Empirical Applications of Multidimensional Inequality Analysis

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Abstract

This paper explores the empirical application of theoretical multidimensional inequality analysis using real household welfare distributions. The paper operationalises recent conceptual developments in multidimensional inequality theory and assesses their usefulness for measurement and policy analysis. Despite the existence of a thriving theoretical literature on multidimensional inequality, empirical applications, particularly at the individual and household levels, are few and far between. This paper compares and contrasts different methodologies for the analysis of multidimensional welfare, including multidimensional inequality indices and stochastic dominance techniques. The results strongly highlight the importance of bringing non-monetary aspects of household welfare into the forefront of inequality analysis since measurements based solely on the distribution of income variables may misrepresent the degree of overall inequality in society. Agreement over the various approaches to the measurement of multidimensional inequality entails, however, non-trivial decisions that may limit the practical usefulness of these measures. We suggest that the use of multidimensional inequality ranges and restrictive dominance criteria may open significant scope for further developments in the empirical analysis of multidimensional inequality.

JEL codes: D31, D63, I19, I29.

Keywords: Multidimensional inequality; inequality indices; income inequality; education inequality; health inequality; stochastic dominance.

1. Introduction

The analysis of inequality has recently taken on a central role in the economics literature (see Atkinson, 1996). Despite this renewed interest, most studies are by and large concerned with inequalities in the distribution of income and other forms of material wealth. Income may not, however, be sufficient for characterising adequately the level of social welfare in a given society, which may also depend on other welfare attributes such as employment conditions, access to land and other assets, use and access to health, education and other social services, rights of access to political power and legal institutions and security from crime and violence. Moreover, income distributions will not fully reflect all individual benefits, needs or abilities, particularly those that cannot be priced as they are non-tradable such as education, health, and so forth (Sen, 1985, 1997; Sen and Nussbaum, 1993; Narayan et al., 2000).

One common mis-perception in the literature is that income inequality is closely related to other forms of inequality and can thus be used as a proxy for the level and changes in overall inequality in any given society. There is, however, no reason to expect a-priori different dimensions of inequality to be determined by the same factors. For instance, while income distribution may be related to employment structures, access to minimum wage, social security provision and so forth, educational choices may depend on different factors such as the public provision of schools, legislation regarding child labour and opportunities available in labour markets (Jensen and Skyt Nielsen, 1997; Justino, Litchfield and Niimi, 2004).

In addition, once it is accepted that well-being depends on characteristics other than income, conventional analyses of income inequality will exhibit unsatisfactory properties. As pointed out by Kanbur (2003), common poverty measures generally register a fall in overall poverty when a poor person dies say of AIDS. Conventional measures of income inequality will register a fall in overall inequality in similar circumstances.

The recognition of these facts has given rise to a large literature on multidimensional welfare. Two parallel developments have taken place. The first, which has become quite popular in the recent literature on the measurement of income inequality, is the use of equivalence scales. These scales 'correct' observed incomes as to allow for possible heterogeneity in individual or household non-income needs by using a common metric across all individuals or households (generally, family size or age and gender composition of the household) based on the notion of 'equivalised income' (Deaton and Muellbauer, 1980, 1986; Atkinson and Bourguignon, 1989; Ebert, 1996). This approach takes into account important differences in needs between households, notably the fact that children need less food than adults to achieve the same level of nutrition, and that larger households benefit from economies of scale in the consumption of certain goods and services. Equivalence scales do not, however, allow for individual or household differences in other aspects of well-being such as disability, mortality, literacy levels, schooling attainments and so forth (see discussion in Anand and Sen, 1993).[1]

The second development examines directly those different individual or household non-monetary welfare attributes that may be relevant in determining overall inequality using an explicit multidimensional framework. We focus in this paper on this latter option. The formal analysis of multidimensional inequality had its origins in a pioneering article by Kolm (1977). This has given rise to two lines of research. The first is concerned with deriving direct indices of multidimensional inequality (Maasoumi, 1986; Tsui, 1995, 1999). The second approach is based on partial orderings and derives stochastic dominance conditions for

[1] Equivalence scales also do take into consideration the fact that households living in regions with higher prices will need more income than household living in lower-price regions. This aspect of household welfare can be accounted for through the use of price indices. Deaton (1997) provides a detailed review of this issue.

comparing multivariate welfare distributions (Atkinson and Bourguignon, 1982, 1987; Muller and Trannoy, 2003).[2]

The theoretical literature on multidimensional welfare has thrived in the last years and has had significant applications in the literature on standards of living. The United Nations Human Development Index (UNDP, 1990) is the most widely used application, combining indicators of PPP GDP per capita, life expectancy at birth, adult literacy and school enrolment ratios into an overall index of standards of living across developed and developing countries.[3] Empirical applications of multidimensional inequality and distribution analysis in the multidimensional context, particularly at the household level, are much scarcer, despite the wealth of existing research on micro-level distribution analysis.[4]

Measuring empirically the distribution of non-monetary dimensions of welfare at the individual or household level entails significant challenges. First, the construction of most conventional indices of inequality is based on the assumption that individuals can be ranked according to their specific endowments of relevant attributes. Ranking individuals along income, consumption or earnings levels is a straightforward exercise as each level can be perfectly matched to a monetary value. However, ranking individuals along educational, health or political outcomes is a more complex exercise since it implies subjective judgements and, hence, interpersonal comparisons of welfare. It also requires quantifiable information on non-monetary attributes, which is often not available at the individual or household level.

Second, identifying relevant dimensions of welfare can involve numerous difficulties. Even if we agree on including say three attributes (for instance, income, education and health), it is not clear what the concepts or ideals of those attributes mean. Individuals are

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[2] See Maasoumi (1999), Weymark (2004) and Trannoy (2004) for extensive surveys of this literature.

[3] For further developments in the construction of the HDI and cross-national empirical applications see Anand and Sen (1993), Hicks (1997) and Foster, Lopez-Calva and Szekely (2005).

[4] Maasoumi (1986), Hirschberg, Maasoumi and Slottje (1991) and Lugo (2004) provide empirical analyses of multidimensional inequality based on national and regional level data. We are not aware of any existing

born under different circumstances, which will determine their health status over their lifetime and their academic achievements. Each individual will have different heights, different propensity to be over or underweight, different metabolism and immune systems, as well as different mental abilities and talents. It is thus not possible to expect society to aim to equalise all these differences and it may be more sensible to define education and health inequalities as those that arise from circumstances or policies that cannot be affected by individual tastes and preferences (Roemer, 1996). Consensus over the choice of appropriate variables to represent those circumstances may not always be possible.

A third related problem is whether to analyse each dimension of welfare separately or to aggregate the various dimensions of inequality into summary indices. If aggregation is considered to be the right route, decisions must be made on how to aggregate attributes in adequate measures that encompass both monetary and non-monetary dimensions of inequality, which weights to use, how to measure the extent of risk aversion in society and what are the levels of correlation or degree of substitution between the various welfare attributes. These are not trivial decisions and choosing particular indicators of welfare and measures may determine research outcomes.

This paper explores the empirical application of existing theoretical approaches to the measurement of multidimensional inequality using household-level data for two countries, Brazil and Vietnam. Brazil and Vietnam were chosen because they entail interesting features for the analysis of multidimensional inequality. Brazil is one of the most unequal countries in the world in monetary terms, though little is known about the distribution of other welfare attributes. Vietnam, on the other hand, is a typical example of a transition economy, where inequalities tend to play a predominant role (see Kuznets, 1955). Vietnam experienced in the mid-1990s one of the largest increases in economic growth amongst developing countries,

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applications of multidimensional inequality using household or individual-level data. In the field of following an intense programme of economic liberalisation. This process was accompanied by a drastic reduction in poverty, but also by an increase in income inequality (Glewwe, Gragnolati and Zaman, 2002; Justino and Litchfield, 2003), which has raised interesting questions in terms of distributive justice.

We make use of data from the 1996 Brazilian household survey, the Pesquisa Nacional de Amostra de Domicilios (PNAD), and the 1992-93 and 1997-98 Vietnam Living Standards Measurement Surveys (VLSS). For the purpose of this paper, we chose three (continuous) welfare attributes that can be derived from these three household surveys.[5] The first is a monetary welfare attribute, represented by per capita income, for Brazil, and per capita consumption expenditure, in the case of Vietnam.[6] The second attribute is education status. This variable refers, in the three surveys, to the number of years spent in school by the head of the household, which have been shown to be a significant influence on social and private welfare (e.g. Haveman and Wolfe, 1984), as well as economic growth (Lucas, 1988). Another important influence on household welfare is the health status of the household members, particularly those that work (e.g. Rosenzweig and Schultz, 1983). We proxy this variable by the number of days of work, within a reference period of four weeks, lost by the head of the household due to illness. Unfortunately, we can only estimate the extent of health inequalities in Vietnam as the Brazilian household survey does not contain comparable health information.

Many other dimensions of welfare could have been chosen (for instance, house ownership, land size, access to infrastructure and so forth).[7] However, the objective of this

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multidimensional poverty analysis see Duclos, Sahn and Younger (2001).

[5] Education and health are of course not strictly continuous as they take discrete values. In addition, education is bounded from above. This should not, however, violate the application of the various measures of inequality used in this paper as these can be easily extended to discrete variables (see Cowell, 1995).

[6] See Litchfield (2001) and Justino and Litchfield (2003), respectively, for details on how the Brazilian and Vietnamese monetary variables have been computed.

[7] Other variables could have been used. In another paper, Justino, Litchfield and Niimi (2004) use the ratio of stillborns per pregnancy in the household as the indicator of health inequality in Brazil and the distribution of the

paper is to illustrate the empirical application of multidimensional inequality analysis methods, not to derive conclusions about welfare distributions in Brazil or Vietnam.[8]

We first analyse the extent of multidimensional inequality in Brazil and Vietnam using an independent ('one-at-a-time') analysis of monetary and non-monetary welfare attributes. This approach has been popularised in recent studies of education and health inequalities but does not constitute a truly multidimensional approach as it does not allow for possible degrees of complementarities between different distributions of household welfare attributes.

In section 3, we contrast this analytical method with approaches that consider the joint distribution of attributes, thereby allowing for differences in the various distributions, as well as possible correlations between the attributes. In this section, we explore the empirical application of aggregate multidimensional indices (Maasoumi, 1986; Tsui, 1999). These allow a truly multidimensional analysis of the distribution of various household welfare attributes. However, multidimensional indices involve several non-trivial decisions and value judgements that limit their usefulness in policy analysis. We argue that rather than trying to focus on point estimates of multidimensional inequality, it may be more useful to calculate ranges of multidimensional inequality values for classes of suitable weights and correlation coefficient scales, which can be conveniently used for comparative analyses between different countries (or regions, provinces, etc) and across time.

One way of avoiding the difficulties entailed by the use of composite indices is to resort to multidimensional stochastic dominance techniques proposed by Atkinson and

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maximum level of schooling achieved by any member of the household as an alternative measure for education inequality. This paper discusses also the measurement of inequalities in political and social participation. Justino and Niimi (2005) explore further health status indicators such as individual life expectancy and weight/height ratios. A large literature concern with the measurement of capabilities and functionings, standard of living and notions of well-being has proposed other welfare vectors (see Allardt, 1993; Lovell et al., 1994; Cummin, 1996; Narayan et al., 2000; Deutsch, 2001). Ramos and Silber (2005) compute several of these approaches using UK household data but do not find significant differences between the various concepts of well-being.

[8] Using these variables raises several other issues, such as the use of variables that are not strictly continuous and the comparison of flow and stock variables, which we presently ignore but address elsewhere (see Justino, Litchfield and Niimi, 2004 and Justino and Niimi, 2005).

Bourguignon (1982). These have the advantage of using partial ranking of distributions without requiring the knowledge of the precise form of the social welfare function. Empirical application of stochastic dominance techniques may, however, become intractable when more than two or three variables are considered. We illustrate this approach in section 4.

Finally, in section 5, we explore the adaptation of the restrictive dominance criteria, proposed by Atkinson and Bourguignon (1989) and extended more recently in Muller and Trannoy (2003) and Trannoy (2004), to the measurement of multidimensional inequality as a form of avoiding the computational complexity of aggregative inequality measures. This method constitutes a promising avenue for further empirical work on multidimensional inequality. Section 6 summarises the results, concludes the paper and identifies areas for future research.

2. Independent distribution of attributes

The last decade has seen a significant increase in the analysis of non-income distributions at the household and individual levels, spurred by significant improvements in the availability and collection of data on non-monetary welfare attributes. Most of this work is based on the individual analysis of non-income distributions and multidimensional features of household welfare are brought into the picture by examining the correlation between the various distributions. This is the case with recent work on non-monetary poverty (Ruggeri Laderchi, Saith and Stewart, 2003), as well as the measurement of education inequalities (Checchi, 2000; Thomas, Wang and Fan, 2000) and health inequality (Gakidou, Murray and Frenk, 2000; Wagstaff, 2000). These studies apply standard techniques used in income distribution analyses, such as inequality indices (Kolm, 1969; Atkinson, 1970; Sen, 1973) and stochastic dominance approaches (Shorrocks, 1983; Atkinson and Bourguignon, 1987; Ok and Lambert, 1999), to distributions of non-monetary attributes. Both inequality indices and stochastic dominance analysis are then used to represent the degree of inequality in the distribution of household welfare attributes and to compare degrees of dispersion between distributions of different welfare attributes across different geographic locations, different population groups and across time.

The analysis of non-monetary distributions can add significant insights to the understanding of household welfare characteristics. In table 1, we illustrate the application of inequality indices to household education and health distributions in Brazil and Vietnam. Figures 1 and 2 exemplify the use of stochastic dominance techniques for non-monetary household welfare attributes. In order to be able to compare distributions, the various welfare attributes have been normalised to the same mean by subtracting each variable by the minimum value of its distribution and dividing that value by the range of the distribution (see UNDP, 1990; Anand and Sen, 1993). We have also re-estimated the results in table 1 using z-scores to normalise all variables as proposed by Hirschberg, Maasoumi and Slottje (1991). We do not report those results as no significant differences were found when applying the two methods.[9] The inequality indices used were the Gini coefficient and three measures of Generalised Entropy (GE(α)) class of inequality measures: the mean logarithmic deviation (also known as the Theil's second measure), the Theil index and one-half of the squared coefficient of variation. This corresponds, respectively, to $\alpha = 0$, $\alpha = 1$ and $\alpha = 2$.[10]

Our empirical results show significant differences in household welfare distributions in both Brazil and Vietnam depending on the underlying welfare variables considered. It is clear that both monetary and education inequalities are higher in Brazil than in Vietnam (table

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[9] Ramos and Silber (2005) provide alternative ways of normalising welfare dimensions using efficiency analysis techniques first proposed by Lovell (1994).

[10] See Cowell (1995) for discussion of the various indices of inequality.

1 and figures 1 and 2).[11] The figures do not allow for conclusive evidence for first-order dominance as the Lorenz curves cross each other at low ends of both monetary and education distributions. Figure 1 shows, however, that the Lorenz curves for the Brazilian distribution of the monetary attribute dominates both Vietnamese distributions at higher ends of the monetary distribution, while figure 2 shows similar evidence for the distribution of educational outcomes. With respect to Brazil, it is clear that education inequality is lower than income inequality. In Vietnam, distributions of different household welfare attributes behave very differently. For instance, while monetary inequality rose by 12% in Vietnam between 1992-93 and 1997-98, education inequality decreased by the same amount during the same period, and health inequalities went up by over 30%.[12]

In table 2, we present the correlation coefficients for each pair of household welfare dimensions. Despite the widespread view that income inequality is generally highly correlated with other types of inequality, table 2 shows a very different picture: neither Vietnam not Brazil show any significant correlation between the various household welfare attributes. This is in line with similar findings in the literature on standards of living (e.g. Lovell et al., 1994; Ramos and Silber, 2005).

These results suggest that the analysis of both monetary and non-monetary welfare distributions may have important normative relevance in societies with a particular concern with the distribution of specific household welfare attributes that are not necessarily correlated with income inequality. One question that follows from the analysis above is whether it would be appropriate to consider possible complementarities between the various dimensions of household welfare. For instance, in the case of Vietnam, policy-makers may be

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[11] The results in this table assume equal weights for each welfare dimension. This is of course an arbitrary decision but has the advantage of minimising interference with the data. Below we test this assumption. For more extensive discussion of weights see Desai and Shah (1988) and Anand and Sen (1993).

[12] We should point out that this significant rise in health inequalities in Vietnam between 1992-93 and 1997-98 may not reflect an increase in inequality but simply the fact that better-off households may forgo workdays due

interested in assessing whether decreases in educational inequalities in Vietnam between 1992-93 and 1997-98 were sufficient to compensate for rises in monetary and health inequalities.

The 'one-at-a-time' analysis exemplified in this section is not, however, sufficient to assess the degree of complementarity or substitution that may exist between different dimensions of household welfare. Determining which of the two distributions (1992-93 or 1997-98) is more equal will depend on the degree to which the decrease in education inequality will balance out increases in income and health inequalities. In other words, determining the extent of joint inequality along the three welfare dimensions will depend on the degree of substitution between the various dimensions of household welfare. It will depend also on the relative weight society attributes to each welfare dimension.

It is also unreasonable to expect that changes in education and health distributions will leave the distribution of income unchanged or vice-versa. For instance, as mentioned above, the propensity to self-report illnesses will vary across levels of income (see Case and Deaton, 2002). In the case of Vietnam illustrated above, policy-makers may therefore be interested in assessing the joint effect of changes in various household welfare attributes.

These aspects of multidimensional welfare analysis are ignored in the one-at-a-time approach, which implies that we must think about attributes as separate and qualitatively 'equal'. In contrast, if we consider possible complementarities between attributes, we will be able to think in terms of the fact that having less of one attribute may be compensated by having more of another (Tsui, 1999; Muller and Trannoy, 2003).[13]

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to illness more frequently than when their levels of income were lower. Several authors have shown that self-reported illness tends to increase with increases in mean incomes (see Case and Deaton, 2002).

[13] See Bourguignon and Chakravarty (2002) for a similar application to the analysis of multidimensional poverty. These authors argue that in the case of two welfare attributes, say income and height, one person should be considered poor if her income falls below the income poverty line or her height falls below a height poverty

3. Multidimensional measures of inequality

The use of indices of inequality, particularly in the multidimensional case, has been subject to intense debate. Composite indices are often criticised for leading to loss of significant information when several vectors of well-being are combined into one scaler measure of inequality, and for the level of arbitrariness involved in the choice of key parameters. However, similarly to the one-dimensional case, indices of multidimensional inequality have the advantage of providing complete orderings, which can be an attractive feature in policy analysis, offer practical use and allow researchers to easily synthesise information on welfare, which is often very complex when more than two or three attributes are considered. Several indices of multidimensional inequality have been developed in the literature. For instance, Kolm (1977) suggested a generalisation of the Atkisnon-Kolm-Sen inequality index, which measures the aggregate amount of each attribute that would be 'destroyed' by the equalisation of each attribute in society (see also Bourguignon, 1999 and List, 1999), whereas Tsui (1995) proposes a measure that takes into account the amount of each attribute that should be taken away from each individual so that we obtain an allocation of attributes that is indifferent to the original distribution. Weymark (2004) provides a comprehensive survey of the state-of-the-art of this normative approach to the measurement of multidimensional inequality.

Despite significant differences in their underlying normative approach, most multidimensional inequality indices are built in a two-stage procedure. In the first stage, a utility or welfare function is used to aggregate welfare attributes for each individual, while in the second stage individual utility or welfare are summed across all individuals. All multidimensional inequality measures require therefore decisions to be made regarding the

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line. Alternatively, we can consider that person to be poor only if she falls below both poverty lines. These functional form of underlying social welfare functions, the weights attributed to different welfare dimensions, the degree of substitution or complementarity between dimensions of welfare and the underlying transfer sensitivity of welfare between different population groups along the multidimensional distribution of welfare. In this paper, we concentrate on the Maasoumi's index of multidimensional inequality. This was one of the first indices proposed in the literature and though not perfect it allows us to illustrate common empirical features of the index approach to the measurement of multidimensional inequality.

Maasoumi (1986) index of multidimensional inequality is constructed in two separate steps. The first step consists in obtaining ideal aggregation functions over desired household welfare attributes, where multivariate inequality is composed of two parts: a weighted sum of attribute inequalities and an adjustment due to the covariation (or trade-offs) between the attributes. The main objective of this first step is to find unanimity among large classes of social welfare functions over the ranking of allocations and considers all attributes to have symmetric roles. The second step uses established fundamental welfare axioms for the univariate framework to generate known index families (such as the GE family of inequality measures, which are ordinarily equivalent to members of the Atkinson class of inequality measures).[14]

The first step consists then in identifying a "well-being" function S_i that aggregates all attributes according to similar characteristics. Consider a measure X_{ij} of attributes j = 1, 2, …, m for household i = 1, 2, …n,[15] and a welfare matrix $X = (X_{ij})$ where X_i represents the i-th row and X^j represents the j-th column. Assume the existence of a scalar function (e.g.

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examples define, respectively, union and intersection definitions of multidimensional poverty.

[14] See Tsui (1995, 1999) for a direct axomatic derivation of multidimensional inequality measures also based on generalised entropy indices.

[15] The welfare unit can also be the household, state, commune, country, etc.

social welfare function) of matrix X. We can then define a generalised multivariate measure of closeness or diversity between *m* densities of *m* attributes:

$$D_{\beta}(S, X; k) = \sum_{j=1}^{m} k_j \left\{ \sum_{i=1}^{n} S_i \left[(S_i / X_{ij})^{\beta} - 1 \right] / \beta(\beta+1) \right\},$$

where k_j are the weights of each household welfare attribute and β is the coefficient of substitution between the various attributes.[16] This coefficient guarantees that changes in inequality take place due not only to changes in rankings but also to changes in the dependence between the various welfare attributes (see also Atkinson and Bourguignon, 1982).[17] This measure obeys axioms of symmetry, continuity, invariance and additive decomposability defined by Bourguigon (1979) and Shorrocks (1980, 1984).[18] Similarly to the one-dimensional case, *D* follows the Pigou-Dalton transfer principle. In the multidimensional context, this principle requires the transfer of a given attribute from a less endowed household to a more endowed household should register as a rise (or at least not as a fall) in multidimensional inequality. In other words, transferring income from an educated to a less

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[16] The Human Development Index uses a similar approach where different welfare dimensions are given equal weights (see UNDP, 1990). The issue of possible degrees of substitution between different attributes is also not considered in the literature on the HDI. Alternative approaches to derive weights are provided by the multivariate method of 'principle components' used in Ram (1982) and the literature on social deprivation (e.g. Bossert and D'Ambrosio, 2004). Bourguignon (1999) proposes a different approach to the normalisation of individual welfare based on the generalisation of a Dalton-type inequality measure to the multidimensional case.

[17] Maasoumi's index (as well as the approach proposed by Tsui, 1999) has been criticised on the grounds that it implicitly assumes attributes to be substitutes, ignoring that attributes can also be complements (see Bourguignon, 1999 and Bourguignon and Chakravarty, 2003; Weymark, 2004 reviews this argument). Effectively, Maasoumi's index averages inequality for a given attribute across its allocation for each individual or household. This averaging assumes implicitly quasi-concavity and has given rise to debate as it essentially implies incorporating properties of individual preferences into dominance analysis (Trannoy, 2004). Complementarity between attributes can be allowed for at the limit of Maasoumi's index when $\beta = 0$. Ideally, degrees of substitution or complementarity should be allowed to change along the distribution as attributes may be regarded as substitutes by the poor but complements by the rich. On this issue, see Garcia-Diaz (2004).

[18] Bourguignon (1999) argues that, contrary to uni-dimensional indices, multidimensional inequality indices need not necessarily be scale invariant in each welfare dimension. For instance, there is no reason to expect the contribution of health inequalities to overall multidimensional inequality to remain the same when incomes are doubled (see also Gajdos and Weymark, 2003). Scale invariance is only maintained in cases where the elasticity

educated household, both having the same level of income, should decrease multidimensional inequality. This is done by imposing a negative sign on the cross derivative of the utility function between two welfare dimensions (see Bourguignon, 1999; Muller and Trannoy, 2003). This principle, also referred to as the majoritization axiom, has been formalised by Koshevoy (1995, 1998), Koshevoy and Mosler (1996) and Tsui (1995, 1999) (see Weymark, 2004 for a review).

Minimising D_β with respect to S_i such as that $\sum S_i = 1$ gives us "optimal aggregation functions":

$$S_i k \left(\sum_{j=1}^{m} k_j X_{ij}^{-\beta} \right)^{-\frac{1}{\beta}}, \quad \beta \neq 0, -1$$

$$S_i k \Pi_j X_{ij}^{kj}, \quad \beta = 0$$

$$S_i k \sum_j k_j X_{ij}, \quad \beta = -1.$$

These are, respectively, the hyperbolic, the generalised geometric and the weighted means of the attributes. The "divergence measure" *D* implies the choice of an aggregate vector *S* with a distribution that is closest to the distribution of the various attributes (Maasoumi, 1999). *D* allows us therefore to measure the divergence between our distribution and a uniform distribution that represents perfect equality. As with uni-dimensional inequality, the difference between the entropies of the two distributions will constitute an

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of substitution between household welfare attributes equals one (i.e. Cobb-Douglas), which is the case being considered in this paper. For alternative approaches see the measure proposed in Bourguignon (1999).

ideal measure of inequality. The generalised entropy (GE) family of inequality indices can thus be extended to the multidimensional approach when applied to the S_i function:

$$M_{\alpha}(S) = \sum_{i=1}^{n} p_i \left[\left(S_i^* / p_i \right)^{1+\alpha} - 1 \right] / \alpha(1+\alpha), \quad \alpha \neq 0, -1,$$

where α is the degree of inequality aversion, p_i is the i-th unit's population share (1/n) and S_i^* is S_i divided by the total $\sum_{j=1}^{n} S_j$. For $\alpha = 0$, we obtain Theil's first index $M_0(S) = \sum S_i^* \log(S_i^* / p_i)$ and, for $\alpha = 1$, we obtain Theil's second index $M_1(S) = \sum p_i \log(p_i / S_i^*)$.[19]

We illustrate the empirical application of Maasoumi's measure of multidimensional inequality in table 3, where we have only included M(0) and M(1) in order to simplify the presentation of the results. As in the previous section, the various welfare attributes have been normalised to the same mean.

Table 3 includes two *S* functions in addition to an income distribution function (*S1*). *S2* assumes that household welfare is determined by two attributes, namely, income and education. *S3* adds a third dimension of household welfare, in the form of household health status. *S2* and *S3* were calculated using equal weights for each household welfare attribute.

A closer comparison between the Brazilian survey and the two Vietnamese surveys reveals a remarkable consistency in inequality rankings between the two countries; independently of the measure used and the assumptions adopted, multidimensional inequality

[19] List (1999) provides a generalisation of the Gini coefficient (see also Gadjos and Weymark, 2003) and the Atkinson family of inequality measures to the multidimensional case (see also Foster, Lopez-Calva and Szekely, 2005).

is always consistently higher in Brazil than in Vietnam. There are, however, considerable differences when examining each country separately.

In Brazil, multidimensional inequality (*S2*) measured by M(0) is higher than income inequality alone (*S1*). This is not true when we consider higher degrees of inequality aversion in society. In the case of $\alpha = 1$, multidimensional inequality in Brazil is lower than income inequality, suggesting that education inequalities lose importance the more weight is given to disturbances between points at the top of the distribution. Conclusions regarding the high levels of inequality in Brazil ought therefore to be weighted more carefully against analyses that test those results along different multidimensional distributions and different levels of inequality aversion.

The most remarkable differences are those observed between the two Vietnamese surveys, where results are highly dependent on the functional form of the social welfare function adopted, and decisions regarding the degree of substitution between attributes and the weights given to each welfare dimension. Based on *S2*, M(0) shows that multidimensional inequality in Vietnam is lower in 1992-93 than in 1997-98. However, once household health status is taken in consideration (*S3*), multidimensional inequality in Vietnam is always consistently higher in 1997-98 than in 1992-93. Furthermore, *S3* shows lower levels of overall inequality than the other two *S*-functions suggesting that households reporting higher levels of illness are also those with lower incomes and/or lower levels of education. This supports the idea proposed by Sen (1999) of 'coupling of disadvantages'. In conclusion, the analysis of multidimensional inequality in Vietnam shows that, although the decrease in education inequality in Vietnam between the two survey years was sufficient to counterweight the increase in monetary inequality between 1992-93 and 1997-98, it was not sufficient to offset simultaneous increases in both monetary and health inequalities between the two time periods.

Welfare comparisons between the two Vietnam surveys are also dependent on the choice of transfer sensitivity coefficients, of degrees of substitution, of weights attributed to each variable and inequality aversion. The issues of the specific degree of correlation between the various welfare attributes and the different weights attributed to the various welfare dimensions do not take a predominant role in the theoretical literature on multidimensional inequality. These considerations are, however, of crucial importance in empirical analyses. Although multidimensional inequality (based on *S2*) is lower in 1992-93 than in 1997-98 when $\alpha = 0$, it becomes higher for $\alpha = 1$. Conclusions regarding the measurement of multidimensional inequality depend also on the choice of correlation coefficients and different weight functions. Table 3 shows further that multidimensional inequality is lower in 1992-93 than in 1997-98 when the two attributes, consumption and education, are perfect attributes (i.e. $\beta = 1$). For lower levels of substitution between the two welfare attributes (i.e. $\beta \neq 1$), multidimensional inequality in Vietnam (based on *S2*) is consistently higher in Vietnam in 1992-93 than in 1997-98.

In table 4, we have calculated how multidimensional inequality (based on S*2*) varies across different degrees of substitution between attributes. The results show that only in the case of $\beta = 1$ is multidimensional inequality in Vietnam higher in 1997-98 than in 1992-93. In table 5, we examine how multidimensional inequality in Vietnam (based on *S2* again) varies across the different weights attributed to monetary and education inequality. The results show that multidimensional inequality is higher in 1997-98 than in 1992-93 up to the point where higher weight is attributed to the distribution of household education outcomes. From then one, multidimensional inequality in Vietnam in 1992-93 dominates that of 1997-98.

Figure 3 illustrates the importance of different weight functions and degrees of substitution between different attributes in determining the extent of multidimensional inequality in Brazil and Vietnam. The three graphs in the left-hand side of figure 3 show the values of multidimensional inequality, based on *S2*, for different degrees of inequality aversion ($\alpha = 0, \alpha = 1$ and $\alpha = 2$, respectively). The curves show that, in general, when $\alpha = 0$, multidimensional inequality increases as less weight is given to the monetary variable. When $\alpha = 1$ or $\alpha = 2$, inequality decreases for monetary weights higher than 0.9 but increases for lower values. This increase is particularly pronounced in Vietnam 1992-93 and only slight for the other two distributions.

The right-hand side of figure 3 shows the relationship between multidimensional inequality estimates and the degree of substitution between the two household welfare attributes, monetary and education status. In general, the graphs show that the larger the degree of substitution between the two attributes, the lower the degree of inequality, indicating that the attributes tend to complement each other in the three distributions (see Bourguignon, 1999).

These results demonstrate that multidimensional inequality measures are very sensitive to the choice of weights and the degree of correlation between the various dimensions. It may, therefore, be more appropriate to calculate the extent of multidimensional inequality in a given society across a range of weights and degrees of correlation rather than arbitrarily choosing values for these parameters.

4. Multidimensional stochastic dominance

One way of avoiding the criticisms inherent to the use of composite indices to characterise multidimensional inequality in any given society is to resort to stochastic dominance analysis, which allows for agreement over classes of welfare functions and over different forms of aggregating dimensions of welfare without previous knowledge of the precise form of the social welfare function.

Atkinson and Bourguignon (1982) have derived first- and second-order dominance conditions for the multidimensional case. They consider dominance conditions for several classes of utility functions defined by the signs of their derivatives up to the fourth-order.

Assume a vector X of two welfare attributes such that $X = (x_1, x_2)$. The objective is to compare bidimensional distributions with cumulative distributions $F^1(X)$ and $F^2(X)$, restricted to a finite range $[0, a_i]$, with i = 1, 2. Their density functions can be explicitly written as $f^1(x_1, x_2)$ and $f^2(x_1, x_2)$. Similarly to the one-dimensional case, that comparison will be based on the difference in expected utility between the two distributions:

$$\Delta W = \int_0^{a_1} \int_0^{a_2} U(x_1, x_2) \Delta f(x_1, x_2) dx_2 dx_1 ,$$

where U is an expected utility function continuously differentiable, and $\Delta f = f^1 - f^2$ (and $\Delta F = F^1 - F^2$).

If we concentrate, as implied in the previous section, on a class of utility functions with expected utility U increasing in both x_1 and x_2 with negative cross-derivative (i.e. $U_{12} \leq 0$),[20] we get that a sufficient condition for first-order stochastic dominance is that $\Delta F(x_1, x_2) \leq 0$ for all x_1 and x_2. This of course implies, as a special case, that $\Delta F_1(x_1) \leq 0$ for all x_1 and $\Delta F_2(x_2) \leq 0$ for all x_2.

[20] Atkinson and Bourguignon (1982) derive also conditions for first- and second-order dominance for a class of utility functions with positive cross-derivatives. We do not analyse that class in this paper as we are particularly interested in the case when having more of one variable compensates for having less of another, as discussed above.

First-order dominance conditions are naturally quite restrictive. By considering the joint distribution of the two variables, Atkinson and Bourguignon (1982) derive conditions for less restrictive second-order dominance. In particular, they derive the important result that, for the case of the class of utilities with negative cross-derivatives, "for two distributions with the same means, that with the higher covariance cannot dominate the other" (pp. 196). When the means differ, only a distribution with higher (or no smaller) means can dominate.

We explore the application of multidimensional stochastic dominance methods in table 6. In the first two columns of table 6, we compare uni-dimensional distributions of consumption expenditure and education in Vietnam in 1992-93 and 1997-98. The first row of table 6 indicates the percentage of cases of each cumulative distribution in which the 1992-93 distribution dominates the 1997-98 distribution. The second row shows the percentage of household pairs for which the 1997-98 distribution dominates that of 1992-93. The first two columns of table 6 show these dominance conditions for one-dimensioned distributions of consumption expenditure and education. The last column in table 6 shows the dominance conditions for household pairs in the bi-dimensional distribution of household welfare, where household welfare is defined as a joint distribution of household consumption expenditure and education. Unfortunately, we could not present a matrix of stochastic dominance for all point estimates, as illustrated in Atkinson and Bourguignon (1982), due to the large sample size in the surveys we have considered.

Based on the results in table 6, we cannot establish any first-order dominance conditions for any individual dimension of welfare. It is also not possible to derive any conclusions for first-order dominance for the multidimensional case (*S2*). The conditions for first-order dominance derived by Atkinson and Bourguignon (1982) for the bi-dimensional case imply that dominance must exist for all dimensions. Second-order dominance criteria provides less restrictive conditions for stochastic dominance. Table 6 shows that covariance is higher in 1997-98 than in 1992-93, indicating that the 1997-98 multidimensional distribution cannot dominate the 1992-93 distribution.

5. The case of discrete welfare attributes

Stochastic dominance techniques are particularly important when several dimensions of inequality cannot be easily reduced to a single index but we are still interested in analysing the joint distribution of the various welfare dimensions under the assumption that there may be different degrees of interdependence between the various attributes. However, computing empirically table 6 or drawing Lorenz curves for more than three variables is extremely complex.

One specialisation of the multidimensional analysis illustrated above is the case in which one welfare attribute is discrete. This attribute can then be used to split the total population into subgroups according to its values, and distribution of the continuous variable is compared within and across population groups.[21]

Let total population be divided into P exhaustive and exclusive population groups such as $\sum_{p=1}^{P}\varphi(p)=1$, where p is the population share of each group. These subgroups can be defined by any discrete welfare attribute such as, for instance, whether the head of the household is literate or not, or whether it has a debilitating illness or not. The main principle is to choose a discrete variable so that there is agreement on the ranking of individuals or households across the values of that variable. For instance, being literate is better than illiterate, being healthy is better than ill, being in the fifth income quintile is better than being in the fourth, third, second or first income quintiles and so forth. Note that those partitions

[21] See Duclos, Sahn and Younger (2002) for a similar application to the measurement of multidimensional poverty.

could also be generated by different family sizes, which would link direct multidimensional frameworks to the literature on needs and equivalence scales.

The motivation for the use of discrete welfare attributes is to determine whether, for instance, having more income will compensate for an individual being illiterate (or ill). In this case, we would use the fact that an individual is literate (or ill) or not as the splitting variable. The key idea, similarly to the analysis in the previous section, is that we can then use variables that are easily transferable, such as income, to compensate for differences in other attributes that do not result from differences in effort, choices or preferences (e.g. born with disability) (Muller and Trannoy, 2003; Trannoy, 2004). The main difference between this approach and the dominance analysis discussed in the previous section concerns the symmetry in the treatment of different welfare dimensions. The treatment proposed by Atkinson and Bourguignon takes a symmetric approach regarding the different dimensions of welfare as the implementation of the dominance criteria does not change when we permute the rows of the allocation matrix. This implies the presence of an anonymity property with respect to the set of attributes (Trannoy, 2004). In this approach, attributes are no longer symmetric as one (continuous) attribute is viewed as compensatory due to its transferability properties. This approach can therefore be very useful to formulate redistributive policies. The approach can of course be extended to the case of *n-1* discrete variables plus one continuous variable.[22]

This approach has the advantage of minimising the problem of choosing adequate weights for different variables, as well as determining the degree of substitution between different welfare dimensions. The rationale behind this approach is to compare distributions

[22] Note that this is akin to decomposing univariate distributions by a set of multidimensional discrete welfare attributes (e.g. decomposing income inequality by education quintiles).

of monetary or non-monetary attributes across 'equals' (for instance, those that are illiterate).[23]

This is akin to the idea proposed by Atkinson and Bourguignon (1989) of a restricted dominance condition, whereby dominance is required only for the bottom *x%* of the population or for people with incomes less than *y* times the mean (see also Rawls, 1971). This implies a concern with certain groups, such as the bottom quintile of the population or those that are below a poverty line, as suggested by Atkinson and Bourguignon (1989), but also, for instance, with those that are illiterate, those that have a disability, those that are illiterate and have a disability, those that do not have a house and so forth.

Assume that the population can be divided into *P* exhaustive and exclusive groups as above, and that these groups can be ranked $i = 1, \ldots,$ P with n_i households in group *i*. We can therefore write the social (additive) welfare function for all groups as:

$$W \equiv \sum_{i=1}^{P} n_i \int_o^A U^i(x) f^i(x) dx \,,$$

where $f^i(x)$ represents the distribution of a given welfare attribute within group *i* defined across different values of a discrete welfare attribute. This distribution is normalised so that $\int_0^A f^i(x) dx = 1$.

If there is agreement on a ranking of household groups so that $U_x^i(x)$ is non-decreasing with *i* for all *x* (for instance, being in the second income quintile is better than in the first but worse than in the third quintile, being literate is better than being illiterate, and so forth), then Atkinson and Bourguignon (1989) demonstrate that a necessary and sufficient

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[23] The idea of 'equal treatment of equals' is also proposed in the literature on horizontal inequality (Jenkins, 1988; Jenkins and Lambert, 1999).

condition for first-order dominance is that $\sum_{i=P-j}^{P} n_i \Delta F^i(x) \leq 0$ for all x and all j = 0, …, P-1. This condition implies the existence of first-order dominance for the 'most deserving' group P.

This approach allows therefore a practical empirical analysis of the joint impact of both monetary and educational outcomes, with important policy applications in societies interested in particular aspects of distributive justice (Rawls, 1971; Roemer, 1996). One example is, for instance, the increase of equity of opportunities for those at the bottom of the income distribution as a form of encouraging changes in the distribution of incomes by improving the access of the poor to better education and health care.

One restriction with the approach above is that it is implicitly assumed that the marginal distribution of needs does not change when comparing two populations. Muller and Trannoy (2003) relax this assumption and at the same time provide a link between Atkinson and Bourguignon (1989) and their original 1982 paper.

Again we focus on the class of utility functions which allows attributes to be substitutes, i.e. a negative sign is imposed on the cross partial derivative $U_{12} \leq 0$. This condition means, as discussed above, that the marginal utility of an attribute decreases with the level of the other. This can be represented by:

$$\bigcup_1 = \{U_1, U_2 \geq 0, U_{12} \leq 0\}.$$

A stronger version of this condition is given by the Auspitz-Lieben-Edgeworth-Pareto (ALEP) substitutability property proposed by Chipman (1977), which requires that, for instance, as a person gets richer, the marginal utility associated to all other welfare dimensions must decrease. This can be written as:

$$\bigcup_{ALEP} = \{U_1, U_2 \geq 0, U_{11} \leq 0, U_{22} \leq 0, U_{12} \leq 0\}.$$

Muller and Trannoy (2003) consider two subsets of ALEP utility functions for the case of two attributes where income is attribute 1 and health is attribute 2. In the first subset, income is the compensating attribute and health is the compensated attribute. In this case,

$$\bigcup_{MT1} = \{U_1, U_2 \geq 0, U_{11} \leq 0, U_{22} \leq 0, U_{12} \leq 0, U_{112} \geq 0\}.$$

In the second subset, income becomes the compensated attribute:

$$\bigcup_{MT2} = \{U_1, U_2 \geq 0, U_{11} \leq 0, U_{22} \leq 0, U_{12} \leq 0, U_{221} \geq 0\}.$$

The important point about the subsets above is the fact that they are no longer anonymous to the set of attributes, contrary to the approach developed by Atkinson and Bourguignon (1982). The first subset captures the case in which society is predominantly interested in assessing the distribution of income among the unhealthy. In particular, the positive sign in the cross third partial derivative U_{112} implies the decrease in marginal utility of income to be smaller among the healthy than among the unhealthy. A useful policy extrapolation of this result is that the unhealthy must take priority in receiving public funds over the healthy (Trannoy, 2004). The second subset implies that differences in marginal utilities of health between the poor are larger than those among the rich. In policy terms, acceptance of $\bigcup_{MT2}$ requires the poor to have priority in public health care. These conditions can be re-interpreted as an extension of the transfer sensitivity condition proposed by Foster and Shorrocks (1988).

These conditions can be used to compare two populations, A and B, where marginal utilities in needs vary. Multidimensional inequality will be said to be higher in A than in B if the distribution of income amongst the unhealthy is worse in A than in B and the distribution of health among the poor is worse in A than in B.

Similarly to the one-dimensional case (Bourguignon, 1979; Shorrocks, 1983), this condition can be expressed by generalised Lorenz curves. Muller and Trannoy (2003) derive explicit Lorenz dominance criteria for this case.

We illustrate the application of this special case of multidimensional inequality in tables 7 and 8. The tables show three measures of the GE family of inequality measures applied to a bidimensional distribution of household welfare, where welfare is assumed to depend on monetary attributes and education status. In table 7, the education variable (number of years spent in school by the head of the household) acts as the compensatory variable, while the monetary variable (household per capita income, in the case of Brazil, and household per capita consumption expenditure in the case in Vietnam) is used to divide the population into five distinct monetary quintiles. In table 8, income is regarded as the compensatory variable. The results in table 7 show that education inequality in Brazil is consistently higher, the lower the level of income of each population group. This clearly suggests that having less income in Brazil is not compensated by the distribution of education outcomes. In addition, income inequality decreases across the income quintiles, and households at the top of the education distribution benefit from lower income inequality than households with low levels of education.

In Vietnam, the relationship is less linear as the distribution of education outcomes is more unequal among households in the fourth income quintile than among households in the third income quintile. Income inequality is also higher in the last two quintiles than in quintiles 2 and 3. In 1997-98, income inequality is the lowest amongst households at the bottom of the education inequality. This indicates that while the compensatory value of the education dimension of welfare (which can be re-interpreted as the distribution of opportunities) is quite low in Vietnam, the compensatory value of income is high particularly in 1997-98. Conclusions regarding changes in inequality in Vietnam between 1992-93 and 1997-98 depends on decisions regarding the 'most deserving' group. If we draw the line at 20%, then multidimensional inequality has decreased in Vietnam between 1992-93 and 1997-98 given that the distribution of income amongst the 20% least educated households improved between 1992-93 and 1997-98, while the distribution of education amongst the 20% poorest households also improved between the two years.

Stochastic dominance techniques can also be used to illustrate the above approach. In order to explore the nature of multidimensional inequality further, we have drawn various multidimensional Lorenz curves, following the restrictive dominance criteria proposed in Atkinson and Bourguignon (1989) for the case of bidimensional inequality, where one attribute is continuous and the other is discrete. As an illustrative example, we compare education Lorenz curves for the first and fifth income quintiles for Brazil and Vietnam (1992-93 and 1997-98) (figures 4 and 5, respectively). Figures 4 and 5 show interesting results. While figure 4 shows a clear dominance of the Brazilian education distribution over the two Vietnamese distributions, figure 5 shows that education inequality in Brazil amongst those better-off in monetary terms is lower than in Vietnam in any of the two years being considered. This adds a new view to the analysis of overall inequality in Brazil: not only Brazil has high levels of income inequality but these are reinforced by low opportunities generated in other sections such as education. This result offers new perspectives for further research on possible complementarities or reinforcement of inequalities not only in Brazil and Vietnam but elsewhere.

6. Conclusions

This paper explored the empirical application of theoretical multidimensional inequality analysis using real household welfare distributions. Its main aim was to provide a practical link between complex developments in the theoretical analysis of multidimensional welfare and useful empirical applications which have thus far remained elusive. The results discussed above demonstrated that there is an important case for considering inequality as a multidimensional phenomenon. Multidimensional inequality approaches can be of considerable interest for policy analysis as they allow the joint assessment of the simultaneous impact of different social policies, and of whether the deterioration in the distribution of some welfare dimension can be compensated by improvements in the distribution of other welfare attributes.

Multidimensional inequality indices require, however, considerable judgements regarding the relative importance of the various attributes, the degree of substitution between them and the degree of inequality aversion in society that weaken their policy application. In particular, a key question remains as to what effectively constitutes household welfare. In this paper, we abstracted from this consideration as our main objective was to analyse the empirical application of theoretical multidimensional inequality measures. However, conclusions regarding the level and changes in the distribution each dimension of welfare will be highly dependent on the choice of the underlying welfare indicator. For instance, in a different paper, Justino, Litchfield and Niimi (2004) showed that the measurement of education inequality in Brazil varies widely with different education indicators. We have also experimented with measuring health inequalities using different indicators of household health status (Justino and Niimi, 2005), and reached similar conclusions. These issues introduce further layers of complexity to the measurement of multidimensional inequality that need to be resolved in order to enable the use of multidimensional inequality analysis in comparative studies.

The paper showed, however, important ways forward in the analysis of multidimensional inequality. In particular, we showed notions of restrictive dominance criteria and compensatory transfers may offer significant scope for further developments in the empirical analysis of multidimensional inequality.

There are of course still many challenges to be faced in the application of multidimensional inequality analysis. However, these do not imply that consensus over the measurement of multidimensional inequality cannot be reached but rather that much more empirical research is urgently needed.

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Table 1: Monetary and non-monetary inequalities in Brazil and Vietnam

	Gini coefficient	GE(0)	GE(1)	GE(2)
Monetary variables				
Brazil 1996	0.603	0.915	0.735	1.721
Vietnam 1992-93	0.357	0.236	0.227	0.311
Vietnam 1997-98	0.401	0.280	0.298	0.464
Education levels				
Brazil 1996	0.491	4.491	0.446	0.395
Vietnam 1992-93	0.408	2.928	0.310	0.260
Vietnam 1997-98	0.360	1.993	0.239	0.200
Health status				
Vietnam 1992-93	0.573	4.730	0.598	0.678
Vietnam 1997-98	0.753	9.718	1.148	1.672

Source: Author's calculations from PNAD 1996 and VLSS 1992-93 and 1997-98.
Notes: All values are weighted using weights in respective surveys. Monetary indices for Brazil refer to per capita household income, while values for Vietnam values refer to per capita household consumption expenditure. In order to calculate the various inequality indices, zero values in the above variables have been replaced by 0.000000001.

Table 2: Correlation between welfare dimensions in Brazil and Vietnam

	Education		Health	
	Pearson correlation coefficient	Spearman rank correlation	Pearson correlation coefficient	Spearman rank correlation
Vietnam 1992-93				
Monetary	0.164	0.149	-0.058	-0.067
Education	1.000	1.000	-0.128	-0.145
Health			1.000	1.000
Vietnam 1997-98				
Monetary	0.241	0.237	-0.032	-0.096
Education	1.000	1.000	-0.084	-0.123
Health			1.000	1.000
Brazil 1996				
Monetary	0.440	0.527		
Education	1.000	1.000		

Source: Author's calculations from PNAD 1996 and VLSS 1992-93 and 1997-98.
Note: The null hypothesis of rank independence of each pair of variables is rejected for all cases.

Table 3: Multidimensional inequality (Maasoumi index)

Variables included in *S*	S1: Income	S2: Income; education	S3: Income; education; health
Brazil 1996			
M(0) β = 1	0.915	1.221	
M(0) β = 1/2		1.272	
M(0) β = 1/3		1.374	
M(1) β = 1	0.735	0.435	
M(1) β = 1/2		0.447	
M(1) β = 1/3		0.467	
Vietnam 1992-93			
M(0) β = 1	0.236	0.319	0.202
M(0) β = ½		0.374	0.260
M(0) β = 1/3		0.456	0.357
M(1) β = 1	0.227	0.208	0.142
M(1) β = ½		0.218	0.176
M(1) β = 1/3		0.234	0.230
Vietnam 1997-98			
M(0) β = 1	0.280	0.320	0.245
M(0) β = 1/2		0.348	0.314
M(0) β = 1/3		0.400	0.438
M(1) β = 1	0.298	0.199	0.169
M(1) β = 1/2		0.199	0.214
M(1) β = 1/3		0.207	0.298

Source: Author's calculations from PNAD 1996 and VLSS 1992-93 and 1997-98.
Note: S2 and S3 assume equal weights for all household welfare attributes.

Table 4: Multidimensional inequality across correlation ranges for S2

β	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
					Vietnam 1992					
M(0)	0.920	0.626	0.485	0.413	0.374	0.35	0.336	0.327	0.321	0.319
M(1)	0.264	0.253	0.239	0.227	0.218	0.213	0.21	0.208	0.208	0.208
M(2)	0.226	0.219	0.211	0.204	0.199	0.195	0.193	0.193	0.193	0.194
					Vietnam 1998					
M(0)	0.699	0.511	0.419	0.373	0.348	0.335	0.327	0.323	0.321	0.320
M(1)	0.228	0.219	0.21	0.203	0.199	0.198	0.197	0.197	0.198	0.199
M(2)	0.213	0.202	0.192	0.185	0.181	0.178	0.177	0.177	0.178	0.179
					Brazil 1996					
M(0)	1.961	1.598	1.412	1.319	1.272	1.248	1.234	1.227	1.223	1.221
M(1)	0.541	0.503	0.474	0.457	0.447	0.442	0.439	0.437	0.436	0.435
M(2)	0.603	0.527	0.473	0.439	0.419	0.408	0.402	0.398	0.395	0.394

Source: Author's calculations from PNAD 1996 and VLSS 1992-93 and 1997-98.

Table 5: Multidimensional inequality across weight ranges for S2

Income weight	1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
Education weight	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
							Vietnam 1992				
M(0)	0.237	0.184	0.199	0.231	0.272	0.318	0.37	0.431	0.508	0.627	2.928
M(1)	0.227	0.159	0.154	0.168	0.187	0.208	0.228	0.249	0.268	0.288	0.31
M(2)	0.311	0.194	0.166	0.168	0.179	0.194	0.209	0.223	0.237	0.249	0.26
							Vietnam 1998				
M(0)	0.28	0.189	0.213	0.248	0.284	0.32	0.359	0.402	0.455	0.535	1.993
M(1)	0.298	0.172	0.167	0.177	0.189	0.199	0.209	0.217	0.225	0.232	0.239
M(2)	0.464	0.208	0.172	0.169	0.173	0.179	0.184	0.189	0.193	0.196	0.2
							Brazil 1996				
M(0)	0.915	0.773	0.924	1.036	1.131	1.221	1.311	1.411	1.531	1.715	4.491
M(1)	0.735	0.411	0.419	0.426	0.432	0.435	0.438	0.44	0.442	0.444	0.446
M(2)	1.721	0.428	0.4	0.395	0.394	0.394	0.394	0.394	0.394	0.395	0.395

Source: Author's calculations from PNAD 1996 and VLSS 1992-93 and 1997-98.

Table 6: Multidimensional stochastic dominance, Vietnam 1992-93 and 1997-98

$\Delta F = F_{98} - F_{92}$	Monetary distribution	Education distribution	$X = (x_1 : income; x_2 : education)$
$F_{92} \succ F_{98}$ (%)	49.13	54.80	54.74
$F_{98} \succ F_{92}$ (%)	50.87	45.20	45.26
Covariance 1992-3			0.033
Covariance 1997-8			0.034

Source: Author's calculations from VLSS 1992-93 and 1997-98.

Table 7: Education inequality coefficients per monetary quintile

	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
Brazil 1996					
GE(0)	8.432	6.048	4.175	2.329	0.839
GE(1)	0.791	0.545	0.392	0.266	0.150
GE(2)	0.779	0.479	0.333	0.278	0.124
Vietnam 1992-93					
GE(0)	4.477	3.464	2.563	2.075	2.032
GE(1)	0.412	0.333	0.274	0.274	0.253
GE(2)	0.336	0.268	0.229	0.240	0.219
Vietnam 1997-98					
GE(0)	3.473	2.127	1.532	1.683	1.092
GE(1)	0.346	0.240	0.205	0.212	0.173
GE(2)	0.285	0.198	0.175	0.177	0.147

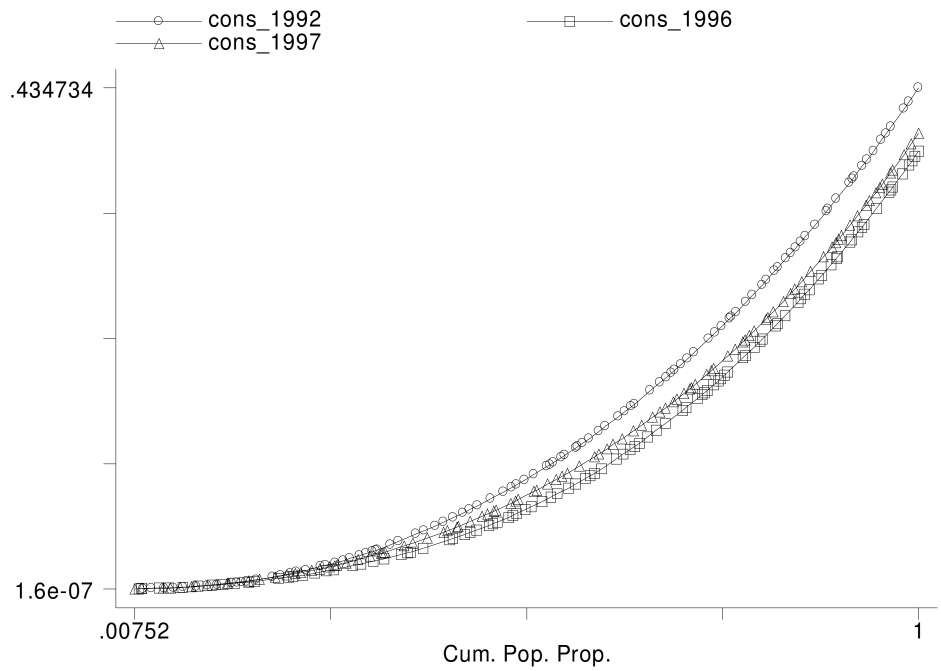
Source: Author's calculations from PNAD 1996 and VLSS 1992-93 and 1997-98.

Table 8: Income inequality coefficients per education quintile

	Quintile 1	Quintile 2	Quintile 3	Quintile 4	Quintile 5
Brazil 1996					
GE(0)	0.948	0.916	0.803	0.857	0.736
GE(1)	0.437	0.503	0.477	0.459	0.515
GE(2)	0.728	1.061	1.029	0.805	0.912
Vietnam 1992-93					
GE(0)	0.189	0.141	0.165	0.172	0.194
GE(1)	0.194	0.181	0.170	0.203	0.213
GE(2)	0.264	0.282	0.204	0.305	0.292
Vietnam 1997-98					
GE(0)	0.155	0.175	0.184	0.158	0.252
GE(1)	0.165	0.206	0.202	0.182	0.282
GE(2)	0.221	0.310	0.264	0.262	0.426

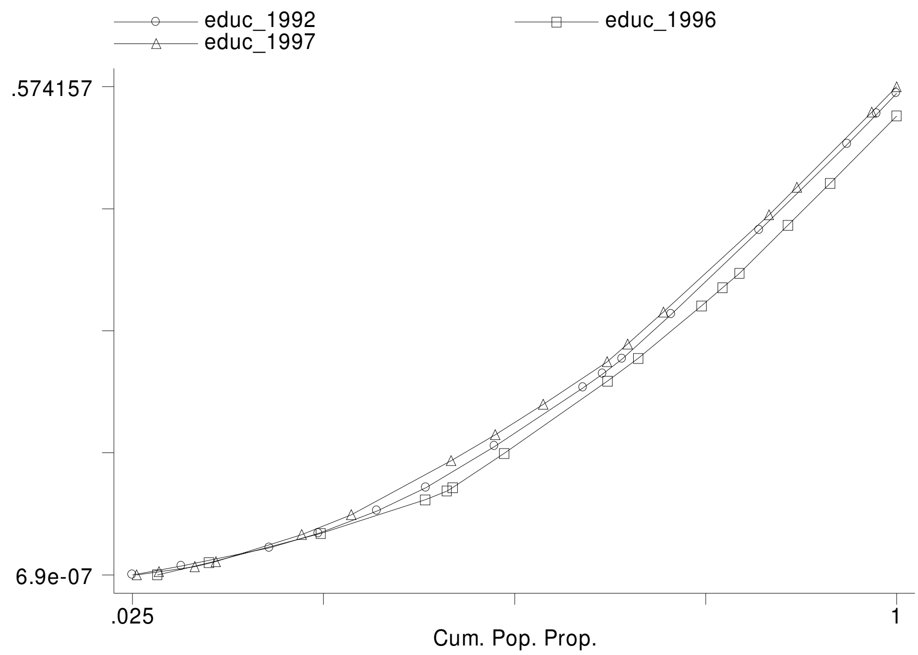
Source: Author's calculations from PNAD 1996 and VLSS 1992-93 and 1997-98.

Figure 1: Monetary Lorenz curves



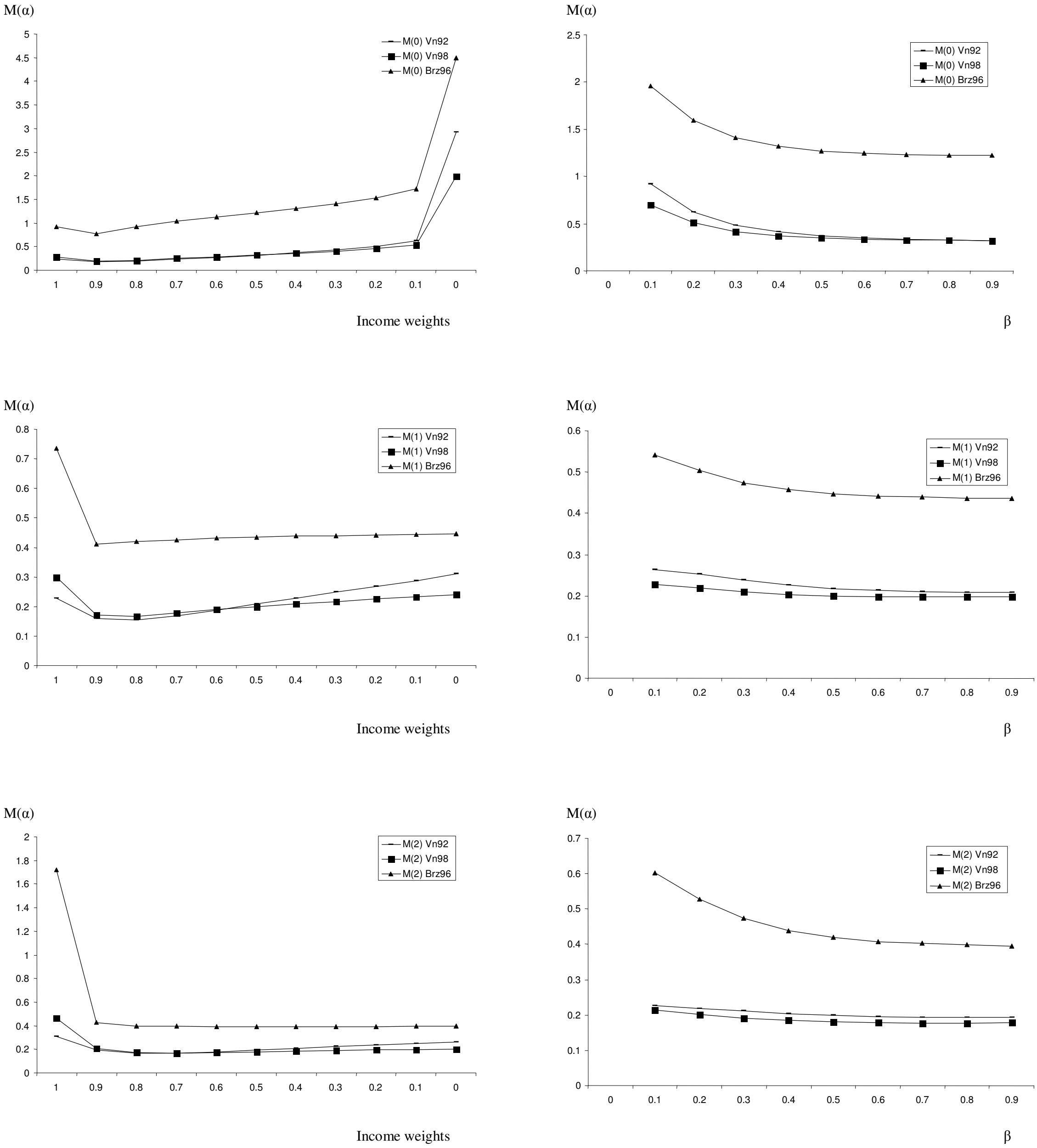
Source: Author's calculations from PNAD 1996 and VLSS 1992-93 and 1997-98.
Notes: cons_1992 refers to household per capita consumption expenditure distribution for Vietnam in 1992-93, cons_1997 refers to household per capita consumption expenditure distribution for Vietnam in 1997-98 and cons_1996 refers to household per capita income distribution for Brazil in 1996.

Figure 2: Education Lorenz curves



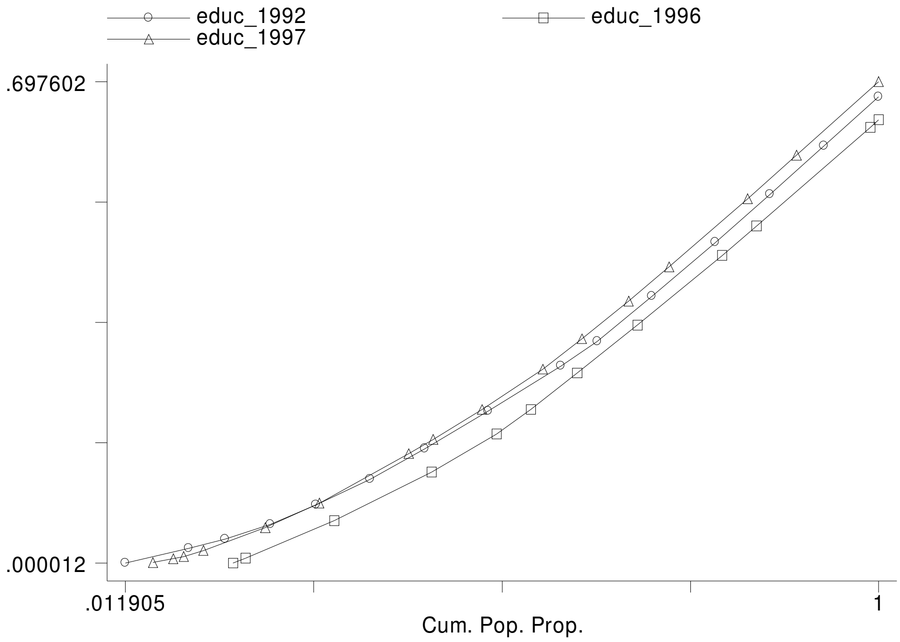
Source: Author's calculations from PNAD 1996 and VLSS 1992-93 and 1997-98.
Notes: educ_1992 refers to household per capita consumption expenditure distribution for Vietnam in 1992-93, educ_1997 refers to household per capita consumption expenditure distribution for Vietnam in 1997-98 and educ_1996 refers to household per capita income distribution for Brazil in 1996.

Figure 3: Change in multidimensional inequality across weight functions and degree of substitution



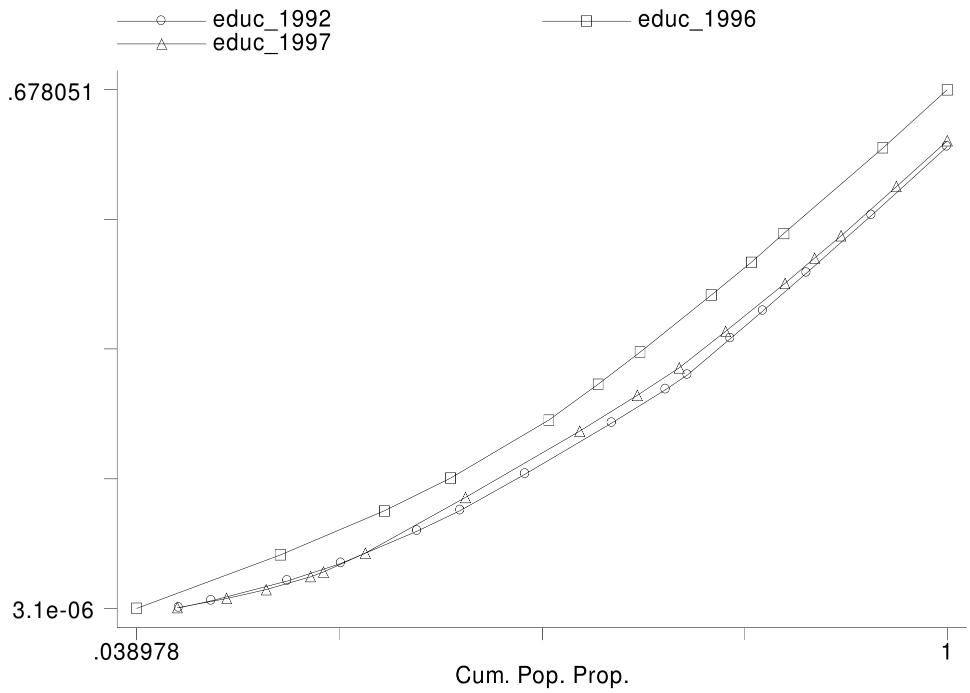
Source: Author's calculations from PNAD 1996 and VLSS 1992-93 and 1997-98.

Figure 4: Education Lorenz curves for first monetary quintile



Source: Author's calculations from PNAD 1996 and VLSS 1992-93 and 1997-98.

Figure 5: Education Lorenz curves for fifth monetary quintile



Source: Author's calculations from PNAD 1996 and VLSS 1992-93 and 1997-98.