An Analysis of the Sectoral Indices of Tokyo Stock Exchange: A Multivariate GARCH Approach with Time-Varying Correlations

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ABSTRACT

Most of the stylized features of volatility dynamics of equity returns are drawn from the aggregate indices of international stock markets. The inference is often based on the class of univariate generalized autoregressive conditional heteroscedasticity (GARCH) models. Owing to computational complexities, only a few studies utilize the multivariate framework, which exploits the possible correlations of volatility across different markets. In this paper, we investigate the applicability of the well-established facts of volatility behaviour of aggregate indices to the sectoral indices. Two competing multivariate (tetravariate) GARCH-type models with time-varying correlations are used to analyze the sectors of the Japanese stock market. The proposed models can parsimoniously capture the stylized features of long-memory, asymmetric conditional volatility, and time-varying correlations associated with stock market returns. In contrast to what is widely documented in the literature, we find that asymmetric effects are not invariably present in the sectoral indices. In addition, the conditional correlations are frequently highly positive and significantly time-varying. We also detect strong evidence of volatility persistence and long memory, and the fractionally integrated models generally outperform those models without long-memory structures in the conditional variance. Our findings not only cast doubts on the "leverage effect" of equity returns, but also have bearing on the strategy of portfolio diversification among various sectors.

Keywords: Constant Correlations; Stock Market Volatility; Fractional Integration; Multivariate Asymmetric Long-Memory GARCH; Varying Correlations

JEL Classification: C12; G15

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1. Introduction

In the past two decades, much research interest has focused on modeling the temporal variation in the volatility of asset returns. Particularly instrumental in capturing the time-varying asset returns volatility is the generalized autoregressive conditional heteroscedasticity (GARCH) model proposed by Bollerslev (1986) and its extensions. Franses and van Dijk (2000) provide an in-depth review of this subject and demonstrate the importance of estimating conditional variance using GARCH-type models in the research of empirical finance. Indeed, based on the class of univariate GARCH-type models, several significant stylized facts pertaining to stock market volatility are well-established in the literature. First, Black (1976) notes the tendency for negative shocks to generate greater volatility in future periods compared with positive shocks of the same magnitude, a phenomenon that he refers to as the "leverage effect". Such asymmetric volatility shocks are mainly detected from the returns of the aggregate stock market indices. For instance, Engle and Ng (1993) employ various model specifications to test for volatility asymmetry in the TOPIX of the Tokyo Stock Exchange; Nelson (1991) applies the exponential GARCH (EGARCH) model to the value-weighted CRSP daily market returns; while Ding, Granger, and Engle (1993) focus on New York's S&P 500 Index to examine the presence of asymmetry. More recent articles on asymmetric conditional volatility of equity returns include Harvey and Shephard (1996), Loudon, Watt, and Yadav (2000), Giot and Laurent (2003), and Asai and McAleer (2003). Despite using different aggregate stock market indices, these studies uniformly conclude that asymmetric effects are detected in the conditional volatility of stock market returns.

Another empirical regularity is that stock market volatility displays very long temporal dependencies and strong persistence. For details, see Baillie (1996), Ding and

Granger (1996), Bollerslev and Mikkelsen (1996), Tse and Tsui (1997), Bollerslev and Jubinski (1999), Andersen, Bollerslev, and Cai (2000), and Beran and Ocker (2001). In particular, Andersen, Bollerslev, and Cai (2000) suggest that high-frequency returns reveal the existence of important long-memory interdaily volatility dependencies. Again, this empirical regularity is mainly established based on international aggregate stock market indices, such as New York's S&P 500 Index, Japan's Nikkei 225, Hong Kong's Hang Seng Index, Singapore's Straits Times Index, and Australia's All Ordinaries Index. The third stylized fact is the rejection of constant conditional correlations of asset returns. Many studies, such as Longin and Solnik (1995), Tsui and Yu (1999), Tse (2000), Bera and Kim (2002), and Engle (2002), and Ledoit, Santa-Clara, and Wolf (2003) use major international stock market indices and find evidence of time-varying correlations of returns.

These stylized facts, however, are based on the aggregate indices of the major international stock markets. Little work has been conducted on the sectoral/component indices of these stock markets. This over-emphasis on aggregate market indices is lop-sided, as the volatility dynamics of the sectoral indices may evolve differently from the aggregate indices. Hence, the stylized facts based on aggregate indices need not be invariably applicable to the individual sectors. In addition, most studies on the conditional volatility dynamics of asset returns either concentrate on the univariate GARCH-type models, which fail to capture correlations of asset returns, or simply assume, for the sake of tractability, that the conditional correlations are time-invariant. This could be partially due to the difficulties associated with the modeling and estimation of the conditional volatility of asset returns in a unified multivariate framework involving time-varying correlations, long-range dependence, and asymmetries. One major challenge is to ensure that the conditional variance-covariance matrix of the multivariate GARCH

(MGARCH) model is positive definite. Several researchers have proposed some multivariate models that require certain parameter restrictions so as to guarantee positive-definiteness of the variance-covariance matrix. For instance, Engle, Granger, and Kraft (1984) have presented the necessary conditions for the matrix of the bivariate ARCH model to be positive definite, but extending this model to higher dimensions is rather intractable. As an alternative, Bollerslev, Engle, and Wooldridge (1988) have proposed the vech-representation, which is the extension of the univariate GARCH representation to the vectorized conditional variance-covariance matrix. However, conditions that guarantee the positive-definiteness of the variance-covariance matrix are not easy to monitor and impose continuously during optimization.

Despite the computational complexities, the multivariate GARCH approach remains important for at least two reasons. First, as many assets are subject to similar information or events, it is expected that their volatilities may be correlated conditional on the given information set. Such conditional correlations can be utilized to design dynamic optimal portfolios comprising different assets. Second, there may be a gain of efficiency by jointly estimating the conditional volatilities of returns of several assets (see, for example, Bera and Higgins (1993)).

To circumvent the obstacles associated with multivariate GARCH models, Engle and Kroner (1995) introduce the Baba-Engle-Kraft-Kroner (BEKK) model, which automatically ensures the positive-definiteness of the variance-covariance matrix once parameter estimates are obtained. Another approach examines the conditional volatilities of different assets as a factor model; see Diebold and Nerlove (1989), Engel and Rodrigues (1989) and Engle, Ng, and Rothschild (1990) for details. However, the main drawback of the BEKK and factor models is that the parameters cannot be easily interpreted, and their net effects on the future variances and covariances are not readily observed. Moreover, since the estimation of the BEKK and factor-GARCH models involves a large number of parameters, especially when the number of assets increases, this lacks parsimony and exacerbates the difficulties of achieving convergence. For example, Bera, Garcia, and Roh (1997) report that the BEKK model does not perform well in the estimation of the optimal hedge ratios, and Lien, Tse, and Tsui (2002) report difficulties in obtaining meaningful estimates for the BEKK model during optimization.

A more manageable alternative is Bollerslev's (1990) constant (conditional) correlations-GARCH approach, which automatically guarantees the positive-definiteness of the variance-covariance matrix once the parameter estimates are obtained. Under the constant-correlation assumption, the maximum likelihood estimate of the correlation matrix is equal to the sample correlation matrix. As the sample correlation matrix is always positive definite, the optimization will not fail as long as the conditional variances are positive. In addition, the parameter estimates are relatively easy to interpret, as the univariate GARCH equations are still retained. Nonetheless, the highly restrictive assumption of constant correlations can adversely affect the reliability of statistical inference if it were violated. Indeed, many studies have highlighted the untenability of this assumption. For details, see Longin and Solnik (1995), Tsui and Yu (1999), Tse (2000), Bera and Kim (2002), and Ledoit, Santa-Clara, and Wolf (2003), respectively.

In this paper, we investigate the applicability of the well-documented facts on volatility behaviour of aggregate indices to the sectoral indices. To ensure consistency in comparison, our study is confined to the multivariate GARCH approach. Specifically, we propose two competing tetravariate GARCH-type models to analyze the volatility dynamics of the sectoral indices. They are the varying-correlations-fractionally integrated asymmetric power ARCH (VC-FIAPARCH) and the VC-FI asymmetric GARCH (VC-FIAGARCH) models. The main reason for considering these models is that they parsimoniously capture the stylized features of volatility asymmetry, long-range persistence in volatility, and time-varying correlations. In addition, these two competing models do not nest each other. Another advantage is that the parameters are relatively easy to interpret, as the univariate GARCH equation is retained for each asset return series. Moreover, once convergence is achieved, the conditional variance-covariance matrix automatically satisfies the positive-definite condition.

The proposed models are applied to four sectoral indices of the TOPIX (Tokyo Stock Price Index) of the Tokyo Stock Exchange (TSE). We detect significant evidence that the asymmetric conditional volatility is not uniformly present in all sectoral indices, even though Engle and Ng (1993) have previously observed the presence of the leverage effect in TOPIX. Apparently our findings cast doubts on the well-established fact that stock market returns exhibit the leverage effect, a phenomenon partially explained by the existence of operating leverage of firms (see Black (1976)). The absence of volatility asymmetry in some sectors may have important bearing on option pricing and on the construction of diversified domestic asset portfolios based on different sectors. In addition, we detect evidence of long-range persistence in volatility for all the sectors, regardless of which GARCH-type model is used. Some sectors apparently share similar degrees of fractional integration. In general, the fractionally integrated models outperform those models without long-memory structures in the conditional variance. Additionally, we also observe that conditional correlations are frequently highly positive and significantly time-varying. Our findings imply that the dynamic nature of sectoral correlations could be further exploited in constructing diversified portfolios over time.

The rest of the paper is organized as follows. Section 2 discusses the methodology adopted in this study. Section 3 describes the nature of the data sets and the estimation results. Section 4 then concludes by highlighting some implications of our findings.

2. Methodology

In this section, we first briefly describe the basic features of the multivariate GARCH(1,1) model with time-varying conditional correlations proposed by Tse and Tsui (2002). We then incorporate the features of asymmetric volatility and long memory into the conditional variance equations by synthesizing Tse and Tsui's (2002) methodology with other models. Two main classes of multivariate GARCH-type models are developed based on this synthesis.

Let $y_t = (y_{1t}, y_{2t}, y_{3t} \ldots y_{kt})'$ be the k-variate vector of variables with time-varying variance-covariance matrix H_t, and let $\mu_{it}(\xi_i)$ be the arbitrary conditional mean functions which depend on ξ_i, a column vector of parameters. A typical k-variate GARCH(1,1) model may be specified as follows:

$$y_{it} = \mu_{it}(\xi_i) + \varepsilon_{it}, \qquad i = 1,2,\ldots,k \tag{1}$$

where $(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, \ldots, \varepsilon_{kt})' \mid \Phi_{t-1} \sim (O, H_t)$ (2)

Note that Φ_t is the σ-algebra generated by all the available information up to time t. The random disturbance terms ε_{it} (which are obtained from equation (1)) and the conditional variance equations h_{iit} are modelled as follows:

$\varepsilon_{it} = \sqrt{h_{iit}}\, e_{it}$, where $e_{it} \sim N(0,1)$ (3)

$$h_{iit} = \eta_i + \alpha_i \varepsilon_{it-1}^2 + \beta_i h_{iit-1} \tag{4}$$

where (4) is the popular Bollerslev's (1986) GARCH(1,1) model.

Denoting the ij-th element (i, j = 1, 2,…,k) in H_t by h_{ijt}, the conditional correlation coefficients are given by $\rho_{ijt} = \dfrac{h_{ijt}}{\sqrt{h_{iit}h_{jjt}}}$. Tse and Tsui (2002) assume that the time-varying conditional correlation matrix $\Gamma_t = \{\rho_{ijt}\}$ is generated by the following recursion

$$\mathbf{\Gamma}_t = (1-\pi_1-\pi_2)\mathbf{\Gamma} + \pi_1\mathbf{\Gamma}_{t-1} + \pi_2\mathbf{\Psi}_{t-1} \tag{5}$$

where $\Gamma = \{\rho_{ij}\}$ is a time-invariant k x k positive-definite correlation matrix, π_1 and π_2 are assumed to be nonnegative and sum up to less than 1, and Ψ_t is a function of the standardised residuals e_{it}.

Denoting $\Psi_t = \{\psi_{ijt}\}$, the elements of Ψ_{t-1} are specified as

$$\psi_{ij,t-1} = \frac{\sum_{a=1}^{M} e_{i,t-a}e_{j,t-a}}{\sqrt{(\sum_{a=1}^{M} e_{i,t-a}^2)(\sum_{a=1}^{M} e_{j,t-a}^2)}}, \quad 1 \le i < j \le k \tag{6}$$

where M is set equal to k. For further details of the model, see Tse and Tsui (2002).

Assuming conditional normality, the log-likelihood function (ignoring the constant term) of the vector of parameters in equations (1), (4), and (5) $\theta = (\xi_1, \xi_2, ..., \xi_k, \eta_1, \eta_2, ..., \eta_k, \alpha_1, ..., \alpha_k, \beta_1, ..., \beta_k, \rho_{ij}, \pi_1, \pi_2)$ is specified as

$$l_t(\theta) = -\frac{1}{2}\log|H_t| - \frac{1}{2}(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, ..., \varepsilon_{kt})H_t^{-1}(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, ,..., \varepsilon_{kt})' \tag{7}$$

where ε_{it} are the random disturbance terms obtained from equation (1). The conditional variance-covariance matrix H_t can be further defined as

$H_t = \{h_{ijt}\} \equiv D_t \Gamma_t D_t$, $D_t = diag\{\sqrt{h_{iit}}\}$, and $\Gamma_t = \{\rho_{ijt}\}$

It can be easily shown that the log-likelihood function can be rewritten as

$$l_t(\theta) = -\frac{1}{2}\log|D_t \Gamma_t D_t| - \frac{1}{2}(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, ..., \varepsilon_{kt}) D_t^{-1} \Gamma_t^{-1} D_t^{-1} (\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, ..., \varepsilon_{kt})' \tag{8}$$

where Γ_t is defined by the recursion in (5). Note that by this formulation, $(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, ..., \varepsilon_{kt}) D_t^{-1}$ represents the standardized residuals $(e_{1t}, e_{2t} ... e_{kt})$.

Equations (1)-(8) summarize the gist of the varying-correlations GARCH (VC-GARCH) model of Tse and Tsui (2002). In particular, when k = 2, the bivariate VC-GARCH(1,1) model is obtained and equations (5)-(7) can be simplified as follows:

$$\rho_{12t} = (1 - \pi_1 - \pi_2)\rho_{12} + \pi_1 \rho_{12,t-1} + \pi_2 \psi_{12,t-1} \tag{5'}$$

$$\psi_{12,t-1} = \frac{\sum_{a=1}^{2} e_{1,t-a} e_{2,t-a}}{\sqrt{(\sum_{a=1}^{2} e_{1,t-a}^2)(\sum_{a=1}^{2} e_{2,t-a}^2)}} \tag{6'}$$

$$l_t(\theta) = -\frac{1}{2}\sum_{i=1}^{2}\log h_{iit} - \frac{1}{2}\log(1 - \rho_{12t}^2) - \frac{e_{1t}^2 + e_{2t}^2 - 2\rho_{12t} e_{1t} e_{2t}}{2(1 - \rho_{12t}^2)} \tag{7'}$$

Note that the VC-GARCH model nests Bollerslev's (1990) constant-correlations GARCH (CC-GARCH) model when $\pi_1 = \pi_2 = 0$. As such, the likelihood ratio test can be readily applied to compare the performance of both models.

In order to incorporate asymmetric volatility and long memory dynamics into the VC-GARCH model, we have to modify the symmetric conditional variance equation in (4). Among the GARCH-type models with asymmetric volatility, we choose two well-established structures: the asymmetric GARCH(1,1) (AGARCH(1,1)) model proposed by Engle (1990) and the asymmetric power ARCH(1,1) (APARCH (1,1)) model of Ding, Granger, and Engle (1993), respectively. Their main features are summarized below.

[a] Engle's (1990) asymmetric GARCH(1,1) (AGARCH(1,1)) model:

$$h_{iit} = \eta_i + \alpha_i(\varepsilon_{it-1} - \gamma_i)^2 + \beta_i h_{iit-1} \tag{9}$$

where γ_i is the asymmetric coefficient. When $\gamma_i = 0$, (9) becomes the GARCH(1,1) model and when $\beta_i = 0$, it becomes the prototype ARCH(1) model.

[b] Ding, Granger, and Engle's (1993) asymmetric power ARCH(1,1) (APARCH (1,1)) model.

$$h_{iit}^{\delta_i/2} = \eta_i + \alpha_i \, (|\,\varepsilon_{it-1}\,| - \gamma_i \varepsilon_{it-1})^{\delta_i} + \beta_i \, h_{iit-1}^{\delta_i/2} \tag{10}$$

where γ_i is the asymmetric coefficient. When $\delta_i = 2$, (10) becomes the leveraged GARCH (LGARCH(1,1)) model, which nests the GJR model of Glosten, Jaganathan and Runkle (1993). When $\delta_i = 1$, it becomes the threshold GARCH(1,1) (TGARCH(1,1)) model, which includes an asymmetric version of the Taylor/Schwert (1986/1989) model and Zakoian's (1994) threshold ARCH (TARCH) model. Ding, Granger, and Engle (1993) show that when δ_i approaches 0, the logarithmic GARCH(1,1) (LOGGARCH(1,1)) model is obtained, which incorporates an asymmetric version of the Geweke/Pantula (1986) model. Although the APARCH structure nests 7 models in total (see Ding, Granger, and Engle (1993) for details), it does not nest the AGARCH model.

As regards the structure of long-memory dynamics in volatility, we may generalise the conditional variance equations in (4), (9), and (10), such that they are fractionally integrated. We adopt the approach of Baillie, Bollerslev, and Mikkelsen (BBM) (1996), which is demonstrated below:

First, consider a GARCH(p,q) model, which is an extension of equation (4):

$$h_{iit} = \eta_i + \alpha_i(L)\varepsilon_{it}^2 + \beta_i(L)h_{iit} \tag{11}$$

where $\alpha_i(L)$ and $\beta_i(L)$ are lag polynomials of order q and p, respectively. Equation (11) may be rewritten in terms of an ARMA(m,p) process in ε_{it}^2:

$$[1-\beta_i(L)-\alpha_i(L)]\varepsilon_{it}^2=\eta_i\ +[1-\beta_i(L)]\upsilon_{it} \quad (12)$$

where m = max(q,p) and $\upsilon_{it}=\varepsilon_{it}^2-h_{iit}$ is the innovation to the variance process. The GARCH(p,q) model is covariance-stationary if all the roots of $1-\beta_i(L)-\alpha_i(L)$ lie outside the unit circle. If a unit root exists, (11) becomes the integrated GARCH (IGARCH) model with a polynomial $\phi_i(\mathrm{L})$ such that $1-\beta_i(L)-\alpha_i(L)=(1-L)\phi_i(L)$, where the characteristic equation $\phi_i(\mathrm{L})=0$ has all the roots outside the unit circle. This model represents an extreme case of persistence in the conditional variance. The BBM's approach replaces the first difference operator in the factorisation with a fractional difference operator to obtain the FIGARCH(p,d,q) model as below:

$$(1-L)^{d_i}\phi_i(L)\varepsilon_{it}^2=\eta_i+[1-\beta_i(L)]\upsilon_{it} \quad (13)$$

where $0\le d_i\le 1$, and $1-\beta_i(L)-\alpha_i(L)=(1-L)^{d_i}\phi_i(L)$

Conceivably, the FIGARCH(p,d,q) model has a more general structure which nests the usual GARCH and the IGARCH models. Alternatively, (12) may be expressed as the following infinite ARCH process:

$$h_{iit}=\frac{\eta_i}{1-\beta_i(1)}+[1-(1-\beta_i(L))^{-1}\phi_i(L)(1-L)^{d_i}]\varepsilon_{it}^2 \quad (14)$$

When both $1\text{-}\beta_i(L)$ and $\phi_i(L)$ are reduced to polynomials of degree 1, we obtain the FIGARCH(1,d,1) model:

$$h_{iit}=\frac{\eta_i}{1-\beta_i}+\lambda_i(L)\varepsilon_{it}^2 \quad (15)$$

where $\lambda_i(L)=\sum_{a=1}^{\infty}\lambda_{ia}L^a=1-(1-\beta_i L)^{-1}(1-\phi_i L)(1-L)^{d_i}$.

However, the FIGARCH(1,d,1) model does not include the feature of asymmetric volatility, whereby negative shocks have a different impact on future volatilities compared with positive shocks of the same magnitude. To remedy the shortcoming of

(15), we may apply the fractionally integrated process to the conditional variance equations specified in (9) and (10).

In what follows we derive the fractionally integrated asymmetric GARCH (FIAGARCH) model using the BBM's approach. Consider the AGARCH(p,q) model:

$$h_{iit} = \eta_i + \alpha_i(L)(\varepsilon_{it} - \gamma_i)^2 + \beta_i(L)h_{iit} \tag{16}$$

By redefining $g(\varepsilon_{it}) \equiv (\varepsilon_{it} - \gamma_i)^2$, and $\tau_{it} \equiv g(\varepsilon_{it}) - h_{iit}$, the fractionally integrated process can be straightforwardly applied to the AGARCH model by rewriting equation (16) as follows:

$$[1 - \beta_i(L) - \alpha_i(L)]g(\varepsilon_{it}) = \eta_i + (1 - \beta_i(L))\tau_{it} \tag{17}$$

After factorizing the lag polynomial $1 - \beta_i(L) - \alpha_i(L) = (1 - L)^{d_i}\phi_i(L)$, and rewriting (17) as an infinite ARCH operation applied to $g(\varepsilon_{it})$, we obtain

$$h_{iit} = \frac{\eta_i}{1 - \beta_i(1)} + [1 - (1 - \beta_i(L))^{-1}\phi_i(L)(1 - L)^{d_i}]g(\varepsilon_{it}) \tag{18}$$

For a particular case of FIAGARCH(1,d,1), we have

$$h_{iit} = \frac{\eta_i}{1 - \beta_i} + \lambda_i(L)(\varepsilon_{it} - \gamma_i)^2 \tag{19}$$

where $\lambda_i(L) = \sum_{a=1}^{\infty} \lambda_{ia} L^a = 1 - (1 - \beta_i L)^{-1}(1 - \phi_i L)(1 - L)^{d_i}$.

Note that (19) is similar to the FIGARCH(1,d,1) model in (15), except that it allows past return shocks to have asymmetric effects on the conditional volatility.

Similarly, we derive the FIAPARCH(p,d,q) model using the BBM's procedure based on an APARCH(p,q) model in (20). Specifically, we now define $g(\varepsilon_{it}) \equiv |\varepsilon_{it}| - \gamma_i \varepsilon_{it}$ and $\tau_{it} \equiv g(\varepsilon_{it})^{\delta_i} - h_{iit}^{\delta_i/2}$, and (20) can be rewritten as (21):

$$h_{iit}^{\delta_i/2} = \eta_i + \alpha_i(L)(|\varepsilon_{it}| - \gamma_i \varepsilon_{it})^{\delta_i} + \beta_i(L) h_{iit}^{\delta_i/2} \qquad (20)$$

$$[1 - \beta_i(L) - \alpha_i(L)] g(\varepsilon_{it})^{\delta_i} = \eta_i + (1 - \beta_i(L)) \tau_{it} \qquad (21)$$

By factorizing $1 - \beta_i(L) - \alpha_i(L)$, (21) can be further rewritten as an infinite ARCH operation applied to $g(\varepsilon_{it})$. Finally, the FIAPARCH(p,d,q) takes the following form:

$$h_{iit}^{\delta_i/2} = \frac{\eta_i}{1 - \beta_i(1)} + [1 - (1 - \beta_i(L))^{-1} \phi_i(L)(1 - L)^{d_i}] g(\varepsilon_{it})^{\delta_i} \qquad (22)$$

In particular, the FIAPARCH(1,d,1) model is specified as:

$$h_{iit}^{\delta_i/2} = \frac{\eta_i}{1 - \beta_i} + \lambda_i(L)(|\varepsilon_{it}| - \gamma_i \varepsilon_{it})^{\delta_i} \qquad (23)$$

where $\lambda_i(L)$ is defined as in (19). Similar to the FIAGARCH(1,d,1) model in (19), (23) allows past shocks to have asymmetric effects on the conditional volatility.

The parameters of the different multivariate fractionally integrated GARCH-type models can be estimated using Bollerslev and Wooldridge's (1992) quasi-maximum likelihood estimation (QMLE) approach. To facilitate convergence in the estimation, we have to make appropriate assumptions for the start-up conditions, including the computation of $\lambda_i(L)$, the number of lags, and the initial values. In particular, to compute the response coefficients, $\lambda_i(L) = \sum_{a=1}^{\infty} \lambda_{ia} L^a = 1 - (1 - \beta_i L)^{-1}(1 - \phi_i L)(1 - L)^{d_i}$, we adopt the following infinite recursions given in Bollerslev and Mikkelsen (1996):

$$\begin{aligned} &\lambda_{i1} = \phi_i - \beta_i + d_i, \\ &\lambda_{ib} = \beta_i \lambda_{ib-1} + [(b - 1 - d_i)/b - \phi_i]\zeta_{ib-1}, \qquad b = 2, \ldots, \infty \end{aligned} \qquad (24)$$

where $\zeta_{ib} = \zeta_{ib-1}(b - 1 - d_i)/b$, with $\zeta_{i1} = d_i$

(The derivation is given in Appendix I).

It can be observed from (24) that since *b* goes to infinity, an appropriate finite truncation is required during estimation. In our calibration, we have used 1000 and 2000 lags, respectively. We find that the parameter estimates obtained by truncating at 1000 lags are reasonably close to those based on 2000 lags. To save the computational time, we truncate $\lambda_i(L)$ after the first 1000 lags.

As regards the choice of initial values, we set the presample observations ε_{it}^2 to the unconditional sample variance for the FIGARCH(1,d,1) model. However, this assumption is inappropriate for the other models, as the infinite ARCH representation affects $g(\varepsilon_{it})$. For the multivariate FIAGARCH(1,d,1) model, we equate the presample observations of $g(\varepsilon_{it}) = (\varepsilon_{it} - \gamma_i)^2$ to the sample mean of $(\hat{\varepsilon}_{it} - \hat{\gamma}_i)^2$, where $\hat{\gamma}_i$ is the estimate of γ_i based on the univariate FIAGARCH(1,d,1) model. As for the multivariate FIAPARCH(1,d,1) model, the presample observations of $g(\varepsilon_{it})^{\delta_i} = (|\varepsilon_{it}| - \gamma_i \varepsilon_{it})^{\delta_i}$ are equated to the sample mean of $(|\hat{\varepsilon}_{it}| - \hat{\gamma}_i \hat{\varepsilon}_{it})^{\hat{\delta}_i}$, where $\hat{\gamma}_i$ and $\hat{\delta}_i$ are the estimates of γ_i and δ_i based on the univariate FIAPARCH(1,d,1) model.

3. Data and Estimation Results

The Tokyo Stock Exchange (TSE) was established in 15 May 1878, but its present form was founded in 1 April 1949. The TSE domestic stock market is divided into two sections - the First and Second Sections. In simple terms, the First Section is the market place for stocks of larger companies, and the Second Section is for those of smaller and newly listed companies. Relative to global stock exchanges, TSE has a market value of 232 trillion yen as of end March 2003, and an average daily trading value of 739 billion yen in the fiscal year 2002. This makes it one of the leading stock exchanges in the world in terms of both size and liquidity. Indeed, the TSE is a major international capital market with trading by non-Japanese investors accounting for nearly one-third of the value of its trading turnover during 2002.

On 1 July 1969, the TSE introduced TOPIX (Tokyo Stock Price Index), a composite index of all the common stocks listed on the First Section of TSE, to provide a comprehensive measure of the market trend for investors who are interested in general market price movements. This composite index is supplemented by subindices for each of the 33 industry groups, which are categorized according to the industrial sectors defined by the Securities Identification Code Conference. These 33 subindices can be classed based on the following broad groups: Fishery, Agriculture, and Forestry; Mining; Construction; Manufacturing; Electric Power and Gas; Transport and Communications; Commerce; Finance and Insurance; Real Estate; and, Services.

The sectors analyzed in this paper are tabulated as follows:

TOPIX Sectoral Index	Category
Air Transportation (ATRN)	Transport and Communication
Electric Power and Gas (EPOW)	Electric Power and Gas
Precision Instruments (PREI)	Manufacturing
Other Products (OPRD)	Manufacturing

As the manufacturing category occupies approximately half the number of sectors in the TOPIX, we select two sectors from this category: precision instruments (PREI) and other products (OPRD). The next largest category is transportation and communication, from which we pick one sector, air transportation (ATRN). The fourth sector is chosen from electric power and gas (EPOW). Our data sets cover the sample period from 4 January 1983 to 21 February 2003, thereby providing 5254 daily observations. These series are obtained from DataStream International.[1]

[Insert Figures 1-2 and Table 1 here]

Figure 1 presents the plots of the four sectoral series. The OPRD and the PREI series apparently move quite closely together, whereas the ATRN series exhibits a significant amount of fluctuation, with peaks occurring in the period from 1987-1990. In contrast, the EPOW series is relatively less volatile, and remains sluggish after 1990. Table 1 displays the descriptive statistics of the return series of all the sectors (calculated on a continuously compounding basis).[2] For a standard normal distribution, the skewness and kurtosis take the values of 0 and 3, respectively. As can be observed from Table 1, all the return series have kurtosis higher than 3. In addition, some of the data series exhibit significant serial correlations, as indicated by the Ljung-Box Q-statistics (Ljung and Box (1978)). Also, the BDS test statistics (Brock, Dechert, and Scheinkman (1996)), which are calculated based on the correlation integral, indicate that the series are not independently and identically distributed. Furthermore, the highly significant ARCH (Engle (1982)) and QARCH (Sentana (1995)) Lagrange Multiplier (LM) test statistics consistently suggest the presence of conditional heteroscedasticity; as such, GARCH-type modeling might be required.

[1] We have also used our models on the rest of the sectoral indices, but the main findings that we highlight in this paper remain largely unchanged. In particular, we have evidence that asymmetric conditional volatility is either weak or absent in sectors such as Land Transportation, Insurance, Mining, Pulp & Paper, Real Estate, and Wholesale. Time-varying (pair-wise) correlations are also detected, and several sectors apparently share a common degree of fractional integration in volatility. The complete results are obtainable from the authors upon request.

[2] All the return series are stationary as indicated by the augmented Dickey-Fuller and Phillips-Perron test statistics (which are not reported here due to space constraints).

To estimate the conditional mean, variance and correlation components of the proposed multivariate GARCH-type models simultaneously, we adopt Bollerslev and Wooldridge's (1992) quasi maximum-likelihood estimation (QMLE) procedure, with all the programmes coded using Gauss Version 5.0. The QMLE approach provides consistent estimators even when the disturbance term follows a thick-tailed distribution. For the mean equation, we find that the parsimonious AR(2) model is a reasonably adequate autoregressive filter, taking into account of the significance of individual parameters, the log-likelihood values and the residual diagnostics. To save space, we shall only report the estimates of the conditional variance and correlation equations from the following models: the VC-GARCH, the VC-AGARCH, the VC-APARCH, the VC-FIGARCH, the VC-FIAGARCH, and the VC-FIAPARCH. In addition, other than the correlation coefficients and the log-likelihood values, most of the parameter estimates from the constant-correlation models are omitted. The complete set of estimation results is available upon request.

[Insert Tables 2-7 here]

Tables 2 and 3 summarize the QMLE of the parameters of the tetravariate VC-GARCH, VC-AGARCH, VC-APARCH, VC-FIGARCH, VC-FIAGARCH and VC-FIAPARCH models for all the sectoral returns, respectively. Quite clearly, the estimated values of the coefficient of asymmetry (γ) vary considerably across the sectors, ranging from -0.0062 to 0.2924. In particular, for the ATRN and EPOW indices, we do not find evidence of asymmetric volatility, and this is robust across different specifications, such as the VC-AGARCH, VC-APARCH, VC-FIAGARCH, and VC-FIAPARCH models. For the OPRD index, there is some evidence of asymmetric volatility, especially based on the AGARCH specification. As for the PREI index, we find significant evidence of asymmetric effects across different models. More specifically, as summarized in the second main column of Table 4, for the VC-APARCH (VC-FIAPARCH) and the VC-AGARCH (VC-FIAGARCH) models, the estimated absolute values of the coefficient of asymmetry γ for PREI are 0.1666 (0.1554) and 0.2924 (0.2528) respectively, and they are significant at the 5% level. In contrast, those estimated values for ATRN (EPOW) are: -0.1871 (-0.0516), -0.0586 (0.0062), -0.2295 (-0.1106), and -0.0620 (-0.0111) for the VC-AGARCH, VC-APARCH, VC-FIAGARCH, and VC-FIAPARCH models, respectively, and they are insignificant even at the 10% level. Apparently the absence of leverage effects in some of the sectors indicates that the widely accepted leverage effects in the aggregate indices of the highly developed stock markets (such as TOPIX (Engle and Ng (1993)), S&P 500 (Ding, Granger, and Engle (1993)) and several other Asia-Pacific counterparts (see, for example, Tse and Tsui (1997)) are not invariably applicable to the sectors. We shall discuss in greater detail some implications of this finding in the conclusion.

The estimated values of the fractional differencing parameter d are reported in the first main column of Table 4. As can be observed, all the estimates are statistically different from zero and one at the 5% significance level, regardless of the sectors and the models. This implies that the impact of shocks on the conditional volatility of the sectoral returns consistently exhibits a hyperbolic rate of decay. In addition, most of the estimates of d are quite similar in magnitude across different models for the same sector; and the sectors ATRN, OPRD, and EPOW seem to share a common degree of fractional integration in the conditional volatility process. For example, the estimated values of d for sectors ATRN, OPRD, and EPOW are 0.3457 (0.3470), 0.3423 (0.3431), and 0.3224 (0.3021) in the FIGARCH (FIAGARCH) models, respectively. Moreover, the likelihood ratio test statistics reported in Table 7 are all significant at the 5% level, thereby indicating that the fractionally integrated models outperform those without the long memory structures.

[Insert Figures 3-4 here]

Figures 3-4 present the plots of the time path of conditional standard deviation for each sector based on the VC-FIAPARCH, VC-APARCH, VC-FIAGARCH, and the VC-AGARCH models, respectively. As can be observed from these plots, the non-FI models seem to under-estimate the magnitude of volatility, and this is more conspicuous during periods in which the conditional standard deviation is relatively high (such as in 1987). At the risk of oversimplification, the under-estimation of the risk premium of assets could be more acute in the non-FI models than the FI models.

We now discuss the conditional correlation dynamics of the four sectors. Under the null hypothesis that both π_1 and π_2 are zero, the likelihood ratio test statistic is asymptotically distributed as a chi-squared with 2 degrees of freedom. As can be gleaned from Tables 2, 3, and 6, all the test statistics indicate that the null hypothesis of no time-varying conditional correlations is rejected. In addition, all the estimates of π_1 and π_2 are individually significant at the 1% level, which further suggest that the conditional correlations are time-varying. Such findings are robust across different model specifications.

Table 5 displays estimates of the (pair-wise) time-invariant component of the conditional-correlation equation from different VC-models. Quite obviously, these pair-wise correlations are all positive and remarkably close, with those obtained from the VC-

FIAPARCH model being slightly higher. Additionally, the estimates of the time-invariant correlation between OPRD and PREI sectors are the highest (regardless of the VC models), probably because these sectors belong to the same industrial category and are therefore influenced by similar factors. In contrast, the pair-wise correlations of these two sectors with EPOW are relatively lower compared with other pair-wise correlations, but they are nonetheless positive. The positive pair-wise correlations we have obtained for all the return series may imply that limited benefits are possible from diversification among the sectors. However, effective diversification among different sectors may still be feasible by changing the optimal portfolio weights in tandem with changes in the correlations over time.

[Insert Figures 5-6 here]

The VC-models allow us to keep track of the evolution of the pair-wise conditional correlations over time. Figures 5-6 plot the time paths of the pair-wise correlations for two selected models: VC-FIAPARCH and VC-FIAGARCH. It can be seen that their patterns are largely similar. Specifically, during the period from 1989-1995, most of the conditional correlations experienced a gradual upward shift. After this, it is particularly evident that there is a drop in the level for the following pairs: ATRN-EPOW, EPOW-OPRD, and EPOW-PREI, respectively. For the other pairs like ATRN-OPRD, ATRN-PREI, and OPRD-PREI, the magnitude of their correlations rebounded after 1999. In addition, there are episodes in which the pair-wise correlations of EPOW-PREI, ATRN-OPRD, and ATRN-PREI are quite low (and occasionally negative); these might be exploited when designing the optimal weights of a diversified portfolio over time.

[Insert Table 8 here]

Finally, we perform some residual diagnostic tests to evaluate the adequacy of the proposed models. Due to space constraints, only the test statistics for the VC-APARCH, VC-FIAPARCH, VC-AGARCH, and VC-FIAGARCH models are reported in Table 8. As can be observed from the summary statistics in Panel A, the kurtosis coefficients of all standardized residuals across sectors and across models are lower than those reported in Table 1. In addition, the Q-statistics, as shown in Panel B of Table 8, indicate no strong evidence of serial correlation in the standardized residuals. Moreover, the McLeod-Li test statistics suggest that the fractionally integrated (FI) models are more adequate compared with the non-FI models. This could be because the FI models are more capable of capturing long-range temporal dependencies in volatility. The adequacy of the FI models is further corroborated by the BDS and the runs test statistics. In particular, most of the BDS test statistics tabulated in Panel E for the VC-FIAPARCH and VC-FIAGARCH models are insignificant at the 5% level. In contrast, the BDS tests for the VC-AGARCH and the VC-APARCH models are still significant at the 5% level.[3]

4. Conclusion

We have investigated the applicability of the stylized facts of volatility behaviour of aggregate indices to the sectoral indices. Two main classes of multivariate GARCH-type models with time-varying correlations are proposed to analyze four sectors of the Japanese stock market. These models can concurrently capture the stylized features of long-memory, asymmetric conditional volatility, and time-varying correlations commonly

[3] Strictly speaking, portmanteau test statistics, such as the Box-Pierce test, the Ljung-Box Q-statistics, and the McLeod-Li test statistics, are not asymptotically distributed as chi-squared variables under the null hypothesis of no misspecification (see Ling and Li (1997)). Nonetheless, it has been suggested that the chi-squared distribution may still be used as an approximation (see, for instance, Bollerslev (1990) and Tse and Tsui (1999)).

associated with equity returns. Besides the possible gains in efficiency through the joint estimation of parameters, such a multivariate approach also has the advantage of providing us with the time-history of the conditional correlations between any two sectoral return series.

In contrast to what is widely documented in the literature, we find strong evidence that asymmetric effects are not invariably present in the sectoral indices. Our result is robust across different models that incorporate asymmetric structures in the conditional volatility. This finding not only casts doubts on the well-established fact that equity returns exhibit the leverage effect, but also affects the strategies for option pricing and portfolio diversification. More specifically, options based on sectoral indices may be wrongly priced if asymmetric effects are falsely assumed for those sectors without such features. Furthermore, in order to make optimal hedging decisions, market practitioners probably have to take into account the existence (or absence thereof) of asymmetric effects in the conditional volatility of different sectors. In addition, although constructing theoretical explanations as to why volatility is not entirely asymmetric across different sectors of the same market is beyond the scope of this paper, this does present a challenging topic for future research.

We also find corroborating evidence that the conditional correlations between sectors are frequently highly positive and significantly time-varying. Highly positive correlations may imply limited advantages from domestic diversification among sectors; however, effective diversification exploiting the time-varying nature of conditional correlations may still be possible by altering the portfolio weights of different sectors over time.

Lastly, we also detect strong evidence of volatility persistence and long memory in all the sectoral indices for different models. Some sectors apparently have a common degree of fractional integration. We conjecture that this may provide some support of fractional co-integration in volatility, an issue which has not been widely studied in the literature to date (see Brunetti and Gilbert (2000) for an exception). This topic is left for future research.

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Appendix I

In this appendix we obtain the response coefficients of the FIGARCH(1,d,1) model. Consider the fractional differencing operator $(1-L)^d$, where L is the lag (backshift) operator, and $d \in [0,1]$ is the fractional differencing parameter. The Maclaurin series expansion is applied to the fractional differencing operator as follows:

$$\begin{aligned}&(1-L)^d \\ &= 1 - dL - \frac{d(1-d)}{2}L^2 - \frac{d(1-d)(2-d)}{6}L^3 - \ldots - \frac{d(1-d)(2-d)\ldots[(n-1)-d]}{n!}L^n - \ldots\end{aligned} \tag{A.1}$$

As noted in the text, the FIGARCH(1,d,1) model of Baillie, Bollerslev, and Mikkelsen (1996) can be expressed as an ARCH(∞) representation:

$$h_t = \frac{\eta}{1-\beta} + \lambda(L)\varepsilon_t^2 \tag{A.2}$$

$$\lambda(L) = \sum_{a=1}^{\infty} \lambda_a L^a = 1 - (1-\beta\, L)^{-1}(1-\phi\, L)(1-L)^d \tag{A.3}$$

Substitute (A.1) into (A.3):

$$\begin{aligned}&\lambda(L) \\ &= 1 - (1-\beta\, L)^{-1}(1-\phi L)[1 - dL - \frac{d(1-d)}{2}L^2 - \frac{d(1-d)(2-d)}{6}L^3 - \ldots] \\ &= -(1-\beta L)^{-1}[-(1-\beta L) + (1-\phi L) - dL + \phi d L^2 - \frac{d(1-d)}{2}L^2 \\ &+ \frac{d(1-d)\phi}{2}L^3 - \frac{d(1-d)(2-d)}{6}L^3 + \frac{d(1-d)(2-d)\phi}{6}L^4 \\ &- \frac{d(1-d)(2-d)(3-d)}{24}L^4 + \frac{d(1-d)(2-d)(3-d)\phi}{24}L^5 - \ldots] \\ &= -(1-\beta L)^{-1}\{-(\phi - \beta + d)L - (\frac{1-d}{2} - \phi)dL^2 \\ &- (\frac{2-d}{3} - \phi)\frac{d(1-d)}{2}L^3 - (\frac{3-d}{4} - \phi)\frac{d(1-d)(2-d)}{6}L^4 - \ldots \\ &- (\frac{b-1-d}{b} - \phi)\frac{d(1-d)(2-d)\ldots[(n-1)-d]}{n!}L^{n+1} - \ldots\}\end{aligned} \tag{A.4}$$

Redefine the fractional differencing operator in (A.1) as below:

$$
\begin{aligned}
&(1-L)^d \\
&=1-dL-\frac{d(1-d)}{2}L^2-...-\frac{d(1-d)(2-d)...[(n-1)-d]}{n!}L^n-... \qquad \text{(A.5)}\\
&\equiv 1-\sum_{b=1}^{\infty}\zeta_b L^b
\end{aligned}
$$

where $\zeta_b \equiv \zeta_{b-1}(b-1-d)b^{-1}, \quad b=2,3,...\infty$ and $\zeta_1 \equiv d$,

The response coefficients $\lambda(L)$ in (A.4) can be calculated by the following recursions:

$$
\begin{aligned}
&\lambda_1=\phi-\beta+d, \\
&\lambda_b=\beta\lambda_{b-1}+[(b-1-d)/b-\phi]\zeta_{b-1}, \qquad b=2,3,...,\infty
\end{aligned}
\qquad \text{(A.6)}
$$

The response coefficients of the FIAPARCH(1,d,1) and FIAGARCH(1,d,1) models can be obtained in a similar fashion.

Table 1 Summary Statistics of the Returns of the TOPIX Sectoral Indices (4 January 1983 – 21 February 2003)

Variable	ATRN	EPOW	OPRD	PREI
Panel A: Moments, Maximum, Minimum				
Mean	0.0004	0.0163	0.0105	0.0121
Median	0.0000	-0.0136	0.0000	0.0000
Maximum	12.3433	12.6845	8.4182	10.8793
Minimum	-15.5253	-15.8122	-13.3764	-16.9355
Std. Dev.	1.7932	1.4598	1.2694	1.5034
Skewness	0.1404	0.6701	-0.2453	-0.1016
Kurtosis	8.8508	14.0606	9.7354	8.8749
Observations	5254	5254	5254	5254
Panel B: Ljung-Box Q-statistic				
5 lags	7.1332	10.1723	28.3657	22.2856
10 lags	11.4545	24.8193	34.4645	31.4357
Panel C: McLeod-Li Test				
5 lags	454.9580	867.3978	458.8415	458.6054
10 lags	507.4569	1070.3095	577.5608	522.4805
Panel D: ARCH LM Test				
5 lags	342.2827	586.3227	300.9401	338.9370
10 lags	352.7244	634.3523	320.2767	352.1614
Panel E: QARCH LM Test				
1 lag	265.3754	518.8192	192.0298	300.8171
4 lags	410.0523	766.5183	429.8772	494.1323
Panel F: BDS Test				
e=1,l=3	18.0955	24.7634	15.7275	13.6280
e=1,l=4	20.3098	28.5376	18.5109	15.8524
e=1,l=5	22.6325	31.7168	21.0787	17.8060
e=1.5,l=3	17.5865	23.5251	17.2398	13.6560
e=1.5,l=4	18.7314	26.0731	19.5668	15.2663
e=1.5,l=5	20.0159	27.9693	21.2810	16.4700
Panel G: Runs Test				
R_1	1.7060	-2.4347	-4.3558	-5.4276
R_2	-7.9305	-12.4141	-6.9193	-5.7632
R_3	-11.1357	-14.7822	-7.8492	-6.8424

Notes:

1. ATRN = Air Transportation, EPOW = Electric Power and Gas, OPRD = Other Products, PREI = Precision Instruments
2. QARCH LM test statistic is due to Sentana (1995) and it is distributed as chi-squared with q(q+3)/2 degrees of freedom, where q is the number of lags.
3. For the BDS Test, e represents the embedding dimension whereas l represents the distance between pairs of consecutive observations, measured as a multiple of the standard deviation of the series. Under the null hypothesis of independence, the test statistic is asymptotically distributed as standard normal.
4. For the Runs Test, R_i for i = 1, 2, 3 denote the runs tests of the series R_t, $|R_t|$, and R_t^2 respectively. Under the null hypothesis that successive observations in the series are independent, the test statistic is asymptotically standard normal.

Table 2 Estimation Results of Tetravariate Varying Correlations (VC) Models

Variable	η	β	α	γ	δ	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
Panel A: VC-GARCH												
ATRN	0.0575	0.9146	0.0699	-	-	ρ_{AE} = 0.6478	0.9861	0.0117	-13897.0341	ρ_{AE} = 0.4042	-14346.6247	899.1812
	(0.0205)	(0.0169)	(0.0131)			(0.0694)	(0.0024)	(0.0019)		(0.0164)		
						ρ_{AO} = 0.7654				ρ_{AO} = 0.4274		
EPOW	0.0256	0.8952	0.0997	-	-	(0.0780)				(0.0170)		
	(0.0120)	(0.0256)	(0.0238)			ρ_{AP} = 0.5902				ρ_{AP} = 0.3703		
						(0.0698)				(0.0172)		
OPRD	0.0312	0.9101	0.0712	-	-	ρ_{EO} = 0.5851				ρ_{EO} = 0.3720		
	(0.0079)	(0.0138)	(0.0111)			(0.0814)				(0.0181)		
						ρ_{EP} = 0.4723				ρ_{EP} = 0.2986		
PREI	0.0426	0.9272	0.0529	-	-	(0.0795)				(0.0184)		
	(0.0162)	(0.0164)	(0.0107)			ρ_{OP} = 0.7958				ρ_{OP} = 0.5961		
						(0.0407)				(0.0152)		
Panel B: VC-AGARCH												
ATRN	0.0615	0.9092	0.0737	-0.1871	-	ρ_{AE} = 0.6584	0.9859	0.0119	-13876.6602	ρ_{AE} = 0.4063	-14327.3010	901.2816
	(0.0222)	(0.0187)	(0.0143)	(0.1499)		(0.0693)	(0.0024)	(0.0019)		(0.0163)		
						ρ_{AO} = 0.7727				ρ_{AO} = 0.4283		
EPOW	0.0261	0.8939	0.1012	-0.0516	-	(0.0795)				(0.0169)		
	(0.0125)	(0.0263)	(0.0245)	(0.0912)		ρ_{AP} = 0.5935				ρ_{AP} = 0.3704		
						(0.0708)				(0.0171)		
OPRD	0.0289	0.9076	0.0708	0.2563	-	ρ_{EO} = 0.5875				ρ_{EO} = 0.3715		
	(0.0081)	(0.0144)	(0.0114)	(0.1041)		(0.0827)				(0.0181)		
						ρ_{EP} = 0.4684				ρ_{EP} = 0.2967		
PREI	0.0393	0.9262	0.0527	0.2924	-	(0.0814)				(0.0182)		
	(0.0179)	(0.0181)	(0.0115)	(0.1215)		ρ_{OP} = 0.7887				ρ_{OP} = 0.5916		

Variable	η	β	α	γ	δ	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
						(0.0434)				(0.0150)		
Panel C: VC-APARCH												
ATRN	0.0542	0.9134	0.0774	-0.0586	1.7328	ρ_{AE} = 0.6571	0.9856	0.0122	-13859.4894	ρ_{AE} = 0.4063	-14316.6415	914.3041
	(0.0188)	(0.0165)	(0.0142)	(0.0544)	(0.1841)	(0.0684)	(0.0023)	(0.0019)		(0.0163)		
						ρ_{AO} = 0.7780				ρ_{AO} = 0.4296		
EPOW	0.0265	0.8958	0.1075	-0.0062	1.7480	(0.0764)				(0.0171)		
	(0.0113)	(0.0231)	(0.0233)	(0.0495)	(0.1771)	ρ_{AP} = 0.6003				ρ_{AP} = 0.3710		
						(0.0702)				(0.0172)		
OPRD	0.0285	0.9203	0.0729	0.1631	1.3736	ρ_{EO} = 0.5868				ρ_{EO} = 0.3719		
	(0.0073)	(0.0135)	(0.0116)	(0.0976)	(0.2245)	(0.0815)				(0.0186)		
						ρ_{EP} = 0.4722				ρ_{EP} = 0.2969		
PREI	0.0306	0.9380	0.0544	0.1666	1.3665	(0.0808)				(0.0185)		
	(0.0127)	(0.0154)	(0.0106)	(0.0716)	(0.2101)	ρ_{OP} = 0.8024				ρ_{OP} = 0.5939		
						(0.0419)				(0.0155)		

Notes:

1. All standard errors (in parenthesis) are the heteroskedastic-consistent standard errors computed based on the Quasi-Maximum Likelihood Estimation (QMLE) technique.
2. Log-likelihood value (VC) and Log-likelihood value (CC) refer to the likelihood values obtained from the VC and CC models respectively.
3. Corr (CC) refer to the conditional correlation coefficient obtained from the CC model.
4. LR is the likelihood ratio statistic for H_0: $\pi_1 = \pi_2 = 0$ in the VC model. It is distributed as chi-squared with 2 degrees of freedom under H_0.

Table 3 Estimation Results of Tetravariate Fractionally Integrated Varying Correlations (FI-VC) Models

Variable	η	ϕ	γ	δ	β	d	Γ		π_1	π_2	LL (VC)		Corr (CC)	LL (CC)	LR
Panel A: VC-FIGARCH															
ATRN	0.1092	0.5231	-	-	0.6998	0.3457	ρ_{AE} =	0.6730	0.9871	0.0110	-13767.3344	ρ_{AE} =	0.4008	-14244.8603	955.0519
	(0.0450)	(0.0932)			(0.0825)	(0.0564)		(0.0697)	(0.0018)	(0.0015)			(0.0158)		
							ρ_{AO} =	0.7982				ρ_{AO} =	0.4237		
EPOW	0.0698	0.2973	-	-	0.4730	0.3423		(0.0745)					(0.0163)		
	(0.0339)	(0.1508)			(0.1628)	(0.0395)	ρ_{AP} =	0.6145				ρ_{AP} =	0.3692		
								(0.0684)					(0.0164)		
OPRD	0.0932	0.2787	-	-	0.5003	0.3224	ρ_{EO} =	0.6153				ρ_{EO} =	0.3707		
	(0.0403)	(0.1136)			(0.1548)	(0.0689)		(0.0859)					(0.0184)		
							ρ_{EP} =	0.4830				ρ_{EP} =	0.2939		
PREI	0.1749	0.4043	-	-	0.5394	0.2428		(0.0811)					(0.0180)		
	(0.0886)	(0.1716)			(0.1785)	(0.0384)	ρ_{OP} =	0.8007				ρ_{OP} =	0.5974		
								(0.0402)					(0.0146)		
Panel B: VC-FIAGARCH															
ATRN	0.1023	0.5194	-0.2295	-	0.6926	0.3470	ρ_{AE} =	0.6857	0.9870	0.0112	-13747.3280	ρ_{AE} =	0.4041	-14225.3266	955.9971
	(0.0485)	(0.0935)	(0.1415)		(0.0854)	(0.0580)		(0.0682)	(0.0018)	(0.0015)			(0.0156)		
							ρ_{AO} =	0.8089				ρ_{AO} =	0.4260		
EPOW	0.0670	0.3111	-0.1106	-	0.4805	0.3431		(0.0738)					(0.0164)		
	(0.0390)	(0.1716)	(0.0940)		(0.1828)	(0.0406)	ρ_{AP} =	0.6234				ρ_{AP} =	0.3706		
								(0.0686)					(0.0164)		
OPRD	0.0778	0.2666	0.2264	-	0.4702	0.3021	ρ_{EO} =	0.6258				ρ_{EO} =	0.3724		
	(0.0340)	(0.1174)	(0.1195)		(0.1627)	(0.0729)		(0.0862)					(0.0184)		
							ρ_{EP} =	0.4890				ρ_{EP} =	0.2942		

Variable	η	ϕ	γ	δ	β	d	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
PREI	0.1627	0.3588	0.2528	-	0.4954	0.2371	(0.0827)				(0.0179)		
	(0.0770)	(0.1546)	(0.1169)		(0.1619)	(0.0362)	ρ_{OP} = 0.8013				ρ_{OP} = 0.5940		
							(0.0414)				(0.0146)		
Panel C: VC-FIAPARCH													
ATRN	0.1170	0.5134	-0.0620	1.9316	0.7036	0.3649	ρ_{AE} = 0.7126	0.9871	0.0111	-13734.3056	ρ_{AE} = 0.4057	-14212.3041	955.9971
	(0.0471)	(0.0882)	(0.0527)	(0.1611)	(0.0812)	(0.0747)	(0.0686)	(0.0018)	(0.0015)		(0.0158)		
							ρ_{AO} = 0.8236				ρ_{AO} = 0.4276		
EPOW	0.0504	0.2097	-0.0111	2.1796	0.3319	0.2933	(0.0725)				(0.0164)		
	(0.0424)	(0.3770)	(0.0428)	(0.1279)	(0.4119)	(0.0593)	ρ_{AP} = 0.6308				ρ_{AP} = 0.3711		
							(0.0686)				(0.0163)		
OPRD	0.1096	0.3151	0.1469	1.5165	0.6226	0.4193	ρ_{EO} = 0.6447				ρ_{EO} = 0.3735		
	(0.0371)	(0.0593)	(0.0905)	(0.1545)	(0.1128)	(0.0853)	(0.0862)				(0.0184)		
							ρ_{EP} = 0.4998				ρ_{EP} = 0.2936		
PREI	0.1953	0.3934	0.1554	1.5542	0.5774	0.2942	(0.0829)				(0.0176)		
	(0.0672)	(0.1112)	(0.0620)	(0.1858)	(0.1234)	(0.0442)	ρ_{OP} = 0.8039				ρ_{OP} = 0.5940		
							(0.0424)				(0.0140)		

Notes: See Notes to Table 2.

Table 4 Fractional Differencing Parameter d and Coefficient of Asymmetry γ

	Fractional Differencing Parameter d			Coefficient of Asymmetry γ			
	VC-FIGARCH	VC-FIAGARCH	VC-FIAPARCH	VC-FIAGARCH	VC-FIAPARCH	VC-AGARCH	VC-APARCH
ATRN	0.3457 (0.0564)	0.3470 (0.0580)	0.3649 (0.0747)	-0.2295 (0.1415)	-0.0620 (0.0527)	-0.1871 (0.1499)	-0.0586 (0.0544)
EPOW	0.3423 (0.0395)	0.3431 (0.0406)	0.2933 (0.0593)	-0.1106 (0.0940)	-0.0111 (0.0428)	-0.0516 (0.0912)	-0.0062 (0.0495)
OPRD	0.3224 (0.0689)	0.3021 (0.0729)	0.4193 (0.0853)	0.2264 (0.1195)	0.1469 (0.0905)	0.2563 (0.1041)	0.1631 (0.0976)
PREI	0.2428 (0.0384)	0.2371 (0.0362)	0.2942 (0.0442)	0.2528 (0.1169)	0.1554 (0.0620)	0.2924 (0.1215)	0.1666 (0.0716)

Notes: ATRN = Air Transportation; EPOW = Electric Power and Gas; OPRD = Other Products; PREI = Precision Instruments. The Bollerslev-Wooldridge (1992) robust standard errors based on the quasi-maximum likelihood estimation (QMLE) technique are reported in parentheses.

Table 5 Comparison of Correlation Coefficients Γ

Sectors	VC-GARCH	VC-AGARCH	VC-APARCH	VC-FIGARCH	VC-FIAGARCH	VC-FIAPARCH
ATRN-EPOW	0.6478 (0.0694)	0.6584 (0.0693)	0.6571 (0.0684)	0.6730 (0.0697)	0.6857 (0.0682)	0.7126 (0.0686)
ATRN-OPRD	0.7654 (0.0780)	0.7727 (0.0795)	0.7780 (0.0764)	0.7982 (0.0745)	0.8089 (0.0738)	0.8236 (0.0725)
ATRN-PREI	0.5902 (0.0698)	0.5935 (0.0708)	0.6003 (0.0702)	0.6145 (0.0684)	0.6234 (0.0686)	0.6308 (0.0686)
EPOW-OPRD	0.5851 (0.0814)	0.5875 (0.0827)	0.5868 (0.0815)	0.6153 (0.0859)	0.6258 (0.0862)	0.6447 (0.0862)
EPOW-PREI	0.4723 (0.0795)	0.4684 (0.0814)	0.4722 (0.0808)	0.4830 (0.0811)	0.4890 (0.0827)	0.4998 (0.0829)
OPRD-PREI	0.7958 (0.0407)	0.7887 (0.0434)	0.8024 (0.0419)	0.8007 (0.0402)	0.8013 (0.0414)	0.8039 (0.0424)

Notes: These figures are the time-invariant component of the conditional-correlation equation $\mathbf{\Gamma}_t = (1-\pi_1-\pi_2)\mathbf{\Gamma} + \pi_1\mathbf{\Gamma}_{t-1} + \pi_2\mathbf{\Psi}_{t-1}$. The Bollerslev-Wooldridge (1992) robust standard errors based on the quasi-maximum likelihood estimation (QMLE) technique are reported in parentheses.

Table 6 Likelihood Ratio Tests: Constant-Correlation (CC) versus Varying-Correlation (VC) Models

	Log-likelihood Value			Log likelihood Value			Log-likelihood Value		
Sectors	CC-FIGARCH	VC-FIGARCH	LR	CC-FIAPARCH	VC-FIAPARCH	LR	CC-FIAGARCH	VC-FIAGARCH	LR
ATRN EPOW OPRD PREI	-14244.8603	-13767.3344	955.0519**	-14212.3041	-13734.3056	955.9971**	-14225.3266	-13747.3280	955.9971**
Sectors	CC-GARCH	VC-GARCH	LR	CC-APARCH	VC-APARCH	LR	CC-AGARCH	VC-AGARCH	LR
ATRN EPOW OPRD PREI	-14346.6247	-13897.0341	899.1812**	-14316.6415	-13859.4894	914.3041**	-14327.3010	-13876.6602	901.2816**

Notes: LR: Likelihood Ratio Statistic. It is asymptotically distributed as chi-squared with 2 degrees of freedom under the null hypothesis that π_1 and π_2 are equal to 0 ($H_0 : \pi_1 = \pi_2 = 0$). ** indicates significance at the 1% level.

Table 7 Likelihood Ratio Tests: Fractionally Integrated (FI) versus Non-fractionally Integrated (Non-FI) Models

	Log-likelihood Value			Log likelihood Value			Log-likelihood Value		
Sectors	VC-GARCH	VC-FIGARCH	LR	VC-APARCH	VC-FIAPARCH	LR	VC-AGARCH	VC-FIAGARCH	LR
ATRN EPOW OPRD PREI	-13897.0341	-13767.3344	259.3994**	-13859.4894	-13734.3056	250.3677**	-13876.6602	-13747.3280	258.6643**
Sectors	CC-GARCH	CC-FIGARCH	LR	CC-APARCH	CC-FIAPARCH	LR	CC-AGARCH	CC-FIAGARCH	LR
ATRN EPOW OPRD PREI	-14346.6247	-14244.8603	203.5288**	-14316.6415	-14212.3041	208.6747**	-14327.3010	-14225.3266	203.9488**

Notes: LR: Likelihood Ratio Statistic. It is asymptotically distributed as chi-squared with 4 degrees of freedom under the null hypothesis that all the fractional differencing parameters are equal to 0 ($H_0 : d_i = 0, \forall i = 1,2,3,4$). ** indicates significance at the 1% level.

Table 8 Standardised Residuals from VC-APARCH, VC-FIAPARCH, VC-AGARCH, and VC-FIAGARCH models

Model	VC-APARCH				VC-FIAPARCH				VC-AGARCH				VC-FIAGARCH			
Variable	ATRN	EPOW	OPRD	PREI	ATRN	EPOW	OPRD	PREI	ATRN	EPOW	OPRD	PREI	ATRN	EPOW	OPRD	PREI
Panel A: Moments, Minimum, Maximum																
Mean	0.0017	0.0018	-0.0279	-0.0220	-0.0011	0.0001	-0.0278	-0.0221	0.0013	0.0012	-0.0283	-0.0219	-0.0012	0.0001	-0.0272	-0.0217
Median	-0.0061	-0.0309	-0.0441	-0.0472	-0.0062	-0.0315	-0.0443	-0.0477	-0.0062	-0.0309	-0.0440	-0.0463	-0.0063	-0.0319	-0.0434	-0.0478
Maximum	5.5904	8.2509	9.2856	5.5168	5.6300	8.8460	9.9386	5.0437	5.3579	8.2443	8.8697	5.4932	5.5366	8.7350	9.8240	5.0489
Minimum	-8.2383	-7.2512	-11.2826	-9.1543	-7.5726	-6.9958	-10.2788	-7.9807	-8.2508	-7.1599	-10.4983	-8.7062	-7.6067	-7.1069	-9.9074	-7.8867
Std. Dev.	0.9992	0.9972	0.9975	1.0041	0.9978	1.0026	1.0011	1.0070	0.9996	0.9973	0.9974	1.0046	0.9988	1.0126	0.9967	1.0047
Skewness	0.0878	0.5857	-0.0888	0.0131	0.0838	0.5206	0.0099	0.0442	0.0717	0.5641	-0.0682	0.0383	0.0702	0.4990	0.0239	0.0502
Kurtosis	7.3153	8.8096	9.0261	6.0904	7.0037	8.2187	8.5325	5.5445	7.2787	8.7898	8.0608	5.9079	6.9909	8.3299	8.2890	5.5480
Panel B: Ljung-Box Q-statistic																
5 lags	5.5329	5.5211	3.0011	3.7201	7.0010	4.9504	3.1603	4.9494	5.4517	5.8709	3.0378	4.1557	7.0465	4.9918	3.2176	5.1843
10 lags	9.6098	15.3000	4.4899	9.0385	11.7092	16.2530	5.1593	11.5990	9.6749	15.5819	4.6195	9.3862	11.8716	16.2282	5.6274	11.8335
Panel C: McLeod-Li Test																
5 lags	25.7561	12.2346	13.3740	37.1043	5.3816	2.4782	4.6076	10.3152	18.9497	8.9143	10.6377	19.6907	4.5597	2.6072	4.5702	6.0207
10 lags	31.6145	15.9472	14.6859	39.5319	9.5329	3.4185	6.0416	12.9071	24.5930	13.0933	12.7323	23.2654	8.3170	3.6938	6.0380	8.4977
Panel D: Runs Test																
R_1	-0.8883	2.5726	1.0964	1.7549	-1.1517	2.5388	1.0964	1.6033	-0.8947	2.5072	1.1430	1.7105	-1.1535	2.5388	1.0987	1.6587
R_2	-2.8884	-2.4353	-1.0785	-1.4865	-0.0029	-1.3239	-0.5493	0.4644	-3.1174	-2.9645	-1.5742	-1.5678	-0.3834	-1.3100	-1.3043	0.2000
R_3	-3.5741	-4.4313	-1.1914	-0.6245	-0.3060	-2.1324	0.4382	0.8346	-3.4581	-4.5043	-2.4068	-1.0039	0.0230	-1.9819	-0.3534	0.4670
Panel E: BDS Test																
e=3,l=1.5	4.7252	6.2059	2.2350	4.2533	0.8289	2.0236	0.5521	1.6580	4.6118	6.1695	1.9308	3.8792	0.7095	2.3129	0.3830	1.6609
e=4,l=1.5	4.2714	5.7840	2.4803	4.5716	1.0660	1.8174	1.0014	2.1826	4.2018	5.7820	2.1956	4.1638	0.9208	2.1844	0.6966	2.0945
e=5,l=1.5	4.0024	5.3064	2.5249	4.8061	1.3171	1.6999	1.1223	2.5266	3.9626	5.3194	2.2802	4.3805	1.1552	2.0490	0.8290	2.4019
e=3,l=1.0	5.4231	5.8944	2.0502	3.6360	1.7520	2.4367	0.3878	1.2373	5.5367	6.0208	2.0771	3.5104	1.6623	2.4661	0.4984	1.3456
e=4,l=1.0	5.2457	5.8125	2.2161	4.0836	2.2502	2.6990	0.7647	1.7894	5.4306	5.9898	2.2927	3.9261	2.1245	2.7527	0.7708	1.8232
e=5,l=1.0	5.0773	5.6216	2.2802	4.4715	2.5713	2.9298	0.9258	2.2639	5.3160	5.8664	2.3952	4.3159	2.4140	2.8982	0.9387	2.2722

Notes: See Notes to Table 1.

Figure 1

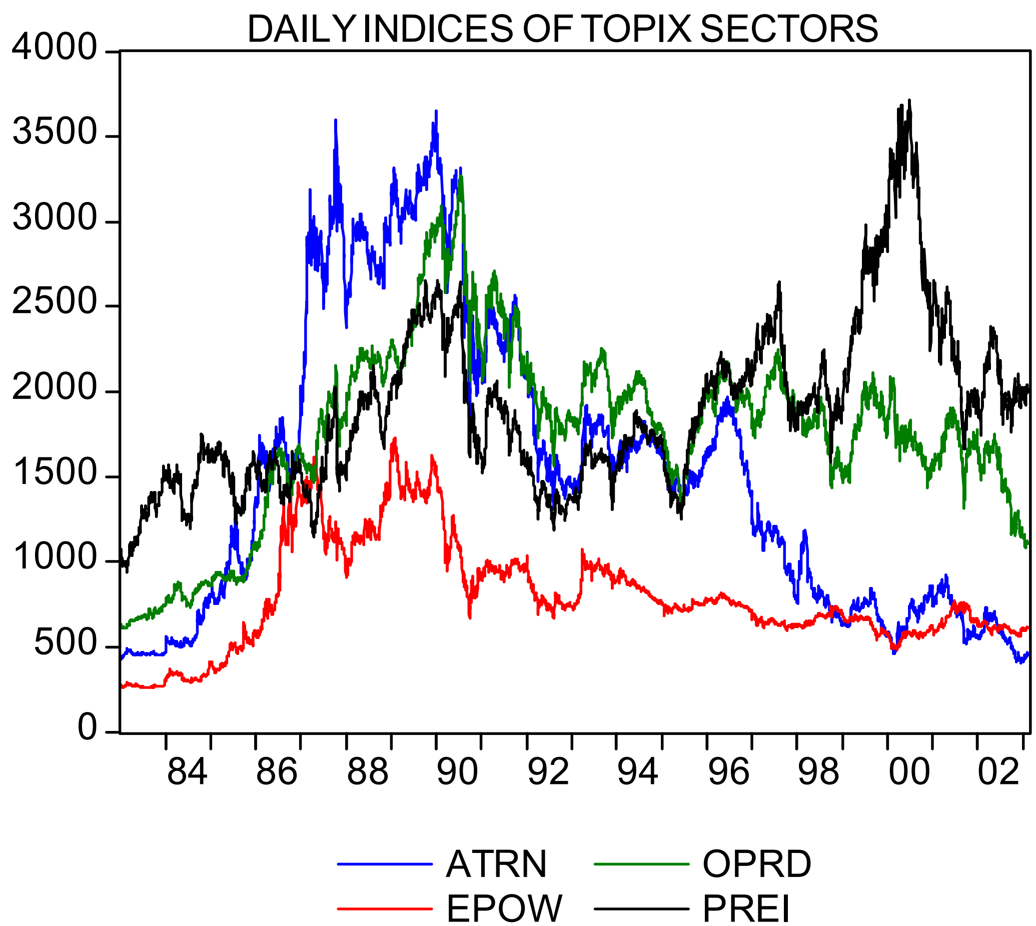

DAILY INDICES OF TOPIX SECTORS
4000
3500
3000
2500
2000
1500
1000
500
0
84
86
88
90
92
94
96
98
00
02
ATRN
OPRD
EPOW
PREI

Figure 2

DAILY RETURNS OF TOPIX SECTORAL INDICES

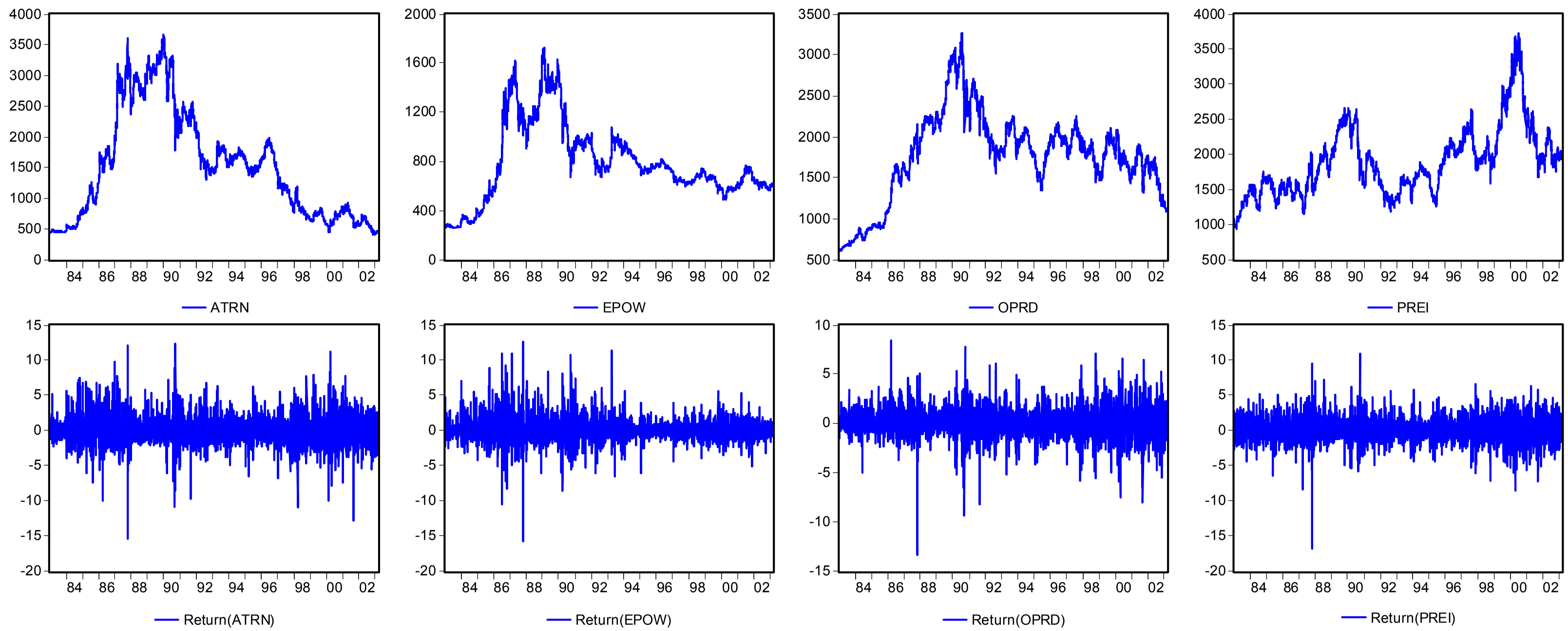


Figure 3 CONDITIONAL STANDARD DEVIATION BASED ON THE APARCH STRUCTURE

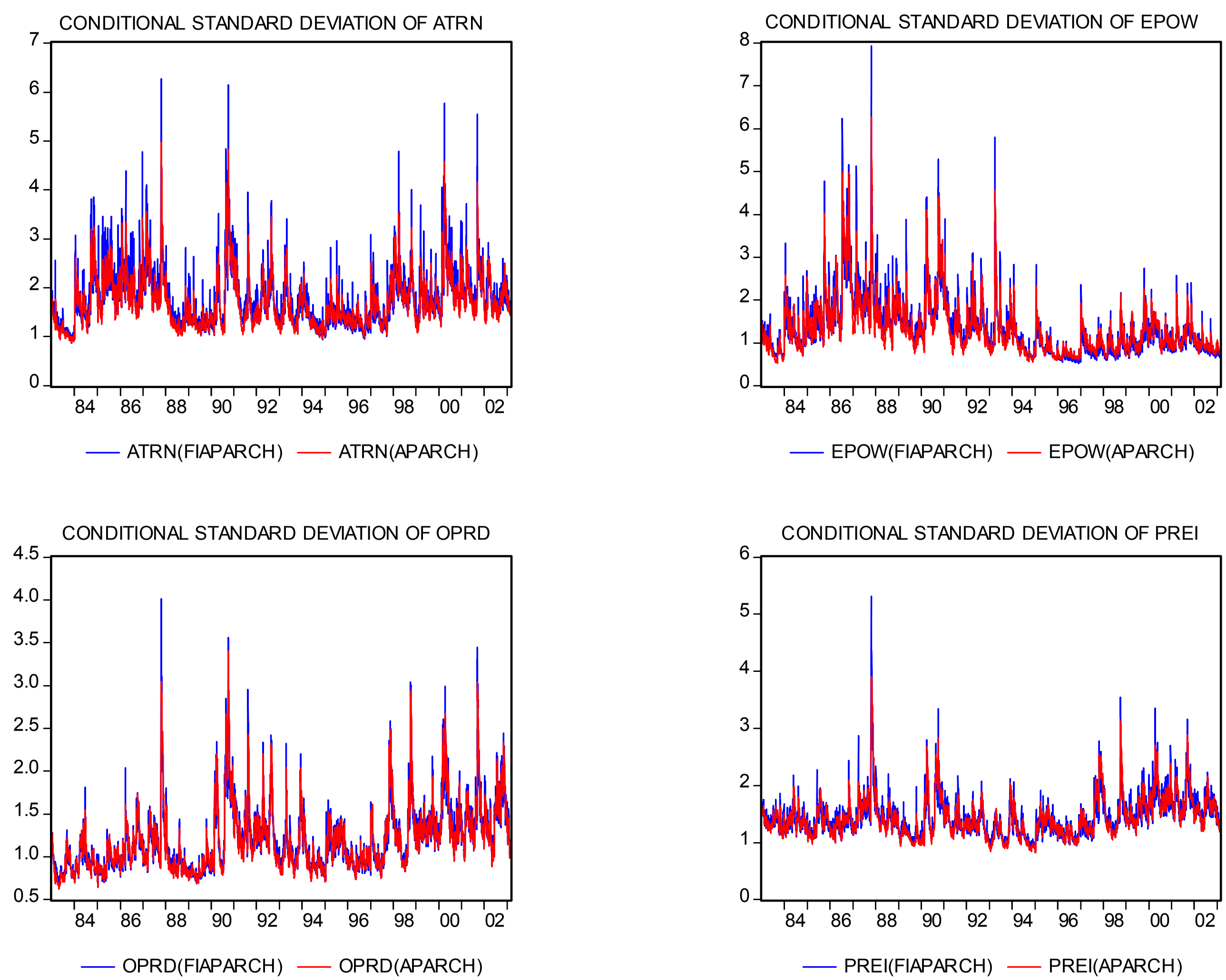


Figure 4 CONDITIONAL STANDARD DEVIATION BASED ON THE AGARCH STRUCTURE

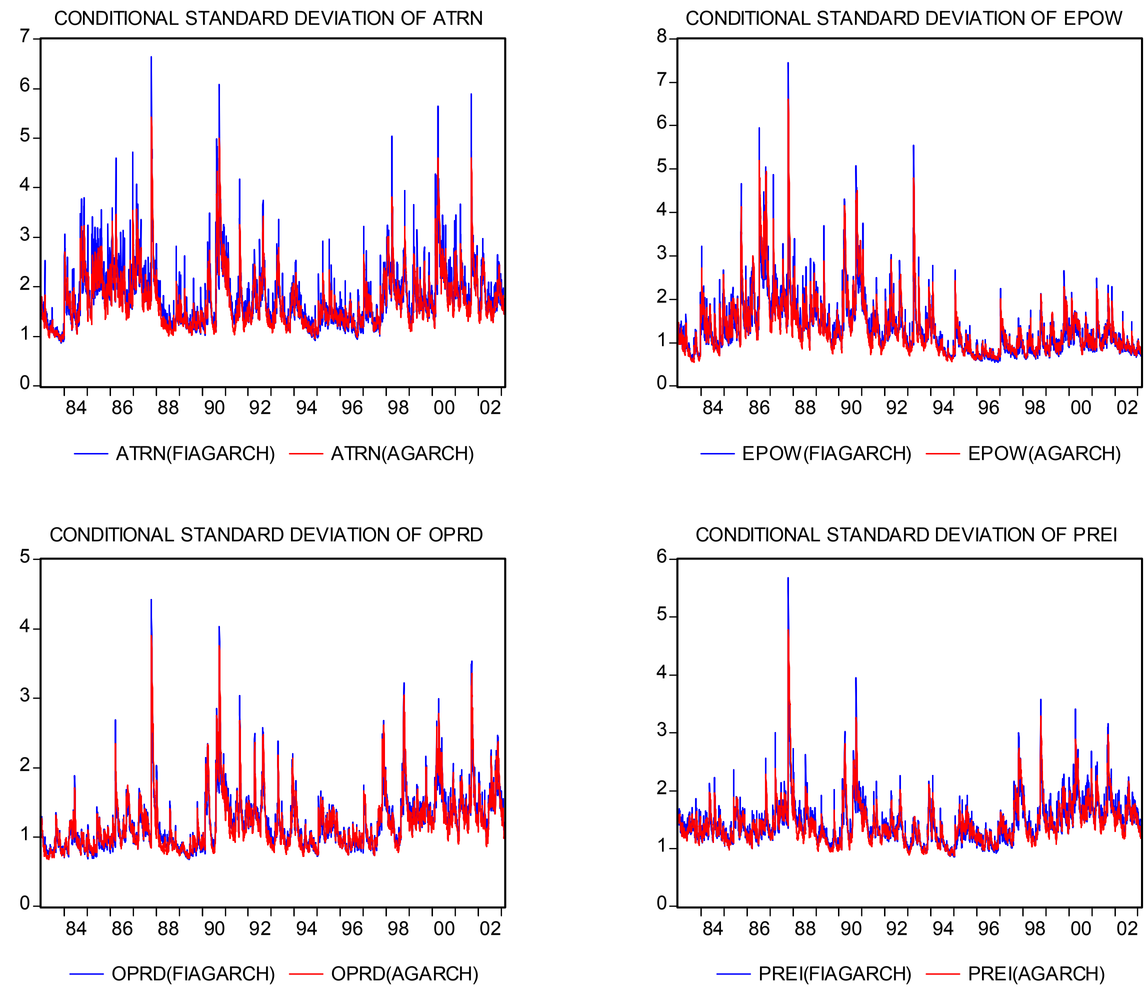


Figure 5

TETRAVARIATE VC-FIAPARCH(1,d,1): CONDITIONAL CORRELATIONS

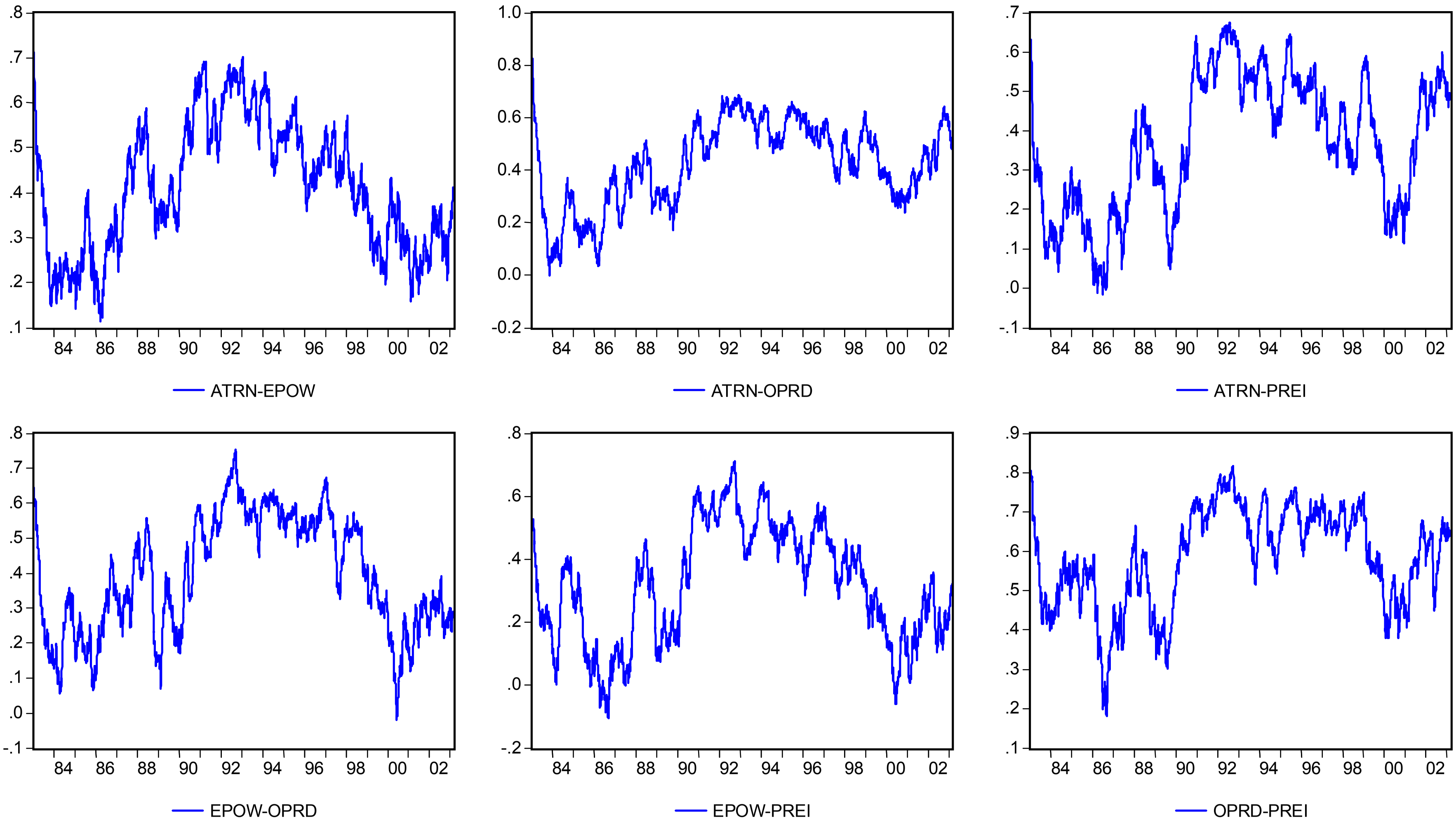
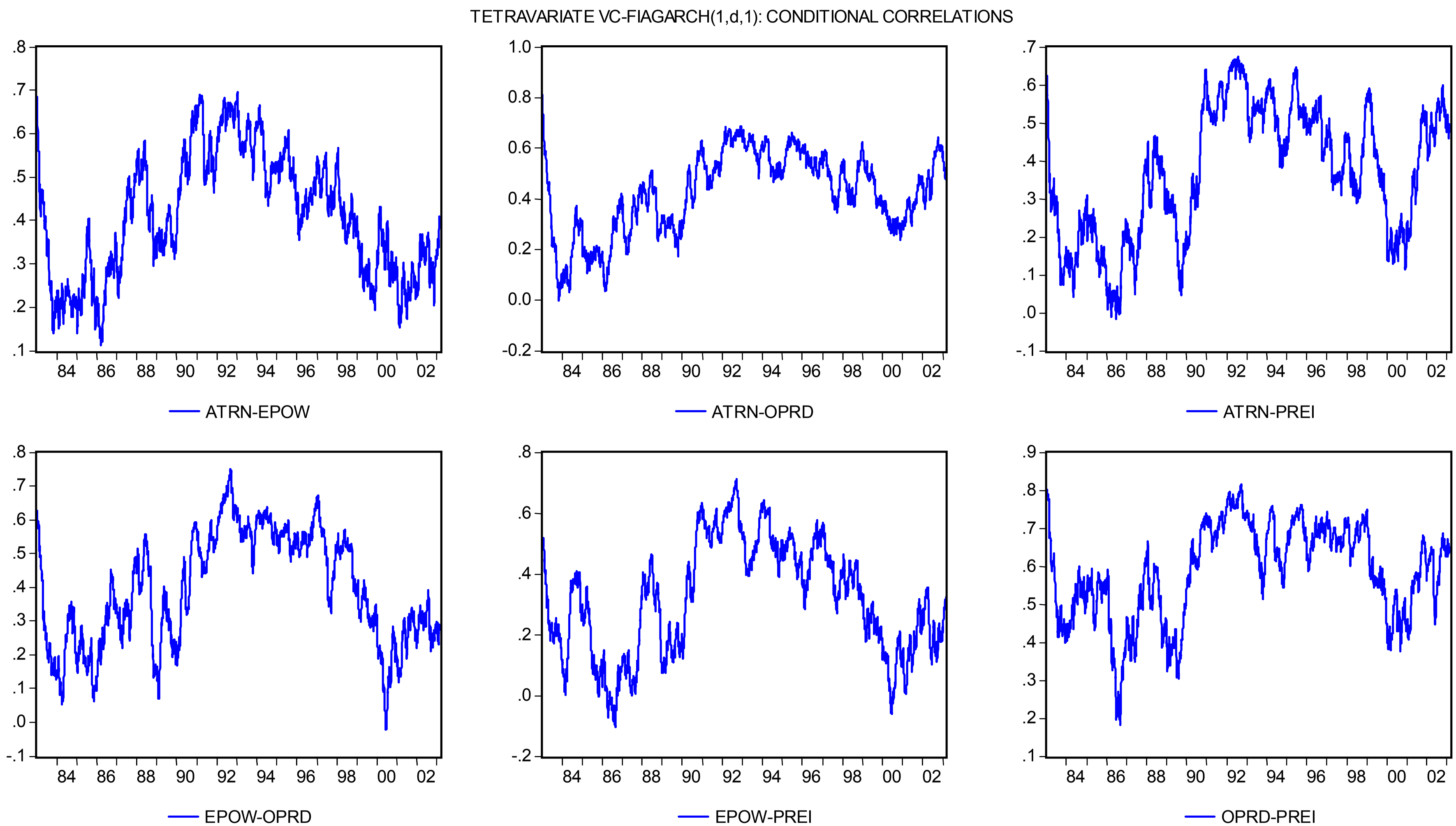


Figure 6


TETRAVARIATE VC-FIAGARCH(1,d,1): CONDITIONAL CORRELATIONS
ATRN-EPOW
ATRN-OPRD
ATRN-PREI
EPOW-OPRD
EPOW-PREI
OPRD-PREI