Central Bank Communication and Interest Rate Rules

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Abstract
The central bank's influence on private sector expectations is an important channel of monetary policy transmission. Therefore, it is important for a central bank to pay attention to its communication. It should also be aware that the public might not be willing or able to process all the information that the central bank provides. In this paper, we explicitly take the public's information processing constraint into account when we design the interest rate rule. We find that if inflationary expectations are important relative to the output gap in determining current inflation, the central bank should be less activist when the public has a tighter information processing constraint.

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1. Introduction

In recent years, central banks have put more and more effort in communicating their policy actions and their policy strategy to the public. By publishing reports and bulletins and by giving press conferences they try to explain what they are doing, why they are doing it and how they are doing it. There are different explanations to be found for this (change in) behavior. One important reason is that the increased legal independence of many central banks has created the legal or moral obligation of the central bank to be accountable to the public for its actions. In this paper, we take the central bank's obligation to communicate as given and analyze the impact of communication on the effectiveness of monetary policy.
Private sector expectations about future macroeconomic developments form one of the channels that the central bank can use to influence inflation and output. Blinder (1998) argues that openness and communication with the public improve the effectiveness of monetary policy as a macroeconomic stabilizer because: "Central banks generally control only the overnight interest rate, an interest rate that is relevant to virtually no economically interesting transactions. Monetary policy has important macroeconomic effects only to the extent that it moves financial market prices that really matter – like long-term interest rates, stock market values, and exchange rates."
A lot of information about the economy is available for free, but the public's capacity to process this information is limited. The central bank plays a central role in informing the public about the economic situation and outlook (see also Amato, Morris and Shin, 2003). When the central bank communicates effectively with the public, it can influence or even shape public expectations. When providing information to the public, the central bank should also take the limited information processing capacity into account. By applying information theory as developed by Shannon (1948), Sims (1998, 2001) has studied the effects of constrained information processing on the behavior of macroeconomic timeseries. Adam (2003) uses the information channel concept to look at optimal monetary policy when firms have private information about shocks hitting the economy. We apply the same concept to expectation formation in a monetary policy framework where the central bank communicates its private information about shocks hitting the economy to the public through an information channel with limited capacity.
We assume that the central bank has private information (a perfect forecast) about next period's demand and inflatio. Based on this information, the central bank communicates its future policy action (in this case future real interest rates) to the public. The public receives this signal through a limited capacity channel and forms its expectations about future output

and inflation. We find that the limitation on information processing induces the central bank to react less strongly to shocks to aggregate demand and inflation if inflationary expectations are important relative to the output gap in determining current inflation.

The structure of the remainder of the paper is as follows. First, we present the basic model of monetary policy. Then, we introduce the concept of the capacity constraint on information transmission. In the next section, we analyze the way the public uses the information provided by the central bank to form its expectations. Then, we look at the consequences for the complexity of the optimal real interest rate rule and finally we conclude.

2. Model

We use the New IS-LM model, as described by King (2000), to illustrate the effect of incorporating information processing capacity constraints in a monetary policy game with a central bank that sets interest rates and the public that forms expectations. Both current aggregate demand and the current inflation rate depend on the public's expectation of future demand and inflation. This feature enables the central bank to influence outcomes by shaping expectations. The model can be built up from microfoundations and is analyzed by using rational expectations.

Our model consists of three equations. We have a forward-looking IS equation that makes current demand (output) y_t dependent on expected future demand $E_t y_{t+1}$, the real interest rate r_t and a demand shock x_{dt}.

IS: $y_t = E_t y_{t+1} - s(r_t - r) + x_{dt}$ **(1)**

We have an expectational New-Keynesian Phillipscurve that relates current inflation $\mathbf{p}_t$ to expected future inflation $E_t\mathbf{p}_{t+1}$, the current output gap $(y_t - \bar{y}_t)$ and an inflation shock x_{pt} (which can be seen as a supply shock).

PC: $\mathbf{p}_t = bE_t\mathbf{p}_{t+1} + j\,(y_t - \bar{y}_t) + x_{pt}$ **(2)**

And, finally, we have the Fisher equation that makes the nominal interest rate R_t equal to the real interest rate plus expected future inflation.

Fischer: $R_t = r_t + E_t\mathbf{p}_{t+1}$ **(3)**

For simplicity and without loss of generality, we normalize potential output and the equilibrium real rate of interest at zero: $\bar{y}_t = 0_t, r = 0$

The central bank sets monetary policy according to a real interest rate rule that is a weighted average of a sophisticated rule and a simple fixed interest rate rule (*r_t=0*) with respective weights *1/(A+1)* and *A/(A+1)* with *A ³ 0*. If *A = 0*, the interest rate rule collapses to the sophisticated rule. For large *A*, the central bank hardly reacts to shocks and comes close to the fixed interest rate rule.

Interest rate rule: $r_t = \frac{1}{s(A+1)}\left(x_{dt} + \frac{x_{pt}}{j}\right)$ **(4)**

The sophisticated rule tries to stabilize the rate of inflation by reacting to demand and inflation shocks[1]. In this case, the parameter for stabilizing inflation shocks is chosen such that inflation is fully stabilized at the expense of output variability.[2] A small A implies therefore a low variability of inflation and a high variability of the output gap.

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[1] Instead of reacting to and communicating about shocks, the model can also be interpreted more generally as the central bank having private information about the state of the economy.

[2] With this rule, the central bank does not use its forecast of future output and inflation to stabilize current output and inflation. It would be optimal for the central bank to use this information but including it at this stage would complicate computations and distract from the focus of the paper. The main mechanism that sending a volatile signal that reacts to all information that reaches the central bank increases noise in the public's expectations if the capacity constraint on information processing is binding will remain valid.

3. Limited capacity for transmission of information

There are many ways for a central bank to communicate with the public. In this paper, we assume that the central bank communicates with the public by releasing information about future monetary policy. Next period's real interest rate r_{t+1}, based on its private (perfect) forecasts[3] of demand shock x_{dt+1} and shock to the Phillipscurve x_{pt+1} (see eq. **(4)**) is communicated. The private sector devotes limited capacity C to observing the information released by the central bank. As a result of this constraint, the private sector knows future monetary policy with an error $\boldsymbol{e}$. According to results derived in information theory, the variance of this error depends negatively on the capacity of the information channel. This can be modeled as a signal r_{t+1} that is sent over a channel with limited capacity C and the receiver (i.e. the public) observes the signal with noise $\boldsymbol{e}_t$. The public observes X_t:

$X_t = r_{t+1} + e_t$, where $r_t \sim N(0, s_r^2)$ and $e_t \sim N(0, s_e^2)$ and r_{t+1} and $\boldsymbol{e}_t$ are independent.

In order to model the information channel with limited capacity, we define a measure of uncertainty of a random variable, called *entropy*. This measure has several attractive properties compared to other measures of uncertainty (see Cover and Thomas, 1991 for a textbook treatment). The entropy for the input r_t is defined as (we have dropped time-subscripts):

$$H(r) = -\int_{-\infty}^{\infty} p(r) \ln p(r) dr = \tfrac{1}{2}(\ln 2pe + \ln s_r^2) \qquad \textbf{(5)}$$

where $p(r)$ is the probability density function of r, which we choose to be normal. The entropy of the stochastic variable r is an increasing function of its variance s_r^2.

Based on this definition we compute the conditional and unconditional entropy of output signal X.

$$H(X|r) = H(e) = \tfrac{1}{2}(\ln 2pe + \ln s_e^2) \qquad \textbf{(6)}$$

$$H(X) = \tfrac{1}{2}(\ln 2pe + \ln(s_r^2 + s_e^2))$$

[3] Introducing forecast errors would complicate the model without changing qualitative results as long as the central bank has some private information that is of interest to the public.

Unless two variables are independent, conditioning reduces the entropy. The information about r obtained by observing X is called the *mutual information*. Using a basic theorem from information theory (see, for instance, Cover and Thomas, 1991) we can write the following expression:

$$I(r,X) = H(r) - H(r|X) = H(X) - H(X|r) \qquad \textbf{(7)}$$

This expression measures how much the uncertainty about input r is reduced by observing output X. Although this interpretation of the first equality in **(7)** makes intuitive sense, we use the last expression because it is computationally more attractive:

$$I(r,X) = H(X) - H(X|r) = \tfrac{1}{2}(\ln 2\pi e + \ln(\sigma_r^2 + \sigma_\varepsilon^2)) - \tfrac{1}{2}(\ln 2\pi e + \ln \sigma_\varepsilon^2) = \tfrac{1}{2}\ln\left(1 + \frac{\sigma_r^2}{\sigma_\varepsilon^2}\right) \qquad \textbf{(8)}$$

As is clear from **(8)**, the mutual information $I(r,X)$ is an increasing function of the signal-to-noise ratio $\frac{\sigma_r^2}{\sigma_\varepsilon^2}$. The larger the variance of the noise, the lower the mutual information.

The *capacity* of the channel is defined as its maximum mutual information. Since communication goes through a channel with limited capacity C, the maximum reduction in entropy that can be achieved by communicating is C:

$$I(r,X) = \tfrac{1}{2}\ln\left(1 + \frac{\sigma_r^2}{\sigma_\varepsilon^2}\right) \le C \qquad \textbf{(9)}$$

When we assume that capacity is used to the maximum, the capacity constraint is binding and the previous equation holds with equality. It follows that

$$\sigma_\varepsilon^2 = \frac{\sigma_r^2}{e^{2C} - 1} \qquad \textbf{(10)}$$

The intuition behind this result is that with infinite capacity of the information channel *($C \to \infty$)* the variance of the noise goes to zero and the receiver observes the central bank's signal about the future real interest rate without noise. If, on the other hand, capacity tends to zero the variance of the noise tends to infinity. In that case, the uncertainty about r after observing X equals the uncertainty of r before observing X, so that the observation of X adds no information at all. With a low capacity, the noise dominates the signal.

4. Expectation formation

As described above, the public receives an output-signal X that indicates next period's interest rate. This signal is received through a channel with limited capacity C.

$$X_t = r_{t+1} + \varepsilon_t = \frac{1}{s(A+1)}\left(x_{dt+1} + \frac{x_{pt+1}}{\varphi}\right) + \varepsilon_t \tag{11}$$

with $\varepsilon_t \sim N\left(0, \frac{\varphi^2\sigma_d^2 + \sigma_p^2}{\varphi^2 s^2 (A+1)^2 (e^{2C}-1)}\right)$ where C is the capacity of the communication channel.

With this signal **(11)**, the agent solves a signal-extraction problem similar to Lucas (1973) to form expectations about the shocks that will hit next period's output and inflation given all the information that is available to him:

$$E_t\left[x_{dt+1} \middle| X_t\right] = \frac{\sigma_d^2 s X_t (A+1)}{\sigma_d^2 + \frac{\sigma_p^2}{\varphi^2} + \sigma_\varepsilon^2 s^2 (A+1)^2} = \frac{\varphi^2 \sigma_d^2 s X_t (A+1)(1-e^{-2C})}{\varphi^2\sigma_d^2 + \sigma_p^2} \tag{12}$$

$$E_t\left[x_{pt+1} \middle| X_t\right] = \frac{\varphi \sigma_p^2 s X_t (A+1)(1-e^{-2C})}{\varphi^2\sigma_d^2 + \sigma_p^2} \tag{13}$$

The higher the information processing capacity C, the more weight the public attaches to output-signal X in forming expectations about future shocks and the larger the central bank's influence on the public's expectations. In the extreme case where the capacity goes to zero, the signal contains no information and the conditional expectation of future shocks equals the unconditional expectation of zero.

Based on the expected shocks and the expected interest rate, it will form expectations of next period's output and inflation. Leading equations **(1)** and **(2)** one period, using signal X for real interest rate *r* and assuming that the public expects the economy to return back to normal (output and inflation equal to zero) at *t+2* we obtain the following expressions:

$$E_t\left[y_{t+1}\middle|X_t\right] = sX_t\frac{\left(j^2 s_d^2 A - s_p^2\right)\left(1-e^{-2C}\right)}{j^2 s_d^2 + s_p^2} \tag{14}$$

$$E_t\left[p_{t+1}\middle|X_t\right] = sAX_t j\,(1-e^{-2C}) \tag{15}$$

5. Optimal rule with information processing constraint

The expectations of inflation and output, conditional on the information received from the central bank through the limited-capacity channel, can be used to compute the central bank's expected loss. Making use of the assumption that demand and inflation shocks are serially uncorrelated, we obtain the following expressions for squared output and inflation:

$$\begin{aligned} Ep_t^2 &= \left(bE_t p_{t+1} + f y_t + x_{pt}\right)^2 = E\left(bE_t p_{t+1} + j\,(E_t y_{t+1} - sr_t + x_{dt}) + x_{pt}\right)^2 \\ &= E\left(bE_t p_{t+1} + jE_t y_{t+1}\right)^2 + E\left(-sj\,r_t + j\,x_{dt} + x_{pt}\right)^2 \end{aligned} \tag{16}$$

$$Ey_t^2 = E\left(E_t y_{t+1}\right)^2 + E\left(-sr_t + x_{dt}\right)^2 \tag{17}$$

These two components form the central bank's quadratic expected loss function *L*

$$EL_t = Ep_t^2 + Ey_t^2 \tag{18}$$

We are interested in the effect of the information processing constraint on the interest rate rule that the central bank prefers. The central bank chooses its interest rate rule by minimizing the stated expected loss with respect to the weight the central bank gives to the simple fixed interest rate rule, *A*. The first-order condition yields for the optimal A:

$$A^* = \frac{s_p^2\left((e^{2C}-1)(1+j^2(1+b)) + e^{2C}\right)}{j^2\left(s_d^2\left((e^{2C}-1)(1+j^2(1+b)^2) + e^{2C}(1+j^2)\right) + s_p^2\left((e^{2C}-1)(b+b^2) + e^{2C}\right)\right)} \tag{19}$$

Now, it is interesting to determine how the information processing capacity constraint influences the optimal interest rate rule (within the class of rules given by eq. **(4)**).
We find that

$$\frac{\partial A^*}{\partial C} \begin{array}{l} \geq 0 \text{ iff } j^2 \geq b - \dfrac{1}{1+b} \\ < 0 \text{ iff } j^2 < b - \dfrac{1}{1+b} \end{array} \qquad \textbf{(20)}$$

The intuition for this result is as follows. Better communication (a larger *C*) gives a higher weight to expectations about future inflation and the future output gap in the determination of current inflation. This is relevant for the optimal interest rate rule. If b is large relative to j, future *inflationary expectations* play an important role in current inflation (see eq. **(2)**). If the central bank communicates with the public through a large-capacity channel (large *C*), a more sophisticated policy rule helps to stabilize inflationary expectations and $\frac{\partial A^*}{\partial C} < 0$. If, on the other hand, b is small relative to j, the current and future *output gaps* play an important role in current inflation. A larger capacity C makes future output gaps important, which are stabilized best by a simple rule, so a large A.
The ability to communicate with the public and influence expectations creates a trade-off for the central bank, comparable to the credibility vs. flexibility discussion back in eighties. When inflationary expectations are important relative to the output gap in determining current inflation, a larger information processing capacity calls for a more sophisticated rule. This effect is driven by the assumption that the sophisticated rule stabilizes inflation (see eq. **(4)**).

6. Concluding remarks

This paper discusses the role of communication in monetary policy. In a standard monetary policy game, based on the New IS-LM Model with a prominent role for expectations, we explicitly incorporate limited information processing capacity by the public. A tight information processing constraint implies that the public finds it difficult to process a very volatile signal sent by the central bank. If the central bank regards its power to influence public expectations about future inflation and output as an important instrument to achieve its goals, it should take this constraint into account when setting monetary policy. We show that, if inflationary expectations play an important role in the inflation process relative to the output gap, the central bank should be less activist in its response to shocks in the economy when the information processing capacity constraint on the side of the public is stricter. We intend to use this model in future analyses of central bank communication and expectations formation. Among other things, we plan to do an empirical analysis, make the capacity of the information channel a decision variable for the public and introduce different information processing capacities for different groups of agents.

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