Credit Frictions, Housing Prices and Optimal Monetary Policy Rules[*]

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Abstract

We try to asses the role of housing price movements in the optimal design of monetary policy rules. Even though the relevance of liquidity constraints for consumption behavior has been well documented in the empirical and theoretical literature little attention has been given to credit frictions at the household level in the monetary business cycle literature.

This paper represents the first attempt of a welfare-based monetary policy evaluation in a model with heterogeneous agents and credit constraints at the household level. In order to evaluate optimal monetary policy we take advantage of the recent advances in computational economics by following the approach illustrated by Schmitt-Grohe and Uribe (2003). Our results show that housing price movements should not be a separate target variable additional to inflation, in an optimally designed simple monetary policy rule

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1 Introduction

The recent rise in housing prices in most of the OECD countries has attracted the attention of policy makers and academics and raised concerns about the macroeconomic implications[1].

What significance do asset prices have for monetary policy? A number of papers have tried to understand the extent to which asset price movements should be relevant for monetary policy[2]. Cecchetti et al. (2000 and 2002) show that reacting to asset prices, reduces the likelihood that bubbles form. On the other hand, Bernanke and Gertler (2001), among others, conclude that inflation-targeting central banks should not respond to asset prices. In fact, conditional on a strong response to inflation, the gain from responding to asset prices is negligible. Both studies employ a financial accelerator framework allowing for credit market frictions and exogenous asset price bubbles. The methodology adopted for evaluating the performance of different monetary policy rules is based on the implied volatility of output and inflation. Different conclusions about the desiderability of including asset prices as an additional argument in the monetary policy rule, depend on different assumptions about the stochastic nature of the model, i.e. the shocks considered.

Directly related to housing prices is the analysis by Iacoviello (2004). He shows the relevance of housing prices in the transmission and amplification of shocks to the real sector. Nevertheless, when computing the inflation-output volatility frontiers it turns out that a response to housing prices does not yield significant gains in terms of output and inflation stabilization.

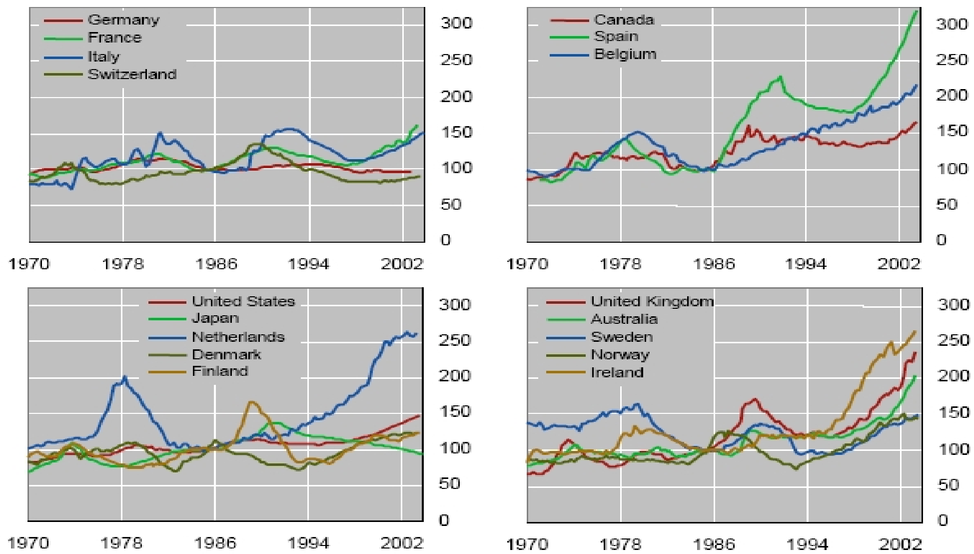
The main shortcoming of all this literature is the absence of welfare considerations in evaluating optimal monetary policy. The only exception is the analysis conducted by Faia and Monacelli (2004). Relying on a welfare-based approach they show that reacting to asset prices is optimal but do not generate relevant welfare improvements. On the other hand, responses to changes in the

[1]See among others Borio and Mc Guire (2004) for the relation between housing and equity prices, Iacoviello (2004) for the relevance of housing prices and credit constraints in the business cycle, Girouard-Blöndal (2001) for the role of housing prices in sustaining consumption spending in the recent downturn of the world economy, Case-Quiugley-Shiller (2001) for empirical evidence on the housing wealth effect.

[2]See e.g. Filardo (2000), Goodhart (2000), Batini and Nelson (2000), Bernanke and Gertler (1999, 2001), Cecchetti, Genberg, Lipsky and Wadhwani (2000), Cecchetti, Genberg and Wadhwani(2003), Taylor(2001), Kontonikas and Montagnoli (2003), Faia and Monacelli (2004).

Inflation-adjusted house prices[1]

Quarterly data; 1985 = 100



[1] Nominal house prices deflated by the personal consumption deflator. For France, Germany, Italy and Japan, quarterly house price data are derived from lower-frequency data using Ginsburgh interpolation techniques.

Sources: Japan Real Estate Institute; Nomisma; national data; BIS calculations. Graph 1

leverage ratio generate more pronounced deviations from a strict price stability policy.

This paper studies optimal monetary policy rules in an economy with credit market frictions at the household level and heterogeneous agents. The aim is to asses the role of household indebtedness and housing prices in designing monetary policy. The paper is related to the large literature on optimal monetary policy in economies with nominal rigidities[3]. This literature assumes that the central bank is a benevolent policy maker, thus, maximizes consumers' welfare[4]. Most of the models consider a dynamic system centered around an efficient non-distorted equilibrium. In practice, the policy maker neutralizes any source of inefficiency present in the economy and not related to the existence of nominal rigidities. Thus, the only duty left to monetary policy is to offset the distortions associated with price rigidities in order to replicate the flexible price equilibrium

[3] See among others, Rotemberg and Woodford (1997), Clarida, Gali and Gertler (1999), King and Wolman (1999), Erceg, Henderson and Levin (2000),

[4] The literature is divided in two streams on the base of a main assumption regarding the deterministic equilibrium around which the model economy evolves.

allocation. The motivation behind this modelling choices is purely technical. In fact, it is sufficient a first order approximation of the equilibrium conditions to approximate welfare up to the second order[5]. Following a method introduced by Rotemberg and Woodford (1997) in these kinds of models it is possible to derive a discounted quadratic loss function from the quadratic approximation of the utility function, and compute optimal policy using a simple linear-quadratic methodology as in the traditional monetary policy theory.

An alternative approach, studies optimal monetary and fiscal policy in models evolving around equilibria that remain distorted[6]. These are models in which different types of distortions, beside price rigidities, proved a rationale for the conduct of monetary policy. In order to get a welfare measure that is accurate to the second order[7] it is necessary to use a higher order approximation of the model's equilibrium conditions. The method suggested by Schmitt-Grohe and Uribe (2003) shows that a second order solution to the model's policy functions is required for the approximation of the welfare function to be accurate up to the second order. Another way of evaluating a welfare measure accurate up to the second order, is proposed by Benigno and Woodford (2003) as an extension of Rotemberg and Woodford's method. On the base of the computation of a second order approximation to the model's structural equations it is possible to substitute out the linear terms in the Taylor approximation to the expected utility and obtain a "pure quadratic" approximation to the welfare function (no linear terms). Once a quadratic function is derived optimal monetary policy can be evaluated using as constraints the first order approximation to the model's equations. Thus, the linear-quadratic methodology is reintroduced again.

Our model economy is characterized by three types of distortions. First, nominal price rigidities, modelled as quadratic adjustment cost on good market price setting are adopted as a source of monetary non neutrality. Second, monopolistic competition in the good market allows for price setting above the marginal cost. Third, credit market imperfections, generated by the assumption that creditors cannot force debtors to repay unless debts are secured by

[5] See Woodford (2003)

[6] See Uribe and Schmitt-Grohe (2004), Benigno and Woodford (2004), Faia and Monacelli (2004).

[7] Up to a first order accuracy the agents' discounted utility function equals its non-stochastic steady state value. Since the monetary policy rules commonly considered do not affect the non-stochastic steady state, it is not possible to rank different rules on the base of first order approximation.

collateral, generate a role for housing prices.

Even though the relevance of liquidity constraints for consumption behavior has been well documented in the empirical and theoretical literature – see Zeldes (1997), Jappelli and Pagano (1997) among others – little attention has been given to credit frictions at the household level in the monetary business cycle literature. In fact, this paper represents the first attempt of a welfare-based monetary policy evaluation in a model with heterogeneous agents and credit constraints at the household level.

The model is built on Kiyotaki and Moore (1997) (KM henceforth). In order to generate a motive for the existence of credit flows, two types of agents are assumed. They differ in terms of discount factors: as a consequence impatient agents are borrowers. Credit constraints arise because lenders cannot force borrowers to repay. Thus, physical assets are used as collateral for loans. As in Iacoviello (2004), we depart from KM's framework from two main features. First, differently from KM we focus on the household sector. In fact, KM's agents are entrepreneurs that produce and consume the same good using a physical asset. Agents are risk neutral and represent two different sectors of the economy - borrowers are "farmers" and lenders are "gatherers". On the contrary, we model households that, apart from getting utility from a flow of consumption and disutility from labor according to a strictly concave function, and consider house holding as a separate argument of their utility function. Housing services are assumed to be proportional to the real amount of housing stock held. In our setup both groups of agents are identical, only difference is the subjective discount factor. Second, we extend the model to include nominal price rigidities and a role for monetary policy. Iacoviello (2004) doesn't distinguish between residencial and commercial properties. Thus, houses are not only a source of direct utility but also an input of production and the asset used in the credit market to secure both firms' and households' debts[8]. These modelling choices are consistent with the aim of showing the importance of financial factors for macroeconomic fluctuation. Instead, being interested in the role of housing prices for the optimal design of monetary policy, we restrict our attention to the household sector. In order to evaluate optimal monetary policy we take

[8] Iacoviello (2004), as Faia and Monacelli (2004),adds collateral constraints tied to firms' real estate holdings (housing) to Bernanke, Gertler and Gilchrist (2000) model. Moreover, he also introduces collater constraits in the household sector.

advantage of the recent advances in computational economics by following the approach illustrated by Schmitt-Grohe and Uribe (2003).

In terms of monetary policy evaluation, the main elements that distinguish our contribution are the use of a welfare-based evaluation of the optimal rules instead of the inflation-output volatility criterion (as in Iacoviello (2004)) and the attention to both lenders' and borrowers' welfare in the implementation of the welfare method. In fact, compared to Faia and Monacelli (2004) we do not focus on the maximization of the lenders' welfare but we adopt as relevant measure the weighted avarege of borrowers' and lenders' welfare. Moreover, we focus on the households' sector in order to understand if housing prices - and not generic asset prices - could be a variable of interest for monetary policy.

The results show that optimally designed simple monetary policy rules should not take into account current housing prices movements. In fact, under normal circumstances, we find out that an explicit objective of housing prices stability is not welfare improving relative to a strict price stability policy.

The remainder of the paper is organized as follows. Section 2 describes the role of housing as a collateral. Section 3 lays out the model and derives the equilibrium conditions. Section 4 turns its attention on the model's calibration. Section 5 describes the welfare measure considered and the methodology to evaluate monetary policy's optimal design. Section 6 comments on the results.

2 Housing Prices and Borrowing Constraint

Why should housing prices be relevant for monetary policy in a bubble-free model? Our main hypothesis is that housing is used as a collateral in the loan market and consequently housing prices are related to consumption and economic activity through both a traditional wealth effect and a mortgage loans market channel. An increas in housing prices contributes to a rise in the value of the collateral that allows households to borrow more. As a consequence, the increased household indebtedness could increase the sensitivity of households to changes in the interest rate and sudden decreases in housing prices themselves. Thus, housing prices movements are relevant to assess how private consumption evolves and the ability of the household sector to smooth different kind of shocks. All this is taken into account by the welfare critirium we use. In fact, it consider households' present and future welfare.

We consider a modified version of the standard business cycle model in which household derive utility from owning houses and using them as collateral in the loan market. We depart from the representative agent framework assuming two groups of agents: borrowers and lenders.

Borrowers face an external borrowing constraint. The constraint is not derived endogenously but it is consistent with standard lending criteria used in the mortgage and consumer loans market. The borrowing constraint is introduced through the assumption that households cannot borrow more than a fraction of the value of their houses. The household borrows (B_{it}) against the value of his housing wealth.

$$B_{it} \leq \gamma E_t[Q_{t+1} h_{it}] \tag{1}$$

where Q_{t+1} is the housing prices and h_{it} is the stock of housing. Mortgage loans refinancing takes place every period and the household repays every new loan after one period. It seems quite realistic that the overall value of the loan cannot be higher than a fraction of the expected value of the collateral. The fraction γ, referred to as *loan to value ratio*, should not exceed one. This can be explained thinking of the overall judicial costs which a creditor incurs in case of the debtor default. Since housing prices affect the collateral value of the houses, fluctuations in the price plays a large role in the determination of borrowing conditions at household level. Borrowing against an higher value of the house is used to finance both investment in housing and consumption. The other source of mortgage equity withdrawal is given by an increase in the value of the collateral due to a rise the loan to value ratio.

3 The Model

Consider a sticky prices model populated by a monopolistic competitive good producing firm, a monetary authority and two types of households. In order to impose the existence of flows of credit in this economy we assume ex-ante heterogeneity at the household level: agents differs in terms of the subjective discount factor. We assume a continuum of households of mass 1: n *Impatient Households* (lower discount rate) that borrow in equilibrium and (1-n) *Patient Households* (higher discount rate) that lend in equilibrium.

3.1 Households

The households derive utility from a flow of consumption and services from house holding - that are assumed to be proportional to the real amount of housing stock held - and disutility from labor:

$$\max_{\{c_{it},h_{it},L_{it}\}} E\sum_{t=0}^{\infty}\beta_i^t U(c_{it},h_{it},L_{it})$$

with $i=1,2$ and $\beta_1>\beta_2$ s.t. a *budget constraint*

$$c_{it}+q_t(h_{it}-h_{it-1})+\frac{b_{it-1}}{\pi_t}=\frac{b_{it}}{R_t}+w_tL_{it}+f_{it}-T_{it}$$

and a *borrowing constraint*

$$b_{it}\leq\gamma E_t[q_{t+1}\pi_{t+1}h_{it}] \quad (2)$$

Except for the gross nominal interest rate, R, all the variables are expressed in real terms. π_t is the gross inflation (P_t/P_{t-1}) and q_t is the price of housing in real terms (Q_t/P_t). The household can borrow (b_t) using as a collateral the next period's expected value of real estate holdings (the stock of housing). This borrowing constraint will hold only for the impatient households since the patient ones will lend in equilibrium. In the budget constraint T_{it} are lump sum taxes from the fiscal authority, and f_{it} are the dividends from firms. We assume that only the patient households own the firms. Thus, $f_{1t} = \frac{1}{(1-N)}(D_t/p_t)$ where D_t are the dividends of the representative firm and $f_{2t} = 0$.

Agents ´optimal choices are characterized by:

$$-U_{L_{it}} = U_{c_{it}} w_t$$

$$\frac{U_{c_{i,t}}}{R_t} \geq \beta_i E_t \frac{U_{c_{i,t+1}}}{\pi_{t+1}}$$

$$U_{c_{i,t}} q_t - \beta_i E_t U_{c_{i,t+1}} q_{t+1} \geq U_{h_{i,t}}$$

The second equation relates the marginal benefit of borrowing to its marginal cost. The third equation states that the opportunity cost of holding one unit of housing, $\left[U_{c_{i,t}} q_t - \beta_i E_t U_{c_{i,t+1}} q_{t+1}\right]$, is bigger or equal to the marginal utility of housing services.

3.1.1 Impatient Households

We can show that Impatient Households borrow up to the maximum in a neighborhood of the steady state. If fact, if we consider the euler equation of the impatient household in steady state

$$\mu_2 = (\beta_1 - \beta_2) U_{c_2} > 0$$

where μ_{2t} is the lagrange multiplier associated to the borrowing constraint[9]. Thus, the borrowing constraint holds with equality in a neighborhood of the steady state

$$b_{2t} = \gamma E_t[q_{t+1}\pi_{t+1}h_{2t}]$$

And we get the following optimal choices for labor, borrowing and housing services

$$-U_{L_{2t}} = U_{c_{2t}} w_t$$

$$\frac{U_{c_{2t}}}{R_t} - \mu_t = \beta_2 E_t U_{c_{2t+1}} \frac{1}{\pi_{t+1}}$$

[9] Once we assume the existence of different discount factors with $-\beta_1 > \beta_2-$ in the deterministic steady state the household caracterized by β_2 is willing to borrow up to the maximum.

Thus, for constrained agents the marginal benefits of borrowing are always bigger than the marginal cost.

$$U_{h_{2t}} + \beta_2 E_t U_{c_{2t+1}} q_{t+1} + \mu_t \gamma E_t q_{t+1} \pi_{t+1} = U_{c_{2t}} q_t$$

Moreover, the marginal benefit of holding one unit of housing is given not only by its marginal utility but also by the marginal benefit of being allowed to borrow more.

3.1.2 Patient Households

Since the patient households' borrowing constraint is not binding in a neighborhood of the steady state it faces a standard problem, only exception is the existence of the housing services in the utility function:

labor supply

$$-U_{L_{1t}} = U_{c_{1t}} w_t$$

borrowing condition

$$U_{c_{1t}} = \beta_1 E_t U_{c_{1t+1}} \frac{R_t}{\pi_{t+1}}$$

housing demand

$$U_{h_{1t}} + \beta_1 E_t U_{c_{1t+1}} q_{t+1} = U_{c_{1t}} q_t$$

3.2 Firms

3.2.1 The final good producing firms

Perfectly competitive firms produce a final good y_t using $y_t(i)$ units of each intermediate good $i \in (0,1)$ adopting a constant return to scale, diminishing marginal product and constant elasticity of substitution technology:

$$y_t \leq \left[\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

with $\theta > 1$. Costs minimization implies

$$\min_{\{y_t(i)\}} \int_0^1 P_t(i) y_t(i) di$$

$$s.t. \;\; y_t \leq \left[\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$$

The price of the intermediate good $y_t(i)$ is denoted by $P_t(i)$ and taken as given by the competitive final good producing firm. The solution yields the following constant price elasticity (θ) demand function for good i that is homogeneous of degree one in the total final output:

$$y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} y_t$$

Combining the demand function with the production function is possible to derive the price index for intermediate goods:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di\right]^{1/(1-\theta)}$$

3.2.2 The intermediate sector

In the wholesale sector there is a continuum of firms indexed by $i \in (0,1)$ and owned by consumers. Intermediate producing firms act on a monopolistic market and produce $y_t(i)$ units of differentiated good i using $L_t(i)$ units of labor according to the following constant return to scale technology

$$Z_t L_t(i) \geq y_t(i)$$

where Z_t is the aggregate productivity shock and follows the autoregressive process

$$\ln(Z_t) = \rho_Z \ln(Z_{t-1}) + \varepsilon_{Zt}, \qquad \varepsilon_{Zt} \backsim^{iid} N(0, \sigma_{\varepsilon Z}), \quad 0 < \rho_Z < 1$$

Cost Minimization Monopolistic competitive firms hire labor from households in a competitive market on period by period basis. Cost minimization implies the following nominal marginal cost s_t^n:

$$\frac{W_t}{Z_t} = s_t^n(i) \tag{7}$$

and thus the total cost could be written in the following way[10]:

$$W_t L_t(i) = s_t^n(i) y_t(i)$$

[10] In equilibrium the firm chooses input such that the marginal product equals the markup times the factor price. In fact, in terms of gross markup $(1 + \eta_t) = \frac{1}{s_t}$:

$$\frac{\bar{y}_t(i)}{L_t^*(i)} = (1 + \eta_t) W_t$$

Price Setting Assume now that intermediate firms set the price of their differentiated good every period, but facing a quadratic cost of adjusting the price between periods[11]. The cost is measured in terms of the final good[12]:

$$\frac{\phi_p}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right]^2$$

where $\phi_p > 0$ represent the degree of nominal rigidity and π is the gross steady state inflation. Each firm faces the following problem:

$$\max_{\{P_t(i)\}} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j}\left[\tfrac{D_t(i)}{P_t}\right]$$
$$s.t.$$
$$y_t(i) = \left[\tfrac{P_t(i)}{P_t}\right]^{-\theta} y_t$$

where $\Lambda_{t,t+j} = \beta_1^j \frac{U_{c1t+j}}{U_{c1t}}$ is the *relevant discount factor*. The firm's profits in real terms are given by :

$$\frac{D_t(i)}{P_t} = \frac{P_t(i)}{P_t} y_t(i) - s_t(i) y_t(i) - \frac{\phi_p}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right]^2$$

Using the results from the cost minimization problem, we replaced the real total costs , $w_t L_t(i)$, with a function of real marginal costs and total output[13]. Thus, substituting for the total costs and the firm's production, the profits maximization problem becomes:

$$\max_{\{P_t(i)\}} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j}\left\{y_t\left[\left(\frac{P_t(i)}{P_t}\right)^{1-\theta} - s_t(i)\left(\frac{P_t(i)}{P_t}\right)^{-\theta}\right] - \frac{\phi_p}{2}\left(\frac{P_t(i)}{P_{t-1}(i)}-1\right)^2\right\}$$

[11] The Calvo setting (most commonly used) and the price adjustment cost setting deliver the same linearized system of necessary conditions up to a reparametrization. For a second order approximation this is not true. The second order term in the resource constraint and in the firms' FOC do not allow to have a one-to-one mapping between the two models.

The second order terms in the Calvo setting are ultimately related to the second order moments of the price distribution - while for the other case they are simply related to the chosen adjustment costs functional form. However, given the demanding assumptions of the re-setting process in a framework a la Calvo, it is hard to tell which of the two set-up are quantitatively more reliable.

For sparing computing time we have preferred the price adjustment cost framework.

[12] See Kim JME 1995

[13]

$$w_t L_t(i) = s_t(i) y_t(i) = \frac{w_t}{Z_t} y_t(i)$$

The derivative with respect to the firm's price, multiplied for the price level P_t, yields:

$$0 = E_t \Lambda_{t,t+1} \left[\phi_p \frac{P_t}{\pi} \frac{P_{t+1}(i)}{P_t(i)^2} \left(\frac{P_{t+1}(i)}{\pi P_t(i)} - 1 \right) \right] +$$
$$+ y_t \left[(1-\theta) \left(\frac{P_t(i)}{P_t} \right)^{-\theta} + \theta s_t(i) \left(\frac{P_t(i)}{P_t} \right)^{-\theta-1} \right] - \phi_p \frac{P_t}{\pi P_{t-1}(i)} \left(\frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right)$$

3.3 The Fiscal Authority

We assume:

$$G_t = T_t$$

where G_t is government consumption of the final good and T_t are lump sum taxes/transfers, where $T_t = (1-n)T_{1t} + nT_{2t}$.Government consumption evolves according to the following exogenous process:

$$(\ln G_t - \ln G) = \rho_G (\ln G_{t-1} - \ln G) + \varepsilon_{Gt} \quad \text{where} \quad \varepsilon_{Gt} \backsim^{iid} N(0, \sigma_{\varepsilon_G}), \quad 0 < \rho_G < 1$$

where G is the steady state share of government consumption.

3.4 Equilibrium and Aggregation

3.4.1 Equilibrium Conditions

In the symmetric equilibrium, all firms make identical decisions, so that:

$$y_t(i) = Y_t \qquad P_t(i) = P_t \qquad L(i) = L_t$$

Consequently, total production becomes

$$Y_t = Z_t L_t \tag{3}$$

and the price setting equation

$$0 = E_t U_{c1t+1} \left[\phi_p \frac{\pi_{t+1}}{\pi} \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \right] + U_{c1t} \left\{ y_t \left[\theta \left(s_t - \frac{\theta - 1}{\theta} \right) \right] - \phi_p \frac{\pi_t}{\pi} \left(\frac{\pi_t}{\pi} - 1 \right) \right\}$$

Market Clearing conditions

$$\begin{array}{cc} (1-n)L_{1t} + nL_{2t} = L_t & (1-n)c_{1t} + nc_{2t} = C_t \\ (1-n)b_{1t} + nb_{2t} = 0 & (1-n)h_{1t} + nh_{2t} = 1 \\ T_t = (1-n)T_{1t} + nT_{2t} & G_t = T_t \end{array}$$

where H_t is in fix supply normalized to 1. Resource constraint

$$Y_t = C_t + \frac{\phi_p}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 + G_t \tag{4}$$

The production of the final sector needs to be allocated to resources costs arising from the prices' adjustment and to private consumption by households and government. This condition together with the household's and firm's first order conditions, the law of motion of the exogenous shocks, the central bank policy rule, the borrowing constraint and one of the two budget constraints constitute a system of non linear difference equations describing the behavior in equilibrium of prices ad quantities. After loglinearizing the system around its steady state we obtain the system of linear difference equations that determine the dynamics of the state and costate variables.

4 Calibration

We set the parameters of the model on the base of quarterly evidence. The households' discount factors are $(\beta_1, \beta_2) = (0.99, 0.98)$. Patient Households' discount factor implies an average annual rate of return of about 4%. Previous estimates of discount factors for poor or young households[14] have been used as a reference in the calibration of β_2. We assume a separable utility function:

$$U(c_{it}, h_{it}, L_{it}) = \frac{c_{it}^{1-\varphi_c}}{1-\varphi_c} + \nu_h \ln h_{it} - \nu_L \frac{L_{it}^{1+\varphi_L}}{1+\varphi_L}$$

For simplicity we assume log-utility for consumption, $\varphi_c = 1$ (risk aversion), and we set $\varphi_L = 2$ (inverse of labor supply elasticity). The weight on labor disutility, ν_L, equals 1, while $\nu_h = 0.019$. This last parameter implies a steady state value of real estate over annual output of 140%. In line with the literature on nominal rigidities, we set the elasticity of substitution, θ, equal 11. The baseline choice for the loan to value ratio[15], γ, is 50% and the fraction of borrowed constraint population is settle to 50%. We calibrate the steady state government consumption value as the 20% of total output. Following Schmitt-Grohe and Uribe (2004) we calibrate the technology and government spending

[14] In fact, Lawrance (1991) and Samwick (1998) estimate discount factors, respectively, for poor and young households in the range (0.97, 0.98).

[15] Using US data from 1974 to 2003, Iacoviello (2004) estimates the households' loan to valio ratio equal to 0.55.

shocks according to standard values in the real business cycle literature[16]. Tab. 1 summarizes the calibrated parameters.

Preferences			
$\beta_1 = 0.99$		$\varphi_c = 1$	$\nu_h = 0.019$
$\beta_2 = 0.98$		$\varphi_L = 2$	$\nu_L = 1$
	Technology	**BOC**	
	$\theta = 11$	$\gamma = 0.5$	
	$\phi_p = 161$		
Shocks			
	$\rho_Z = 0.95$	$\sigma_Z = 0.0056$	
	$\rho_G = 0.9$	$\sigma_G = 0.0074$	
Table 1			

5 Computation and Welfare Measure

5.1 Computation

Since Kydland and Prescott (1982)[17] the first-order approximation approach is the most popular numerical approximation method for solving models too complex to deliver an exact solution. However, first order approximations may produce clearly erroneous results[18]. Comparing welfare among implementable policy rules that have no first-order effects on the model's deterministic steady state, we need to rely on higher order approximation methods.

As shown by Kim and Kim (2003)[19], in this context first order approximation methods are not locally accurate. In general a second-order accurate approximation to the welfare function requires a second-order expansion to the model's equilibrium conditions. The first order approximation solution, is not always accurate enough due to the certainty equivalence property, i.e. the coincidence of the first order approximation to the unconditional means of endogenous variables with their non stochastic steady state values. This neglects important effects of uncertainty on the average level of households' welfare. A first or-

[16] For the technology shock see, Cooley & Prescott (1995, chapter 1 in Cooley's book), or Prescott 1986.

[17] They applied to a real business cycle model a special case of the method of linear approximation around deterministic steady states developed in Magill (1977).

[18] See e.g. Tesar (1992) for an example where completing asset markets will make all agents worse off, Kim and Kim (2003) for stressing the same results in a two agents stochastic model.

[19] They show that a welfare comparison based on linear approximation to the policy functions of a simple two-countries economy, may yield the odd result that welfare is higher under autarky than under full risk-sharing.

der approximation to the policy functions would give an incorrect second order approximation of the welfare function [20].

To overcome this limitation and obtain a second-order accurate approximation, we adopt a perturbation technique introduced by Fleming (1971) and applied to various types of economic models by Judd and coauthors[21] and recently generalized by Schmitt-Grohe and Uribe (2002)[22] (SU henceforth). Second order approximations are quite convenient to implement since, even capturing the effects of uncertainty, do not suffer from the "curse of dimensionality"[23]. In fact, following SU, given the first-order terms of the Taylor expansions of the functions expressing the model's solution, the second-order terms can be identified by solving a linear system of equations whose terms are the first order terms and the derivatives up to the second order of the equilibrium conditions evaluated at the non-stochastic steady state.

5.2 Welfare Measure and Optimal Rules

How should monetary policy be conducted in a world economy with credit frictions at the household level? In order to answer this question, we rely on utility-based welfare calculations, assuming that the benevolent monetary authority maximize the utility of the households subject to the model's equilibrium conditions. Formally, the optimal policy maximizes the household's life-time utility:

$$V_t \equiv E_t \left[\sum_{i=1}^{2} \eta_i \sum_{j=0}^{\infty} \beta_i^j U(c_{i,t+j}, h_{i,t+j}, L_{i,t+j}) \right]$$

[20] See Woodford (2002) and Kim et al. (200?) for a discussion of situations in which second-order accurate welfare evaluations can be obtained using first-order approximations to the policy functions.

[21] See Judd and Guu (1993,1997) for applications to deterministic and stochastic, continuous and discrete-time growth models in one state variable, Gaspar and Judd (1997) for multidimentional stochastic models in continuous time approximated up to the fourth-order, Judd (1998) presents the general method , Jin and Judd (2001) extended these methods to more general rational expectations models .

[22] They derive a second-order approximation to the policy function of a general class of dynamic, discrete-time, rational expectations models. They show that in a second-order expansion of the policy functions, the coefficients on the linear and quadratic terms in the state vector are independent of the volatility of the exogenous shocks. Thus, only the constant term is affected by uncertainty.

[23] Models with large numbers of state variables can be solved without much computational effort.

where η_i are the weights on households' utilities. We choose η_1=(1-β_1) and η_2=(1-β_2).

We measure welfare as the conditional expectation at time zero ($t = 0$), time in which all state variables of the economy equal their steady state values. Since different policy regimes, even not affecting the non-stochastic steady state, are associated with different stochastic steady states, in order to not neglect the welfare effects during the transition from one to another steady state, we use a conditional welfare criterion. Thus, we evaluate welfare conditional on the initial state being the non stochastic steady state[24].

We evaluate the optimal setting of monetary policy in the constrained class of simple interest rate rules. Thus, we assume that the central bank follows an interest rate rule of the form

$$R_t = \Theta(X)$$

Where X represents easily observable macroeconomic indicators tested as possible arguments of the rule

$$X = \left[R_{t-1}, \frac{\pi_{t-s}}{\pi_{ss}}, \frac{y_{t-s}}{y_{ss}}, \frac{q_{t-s}}{q_{ss}} \right]$$

with s=$\{0, 1\}$ .As implementability condition is required policies to deliver local uniqueness of the rational expectations equilibrium. Following SU we require that the associate equilibrium be locally unique. We also exclude bifurcation points. The configuration of parameters satisfying the requirements and yielding the highest welfare gives the optimal implementable rule. In characterizing optimal policy we search over a grid considering different ranges of the parameters. Then, we compute the welfare level - V_0^* - associated with the optimal rule:

$$V_0^* \equiv E_0 \left[\sum_{i=1}^{2} \eta_i \sum_{j=0}^{\infty} \beta_i^j U(c_{i,j}^*, h_{i,j}^*, L_{i,j}^*) \right]$$

where $c_{i,j}^*, h_{i,j}^*$ and $L_{i,j}^*$ denote the contingent planes for consumption, housing and labor under the optimal policy regime.

[24] An alternative to condition on a particular initial state could be to condition on a distribution of values for the initial state. Anyway, when there is a time-inconsistency problem, the optimality of the rule may depens on the initial conditions. A way to overcome this problem could be to find the rule that would prevail under commitment from a "timeless perspective" see Giannoni and Woodford (2002).

In order to compare different rules, we relate the deviations of the welfare associated to the different rules from the steady state welfare.

6 Optimal Simple Rules

In order to investigate how monetary policy should be optimally designed in a model with housing prices we maximize the households' total welfare with respect to the coefficients of a simple monetary policy rule. As in the monetary business cycle literature, we assume that the nominal interest rate responds to inflation and output and lagged interest rate. Following the literature on asset prices and monetary policy we also consider the optimality of responding to current housing prices movements. Thus, we search over the coefficient of an implicit interest rate rule - α_π, α_y, α_R and α_q - using a grid [1,3] for α_π, [0,0.9] for α_R, [0,2] for α_y and α_q[25]. Table 2 summarize the main findings. We report the welfare loss with respect to the steady state's welfare.

Optimization over this simple rule shows that the central bank should not take into account variations of housing prices from the steady state level. This means that housing prices are not the right variable to optimally design a simple monetary policy rule in this economy. The optimal rule is instead characterized by a positive strong response to inflation deviations from its target. In fact, α_π equals the upper limit of its parameter space. On the contrary, it's not optimal to react to output. These results are consistent with the one obtained by Schmitt-Grohe and Uribe (2004). They also show that it is optimal to respond to deviations of output from potential output but not to output variations. While the concept of "output gap" is well understood in models characterized only by inefficiencies related to price stickiness, the definition of potential output in our economy is not clear. Interest rate smoothing turns out to be also not optimal. Being our model economy cashless, in absence of capital, the only motive for having a smoothing on the interest rate would come from the existence of credit friction. However, it turns out that targeting the lagged interest rate it is not optimal.

Figure1 shows the combination of parameters α_π and α_q for the implicit interest rule, under which the equilibrium is determinate.

[25] We consider 25 linearly spaced points for each coefficient.

Optimal Simple Rule			
$\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1-\alpha_R)\alpha_\pi \hat{\pi}_t + (1-\alpha_R)\alpha_y \hat{y}_t + (1-\alpha_R)\alpha_q \hat{q}_t$			
$\alpha_R = 0$	$\alpha_\pi = 3$	$\alpha_y = 0$	$\alpha_q = 0$
$Welfare\ Loss = 0.00937003$			

Table 2
The welfare loss represent the loss in terms of consumption with respect to the steady state's welfare.

As it is often argued in the monetary policy literature the implicit rules are not implementable in practise. For this reason we adopt a simple rule according to which the nominal interest rate reacts to last period inflation, output and housing prices. The result turns out to be the same: targeting housing prices is not optimal.

Lagged Interest Rate Rule			
$\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1-\alpha_R)\alpha_\pi \hat{\pi}_{t-1} + (1-\alpha_R)\alpha_y \hat{y}_{t-1} + (1-\alpha_R)\alpha_q \hat{q}_{t-1}$			
$\alpha_R = 0$	$\alpha_\pi = 3$	$\alpha_y = 0$	$\alpha_q = 0$
$Welfare\ Loss = 0.00937858$			

Table 3
The welfare loss represent the loss in terms of consumption with respect to the steady state's welfare.

Figure 2-3 show the differences in the response to shocks between economies targeting or not housing prices. As standard result in these kind of models a positive (negative) transitory technology shock is shifting (increasing/decreasing) the aggregate supply having, ceteris paribus, a negative (positive) effect on inflation and a negative (positive) effect on total labor supply. Using a Taylor kind rule the CB is loosening (tightening) monetary policy. However housing prices (given the inelastic supply of housing) are positively related to total consumption – which in the model is strictly related to output, hence they will track the shock. So a CB reacting to housing price conditionally to a technology shock is partly offsetting the weight given to inflation. In other words it is like targeting the shock itself. So the distortion coming from price dispersion is much higher than the ones coming from redistribution.

From fig 2 We can see how a rule without any target on housing price reduces the impact on prices. On the other hand, there is almost no difference for housing prices. This confirms the interpretation that, being housing prices strictly related to total consumption which in turn is driven by the shock, a target on "q" is very similar to a direct reaction to "Z_t".

What is interesting is that a reaction to "q" is smoothing the impatient's expenditures, but housing, and labor effort response (viceversa for the patient). From the impatient Euler equation we can see that a full stabilization of his consumption is possible only if the lagrange multiplier that measure the degree of the credit friction (and housing) is absorbing all the variations in the nominal rate and expected inflation[26]. This means that the higher the CB reaction to inflation the higher is the adjustment weight borne by the impatient.

Also in the case of a government shock, the conditional correlation between inflation and housing prices is negative. However now the reasons are different. Total consumption, given ricardian consumers, is falling while output (total labor) is increasing. So a positive aggregate demand shock has a positive impact on inflation but a negative one on housing prices (this depends also clearly from the assumption that the government does not buy houses). Again a central bank's reaction to housing prices is offsetting the one to inflation. However, also in this case, impatient consumption and labor effort response is much more smoothed.

Table 4 compares the optimal implicit simple rule with a number of different *ad hoc* rules using the welfare based approach. As already explained in section 5.2 in order to compare different rules we relate the deviations of the welfare associated to the different rules from the steady state welfare.

[26] μ_t =-$(\beta_1 R_t - \beta_2\ \pi_{t+1})/(\beta_1 - \beta_2)$

Rule	**Welfare Loss**	**% Loss relative to optimal**
No Interest Rate Smoothing		
$\hat{\mathbf{R}}_t = \boldsymbol{\alpha}_\pi \hat{\boldsymbol{\pi}}_t + \boldsymbol{\alpha}_y \hat{\mathbf{y}}_t$		
$\alpha_\pi = 3 \quad \alpha_y = .5$	0.13115	0.1218
$\alpha_\pi = 1.5 \quad \alpha_y = .5$	1.07945	1.0701
$\hat{\mathbf{R}}_t = \boldsymbol{\alpha}_\pi \hat{\boldsymbol{\pi}}_t + \boldsymbol{\alpha}_q \hat{\mathbf{q}}_t$		
$\alpha_\pi = 3 \quad \alpha_q = 1$	0.98708	0.9777
$\hat{\mathbf{R}}_t = \boldsymbol{\alpha}_\pi \hat{\boldsymbol{\pi}}_t + \boldsymbol{\alpha}_y \hat{\mathbf{y}}_t + \boldsymbol{\alpha}_q \hat{\mathbf{q}}_t$		
$\alpha_\pi = 3 \quad \alpha_y = .5 \quad \alpha_q = 1$	1.47957	1.4702
$\alpha_\pi = 2 \quad \alpha_y = .5 \quad \alpha_q = 1$	5.20375	5.1944
Interest Rate Smoothing		
$\hat{\mathbf{R}}_t = \boldsymbol{\alpha}_R \hat{\mathbf{R}}_{t-1} + (\mathbf{1} - \boldsymbol{\alpha}_\mathbf{R}) \boldsymbol{\alpha}_\pi \hat{\boldsymbol{\pi}}_t + (\mathbf{1} - \boldsymbol{\alpha}_\mathbf{R}) \boldsymbol{\alpha}_y \hat{\mathbf{y}}_t$		
$\alpha_R = .9 \quad \alpha_\pi = 3$	0.01551353	0.0061
$\alpha_R = .6 \quad \alpha_\pi = 3$	0.00967056	$3.0053e-004$
$\alpha_R = .9 \quad \alpha_\pi = 1.5$	0.08176999	0.0724
$\hat{\mathbf{R}}_t = \boldsymbol{\alpha}_R \hat{\mathbf{R}}_{t-1} + (\mathbf{1} - \boldsymbol{\alpha}_\mathbf{R}) \boldsymbol{\alpha}_\pi \hat{\boldsymbol{\pi}}_t + (\mathbf{1} - \boldsymbol{\alpha}_\mathbf{R}) \boldsymbol{\alpha}_y \hat{\mathbf{y}}_t$		
$\alpha_R = .9 \quad \alpha_\pi = 3 \quad \alpha_y = .5$	0.16378627	0.1544
$\alpha_R = .9 \quad \alpha_\pi = 1.5 \quad \alpha_y = .5$	1.04395526	1.0346
$\hat{\mathbf{R}}_t = \boldsymbol{\alpha}_R \hat{\mathbf{R}}_{t-1} + (\mathbf{1} - \boldsymbol{\alpha}_\mathbf{R}) \boldsymbol{\alpha}_\pi \hat{\boldsymbol{\pi}}_t + (\mathbf{1} - \boldsymbol{\alpha}_\mathbf{R}) \boldsymbol{\alpha}_q \hat{\mathbf{q}}_t$		
$\alpha_R = .9 \quad \alpha_\pi = 3 \quad \alpha_q = 1$	2.10236651	2.0930
$\hat{\mathbf{R}}_t = \boldsymbol{\alpha}_R \hat{\mathbf{R}}_{t-1} + (\mathbf{1} - \boldsymbol{\alpha}_\mathbf{R}) \boldsymbol{\alpha}_\pi \hat{\boldsymbol{\pi}}_t + (\mathbf{1} - \boldsymbol{\alpha}_\mathbf{R}) \boldsymbol{\alpha}_y \hat{\mathbf{y}}_t + (\mathbf{1} - \boldsymbol{\alpha}_\mathbf{R}) \boldsymbol{\alpha}_q \hat{\mathbf{q}}_t$		
$\alpha_R = .9 \quad \alpha_\pi = 3 \quad \alpha_y = .5 \quad \alpha_q = 1$	2.10296440	2.0936
$\alpha_R = .9 \quad \alpha_\pi = 2 \quad \alpha_y = .5 \quad \alpha_q = 1$	8.95578571	8.9464

Table 4

The welfare loss represent the loss in terms of consumption with respect to the steady state's welfare. The % *Loss* is the welfare loss with respect to the optimal rule

Introducing a reaction to housing prices movements in the optimal simple rule turns out to be welfare reducing. In fact, a unitary response to current housing prices implies a 1% welfare loss with respect to the optimal rule. Differently from the most recent literature on asset prices and monetary policy we do not find that there are gains, even if often negligible, from having a target on asset prices movements. Our results clearly show that targeting housing prices is welfare reducing[27].

It is worthy to notice that a reaction to output is welfare reducing as well.

[27] Iacoviello (2004) shows that, in a model characterized by housing as a collateral for firms and households, a response to housing prices yields gains in terms of output and inflation stabilization even if not quantitatively significant. Looking at optimal simple rules in a world characterized by credit frictions at the firms' level, Faia and Monacelli (2004) show that reactions to asset prices is optimal but do not generate relevant welfare improvements.

The percentage loss is higher, the lower is the response to inflation. In fact, a 0.5 response to output, reduce welfare of about 0.1% when the response to inflation is 3 and about 1% when $\alpha_\pi$is set to 1.5. Even worse the case in which the interest rate also responds to housing prices. The welfare loss is of 1.5% in the first case and 5% in the second one in absence of interest rate smoothing and respectively of about 2% and 8% in presence of a target on lagged interest rate in addition to inflation, housing prices and output. A positive interest rate smoothing makes worse the welfare performance of the simple rules considered.

6.1 Access to the credit market and optimal monetary policy

Now we check the robustness of the results under different values for the loan-to-value ratio. In the baseline model we assume that households can borrow up to the 50% of the expected next period value of their house[28]. Independently from the value for γ the optimal result is unchanged (See Table5). Thus, the degree of access to the credit market doesn't affect the design of optimal monetary policy. The welfare loss with respect to the steady state's welfare decreases with γ.

Optimal Simple Rules				
rule	$\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1-\alpha_R)\alpha_\pi \hat{\pi}_t + (1-\alpha_R)\alpha_y \hat{y}_t + (1-\alpha_R)\alpha_q \hat{q}_t$			
optimal weights	$\alpha_R = 0$	$\alpha_\pi = 3$	$\alpha_q = 0$	$\alpha_y = 0$
γ	$\gamma = .001$	$\gamma = .3$	$\gamma = .4$	$\gamma = .6$
Welfare Loss	0.00978993	0.00962163	0.00951627	0.00917090

Table 5
The welfare loss represent the loss in terms of consumption with respect to the steady state's welfare

However, as Table 6 shows, the welfare cost of deviating from the optimal rule, increases with γ. In fact, the welfare cost of adding a target to housing prices, last period interest rate or output additional to inflation is higher the higher the degree of access to the credit market.

[28] In Italy for instance, until the mid-80 a maximum loan to value ratio of 50% was imposed by regulation. Following the process of deregulation it was increased to 75% in 1986 and to 100% in 1995

Deviating From Optimality				
γ	$\gamma = 0.001$	$\gamma = 0.3$	$\gamma = 0.4$	$\gamma = 0.6$
weights	$\alpha_\pi = 3$	$\alpha_q = 1$		
welfare loss	0.93544628	0.95492812	0.96848473	1.01134530
% *Loss relative*	0.9257	0.9453	0.9590	1.0022
weights	$\alpha_\pi = 3$	$\alpha_q = 1$	$\alpha_y = 0.5$	
welfare loss	1.39750869	1.42845289	1.44974618	1.52065058
% *Loss relative*	1.3877	1.4188	1.4402	1.5115
weights	$\alpha_\pi = 3$	$\alpha_R = 0.9$		
welfare loss	0.01464221	0.01505223	0.01525402	0.01586302
% *Loss relative*	0.0049	0.0054	0.0057	0.0067

Table 6
The welfare loss represent the loss in terms of consumption with respect to the steady state's welfare. The % *Loss* is the welfare loss with respect to the optimal rule

We now look at inflation's volatility under different rules. As expected the optimal rule, independently of γ, implies the lowest volatility. If more variables then inflation are targeted, the volatility of inflation increases. As already shown in the impulse-responses, a target on housing prices reduces the effectiveness of the target on inflation. The same holds for a target on output. The contribution of targeting lagged interest rate to inflation volatility is, instead, negligible. Consistently with the results on the cost of deviating from the optimal rule, over the different rules considered, inflation volatility slightly increases with γ Only exception is the optimal rule's case. Thus, unless the central bank follows the optimal rule, increasing the access to the credit market, and thus reducing the inefficiency, implies an increase in inflation's volatility.

Simple Rules and Inflation Volatility $\hat{R}_t = \alpha_R \hat{R}_{t-1} + (1-\alpha_R)\alpha_\pi \hat{\pi}_t + (1-\alpha_R)\alpha_y \hat{y}_t + (1-\alpha_R)\alpha_q \hat{q}_t$				
γ	$\gamma = 0.001$	$\gamma = 0.3$	$\gamma = 0.6$	$\gamma = 0.75$
$\alpha_\pi = 3$ (*optimal simple rule*)	5.1381e-004	4.9396e-004	4.4721e-004	4.0620e-004
$\alpha_\pi = 3, \alpha_q = 1$	0.0106	0.0106	0.0108	0.01082
$\alpha_\pi = 3, \alpha_R = 0.9$	7.9057e-004	7.9812e-004	8.0498e-004	7.9310e-004
$\alpha_\pi = 1.5, \alpha_y = 0.5$	0.0134	0.0135	0.01356	0.01356
$\alpha_\pi = 3, \alpha_q = 1, \alpha_y = 0.5$	0.0137	0.01378	0.0140	0.01417
$\alpha_R = 0.9, \alpha_\pi = 3, \alpha_q = 1, \alpha_y = 0.5$	0.01445	0.014866	0.01612	0.01822

Table 7

7 Conclusions

We study optimal monetary policy rules in an economy with credit market frictions at the household level and heterogeneous agents. In order to asses the role of housing prices in designing monetary policy we rely on a model built on Kiyotaki and Moore (1997) (KM henceforth).Thus, two types of agents, differing in terms of discount factors, are assumed and credit constraints arise because lenders cannot force borrowers to repay. Physical assets are then used as collateral for loans.

As a result housing prices' movements should not be a separate target variable additional to inflation in an optimally designed simple monetary policy rule. In fact, an explicit objective of housing prices stability is welfare reducing w.r.t. a strict price stability policy. Our results are in line with the idea that under normal circumstances asset prices should not be considered a target of monetary policy as already stressed by Svensson (2004)[29].

The introduction of an housing prices' target in the reaction function of the central bank implies a welfare loss that becomes quantitatively more significant the higher the degree of access to the credit market. In fact, reducing the credit market imperfections implies a decrease in inflation's volatility and a welfare improvements if and only if the central bank follows an optimally designed simple rule.

[29] Svensson argues that performing a *flexible inflation targeting* there is no need for the ECB to take asset prices movements into account.

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8 Appendix

8.1 Steady State

The real wage in steady state equals the real marginal cost:

$$w = s = \frac{\theta - 1}{\theta} \tag{ss.1}$$

Given β_1 and assuming $\pi_{ss} = 1$, we find the following steady state value for the interest rate:

$$R = \frac{1}{\beta_1} \tag{ss.2}$$

Since the deterministic steady state for the other variables is not solvable analytically, a *nonlinear rootfinding problem* arises. In a nonlinear rootfinding problem, a function f mapping $\mathbb{R}^n$ to $\mathbb{R}^n$ is given and one must compute an n-vector x, called a *root* of f, that satisfies $f(x) = 0$. In our problem the $f(x)$ is represented by the following equations:

$$-U_{L_1} = U_{c_1} w \qquad -U_{L_2} = U_{c_2} w$$

$$\tfrac{U_{h_1}}{q} = U_{c_1} (1 - \beta_1) \qquad \tfrac{U_{h_2}}{q} = U_{c_2} (1 - \beta_2) - \gamma\mu$$

$$\mu = U_{c_2} (\beta_1 - \beta_2)$$

$$c_2 = b_2 \left(\frac{1}{R} - 1 \right) + wL_2$$

$$b_2 = \gamma q h_2 \qquad b_1 = \tfrac{nb_2}{(1-n)}$$

$$qh = q(1 - n)h_{1t} + nh_{2t}$$

$$h_1 = \tfrac{qh_1}{q} \qquad h_1 = \tfrac{qh_2}{q}$$

$$h = 1$$

$$c = (1 - n)c_1 + nc_2 \qquad L = (1 - n)L_1 + nL_2$$

$$y = c \qquad c = L$$

Where

$$U_{c_i} = c_i^{-\varphi_c} \qquad U_{L_i} = -\nu_L L_i^{+\varphi_L} \qquad U_{h_i} = \tfrac{\nu_h}{h_i}$$

We implement a numerical algorithms for solving the system quickly and accurately.

$$U(c_{it}, h_{it}, L_{it}) = \frac{c_{it}^{1-\varphi_c}}{1-\varphi_c} + \nu_h \ln h_{it} - \nu_L \frac{L_{it}^{1+\varphi_L}}{1+\varphi_L}$$

8.2 Solution Method

The set of equilibrium conditions and the welfare function of the model can be written as:

$$E_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0$$

where E_t is the expectation operator,y_t is the vector of non-predetermined variable and x_t of predetermined variables. This last vector constists of x_t^1 endogenous predetermined state variables and x_t^2 exogenous state variables. In the baseline case of our model we have:

$y_t = [\pi_t, q_t, w_{t,} y_t, L_t, c_t, s_t, V_{1t}, V_{2t}]'$

$x_t^1 = [b_{2t}, h_{2t}, R_t]'$ $\qquad x_t^2 = [Z_t, G_t]'$

The welfare function is given by the conditional expectation of lifetime utility as of time zero: $V_{it} \equiv \max E_t \left[\sum_{j=0}^{\infty} \beta_i^j U(c_{i,t+j}, h_{i,t+j}, L_{i,t+j})\right]$.Thus, in the optimum it will be: $V_{it} = U(c_{i,t}, h_{i,t}, L_{i,t}) + \beta_i E_t V_{it+1}$.We add to the system of equilibrium conditions, two equations in two unknons: V_{1t} and V_{2t}.

The vector of exogenous state variables follows a stochastic process:

$$x_{t+1}^2 = \Delta x_t^2 + \eta \varepsilon_{t+1} \qquad \varepsilon_t \sim iidN(0, \Sigma)$$

where η a matrix of known parameters[30].

The solution of the model is given by the policy function and the transition function:

$y_t = g(x_t, \sigma)$ $\qquad x_t = h(x_t, \sigma) + \eta \varepsilon_{t+1}$ where σ^2 is the variance of the shocks.

Following Schmitt-Grohe and Uribe (2003), we compute numerically the second order approximation of the functions g and h around the non-stochastic steady state $x_t = x$ and $\sigma = 0$. The solution of the system gives an evolution of the original variables of the form

$$y_t = \alpha_1 x_t^1 + \alpha_2 x_t^2 + \alpha_3 \left(x_t^1\right)^2 + \alpha_4 \left(x_t^2\right)^2 + \alpha_5 x_t^1 x_t^2 + \eta \sigma^2$$

[30] In our model, since the shocks ar uncorrelated, η is a vector.

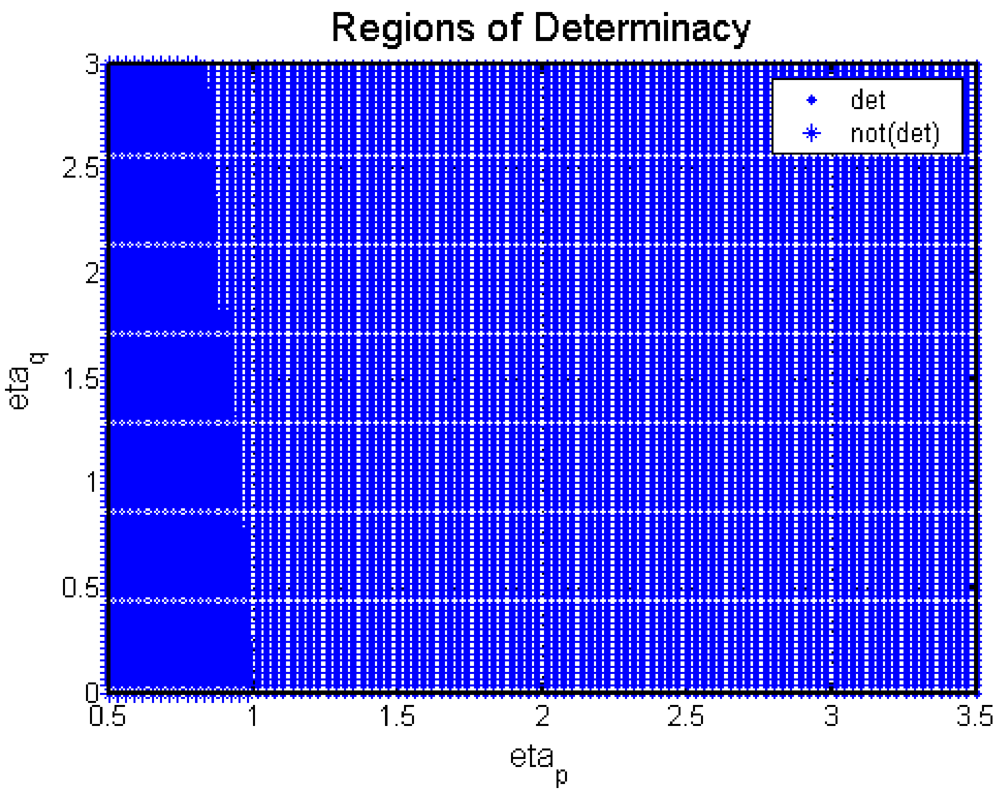
where all the variables are expressed in log deviations. The solution also depends on the variance of the shocks.

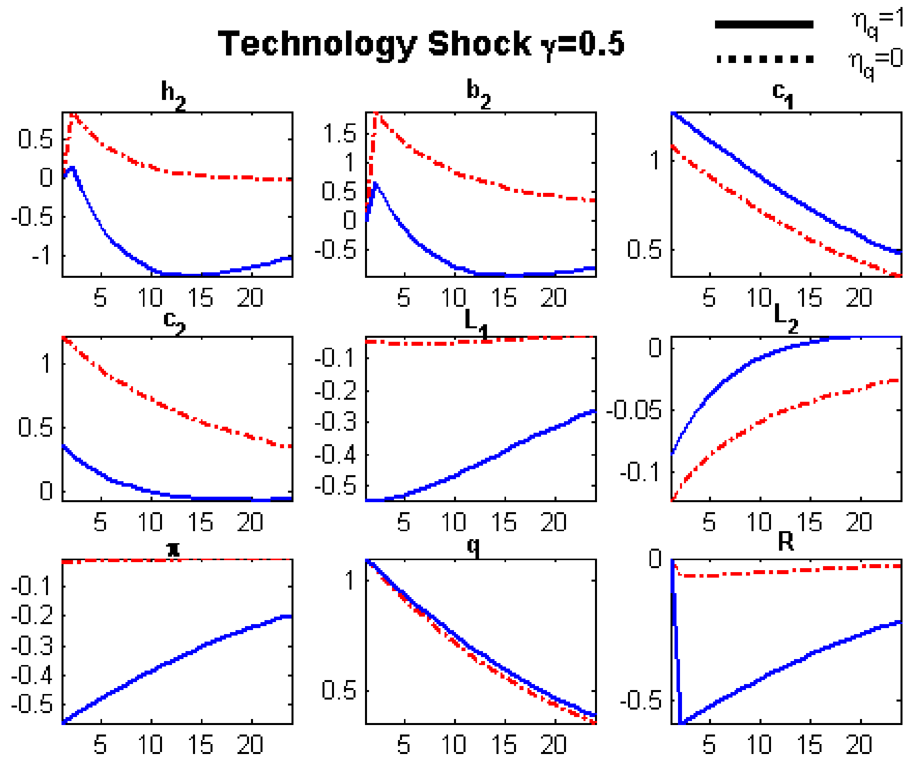
Since we evaluate the welfare functions conditional on having at t=0 all the variables of the economy equal to their steady state values, the second order approximate solution for the welfare functions is given by[31]:

$$V_{it} = \eta_{V_i} \sigma^2$$

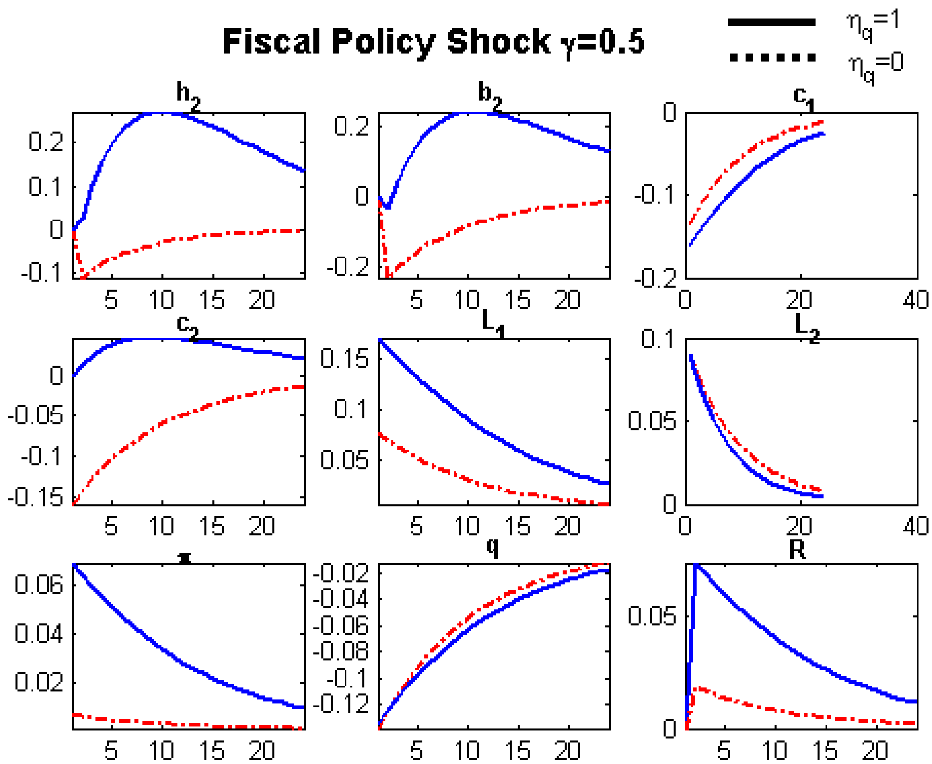
where η_{V_i} is a vector of known parameters that depends on the monetary policy used and σ^2 is the variance of the shocks

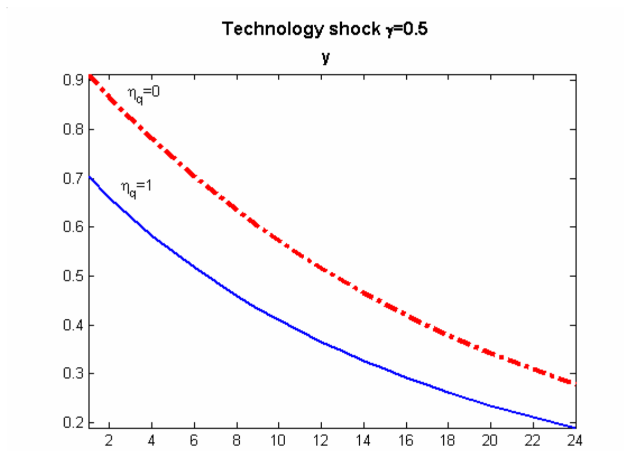
[31] Since in the system all the variables are in log-deviation from their steady state values, they equals zero.


Regions of Determinacy
det
not(det)
eta_q
eta_p
0
0.5
1
1.5
2
2.5
3
3.5

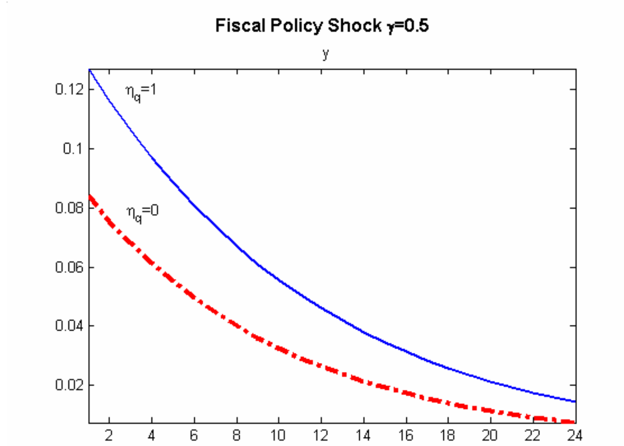


Technology Shock γ=0.5
ηq=1
ηq=0
h2
b2
c1
c2
L1
L2
π
q
R


Fiscal Policy Shock γ=0.5
η_q=1
η_q=0
h_2
b_2
c_1
c_2
L_1
L_2
π
q
R



Technology shock γ=0.5
y
η_q=0
η_q=1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
2 4 6 8 10 12 14 16 18 20 22 24



Fiscal Policy Shock γ=0.5
y
η_q=1
η_q=0
0.12
0.1
0.08
0.06
0.04
0.02
2 4 6 8 10 12 14 16 18 20 22 24