)}\end{aligned}$$

using (A3), (A4), (29) and that $\phi' = (1-\alpha)\phi/\tau$.

Notice that the magnitude of the numerator of (A5) is less than the denominator. Hence $|dn_{mt+1}/dn_{mt}| < 1$ and the dynamic path is stable.

We prove next that the numerator of (A5) is positive. We have

$$\begin{aligned}\kappa &= \tilde{r}(\tau)(1-\theta)^{1-\alpha} - \tau = \tau\frac{\alpha - (1-\alpha)x}{(1-\alpha)x} \\ \phi(\tau)(1-\theta)^{1-\alpha}\mu(\bar{L} - L_m/n_m) &= \tau\frac{\mu[1 - L_m/(n_m\bar{L})]}{x} = \tau\frac{\mu(1-x)}{(1-\alpha)x},\end{aligned}$$

where $x = \frac{(1-\alpha)L_m}{n_m\bar{L}} + \alpha$, and, we have made use of (28) and the definition of ϕ. Hence the numerator of (A5) equals

$$1 + \frac{\tau[\alpha - (1-\alpha)x - \mu(1-x)]}{(1-\alpha)x} > 1$$

since $\alpha > 1 - \alpha > \mu$ and $0 < x < 1$. The denominator of (A5) is then obviously positive. Hence $dn_{mt+1}/dn_{mt} > 0$, implying that the adjustment path is monotonic.

Appendix 2

It is proven that $dn_m/d\theta|_{\text{short run}} < dn_m/d\theta|_{\text{long run}}$. Note first that the equation (35) can be expressed as

$$\theta = \frac{n_m - L_m/\bar{L}}{(1-n_m)x_0}. \tag{35$'$}$$

Substituting this into (36) and eliminating $1 - n_{m0}$ give:

$$\theta \frac{dn_{m1}}{d\theta} = \frac{n_{m0} - L_m/\bar{L}}{\kappa_0 x_0/\tau_0 + 1 + \theta x_0} > 0. \tag{A6}$$

This expression will be compared with the long-run effect.

From equation (35), the long-run effect of an increase in θ on n_m is given by

$$\begin{aligned} \frac{d\theta}{dn_m} &= \frac{\theta}{n_m - L_m/\bar{L}} + \frac{\theta\left(\alpha + \frac{1-\alpha}{n_m^2}\cdot\frac{L_m}{L}\right)}{(1-n_m)x_0} \\ \Rightarrow \theta\frac{dn_m}{d\theta} &= \frac{n_m - L_m/\bar{L}}{1+\theta[\alpha + (x_0-\alpha)/n_m]} \end{aligned} \tag{A7}$$

In obtaining the last expression we have made use of (35$'$) and the definition of x_0.

Now comparing (A6) and (A7), the short-run effect falls short of or exceeds the long-run effect according as:

$$\frac{\kappa_0}{\tau_0} \gtrless \theta\left(\frac{1}{n_{m0}} - 1\right)\left(1 - \frac{\alpha}{x_0}\right) = \frac{1-\alpha/x_0}{1-\alpha}\left(\frac{1}{x_0} - 1\right). \tag{A8}$$

The l.h.s. is equal to $1/\tau_0 + \alpha/[(1-\alpha)x_0] - 1$, which exceeds $1/x_0 - 1$ under the regularity condition (R1). The r.h.s. is obviously less than $1/x_0 - 1$ and hence less than the l.h.s., implying that the short-run effect is smaller in magnitude.

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