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Non-cooperative Monetary and Fiscal Policy: The Value of Leadership*

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May 14, 2004

Abstract

We consider leadership equilibria in dynamic linear rational expectations models with specific reference to monetary and fiscal policy interactions. We pay particular attention to the strategic relationships between policymakers and the private sector. We find that in a dynamic game that although leadership may not matter much, giving leadership to a constrained policymaker should be avoided. Key

Words: Time Consistency, Commitment, Leadership Equilibria, Monetary and Fiscal Policy JEL References: E31, E52, E58, E61, C61

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1 Introduction

The question of how efficiently fiscal policy can be used for the stabilisation of an economy has become a question of great importance for the European Monetary Union. There is a growing body of literature to answer the call for a theoretical framework; see Benigno and Woodford (2003), Dixit and Lambertini (2003), Lambertini and Rovelli (2003), Beetsma and Jensen (2002, 2003) amongst others. Some authors study the question in a simplified, non-strategic setup, or with complete cooperation between policymakers; however it is clear that non-cooperative, leadership equilibria can be very different and more realistic. Such a regime is examined by Dixit and Lambertini (2003) using a static model.

A weakness of the static approach is that the stabilisation problem is intrinsically dynamic, as the role of fiscal policy at least partly depends on debt accumulation. The treatment of dynamic leadership equilibria in the rational expectations literature has not always been either consistent or satisfactory. For example, most of the voluminous literature on policy coordination only considers Nash games. Much of the early analysis relied on static albeit repeated games (e.g. Canzoneri and Henderson, 1996). In this work the focus is on an equilibrium bias, where the average level of inflation is permanently above the optimum. In dynamic games, the level bias is less of an issue: we should rather be worried about so-called stabilisation bias where it takes longer to reach the long run (or target) equilibrium. This is manifested in increased unconditional variances of target variables such as inflation or growth. However, any moderately complicated dynamic model needs to be solved using numerical methods and these methods are neither readily available nor well articulated for models with rational expectations.[1]

Therefore, the purpose of this paper is twofold. Firstly, we wish to discuss the results of recent research on monetary and fiscal interactions and leadership, in particular of Dixit and Lambertini (2003), in a dynamic setting. We show that the form of the game and the leadership assumed matters considerably: not all Dixit and Lambertini's results are transferrable to the analogous dynamic game. We discuss different leaderships regimes and the consequent stabilisation benefits of fiscal policy in a single economy.

The first part is impossible to fulfill without a development of an appropriate modelling framework. Therefore, we discuss in details the concepts of (and solution algorithms for) a leadership discretionary equilibrium for dynamic linear rational expectations macroeconomic models where we make the role of leadership in this context explicit. This is best done if one solves the problem by using Lagrange multipliers. This method conveys the underlying information structure and allows the interpretation of the discretionary

[1] de Zeeuw and van der Ploeg (1991) provides an excellent discussion of discrete dynmaic games and can be compared with our analysis.

optima as feedback Stackelberg equilibria. This method, however, allows to solve problems which are not indeterminate, i.e. where there is a unique solution. Clearly, the initial approximation should deliver determinacy too. It is often not the case in the current mainstream macromodels, at least for initial conditions. Therefore, we then discuss how to find the same solution with easy-to-use numerical procedures based on the principle of dynamic programming, which can pickup a solution for indeterminate problem, but has some other drawbacks instead. Although this framework is a contribution to the literature and a necessary preliminary step before we investigate any real problem, we put detailed discussion of the concept and solutions into Appendix.

In all that follows we are unashamedly game-theoretic in our approach, and adopt the terminology of dynamic game theory, exemplified in Basar and Olsder (1999), even when the game is in some sense implied rather than explicit. This is somewhat at odds with much of the recent monetary policy literature which constructs consistent equilibria with little regard to the underlying strategic behaviour. However, we feel that correct treatment of any potential interactions is vital to the understanding of the resulting policy regimes.

2 Monetary and Fiscal Policy Interaction

In order to investigate monetary and fiscal policy interactions we use the Dixit and Lambertini (2003) model modified to a dynamic context. We aim to compare different leadership equilibria. We also comment on how effective fiscal policy is in stabilising a single economy.

As it is discussed in Appendix A.6, unilateral commitment of one of authorities is not possible in a dynamic game, so we only consider discretionary game.

We consider a closed economy with two policymakers, the fiscal and monetary authorities. Fiscal policy is allowed to support monetary policy in stabilisation of the economy. As it is common in the recent literature, we abstract from the problem of fiscal solvency, and consider short-run stabilisation only[2].

It has been shown in the literature (Kollmann (2003), Schmitt-Grohe and Uribe (2003) and others) that in a single economy a stabilising fiscal policy can do very little – the consumption gain from stabilisation efforts does not exceed 0.02% of a steady state consumption level. In a Monetary Union, however Kirsanova et al. (2003) conclude that a stabilising fiscal

[2]It was shown in Kirsanova, Satchi and Vines (2004) that it is enough for the fiscal authorities to feed back on debt with a small coefficients in order to deliver sustainability of the debt. All quantitative results are then almost identical for a model with debt and without it.

policy greatly improves welfare when economy faces asymmetric shocks, especially if there is substantial persistence. Therefore we keep a potentially important property of inflation persistence in order to both have truly dynamic model[3], and investigate welfare improvements from fiscal stabilisation in a single country with persistence. Despite small stabilisation effect, the leadership issues can be still worth studying – they might be important for a policy design in a monetary union where the fiscal policy is more welfare improving.

3 The Model

We consider the now-mainstream macro model, discussed in Rotemberg and Woodford (1997), and slightly modified to give account to the effects of fiscal policy, in a spirit of Beetsma and Jensen (2002). Here, we only briefly discuss the main assumptions of the model and the reader is referred to the Additional Appendix for a derivation of all equations[4].

Our economy is inhabited by a large number of individuals, and there are two policymakers: monetary and fiscal authorities. Each representative individual is a yeoman-farmer, who specialises in the production of one differentiated good, denoted by z, and spends $h(z)$ of effort on its production. An individual also consumes a consumption basket C, and ξ are technology/taste shocks. Preferences are assumed to be:

$$\max_{\{C_s,h_s\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}[u(C_s,\xi_s)+f(G_s,\xi_s)-v(h_s(z),\xi_s)] \tag{1}$$

An individual chooses optimal consumption and work effort to maximise the criterion (1) subject to the demand system and the intertemporal budget constraint. We have assumed that utility is separable in private and government consumption.

The first order conditions with respect to consumption, leads to the familiar Euler equation (intertemporal IS curve), where c_t denotes consumption and all variables are log-deviations from the efficient equilibrium:

$$c_t = c_{t+1} - \sigma(i_t - \pi_{t+1}) + \eta_t. \tag{2}$$

In order to describe price setting decisions we follow Rotemberg and Woodford (1997) as extended by Steinsson (2003) to get:

$$\pi_t = (1-\chi)\beta\pi_{t+1} + \chi\pi_{t-1} + \kappa_c c_t + \kappa_{x0} x_t + \kappa_{x1} x_{t-1} + \varepsilon_t \tag{3}$$

[3]Despite ignoring the dynamic debt accumulation equation, our model is highly dynamic: an appropreate treatment of household behaviour leads to the dynamic consumption process and inflation process, which we considet to be highly persistent.

[4]Available from www.ex.ac.uk/~tkirsano/AppLeadership.pdf

where π denotes inflation and x denotes output. All coefficients can be derived from microfoundations and given in Appendix B.

The system (2) and (3) is formally equivalent to the optimising behaviour of a representative agent who maximises (1) subject to an aggregate 'law of motion' of the economy (the demand system, the intertemporal budget constraint and pricing decisions) when policymaker's behaviour is taken to be an exogenous process, independent of the individual's actions.

Apart from the private sector's behaviour, explained with (2) and (3), the evolution of the economy, as observed by the policymakers, includes the aggregate demand equation (4):

$$x_t = \theta c_t + (1 - \theta) g_t. \tag{4}$$

Both policymakers are trying to minimise their loss functions. If they are benevolent, each of them tries to minimise social loss. The one-period social loss function can be derived as second-order approximation to consumer's utility, written in terms, which can be affected by policies. We follow the approach discussed in Rotemberg and Woodford (1997) to derive it, see the Additional Appendix. We assume that monopolistic distortions are offset with subsidy, which is financed by a lump-sum taxation, so the social loss only contains quadratic terms:

$$W_t = \lambda_c c_t^2 + \lambda_g g_t^2 + \lambda_x x_t^2 + \pi_t^2 + \mu_\pi (\Delta \pi_t)^2 + \mu_x x_{t-1}^2 + \mu_{\pi x} \Delta \pi_t x_{t-1}. \tag{5}$$

The last three terms in the loss function ($\mu-$ terms) are due to inflation persistence, while the first four terms are more conventional and reflect static functional form of household utility. Since our utility is separable in the private and public consumption, we cannot collapse the three terms with $\lambda-$ coefficients into a single quadratic term in output.

If both authorities are benevolent, then the same social loss function should be given to both of them, it implies that the costs of volatility of the fiscal adjustment are important for the monetary authorities too.

The monetary authorities (MA) use the short-term interest rate to minimise the 'cost-to-go' with one-period social loss function:

$$\min_{\{i\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} W_s. \tag{6}$$

The fiscal authorities (FA) are given the same objective but use government spendings as an instrument.

Although we start with identical loss functions, we aim to investigate some implications of different objectives too. The two policymakers solve their optimisation problems each period, given initial conditions and time preferences. The resulting optimal policy reactions lead to stochastic equilibria that should be compared across a suitable metric, independent of initial conditions. The obvious choice of this metric is the microfounded social loss, which on the convenient assumption that social planner does not discount the future, is a sum of unconditional variances with microfounded weights:

$$W = \lambda_c \text{var}(c) + \lambda_g \text{var}(g) + (\lambda_x + \mu_x) \text{var}(x) + \text{var}(\pi) + \mu_\pi \text{var}(\Delta\pi) + \mu_{\pi x} \text{cov}(x_{-1}, \Delta\pi). \quad (7)$$

Despite that monetary and fiscal authorities both affect demand, they affect it in a very non-symmetric way. The fiscal authorities can change it directly by means of government purchases, while the monetary authorities can change intertemporal allocation of consumption and affect the demand via consumption. Consumption constitutes the biggest part of the aggregate demand, and we intentionally chose a substantial equilibrium ratio of the public consumption to output, in order to increase the power of the fiscal policy.

4 A Strategic Discretionary Game

Our problem can be formalised as follows. We have three players in the game: two explicit players, monetary and fiscal authorities, whose objective functions can be written as

$$\min_{\{U_s^L\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} W_s^L, \quad \min_{\{U_s^F\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} W_s^F \quad (8)$$

and one implicit player, the private sector, whose optimisation problem is solved out and can be presented by a difference equation

$$X_{s+1} = a_{21} Y_s + a_{22} X_s + b_{21} U_s^L + b_{22} U_s^F. \quad (9)$$

Here U denotes policy instruments of the authorities (either leader L or follower F – interest rate and government expenditures) and X denotes instruments of the private sector, inflation (inflation expectations) and consumption, which are non-predetermined, or jump, variables. Additionally, the can be predetermined state variables, Y, which evolution can be explained as

$$Y_{s+1} = a_{11} Y_s + a_{12} X_s + b_{11} U_s^L + b_{12} U_s^F \quad (10)$$

The example of such variable is output as explained by equation (4).

The monetary and fiscal authorities can either move first (leader) or second (follower), but the private sector is an ultimate follower in the policy game: it moves third and treats policy instruments parametrically.

We discuss in the Appendix the information structure of the game and demonstrate that in a dynamic setup with two policymakers we are limited to considering discretionary equilibria only: commitment of one of the authorities is impossible unless the other authority precommit to the same target. We discuss two numerical algorithms for a solution of a discretionary problem in the Appendix, and here we only emphasize that the optimal solution for the monetary and fiscal instruments can be written in the form:

$$U_s^L = -F^L Y_s, \tag{11}$$
$$U_s^F = -F^F Y_s - L U_s^L. \tag{12}$$

Here F denotes feedback coefficients on the predetermined state and L is the leadership parameter. The leader feeds back on the predetermined state variables and the follower takes into account the leader's actions. Thus, the leader can manipulate the follower by changing its instrument.

5 Discretionary Leadership Equilibria

To compute equilibria, we use numerical algorithm described in Appendix. We run the following four scenarios, most commonly discussed in the literature.

In the first scenario both authorities are benevolent and use the social loss function (5) to stabilise the economy. They still act non-cooperatively under either monetary or fiscal leadership. We use this scenario as a benchmark for further investigations.

In the second scenario we, following Dixit and Lambertini (2003), we investigate the case where the monetary authorities are more conservative than fiscal authorities. The fiscal authorities still minimise social 'cost-to-go'.

In the third scenario, the monetary authorities minimise the social loss function while the fiscal authorities also use the same function (they support monetary policy) but, additionally, they are penalised for excessive volatility of the primary deficit/surplus. Such constraint models restrictions similar to those imposed by the Stability and Growth Pact.

In the fourth scenario the fiscal authorities minimise the social loss function, and the monetary authorities do the same, but they are also required to change interest rate smoothly – there is a penalty on change in the interest rate. A sluggishness of interest rate adjustment can be motivated by the requirement of financial stability.

We vary these penalties (or conservatism parameter) to see robustness of the results. Table 1 presents the results of welfare loss evaluation. We keep realistic calibration of the Phillips curve with inflation persistence[5].

[5] We calibrate the parameters as $\beta = 0.99$, $\sigma = 0.5$, $\epsilon = 5$, $\psi = 2$, $\theta = 0.6$ and $\gamma = 0.75$. The standard deviation of all shocks is 0.5%.

5.1 Benevolent Policymakers

As well as Dixit and Lambertini (2003) we obtain that the leadership does not matter for the two benevolent policymakers, see Table 1, the second column. In a dynamic setup all components of the loss are also identical for both leadership regimes. We also include column with non-strategic behaviour by the fiscal authorities where their role is to keep government expenditure constant, so monetary policy is left to stabilise shock alone[6]. A comparison of the two columns reveals that if fiscal policy is allowed to be stabilising, and even in a non-cooperative setup, both monetary and fiscal policy together can do better than the monetary policy can do alone.

It is useful to note that with our preferred calibration of the model, the optimal solution for a monetary and fiscal policy can be presented as follows. For the monetary leadership

$$\begin{aligned} i_t^L &= 12.23\varepsilon_t + 2.00\eta_t + 4.05\pi_{t-1} + 0.80x_{t-1} \\ g_t^L &= 0.03\varepsilon_t - 0.94\eta_t + 0.41\pi_{t-1} - 0.02x_{t-1} + 0.47i_t^L \end{aligned}$$

and for the fiscal leadership

$$\begin{aligned} g_t^F &= 5.76\varepsilon_t + 0.00\eta_t + 2.31\pi_{t-1} + 0.35x_{t-1} \\ i_t^F &= 6.33\varepsilon_t + 2.00\eta_t + 1.69\pi_{t-1} + 0.44x_{t-1} + 1.02g_t^F. \end{aligned}$$

It is seen that in both cases the leadership coefficient is positive, so contractionary monetary policy goes along with an expantionary fiscal policy.

5.2 Conservative Central Banker

We then increase conservatism of the monetary authorities. The fiscal authorities are minimising 'cost-to-go' with social one-period loss function, and the monetary authorities do the same, except their weight on output stabilisation (all $\lambda-$ coefficients) are multiplied by a common multiplier ρ_c which is decreasing from one (benevolent monetary authorities) to almost 0.5 (conservative monetary authorities), see notes to Table 1. We rank the outcomes using social metric (7). We find that

(i) a slight conservatism of monetary authorities delivers better stochastic equilibrium than the two benevolent policymakers,

(ii) only with higher conservatism of the monetary authorities, the fiscal leadership is preferable to the monetary one, and

(iii) if the conservatism is large enough, then both leadership regimes are worse than the benevolent regime.

[6] Automatic stabilisers still operate via taxation.

To understand these results it might be helpful to look at impulse responses to supply shocks[7] in Figure 1. The benchmark solid line denotes responses under benevolent authorities, they are identical for the two leadership regimes. With an increase in the central bank conservatism, the monetary authorities concentrate more on inflation stabilisation, and are prepared to pay for this with higher output variability. If the monetary authorities are a leader, they are able to manipulate the fiscal authorities to help them to reach their conservative target. This requires an aggressive reaction of the interest rate (with a consequent consumption volatility) and it results in higher variability of a fiscal instrument too. The higher output variability (and all its components: terms of (5) with $\lambda-$ coefficients) has smaller effect on the monetary autorities' loss due to the reduced weight of it. However, with increased conservatism the output costs soon become substantial component in the social costs, so the regime stops being superior to all other regimes. When the fiscal authorities lead, they try to manipulate monetary authorities to use their instrument to affect demand to help them to minimise *their* costs. However, the fiscal authorities are more concerned with variability of the fiscal instrument, so they are less able to manipulate the monetary authorities, as their instrument becomes less volatile. So the monetary authorities are still successful in being tough on inflation. This still improves the stochastic equilibrium. Because the biggest relative penalty in the loss function is on inflation variability, this determines the superiority of conservative regimes relative to the benevolent regime. This effect is eliminated when it is paid for with high variability of the fiscal expenditures.

The analysis of these interactions might suggest that a constrained policymaker performs worse when being a leader, as it is not only constrained in optimisation, but also in manipulation of the follower, see formulae (11) and (12). If the follower is penalised, the leader could still be flexible enough to (partly) compensate for such constraint. However, the state of being constrained is difficult to define. In what follows we take examples with additional quadratic terms (with positive weights) in the loss function of one of the policymakers. This determines constraint and we might expect that this should increase social loss of a resulting stochastic equilibrium[8]. We next look at two examples.

[7] Due to entirely forward-looking structure of the Euler consumption equation, demand shocks are immediately suppressed with interest rate reaction, this reaction is identical for all leadership regimes and does not contribute to the difference in losses.

[8] This is not necessarily a general case, as the stochastic metric does not assume discounting, for example.

5.3 Stability and Growth Pact

There is much discussion in the literature of how restrictive is the Stability and Growth Pact (SGP) for the member countries of the European Monetary Union. By imposing penalty for excessive public deficit, the Pact reduces the stabilisation ability of the fiscal policy and reduces union-wide welfare[9]. Here we look at the leadership issue in a single country. It is a common view that in a single country the fiscal authorities are the leader, although it is also common that the fiscal authorities are not supposed to support the stabilisation efforts of the monetary authorities – the priority is given to stabilisation of the domestic debt. In the monetary union the question about whether it is desirable to allow for stabilisation function of the fiscal authorities is not solved yet. Additionally, there is no consensus on the resulting information structure and how the fiscal policy should be organised: it is only clear that the current situation can be improved but rules vs. institution question is still open. We look here whether the leadership issue might be important if we want to design an institutional structure.

Columns (4) and (5) of Table 1 suggest that under the SGP, the regime of fiscal leadership is not only worse than the regime of monetary leadership, but it is also often worse than the regime with non-strategic fiscal behaviour with automatic stabilisers (compare columns (3) and (5)). Figure 2 illustrates that under the fiscal leadership, when the fiscal authorities try to manipulate monetary authorities, inflation is less controlled. Apparently, this is enough to ensure inferiority of the fiscal leadership, as inflation variability constitutes the main component of the social loss. This component can also be big enough so that the fiscal leadership becomes worse than the regime with automatic stabilisers. This example supports the conjecture that a non-constrained leader performs better. However, in this case, we imposed a constraint on a policymaker whose participation in a stabilisation game improves welfare only marginally. In the next example we look at a constrained monetary authorities.

5.4 Financial Stability

We now require the monetary authorities would change interest rate smoothly, in order to protect financial stability. In this case the monetary policy is not able to offset the demand shocks completely, so the costs will be higher than under benevolent authorities simply because of the extra losses due to demand shocks. As demand shocks cannot be eliminated by means of monetary policy, the fiscal policy has to intervene thus raising costs of change in its instrument. This cost is higher under monetary leadership regime as monetary authorities still try to manipulate fiscal authorities inducing extra volatility. For supply shocks, when monetary policy is unable to react

[9] See Kirsanova et al. (2003)

aggressively, the fiscal policy has to compensate for this lack of interest rate adjustment, so it becomes contractionary. The solution for monetary leadership looks like (past interest rate becomes another predetermined variable):

$$\begin{aligned} i_t^L &= 0.71\varepsilon_t + 0.04\eta_t + 0.41\pi_{t-1} + 0.03x_{t-1} + 0.70i_{t-1} \\ g_t^L &= -5.28\varepsilon_t - 1.23\eta_t - 3.00\pi_{t-1} - 0.27x_{t-1} + 0.00i_{t-1} + 2.39i_t^L \end{aligned}$$

Apparently, this may not be enough to eliminate shocks efficiently. Under the fiscal leadership, it economises on the volatility of the fiscal instrument and delivers slightly higher volatility of inflation, but lower volatility of output. Fiscal policy is also unsuccessful in efficient manipulation of the monetary policy so it is contractionary as well:

$$\begin{aligned} g_t^F &= -2.28\varepsilon_t - 1.04\eta_t - 1.26\pi_{t-1} - 0.12x_{t-1} + 1.30i_{t-1}^F \\ i_t^F &= 0.88\varepsilon_t + 0.10\eta_t + 0.52\pi_{t-1} + 0.04x_{t-1} + 0.062i_{t-1}^F + 0.05g_t^F. \end{aligned}$$

Although there is higher volatility of output due do demand shocks, the relative weight of demand shocks in the welfare function is relatively small. Thus both leadership regimes lead to increase of volatility of inflation. Our numerical experiment shows that the fiscal leadership dominates, see also Figures 3 and 4

To summarise, under our calibration of the model, when monetary policy is constrained, then fiscal leadership delivers better results then monetary leadership and, additionally, the participation of fiscal policy in the stabilisation process under both leadership regimes improves welfare.

5.5 Robustness of Results

In the analysis above we assumed a particular form of the Phillips curve with substantial persistence, which we find realistic. Therefore, our conclusions are built on the analysis of transmission of supply shocks. How results of our analysis depend on the degree of inflation persistence? In Table 2, we evaluate the welfare loss as a function of ω, which is a proportion of backward-looking individuals in the economy. When $\omega \to 1$ the population becomes more and more backward-looking, $\chi \to 1$.

When the monetary authorities are slightly conservative, $\rho_c = 0.9$, the fiscal leadership is inferior to the monetary leadership everywhere except for very backward looking and the very forward-looking specifications. For the very backward-looking population, the level of inflation is very much determined by its past values, rather than by the impact of a policy. So there is less difference in inflation variability under the two different regimes and lower variability of the fiscal instrument ensures superiority of the fiscal leadership. Similarly, for the very forward-looking consumers, the shocks are quickly eliminated from the system so the implied volatility of output and all its components, including the fiscal instrument, is small. Here the fiscal leadership becomes superior due to lower variability of fiscal expenditures, see also Figure 5.

For the 'Stability and Growth Pact' scenario, the fiscal leadership is dominated by a monetary leadership for any degree of persistence.

The last case with constrained monetary authorities is different. For our preferred calibration with $\rho_i = 0.5$ we have that superiority of monetary leadership is changing several times with increase in persistence. For the very backward-looking population the difference in variability of inflation and output falls, so the role of variability of the fiscal instrument seem to determine overall superiority of the fiscal leadership, despite that monetary policy is constrained. For sufficiently forward-looking population, however, we have a mixture of effects and apriori it is not clear which dominates. Figure 6 plots the difference between cost components of fiscal leadership regime and monetary policy regime, a weight multiplied by the difference in variances. With diminishing χ, the difference between inflation, output costs all fall to zero. This should ensure the priority of the fiscal leadership as the one with lower instrument variability. However, for the very forward-looking consumers, the fiscal instrument is more volatile for the fiscal leadership, than for the monetary leadership, namely this creates inferiority of the fiscal leadership for the very small χ.

Table 1: Welfare Losses for different penalty

	Conservative CB			SGP		Financial Stability		
	M (W_t^c)			M (W_t)		M ($W_t + \rho_i(\Delta i_t)^2$)		
	F (W_t)		F ($\overline{G}$)	F ($W_t + \rho_d d_t^2$)		F (W_t)		F ($\overline{G}$)
	ML	FL	n/s	ML	FL	ML	FL	n/s
w	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0	1.4748	1.4748	1.5490	1.4748	1.4748	1.4748	1.4748	1.5490
1	1.4721	1.4726	1.5465	1.4762	1.4867	1.601	1.596	1.660
2	1.4701	1.4709	1.5446	1.4821	1.5014	1.652	1.647	1.719
3	1.4690	1.4698	1.5434	1.4905	1.5159	1.687	1.682	1.762
4	1.4691	1.4696	1.5430	1.4998	1.5292	1.714	1.709	1.798
5	1.4708	1.4703	1.5437	1.5094	1.5411	1.736	1.731	1.829
6	1.4744	1.4722	1.5456	1.5188	1.5517	1.754	1.750	1.856
7	1.4804	1.4756	1.5491	1.5279	1.5612	1.771	1.767	1.882
8	1.4897	1.4807	1.5543	1.5364	1.5695	1.786	1.782	1.905
9	1.5030	1.4880	1.5618	1.5444	1.5770	1.799	1.796	1.926

Notes: ML – Monetary leadership; FL – Fiscal leadership; n/s – non-strategic;
$\rho_i = w/10; \rho_d = w/100; \rho_c = 1 - w/2;$
$\omega = 0.5,\ \chi = 0.7.$

Table 2: Table Caption

	Benevolent		Conservative CB			SGP		Financial Stability		
	M(W_t)	M(W_t)	M (W_t^c)			M(W_t)		M($W_t + \rho_i(\Delta i_t)^2$)		
	F($\overline{G}$)	F(W_t)	F($\overline{G}$)	F(W_t)		F($W_t + \rho_d d_t^2$)		F(W_t)		F($\overline{G}$)
χ	n/s	ML&FL	n/s	ML	FL	ML	FL	ML	FL	n/s
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0.0	0.226*	0.207*	0.226*	0.207*	0.207*	0.211*	0.215*	0.220*	0.220*	0.230*
0.1	0.310*	0.280*	0.311*	0.281*	0.281*	0.287*	0.294*	0.304*	0.304*	0.316*
0.2	0.418*	0.373*	0.417*	0.372	0.372	0.383*	0.395*	0.417*	0.417*	0.432*
0.3	0.573	0.506	0.571	0.504	0.504	0.523	0.541	0.598*	0.597*	0.617*
0.4	0.745	0.657	0.740	0.654	0.655	0.681	0.707	0.820*	0.820*	0.853*
0.5	0.975	0.875	0.969	0.869	0.871	0.907	0.939	1.122*	1.123*	1.190*
0.6	1.227	1.132	1.221	1.126	1.127	1.168	1.203	1.412	1.410	1.507
0.7	1.549	1.475	1.545	1.470	1.471	1.509	1.541	1.736	1.731	1.829
0.8	2.020	1.976	2.017	1.973	1.974	2.002	2.027	2.186	2.180	2.257
0.9	3.086	3.067	3.084	3.067	3.066	3.079	3.095	3.207	3.202	3.254

Notes:
ML –Monetary leadership; FL – Fiscal leadership; n/s – non-strategic;
$\rho_d = 0.05;\ \rho_i = 0.5;\ \rho_c = 0.9;$
* – shows cases where the economy under control is indeterminate

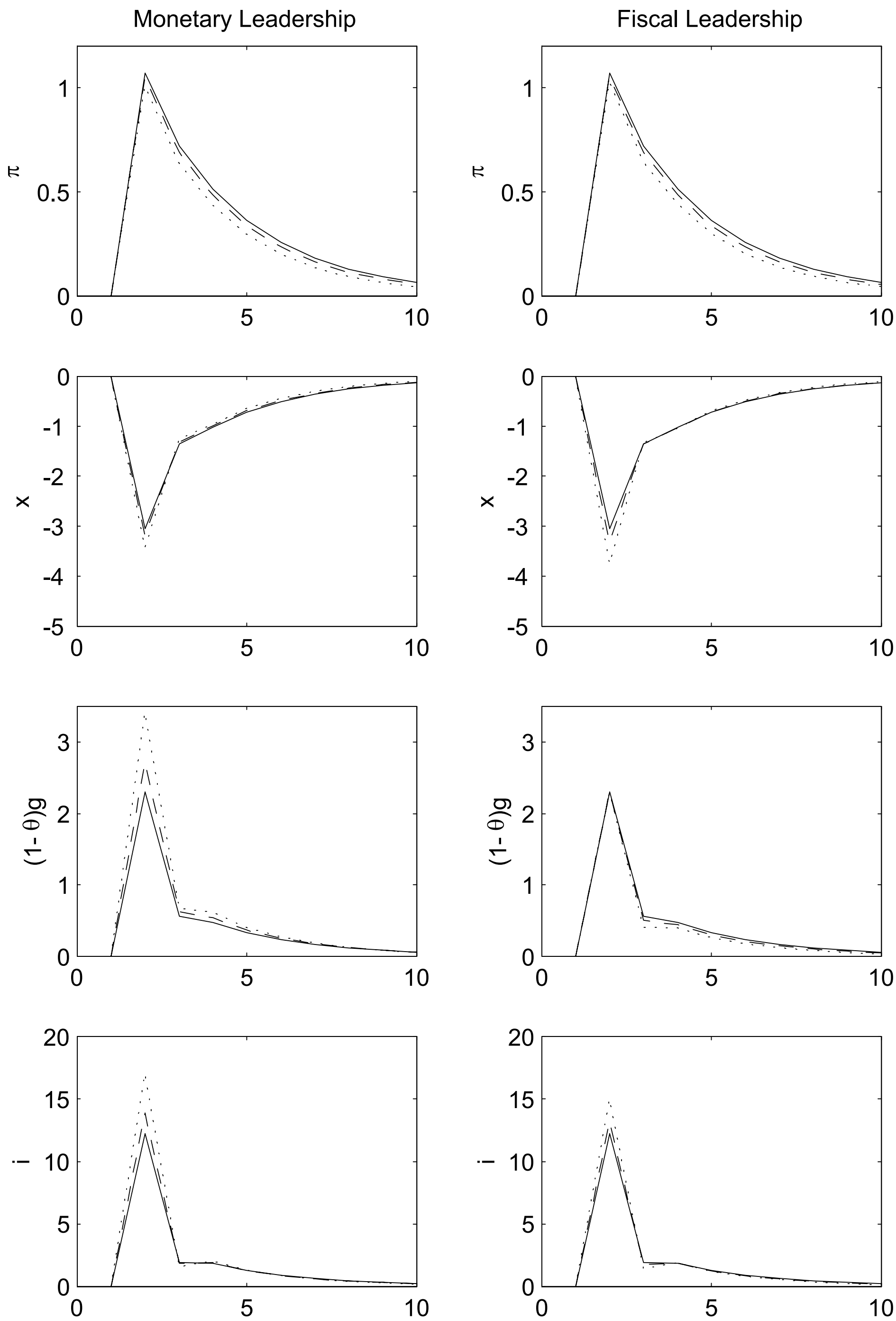

Figure 1: Supply shock for the 'conservative central bank' scenario. Solid line denotes responses under the benevolent authorities, the dashed line denotes responses under slightly conservative central bank, and the dotted line denotes responses under more conservative central bank.

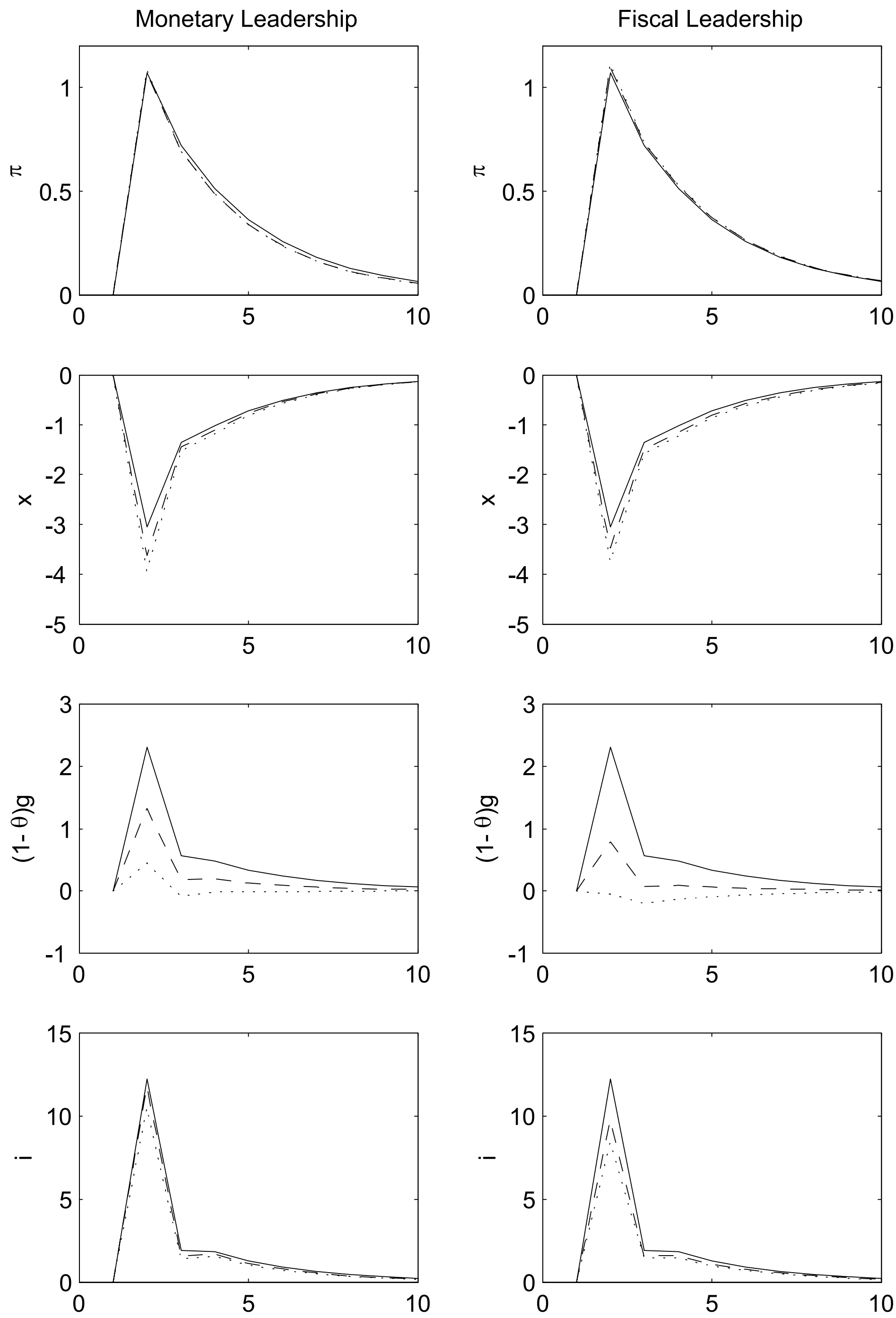

Figure 2: Supply shock for the ‘Stability and Growth Pact’ scenario. Solid line denotes responses under the benevolent authorities, the dashed line denotes responses under small penalty on fiscal deficit, and the dotted line denotes responses under bigger penalty on the fiscal deficit.

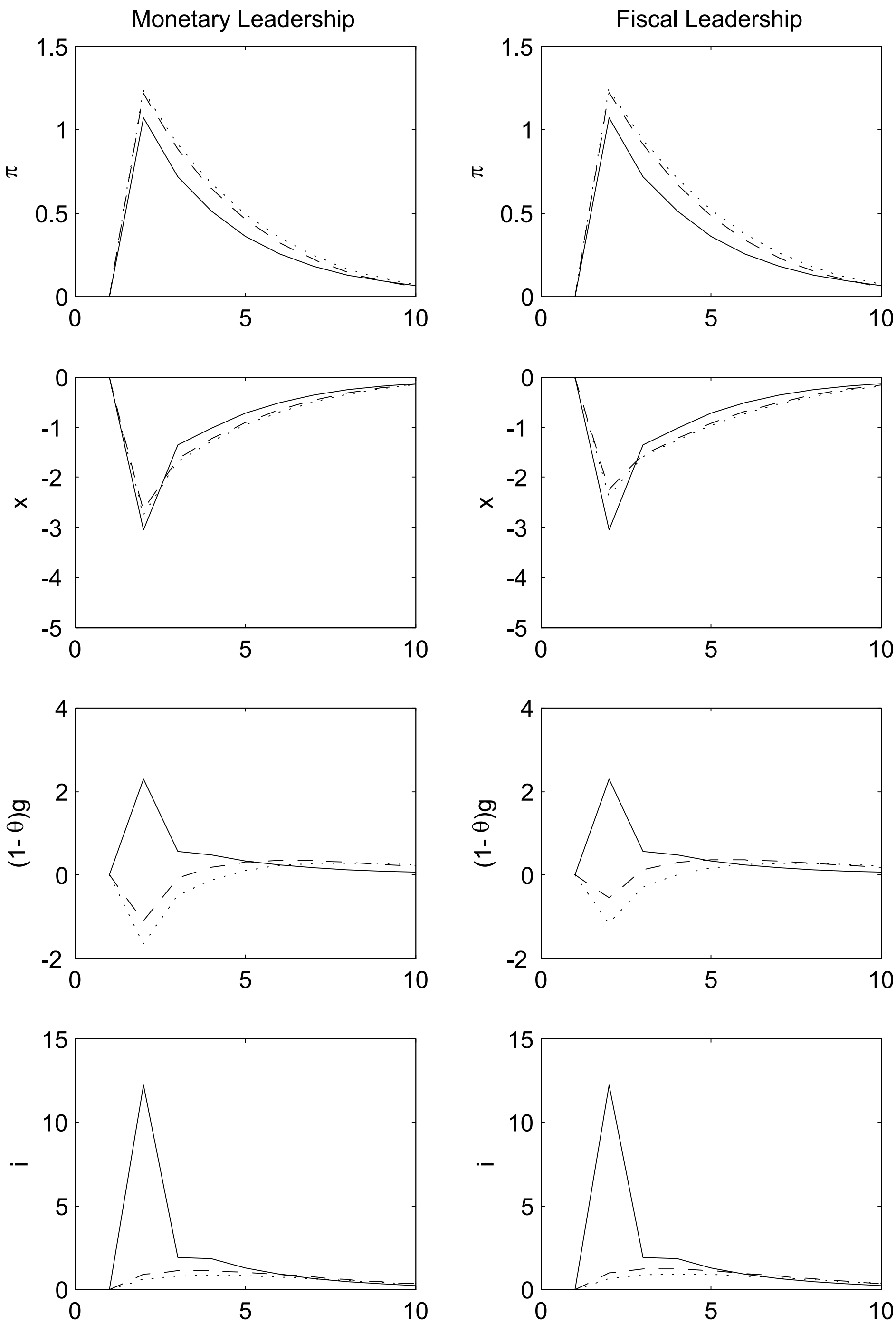

Figure 3: Supply shock for the 'Financial Stability' scenario. Solid line denotes responses under the benevolent authorities, the dashed line denotes responses under small penalty on change in interest rate, and the dotted line denotes responses under bigger penalty on change in interest rate.

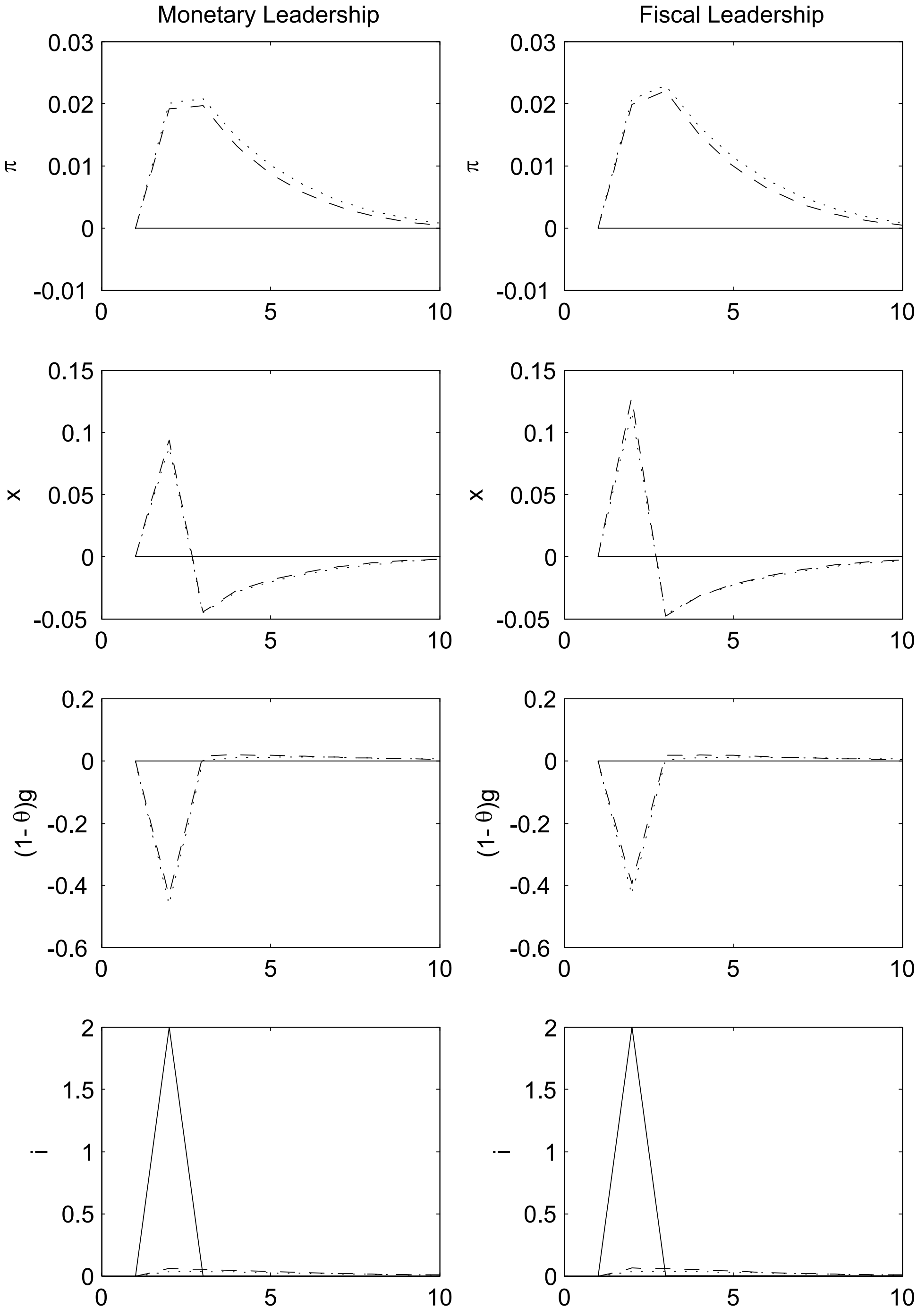

Figure 4: Demand shock for the 'Financial Stability' scenario. Solid line denotes responses under the benevolent authorities, the dashed line denotes responses under small penalty on change in interest rate, and the dotted line denotes responses under bigger penalty on change in interest rate.

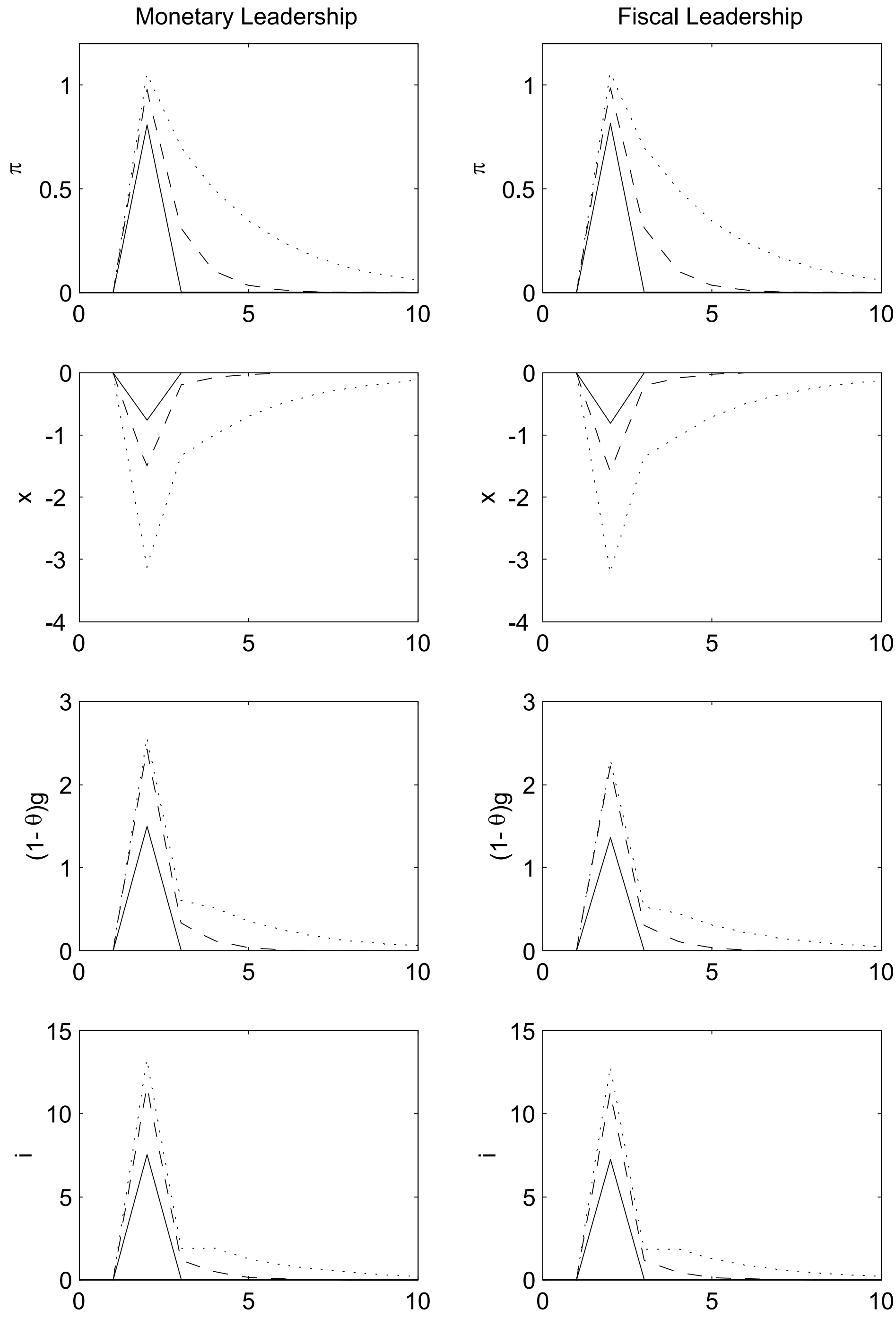

Figure 5: Supply shocks for 'conservative central bank' scenario. The solid line presents no persistence case, the dashed line presents increased persistence and the dotted line shows case with high persistence.

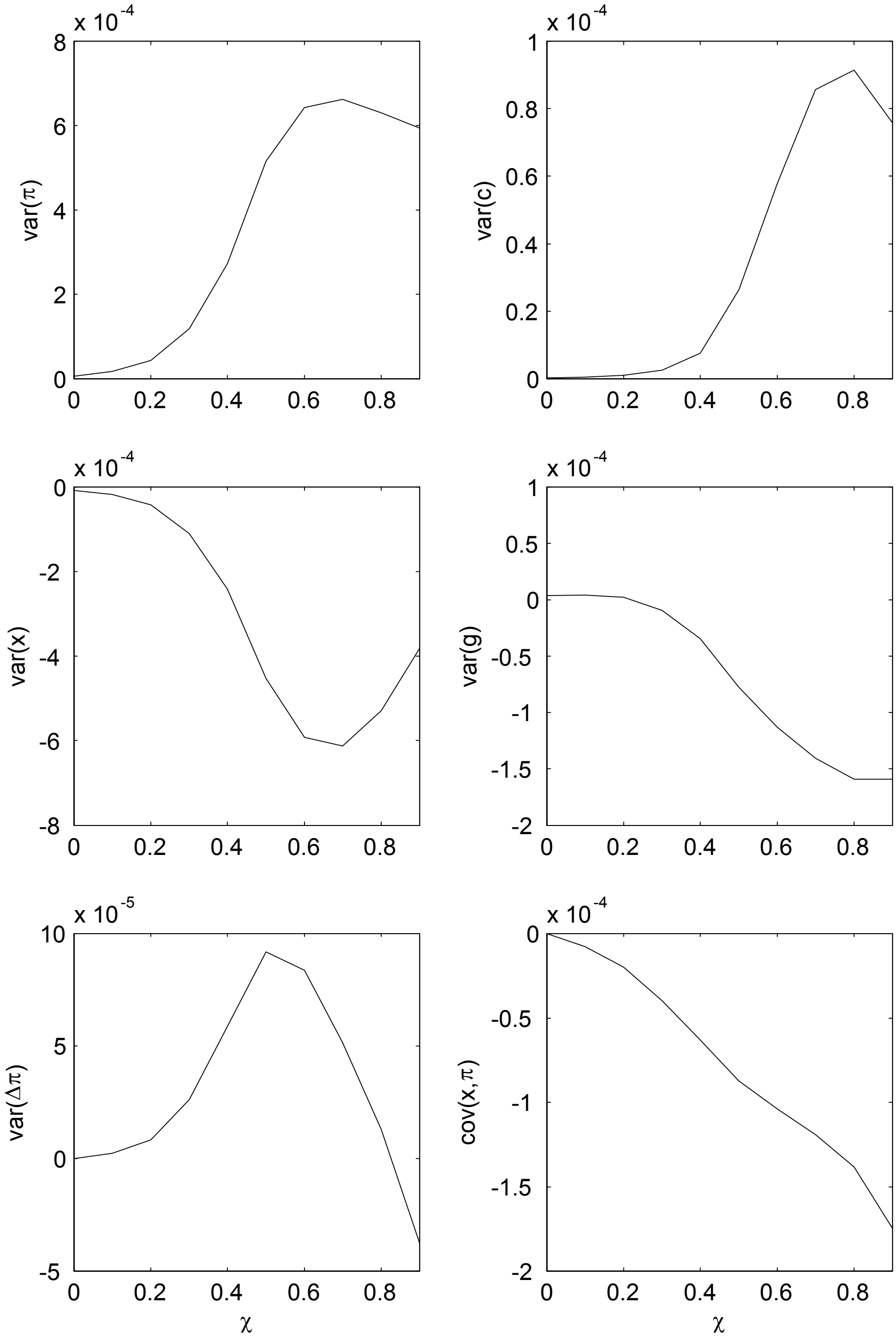

Figure 6: Difference in costs components between fiscal leadership regime and monetary leadership regume.

A Leadership equilibria under discretion

What role is there for leadership under discretion? This question was posed (and answered) by Cohen and Michel (1988). They made a distinction between the ability to lead within the periodicity of the model and over all time. In their model, a continuous time one, the periodicity of the model is infinitesimal. There is still a gain from leadership.[10] Acting as a Stackelberg leader for all time is akin to being able to adopt a time-inconsistent commitment strategy (see the discussion in Fudenberg and Tirole, 1991, p. 74–77). We eschew this, but indicate how a leadership role can still be modelled.[11] In this section we discuss the forms of model we are interested in and set up some useful relationships before discussing the policy equilibria. We pay careful attention to the key relationships that we use in deriving our leadership equilibria.

A.1 A class of models

We need to set up an analytical framework. We assume a nonsingular linear stochastic rational expectations model of the type described by Blanchard and Kahn (1980) augmented by a vector of control instruments. Specifically, the evolution of the economies is explained by the following system:

$$\begin{bmatrix} Y_{t+1} \\ X_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} U_t^L \\ U_t^F \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix} \tag{13}$$

where Y_t is an n_1-vector of predetermined variables with initial conditions Y_0 given, X_t is n_2-vector of non-predetermined (or jump) variables, U_t^F and U_t^L are two vectors of policy instruments of two policymakers, F and L, of size k_F and k_L respectively, and ε_{t+1} is vector of innovations to predetermined variables with covariance matrix Σ. Some of elements in Y_t could be exogenous variables, like the level of domestic debt. For notational convenience we define the n-vector $Z_t = (Y_t', X_t')'$ where $n = n_1 + n_2$ and a vector of control variables $U_t = (U_t^{L\prime}, U_t^{F\prime})'$.

Typically, this system represents the solved out optimisation problem for the ultimate follower in the policy game. This player also has 'instruments', represented by X_t. Rational agents when solving their optimisation problem treat the instruments of other players parametrically. Additionally, there is an equation explaining evolution of predetermined variable Y, so these two equations together describe the 'evolution of the economy' (13). In a canonical representation of such a system the first equation explains the evolution of Y and the second equation describes the reaction function X. In what follows, we will treat the second equation in this system as to an one

[10] Of course, in the original formulation of static policy games, this form of commitment is all (Barro and Gordon, 1983).

[11] For completeness we discuss the implications for solution in Section A.6.

explaining the behaviour of the *third* player. One can draw the analogy with the behaviour of the private sector in macroeconomic models, presented by the Euler consumption equation and the Phillips curve, see Section 2 below.

The two policymakers have the following loss functions:

$$J_t^F = \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}(G_s^{F\prime} Q^F G_s^F) = \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}(\widetilde{Z}_s^{\prime} \mathcal{K}^F \widetilde{Z}_s) \tag{14}$$

$$= \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}(Z_s^{\prime} \mathcal{Q}^F Z_s + 2Z_s^{\prime}\mathcal{P}^F U_s + U_s^{\prime}\mathcal{R}^F U_s)$$

$$J_t^L = \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}(G_s^{L\prime} Q^L G_s^L) = \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}(\widetilde{Z}_s^{\prime} \mathcal{K}^L \widetilde{Z}_s) \tag{15}$$

$$= \frac{1}{2}\mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}(Z_s^{\prime} \mathcal{Q}^L Z_s + 2Z_s^{\prime}\mathcal{P}^L U_s + U_s^{\prime}\mathcal{R}^L U_s)$$

The vectors G_s^F and G_s^F are the goal variables of policymakers F and L correspondingly, $G_s^j = C\widetilde{Z}_s^{\prime}$ so $\mathcal{K}^j = C^{j\prime} Q^j C^j$, $j = \{L, F\}$, where:

$$\mathcal{K}^j = \begin{bmatrix} \mathcal{Q}^j & \mathcal{P}^j \\ \mathcal{P}^{j\prime} & \mathcal{R}^j \end{bmatrix}, \qquad \widetilde{Z}_s^{\prime} = (Y_t^{\prime}, X_t^{\prime}, U_t^{L\prime}, U_t^{F\prime})^{\prime} \tag{16}$$

and the matrices $\mathcal{K}^F$ and $\mathcal{K}^L$ are symmetric (without loss of generality) and contain weights on each goal. The loss function of each player can include instrument costs of both players, but no assumptions of invertibility of $\mathcal{R}^j$ are made.

A.2 Useful Relationships

In a linear-quadratic setup the optimal solution of a time-consistent feedback policy is necessarily a *linear* rule. Therefore, when imposing this functional form we do not narrow the class of possible solutions. We now derive several useful relationships between parameters of our model.

In a leadership equilibrium, the follower treats the leader's policy instrument parametrically. We denote:

$$L = -\frac{\partial U_t^F}{\partial U_t^L}$$

Therefore, the follower's reaction function will necessarily be a rule of the following form:

$$U_t^F = -F^F Y_t - L U_t^L \tag{17}$$

and the leader's reaction will be:

$$U_t^L = -F^L Y_t. \tag{18}$$

The evolution of the system under control can be written as:

$$\begin{bmatrix} Y_{t+1} \\ X_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} - B_{12}(F^F - LF^L) - B_{11}F^L & A_{12} \\ A_{21} - B_{22}(F^F - LF^L) - B_{21}F^L & A_{22} \end{bmatrix} \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}. \tag{19}$$

Therefore, using either Blanchard and Kahn (1980) formula or a generalised Schur decomposition (see, e.g., Söderlind, 1999) it is easy to find the current value of jump variables (or the reaction of the third player):

$$X_t = -NY_t. \tag{20}$$

We can bring this representation into an equivalent form in terms of predetermined variables and controls (as did Oudiz and Sachs, 1985).

This implies that the following relationships always hold:

$$\begin{aligned} X_{t+1} = -NY_{t+1} &= -N(A_{11}Y_t + A_{12}X_t + B_1U_t^F + B_{11}U_t^L) \\ &= A_{21}Y_t + A_{22}X_t + B_2U_t^F + B_{21}U_t^L \end{aligned} \tag{21}$$

from where we can obtain:

$$\begin{aligned} X_t &= -(A_{22} + NA_{12})^{-1}[(A_{21} + NA_{11})Y_t \\ &\quad + (B_{22} + NB_1)U_t^F + (B_{21} + NB_{11})U_t^L] \\ &= -JY_t - K^F U_t^F - K^L U_t^L. \end{aligned} \tag{22}$$

In the last formula:

$$J = (A_{22} + NA_{12})^{-1}(A_{21} + NA_{11}) \tag{23}$$

$$K^F = (A_{22} + NA_{12})^{-1}(B_{22} + NB_{12}) \tag{24}$$

$$K^L = (A_{22} + NA_{12})^{-1}(B_{21} + NB_{11}) \tag{25}$$

therefore the matrix N can also be determined from:

$$N = J - K^F(F^F - LF^L) - K^L F^L. \tag{26}$$

This is an important implication of the system under control. N obtained from (20) should coincide with that from (26).

So far we have discussed properties of policy equilibria without discussing the optimisation problems explicitly. We note that the treatment of rational agents is that they have essentially solved an optimisation problem already treating the behaviour of all other decision makers as parametric. However we are able to describe their reactions to potential changes in policies by those agents using (22).

A.3 Method of Lagrange Multipliers

We begin by solving the relevant optimisation problems using the method of Lagrange multipliers.

A.3.1 Follower's optimisation problem

The follower is maximising its objective function with respect to U_t^F, taking X_t as given, but recognising dependence of X_t on U_t^F. The instrument of the leader, U_t^L is given too and treated parametrically. We define a constrained welfare loss function as:

$$w = \mathcal{E}_t \sum_{s=t}^{\infty} H_s^F$$

where:

$$\begin{aligned} H_s^F &= \frac{1}{2}\beta^{s-t}(Z_s'\mathcal{Q}^F Z_s + 2Z_s'\mathcal{P}^F U_s + U_s'\mathcal{R}^F U_s) \\ &+ \lambda_{s+1}'(A_{11}Y_s + A_{12}X_s + B_1 U_s - Y_{s+1}) \end{aligned}$$

with λ_{s+1} is a vector of (non-predetermined) Lagrange multipliers. The first-order conditions are written as:

$$\frac{\partial H_s^F}{\partial U_s^F} + \frac{\partial H_s}{\partial X_s}\frac{\partial X_s}{\partial U_s^F} = 0 \tag{27}$$

$$\frac{\partial H_s^F}{\partial Y_s} + \frac{\partial H_s^F}{\partial X_s}\frac{\partial X_s}{\partial Y_s} + \frac{\partial H_{s-1}^F}{\partial Y_s} = 0 \tag{28}$$

$$\frac{\partial H_s}{\partial \lambda_{s+1}} = 0 \tag{29}$$

where:

$$\begin{aligned} \frac{\partial H_s^F}{\partial U_s^F} &= B_{12}'\lambda_{s+1} + \beta^{s-t}(\mathcal{P}_{12}^{F\prime}Y_s + \mathcal{P}_{22}^{F\prime}X_s + \mathcal{R}_{21}^F U_s^L + \mathcal{R}_{22}^F U_s^F), \\ \frac{\partial H_s^F}{\partial X_s} &= A_{12}'\lambda_{s+1} + \beta^{s-t}(\mathcal{Q}_{21}^F Y_s + \mathcal{Q}_{22}^F X_s + \mathcal{P}_{21}^F U_s^L + \mathcal{P}_{22}^F U_s^F) \\ \frac{\partial H_s^F}{\partial Y_s} &= A_{11}'\lambda_{s+1} + \beta^{s-t}(\mathcal{Q}_{11}^F Y_s + \mathcal{Q}_{12}^F X_s + \mathcal{P}_{11}^F U_s^L + \mathcal{P}_{12}^F U_s^F) \\ \frac{\partial H_{s-1}}{\partial Y_s} &= -\lambda_s \end{aligned}$$

and matrices $\mathcal{Q}$, $\mathcal{P}$ and $\mathcal{R}$ were partitioned conformably with $Z_t = (Y_t', X_t')'$ and U_s^F correspondingly. However, the non-predetermined variable X_t is chosen given the information about the behaviour of the policymakers (26) and the state of the economy:

$$\frac{\partial X_s}{\partial Y_s} = -J', \quad \frac{\partial X_s}{\partial U_s^F} = -K^{F\prime}, \quad \frac{\partial X_s}{\partial U_s^L} = -K^{L\prime}$$

This system must be solved for $U_t^F = -F^F Y_t - LU_t^L$ as function of state variables Y_t and U_t^L. Then it will be substituted back into the evolution of the system and the leader will chose its instrument optimally. We now demonstrate how it can be done.

We first rewrite the first order conditions (27)–(29) as:

$$\begin{aligned} 0 &= (\mathcal{P}_{12}^{F\prime} - K^{F\prime}\mathcal{Q}_{21}^F)Y_s + (\mathcal{P}_{22}^{F\prime} - K^{F\prime}\mathcal{Q}_{22}^F)X_s \qquad (30)\\ &\quad + (\mathcal{R}_{21}^F - K^{F\prime}\mathcal{P}_{21}^F)U_s^L + (\mathcal{R}_{22}^F - K^{F\prime}\mathcal{P}_{22}^F)U_s^F + \beta(B_{12}' - K^{F\prime}A_{12}')\mu_{s+1}\\ 0 &= (\mathcal{Q}_{11}^F - J'\mathcal{Q}_{21})Y_s + (\mathcal{Q}_{12}^F - J'\mathcal{Q}_{22}^F)X_s \qquad (31)\\ &\quad + (\mathcal{P}_{11}^F - J'\mathcal{P}_{21}^F)U_s^L + (\mathcal{P}_{12}^F - J'\mathcal{P}_{22}^F)U_s^F + \beta(A_{11}' - J'A_{12}')\mu_{s+1} - \mu_s\\ 0 &= A_{11}Y_s + A_{12}X_s + B_{11}U_s^L + B_{12}U_s^F - Y_{s+1} = 0 \qquad (32)\end{aligned}$$

here we used $\mu_s = \beta^{s-t}\lambda_s$. We now substitute $X_t = -JY_t - K^F U_t^F - K^L U_t^L$ into system (30)–(32). Then it will have three variables, predetermined Y_t and non-predetermined μ_t and U_t^F. In matrix form, the system can be written as:

$$\begin{bmatrix} I & 0 \\ 0 & \Phi_{22}^F \end{bmatrix} \begin{bmatrix} Y_{t+1} \\ \widetilde{U}_{t+1}^F \end{bmatrix} = \begin{bmatrix} \Psi_{11}^F & \Psi_{12}^F \\ \Psi_{21}^F & \Psi_{22}^F \end{bmatrix} \begin{bmatrix} Y_t \\ \widetilde{U}_t^F \end{bmatrix} + \begin{bmatrix} \Omega_1^F \\ \Omega_2^F \end{bmatrix} \begin{bmatrix} U_t^L \end{bmatrix} \qquad (33)$$

where $\widetilde{U}^F = (U^{F\prime}, \mu')'$. This system is similar to (13) but with singular matrix Φ^F, because Φ_{22}^F contains columns of zeros. Now we repeat the same procedure as we did in equations (13)–(22). We assume that $U_t^L = -F^L Y_t$, so we substitute it into equation (33) to obtain:

$$\begin{bmatrix} I & 0 \\ 0 & \Phi_{22}^F \end{bmatrix} \begin{bmatrix} Y_{t+1} \\ \widetilde{U}_{t+1}^F \end{bmatrix} = \begin{bmatrix} \Psi_{11}^F - \Omega_1^F F^L & \Psi_{12}^F \\ \Psi_{21}^F - \Omega_2^F F^L & \Psi_{22}^F \end{bmatrix} \begin{bmatrix} Y_t \\ \widetilde{U}_t^F \end{bmatrix} \qquad (34)$$

As before, this equation can now be solved using the generalised Schur decomposition, which leads to the following ralationships:

$$Y_{t+1} = MY_t \qquad (35)$$

$$\widetilde{U}_t = -SY_t \qquad (36)$$

from where it follows that:

$$\begin{aligned} \Phi_{22}^F \widetilde{U}_{t+1}^F &= -\Phi_{22}^F S^F Y_{t+1} = -\Phi_{22}^F S(\Psi_{11}^F Y_t + \Psi_{12}^F \widetilde{U}_t^F + \Omega_1^F U_t^L)\\ &= \Psi_{21}^F Y_t + \Psi_{22}^F \widetilde{U}_t^F + \Omega_2^F U_t^L \end{aligned}$$

Therefore, we can solve for $\widetilde{X}_t^F$ and obtain:

$$\widetilde{U}_t^F = -(\Phi_{22}^F S \Psi_{12}^F + \Psi_{22}^F)^{-1}((\Psi_{21}^F + \Phi_{22}^F S \Psi_{11}^F)Y_t + (\Omega_2^F + \Phi_{22}^F S \Omega_1^F)U_t^L) \qquad (37)$$

and if matrix $\Phi_{22}^F S \Psi_{12}^F + \Psi_{22}^F$ is invertible, we get a decomposition of the follower's reaction on the predetermined state and the leader's instrument:

$$\begin{bmatrix} U_t^F \\ \mu_t \end{bmatrix} = \begin{bmatrix} -F^F \\ -F^\mu \end{bmatrix} Y_t + \begin{bmatrix} -L \\ -L^\mu \end{bmatrix} U_t^L \qquad (38)$$

Therefore, from the final formula (38) we have $U_t^F = -F^F Y_t - LU_t^L$ and this reaction function should be substituted in the optimisation problem for the leader. Note that matrices K^F, K^L and J in system (30)–(32) are unknown. They must be found from the iterative procedure which we will discuss later in the text.

A.3.2 Leader's optimisation problem

Leader takes into account the follower's reaction function. Thus, we define constrained loss function as:

$$\begin{aligned} H_s^L = & \frac{1}{2}\beta^{s-t}(Z_s'\widetilde{\mathcal{Q}}^L Z_s + 2Z_s'\widetilde{\mathcal{P}}^L U_s^L + U_s^{L\prime}\widetilde{\mathcal{R}}^L U_s^L) \\ & + \kappa_{s+1}'((A_{11} - B_{12}F^F)Y_s + A_{12}X_s + (B_{11} - B_{12}L)U_s^L - Y_{s+1}) \end{aligned}$$

with κ_{s+1} is a vector of (non-predetermined) Lagrange multipliers. Since the feedback rule (17) is essentially static relationship, we substitute it into objective function (15) in order to obtain matrices with tilda . Namely, matrix (16) can be found as:

$$\widetilde{\mathcal{K}}^L = \mathcal{C}'\mathcal{K}^L\mathcal{C}$$

where matrix $\mathcal{C}$ translates the vector of all variables into the vector of variables relevant for the leader's problem:

$$\begin{bmatrix} Y_t \\ X_t \\ U_t^L \\ U_t^F \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ -F^F & 0 & -L \end{bmatrix} \begin{bmatrix} Y_t \\ X_t \\ U_t^L \end{bmatrix}$$

The feedback rule is also substituted into constraint.

Thus, our problem has collapsed to the standard problem of finding discretionary equilibrium, see e.g., Söderlind (1999).

The first-order conditions can be written as:

$$\frac{\partial H_s^L}{\partial U_s^L} + \frac{\partial H_s^L}{\partial X_s}\frac{\partial X_s}{\partial U_s^L} = 0 \tag{39}$$

$$\frac{\partial H_s^L}{\partial Y_s} + \frac{\partial H_s^L}{\partial X_s}\frac{\partial X_s}{\partial Y_s} + \frac{\partial H_{s-1}^L}{\partial Y_s} = 0 \tag{40}$$

$$\frac{\partial H_s^L}{\partial \kappa_{s+1}} = 0. \tag{41}$$

Here:

$$\frac{\partial H_s^L}{\partial U_t^L} = \beta^{s-t}(\widetilde{\mathcal{R}}^L U_s^L + \widetilde{\mathcal{P}}_1^{L\prime} Y_s + \widetilde{\mathcal{P}}_2^{L\prime} X_s) + (B_{11} - B_{12}L)'\kappa_{s+1}$$
$$\frac{\partial H_s^L}{\partial X_t} = A_{12}'\kappa_{s+1} + \beta^{s-t}(\widetilde{\mathcal{Q}}_{21}^L Y_s + \widetilde{\mathcal{Q}}_{22}^L X_s + \widetilde{\mathcal{P}}_2^L U_s^L)$$
$$\frac{\partial H_s^L}{\partial Y_s} = \beta^{s-t}(\widetilde{\mathcal{Q}}_{11}^L Y_s + \widetilde{\mathcal{Q}}_{12}^L X_s + \widetilde{\mathcal{P}}_1^L U_s^L) + (A_{11} - B_{12}F^F)'\kappa_{s+1}$$
$$\frac{\partial H_{s-1}^L}{\partial Y_s} = -\kappa_s,$$
$$\frac{\partial X_s}{\partial U_s^L} = -(K^L - K^F L)', \frac{\partial X_s}{\partial Y_s} = -(J - K^F F^F)'$$

We substitute these matrices into system (39)–(41) and obtain the following system:

$$0 = (\widetilde{\mathcal{R}}^L - (K^L - K^F L)'\widetilde{\mathcal{P}}_2^L)U_s^L + (\widetilde{\mathcal{P}}_1^{L\prime} - (K^L - K^F L)'\widetilde{\mathcal{Q}}_{21}^L)Y_s \quad (42)$$
$$+ (\widetilde{\mathcal{P}}_2^{L\prime} - (K^L - K^F L)'\widetilde{\mathcal{Q}}_{22}^L)X_s + \beta((B_{11} - B_{12}L)' - (K^L - K^F L)'A_{12}')\nu_{s+1}$$
$$0 = (\widetilde{\mathcal{Q}}_{11}^L - (J - K^F F^F)'\widetilde{\mathcal{Q}}_{21}^L)Y_s + (\widetilde{\mathcal{Q}}_{12}^L - (J - K^F F^F)'\widetilde{\mathcal{Q}}_{22}^L)X_s \quad (43)$$
$$+ (\widetilde{\mathcal{P}}_1^L - (J - K^F F^F)'\widetilde{\mathcal{P}}_2^L)U_s^L + \beta((A_{11} - B_{12}F^F)' - (J - K^F F^F)'A_{12}')\nu_{s+1} - \nu_s$$
$$0 = (A_{11} - B_{12}F^F)Y_s + A_{12}X_s + (B_{11} - B_{12}L)U_s^L - Y_{s+1} \quad (44)$$

where we used notation $\nu_s = \beta^{s-t}\kappa_s$. Additionally, we have the feedback rule:

$$X_s = -JY_s - K^F U_s^F - K^L U_s^L = -(J - K^F F^F)Y_s - (K^L - K^F L)U_s^L$$

that should be substituted into equations (42)–(44) which can be written in a matrix form as:

$$\begin{bmatrix} I & 0 \\ 0 & \Phi_{22}^L \end{bmatrix} \begin{bmatrix} Y_{s+1} \\ \widetilde{U}_{s+1}^L \end{bmatrix} = \begin{bmatrix} \Psi_{11}^L & \Psi_{12}^L \\ \Psi_{21}^L & \Psi_{22}^L \end{bmatrix} \begin{bmatrix} Y_s \\ \widetilde{U}_s^L \end{bmatrix} \quad (45)$$

where $\widetilde{U}^L = (U^{L\prime}, \nu')'$, and solved, using Schur decomposition, as:

$$Y_{s+1} = MY_s \quad (46)$$
$$\begin{bmatrix} U_s^L \\ \nu_s \end{bmatrix} = \begin{bmatrix} -F^L \\ -F^\nu \end{bmatrix} Y_s. \quad (47)$$

Equation (47) gives the optimal feedback rule $U_t^L = -F^L Y_t$.

A.3.3 Iterative Procedure

We start with initial approximation for policy rules, with $F_{(0)}^F$, $F_{(0)}^L$ and $L_{(0)}$ and solve the follower's problem, using formulae (36), (38) in turn. We will improve $F^F_{(1)}$ and $L_{(1)}$ but not $F^L_{(0)}$. We then update matrices in equations (23)-(25) and solve the leader's problem. This will give us new best reaction $F^L_{(1)}$. Then we again solve the problem for the follower to update $F^F_{(2)}$ and $L_{(2)}$ and so on.

A.4 Dynamic Programming Approach

We turn to an alternative characterisation using dynamic programming.

A.4.1 The Follower's Optimisation Problem

As discussed above, both policymakers implement policy using feedback rules. The leader feeds back on the predetermined state knowing the follower's reaction, $U^L_t = -F^L Y_t$. The follower observes the leader's action and treats it parametrically, $U^F_t = -F^F Y_t - LU^L_t = -(F^F - LF^L)Y_t$.

The cost-to-go from time t satisfies the following dynamic programming equation:

$$W_t = \frac{1}{2}(Z_t' \mathcal{Q}^F Z_t + 2Z_t' \mathcal{P}^F U_t + U_t^{F\prime} \mathcal{R}^F U_t) + \beta W_{t+1} \tag{48}$$

There is only one state variable, Y, so the welfare loss from time t should be given by:

$$W_t = \frac{1}{2}(Y_t' \mathcal{S}^t Y_t). \tag{49}$$

We can substitute this into (48) and using the fact that $X_t = -JY_t - K^F U^F_t - K^L U^L_t$ and $Y_{t+1} = (A_{11} - A_{12}J)Y_t + (B_1 - A_{12}K^F)U^F_t + (D_1 - A_{12}K^L)U^L_t$ we obtain:

$$\begin{aligned}
Y_t' \mathcal{S}^t Y_t = {} & Y_t'(Q^s + \beta(A_{11}' - J'A_{12}')\mathcal{S}^{t+1}(A_{11} - A_{12}J))Y_t \\
& + U_t^{F\prime}(U_F^{s\prime} + \beta(B_1' - K^{F\prime}A_{12}')\mathcal{S}^{t+1}(A_{11} - A_{12}J))Y_t \\
& + Y_t'(U_F^s + \beta(A_{11}' - J'A_{12}')\mathcal{S}^{t+1}(B_1 - A_{12}K^F))U_t^F \\
& + U_t^{L\prime}(U_L^{s\prime} + \beta(D_1' - K^{L\prime}A_{12}')\mathcal{S}^{t+1}(A_{11} - A_{12}J))Y_t \\
& + Y_t'(U_L^s + \beta(A_{11}' - J'A_{12}')\mathcal{S}^{t+1}(D_1 - A_{12}K^L))U_t^L \\
& + U_t^{F\prime}(\beta(B_1' - K^{F\prime}A_{12}')\mathcal{S}^{t+1}(B_1 - A_{12}K^F) + R^s)U_t^F \\
& + U_t^{F\prime}(\beta(B_1' - K^{F\prime}A_{12}')\mathcal{S}^{t+1}(D_1 - A_{12}K^L) + P^{s\prime})U_t^L \\
& + U_t^{L\prime}(\beta(D_1' - K^{L\prime}A_{12}')\mathcal{S}^{t+1}(B_1 - A_{12}K^F) + P^s)U_t^F \\
& + U_t^{L\prime}(\beta(D_1' - K^{L\prime}A_{12}')\mathcal{S}^{t+1}(D_1 - A_{12}K^L) + T^s)U_t^L
\end{aligned} \tag{50}$$

where:

$$
\begin{aligned}
Q^s &= \mathcal{Q}_{11}^F - \mathcal{Q}_{12}^F J - J'\mathcal{Q}_{21}^F + J'\mathcal{Q}_{22}^F J,\\
U_F^s &= J'\mathcal{Q}_{22}^F K^F - \mathcal{Q}_{12}^F K^F + \mathcal{P}_{12}^F - J'\mathcal{P}_{22}^F,\\
U_L^s &= J'\mathcal{Q}_{22}^F K^L - \mathcal{Q}_{12}^F K^L + \mathcal{P}_{11}^F - J'\mathcal{P}_{21}^F,\\
R^s &= K^{F\prime}\mathcal{Q}_{22}^F K^F + \mathcal{R}_{22}^F - K^{F\prime}\mathcal{P}_{22}^F - \mathcal{P}_{22}^{F\prime} K^F,\\
P^s &= K^{L\prime}\mathcal{Q}_{22}^F K^F - K^{L\prime}\mathcal{P}_{22}^F + \mathcal{R}_{12}^F - \mathcal{P}_{21}^{F\prime} K^F,\\
T^s &= K^{L\prime}\mathcal{Q}_{22}^F K^L - K^{L\prime}\mathcal{P}_{21}^F - \mathcal{P}_{21}^{F\prime} K^L + \mathcal{R}_{11}^F.
\end{aligned}
$$

Now, we can substitute the reaction rules (17) and (18) in (50) to obtain recursive equations for $\mathcal{S}^t$:

$$
\mathcal{S}^t = T_0 + \beta T'\mathcal{S}^{t+1} T \tag{51}
$$

where:

$$
\begin{aligned}
T_0 &= Q^s + F^{F\prime} R^s F^F + F^{L\prime} L' R^s L F^L + F^{L\prime} T_s F^L\\
&\quad - F^{F\prime} U_F^{s\prime} - U_F^s F^F - U_L^s F^L - F^{L\prime} U_L^{s\prime} + U_F^s L F^L + F^{L\prime} L' U_F^{s\prime}\\
&\quad - F^{L\prime} L' P_s' F^L - F^{L\prime} P_s L F^L - F^{F\prime} R^s L F^L - F^{L\prime} L' R^s F^F\\
&\quad + F^{F\prime} P_s' F^L + F^{L\prime} P_s F^F\\
T &= (A_{11} - A_{12}J) - (B_{12} - A_{12}K^F)F^F\\
&\quad + ((B_{12} - A_{12}K^F)L - (B_{11} - A_{12}K^L))F^L
\end{aligned}
$$

while the feedback rule can be determined from (50) by differentiating the loss function with respect to U_t^F:

$$
\begin{aligned}
U_t^F &= -(R^s + \beta(B_{12}' - K^{F\prime} A_{12}')\mathcal{S}^{t+1}(B_{12} - A_{12}K^F))^{-1}(U_F^{s\prime}\\
&\quad + \beta(B_{12}' - K^{F\prime} A_{12}')\mathcal{S}^{t+1}(A_{11} - A_{12}J))Y_t\\
&\quad - (R^s + \beta(B_{12}' - K^{F\prime} A_{12}')\mathcal{S}^{t+1}(B_{12} - A_{12}K^F))^{-1}(P^{s\prime}\\
&\quad + \beta(B_{12}' - K^{F\prime} A_{12}')\mathcal{S}^{t+1}(B_{11} - A_{12}K^L))U_t^L\\
&= -F^F Y_t - L U_t^L
\end{aligned}
$$

from where:

$$
\begin{aligned}
F^F &= (R^s + \beta(B_{12}' - K^{F\prime} A_{12}')\mathcal{S}^{t+1}(B_{12} - A_{12}K^F))^{-1}(U_F^{s\prime} \qquad (52)\\
&\quad + \beta(B_{12}' - K^{F\prime} A_{12}')\mathcal{S}^{t+1}(A_{11} - A_{12}J))\\
L &= (R^s + \beta(B_{12}' - K^{F\prime} A_{12}')\mathcal{S}^{t+1}(B_{12} - A_{12}K^F))^{-1}(P^{s\prime} \qquad (53)\\
&\quad + \beta(B_{12}' - K^{F\prime} A_{12}')\mathcal{S}^{t+1}(B_{11} - A_{12}K^L))
\end{aligned}
$$

These two formulae give an update on F^F and L.

These three formulae (51), (52) and (53) are an analogue of the Oudiz and Sachs (1985) recursive procedure which can be easily programmed.

A.4.2 The Leader's Optimisation Problem

This part of optimisation is the standard Oudiz and Sachs (1985) procedure for two players with the optimisation problem of where the system under control evolves:

$$\begin{bmatrix} Y_{t+1} \\ X_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} - B_{12}F^F & A_{12} \\ A_{21} - B_{22}F^F & A_{22} \end{bmatrix} \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + \begin{bmatrix} B_{11} - B_{12}L \\ B_{21} - B_{22}L \end{bmatrix} [U_t^L] + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix} \tag{54}$$

and the loss function is determined by:

$$\widetilde{\mathcal{K}}^L = \mathcal{C}'\mathcal{K}^L\mathcal{C}$$

where matrix $\mathcal{C}$ translates the vector of all variables into the vector of variables relevant for the leader's problem:

$$\begin{bmatrix} Y_t \\ X_t \\ U_t^L \\ U_t^F \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ -F^F & 0 & -L \end{bmatrix} \begin{bmatrix} Y_t \\ X_t \\ U_t^L \end{bmatrix}$$

and $\widetilde{\mathcal{K}}^L$ is partitioned conformally with $(Y_t', X_t', U_t^{L\prime})'$. This problem can be solved using the dynamic programming procedure explained in detail in Söderlind (1999).

A.4.3 Iterative Procedure

We start with an initial guess of $F_{(0)}^F$, $F_{(0)}^L$ and $L_{(0)}$ and solve the optimisation problem for the follower. We also need initial approximation for $N_{(0)}$ and $\mathcal{S}_{(0)}$. The iteration involves computing reaction function $N_{(1)}$ and the value function $\mathcal{S}_{(1)}$. This improves $F_{(1)}^F$ and $L_{(1)}$ but not, of course, $F_{(0)}^L$. We then compute new matrices using (54) and solve the problem for the leader that takes into account the reaction of the follower and the evolution of the economy. This will give us new best reaction $F_{(1)}^L$. Then we again solve the problem for the follower to update $F_{(2)}^F$ and $L_{(2)}$ and so on.

A.5 Lagrange Multipliers vs. Dynamic Programming

This iterative procedure which uses Lagrange multipliers involves finding N and F^F, F^L, L using (36), (38) and (47). Different from the dynamic programming algorithm, the value function $\mathcal{S}$ is not computed. As this algorithm involves eigenvalue decomposition such as Schur decomposition, it is important that all approximations of policy rules deliver the saddle-path stability of the system, otherwise there is a wrong number of stable and unstable roots and the algorithm will stop. Clearly, it is not always possible to ensure. However, in our experiments the Lagrange multipliers algorithm was converging faster than the dynamic programming algorithm in terms of computational time. In fact, in many cases, the dynamic programming algorithm also requires a reasonable starting values for N and $\mathcal{S}$. While $\mathcal{S}$ can be computed iteratively when solving Riccati equation, N still has to be obtained using matrix inversion as in Blanchard and Kahn (1980), using initial approximations to F^F, F^L and L. Therefore, the same problem of finding a good prior which ensures saddle-path stability also remains for the dynamic programming approach. There is an advantage of Lagrange multipliers approach in that it conveys the underlying information structure, and allows us to interpret discretionary the equilibrium as a feedback Stackelberg one, with a very clear treatment of the third player as the ultimate follower. For most complicated problems these two approaches should probably be used together, especially in order to check that both of them converge to the same equilibrium. We have seen that for some problems the dynamic programming algorithm was converging to the wrong point (not welfare maximising). The Lagrange multipliers approach was necessary to check the solution.

A.6 Discretion, Commitment and Leadership

Dixit and Lambertini (2003) discuss two leadership equilibria when one of the players can precommit and the other acts under discretion. They find that the solution is exactly the same as when both players act with discretion. However, they consider a static game. In the dynamic game which we consider, the equivalent equilibria cannot be calculated.

Indeed, suppose the leader can precommit for all periods. In our setup this mean that the optimal policy for the leader will be to feed back on the predetermined state Y_t and the state of *predetermined* Lagrange multipliers, say Λ_t. The follower observes the leader's decisions and should react with $U_t^F = -F^F Y_t - LU_t^L$. Then the evolution of the system under control should be written:

$$\begin{bmatrix} Y_{t+1} \\ \Lambda_{t+1} \\ X_{t+1} \end{bmatrix} = \begin{bmatrix} \tilde{A}_{11} & (B_{12}LF^\Lambda - B_{11}F^\Lambda) & A_{12} \\ S_1 & S_2 & 0 \\ \tilde{A}_{21} & (B_{22}LF^\Lambda - B_{21}F^\Lambda) & A_{22} \end{bmatrix} \begin{bmatrix} Y_t \\ \Lambda_t \\ X_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \\ 0 \end{bmatrix}$$

where $\tilde{A}_{11} = A_{11} - B_{12}(F^F - LF^M) - B_{11}F^M$ and $\tilde{A}_{21} = A_{21} - B_{22}(F^F - LF^M) - B_{21}F^M$ and the second line explains evolution of the predetermined Lagrange multipliers. Immediately from here it is clear that if one of the followers acts in a time-consistent way it should feed back on both sets of predetermined variables, $X_t = -N_1Y_t - N_2\Lambda_t$ or $U_t^F = -F^F Y_t - LU_t^L = -(F^F - LF^L)Y_t + LF^\Lambda\Lambda_t$. The welfare function (49) should depend on $(Y_t', \Lambda_t')'$. Thus, any followers necessarily also react to Λ_t as this reflects the leader's behaviour.

The importance of this is illustrated by what could sometimes described as the difference between a levels bias and so-called stabilisation bias. In the static game set up (for example the two 'player' Barro and Gordon (1983) one) the resulting equilibrium manifests a suboptimal level solution (in that case an inflationary bias) whereas in dynamic games there is a different source of suboptimality, such that disturbances are rejected less well than under the optimal plan, the stabilisation bias. But the optimal plan is also time inconsistent and so would usually be ruled out. However, it is completely different to the Dixit and Lambertini (2003) example. The ability to precommit in our example would generate a completely different optimisation problem, and the resulting equilibrium would not coincide with the 'all players act with discretion' case.

B Model Parameters

The Phillips curve can be derived as :

$$\pi_t = (1-\chi)\beta\pi_{t+1} + \chi\pi_{t-1} + \kappa_c c_t + \kappa_{x0} x_t + \kappa_{x1} x_{t-1} + \varepsilon_t \tag{55}$$

where

$$\chi = \frac{\omega(1+\gamma\beta)}{\gamma(1-\omega)+\omega(1+\gamma\beta)}, \quad \kappa_{x0} = \frac{(1-\omega)(1-\gamma\beta)(1-\gamma)}{(\gamma(1-\omega)+\omega(1+\gamma\beta))(\psi+\epsilon)},$$
$$\kappa_{x1} = \frac{\omega(1+\gamma\beta)(1-\gamma)}{\gamma(1-\omega)+\omega(1+\gamma\beta)}\delta, \quad \kappa_c = \frac{(1-\omega)(1-\gamma\beta)(1-\gamma)\psi}{(\gamma(1-\omega)+\omega(1+\gamma\beta))(\psi+\epsilon)\sigma}$$

where γ is probability that wage contract is not renewed, ω is proportion of rule-of thumb price-setters (Steinsson (2003)) and δ is coefficient on demand pressure in price-setting rule for the rule of thumb price-setters. Coefficients σ and ψ are parameters of utility terms $u(C_s, \xi_s)$ and $v(h_s(z), \xi_s)$ correspondingly (see Steinsson (2003), the notation is standard).

The one-period social loss function can be derived as:

$$W_t = \frac{\psi(1-\gamma\beta)(1-\gamma)}{\epsilon(\epsilon+\psi)\gamma}\left(\theta\frac{1}{\sigma}c_t^2 + (1-\theta)\frac{1}{\sigma}g_t^2 + \frac{1}{\psi}x_t^2\right) + \pi_t^2 \tag{56}$$
$$+ \frac{\omega}{(1-\omega)\gamma}(\Delta\pi_t)^2 + \frac{\omega(1-\gamma)^2\delta^2}{\gamma(1-\omega)}x_{t-1}^2 + 2\frac{(1-\gamma)\omega\delta}{(1-\omega)\gamma}x_{t-1}\Delta\pi_t$$

where $1-\theta$ is the size of the government sector in the economy, G/Y.

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