The Purchasing Power Parity Persistence Paradigm: Evidence from Black Currency Markets

Mario Cerrato[†], Neil Kellard[‡] and Nicholas Sarantis[†]

[†]London Metropolitan University, UK
[‡]University of Essex, UK

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Corresponding author: Mario Cerrato, Department of Economics, Finance & International Business, London Metropolitan University, 84 Moorgate, London EC2M 6SQ. *Tel.: 020 7320 1551. E-mail:m.cerrato@londonmet.ac.uk*

Abstract

This paper provides an extensive analysis of the PPP persistence puzzle by using a unique data set of black market real exchange rates for thirty-six emerging market economies. In estimating PPP persistence the problems of small sample bias and serial correlation are addressed by using exact and approximate median unbiased estimation methods. We construct bootstrap confidence intervals for the half-lives, as well as exact quantiles of the median function for different significance levels using Monte Carlo simulation. Strikingly, it is found that point estimates of half-lives are much lower than the 'celebrated' 3-5 year range suggested by Rogoff (1996). The confidence intervals for some countries are quite narrow, a corollary being, in the emerging market context at least, there is not a PPP persistence puzzle after all!

Keywords: Exchange rate persistence; Half-lives; Black markets; Median unbiased estimation; Bootstrap confidence intervals

JEL Classification: F31; C22; O11

1. Introduction

An important issue in the literature on purchasing power parity (PPP) is the slow speed of adjustment of the real exchange rate to its PPP level (generally 3-5 years half-lives[1]). The length of these half-lives, considered too long to be explained by nominal rigidities given the high short-term volatility of the real exchange rate, has been described by Rogoff (1996) as the "Purchasing Power Parity Puzzle."

In a recent paper, Murray and Papell (2002) [hereafter known as MP] note that most of the empirical evidence is derived from Dickey-Fuller unit root tests, where the half-life is calculated from the coefficient on the lagged real exchange rate[2]. MP identify three crucial weaknesses in this approach. Firstly, it is not appropriate if there is autocorrelation and the real exchange rate actually follows an autoregressive (AR) process with an order greater than one. Secondly, most previous studies provide only point estimates of half-lives which give an incomplete picture of the speed of convergence; these point estimates need to be supplemented with confidence intervals. Finally, least squares (LS) estimates of the half-lives are biased downwards in the small samples encountered in practice.

To address the above econometric issues MP apply exact and approximate median unbiased estimators to two different data sets; an annual data set collected by Lee (1976) and consisting of six US dollar real exchange rates for industrial countries spanning 1990-1996, and a second quarterly data set consisting of twenty US dollar real exchange rates for industrial countries spanning 1973:01 to 1998:02. Strikingly, they report point estimates for half-lives that concur with the consensus in previous

[1] Half-Life' refers to the number of years it takes for at least fifty percent of the deviation from PPP to be eliminated, following a real exchange rate shock.
[2] See, for example, Abuaf and Jorion, 1990; Cheung and Lai, 1994; Wu, 1996; Lothian and Taylor, 1996, Papell, 1997.

literature; unfortunately, confidence interval estimates are too wide to allow the point estimates to be of any use.

Of course in examining the half-life of real exchange rates, previous literature has already made the assumption that a long-run PPP level exists. However, in MP the upper bound of the confidence interval is generally infinity suggesting the possibility that the real exchange rate is possibly a unit root process and that long-run PPP doesn't hold. It is this indeterminacy which drives the wide confidence interval for the half-life point estimates.

In this paper we extend the MP study on the PPP persistence puzzle by estimating half-lives for the black market real exchange rates of thirty-six emerging markets economies. This is an important contribution for three reasons. Firstly, Cheung and Lai (2000b) compare PPP half-lives in emerging and industrial countries and suggest that the former generally show less persistence. However, the methodology used is vulnerable to many of the criticisms made by MP. Secondly, empirical work has often found that the real exchange rate *appears* more likely to be a stationary variable in emerging rather than in industrial countries (see Cheung and Lai, 2000b). This may allow for smaller confidence intervals for half-life estimates. Thirdly, we employ a unique data set[3] that has not been used previously in this literature. In emerging market economies, fixed exchange rate systems combined with foreign trade restrictions, capital controls, high inflation and external deficits have led to the development of thriving black markets for foreign exchange (see, Agenor,

[3] Very few studies have investigated the PPP hypothesis using this major source of information from emerging market economies, and these cover only a small number of countries (typically 1-7) using data up to the late 1980s (e.g. Phylaktis and Kassimatis, 1994; Baghestani, 1997; Luintel, 2000; and Diamandis, 2003). But none of these studies has investigated the issue of half-lives of PPP deviations.

1992; Kiguel and O'Connell, 1999). In most of these countries, black currency markets have a long tradition, are supported by governments and their volume of transactions is very large. So these black markets play an important role in the economies of emerging market countries and it might be argued that the black market exchange rates reflect the true value of domestic currency much better than the official exchange rates.

The rest of the paper is organized as follows: Section 2 describes the econometric methodologies employed, while section 3 explains the data. Section 4 presents the empirical results on half-lives. Finally, conclusions are reported in Section 5.

2. Modeling persistence

2.1 Least squares approach

Consider the following AR(1) model for the real exchange rate, q:

$$q_t = a + bq_{t-1} + u_t \tag{1}$$

with $u_t \sim iidN(0,\sigma^2)$, initial value $q_0 \sim N(0,\sigma^2/(1-b^2))$ and the AR parameter lying within the interval (-1,1). Define b_{LS} as the least square estimator of b and note that the half-life is calculated as $\ln(0.5)/\ln(b)$.

An alternative way of obtaining point estimates of b_{LS} consists of using bootstrap methods. Furthermore, by extending such a methodology one can also construct confidence intervals. For example, suppose that b_{LS} is a consistent estimator of b and $b_{LS}^{\oplus}$ is the bootstrap estimator. Assume also that B is the number of bootstrap replications. We generate the bootstrap distribution of $b_{LS}^{\oplus}$ by drawing repeated samples with replacement from $(q_1, q_2, ... q_T)$ - i.e. by applying the residual based re-sampling method discussed in Li and Maddala (1996)[4].

We use the bootstrap sample to obtain $b_{LS}^{\oplus}$. Finally, repeating this B times we obtain the bootstrap distribution. A two-sided (100-2α) confidence interval for b is $b_{LS} - L_{1-\alpha}^{*} . b_{LS} + L_{\alpha}^{*}$ where L_{α}^{*} is the 100α percentile. We construct confidence intervals using the methodology described above. One major problem of the bootstrap approach is that the bootstrap confidence interval is asymptotically invalid in the case where $b_{LS} = 1$ and $a = 0$ (see Basawa *et al.,* 1991).

Another major drawback with the methodology described above is that it ignores serial correlation. Of course, the presence of significant serial correlation should be taken into account by using higher order AR processes. In this case, model (1) is replaced by the AR(*p*) model:

$$q_t = a + bq_{t-1} + \sum_{i=1}^{p} \theta_i \Delta q_{t-1} + u_t \tag{2}$$

where if $p > 0$, the distribution of the LS estimator of *b* depends not only on the true value of *b*, but also on the true values of the θ_i terms in (2). It should be noted that half-lives calculated directly from an estimate of b assume shocks to real exchange rates decay monotonically. MP point out that while this is appropriate in the case of an AR(1) model, it is no longer so in the case of an AR(*p*) model where shocks do not decay at a constant rate. Following Inoue and Kilian (2002), MP suggest obtaining

[4] It should be noted that this is a non-parametric bootstrap method and contrasts with MP who use a parametric bootstrap approach based on generating artificial time series from an i.i.d. normal distribution. In fact MP point out that research on the sensitivity of their results with respect to departures from normality is needed. Employing a non-parametric procedure avoids such issues.

point estimates of half-lives directly from the relevant impulse response function (IRF).

The levels representation of (2) can be expressed as:

$$q_t = a + \sum_{i=1}^{p+1} \phi_i q_{t-i} + u_t \tag{3}$$

where $\phi_1 = b + \theta_1$ and $\phi_l = \theta_l - \theta_{l-1}$ for $l = 2,...,p$ and $\phi_{p+1} = -\theta_p$. Inoue and Kilian (2002) show that although the bootstrap is not valid for the unit root parameter b in (2) when $b = 1$ and $a = 0$, nevertheless, it is asymptotically valid for the slope parameters ϕ_i. Thus, half lives and their corresponding confidence intervals constructed from an IRF (i.e. from the ϕ_i coefficients), are also asymptotically valid.

2.2 Median unbiased approach

It is well known that, in small samples, b_{LS} is biased downward with the size of this bias increasing for large values of b (see, for example, Andrews, 1993). The problem of small sample bias of b_{LS} is of particular relevance, especially in empirical works dealing with half-lives, since the calculation of the latter relies on the biased parameter. Different methodologies have been proposed in the literature. For example, it is well known that the jackknife estimator of b is mean unbiased of order $1/T$ for $T \rightarrow \infty$. One problem with this estimator is that it is not clear if the result holds for values of the AR parameter lying in the region of a unit root.

Another way of approaching the problem is by using median unbiased (MU) estimation. Following Andrews (1993) we define the median z of a random variable X as:

$$P(X \geq z) \geq 1/2 \text{ and } P(X \geq z) \leq 1/2 \tag{4}$$

Assume that b^* is an estimator of b. By definition b^* is an MU estimator of b if the true parameter b is a median of b^* for each b in the parameter space. In other words b^* is a MU estimator if the distance between b^* and the true parameter being estimated is on average the same as that from any other value in the parameter space. Suppose there are two candidates as population parameters b and $b^{'}$ then $E_b \mid b^* - b \mid \leq E_b \mid b^* - b^{'} \mid$ for all b and $b^{'}$ in the parameter space. In this way, the probability that b^* will overestimate the true parameter is the same to that it will underestimate it. Therefore, b_U^*, the exact MU estimator of b in (1), is given by:

$$b_U^* = 1 \qquad \text{if } b_{LS} > z(1)$$

$$b_U^* = z^{-1}(b_{LS}) \qquad \text{if } z(-1) < b_{LS} \leq z(1)$$

$$b_U^* = -1 \qquad \text{if } b_{LS} \leq z(-1) \tag{5}$$

where $z(-1) = \lim_{b \to -1} z(b)$ and z^{-1} is the inverse function of $z(.) = z_T(.)$ so that $z^{-1}(z(b)) = b_{LS}$. In other words, if $b_{LS} = 0.85$, this is not used as the estimate of b. Instead, to calculate the MU estimate, we locate the value of b that generates the LS estimator to have a median of 0.85.

Appendix 1 shows quantiles of the median function $z(b)$ for different values of $b \in$ {-1,1} and significance levels for our particular sample size obtained by Monte Carlo simulation as in Andrews (1993). The appendix has been constructed using a simple AR(1) model as a DGP and increasing the value of b by 0.01. The number of Monte Carlo replicates was set to 3000. In what follows, we report a simple example demonstrating how to use the tables in the Appendix 1. Suppose that $z(1) = 0.9968$, then any values of $b_{LS} \geq 0.9968$ corresponds to $b_U^* = 1$. In the same way we calculate b_U^* when $z(-1)$. For example, if $z(-1) = -0.9955$, then, for any values of $b_{LS} \leq -0.9955$, $b_U^* = -1$. Finally if $-0.9955 \leq b_{LS} \leq 0.9968$ one finds b_U^* by looking at the 0.5 quantile column as follows: $b_{LS} = 0.7482$, then $b_U^* = 0.75$. For values of b_{LS} not contained in the 0.5 quantile column, interpolation is required.

Again using the same approach an in Andrews (1993), we can also construct confidence intervals for the median unbiased estimator. The $100(1-p)\%$ confidence interval can be constructed as follows:

$$c_u^L = 1 \qquad \text{if } b_{LS} > lu(1)$$

$$c_u^L = lu^{-1}(b_{LS}) \quad \text{if } lu(-1) < b_{LS} \leq lu(1)$$

$$c_u^L = -1 \qquad \text{if } b_{LS} \leq lu(-1) \tag{6}$$

where c_u^L is the lower confidence interval and $lu(.)$ is the upper quantile. Employing the same approach we can also construct upper confidence interval as follows:

$$c_u^u = 1 \qquad \text{if } b_{LS} > ll(1)$$

$$c_u^u = ll^{-1}(b_{LS}) \quad \text{if } ll(-1) < b_{LS} \leq ll(1)$$

$$c_u^u = -1 \qquad \text{if } b_{LS} \leq ll(-1) \tag{7}$$

where c_u^u is the upper confidence interval and $ll(.)$ the lower quantile. For example, consider the two-sided 95% confidence interval for b_U^*. Assuming that $b_{LS} = 0.9943$ then, using the 0.975 quantile column, $Il = 0.98$, while $lu = 1$, using the 0.025 quantile column.

2.2 Approximate median unbiased approach

The major drawback with the exact MU methodology described above is that it is only appropriate when the data is well represented by an AR(1) model. Of course, the presence of serial correlation should be taken into account by using higher order processes. In this case, model (1) is replaced by model (2). As the true values of the θ_i terms in (2) are unknown in practice, the bias correction method in (5) cannot be applied. Instead, Andrews and Chen (1994) posit an iterative procedure that generates an approximately median unbiased [5] (AMU) estimate, b_{AMU}. Firstly, estimate (2) using LS and, treating the estimated values of the θ_i terms as true, compute the MU estimator of *b*, $b_{1,AMU}$, using (5). Secondly, conditional on $b_{1,AMU}$, generate a second set of estimates for θ_i and based on these compute second MU estimator of *b*, $b_{2,AMU}$. The final AMU estimate, b_{AMU}, is achieved when convergence is reached. Approximate confidence intervals can be obtained in an analogous manner.

3. Data

We employ monthly data on the black market exchange rates for a panel of thirty-six emerging market countries over the period 1973M1-1998M12. The US Dollar is used as numeraire currency. The black market exchange rates are obtained from *Pick's World Currency Yearbook* (various publications). The consumer price index (CPI) is used as the price index. The countries in our panel are highly heterogeneous, varying from poor developing to semi-industrial countries, with different growth experiences and levels of per capita income.

[5] Andrews and Chen (1994) show Monte Carlo evidence demonstrating the accuracy of the AMU method.

4. Empirical results

4.1 Least squares estimates

To establish a benchmark we begin by estimating least squares point and 95% bootstrap confidence interval estimates of half-lives (in years) from an AR(1) model. These results are reported in Table 1.

[Table 1 around here]

The median half-life is 2.42 years and thus falls outside Rogoff's 3-5 year consensus[6]. It is interesting to note that MP, using 20 OECD countries and post-1973 quarterly data, report a median value of half lives for their point estimate that is very similar to ours (i.e. 2.52 years). We support our point estimates of half-lives with interval estimates obtained using bootstrap. We use the non-parametric bootstrap method described in Section 3 to construct a 95% confidence interval with B set equal to 3000. The median lower and upper bounds of our confidence interval are 0.68 and 4.41 years respectively. Again this is similar to MP where the lower bound is around 0.64 and the upper bound is 4.95 years.

To account for serial correlation in the data, LS estimates for relevant ADF regressions are given in Table 2.

[Table 2 around here]

The number of lags in the ADF regressions are selected by using the general-to-specific lag[7] selection criterion suggested by Ng and Perron (1995). For comparison purposes, we calculate point estimates and confidence intervals of half-lives by using two different methods: first, point estimates of the unit root parameter

[6] We focus on the median rather than the average half-lives, because the average can provide a distorted picture due to the presence of outliers. Murray and Papell (2002) adopt the same approach.
[7] Maximum lag set to 8.

in the AR(p) model; second, the IRF as in MP. In both cases we construct confidence intervals using the non-parametric bootstrap method described in Section 2.

The median point estimate of half-lives is 2.51 years, which is similar to that obtained from the DF test on the AR(1) model. In MP the median point estimate for OECD countries is 1.77. The median lower bound is 0.64 which is slightly lower than the median lower bound in the case of an AR(1) model. On the other hand the upper bound is now 5.22 years, which is much higher than the DF-based estimate. In MP, the median lower bound of 0.64 is very close to our estimate, while the median upper bound is 3.12.

However, these estimates are of little use since they are based on the assumption that shocks to the real exchange rate decay monotonically. But shocks to an AR(p) model will not in general decay at a constant rate (MP make the same point). Furthermore, Inoue and Kilian (2002) show that although the bootstrap method is asymptotically invalid for the unit root parameter in the AR(p) model, it is valid for the individual slopes in the level representation (6). Hence, in what follows we shall use the IRF based on these slope estimates to calculate half-lives, and the non-parametric bootstrap method to construct their respective confidence intervals. The median of half-lives calculated from the IRF is 3.24 years, which is much higher than that obtained from the DF method. MP report median estimates of half-life of 2.15 for OECD countries. The median lower and upper bounds of our confidence interval are 1.80 and 4.17 years respectively. This is wider than MP where the lower bound is around 1.14 and the upper bound is 4.04 years.

4.2 Median unbiased estimates

Exact median unbiased point estimates, reported in Table 3, are generally higher than the ones presented in Table 1, which probably reflects the correction for LS small sample bias.

[Table 3 around here]

The median point estimate of half-life is 2.86. Contrary to the findings reported in MP, we do not note a very significant increase in the median point estimate. In fact, MP report a median estimate of 5.69 that is much larger than their least squares estimate. Our median lower bound is 1.41 and the upper bound is infinite. These bounds are in line with MP, though they find an infinite upper bound in all OECD countries while our results show a finite upper bound for eight emerging market countries.

Table 4 reports point estimates and 95% confidence intervals employing approximately median unbiased estimation, which corrects both for the LS small sample bias and serial correlation.

[Table 4 around here]

The median point estimate of IRF calculated half-life is 2.80 years. As with the MU estimates, we only report a slight increase in the median point estimate over the DF estimate. On the other hand, this is lower than both the MU and ADF median point estimates. MP report a median estimate of 3.07 that again is much larger than their LS estimate. Our median lower bound is 0.74 and the upper bound is infinite. MP have a lower bound of 1.24 and an infinite upper bound.

There are several points to note here. Firstly, results confirm that there exists *different* behavior in the degree of persistence of the real exchange rate in OECD countries and emerging market economies. For example, individual country estimates (based on our preferred AMU specification and the impulse response function) suggest a half-life below two years in twelve countries out of thirty-six[8], and a half-life below three years in twenty countries. Furthermore, only one country [Morocco] presents a median half-life that falls within Rogoff's range of 3-5 years. Our finding is in line with Cheung and Lai (2000b) who discover average half-lives for developing countries less than three years and in many cases in the range 0-2 years. Strikingly, it is also interesting to note that the lower bound is below two years for 31 countries. And although the upper bound is [∞] for 28 countries, this leaves 8 countries with finite upper bounds below 7 years. By comparison only four of the twenty countries considered by Murray and Papell had a finite upper bound and none were estimated to be below 9 years.

Secondly, the results from point estimates and lower bound estimates of half-lives are consistent with the view that deviations from PPP in emerging market economies can be explained by financial shocks and nominal rigidities.

Thirdly, when the upper bound of interval estimates are considered, these suggest that for 28 countries (i.e. those with an upper bound of [∞]) estimates of half-lives are consistent with anything, even unit root processes[9] where there is no convergence to PPP. Given such uninformative intervals this tempers the conclusions in the paragraph above and suggests that no conclusions can be drawn in such cases as to the persistence of PPP. The countries considered by MP, as already mentioned, mainly have infinite upper bounds and thus, in terms of the OECD context they conclude that it cannot even be shown whether the PPP paradigm exists.

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[8] In MP there is no OECD country for which point estimates indicate half-life below two years (for their preferred AMU specification).

Fourthly, given the eight emerging market countries in our sample with finite upper bounds, we can, in contrast to MP, make some inference as to the nature of the PPP paradigm. Given that the PPP paradigm involves the speed of adjustment to a long-run equilibrium, it makes sense to predicate any conclusions on the PPP persistence paradigm only where this equilibrium is detected. Table 5 isolates these cases from Table 4.

[Table 5 around here]

The median point estimate of IRF calculated half-life is 1.01 years and the average half-live is 1.08 years. Our median lower bound is 0.38 and the upper bound is 2.59. Such results show that if we restrict half-life estimation to those cases where we are statistically sure that long-run PPP exists then reversion to that equilibrium is within the region that can be explained by nominal rigidities. In other words, when it is established that mean reversion holds, strong evidence against the PPP persistence puzzle materializes in emerging markets!

5. Conclusions

Previous studies on the persistence of deviations from PPP, using data for industrial economies, have generally obtained half-lives falling within Rogoff's (1996) 'consensus range' of 3 to 5 years. Given that the length of these deviations is considered too long to be explained by nominal rigidities, the so-called PPP puzzle or paradigm emerges. However, Murray and Papell (2002) have questioned the results of the literature, arguing econometric issues such as small sample bias, serial correlation and confidence intervals have generally been ignored. Employing exact and

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[9] This is consistent with the finding by Cerrato and Sarantis (2003). Using the same panel of data and various panel unit root tests, the authors fail to reject the unit root

approximate median unbiased estimation they show that 95% confidence intervals for the degree of persistence of the real exchange rate are very wide. For example, although many lower bounds are less than 2 years, upper bounds commonly equal infinity. Murray and Papell (2002) thus suggest given the uninformative nature of the results that there is no evidence that a puzzle even exists!

This paper provides an extensive analysis of the PPP persistence issue by examining a unique data set of black market real exchange rates for thirty-six heterogeneous emerging market economies. Previous studies have posited that emerging country exchange rates may be less persistent than those from industrial countries. As such, this may allow narrower confidence intervals to be obtained when employing estimation techniques that address the econometric problems highlighted by Murray and Papell (2002). We calculate the half-life point estimates from the impulse response function and construct bootstrap confidence intervals for the half-lives.

Based on our preferred specification of the approximate median unbiased estimation, a median point estimate for half-lives of 2.80 years is obtained for the 36 emerging market economies. This is below both Rogoff's (1996) 'consensus' of 3-5 year half-lives and the estimate reported by Murray and Papell (2002) for the OECD countries. The 95% confidence interval has a median lower bound of 0.74 years and an upper bound of infinity. At an aggregated level therefore results for emerging countries are shown to be as uninformative as those from industrial countries. However, at a disaggregated level, it is found that far more finite upper bounds are found for individual emerging countries than previous studies indicate for industrial

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hypothesis for the full panel of emerging market economies.

countries. These results confirm the hypothesis put forward in previous literature that emerging countries have relatively less persistent real exchange rates.

Finally, we note that it is only when a finite upper bound is obtained that statistically evidence of convergence to long-run PPP exists. Given that the PPP puzzle involves the speed of adjustment to this equilibrium, we suggest that unlike all previous studies, inference on the PPP persistence issue should only be gleaned when mean reversion is detected. For those relevant countries the median point estimate for half-life is 1.01 years, the lower bound is 0.38 years and the upper bound is only 2.59 years. Put simply, when it is established that PPP holds, the PPP persistence paradigm does not.

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Table 1: OLS half-lives in DF regressions

Country	b_{LS}	95% CI	HL_{LS}	95% CI
Algeria	0.9828	[0.926 0.991]	3.33	[0.750 6.390]
Argentina	0.9612	[0.899 0.919]	1.46	[0.540 0.680]
Bolivia	0.9189	[0.849 0.945]	0.68	[0.350 1.000]
Brazil	0.9495	[0.865 0.970]	1.11	[0.400 1.900]
Chile	0.9644	[0.900 0.979]	1.59	[0.550 2.720]
Colombia	0.9929	[0.936 0.997]	8.11	[0.870 19.20]
C.Rica	0.9852	[0.928 0.992]	3.87	[0.770 7.200]
D.Republic	0.9828	[0.927 0.991]	3.33	[0.760 6.390]
Ecuador	0.9886	[0.931 0.995]	5.04	[0.810 11.50]
Egypt	0.9356	[0.870 0.961]	0.87	[0.410 1.450]
El Salvador	0.9789	[0.921 0.988]	2.71	[0.700 4.780]
Ethiopia	0.9672	[0.910 0.982]	1.73	[0.610 3.180]
Hungary	0.9759	[0.916 0.987]	2.37	[0.660 4.410]
Ghana	0.8143	[0.730 0.860]	0.28	[0.180 0.380]
India	0.9915	[0.935 0.996]	6.77	[0.860 14.40]
Indonesia	0.9919	[0.936 0.996]	7.11	[0.870 14.40]
Kenya	0.9651	[0.903 0.981]	1.63	[0.570 3.010]
Korea	0.4769	[0.397 0.543]	0.08	[0.060 0.090]
Kuwait	0.9764	[0.918 0.987]	2.42	[0.680 4.410]
Malaysia	0.9988	[0.943 1.000]	48.11	[0.980 ∞]
Mexico	0.9758	[0.922 0.987]	2.36	[0.710 4.410]
Morocco	0.9849	[0.926 0.992]	3.8	[0.750 7.190]
Nepal	0.9804	[0.921 0.989]	2.92	[0.700 5.220]
Nigeria	0.9823	[0.926 0.991]	3.23	[0.750 6.390]
Pakistan	0.9892	[0.932 0.995]	5.32	[0.820 11.50]
Paraguay	0.9922	[0.944 0.996]	7.38	[1.000 14.40]
Philippines	0.9494	[0.887 0.968]	1.11	[0.480 1.780]
Poland	0.9738	[0.846 0.988]	2.18	[0.350 4.790]
Singapore	0.9722	[0.920 0.984]	2.05	[0.690 3.580]
S.Africa	0.922	[0.855 0.951]	0.71	[0.370 1.500]
S.Lanka	0.9789	[0.919 0.989]	2.71	[0.680 5.220]
Thailand	0.9651	[0.904 0.979]	1.63	[0.570 2.720]
Tunisia	1.0000	[0.955 1.000]	∞	[1.25 ∞]
Turkey	0.9677	[0.906 0.982]	1.76	[0.590 3.180]
Uruguay	0.9779	[0.919 0.988]	2.58	[0.680 4.780]
Venezuela	0.9883	[0.932 0.994]	4.91	[0.820 9.600]

Note: 95% CI represents the 95% bootstrap confidence interval. HL_{LS} indicates half-lives based on OLS estimates of b.

Table 2: OLS half-lives in ADF regressions

Country	k	b_{LS}	95% CI	HL_{LS}	95% CI	HL_{IRF}	95% CI
Algeria	5	0.9917	[0.932 0.997]	6.930	[0.82 19.23]	7.01	[2.80 9.80]
Argentina	4	0.9729	[0.909 0.985]	2.100	[0.61 3.82]	6.60	[6.30 6.90]
Bolivia	7	0.9507	[0.871 0.971]	1.140	[0.42 1.96]	3.30	[3.00 3.80]
Brazil	3	0.9505	[0.859 0.971]	1.138	[0.38 1.96]	4.90	[4.80 6.30]
Chile	5	0.9368	[0.861 0.961]	0.880	[0.39 1.45]	1.80	[1.50 2.00]
Colombia	6	0.9921	[0.929 0.997]	7.280	[0.78 19.23]	5.50	[5.00 5.70]
C.Rica	6	0.9828	[0.916 0.991]	3.300	[0.66 6.39]	7.80	[7.60 7.90]
D.Republic	6	0.9777	[0.913 0.989]	2.560	[0.63 5.22]	3.50	[2.10 6.20]
Ecuador	4	0.9845	[0.923 0.993]	3.700	[0.72 8.22]	0.92	[0.58 1.40]
Egypt	5	0.9060	[0.832 0.936]	0.580	[0.31 0.87]	1.70	[1.50 1.90]
El Salvador	1	0.9889	[0.931 0.994]	5.170	[0.81 9.60]	6.50	[1.63 9.90]
Ethiopia	4	0.9633	[0.896 0.979]	1.540	[0.53 2.72]	0.80	[0.50 1.60]
Hungary	6	0.9821	[0.916 0.992]	3.200	[1.45 7.19]	2.80	[1.80 7.90]
Ghana	6	0.8168	[0.693 0.875]	0.280	[0.16 0.43]	1.30	[0.80 1.90]
India	0	0.9915	[0.935 0.996]	6.770	[0.86 14.4]	6.70	[0.86 14.40]
Indonesia	5	0.9869	[0.923 0.993]	4.380	[0.72 8.22]	5.08	[4.80 6.25]
Kenya	7	0.9568	[0.88 0.975]	1.310	[0.45 2.28]	3.00	[2.80 3.25]
Korea	7	0.4578	[0.25 0.598]	0.070	[0.04 0.11]	0.92	[0.75 1.16]
Kuwait	4	0.9816	[0.919 0.991]	3.110	[0.68 6.38]	4.80	[2.17 6.75]
Malaysia	7	0.9939	[0.933 0.999]	9.400	[0.83 57.7]	∞	[2.20 ∞]
Mexico	6	0.9740	[0.909 0.986]	2.190	[0.61 4.09]	3.00	[2.75 4.17]
Morocco	2	0.9874	[0.929 0.994]	5.560	[0.78 9.59]	2.10	[1.75 2.10]
Nepal	5	0.9801	[0.913 0.989]	2.870	[0.63 5.22]	3.17	[3.00 3.42]
Nigeria	1	0.9852	[0.925 0.993]	3.870	[0.74 8.22]	0.50	[0.42]
Pakistan	5	1.0001	[0.943 1.003]	∞	[0.98 ∞]	∞	[4.8 ∞]
Paraguay	0	0.9922	[0.944 0.996]	7.380	[1.01 14.4]	7.38	[1.00 14.4]
Philippines	1	0.9550	[0.894 0.973]	1.260	[0.51 2.11]	1.050	[0.97 1.13]
Poland	0	0.9738	[0.846 0.988]	2.180	[0.34 4.78]	2.18	[0.35 4.75]
Singapore	7	0.9768	[0.908 0.988]	2.460	[0.60 4.78]	3.92	[3.50 4.17]
S.Africa	2	0.9374	[0.867 0.959]	0.890	[0.40 1.38]	1.60	[0.80 2.90]
S.Lanka	3	0.9759	[0.916 0.987]	2.380	[0.66 4.41]	1.50	[1.25 1.60]
Thailand	1	0.9705	[0.909 0.984]	1.930	[0.61 3.58]	0.33	[0.25 0.67]
Tunisia	6	1.0057	[0.953 1.008]	∞	[1.20 ∞]	∞	[2.60 ∞]
Turkey	4	0.9739	[0.911 0.986]	2.180	[0.62 4.09]	3.40	[0.89 4.90]
Uruguay	1	0.8685	[0.924 0.991]	0.410	[0.73 6.38]	1.970	[1.80 2.05]
Venezuela	3	0.9860	[0.927 0.993]	4.110	[0.76 8.22]	3.75	[3.58 4.17]

Note: HL_{LS} and HL_{IRF} represent, respectively, point estimates of half-lives (in years) from OLS estimates of b and the impulse response function. Their respective 95% bootstrap confidence intervals are presented in columns six and eight.

Table 3: Exact median unbiased half-lives in DF regressions

Country	b_U^*	95% CI	HL_{MU}	95% CI
Algeria	0.986	[0.970 1.000]	4.097	[1.900 ∞]
Argentina	0.966	[0.940 1.000]	1.670	[0.930 ∞]
Bolivia	0.921	[0.880 0.966]	0.702	[0.450 1.670]
Brazil	0.956	[0.916 0.986]	1.284	[0.660 4.100]
Chile	0.971	[0.946 1.000]	1.963	[1.040 ∞]
Colombia	0.995	[0.986 1.000]	11.524	[4.100 ∞]
C.Rica	0.991	[0.976 1.000]	6.389	[2.380 ∞]
D.Republic	0.991	[0.976 1.000]	6.389	[2.380 ∞]
Ecuador	0.993	[0.980 1.000]	8.223	[2.860 ∞]
Egypt	0.941	[0.906 0.986]	0.950	[0.590 4.100]
El Salvador	0.985	[0.966 1.000]	3.822	[1.670 ∞]
Ethiopia	0.970	[0.950 1.000]	1.896	[1.130 ∞]
Hungary	0.976	[0.956 1.000]	2.378	[1.280 ∞]
Ghana	0.816	[0.756 0.886]	0.284	[0.210 0.480]
India	0.994	[0.986 1.000]	9.598	[4.100 ∞]
Indonesia	0.994	[0.986 1.000]	9.598	[4.100 ∞]
Kenya	0.971	[0.946 1.000]	1.963	[1.040 ∞]
Korea	0.477	[0.386 0.576]	0.078	[0.060 0.105]
Kuwait	0.981	[0.966 1.000]	3.011	[1.670 ∞]
Malaysia	1.000	[0.996 1.000]	∞	[14.40 ∞]
Mexico	0.976	[0.926 0.996]	2.378	[0.750 14.41]
Morocco	0.986	[0.970 1.000]	4.097	[1.900 ∞]
Nepal	0.986	[0.970 1.000]	4.097	[1.900 ∞]
Nigeria	0.986	[0.970 1.000]	4.097	[1.900 ∞]
Pakistan	0.991	[0.976 1.000]	6.389	[2.380 ∞]
Paraguay	0.993	[0.980 1.000]	8.223	[2.860 ∞]
Philippines	0.950	[0.916 0.990]	1.126	[0.660 5.750]
Poland	0.976	[0.950 1.000]	2.378	[1.130 ∞]
Singapore	0.976	[0.956 1.000]	2.378	[1.280 ∞]
S.Africa	0.926	[0.886 0.976]	0.751	[0.480 2.380]
S.Lanka	0.980	[0.960 1.000]	2.859	[1.410 ∞]
Thailand	0.970	[0.946 1.000]	1.896	[1.040 ∞]
Tunisia	1.004	[0.997 1.000]	∞	[19.22 ∞]
Turkey	0.970	[0.946 1.000]	1.896	[1.040 ∞]
Uruguay	0.980	[0.960 1.000]	2.859	[1.410 ∞]
Venezuela	0.994	[0.986 1.000]	9.598	[4.100 ∞]

Note: 95% CI represents the 95% bootstrap confidence interval. HL_{MU} indicates half-lives based on exact median unbiased estimates of b.

Table 4: Approximately median unbiased half-lives in ADF regressions

Country	k	b_{LS}	95% CI	HL_{LS}	95% CI	HL_{IRF}	95% CI
Algeria	5	1.00	[0.88 1.00]	∞	[0.45 ∞]	∞	[2.40 ∞]
Argentina	4	1.00	[0.75 1.00]	∞	[0.20 ∞]	∞	[1.80 ∞]
Bolivia	7	0.92	[0.75 1.00]	0.69	[0.20 ∞]	0.13	[0.08 ∞]
Brazil	3	0.96	[0.89 0.99]	1.41	[0.50 5.7]	1.13	[0.38 5.30]
Chile	5	0.97	[0.94 0.99]	1.90	[0.93 ∞]	1.98	[0.96 6.13]
Colombia	6	1.00	[0.96 1.00]	∞	[1.45 ∞]	∞	[1.55 ∞]
C.Rica	6	0.99	[0.95 1.00]	5.75	[1.13 ∞]	6.22	[1.47 ∞]
D.Republic	6	0.99	[0.95 1.00]	5.75	[1.13 ∞]	6.22	[0.76 ∞]
Ecuador	4	1.00	[0.97 1.00]	∞	[1.90 ∞]	∞	[2.13 ∞]
Egypt	5	0.94	[0.89 0.98]	0.93	[0.50 ∞]	0.97	[0.38 2.38]
El Salvador	1	0.98	[0.92 1.00]	2.86	[0.69 ∞]	2.55	[0.55 ∞]
Ethiopia	4	0.97	[0.93 1.00]	1.90	[0.80 ∞]	2.05	[0.97 ∞]
Hungary	6	0.98	[0.91 1.00]	2.86	[0.61 ∞]	1.88	[0.30 ∞]
Ghana	6	0.82	[0.75 0.88]	0.29	[0.20 ∞]	0.38	[0.22 0.47]
India	0	0.99	[0.99 1.00]	9.60	[4.10 ∞]	9.60	[4.10 ∞]
Indonesia	5	1.00	[0.97 1.00]	∞	[1.90 ∞]	∞	[16.3 ∞]
Kenya	7	0.97	[0.93 0.99]	1.90	[0.79 ∞]	2.30	[1.13 6.38]
Korea	7	0.75	[0.70 0.77]	0.20	[0.16 ∞]	0.22	[0.05 0.40]
Kuwait	4	0.98	[0.93 1.00]	2.86	[0.80 ∞]	2.55	[0.22 ∞]
Malaysia	7	1.00	[0.96 1.00]	∞	[1.41 ∞]	∞	[1.72 ∞]
Mexico	6	0.98	[0.90 1.00]	2.86	[0.55 ∞]	1.50	[0.38 ∞]
Morocco	2	0.99	[0.92 1.00]	5.75	[0.69 ∞]	4.88	[0.55 ∞]
Nepal	5	0.99	[0.94 1.00]	5.75	[0.93 ∞]	5.30	[0.88 ∞]
Nigeria	1	0.99	[0.94 1.00]	5.75	[0.93 ∞]	5.05	[0.80 ∞]
Pakistan	5	0.99	[0.93 1.00]	5.75	[0.80 ∞]	4.88	[0.22 ∞]
Paraguay	0	0.99	[0.98 1.00]	8.22	[2.86 ∞]	8.22	[2.86 ∞]
Philippines	1	0.95	[0.89 0.98]	1.13	[0.50 ∞]	1.05	[0.47 2.80]
Poland	0	0.98	[0.95 1.00]	2.38	[1.13 ∞]	2.38	[1.13 ∞]
Singapore	7	0.98	[0.93 1.00]	2.86	[0.80 ∞]	2.97	[0.97 ∞]
S.Africa	2	0.92	[0.86 0.96]	0.69	[0.38 1.4]	0.63	[0.3 1.38]
S.Lanka	3	0.98	[0.93 1.00]	2.86	[0.80 ∞]	2.97	[0.88 ∞]
Thailand	1	0.97	[0.92 1.00]	1.90	[0.69 ∞]	1.75	[0.67 ∞]
Tunisia	6	1.00	[0.95 1.00]	∞	[1.13 ∞]	∞	[0.47 ∞]
Turkey	4	0.97	[0.91 1.00]	1.90	[0.61 ∞]	1.55	[0.30 ∞]
Uruguay	1	0.98	[0.93 1.00]	2.86	[0.80 ∞]	2.63	[0.72 ∞]
Venezuela	3	1.00	[0.96 1.00]	∞	[1.41 ∞]	∞	[0.17 ∞]

Note: HL_{LS} and HL_{IRF} represent, respectively, point estimates of half-lives (in years) from OLS estimates of b and the impulse response function. Their respective 95% bootstrap confidence intervals are presented in columns six and eight.

Table 5: Selected approximately median unbiased half-lives in ADF regressions

Country	k	b_{LS}	95% CI	HL_{LS}	95% CI	HL_{IRF}	95% CI
Brazil	3	0.96	[0.89 0.99]	1.41	[0.50 5.75]	1.13	[0.38 5.30]
Chile	5	0.97	[0.94 0.99]	1.90	[0.93 5.75]	1.98	[0.96 6.13]
Egypt	5	0.94	[0.89 0.98]	0.93	[0.50 2.86]	0.97	[0.38 2.38]
Ghana	6	0.82	[0.75 0.88]	0.29	[0.20 0.45]	0.38	[0.22 0.47]
Kenya	7	0.97	[0.93 0.99]	1.90	[0.80 5.75]	2.30	[1.13 6.38]
Korea	7	0.75	[0.70 0.77]	0.20	[0.16 0.22]	0.22	[0.05 0.40]
Philippines	1	0.95	[0.89 0.98]	1.13	[0.50 2.86]	1.05	[0.47 2.80]
S.Africa	2	0.92	[0.86 0.96]	0.69	[0.38 1.41]	0.63	[0.30 1.38]

Note: HL_{LS} and HL_{IRF} represent, respectively, point estimates of half-lives (in years) from OLS estimates of b and the impulse response function. Their respective 95% bootstrap confidence intervals are presented in columns six and eight.

Appendix

Quantiles of the Median Function $z(b_{LS})$ for T+1=312

b_{LS}	0.025	0.05	0.5	0.95	0.975
-0.99	-0.9988	-0.9975	-0.9869	-0.9589	-0.951
-0.98	-0.9931	-0.9914	-0.9769	-0.946	-0.9367
-0.97	-0.9869	-0.9847	-0.9668	-0.9321	-0.9227
-0.96	-0.9806	-0.9778	-0.9571	-0.9195	-0.9101
-0.95	-0.9738	-0.9706	-0.9469	-0.9066	-0.8968
-0.94	-0.9667	-0.9632	-0.9369	-0.8948	-0.8845
-0.93	-0.9594	-0.9554	-0.927	-0.8822	-0.8712
-0.92	-0.9525	-0.9478	-0.917	-0.8708	-0.8592
-0.91	-0.9449	-0.9401	-0.907	-0.8588	-0.8473
-0.9	-0.9364	-0.9314	-0.8973	-0.8471	-0.8359
-0.89	-0.9288	-0.9235	-0.8871	-0.8347	-0.8232
-0.88	-0.9215	-0.9159	-0.8768	-0.8228	-0.8106
-0.87	-0.9133	-0.9073	-0.8672	-0.813	-0.8
-0.86	-0.9051	-0.8987	-0.857	-0.8016	-0.7887
-0.85	-0.8963	-0.8897	-0.8472	-0.7904	-0.7768
-0.84	-0.8898	-0.8818	-0.8375	-0.7785	-0.7667
-0.83	-0.8814	-0.8735	-0.8269	-0.7665	-0.7536
-0.82	-0.8708	-0.864	-0.8171	-0.7554	-0.742
-0.81	-0.8636	-0.8555	-0.8079	-0.7456	-0.7325
-0.8	-0.855	-0.8467	-0.7968	-0.7336	-0.7194
-0.79	-0.8471	-0.8385	-0.7868	-0.7209	-0.7074
-0.78	-0.85	-0.8402	-0.7759	-0.691	-0.6738
-0.77	-0.8416	-0.8322	-0.7664	-0.6795	-0.6611
-0.76	-0.8336	-0.8232	-0.7564	-0.6705	-0.6503
-0.75	-0.8256	-0.8142	-0.7457	-0.6586	-0.6388
-0.74	-0.8161	-0.8057	-0.7366	-0.6477	-0.6286
-0.73	-0.8088	-0.7967	-0.7265	-0.637	-0.6178
-0.72	-0.8001	-0.7881	-0.7172	-0.6272	-0.6074
-0.71	-0.7919	-0.7806	-0.7067	-0.615	-0.5944
-0.7	-0.7831	-0.7707	-0.6965	-0.6017	-0.5838
-0.69	-0.7749	-0.7625	-0.6865	-0.593	-0.5733
-0.68	-0.7666	-0.7536	-0.6763	-0.5813	-0.5603
-0.67	-0.7561	-0.7437	-0.666	-0.5692	-0.5483
-0.66	-0.7493	-0.7362	-0.6573	-0.5599	-0.5387
-0.65	-0.7412	-0.7271	-0.6471	-0.5483	-0.5272
-0.64	-0.7316	-0.7175	-0.6363	-0.5383	-0.5177
-0.63	-0.7235	-0.71	-0.6269	-0.5269	-0.5064
-0.62	-0.7144	-0.7015	-0.6165	-0.5192	-0.4983
-0.61	-0.7066	-0.6927	-0.6063	-0.5056	-0.4844
-0.6	-0.6973	-0.6832	-0.5975	-0.4964	-0.4756
-0.59	-0.688	-0.6736	-0.5868	-0.4862	-0.4655
-0.58	-0.6806	-0.661	-0.5772	-0.4733	-0.4513
-0.57	-0.67	-0.6555	-0.5669	-0.4633	-0.4411
-0.56	-0.6631	-0.6476	-0.557	-0.4538	-0.433
-0.55	-0.6515	-0.6372	-0.5479	-0.4417	-0.4206
-0.54	-0.6446	-0.6277	-0.5362	-0.4313	-0.4108
-0.53	-0.6357	-0.6197	-0.527	-0.4219	-0.4002
-0.52	-0.625	-0.6092	-0.517	-0.4097	-0.3899
-0.51	-0.6157	-0.5859	-0.5079	-0.4029	-0.3805
-0.5	-0.6086	-0.5922	-0.4976	-0.3886	-0.3656

b_{LS}	**0.025**	**0.05**	**0.5**	**0.95**	**0.975**
-0.49	-0.599	-0.5819	-0.4874	-0.3806	-0.3589
-0.48	-0.5908	-0.574	-0.4778	-0.3705	-0.3469
-0.47	-0.5815	-0.5646	-0.4683	-0.3585	-0.3369
-0.46	-0.5736	-0.5558	-0.4578	-0.3484	-0.3269
-0.45	-0.5637	-0.5465	-0.4477	-0.3384	-0.3164
-0.44	-0.5541	-0.5365	-0.4384	-0.3272	-0.3048
-0.43	-0.5428	-0.5257	-0.4273	-0.3171	-0.2961
-0.42	-0.5361	-0.5166	-0.4181	-0.3094	-0.2858
-0.41	-0.5245	-0.5078	-0.4084	-0.2971	-0.2747
-0.4	-0.5173	-0.4982	-0.3963	-0.2858	-0.2629
-0.39	-0.5073	-0.4886	-0.3878	-0.2759	-0.2529
-0.38	-0.4997	-0.481	-0.3784	-0.2645	-0.2419
-0.37	-0.4894	-0.4708	-0.368	-0.2576	-0.2361
-0.36	-0.4804	-0.4617	-0.3574	-0.2448	-0.2219
-0.35	-0.4698	-0.4518	-0.3473	-0.2342	-0.2143
-0.34	-0.4637	-0.4421	-0.3386	-0.2233	-0.2016
-0.33	-0.4526	-0.4334	-0.3286	-0.2149	-0.1931
-0.32	-0.443	-0.4246	-0.3189	-0.2043	-0.187
-0.31	-0.4328	-0.414	-0.3084	-0.1928	-0.1711
-0.3	-0.4257	-0.4055	-0.299	-0.1841	-0.1601
-0.29	-0.4143	-0.3947	-0.2895	-0.1748	-0.1512
-0.28	-0.4063	-0.3861	-0.2786	-0.164	-0.1413
-0.27	-0.3964	-0.3765	-0.2679	-0.1529	-0.131
-0.26	-0.3867	-0.3666	-0.2588	-0.144	-0.1216
-0.25	-0.3525	-0.3365	-0.2483	-0.1563	-0.1377
-0.24	-0.3431	-0.3266	-0.2394	-0.1471	-0.1301
-0.23	-0.3335	-0.3171	-0.2294	-0.1362	-0.1183
-0.22	-0.3246	-0.3091	-0.2191	-0.1264	-0.1078
-0.21	-0.3158	-0.299	-0.2089	-0.1177	-0.0986
-0.2	-0.3077	-0.2898	-0.2	-0.1071	-0.0907
-0.19	-0.296	-0.2779	-0.1889	-0.0963	-0.0776
-0.18	-0.2878	-0.2699	-0.179	-0.0866	-0.0699
-0.17	-0.2764	-0.2596	-0.1692	-0.0763	-0.0587
-0.16	-0.2671	-0.2496	-0.1591	-0.0669	-0.0488
-0.15	-0.2584	-0.2404	-0.1493	-0.0565	-0.038
-0.14	-0.2475	-0.2299	-0.1391	-0.0468	-0.0275
-0.13	-0.2392	-0.2227	-0.13	-0.0367	-0.0194
-0.12	-0.2297	-0.2125	-0.1193	-0.0259	-0.0083
-0.11	-0.2181	-0.2013	-0.1099	-0.0178	-0.00018
-0.1	-0.2074	-0.1909	-0.0998	-0.0064	-0.0109

b_{LS}	**0.025**	**0.05**	**0.5**	**0.95**	**0.975**
0.1	-0.0105	0.0078	0.0994	0.1895	0.2063
0.11	-0.0011	0.0173	0.1111	0.2019	0.219
0.12	0.0084	0.0268	0.12	0.2114	0.2287
0.13	0.0175	0.0353	0.1297	0.2215	0.2393
0.14	0.00285	0.0469	0.1397	0.2313	0.2483
0.15	0.0375	0.0549	0.1495	0.2396	0.2553
0.16	0.0475	0.0647	0.1594	0.2501	0.2667
0.17	0.0579	0.0763	0.1696	0.2592	0.2763
0.18	0.0694	0.0866	0.1795	0.2695	0.2853
0.19	0.0778	0.0966	0.1895	0.2779	0.294
0.2	0.0876	0.1059	0.2	0.2887	0.306
0.21	0.0989	0.1158	0.2086	0.2979	0.3143
0.22	0.1092	0.1264	0.2194	0.3079	0.325
0.23	0.1183	0.1366	0.2287	0.3175	0.3348
0.24	0.128	0.1463	0.2387	0.3267	0.3427
0.25	0.1405	0.1581	0.2492	0.3371	0.3532
0.26	0.1483	0.1664	0.2588	0.3467	0.3638
0.27	0.1591	0.1779	0.2685	0.3553	0.3716
0.28	0.1679	0.187	0.2793	0.3656	0.381
0.29	0.1786	0.1965	0.2901	0.3761	0.3922
0.3	0.1886	0.2075	0.2986	0.3844	0.4001
0.31	0.2014	0.2199	0.3094	0.3956	0.4113
0.32	0.2087	0.2271	0.3183	0.4035	0.42
0.33	0.2193	0.2375	0.3281	0.4137	0.4285
0.34	0.2317	0.25	0.3392	0.4244	0.4382
0.35	0.2413	0.2576	0.3486	0.4325	0.4475
0.36	0.25	0.2673	0.3586	0.4429	0.4575
0.37	0.2597	0.2789	0.369	0.4515	0.466
0.38	0.2702	0.2899	0.3784	0.461	0.4746
0.39	0.2833	0.3006	0.3882	0.4704	0.4868
0.4	0.2926	0.3101	0.3988	0.4803	0.495
0.41	0.303	0.3203	0.4085	0.4904	0.5051
0.42	0.3114	0.3299	0.419	0.5005	0.5146
0.43	0.3221	0.3402	0.428	0.5083	0.5222
0.44	0.3347	0.3513	0.4388	0.5177	0.5316
0.45	0.3438	0.3613	0.4483	0.527	0.5413
0.46	0.3547	0.373	0.4587	0.5381	0.5516
0.47	0.3652	0.3821	0.4688	0.5462	0.5608
0.48	0.3759	0.3925	0.4785	0.556	0.5703
0.49	0.3842	0.4022	0.4889	0.5662	0.5798
0.5	0.39	0.4125	0.4977	0.5741	0.5868

b_{LS}	**0.025**	**0.05**	**0.5**	**0.95**	**0.975**
0.51	0.4077	0.4241	0.5083	0.5833	0.5971
0.52	0.4162	0.4338	0.5183	0.5927	0.6061
0.53	0.4262	0.4435	0.5285	0.6043	0.6165
0.54	0.4375	0.4559	0.5384	0.6127	0.6253
0.55	0.4484	0.4656	0.5482	0.6213	0.6253
0.56	0.4576	0.4749	0.5579	0.6305	0.6438
0.57	0.4699	0.4871	0.5682	0.6401	0.6527
0.58	0.4798	0.497	0.5784	0.6497	0.6622
0.59	0.4896	0.5067	0.5881	0.6578	0.6705
0.6	0.5019	0.518	0.5981	0.6668	0.6793
0.61	0.514	0.05294	0.6084	0.9765	0.689
0.62	0.5229	0.5396	0.6174	0.6857	0.6974
0.63	0.5338	0.5508	0.6286	0.695	0.7076
0.64	0.5451	0.5617	0.6378	0.7032	0.7143
0.65	0.555	0.5701	0.6478	0.7135	0.7247
0.66	0.5657	0.582	0.6577	0.7217	0.7332
0.67	0.5783	0.5931	0.6679	0.7321	0.7434
0.68	0.587	0.6022	0.6774	0.7407	0.751
0.69	0.5975	0.6131	0.6877	0.7493	0.7595
0.7	0.6104	0.6259	0.6978	0.7587	0.7691
0.71	0.6189	0.6353	0.7077	0.7672	0.7769
0.72	0.6305	0.6469	0.7185	0.7774	0.7879
0.73	0.6411	0.6564	0.7272	0.7857	0.7948
0.74	0.6529	0.6683	0.7381	0.7946	0.8044
0.75	0.6642	0.6787	0.7479	0.8029	0.8126
0.76	0.6759	0.6905	0.7576	0.8122	0.8209
0.77	0.6848	0.6996	0.7674	0.8214	0.8305
0.78	0.697	0.712	0.7771	0.8296	0.8379
0.79	0.7095	0.7226	0.7874	0.8384	0.8472
0.8	0.7208	0.7335	0.7971	0.847	0.8551
0.81	0.7311	0.7447	0.8076	0.8561	0.8644
0.82	0.7417	0.7552	0.8171	0.8651	0.8728
0.83	0.754	0.7667	0.8271	0.873	0.8802
0.84	0.7651	0.7785	0.8373	0.8813	0.8887
0.85	0.7779	0.7893	0.8474	0.8902	0.8972
0.86	0.7891	0.801	0.8574	0.8981	0.9048
0.87	0.7997	0.8119	0.8672	0.9066	0.9128
0.88	0.811	0.824	0.8772	0.9157	0.9213
0.89	0.8227	0.8351	0.8871	0.9238	0.929
0.9	0.8351	0.8472	0.8972	0.9316	0.9367
0.91	0.8474	0.8594	0.9072	0.9402	0.9451
0.92	0.8586	0.8704	0.9172	0.948	0.9526
0.93	0.87	0.8832	0.9271	0.9554	0.9593
0.94	0.8848	0.8945	0.9368	0.9631	0.9665
0.95	0.8976	0.9073	0.947	0.9704	0.9737
0.96	0.9106	0.92	0.9571	0.9777	0.9805
0.97	0.9231	0.9327	0.9671	0.9847	0.9869
0.98	0.9373	0.9461	0.9771	0.9914	0.9929
0.99	0.9512	0.9596	0.9869	0.9974	0.9987
1	0.9668	0.9742	0.9973	1.0042	1.0052