Linking Real Activity and Financial Markets:

BEAM model[1]

Céline Gauthier and Fu Chun Li[2]

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Abstract

A small monthly macroeconometric model of the Canadian economy built around three cointegrating relationships linking financial and real variables is estimated over the 1975-2002 period. One of the cointegrating relationships allows the identification of a supply shock as the only shock affecting permanently the stock market and a demand shock leading to important transitory stock market overvaluation. We also suggest a monetary policy reaction function using the estimated impact over time of the typical permanent inflation shock to modify the historical reaction function in such a way to eliminate any forecast persistent deviation from target. In-sample simulations over the last ten years suggest that following BEAM's advice would have cut by more than half the root-mean squared deviation of inflation from target over the period. A technical innovation of the paper consists in showing under which conditions permanent shocks can be identified in a VECM with exogenous variables.

Keywords:

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1. BEAM stands for Bonds, Equity and Money.
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1. Introduction

The most famous macroeconomic model aggregates all financial markets into only two: the market for money and the market for everything else. This allows summarizing asset market equilibrium in a single LM curve but hides the needed structure to achieve a good understanding of how monetary policy, financial markets and the real economy are interrelated. Some important theoritical contributions linking the real and financial sectors have been made since Hicks famous exposition of the Keynesian system (Tobin [1969] and Blanchard [1981] being among the most well known). But the recent stock market collapse and the subsequent loosening of monetary policy toward historically low levels of interest rates emphasized the lack of empirical “financial” models to guide monetary policy makers in such circumstances.

Over the past two decades, there has been a growing interest in developing tractable empirical macroeconomic models with transparent theoretical foundations, but few have focussed on financial market behaviours. Our aim is to contribute in filling this gap. As written in Garratt, Pesaran and Shin (2001), there are two main theoretical approaches to the derivation of long-run, steady state relations of a core macroeconomic model. One possibility is to start with the inter-temporal optimization problems faced by “representative” agents and solve for the long-run relations. The strength of this approach lies in the explicit identification of macroeconomic disturbances as innovations (shocks) to processes generating tastes and technology. However, this is achieved at the expense of often strong assumptions concerning the form of the underlying utility and production functions. Consequently, despite the enormous progress recently seen in the DGE literature, there is still a lot of work to be done before a general equilibrium model incorporates in a satisfying way the real, nominal, and financial sectors of the economy. An alternative approach is to work directly with the arbitrage conditions which provide inter-temporal links between prices and assets returns in the economy as a whole. This latter approach, by focussing on long-run theory restrictions and leaving the short-run dynamics largely unrestricted (in the context of a VECM model), provides a much more flexible modelling strategy.

We propose a small model for Canada combining Garratt *et al.* (2001)’s approach with King, Plosser, Stock and Watson (1991)’s methodology allowing the identification of permanent shocks in a cointegrated system. Crowder, Hoffman and Rasche (1999), Dhar, Pain and Thomas (2000), Jacobson, Jansson, Vredin and Warne (2001), and Cassola and Morana (2002) all follow that *route*, respectively for the U.S., the U.K., Sweden and Europe, and show the degree of “structure" that may be assigned to a simple VAR framework characterized by cointegration if one embraces sufficient identifying restrictions. Following Dhar *et al.* (2000) and Cassola and Morana (2002), we focus on the interactions between different marketable asset values and the real economy. One technical contribution of this paper is to include exogenous variables and to show under which conditions King *et al.* (1991)'s identification procedure can be applied to a VECM with weakly exogenous I(1) variables.

The building blocks of our model consist in three cointegrating relations: (1) a money market equilibrium relation, (2) an arbitrage relation between short- and long-term bonds, (3) a long-run relation between the stock market and real output. This last relation allows the identification of a supply shock as the only shock affecting permanently the stock market and a demand shock leading to important transitory stock market overvaluation.

A weakness of most models that purport to describe the transmission mechanism is their failure to pass the simple test of generating a different steady-state rate of inflation in response to a series of monetary policy actions.[3] Such models with an unique steady state rate of inflation are very difficult to reconcile with the unit-root test results found in the empirical literature.[4] In this paper, we identify permanent shocks causing inflation to reach a new steady state rate of growth as the only shocks having a permanent impact on the level of inflation. We then suggest a monetary policy reaction function consisting in reversing any identified nominal shock causing inflation to permanently deviate from the target.

Our paper is organised as follows. The theoretical foundations of the model are presented in Section 2. The cointegration analysis and specification tests' results are given in Section 3. Section 4 presents the econometric formulation of the core model. Section 5 analyses the impulse response functions. Section 6 proposes a monetary policy reaction function. A conclusion follows.

2. The theoretical foundations of the model

In this section, we describe the long-run relations used as building blocks of our model. We "loosely" base our core model on Blanchard (1981) who develops a simple model of the

3. More details on this point are made in Selody (2001).
4. This is also a very difficult issue as inflation is expected to become stationary, or at least more stable, in a successful inflation targeting environment.

determination of output, the stock market and the term structure of interest rates. The model is an extension of the IS-LM model. However, whereas the IS-LM model emphasizes the interaction between “the interest rate” and output, Blanchard’s model emphasizes the interactions between output and four marketable asset values. These are shares which are titles to the physical capital, private short and long-term bonds issued and held by individuals, and money.

Linking the real economy and the stock market We assume that there are two main determinants of spending.[5] The first is the value of shares in the stock market. It may affect spending directly through the wealth effect on consumers, or indirectly through its impact on the borrowing capacity of consumers and investors (the credit channel effect); determining the value of capital in place relative to its replacement costs, it affects investment (see James Tobin). The second is current income which may affect spending independently of wealth if consumers are liquidity constrained. Total spending is expressed as:

$$d_t = \alpha sm_t + \beta y_t; \qquad \alpha > 0; \quad \beta > 0 \tag{1}$$

All variables are real, d denotes spending, sm is the stock market value, y is income.[6] We can see equation (1) as a forward-looking aggregate spending curve with sm being a function of expected actualised future profits, the latter being a function of expected future output. Hence, aggregate spending is implicitly a negative function of actual and expected interest rates and a positive function of actual and future expected output. Output adjusts to spending over time:

$$\dot{y}_t = \sigma(d_t - y_t) = \sigma(\alpha sm_t - b y_t); \qquad \sigma > 0; \quad \mathrm{b} \equiv 1 - \beta \tag{2}$$

where a dot denotes a time derivative. Since output growth is a stationary variable and the level of output and the stock market price are both I(1) variables, equation (2) can be seen as an error-correction equation linking positively the short-run dynamics of output to deviations of the stock market from the real economy. Such a long-run relation between output and the stock market implies that transitory changes in output cannot permanently affect the level of the stock market.

5. Blanchard also includes a balanced budget change in public spending as a third determinant of total spending.
6. No stochastic error term are included in this section to simplify the presentation.

Money market equilibrium Portfolio balance is characterized by a long-run relation between money, output, interest rate and inflation:

$$M_t - p_t = cy_t - hi_t - \beta\pi_t \;; \qquad c > 0;\, h > 0; \beta > 0 \tag{3}$$

where i denotes the short-term nominal rate, y is real income, M and p denote the logarithms of nominal money and the price level and π is the level of inflation. The parameter c is positive because an increase in output shifts the money demand for transactions purpose upward; an increase in interest rate and an increase in inflation both increases the opportunity cost of holding money which decreases real balance. Given all the variables in equation (3) are better characterized as I(1) variables, if deviations of real money from its determinants are transitory, then this equation represents a cointegrating relationship.

Arbitrage between short- and long-term bonds The expectations hypothesis is perhaps the best known and most intuitive theory of the term structure of interest rates. If lr_t is the nominal yield to maturity of a discount bound and i_t is the period-t one-period rate, the expectations hypothesis in the absence of uncertainty implies that

$$(1 + lr_t)^n = \prod_{i=0}^{n-1} (1 + i_{t+i}) \tag{4}$$

This is an arbitrage condition ensuring that the holding-period yield on the n-period bond is equal to the yield from holding a sequence of one-period bonds. Taking logs of both sides and recalling that $\ln(1 + x) \sim x$ for small x, yields a common approximation:

$$lr_t = \frac{1}{n} \sum_{i=0}^{n-1} i_{t+i} \tag{5}$$

The long-term yield is equal to the average of one-period yields. Hence, a permanent shock to the short-term yield will, in the long-run, be reflected one-for-one on the long-term yield, once the shock is correctly perceived as permanent by the financial markets. Cointegration between short and long term interest rates is thus a necessary condition for the expectation hypothesis to hold.

3. Cointegration analysis

We estimate a monthly VECM over the 1975-2002 period with 6 endogenous and 1 exogenous variables and 2 lags[7]. The endogenous variables are the following Canadian variables: real GDP at basic prices[8], the over 10-year marketable bond rate, the overnight rate[9], a broad money aggregate (real CPI deflated M2++), the real stock market price (the TSE 300 divided by CPI inflation) and the CPI year over year inflation rate. M2++ includes mutual funds, whose importance has increased continuously in consumer portfolios over the nineties, and are relatively liquid. Using a broad aggregate like M2++ in the model avoids interpreting a precautionary portfolio adjustment from mutual funds to money as inflationary.[10] Given the strong economic links between Canada and the U.S. we incorporate as an exogenous variable the real US industrial production index, an available monthly proxy for U.S. activity. This allows simulation of different U.S. scenarios. Unit-root tests indicate that all variables can be treated as I(1) variables.[11] We add a dummy equalling one from 1993 onward and zero before to capture the change in the trend of inflation apparent after the adoption of the inflation target in 1991. We are aware of the possibility that inflation might have become stationnary since the adoption of an inflation targeting regime in 1991. However, the evidence on that point is not clear-cut. Cogley and Sargent (2001) have argued that there has been a downward shift in the degree of persistence in the inflation process in the US. Others (see Stock, 2001) have countered that the statistical evidence in favour of such a break is weak. But even if there was no doubt that inflation has become stationary, the treatment of variables whose degree of integration changes over the estimated sample is still unknown. Moreover, Coenen (2002) and Angeloni, Coenen and Smets (2003) show that when there is uncertainty about inflation persistence, it is better for monetary policy-makers to work on the assumption that the economy is characterized by a high degree of inflation persistence.

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7. Two lags minimizes the Hannan-Quinn and Schwartz information criterias and are sufficient to remove the correlation in residuals. We use monthly datas because the Bank of Canada has adopted a fixed action date schedule eight times a year. Other specification tests will be included in another working-paper together with the forecasting performance of the model.
8. This series has been merged with real gdp at factor cost for the period 1975-1980.
9. As noticed in Selody (2001), a good monetary policy instrument must be under the direct or close control of the central bank.
10. Moreover, Longworth (2003) finds that since 1992 both core inflation and M2++ have been remarkably stable.
11. Unit root tests results are available upon request.

Based on the theoretical foundations of the core model described in the above section, we expect to find three cointegrating relations in the estimated VECM [as described by equation (2), (3) and (5)]. The cointegration tests corrected for the presence of one exogenous variable, as proposed by Pesaran, Shin and Smith (2000), are presented in Table 1. Both the L-max and the Trace tests indicates the presence of 2 cointegration vectors but the L-max test marginally reject the presence of a third cointegrating vector, which would support our *a priori* expectations.

Table 1: Cointegration Tests[1]

L-max	Trace	H0:r=	L-max (.10)	Trace (.10)
63.12	151.48	0	40.2	104.4
46.36	88.36	1	34.1	76.9
26.84	42.00	2	28.3	54.8
10.39	16.17	3	22.2	35.9
2.97	5.78	4	15.9	20.8
2.81	2.81	5	9.5	9.5

1. The critical values corrected for the presence of one exogenous variable are taken from Table T.3 in Pesaran *et al.* (2000).

Given the borderline results of our cointegration tests, we looked at the t-values of the α coefficients for the third vector, as suggested in Hendry and Juselius (2001); when these are small, say less than 3.0, then one would not loose greatly by excluding that vector as a cointegration relation in the model. Given that some of these t-values are greater than 3.0 for all three vectors and that our theoretical model also suggests three vectors, we proceed under the assumption that there are three cointegrating vectors in our model.

The Johansen (1992) procedure allows us to identify the number of cointegrating vectors. However, in the case of existence of multiple cointegrating vectors, an interesting problem arises: α and β are only determined up to the space spanned by them. Thus for any non-singular matrix ζ comformable by product:

$$\Pi = \alpha\beta' = \alpha\zeta\zeta^{-1}\beta' .$$

In other words, β and $\beta'\zeta$ are two observationally equivalent bases of the cointegrating space. The obvious implication is that before solving such an identification problem no meaningful economic interpretation of coefficients in cointegrating space can be proposed. The solution is imposing a sufficient number of restrictions on parameters such that the matrix satisfying such restrictions in the cointegration space is unique. Such a criterion is derived in Johansen (1992) and discussed in Hamilton (1994). We base our restrictions on Blanchard's model which suggest more than a sufficient number of constraints to the cointegration space. The over-identification restrictions can therefore be tested. The results are in Table 2.

Table 2: Testing restrictions on the cointegrating vectors[a]

	The LR test, $\chi^2(10) = 7.02$, p-value = 0.72					
inf	y	onr	m	sm	lr	y^{us}
2.41 (.27)	-1.18 (.08)	2.41 (.27)	1	0	0	0
0	0	1	0	0	-1	0
0	-1	0	0	1	0	0

a. Standard errors are given within parentheses.

The restricted core model is easily accepted with a p-value of 0.72. In comparison, Dhar *et al.* (1999) did not find a significant core model while Cassola and Morana just slightly accepted theirs with a p-value of 0.11. Our results are consistent with the theoretical foundations presented in Section 2. The first cointegrating relation corresponds to the money market equilibrium, the second to an approximation of the pure expectations hypothesis based on an arbitrage relation between short and long term bonds, while the third relation links real activity with the real stock market. The coefficients of the cointegrating relation cannot usually be interpreted as elasticities even if the variables are in logs, since a shock to one variable implies a shock to all variables in the long run. Hence the coefficients do not in general allow for a *ceteris paribus* interpretation (see Lutkepohl [1994]). Interpreting the coefficients in the first cointegrating relation is thus meaningless. However, given the last two cointegrating relations involve only two variables, we do not need the *ceteris paribus* interpretation. The second long-run relation specifies that a permanent 1% increase in the overnight rate is associated with the equivalent increase in the long-run interest rate. This is consistent with a stationnary term spread and the expectation hypothesis of the term structure of interest rates. The third cointegrating relation suggests that a 1% permanent increase in output (or a 1% increase in potential output) is associated with a permanent 1% increase in the stock market. Given the ratio of the TSX to output has been hoovering around a constant value for most of the last 25 years, unit coefficients in this cointegration relationship are not surprising. Interestingly, this last relation also implies that transitory changes in real output can only lead to transitory changes in the level of the stock market. The second and third cointegrating vector are similar to those found in Cassola and Morana (2002). However, they found a Fisher relation which was impossible to find over our sample, and their money demand relationship includes only the level of real output which is not standard.

The economy is in a long-run equilibrium when those three cointegrating relationships are respected, that is when there is no gap between money, output, inflation and the overnight rate (or no money gap), the overnight rate is equal to the long rate, up to an unidentified constant (no interest rate gap), and the stock market level is consistent with potential output (no stock market gap).[12]

Chart 1 illustrates the money gap simply defined as the error-correction term from the money demand cointegrating relationship.[13] The two surges in inflation, in 1981 and 1991, were preceeded by increasing money gap around 2 years before. It is also interesting to notice that since the Bank of Canada has adopted explicit inflation target in 1991, the money gap has been much more stable, deviating only slightly from equilibrium and for short periods of time in 1995 and 2000. This is in line with the results in Longworth (2003) who reports that since 1992 both core inflation and M2++ have been remarkably stable.

The stock market gap defined as the error-correction term of the third cointegration relationship (Chart 2) illustrates periods of “mis-valuation” of the stock market and/or correct anticipation by the stock market of the direction of output going forward. The transitory and permanent components of the variables have to be identified in order to discriminate between those two. This is done below.

5. Shock analysis

The impact of a change in U.S. industrial production The response functions to a permanent increase of 1 percent in U.S. industrial production are shown in Chart 4. Small inflation pres-

12.Notice there are constant terms in the three cointegrating vectors.

13. Gaps based on permanent components of the variables will be presented in section 6 below.

sures are generated as output is boosted by almost .2 percent on impact. Interest rates are increased by around 25 basis points to keep demand in line with short-run supply. The Canadian stock market is temporarily hurt by the higher interest rate. It nevertheless increases by .12 percent in the long-run, in line with the permanent increase in output.[14] Broad aggregate money is negatively affected in the short-run by the slight increases in inflation and real interest rates. Only output is significantly affected in the long-run.

Identification of the permanent shocks Given the presence of three cointegrating vectors and 6 endogenous variables, there are three stochastic trends or permanent shocks to be identified. Appendix A shows that KPSW's identification methodology can be used provided the exogenous variable does not cointegrate with the endogenous variables. The first permanent shock, $\varepsilon_{\pi t}$, labeled an inflation shock, is the only shock having a permanent impact on inflation. According to the "monetarist" view, the long-run money growth and inflation rate are ultimately set exogenously by monetary authorities. So the inflation shock relates to central bank monetary policy. A positive inflation shock reflects the central bank's decision to permanently increase the inflation rate. Hence, the structural inflation shock is identified by assuming that the long-run system has the following recursive structure:

$$\lim_{s \to \infty} \begin{bmatrix} inf_{t+s} \\ y_{t+s} \\ onr_{t+s} \\ m_{t+S} \\ sm_{t+S} \\ lr_{t+S} \end{bmatrix} = \begin{bmatrix} \tau_{11} & 0 & 0 \\ \tau_{21} & \tau_{22} & 0 \\ \tau_{31} & \tau_{32} & \tau_{33} \\ \tau_{41} & \tau_{42} & \tau_{43} \\ \tau_{51} & \tau_{52} & \tau_{53} \\ \tau_{61} & \tau_{62} & \tau_{63} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{\pi t} \\ \varepsilon_{yt} \\ \varepsilon_{dt} \end{bmatrix}$$

Note that τ_{ij} is the long-run response of the ith endogenous variable to the j element in the vector of structural disturbances ε_t. The restrictions $\tau_{12} = 0$ and $\tau_{13} = 0$ mean that only an inflation shock, $\varepsilon_{\pi t}$, affects the long-run level of inflation. The mainstream view would predict that the decision to change inflation permanently has no permanent impact on real variables and

14. US industrial production represents about 15% of US total GDP. Under the assumption that a permanent increase of 1% in US industrial production translate in an increase of .15% in US total GDP, our results suggest that a .15% increase in US GDP is associated with an increase of about .12% of Canadian GDP.

thus that $\begin{bmatrix}\tau_{21} & \tau_{41} & \tau_{51}\end{bmatrix} = 0$. However economic theory provides no clear-cut predictions on that question. In several theoritical models, the superneutrality result due to Sidrausky (1967) breaks down as inflation can have either positive or negative effects on real variables such as consumption and investment, depending on the exact assumptions concerning preferences. Additionally, in these models the real interest rate may or may not be independent of inflation in the long-run (see Orphanides and Solow (1999) for a survey). Some recent empirical results (see for example Rapach [2003] and Gauthier, Pelgrin and Schweisguth [2003]) find support to the Mundell-Tobin effect suggesting that an unexpected increase in inflation has a permanent negative impact on real interest rate. We let the data talk on this point by leaving unconstrained the parameters in $\begin{bmatrix}\tau_{21} & \tau_{31} & \tau_{41} & \tau_{51} & \tau_{61}\end{bmatrix}$.

Most theoretical models define supply shocks as being governed by technology innovations determining the technical capacity of the economy. We thus identify a supply shock as a shock allowed to have a permanent effect on output but not on inflation. The long run effects on all the other real variables are left unconstrained. Notice that all shocks are allowed to impact all the variables in the short-run. In particular, a supply shock is expected to decrease inflation in the short-run.

The third structural shock is a shock having no permanent impact neither on output nor on inflation. This shock is labelled a demand shock. Our interpretation of disturbances with permanent effects as supply disturbances, and of disturbances with transitory effects as demand disturbances is motivated by a traditional Keynesian view of fluctuations (see Blanchard and Quah (1989) for a simple model which delivers those implications).

The inflation shock A positive inflation shock reflects the central bank's decision to permanently increase the inflation rate.[15] Given the instrument used by the central bank, this can only be achieved by decreasing the overnight rate. Chart 5 shows that our results are consistent with this view. To achieve a typical unexpected inflation increase of around .3 percent in the long-run, the central bank has to decrease the overnight rate by about 25 basis points. Given the expectations hypothesis of the term structure in our core model, the long rate is persistently depressed

15. Such a shock can always be reversed by a negative inflation shock of the same size, if the central bank decides to do so.

as well. The bank's intervention leads to a small output stimulus in the short-run. The shock also hurts significantly the stock market and decreases *real* broad aggregate money in the short run.

The permanent significant negative effect of inflation on interest rates may be explained through the Mundell effect: An *unexpected* increase in inflation decreases real wealth, which increases savings. Real interest rates must then fall to restore goods market equilibrium. Risk averse agents prefer to smooth consumption and thus savings, which implies a higher *path* of savings and persistently lower real interest rates. This is consistent with the long-run growth theory models in which any permanent increase in the share of output going into investment (or savings) is associated with permanently lower real interest rates. Our results are also in line with the need to increase persistently the interest rate in disinflation periods and in the first years of inflation targeting in order to gain credibility. Rapach (2003) also finds that an unexpected permanent increase in inflation is associated with permanently lower long-run real interest rates in every industrialized country of a sample of 14, including Canada, Germany, France and Italy.[16] This contradicts Cassola and Morana (2002) in which an unexpected increase in inflation increases interest rates in the long-run. Their result may be driven by the absence of shift in the deterministic trend of nominal variables that would have taken into account the more stable inflation of the nineties. That period of stable inflation is then interpreted as a period dominated by positive inflation shocks (relative to the negative trend estimated over the full sample) which are associated with higher nominal interest rates over the period. Since the nineties represents half of their sample, it might very well have distorted their results.

When inflation is forecast to deviate permanently from the actual target of 2 percent, the historical estimated reaction function (the equation for the overnight rate) may be adjusted using the estimated impact over time of the typical permanent inflation shock in such a way to eliminate the expected long-run deviation from target. This is the basics of the reaction function proposed in section 6.

The supply shock The typical supply shock increases the productive capacity of the economy by around .9% in the long-run. Inflation is pushed downward in the short-run as pro-

16. Notice that a permanent inflation shock represents an *unexpected* persistent deviation of inflation from its deterministic trend. This source of increase in inflation is associated in the long-run with a decrease in interest rates. That, of course, does not mean that *expected* changes in inflation have the same effect on interest rates.

duction costs are decreased (see Chart 6) but goes back to its initial level in the long-run. The central bank has, over the sample, accomodated the shock by decreasing interest rates to eliminate the excess supply in the good market and bring inflation back to target.[17] Interstingly, interest rates are not affected in the long-run. This is consistent with the model of Ramsey in which the interest rate is determined by the rate of time preferences and technology determines the level of capital such that marginal product of capital is equal to the interest rate. The stock market leads output and overshoots somewhat. Broad money is higher in the short-run because of the accomodative stance of monetary policy and remains higher in the long run because of both higher money demand for transaction purposes and higher real value of the stock market. These results are similar to Cassola and Morana (2003) except that in their model output decreases in the short-run which is kind of a puzzle.

A demand shock[18] The demand shock increases inflation, output and the stock market in the short-run. Short and long interest rates increase in the short-run as expected. This can be seen as the result of a standard textbook open market operation with a disinflationary objective. When inflation and output turn out to be higher than expected, an inflation targeting central bank has to increase interest rates. It is interesting to notice that since a demand shock has no permanent impact on output, the important stock market surge in the first months following the shock slowly dissipates as investors realise that higher profits cannot be sustained without a permanent increase in productivity.

The permanent positive impact on the overnight rate implies that the so-called demand shock induces, on average, a higher equilibrium interest rate. According to the model of Ramsey, this would correspond to a rate of time preference shock. Furthermore, as predicted by the long-run theory of growth models, any shock that lowers persistently the share of product going into investment is associated with higher real interest rate in the long-run. King *et al*. (1991) estimate a significant cointegration relationship linking negatively the ratio of investment over output and the real interest rate in the U.S. and identify what they call a “real interest rate shock” with long-run properties very similar to our “demand” shock. They also identify what they call a “balanced-

17. In some SDGE models with adjustment costs on capital (see Neiss and Nelson [2001], page 23 for an example), productivity shocks would decrease the *neutral* rate in the short run. This provides further incentives to decrease the actual interest rate after a productivity shock.

18. Other demand shocks having only transitory effects may also be identified.

growth” shock which is very similar to our supply shock, increasing output permanently while leaving the ratios of investment and consumption over output and the real interest rate and level of inflation unchanged in the long-run. For example, a fiscal shock crowding out investment persistently would be associated with persistently higher interest rates.

Output gap An output gap is easily obtained from our model as the difference between actual output and the historical contribution of permanent shocks on output (determining potential output). Potential output and the output gap are graphed in Chart 8 and 9 respectively. According to these results, the Canadian economy was in excess demand before both the 1982 and the 1991 recessions and was in excess supply for most of the nineties. The gap was closed at the end of 1999 and the economy turned in excess demand for the following two years. The economy was back in excess supply (though close to zero) at the end of 2002. These results are largely consistent with the estimations done at the Bank of Canada with other methodologies. What may be more surprising is the period over which supply shocks contributed to increase output permanently. Chart 8 suggests that it started around 1985 and last until 1996, the year Greenspan first talks of irrational exuberance. From 1996 until the end of 2000 and the strong stock market correction, the economy was demand-driven and potential would have been growing at a rate lower then the deterministic rate.[19] This result, in line with Dueker and Nelson (2002) and the latest economic developments, casts some doubts on the purported New Economy in the second half of the nineties.

The deterministic shift in output in 1993 implies a quite different stock market gap than the one simply defined above as the error-correction term of the third cointegrating relationship. Actually, with a definition of the stock market gap based on permanent component of the variables, the Canadian stock market would have been overvalued by around 20% in 1999 and was undervalued by almost 20% at the end of 2002. This is consistent with the observed increase of

19. It shoud be noticed however that a shift in the deterministic trend in output is estimated in 1993. Hence, the growth of potential in the second half of the nineties is lower compared with a relatively higher growth in trend. Depending on our judgment on the source of this shift, the story can be completely different. If the higher deterministic output growth is attributed to supply shocks, then potential output would have increased continuously in the nineties and the Canadian economy would currently be in considerable excess supply. Nevertheless, given the deterministic nature of this shift and the recent economic developments, we proceed under the assumption that this change in trend should be considered as demand-driven implying that potential output and the output gap are well approximated by Charts 8 and 9. The fact that potential has been below the higher growth trend for the last seven years is also an indication that the higher trend should be seen as transitory.

the TSX since then. These results are in line with Dupuis, Tessier (2003) who estimate a three-variables VECM linking the U.S. stock market to dividends and long-term interest rate.

6. BEAM's proposed reaction function

When inflation is forecast to deviate permanently from the target, the central bank's reaction must differ from the historical estimated reaction function (the equation for the overnight rate) in order to prevent the unwanted deviation. Only permanent shocks to inflation can reverse a permanent deviation from target. We thus simply propose to simulate the impact of the necessary permanent inflation shock on the overnight rate and adjust the future path of the overnight rate accordingly. For example, if the difference between the long-run forecast of inflation and the target is 1%, we know from the long-run matrix in Table 3 that an inflation shock of size -(1/0.32) times the typical inflation shock will bring inflation back to the target. We also know the overnight rate's response to such a shock, so we can adjust the forecast reaction function accordingly.

Table 3: Long-run impact of permanent shocks ($\Gamma(1)$)

	ε_π	ε_y	ε_d
inf	0.32	0	0
y	-0.05	0.89	0
onr	-0.24	0.01	0.25
m	-0.26	1.03	-0.61
sm	-0.05	0.89	0
lr	-0.24	0.01	0.25

One might question the impact on BEAM's reaction function of assuming inflation non stationnary in the actual inflation-targeting environment, which has rendered inflation at least more stable. Since BEAM's reaction function is based on an average degree of persistence of inflation and an average level of credibility of the Bank of Canada over the sample, it should be seen as more aggressive than what is probably needed in the actual environment. Nevertheless, Coenen (2002) and Angeloni, Coenen and Smets (2003) show that when there is uncertainty about inflation persistence, it is better for monetary policy-makers to work on the assumption that the economy is characterized by a high degree of inflation persistence. We next analyse the overnight rate paths recommendations that would have came out from BEAM's reaction function in an out-of-sample exercise at the end of 2002 and the end of 2001 and in an in-sample simulation over the nineties.

Out-of-sample simulations at the end of 2002 and 2001 At the end of 2002, inflation was at 3.8% and the overnight rate was 2.75% (-1% in real terms). According to the stock market and money gaps based on the estimated permanent components (Chart 10 and 11 respectively), the stock market was undervalued by almost 20% at the time, and the stock of broad money was a little lower than its long-run equilibrium. BEAM forecasts that inflation would increase further in the long-run (see Chart 13). This forecast is easily explained by the fact that the surge in inflation at the end of 2002 was seen as permanent (see the estimated transitory component of inflation in Chart 12). BEAM's reaction function adjusting the implicit estimated reaction function to bring back forecast inflation to 2% would have suggested at the time an increase in the real overnight rate of about 100 basis points in the short-run and a long-run nominal overnight rate of 3.2% (see Chart 14). The low level of the neutral rate must be interpret as the need for a quite sustained stimulative monetary stance.

At the end of 2001, the situation was quite different: the short rate was at 2.25% and inflation at 0.7% (a real rate of 1.5%). The stock market was about fair valued and broad money a little bit high. Chart 15 shows that the model forecasts that inflation would slowly increase toward the lower band of the inflation target at 1.2%. BEAM's adjusted reaction function (see chart 16) suggests the real overnight rate should be decreased by around 50 basis points in the short-run for inflation to reach 2% in the medium term. BEAM's reaction function also suggest that the long-run neutral nominal interest rate was around 3.4% at the end of 2001. It is interesting to notice that BEAM's recommendations are quite similar with the Bank's view at both points in time.

In-sample simulations Finally, we did an in-sample simulation exercice to study what BEAM's recommendations would have been over the last 10 years. In the simulation, the model is shocked every period to bring back inflation to target according to BEAM's forecast and response functions of the identified nominal shock. BEAM would have make recommendations not that far from what the bank of Canada did (see Chart 17). There would have been noticeable differences in 1994 where BEAM would have suggested to be less aggressive and in 2001 where BEAM would have proposed a faster and more aggressive tightening. However, despite the similar interest rate paths, the differences in stance would have been sufficient to diminish by more than half the deviation of inflation from 2 percent over the period.

7. Conclusion

We have estimated a small monthly VECM to study the interactions between the real and financial sectors of the Canadian economy. To take into account the high degree of economic integration between Canada and the U.S., the U.S. industrial production index has been included as an exogenous variable. Identification of permanent shocks in a VECM with exogenous variables represents a technical contribution to the literature.

Our principal results are: (1) The identification of a long-run relation between the stock market and real output which allows the identification of a supply shock as the only shock affecting permanently the stock market and a demand shock leading to important transitory stock market overvaluation. (2) The money gap defined as the error-correction term from the first cointegrating relation has been much more stable since the adoption of inflation-targets in Canada. (3) Out-of-sample simulations based on BEAM's reaction function at the end of 2001 and 2002 lead to overnight rate recommendations similar to the Bank of Canada's decisions at the time. However, in-sample simulations over the last ten years suggest that following BEAM's advice would have cut by more than half the root-mean squared deviation of inflation from target.

An important question is the impact on BEAM's reaction function of assuming inflation non stationnary in the actual inflation-targeting environment, which has rendered inflation at least more stable. Since BEAM's reaction function is based on an average degree of persistence of inflation and an average level of credibility of the Bank of Canada over the sample, it should be seen as more aggressive than what is probably needed in the actual environment. Nevertheless, Coenen (2002) and Angeloni, Coenen and Smets (2003) show that when there is uncertainty about inflation persistence, it is better for monetary policy-makers to work on the assumption that the economy is characterized by a high degree of inflation persistence.

Another remaining question is what determine interest rates in the long-run. Are the so called great ratios coming from the long-run growth theory the main determinant? This is left for future research.

The model could possibly be used to build a financial condition index for Canada using the stock market and money gaps from the core model together with the deviation of actual real interest rate from the neutral interest rate recommended by the proposed reaction function. This index could eventually be completed with the deviation of the Canadian exchange rate from equilibrium provided in Gauthier and Tessier (2002) and tested against those proposed in Gauthier, Graham and Liu (2003). This is left for future research.

Appendix A: Identification of permanent shocks in a model with exogenous variables.

In a non-cointegrated VAR model, the structural shocks' identification procedure (Blanchard and Quah [1989] for example) is clearly invariant to the presence or not of exogenous variables in the model. However, in presence of cointegration, this is not obvious as the common stochastic trends must be consistent with the cointegrating relations which possibly include exogenous variables. Wickens and Motto (2001) has shown how to identify the shocks when the following restrictions are made: the variables can be classified as endogenous or exogenous, there are as many cointegrating relations as endogenous variables, the cointegrated vectors are identified and they contain at least one exogenous variable. In Wickens and Motto (2001) the complete model need to be estimated. In this section, we show how King *et al.* (1991)'s identification procedure can be applied to a VECM with weakly exogenous I(1) variables restricted not to be in the cointegrating relations.

Consider a structural model of the form:

$$\Delta y_t = \mu + C_x(L)\Delta x_t + \Gamma(L)\eta_t \tag{4}$$

where $\eta_t \sim IN(0, \Omega_\eta)$ is a $n \times 1$ vector of serially uncorrelated disturbances independent of Δx_t (being a linear combination of u_t), and where the endogenous variables' response to a change in the exogenous variables is given by $C_x(L)$.

The identifying procedure documented in King *et al.* (1991) is based on the infinite moving average (MA) form obtained by inverting the estimated VECM. This inversion cannot be directly made because of the presence of cointegration. An easier way to invert a VECM than those commonly suggested in the literature (see Yang [1998] for example) is proposed in Appendix B. The inverted reduced form model obtained is:

$$\Delta y_t = \mu + C_x(L)\Delta x_t + C(L)u_t \tag{5}$$

where all the parameters are defined in Appendix A. Notice that, since u_t is independent of e_{xt}, u_t is independent of Δx_t.

The identifying problem consist in identifying the individual components in η_t from the estimated reduced form model given by (5) and can be described as follows. There are $s = n - r$ identifiable common stochastic trends driving the $n \times 1$ vector y_t where $r = Rank[\Pi_y]$.[20] We express $\Pi_y = \alpha_y\beta'$ where the $n \times r$ loading matrix α_y, and the $m \times r$ matrix of cointegrating vector β are each full column rank and identified up to an arbitrary $r \times r$ non-singular matrix.[21] Partition β comformably with z_t as $\beta = (\beta_y',\beta_x')'$ where β_y and β_x are respectively $n \times r$ and $k \times r$, and partition the vector of structural disturbances η_t into two components, $(\eta_t^{1'},\eta_t^{2'})'$, where η_t^1 contains the s disturbances that have permanent effects on the components of y_t and where η_t^2 contains $n - s$ elements that have only temporary effects.

Partition the matrix of long-run multipliers, $\Gamma(1)$, comformably with η_t as $\Gamma(1) = [\Theta, 0]$, where Θ is the $n \times s$ matrix of long-run multipliers for η_t^1 and 0 is a $n \times (n - s)$ matrix of zeros corresponding to the long-run multipliers of η_t^2.

Assumption 3.1 $\beta_x' = 0$

Under Assumption 3.1 $\beta' z_t$ being stationnary implies that $\beta'_y y_t$ is stationnary, which implies $\beta_y'\Gamma(1) = 0$. So under Assumption 3.1, the matrix of long-run multipliers is determined by the condition that its columns are orthogonal to β_y', and $\Theta\eta_t^1$ represents the innovations in the long-run components of y_t. While the cointegration restrictions identify the permanent innovations

20.We implicitly make the assumption that s is strictly positive. Wickens (1996) has shown that if $rank(\Pi) = n$, then the full model has to be estimated and the common stochastic trends can be equated with the non-stationary component of the exogenous variables.

21.That is, $(\alpha_y K^{-1})(K\beta') = (\alpha_y\beta')$ for any (r,r) non-singular matrix K.

$\Theta\eta_t^1$, they fail to identify η_t^1 because $\Theta\eta_t^1 = (\Theta P)(P^{-1}\eta_t^1)$ for any non-singular matrix P. To identify the individual elements of η_t^1, we need the following identifying restrictions:

Assumption 3.2. $u_t = \Gamma_0\eta_t$ where Γ_0^{-1} exists.

Under assumption 3.2, the structural disturbances are in the space spanned by current and lagged values of z_t and that there are no singularities in the structural model.

Assumption 3.3. Θ is assumed triangular which permits writing $\Gamma(1) = [\tilde{\Theta}\Pi, 0]$ where $\tilde{\Theta}$ is a $n \times s$ matrix with no unknown parameters whose columns are orthogonal to β_y', and Π is a $s \times s$ lower triangular matrix with full rank and 1's on the diagonal.[22]

The covariance matrix of the structural disturbances is partitioned comformably with $\eta_t = (\eta_t^{1'}, \eta_t^{2'})'$ and is assumed to be

Assumption 3.4. $\Omega_\eta = \begin{bmatrix} \Omega_{\eta^1} & 0 \\ 0 & \Omega_{\eta^2} \end{bmatrix}$ where Ω_{η^1} is diagonal.

That is, the permanent shocks, η_t^1, are assumed to be uncorrelated with the transitory shocks, η_t^2, and the permanent shocks are assumed to be mutually uncorrelated.

The permanent innovations, η_t^1, can be determined from the reduced form (7) as follows. From equations (7) and (8) and Assumption 3.2, $C(L) = \Gamma(L)\Gamma_0^{-1}$ and $C(1) = \Gamma(1)\Gamma_0^{-1}$. Let D be any solution of $C(1) = \tilde{\Theta}D$. Thus, $\tilde{\Theta}Du_t = \tilde{\Theta}\Pi\eta_t^1$ and $D\Omega_u D' = \Pi\Omega_{\eta^1}\Pi'$. Let

22.The diagonal elements of Π are normalised to unity without loss of generality, since the variances of η_t^1 in Assumption 4.3 are unrestricted.

$\bar{\Pi} = chol(D\Omega_u D') = \Pi\Omega_{\eta^1}^{1/2}$. Since Π is a triangular matrix, and Ω_{η^1} is diagonal, there is a unique solution for Π and Ω_{η^1}. We can thus identify the permanent shocks $\eta_t^1 = \Pi^{-1}Du_t$. Defining $G = \Pi^{-1}D$, it is then easy to show that the dynamic multipliers associated with η_t^1 are $C(L)\Omega_u G'\Omega_{\eta^1}^{-1}$.

Appendix B. A simple way to invert a VECM with exogenous variables.

The identifying procedure documented in King *et al.* (1991) is based on the infinite moving average (MA) form obtained by inverting the estimated VECM. This inversion cannot be directly made because of the presence of cointegration. In this section, we propose an easier way to invert a VECM than those commonly suggested in the literature (see Yang [1998] for example).

By partitionning Π_y and Ψ_i conformably with $z_t = (y_t', x_t')'$ as $\Pi_y = (\Pi_y^{y}{}', \Pi_y^{x}{}')'$ and $\Psi_i = (\psi_i^{y}{}', \psi_i^{x}{}')'$, where Π_y^y and ψ_i^y are $n \times n$ and Π_y^x and ψ_i^x are $n \times k$ constant coefficient matrices, we can rewrite (3) as:

$$y_t = c + B_0 x_t + \sum_{i=1}^{p} A_i y_{t-i} + \sum_{i=1}^{p} B_i x_{t-i} + u_t \tag{A1}$$

where $B_0 = \Lambda$, $B_1 = -(\Lambda - \Pi_y^x - \psi_1^y)$, $B_i = (\psi_i^x - \psi_{i-1}^x)$ for $i = 2, \ldots, p-1$, $B_p = -\psi_{p-1}^x$, $A_1 = (\psi_1^y + \Pi^y + I_n)$, $A_i = (\psi_i^y - \psi_{i-1}^y)$ for $i = 2, \ldots, p-1$ and $A_p = -\psi_{p-1}^y$.

We then write (4) as the following VARX(1):

$$\mathcal{Y}_t = \mathcal{C} + \mathcal{A}\mathcal{Y}_{t-1} + \mathcal{B}x_t + \mathcal{U}_t \tag{A2}$$

where $\mathcal{Y}_t \equiv (y_t', y'_{t-1}, \ldots, y'_{t-p+1}, x_t', x'_{t-1}, \ldots, x'_{t-p+1})'$, $\mathcal{U}_t \equiv (u_t', 0, 0, \ldots, 0)'$ and $\mathcal{C} \equiv (c', 0, 0, \ldots, 0)'$ are $mp \times 1$ matrices. Matrices $\mathcal{A}$ and $\mathcal{B}$, respectively of dimension

$mp \times mp$ and $mp \times k$, are defined accordingly to Y and x following Luktepohl (p.335). Assuming that the process starts at a finite time $t = 0$, it is straightforward to obtain the inverted form:[23]

$$y_t = A^t y_0 + \sum_{i=0}^{t-1} A^i c + \sum_{i=0}^{t-1} A^i B x_{t-i} + \sum_{i=0}^{t-1} A^i U_{t-i} \tag{A3}$$

Taking the first difference of (A3), assuming for simplicity that $U_0 = x_0 = y_0 = 0$, and extracting the endogenous variables with the appropriate $nm \times p$ matrix $J = [I_n, 0, \ldots, 0]$, we get:

$$\Delta y_t = \mu + C_x(L)\Delta x_t + C(L)u_t \tag{A4}$$

where $\mu = JA^{t-1}c$, $C_x(L) = \sum_{i=0}^{t-1} JA^i B L^i$, $C(L) = \sum_{i=0}^{t-1} C_i L^i$,

$$C_i = J(A^i - A^{i-1})J'L^i \quad \text{for } i = 1, \ldots, t-1 \text{ and } C_0 = I_n.$$

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23. In this unstable system, a one time impulse may have a permanent effect in the sense that it shifts the system to a new equilibrium, but the impulse responses may be calculated just as in the stable case. See Lutkepohl, Reimers (1992) for further details on this point.

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CHART 1. Money gap

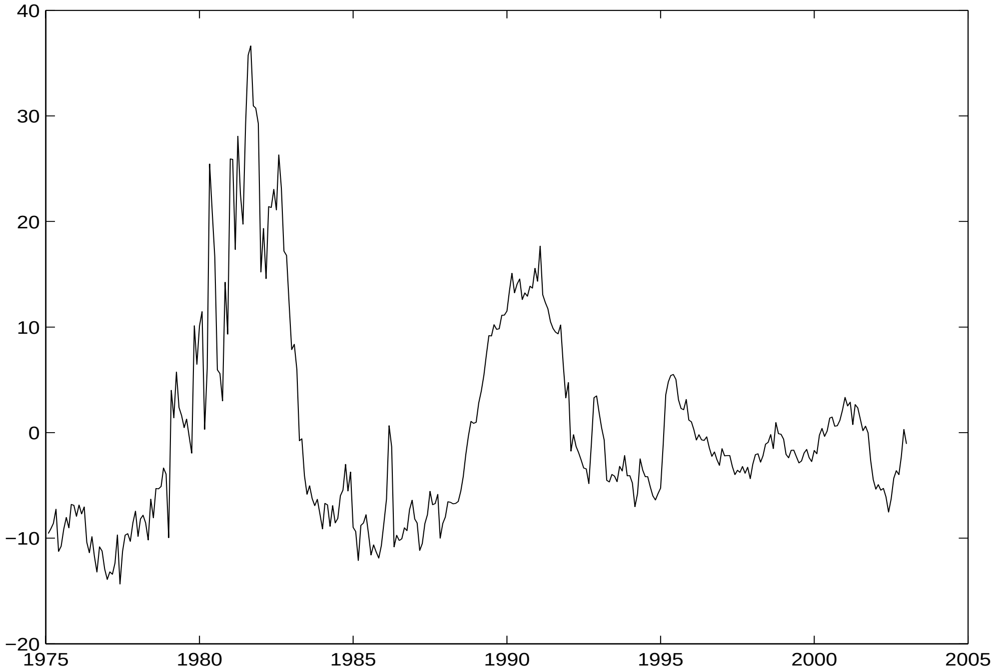
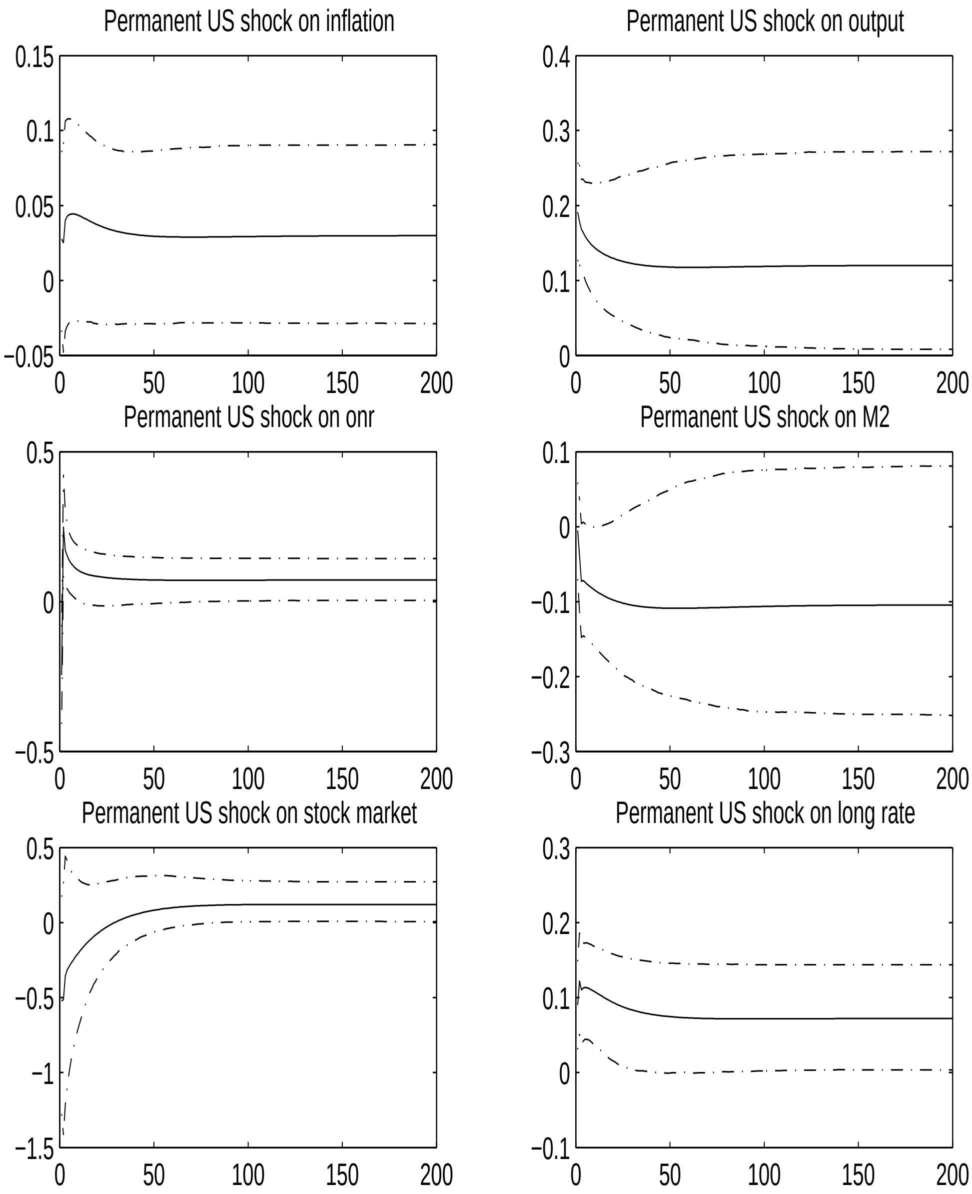


CHART 2. Stock Market gap

CHART 4. Responses to a permanent increase in U.S. industrial production[1]



1. The confidence bands are calculated by non-parametric bootstrap.

CHART 5. Impulse responses to an inflation shock

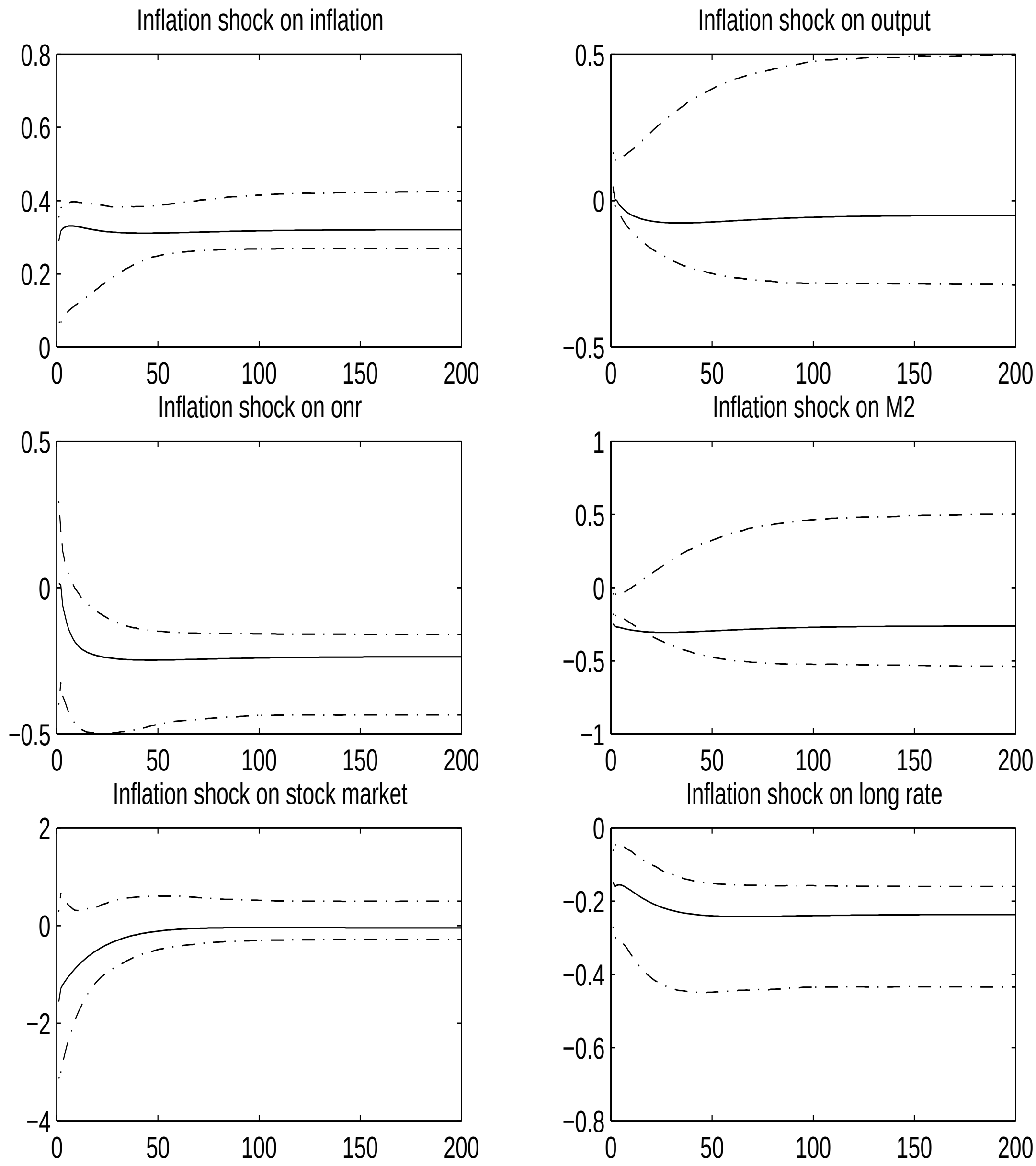


CHART 6. Impulse responses to a supply shock

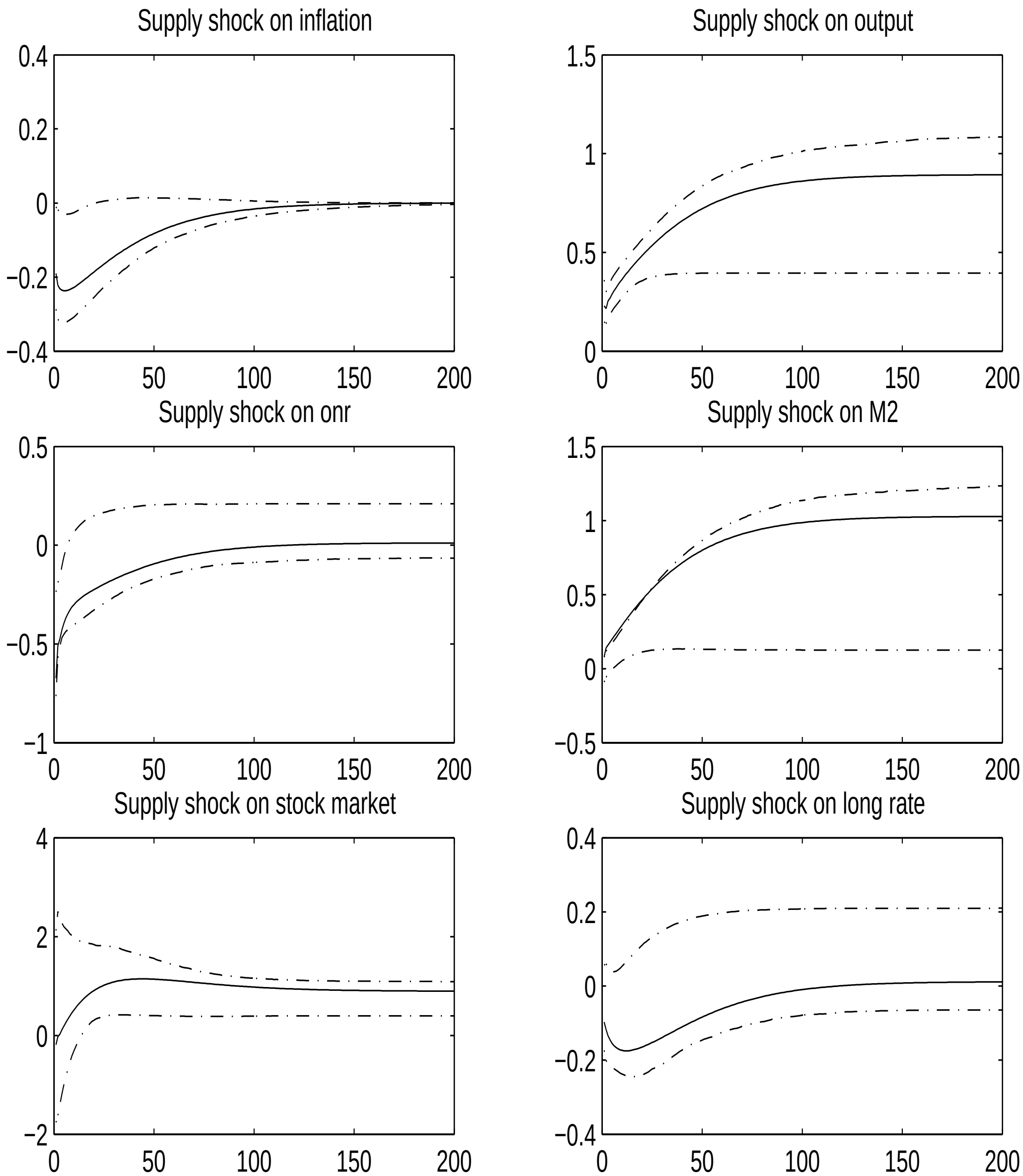


CHART 7. Impulse response to a demand shock

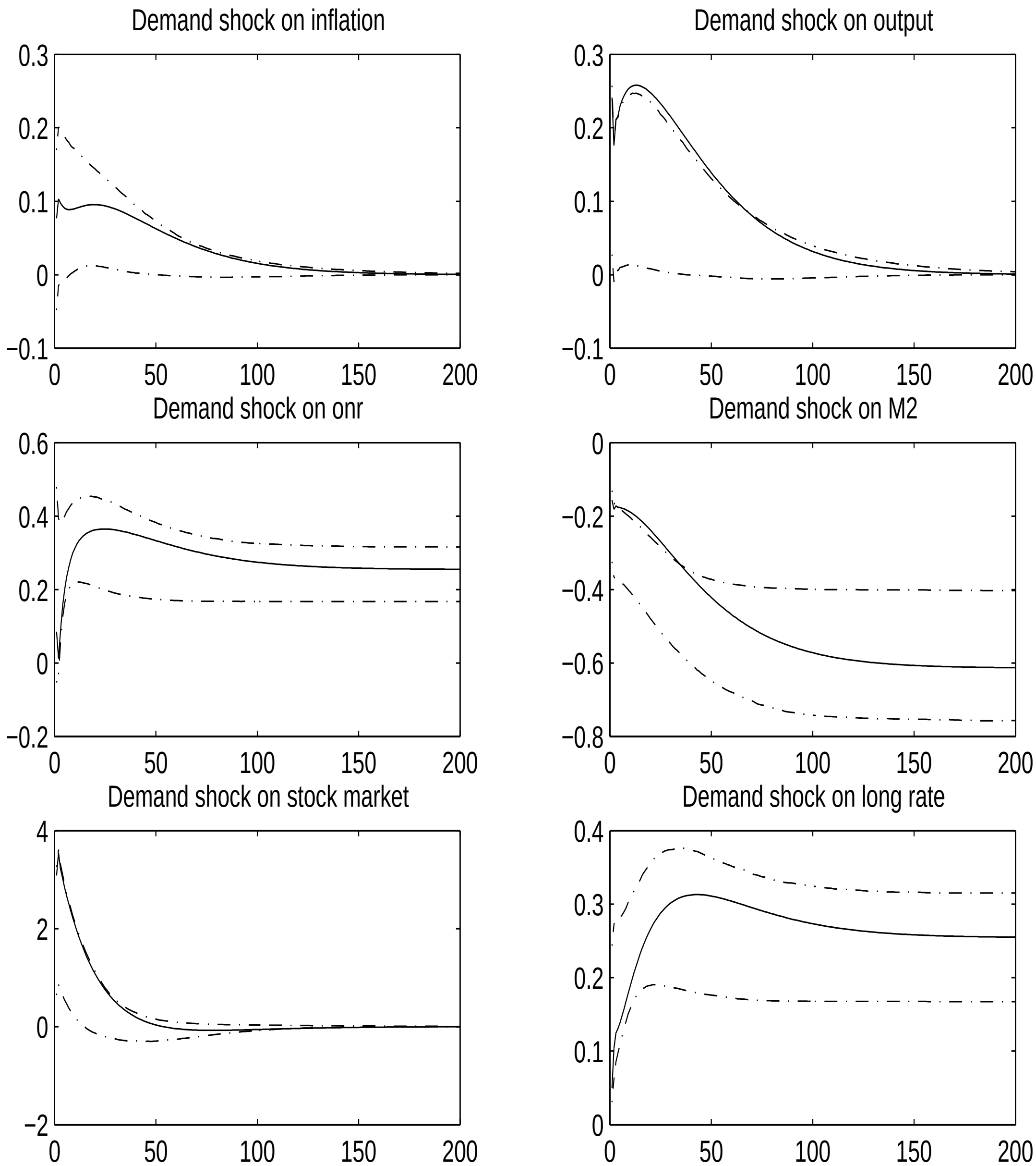


CHART 8. Potential output

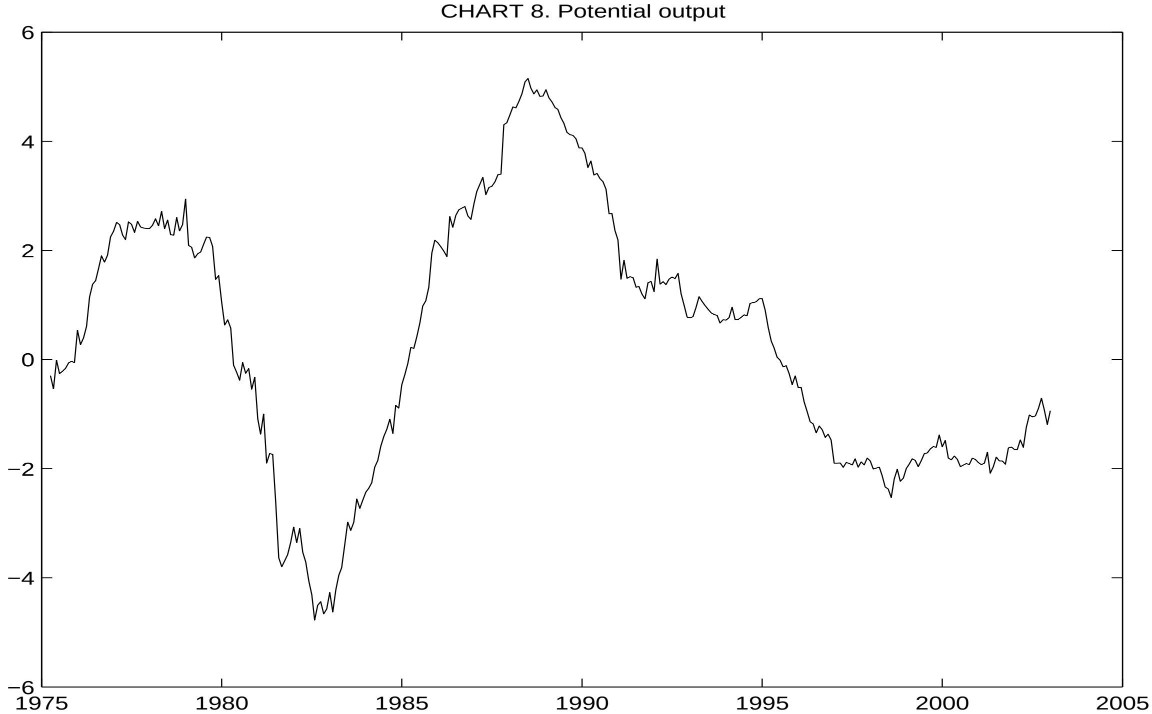


CHART 9. Output gap

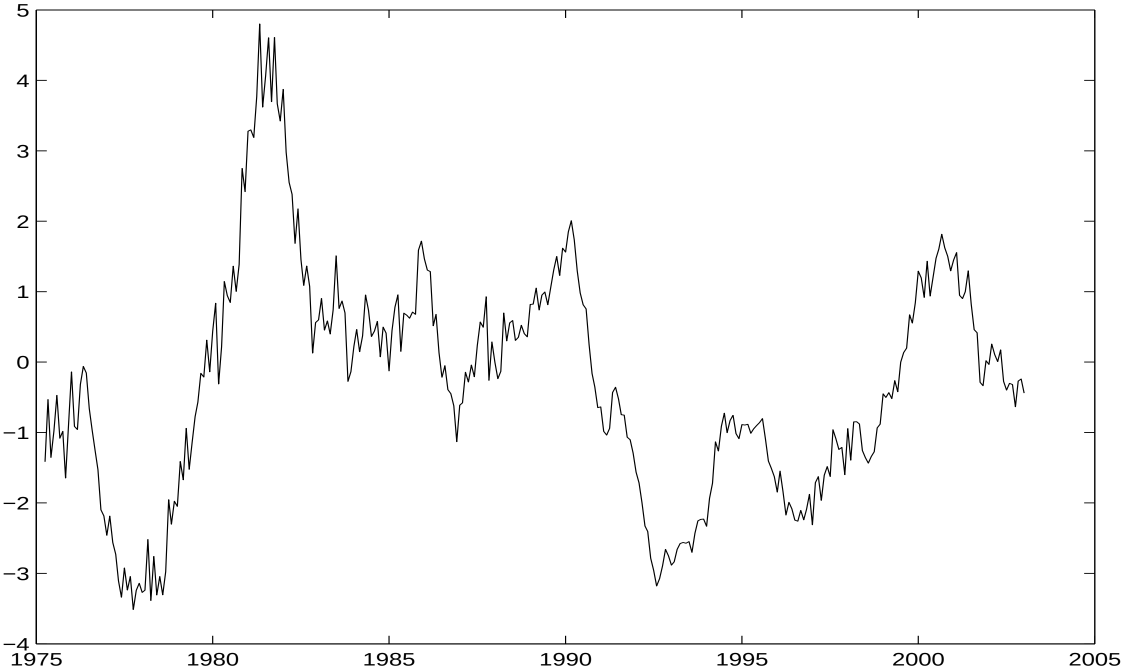


CHART 10. Money gap (based on permanent components)

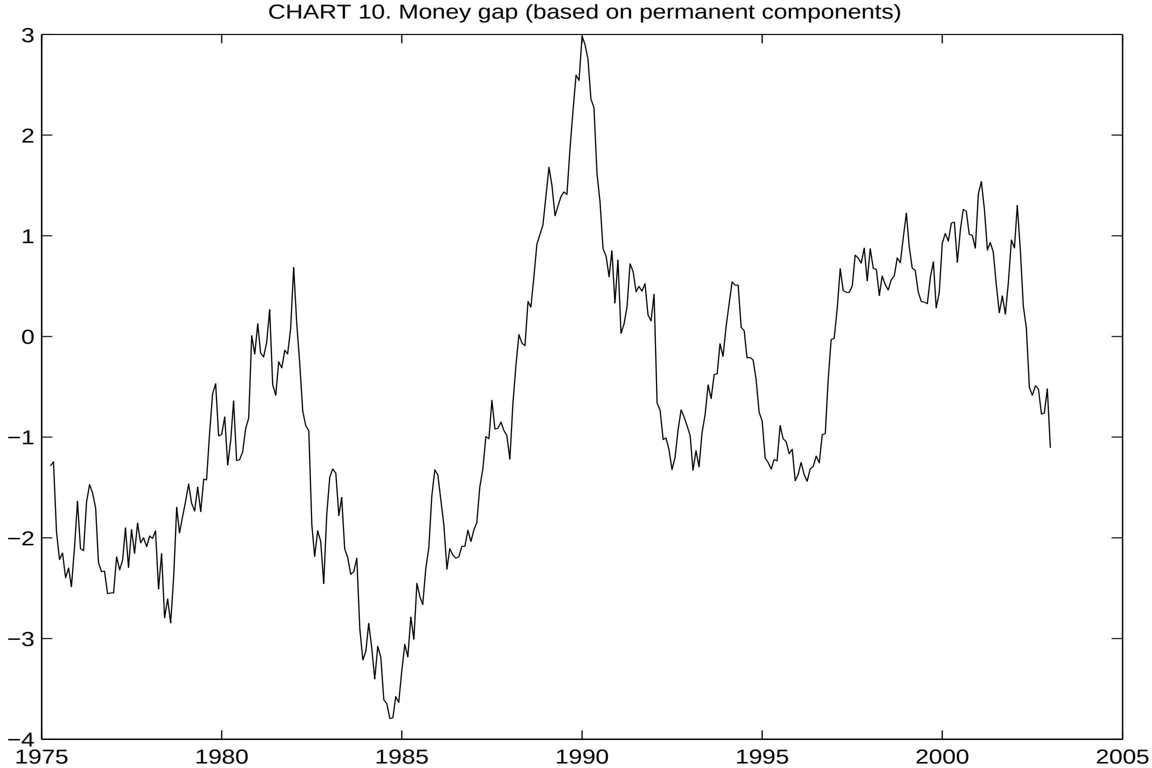


CHART 11. Stock Market gap (based on permanent components)

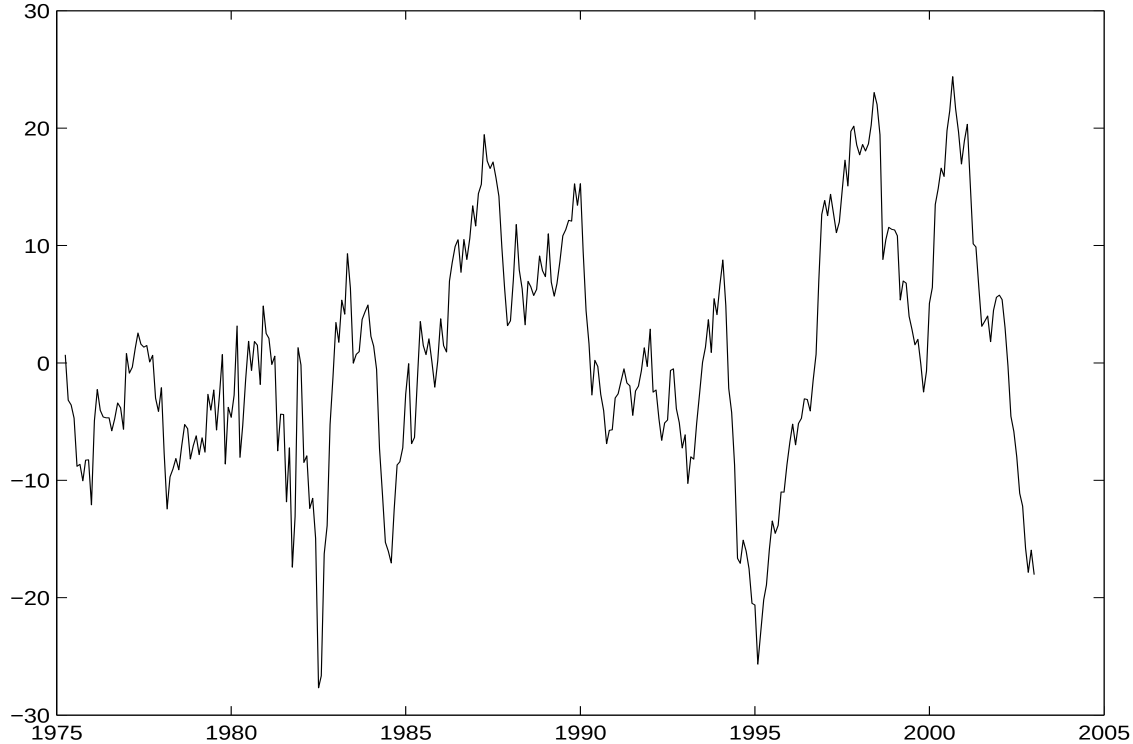


CHART 12. Transitory component of inflation

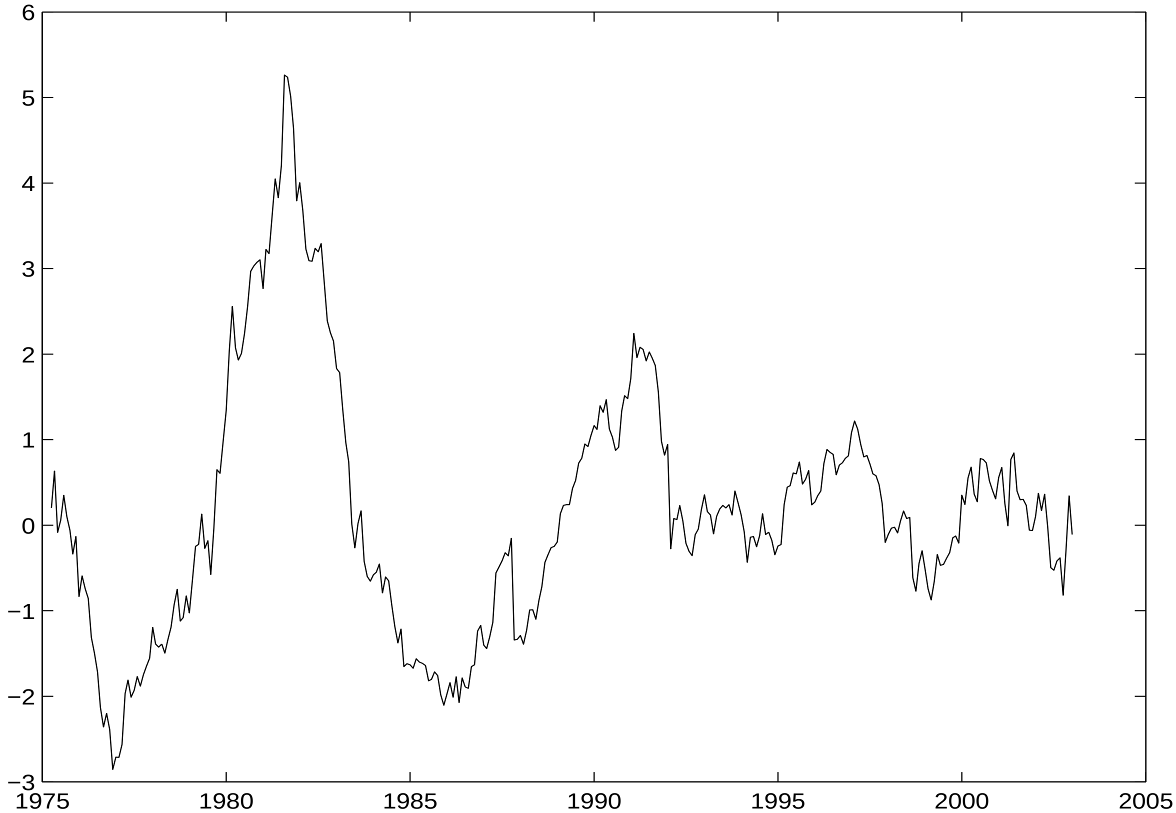


CHART 13. Forecast inflation

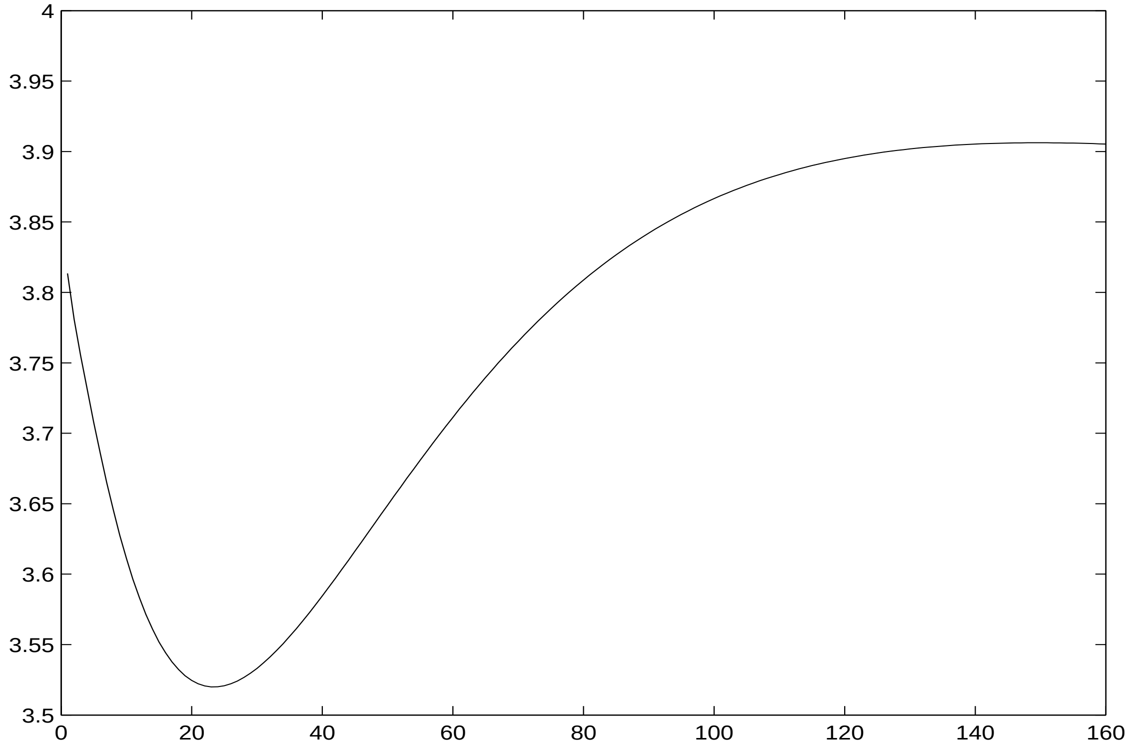


CHART 14. Real reaction function

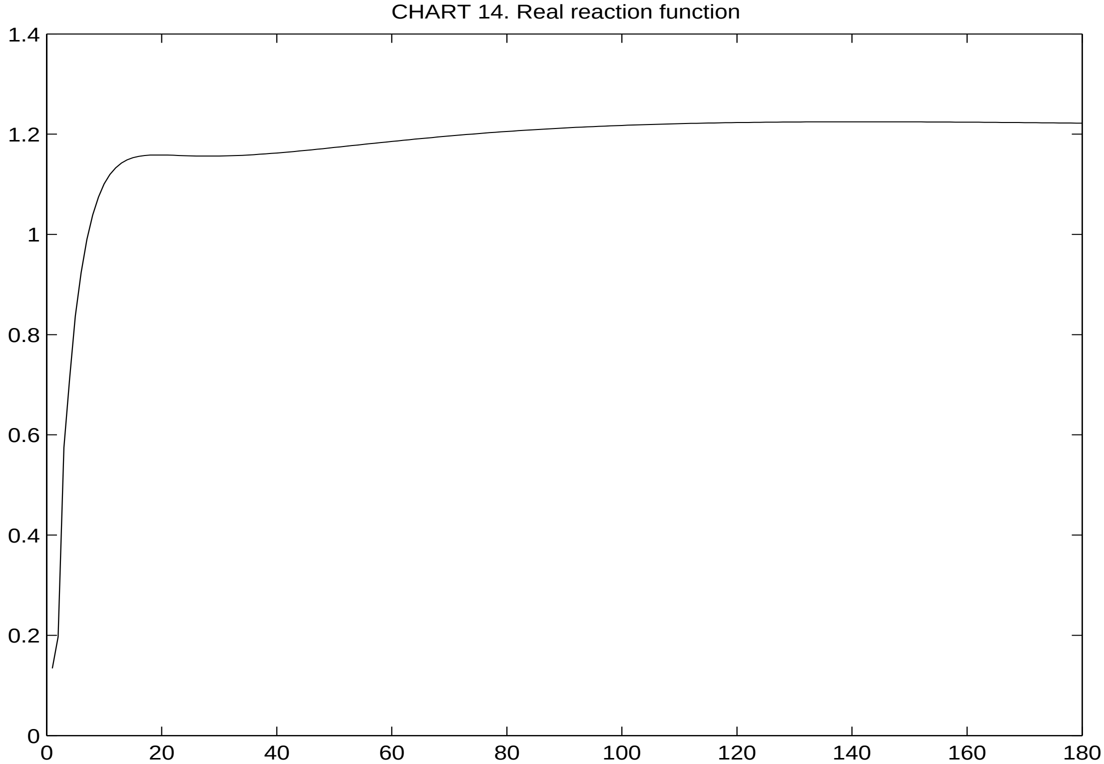


CHART 15. Forecast inflation

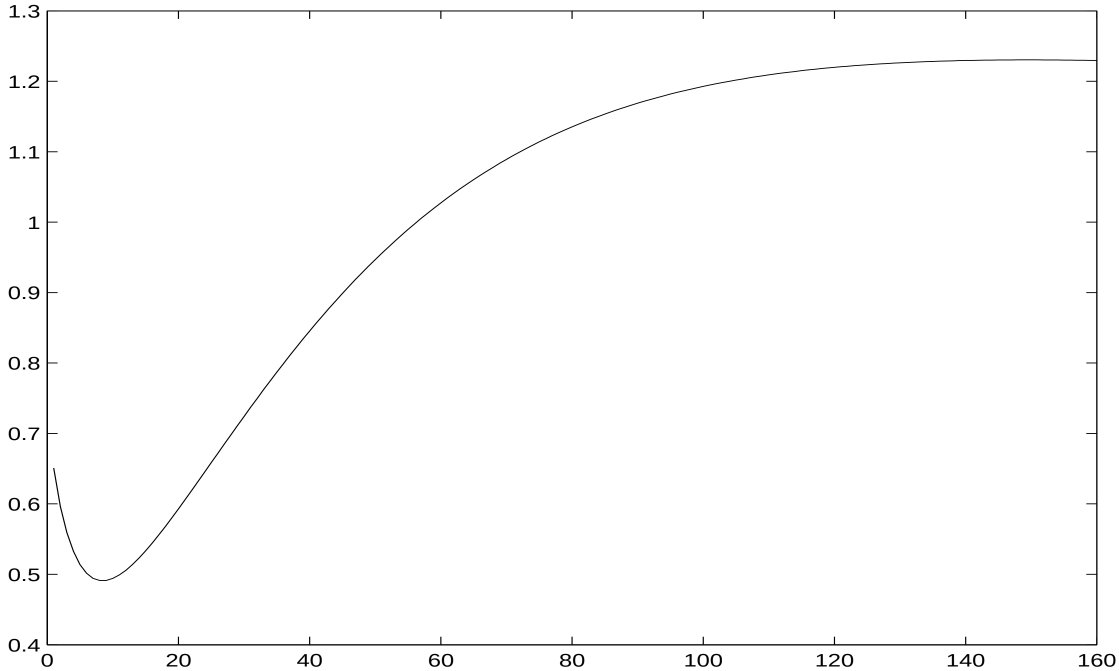


CHART 16. Real reaction function

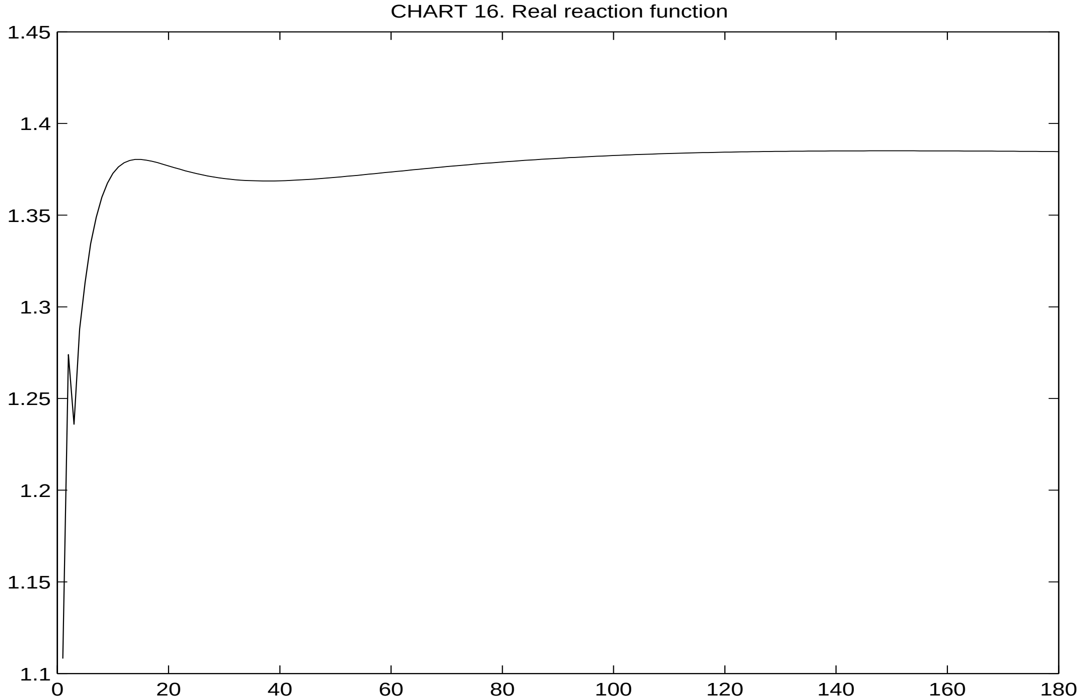


CHART 17. ron (+−) and BEAM's reaction function (−−)