Net foreign assets, interest rate policy, and macroeconomic stability

Ludger Linnemann*and Andreas Schabert†

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Abstract: We examine the role of foreign debt for the requirements of saddle path stability in a sticky-price small open economy model where the central bank sets the nominal interest rate and home residents are net borrowers on the international capital market. Uncovered interest rate parity does not hold as the risk of defaulting on foreign debt is increasing in its real value. Under this asset market imperfection, a monetary policy strategy of letting the nominal interest rate increase strongly in response to domestic inflation (which would be stabilizing with perfect asset markets) entails the risk of setting the economy on an explosive path with unbounded foreign debt accumulation. However, the central bank can restore macroeconomic stability if it takes current account dynamics into consideration and reduces the interest rate when indebtedness rises, or alternatively if it refrains from aggressively reacting on inflation – e.g. by pegging the interest rate.

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*Corresponding author. University of Cologne, Department of Economics (Staatswiss. Seminar), D-50923 Koeln, Germany, email: linnemann@wiso.uni-koeln.de, fax: +49/221/470-5077, tel: +49/221/470-2999.

†University of Cologne, Department of Economics (Staatswiss. Seminar), D-50923 Koeln, Germany, email: schabert@wiso.uni-koeln.de, fax: +49/221/470-5077, tel: +49/221/470-4532.

1 Introduction

The role of current account deficits and a country's net foreign asset position for macroeconomic stability is a subject of ongoing debate. Traditionally, the intertemporal optimizing view of the current account (as summarized in Obstfeld and Rogoff, 1996) has been interpreted as implying that foreign debt accumulation should not be seen as a macroeconomic problem since it reflects optimal consumption smoothing over time. However, this view is being debated in the literature on currency crises (see the survey in Edwards, 2002) mainly on empirical grounds. The recent theoretical literature featuring the intertemporal optimizing model coupled with short-run nominal price rigidities, often referred to as the 'New open economy macroeconomics' (see Lane, 2001) tends to avoid explicit modelling of foreign assets, presumably because its consideration can lead to indeterminacy of the steady state and unit root dynamics which defy the study of local equilibrium dynamics based on log-linear approximations. Thus, current account dynamics are often excluded by using specific assumptions on preferences or the structure of asset markets (e.g. Corsetti and Pesenti, 2000, Schmitt-Grohé and Uribe, 2002).

The present paper combines elements from both strands of the literature in analyzing the role of net foreign assets for macroeconomic stability in an indebted small open economy with short-run price stickiness. We derive the conditions under which a central bank that sets the short-run nominal interest rate on domestic debt ensures stability in the sense that explosive or self-fulfilling equilibria are prevented from occuring. Interest rate setting rules in the presence of current account dynamics have also recently been studied by Cavallo and Ghironi (2002). They use an overlapping generations model and derive the welfare properties of Taylor (1993)-style rules. The stability analysis carried out in the present paper can be seen as complementary to their welfare analysis, although the reason why net foreign assets matter is different here.

In accordance with empirical evidence that international interest rate differentials reflect the distribution of net foreign assets (Lane and Milesi-Ferretti, 2001), we assume an asset market imperfection that consists of the risk that residents of the home country may default on their external debt obligations with a probability that depends positively on the stock of foreign debt. There is thus a default risk premium on domestic interest rates (similar to Turnovski, 1997) that prevents the standard uncovered interest rate parity condition from being fulfilled. As a consequence, a real depreciation lowers the average real return on domestic bonds due to the implied increase in foreign indebtedness, and thus in the probability of default. Given that arbitrage freeness requires a future appreciation in this situation, the real exchange rate will return to its steady state over time. This is what is required to prevent the real value of debt from exploding. However, this stabilizing debt feedback mechanism can be disturbed by the central bank's interest rate setting policy. A depreciated real exchange rate will be associated high aggregate demand and thus rising inflation. If the central bank aims to target inflation using a simple Taylor (1993)-style rule with a high coefficient on inflation, it will then raise the real return on domestic bonds, which in equilibrium is associated with a future real exchange rate depreciation, and thus future growth in inflation and real foreign debt. Thus, a central bank behavior which is known to result in a uniquely determined and stable equilibrium in closed economy models (Clarida et al., 2000; Benhabib et al., 2001, Woodford, 2001), or in open economy models where perfect capital markets imply that interest bearing assets are irrelevant for the determination of output and inflation (Linnemann and Schabert, 2001), can entail explosiveness in the model presented here, even for a very slight dependence of default risk on debt.

Based on these results, it can be conjectured that the net foreign asset position can potentially be a useful monetary policy indicator, in that a policy which takes the information content of foreign assets in this model into consideration might be able to target inflation and at the same time to avoid destabilizing the economy via the aforementioned debt spiral. In particular, we show that the central bank can restore macroeconomic stability – in the sense of a saddle stable equilibrium path – even for highly inflation-reactive interest rate policies, if it lowers the nominal interest rate in face of an increase in foreign debt. Put differently, central bank actually raises the likelihood that the economy is set on an explosive debt path if it tackles higher foreign debt by a contractionary monetary policy measure. Thus, the analysis presented in this paper reveals the potential stability gain of considering the current account dynamics for a central bank, which actively aims to target macroeconomic variables such as inflation and output through its interest rate policy. Alternatively, our results imply that an interest rate peg – which in many models is found to be associated with indeterminacy of prices and real aggregates – can be a sensible strategy for a central bank which predominantly fears the emergence of a debt crisis.

The remainder is organized as follows. Section 2 develops the model. In section 3, we examine the local dynamics of the model allowing for perfect and imperfect asset markets. Section 4 shows how foreign debt as an monetary policy indicator alters the results. Section 5 concludes.

2 The model

The model extends a continuous time version of a small open economy model with staggered prices closely related to Parrado and Velasco (2002), Gali and Monacelli (2002), and Kollmann (2001). Following the former, we assume that there is an integrated world asset market that allows consumption risk to be shared internationally. However, the asset market is imperfect in the sense that there is an uninsurable risk of capital loss due to debtor default associated with holding domestic bonds. The crucial assumption is that the risk of domestic debtors defaulting on their bonds is increasing in the level of external indebtedness.

Time arguments are suppressed wherever possible to lighten the notation. Lower case letters denote real variables, upper case letters denote nominal variables. A dot over a variable denotes a time derivative, a bar over a variable denotes a steady state value. Asterisks are used to mark foreign variables. The subscript H (F) characterizes variables of home (foreign) origin. Thus, for example, c_F means consumption of foreign goods at home (i.e., imports); while P_F denotes their price (in home currency) at home, P_F^* is the corresponding foreign currency price. The small open economy assumption implies, among other things, that starred variables are exogenous to the home economy.

Households The economy is populated by a continuum of identical and infinitely lived households of measure one. Households' instantaneous utility u is defined over consumption and leisure, and their objective is to maximize

$$\int_0^\infty \exp\{-\rho t\} \left(\frac{c^{1-\sigma}}{1-\sigma} - l^\gamma \right) dt, \qquad \rho > 0,\ \sigma > 0,\ \gamma \geq 1, \tag{1}$$

where c is consumption, and l is labor supply. Households are endowed with an initial stock of nominal financial wealth $A_0 > 0$. They have access to two types of assets: domestic currency denominated bonds, B, which are of infinitesimal maturity and pay a risky nominal return R, and foreign currency denominated bonds, where $\mathfrak{B}$ denotes the stock of foreign bonds held by domestic residents. The average probability that a domestic household defaults on its bond emissions is $\delta(d) \in (0,1)$, where $d \equiv D/P$ and D is aggregate nominal external debt and P is the consumption based priced level. External debt is defined as net foreign liabilities, i.e. domestic bonds held by foreigners (called B^f) less foreign bonds held by domestic residents, $e\mathfrak{B}$, where e is the nominal exchange rate. Thus, real foreign debt is

$$d \equiv \frac{B^f - e\mathfrak{B}}{P} = b^f - x\mathfrak{b}, \tag{2}$$

where $b^f \equiv B^f/P$, $\mathfrak{b} \equiv \mathfrak{B}/P^*$ (with P^* the foreign price level), and

$$x \equiv \frac{eP^*}{P} \tag{3}$$

is the real exchange rate. As an implication of the assumption that the home country is a small economy, we assume that B^f is of a negligible magnitude, and use $d = -x\mathfrak{b}$ henceforth. It is assumed that $\delta'(d) \geq 0$, and the analysis is limited to the case of $d > 0 \Leftrightarrow \mathfrak{b} < 0$ to exclude corner solutions.

When making his optimal decisions, each household takes d, which is an economywide aggregate variable, as given and constant, although in the aggregate d will be determined endogenously by the optimal choices of all households. The flow budget constraint is

$$e\dot{\mathfrak{B}} + \dot{B} = Pwl + P\kappa_H + eR^*\mathfrak{B} + R[1-\delta(d)]B - Pc,$$

or in terms of real financial wealth $a \equiv A/P$ with $A \equiv B + e\mathfrak{B}$,

$$\dot{a} = \{R[1-\delta(d)] - \pi\}a - \{R[1-\delta(d)] - R^* - \frac{\dot{e}}{e}\}x\mathfrak{b} - c + wl + \kappa_H, \tag{4}$$

where $\pi \equiv \dot{P}/P, w, R^*$, and κ_H denote the (consumption price) inflation rate, the real wage, the foreign nominal interest rate, and real dividends from domestic firms, respectively. The assumption of imperfect asset markets leads to the appearance of $R[1-\delta(d)]$, which is the nominal home bond interest rate adjusted for the average risk of default; with positive foreign debt, $\delta(d) > 0$ and in equilibrium there must be a risk premium on the home interest rate to exclude arbitrage opportunities (as e.g. in Turnovsky, 1997). No such default risk premium is associated with foreign assets. Ponzi games are ruled out through

$$\lim_{t\to\infty} a(t) \exp\left[-\int_0^t (R(v)[1-\delta(d)] - \pi(v))\, dv\right] \geq 0.$$

The household's first order conditions for consumption, labor supply, foreign bonds, and real wealth are, denoting the shadow price of wealth as λ,

$$\lambda = c^{-\sigma}, \tag{5}$$

$$w\lambda = \gamma l^{\gamma-1}, \tag{6}$$

$$0 = \lambda\left\{R[1-\delta(d)] - R^* - \frac{\dot{e}}{e}\right\}x, \tag{7}$$

$$-\frac{\dot{\lambda}}{\lambda} = R[1-\delta(d)] - \pi - \rho. \tag{8}$$

Additionally, the flow budget constraint (4) and following transversality condition hold in the optimum:

$$\lim_{t\to\infty} a(t) \exp\left[-\int_0^t (R(v)[1-\delta(d(v))] - \pi(v))\, dv\right] = 0. \tag{9}$$

Note that (7) is a modified version of an uncovered interest rate parity condition, where the modification consists of the fact that the level of external debt is relevant in determining whether domestic and foreign nominal interest rates satisfy arbitrage freeness, given expectations of future nominal currency depreciation.

The consumption basket c is a CES aggregate of goods of domestic origin, c_H, and of foreign origin, c_F,

$$c = \left[(1-\vartheta)^{\frac{1}{\eta}} c_H^{\frac{\eta-1}{\eta}} + \vartheta^{\frac{1}{\eta}} c_F^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \qquad \eta > 1, \qquad 0 < \vartheta < 1.$$

Given aggregate consumption c, the demands for goods of home and foreign origin are

$$c_H = (1-\vartheta) \left(\frac{P_H}{P} \right)^{-\eta} c, \tag{10}$$

$$c_F = \vartheta \left(\frac{P_F}{P} \right)^{-\eta} c, \tag{11}$$

where P_H and P_F are the price indices of the domestically produced and foreign produced consumption good, respectively, and the overall price index of consumption goods P at home (the CPI, henceforth) is

$$P = \left[(1-\vartheta) P_H^{1-\eta} + \vartheta P_F^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{12}$$

Firms Intermediate production in the home country is conducted by a continuum of monopolistically competitive firms each producing a differentiated intermediate good being indexed on $i \in [0, 1]$. Technology is linear in labor l,

$$y_i = y_{H,i} + y_{H,i}^X = l_i, \tag{13}$$

where y_i is production of firm i, $y_{H,i}$ is production for the home market, and $y_{H,i}^X$ is exports. Final goods producers are perfectly competitive and combine the differentiated intermediate inputs using a CES aggregation technology. The aggregators for total production for the home market, y_H, and total exports y_H^X, are

$$y_H = \left[\int_0^1 (y_{H,i})^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \qquad y_H^X = \left[\int_0^1 (y_{H,i}^X)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \qquad \epsilon > 1.$$

Intermediate firm i sets the price for its good $P_{H,i}$ in home currency (there is no pricing to market with respect to the export market), taking into account that the final producer's cost minimizing demand for the individual good is

$$y_i = \left(\frac{P_{H,i}}{P_H} \right)^{-\epsilon} (y_H + y_X) = \left(\frac{P_{H,i}}{P_H} \right)^{-\epsilon} y. \tag{14}$$

Zero profits in the final goods market then imply that the price index of home produced goods is

$$P_H = \left[\int_0^1 P_{H,i}^{(1-\epsilon)} di \right]^{\frac{1}{1-\epsilon}} . \quad (15)$$

The optimal price setting decision of an intermediate producer is modelled as in Calvo [?] and Yun [?]; the continuous time version used here is due to Benhabib et al. [?]. Firms set prices to maximize a discounted stream of current and future real profits. The nominal rigidity is that firms may freely adjust prices in any given point in time only when they receive a random signal that allows them to do so; otherwise, they must let their prices mechanically grow with the steady state rate of domestic producer price inflation $\overline{\pi}_H$, where $\pi_H \equiv \dot{P}_H/P_H$. The waiting time interval until the arrival of a random price-change signal is assumed to follow an exponential distribution, such that the probability of not being allowed to change prices between dates t and $s > t$ is $\exp(-\xi[s-t])$, where $\xi > 0$ is a parameter. Let Q_t be the nominal price that firm i sets in period t if it receives the signal permitting to freely adjust its price. Note that we write the problem for a general constant returns to scale production function, which implies that total costs can be separated in marginal costs and output, and perfect factor mobility ensures that marginal cost is a function of aggregate nominal factor prices only and hence independent of firm specific variables. Thus, all firms being allowed to adjust their prices will choose the same price, so that we abstain from indexing Q_t with a firm index from the outset. The firm's problem then is

$$\max_{Q_t} \int_t^\infty \exp\{-(\xi+\rho)(s-t)\}\lambda_s \left[(Q_t \exp\{\overline{\pi}_H(s-t)\} y_{is}(Q_t) - MC_s y_{is}(Q_t))/P_{Hs} \right] ds, \quad (16)$$

where MC is nominal marginal cost, given the initial price level $P_{H0} > 0$ and the demand constraint (14). Note that the term in square brackets in (16) is real profits as of time s if the firm has last adjusted in time t, which is discounted with the probability of not adjusting, and the pricing kernel $\lambda_s \exp\{-\rho(s-t)\}$ derived from the consumer's maximization problem; the maximization is subject to the firm's demand constraint (14), giving $y_{is}(Q_t) = (Q_t \exp\{\overline{\pi}_H(s-t)\})^{-\varepsilon} P_{Hs}^{\varepsilon} y_s$. The first order condition is

$$Q_t = \frac{\epsilon}{\epsilon - 1} \frac{\int_t^\infty \exp\{-(\xi+\rho)(s-t)\}\lambda_s \widetilde{P}_{Hs}^{\varepsilon-1} y_s \widetilde{MC}_s ds}{\int_t^\infty \exp\{-(\xi+\rho)(s-t)\}\lambda_s \widetilde{P}_{Hs}^{\varepsilon-1} y_s ds},$$

where we define $\widetilde{X}_s \equiv X_s / \exp\{\overline{\pi}_H(s-t)\}, \quad X = P_H, MC$. This first order condition, together with the price index (15), can be manipulated to give an approximate linear differential equation for the evolution of aggregate home price inflation in the neighborhood of a steady state, which we assume to exist and to have the property that home prices grow at the rate $\overline{\pi}_H$ while all real variables are constant; in particular, real marginal cost in the steady state will be the constant $\overline{mc}_H \equiv \overline{MC}/\overline{P_H} = (\varepsilon - 1)/\varepsilon < 1$. Details of the calculation can be found in appendix 5.1. The result is the linearized economy's domestic inflation equation, or Phillips curve, linking domestic producer price inflation π_H to real marginal costs deflated by home prices, $mc_H \equiv MC/P_H$,

$$\dot{\pi}_H = \rho(\pi_H - \overline{\pi}_H) - \xi(\xi + \rho)\frac{\varepsilon}{\varepsilon - 1}(mc_H - \overline{mc}^H). \tag{17}$$

Finally, the labor demand schedule in a symmetric equilibrium is

$$w = \frac{P_H}{P} mc_H, \tag{18}$$

and the symmetric aggregate production function is

$$y = y_H + y_H^X = l. \tag{19}$$

Exchange rates and foreign demand Following Gali and Monacelli (2002), the home country is assumed to be small in the sense that its exports to foreign are negligible in the foreign price indices; thus, the foreign producer price level P_F^* is identical to the foreign consumption price index P^*,

$$P^* = P_F^*.$$

The law of one price is assumed to hold for every good, and the foreign country's aggregators are assumed to have the same structure as the home country ones, giving rise to the relations

$$P_H = eP_H^*, \qquad P_F = eP_F^*,$$

where P_H^* is the price of home produced goods expressed in foreign currency. The terms of trade t are defined as

$$t \equiv \frac{P_H}{P_F}. \tag{20}$$

Due to the assumptions of smallness and the law of one price, the relation of domestic producer prices to the consumer price index, P_H/P, can be expressed as a function of the terms of trade and the real exchange rate,

$$\frac{P_H}{P} = x \cdot t. \tag{21}$$

Following Kollmann (2001), we assume that the rest of the world has a demand for the home country's exports that can be expressed analogously to the domestic goods demand functions. Specifically, let $\vartheta^* > 0$ be the weight of home produced goods in foreign's consumption basket and $\eta^* > 1$ be foreign's demand elasticity (of course, ϑ^* should be 'small' in the sense that foreign variables can still safely be regarded as exogenous by the home country). Foreign's demand is then assumed to be

$$y_H^X = \vartheta^* \left(\frac{P_H^*}{P^*}\right)^{-\eta^*} c^*,$$

where c^* is aggregate foreign consumption, which can be rewritten using the above assumptions on smallness and the law of one price as

$$y_H^X = \vartheta^* t^{-\eta^*} c^*. \tag{22}$$

Given frictionless international trade in bonds and assuming that households in the rest of the world behave analogously to the domestic households leads to

$$\begin{aligned} \lambda^* &= u_{c^*}^*, \\ -\frac{\dot{\lambda}^*}{\lambda^*} &= R^* - \pi^* - \rho. \end{aligned} \tag{23}$$

Using the uncovered nominal interest rate parity condition (7), the domestic household's intertemporal first order condition for bond accumulation (8), and the growth rate of the real exchange rate (from 3) $\frac{\dot{x}}{x} = \frac{\dot{e}}{e} + \pi^* - \pi$, we obtain

$$\frac{\dot{x}}{x} = \frac{\dot{\lambda}^*}{\lambda^*} - \frac{\dot{\lambda}}{\lambda}. \tag{24}$$

Inserting the foreign and domestic households' intertemporal first order conditions on the right hand side of (24) delivers a modified version of real interest rate parity,

$$\frac{\dot{x}}{x} = \widetilde{r} - r^*, \tag{25}$$

where the home and foreign real interest rates are defined as

$$\widetilde{r} \equiv R[1 - \delta(d)] - \pi \tag{26}$$

and $r^* \equiv R^* - \pi^*$. This condition states that the *default risk adjusted* home real interest rate, $\widetilde{r}$, in the small open economy can be higher than the world real interest rate only if a future real depreciation ($\dot{x}/x > 0$) is impending. The *unadjusted* home real interest rate, $r \equiv R - \pi$, will, however, be larger than r^* even for zero future real exchange rate growth, because it positively depends on δ, and therefore on real debt d. Noteworthily, the implied negative relationship between real net foreign assets and the home real interest rate (or its difference with respect to the world interest rate) is precisely what is found empirically by Lane and Milesi-Ferretti (2001) in their cross-country panel data set.

Central bank The central bank is assumed to set the nominal interest rate in reaction to the domestic producer price inflation rate $\pi_H \equiv \dot{P}_H/P_H$ (domestic inflation, henceforth). Furthermore, the central bank bases its interest rate setting decisions on the real exchange rate x and the level of real net foreign assets $\mathfrak{b}$, such that its policy rule reads

$$R = R(\pi_H, x, \mathfrak{b}) > 0, \qquad R_1 \geq 0, \quad R_2, R_3 \gtreqless 0, \tag{27}$$

where R_j $(j = 1, \ldots, 3)$ is the first partial derivative of the interest rate rule with respect to its j-th argument. Furthermore, the interest rate rule in (27) is restricted such that the steady state condition $\overline{R}(1 - \delta(\overline{d})) = \rho + \overline{\pi} > 0$ has a solution for a positive nominal interest rate.

Perfect foresight equilibrium In equilibrium all markets clear, implying $c_H = y_H$, $c_F = y_F$, and $A = e\mathfrak{B}$. The aggregate resource constraint is then

$$y = c + x[\dot{\mathfrak{b}} - r^* \mathfrak{b}]. \tag{28}$$

A *perfect foresight equilibrium* is a set of sequences $\{c$, l, λ, R, $\frac{\dot{e}}{e}$, π, π_H, mc_H, w, x, y_H, y_H^X, y, t, $\frac{P_H}{P}$, $\frac{P_F}{P}$, $\mathfrak{b}\}_0^\infty$ satisfying the households' first order conditions (5) to (8), the optimality condition of domestic firms (17) and (18), the aggregate domestic production function (19), the optimality conditions for domestic and foreign demand for domestically produced goods (10) and (22), the smallness implication for the foreign price level ($P_F = eP^*$), the interest rate rule (27), the foreign first order condition for bonds (23), the domestic budget constraints consolidated to the aggregate resource constraint (28), and the transversality condition (9), given the definitions of the CPI price level (12), the real exchange rate (3), the terms of trade (7), real foreign debt given (2), and given initial values of households' financial wealth, A_0, and the price level of domestically produced goods, P_{H0}.

Note that international risk sharing implies that the steady state current account $\overline{y} - \overline{c} = -\overline{x}\overline{r}^*\overline{\mathfrak{b}}$ is constant, since it implies that domestic consumption is proportional to foreign consumption, and therefore to the real exchange rate and thus output (see also Schmitt-Grohé and Uribe, 2002). As we want to present results for a version with perfect international capital markets (i.e. no default risk) as a background for comparison, we need to make sure that even in that case the household transversality condition (9) is not violated, which implies that the discounted stock of real foreign bonds held by domestic households must asymptotically converge to zero. Given that $\mathfrak{b}$ grows asymptotically at the rate r^*, as implied by the aggregate resource constraint (28), it is sufficient to assume that the initial value for the stock of domestically held foreign bonds equals zero ($\mathfrak{B}_0 = 0$), which together with the risk-sharing implication that the current account is asymptotically finite ensures that the discounted stock of foreign bonds converges asymptotically to zero. Note that in the model with imperfect capital markets, i.e. with a non-zero default risk as presented so far, no such assumption is needed, since a stable equilibrium path implies a finite solution for $\mathfrak{b}$ anyway. The assumption does, however, not limit the generality of the results.

In order to analyze the local dynamics, the model is linearized around the steady state (see appendix 5.2 for details). The real exchange rate is normalized to equal one in the steady state, implying, together with the smallness assumption ($P_F = eP^*$), that all home currency price levels are equal in the steady state ($\overline{P}_H = \overline{P}_F = \overline{P}$), such that $\overline{y}_H = \overline{c}_H = (1-\vartheta)\,\overline{c}$, and $\overline{y}_H^X = \overline{y} - \overline{y}_H = \{\overline{y} - (1-\vartheta)\,\overline{c}\}$.

The precise steps of the linearization are given in appendix 5.3 for convenience. The result is the linearized three-dimensional system of differential equations in $(x, \pi_H, \mathfrak{b})$ given by

$$\dot{x} = (1-\vartheta)\left\{\overline{x}[1-\overline{\delta(d)}](R-\overline{R}) - \overline{x}(\pi_H - \overline{\pi}_H) + \overline{R}\overline{\delta'}\overline{x}\overline{\mathfrak{b}}(x-\overline{x}) + \overline{R}\overline{\delta'}\overline{x}^2(\mathfrak{b}-\overline{\mathfrak{b}})\right\}, \quad (29)$$

$$\dot{\pi}_H = \rho(\pi_H - \overline{\pi}_H) - \frac{\psi}{\overline{x}}(x-\overline{x}), \quad (30)$$

$$\dot{\mathfrak{b}} = \frac{1}{\overline{x}^2}\left[(\varphi-1)\overline{y} + (1-1/\sigma)\overline{c}\right](x-\overline{x}) + \rho(\mathfrak{b}-\overline{\mathfrak{b}}), \quad (31)$$

where $\psi \equiv \xi(\xi+\rho)\frac{1}{1-\vartheta} + (\gamma-1)\left[\frac{1-\vartheta}{\sigma}\frac{\overline{c}}{\overline{y}} + \left\{\frac{\overline{c}}{\overline{y}}\eta\vartheta + \left[\frac{1}{1-\vartheta} - \frac{\overline{c}}{\overline{y}}\right]\eta^*\right\}\right] > 0$, and the steady state condition $r^* = \rho$ has been used. The deviation from steady state of the nominal interest rate, $(R-\overline{R})$, is to be substituted by the linearized version of the central banks monetary policy rule from (27).

Definition 1 *A perfect foresight equilibrium of the linear approximation to the model is a set of sequences* $\{x, \pi_H, \mathfrak{b}\}$ *satisfying (29), (30), and (31), a linearized version of (27), and (9), given* $P_{H0} > 0$, $\mathfrak{B}_0 = 0$.

3 Results

3.1 Perfect asset markets

For comparison, we first present a model version where there are no capital market imperfections, and hence no default risk on bonds, i.e. $\delta = \delta' = 0$.[1] The assumption of international risk sharing allows to solve separately for the accumulation of foreign bonds,

[1] This version of the model is identical to the one in Gali and Monacelli (2002). The stability result together with the one for a variant where the CPI inflation rate $\pi \equiv \dot{P}/P$ enters the policy rule is discussed in Linnemann and Schabert (2001).

since with perfect capital markets $\mathfrak{b}$ does not affect the other endogenous variables of the system (29) to (30). Assuming that the central bank only considers domestic inflation when formulating its interest rate policy[2], the relevant linearized version of (27) is

$$R - \overline{R} = \alpha(\pi_H - \overline{\pi}_H), \tag{32}$$

where $\alpha > 0$ is the central bank's reaction coefficient. Inserting this into (29) and applying $\overline{\delta} = \overline{\delta}' = 0$ gives

$$\dot{x} = (1 - \vartheta)\,\overline{x}(\alpha - 1)(\pi_H - \overline{\pi}_H). \tag{33}$$

The approximate equilibrium system then consists of the two jump variables x, π_H, and is given by (33) and (30). It turns out that as in the case of a closed economy, equilibrium determinacy requires interest rate policy to react more than one-to-one to inflation (i.e. to be 'active', see e.g. Woodford, 2001). The following proposition summarizes the result, which can also be found in Linnemann and Schabert (2001).

Proposition 1 *When capital markets are perfect, the equilibrium is locally saddle path stable if $\alpha > 1$.*

Proof. The model (33) and (30) can be written as

$$\begin{pmatrix} \dot{x} \\ \dot{\pi}_H \end{pmatrix} = A \begin{pmatrix} x - \overline{x} \\ \pi_H - \overline{\pi}_H \end{pmatrix}, \qquad \text{with } A \equiv \begin{pmatrix} 0 & (1-\vartheta)(\alpha - 1)\overline{x} \\ -\psi/\overline{x} & \rho \end{pmatrix}.$$

Since both variables can jump, a uniquely determined saddle path stable equilibrium requires that A has two unstable (positive) eigenvalues (see Blanchard and Kahn, 1980). Since trace$(A) = \rho > 0$ and det$(A) = (1 - \vartheta)\,\psi(\alpha - 1)$, this is fulfilled with $\alpha > 1$, while in the opposite case ($\alpha < 1$) there is one stable and one unstable eigenvalue.■

Thus, active policy ($\alpha > 1$) implies a unique perfect foresight equilibrium path of (x, π_H) converging to the steady state, namely the steady state itself. For a passive policy rule ($\alpha < 1$), there are infinitely many perfect foresight equilibrium paths. This result is familiar from the closed-economy literature, where it has been named the Taylor principle (in honor of Taylor, 1993; see e.g. Woodford, 2001). Its essence is that the central bank is stabilizing the economy if it ties inflation to the real interest rate, by raising the nominal rate more than one-to-one when inflation changes. Thereby, it also stabilizes the real exchange rate and aggregate demand. To see why, assume that the exchange rate were initially undervalued. Since prices are sticky temporarily, this implies that the real

[2] See below section 3.3 for the case when also the exchange rate and foreign assets appear in the policy rule.

exchange rate, too, is above its long run steady state, $x > \overline{x}$. Perfect foresight informs agents that there is a future real appreciation needed to bring x back into the steady state. As this will imply a future drop in consumer prices through the decrease in the prices of imported items, there is negative future growth in real wage costs of firms and thus in domestic inflation. Along a stable equilibrium path, domestic inflation must thus be currently high according to the Phillips curve. If the central bank reacts strongly to the domestic inflation increase in the sense $\alpha > 1$, the home real interest rate (both in the sense $R - \pi_H$ and $R - \pi$) is raised.[3] Thus, given the constant world real interest rate r^*, a future real depreciation ($\dot{x}/x > 0$) is implied by the real interest rate parity condition for arbitrage freeness, which with perfect international capital markets is different from (25) in that it reads $R - \pi = r^* + \frac{\dot{x}}{x}$ in this case. The real exchange rate would thus explode, which is exactly what is required for saddle path stability: since the system contains only jump variables, and thus exhibits no sluggishness, the adjustment takes place in an instant, and x could never leave the steady state. In the opposite case if the central bank were passive, i.e. $\alpha < 1$, any arbitrary deviation of the real exchange rate from its steady state is compatible with a perfect foresight equilibrium, since the real interest rate is lowered through inflation and return of the x to the steady state is supported from arbitrary intitial positions. In this case, there is equilibrium indeterminacy with the associated possibility of arbitrary fluctuations through self-fulfilling prophecies.

3.2 Imperfect asset markets

We now return to the imperfect capital market case with $0 < \delta < 1$ and $\delta' \geq 0$. The maintained assumption that the home country is a net debtor to the world capital market, in the sense that its stock of net foreign assets $\mathfrak{b}$ is negative. We begin with the case where the central bank only looks at the domestic inflation rate π_H when formulating its interest rate policy, and defer an alternative specification to a later section. In what follows, it turns out to be useful to restrict the elasticity of the probability $(1 - \delta)$ of not defaulting on private debt with respect to the stock of foreign debt, which is defined as $\varepsilon^d \equiv -\frac{\partial(1-\overline{\delta})}{\partial d}\frac{d}{1-\overline{\delta}}$. Assuming that $\varepsilon^d < \widetilde{\varepsilon}$ where $\widetilde{\varepsilon}$ is the upper bound $\widetilde{\varepsilon} \equiv \frac{2}{(1-\vartheta)(\overline{\pi}/\rho+1)}$ not only rules out inessential complications, but also serves to highlight our argument that the impact of foreign debt on macroeconomic stability is a qualitative one, and the theory does not at all rely on quantitively large changes in debt. The following proposition states the main stability result.

[3] Proof: Using (44) and (46) from appendix 5.3 in (25) shows that $R - \pi$ and $R - \pi_H$ differ only by a constant.

Proposition 2 *Suppose capital markets are imperfect, the home country is a net debtor* ($d > 0$) *and the variability of the default probability is sufficiently low to satisfy* $\varepsilon^d < \widetilde{\varepsilon}$. *Then the equilibrium is locally saddle path stable if and only if*

$$\alpha < \frac{1+\Gamma}{1-\overline{\delta}}, \qquad \textit{where} \quad \Gamma \equiv \overline{\delta'}\frac{\overline{R}}{\psi}\left[\varphi\overline{y} - (1/\sigma)\overline{c}\right] > 0.$$

Otherwise, there is no equilibrium path converging to the steady state.

Proof. From (29) to (31) upon substitution of (32), the approximate equilibrium system is

$$\begin{pmatrix} \dot{x} \\ \dot{\pi}_H \\ \dot{\mathfrak{b}} \end{pmatrix} \simeq \begin{pmatrix} \overline{\mathfrak{b}}\Phi_1 & \Phi_2 & \overline{x}\Phi_1 \\ -\psi/\overline{x} & \rho & 0 \\ \Phi_3 & 0 & \rho \end{pmatrix} \begin{pmatrix} x-\overline{x} \\ \pi_H - \overline{\pi}_H \\ \mathfrak{b}-\overline{\mathfrak{b}} \end{pmatrix} = F \begin{pmatrix} x-\overline{x} \\ \pi_H - \overline{\pi}_H \\ \mathfrak{b}-\overline{\mathfrak{b}} \end{pmatrix},$$

where $\Phi_1 \equiv (1-\vartheta)\overline{R}\overline{\delta'}\overline{x} \geq 0$, $\Phi_2 \equiv (1-\vartheta)\overline{x}\{[1-\overline{\delta(d)}]\alpha - 1\}$, $\Phi_3 \equiv \frac{1}{\overline{x}^2}[(\varphi-1)\overline{y} + (1-1/\sigma)\overline{c}]$, and the second equality sign defines the fixed coefficients matrix F.

Since the model consists of two jump variables x, π_H and the predetermined state variable $\mathfrak{b}$, a uniquely determined saddle path stable equilibrium requires that one eigenvalue of the coefficient matrix F is stable (negative), while the two others are unstable (positive) (see Blanchard and Kahn, 1980). A sufficient condition for this to hold is trace$(F) > 0$ and det$(F) < 0$. Using the steady state condition $\overline{R} = \frac{\overline{\pi}+\rho}{1-\overline{\delta}}$ and $\overline{d} = -\overline{x}\overline{\mathfrak{b}}$, we can write trace$(F) = \overline{\mathfrak{b}}(1-\vartheta)\overline{R}\overline{\delta'}\overline{x} + 2\rho = -\varepsilon^d(1-\vartheta)(\overline{\pi}+\rho) + 2\rho$, such that the assumed upper bound $\varepsilon^d < \frac{2}{(1-\vartheta)(\overline{\pi}/\rho+1)}$ ensures that trace$(F) > 0$. The determinant of F, which is given by det$(F) = -(1-\vartheta)\{\overline{R}\overline{\delta'}\rho\left[\varphi\overline{y} - (1/\sigma)\overline{c}\right] - \rho\psi\left([1-\overline{\delta}]\alpha - 1\right)\}$, is strictly negative if

$$\det(F) < 0 \Leftrightarrow \alpha[1-\overline{\delta}] < 1 + \frac{\overline{R}\overline{\delta'}}{\psi}\left[\varphi\overline{y} - (1/\sigma)\overline{c}\right]. \tag{34}$$

The term in square brackets on the right hand side of (34) is positive, since the assumption of a net debtor country implies a positive current account $\overline{ca} = \overline{y} - \overline{c} = -\rho\overline{x}\overline{\mathfrak{b}}$ in the steady state, such that $\overline{y} > \overline{c}$, and the coefficient φ can be shown to satisfy $\varphi > \frac{1}{\sigma}$. Thus, the right hand side of (34) is positive and larger than one. Hence, if α fulfills (34), there is exactly one stable eigenvalue, and the equilibrium is saddle path stable.

If, in contrast, α is such that det$(F) > 0$, there are either two or zero stable eigenvalues. The latter case obtains if det$(-\mathcal{F}) < 0$, where $\mathcal{F}$ is the matrix

$$\mathcal{F} \equiv \begin{pmatrix} f_{11}+f_{22} & f_{23} & -f_{13} \\ f_{32} & f_{11}+f_{33} & f_{12} \\ -f_{31} & f_{21} & f_{22}+f_{33} \end{pmatrix} = \begin{pmatrix} \overline{\mathfrak{b}}\Phi_1 + \rho & 0 & -\overline{x}\Phi_1 \\ 0 & \overline{\mathfrak{b}}\Phi_1 + \rho & \Phi_2 \\ -\Phi_3 & -\psi/\overline{x} & 2\rho \end{pmatrix}$$

(with f_{ij} the (i,j)-element of F). Obviously, this requires

$$[1-\overline{\delta}]\alpha > 1 + \frac{\overline{R}\overline{\delta'}}{\psi}(\varphi\overline{y} - \overline{c}/\sigma) - \frac{1}{\psi}\left[\rho\overline{\mathfrak{b}}\overline{R}\overline{\delta'} + \frac{2\rho^2}{1-\vartheta}\right] \tag{35}$$

where we used $\overline{y} - \overline{c} = -\rho\overline{\mathfrak{b}}$ and $\overline{x} = 1$. Given that the term in the square brackets equals $\frac{\rho}{1-\vartheta}\text{trace}(F) > 0$, (35) is implied by $\det(F) > 0$. Hence, if α violates (34), there are zero stable eigenvalues, and the equilibrium is explosive. ■

The proposition states that the central bank's interest rate policy must not be too aggressively trying to fight inflation with higher nominal interest rates if the economy's equilibrium is to be (saddle path) stable. Otherwise, with a high inflation reaction coefficient α, the equilibrium will be explosive, in the sense that there is no path of the endogenous variables converging to the steady state. While in general the critical value for α depends on a host of parameters, there is a more easily interpretable conclusion for the intuitively appealing case that the steady state default risk probability $\delta(\overline{d})$ is only marginally, or asymptotically not at all, changing with the level of foreign debt, i.e. $\overline{\delta'} \to 0$. This is summarized in the following corollary.

Corollary 3 *If the default risk approaches a constant, $\overline{\delta'} \to 0$, the equilibrium is saddle path stable if $\alpha < 1/[1-\overline{\delta}]$, and explosive otherwise.*

This result can easily be linked to the one stated above in the context of perfect capital markets, since if we assume that the steady state default risk is not large, the critical threshold for α should be close to one, hence to the borderline that subdivides the outcomes of saddle path stability ($\alpha > 1$) and indeterminacy ($\alpha < 1$) in the perfect capital markets case. Viewed in this way, the main result is that the presence of imperfect capital markets implies that stability requires a departure from the Taylor principle in that weak interest rate reactions to inflation on the part of the central bank are associated with saddle path stability, and strong reactions lead to explosiveness, while indeterminacy cannot occur.

Why does the attempt of the central bank to stabilize inflation through higher nominal interest rates result in destabilization in the present model? The reason is the nature of the impact of net foreign assets on the economy. Assume an initial exchange rate undervaluation, i.e. a real exchange rate larger than its steady state value, $x > \overline{x}$. Note that the default risk depends on x through the definition of foreign debt, $\delta(d) = \delta(-x\mathfrak{b})$, such that when linearized at the steady state

$$\delta - \overline{\delta} = -\overline{\mathfrak{b}}\overline{\delta'}(x - \overline{x}) - \overline{x}\overline{\delta'}(\mathfrak{b} - \overline{\mathfrak{b}}).$$

Since we are considering the case of an indebted country, net foreign assets $\overline{\mathfrak{b}}$ are assumed to be negative at the steady state, such that a larger than steady state real exchange rate $x - \overline{x} > 0$ raises the current burden of real foreign indebtedness, and thus the default risk probability δ. This means that, ceteris paribus, the risk adjusted real home interest rate $\widetilde{r} \equiv R[1 - \delta(d)] - \pi$ is currently relatively low, compared to its steady state value. From the modified real interest rate parity conditon (25), given the foreign real interest rate, a lower than steady state value of $\widetilde{r}$ must (to satisfy arbitrage freeness) be associated with an anticipation of a future decline in the real exchange rate x, i.e. a future real appreciation ($\dot{x}/x < 0$). This is precisely what is required for the real exchange rate to return to the steady state from above.

The important issue to note is that this stabilizing mechanism of the role of net foreign assets works without the interference of interest rate policy. In the present model, there is an inherently stabilizing negative feedback in the real exchange rate, in the sense that $\partial \dot{x}/\partial(x - \overline{x}) < 0$ as just described. As the real exchange rate and net foreign assets affect the modified real interest rate parity condition (25), it is possible that stability is brought about by these variables only, without any need for the central bank to adjust the nominal interest rate. Since the model (in contrast to the version with perfect capital markets discussed in an earlier section) contains a predetermined state variable, namely foreign assets $\mathfrak{b}$, this return to steady state of the real exchange rate is precisely what is needed for stability: any time when x is out of steady state real debt is, too, there must be an endogenous mechanism that brings back x to the steady state (with an implied overshooting to ensure a constant steady state value $\overline{\mathfrak{b}}$), because otherwise $\mathfrak{b}$ would shrink indefinitely with the rate of the world real interest rate.

To clarify the role of interest rate policy in the present model, we continue the example of initial real exchange rate undervaluation $x - \overline{x} > 0$. From the supply side of the economy, expressed in the Phillips curve, a higher than steady state real exchange rate is associated with currently high domestic inflation, $\pi_H - \overline{\pi}_H > 0$. The reason is the usual aggregate demand effect of a high real exchange rate, which promotes export demand and consumption, raising employment, labor costs, and thus domestic producer prices. Now assume the central bank were actively trying to counteract the inflationary pressure by raising the nominal interest rate strongly. The precise meaning of a 'strong' interest rate reaction is that the central bank policy parameter α is high enough to raise the real interest rate and thus violate the condition in proposition 2. By implication, the risk adjusted real interest rate $\widetilde{r}$ is increased *ceteris paribus*, too. From the modified real interest rate parity condition (25), in equilibrium there must be an anticipated future growth in x, i.e. a future real depreciation. Since the starting point was that x was higher than steady state, this will remove x even farther from steady state. Hence, the central bank reaction would set the economy on an explosive path where an initial real overvaluation is perpetually reinforced, which feeds an unbounded growth of foreign debt.

If, on the other hand, the central bank were not aggressively raising the interest rate in the wake of inflation, the real interest rate would decline. This would allow the real exchange rate to appreciate over time, and thus return to the steady state. It is, from the pure point of view of stability, therefore not necessary for the central bank to react at all to changes in inflation. Put differently, a situation of high aggregate demand cures itself without change in the nominal interest rate. As usual, high aggregate demand implies positive inflation and a low real interest rate. In the present model, however, this also implies that the high export demand generates current account surpluses that help to reduce foreign indebtedness. This makes domestic debt less risky, such that international capital markets will accept require a lower rate of return for holding it. With lower default risk, the risk adjusted real interest rate $\widetilde{r}$ is higher for any given nominal interest rate and inflation rate, which brings aggregate demand back to normal through the associated real appreciation.

3.3 When the central bank reacts on net foreign assets

So far, the central bank has been assumed only to react to inflation changes according to (32). On the other hand, the model's essence is that foreign debt is crucial to stability. A well informed central bank would thus probably be inclined to base its interest rate setting decisions on the whole set of relevant variables. This is what we assume in this section by postulating the general version of the policy rule from (27), which in linearized form reads

$$\begin{aligned}(R-\overline{R}) &= \alpha(\pi_H-\overline{\pi}_H)+\kappa(d-\overline{d}) \\ &= \alpha(\pi_H-\overline{\pi}_H)-\kappa\overline{\mathfrak{b}}(x-\overline{x})-\kappa\overline{x}(\mathfrak{b}-\overline{\mathfrak{b}}).\end{aligned} \tag{36}$$

Note that we leave the sign of κ open; since $\overline{\mathfrak{b}}<0$, an assumption $\kappa>0$ would imply that the nominal interest rate is raised if real foreign debt rises, i.e. if the exchange rate has depreciated in real terms or if net foreign assets have decreased.

Inserting (36) into equilibrium condition (29) leads to

$$\dot{x}=\overline{\mathfrak{b}}\Phi_4(x-\overline{x})+\Phi_2(\pi_H-\overline{\pi}_H)+\overline{x}\Phi_4(\mathfrak{b}-\overline{\mathfrak{b}}), \tag{37}$$

with $\Phi_4\equiv(1-\vartheta)\overline{x}\left[\overline{R}\overline{\delta'}-\kappa[1-\overline{\delta(d)}]\right]=\Phi_1-\kappa\overline{x}(1-\vartheta)[1-\overline{\delta(d)}]$. The basic intuition for the effect of the more general policy rule can be seen from this expression. Recall that above it has been argued that the model's stability properties depend crucially on the feedback in the real exchange rate which in the case where the central bank only targets inflation ensures $\partial\dot{x}/\partial(x-\overline{x})<0$, and thus ensures a return of the real exchange rate to its steady state. If, however, the central bank targets foreign debt, it reacts directly to the real exchange rate by implication, such that it can break this stabilizing link between the level of x and its change, since $\partial\dot{x}/\partial(x-\overline{x}) = \overline{\mathfrak{b}}\Phi_4$ can be of either sign. In particular, since $\overline{\mathfrak{b}} < 0$, a large positive value of κ can render this expression positive, which according to the intuition set out above should result in an unstable outcome. The following proposition summarizes the main stability result.

Proposition 4 *If capital markets are imperfect and the home country is a net debtor ($d > 0$), the likelihood that the necessary condition for saddle path stability is satisfied declines with κ.*

Proof. The system consisting of (37), (30), and (31) can be written as

$$\begin{pmatrix} \dot{x} \\ \dot{\pi}_H \\ \dot{\mathfrak{b}} \end{pmatrix} \simeq \begin{pmatrix} \overline{\mathfrak{b}}\Phi_4 & \Phi_2 & \overline{x}\Phi_4 \\ -\psi/\overline{x} & \rho & 0 \\ \Phi_3 & 0 & \rho \end{pmatrix} \begin{pmatrix} x-\overline{x} \\ \pi_H - \overline{\pi}_H \\ \mathfrak{b}-\overline{\mathfrak{b}} \end{pmatrix} = F^d \begin{pmatrix} x-\overline{x} \\ \pi_H - \overline{\pi}_H \\ \mathfrak{b}-\overline{\mathfrak{b}} \end{pmatrix},$$

where the second equality sign defindes the fixed coefficients matrix F^d. A uniquely determined saddle path stable equilibrium again requires that F^d has one negative and two positive eigenvalues. A necessary condition for this to hold is that $\det(F^d) < 0$, which is the case if and only if

$$[1-\overline{\delta}]\alpha < 1 + \Gamma - \kappa(1-\overline{\delta(d)})\left(\varphi\overline{y} - \overline{c}/\sigma\right)/\psi,$$

with $\Gamma \equiv \overline{\delta'}\frac{\overline{R}}{\psi}\left[\varphi\overline{y} - (1/\sigma)\overline{c}\right] > 0$ defined as above. Since $(1-\overline{\delta(d)})\frac{1}{\psi}\left(\varphi\overline{y} - \overline{c}/\sigma\right) > 0$, the right hand side is declining in κ. ■

Thus, a positive reaction to an increase in foreign debt makes the range of values of the inflation targeting coefficient α that are compatible with stability smaller. The intuition for this result again builds on the role of foreign debt in the real interest rate parity condition (25): if the real exchange rate is higher than in steady state, the real value of foreign debt is high, the default risk δ of domestic creditors is accordingly high, and the risk adjusted real interest rate $\widetilde{r}$ is low, for any given nominal interest rate. While this alone would imply that a stabilizing future real apprection ($\dot{x}/x < 0$) is impending that would return x to its steady state, the central bank may break this chain of events by targeting foreign debt with a positive reaction coefficient $\kappa > 0$. If it does so, namely, it would raise the nominal interest rate R when x is higher than steady state, which ceteris paribus would raise the risk adjusted real interest rate $\widetilde{r}$. If the effect is strong enough, the stabilizing real appreciation that in the absence of central bank action would be induced through a high real value of foreign debt is counteracted, and the model may become unstable.

4 Conclusion

This paper extends the line of research on the local dynamic properties of sticky-price models with monetary policy characterized by nominal interest rate rules for the case of small open economies. We consider the case of imperfect asset markets due to the the existence of default risk on debt emissions, where the probability of default is assumed to depend on the aggregate stock of foreign debt. While in the absence of this capital market imperfection saddle path stability requires interest rate policy to satisfy the Taylor-principle (as in the case of closed economies), the same priniciple of aggressive inflation targeting bears the danger of unbounded debt growth if default risk depends on indebtedness. Hence, when asset markets are imperfect the central bank should either refrain from actively targeting domestic inflation, e.g., by choosing a nominal interest rate peg, or consider the evolution of net foreign assets as an argument of its interest rate policy rule. In particular, it is shown that the central bank can raise the likelihood for the equilibrium to be saddle path stable if it allows the nominal interest rate to respond negatively to changes in net foreign debt.

5 Appendix

5.1 Derivation of the Phillips curve

The firm's optimal price setting problem is

$$\max_{Q_t} \int_t^\infty e^{-(\xi+\rho)(s-t)} \lambda_s [(Q_t e^{\overline{\pi}_H (s-t)} y_{is}(Q_t) - MC_s y_{is}(Q_t))/P_{Hs}] ds. \tag{38}$$

subject to given initial prices and to (14) and the production function (13). The first order condition is

$$\int_t^\infty e^{-(\xi+\rho)(s-t)} \frac{\lambda_s}{P_{Hs}} [(1-\varepsilon)(Q_t e^{\overline{\pi}_H (s-t)})^{-\varepsilon} P_{Hs}^{\varepsilon} y_s e^{\overline{\pi}_H (s-t)} +$$
$$\varepsilon \left(Q_t e^{\overline{\pi}_H (s-t)}\right)^{-\varepsilon-1} MC_s P_{Hs}^{\varepsilon} y_s e^{\overline{\pi}_H (s-t)}] ds = 0.$$

Simplifying and rearranging, this is equivalent to

$$\int_t^\infty e^{-(\xi+\rho)(s-t)} \lambda_s \widetilde{P}_{Hs}^{\varepsilon-1} y_s Q_t ds = \frac{\varepsilon}{\varepsilon-1} \int_t^\infty e^{-(\xi+\rho)(s-t)} \lambda_s \widetilde{P}_{Hs}^{\varepsilon-1} y_s \widetilde{MC}_s ds,$$

where we define $\widetilde{X}_s \equiv X_s / e^{\overline{\pi}_H (s-t)}, \quad X = P_H, MC$. Dividing both sides by P_{Ht} and letting $q_t \equiv Q_t / P_{Ht}$, we have

$$\int_t^\infty e^{-(\xi+\rho)(s-t)} \lambda_s \widetilde{P}_{Hs}^{\varepsilon-1} y_s q_t ds = \frac{\varepsilon}{\varepsilon-1} \int_t^\infty e^{-(\xi+\rho)(s-t)} \lambda_s \widetilde{P}_{Hs}^{\varepsilon-1} y_s \widetilde{MC}_s \frac{1}{P_{Ht}} ds.$$

Linearizing this expression around the steady state, we obtain

$$\int_t^\infty e^{-(\xi+\rho)(s-t)} \overline{\lambda}_s \overline{\widetilde{P}}_{Hs}^{\varepsilon-1} \overline{y}_s \overline{q}_t \left[\frac{\lambda_s - \overline{\lambda}_s}{\overline{\lambda}_s} + (\varepsilon-1) \frac{\widetilde{P}_{Hs} - \overline{\widetilde{P}}_{Hs}}{\overline{\widetilde{P}}_{Hs}} + \frac{y_s - \overline{y}_s}{\overline{y}_s} + \frac{q_t - \overline{q}_t}{\overline{q}_t} \right] ds$$
$$= \frac{\varepsilon}{\varepsilon-1} \int_t^\infty e^{-(\xi+\rho)(s-t)} \overline{\lambda}_s \overline{\widetilde{P}}_{Hs}^{\varepsilon-1} \overline{y}_s \overline{\widetilde{MC}}_s \frac{1}{\overline{P}_t} \left[\frac{\lambda_s - \overline{\lambda}_s}{\overline{\lambda}_s} + (\varepsilon-1) \frac{\widetilde{P}_{Hs} - \overline{\widetilde{P}}_{Hs}}{\overline{\widetilde{P}}_{Hs}} + \right.$$
$$\left. \frac{y_s - \overline{y}_s}{\overline{y}_s} + \frac{\widetilde{MC}_s - \overline{\widetilde{MC}}_s}{\overline{\widetilde{MC}}_s} - \frac{P_{Ht} - \overline{P}_{Ht}}{\overline{P}_{Ht}} \right] ds, \tag{39}$$

where, as usual, bars over variables denote the respective steady state values. Note that, in steady state, we have the following relations: $\overline{P}_{Hs}$ grows with the rate $\overline{\pi}$, whereas $\overline{\widetilde{P}}_{Hs}$ is constant (as are $\overline{\lambda_s}$ and $\overline{y_s}$). Further, the price chosen by an adjusting firm must equal the aggregate price index, such that $\overline{q}_t = 1$. The constant elasticity property of the demand function implies that the steady state price level is a constant markup over nominal marginal costs, $\overline{P}_{Hs} = \varepsilon/(\varepsilon-1)\overline{MC}_s$. Therefore, as $\overline{P}_{Hs} = \overline{P}_{Ht} e^{\overline{\pi}_H (s-t)}$, we have that $\varepsilon/(\varepsilon-1)\overline{\widetilde{MC}}_s / \overline{P}_{Ht} = 1$, and the coefficients on the left and right hand sides of (39) are the same. Hence, the equation simplifies to

$$\int_t^\infty e^{-(\xi+\rho)(s-t)}\frac{q_t-\overline{q}_t}{\overline{q}_t}ds=\int_t^\infty e^{-(\xi+\rho)(s-t)}\left[\frac{\widetilde{MC}_s-\overline{\widetilde{MC}}_s}{\overline{\widetilde{MC}}_s}-\frac{P_{Ht}-\overline{P}_{Ht}}{\overline{P}_{Ht}}\right]ds.$$

Noting that $(\widetilde{MC}_s-\overline{\widetilde{MC}}_s)/\overline{\widetilde{MC}}_s=(MC_s-\overline{MC}_s)/\overline{MC}_s$ and defining real marginal costs as $mc_{Hs}=MC_s/P_{Hs}$, this can be written as

$$\frac{q_t-\overline{q}_t}{\overline{q}_t}=(\xi+\rho)\int_t^\infty e^{-(\xi+\rho)(s-t)}\left[\frac{mc_{Hs}-\overline{mc}_{Hs}}{\overline{mc}_{Hs}}+\frac{P_{Hs}/P_{Ht}-\overline{P_{Hs}/P_{Ht}}}{\overline{P_{Hs}/P}_{Ht}}\right]ds. \qquad (40)$$

The last term in square brackets in the preceding expression is a function of the deviations of the inflation rates between t and s from steady state inflation, as from $P_{Hs}/P_{Ht}=\exp(\int_t^s\pi_{Hr}dr)$ it follows that $(P_{Hs}/P_{Ht}-\overline{P_{Hs}/P_{Ht}})/\overline{P_{Hs}/P_{Ht}}=\int_t^s(\pi_{Hr}-\overline{\pi}_H)dr$. Using this and differentiating (40) with respect to t we obtain

$$\begin{aligned}\frac{d}{dt}\frac{q_t-\overline{q}_t}{\overline{q}_t}&=-(\xi+\rho)\frac{mc_{Hs}-\overline{mc}_{Hs}}{\overline{mc}_{Hs}}+e^{-(\xi+\rho)(s-t)}\left[-(\pi_{Ht}-\overline{\pi}_{Ht})\right]ds\\&\quad+(\xi+\rho)\int_t^\infty(\xi+\rho)e^{-(\xi+\rho)(s-t)}\left[\frac{mc_{Hs}-\overline{mc}_{Hs}}{\overline{mc}_{Hs}}+\int_t^s(\pi_{Hr}-\overline{\pi}_{Hr})dr\right]\\&=(\xi+\rho)\left[\frac{q_t-\overline{q}_t}{\overline{q}_t}-\frac{mc_{Hs}-\overline{mc}_{Hs}}{\overline{mc}_{Hs}}\right]-(\pi_{Ht}-\overline{\pi}_{Ht}). \qquad (41)\end{aligned}$$

This can be converted into a differential equation in π_H by finding the relation between, respectively, the steady state deviations and the growth rates of inflation and the real reset price. First, the price index (15) can be expressed as a function of past reset prices, where each historical reset price has to be weighted by the probability that a price set at time s is not adjusted in time t, which is given by $\xi\exp\{-\xi(t-s)\}$ (see Calvo, 1983, Benhabib et al., 2001). Therefore, the price index can be written as

$$P_{Ht}^{1-\varepsilon}=\int_{-\infty}^t\xi e^{-\xi(t-s)}Q_s^{1-\varepsilon}ds.$$

Differentiating with respect to t, we get

$$\pi_{Ht}=\xi(q_t-1),$$

which when linearized around the steady state implies

$$\pi_{Ht}-\overline{\pi}_{Ht}=\xi(q_t-\overline{q_t}). \qquad (42)$$

Using (42) in (41) and noting that $\overline{q}_t = 1$ and $\overline{mc}_{Ht} = (\varepsilon - 1)/\varepsilon$, this finally results in

$$\dot{\pi}_H = \rho(\pi_{Ht} - \overline{\pi}_{Ht}) - \frac{\varepsilon\xi(\xi + \rho)}{\varepsilon - 1}(mc_{Ht} - \overline{mc}_{Ht}).$$

This is the linearized economy's domestic inflation equation.

5.2 Steady state

Imposing stationarity for endogenous variables, the steady state values for consumption, output, foreign debt, the real exchange rate, and CPI inflation satisfy

$$\begin{aligned}
\gamma\overline{l}^{\gamma-1} &= [(\epsilon - 1)/\epsilon]\overline{c}^{-\sigma}, \\
\overline{y} &= \overline{c} - \overline{x}\rho\overline{\mathfrak{b}}, \\
\rho &= \overline{R}(\overline{\pi}_H)[1 - \delta(-\overline{x}\overline{\mathfrak{b}})] - \overline{\pi} = r^*,
\end{aligned}$$

Assuming that $\overline{x} = 1$, which implies $R(\overline{\pi}_H) = R(\overline{\pi})$ and $\frac{\dot{\lambda}^*}{\lambda^*} = \frac{\dot{\lambda}}{\lambda} \Rightarrow \overline{c} = \zeta\overline{c}^*$, where ζ denotes an arbitary constant, output is pinned down by the first condition, $\overline{y} = \{[(\epsilon - 1)/\epsilon\gamma](\zeta\overline{c}^*)^{-\sigma}\}^{\frac{1}{\gamma-1}}$, foreign debt by the resource constraint, $\overline{\mathfrak{b}} = (\zeta\overline{c}^* - \overline{y})/\rho$, while the modified UIP condition delivers the inflation rate through $\overline{\pi}/R(\overline{\pi}) = [1 - \delta(-\overline{\mathfrak{b}})] - \rho$.

5.3 Linearization

From the definitions of the terms of trade (20) and the CPI (12), the steady state deviations of the price level ratio P_H/P can be approximated linearly as

$$\frac{P/P_H - \overline{P/P_H}}{\overline{P/P_H}} = -\vartheta\frac{t - \overline{t}}{\overline{t}}. \tag{43}$$

Differentiating (43) and the terms of trade definition (20) with respect to time allows to find an approximate linear relation between the growth in the terms of trade t and the domestic inflation rate π_H and the CPI inflation rate π,

$$\pi = \pi_H - \vartheta\frac{\dot{t}}{t}. \tag{44}$$

Using $P_F/P = x$, which follows from the real exchange rate definition (3) considering smallness of the economy ($P^* = P_F^*$) and the law of one price ($P_F = eP_F^*$), we obtain together with (43) the following linearized relation between the real exchange rate and the terms-of-trade

$$\frac{x - \overline{x}}{\overline{x}} = -(1 - \vartheta)\frac{t - \overline{t}}{\overline{t}}. \tag{45}$$

Differentiating with respect to time gives

$$\frac{\dot{x}}{x} = -(1-\vartheta)\frac{\dot{t}}{t}. \tag{46}$$

Linearizing the aggregate production function (19) gives

$$\frac{l-\overline{l}}{\overline{l}} = \frac{y-\overline{y}}{\overline{y}} = \frac{(1-\vartheta)\overline{c}}{\overline{y}}\frac{y_H - \overline{y_H}}{\overline{y_H}} + \left[1 - \frac{(1-\vartheta)\overline{c}}{\overline{y}}\right]\frac{y_H^X - \overline{y_H^X}}{\overline{y_H^X}}. \tag{47}$$

Using the linearized demand conditions for domestic goods (10) and for exported goods (22) we can write (47) as

$$\frac{l-\overline{l}}{\overline{l}} = \frac{(1-\vartheta)\overline{c}}{\overline{y}}\frac{c-\overline{c}}{\overline{c}} - \left\{\frac{(1-\vartheta)\overline{c}}{\overline{y}}\eta\vartheta + \left[1-\frac{(1-\vartheta)\overline{c}}{\overline{y}}\right]\eta^*\right\}\frac{t-\overline{t}}{\overline{t}} + \left[1-\frac{(1-\vartheta)\overline{c}}{\overline{y}}\right]\frac{c^*-\overline{c^*}}{\overline{c^*}}. \tag{48}$$

Integrating (24), which after inserting the static first order condition for consumption, (5), gives in linearized form

$$\frac{1}{\sigma}\frac{x-\overline{x}}{\overline{x}} = \frac{c-\overline{c}}{\overline{c}} - \frac{c-\overline{c}^*}{\overline{c}^*} \tag{49}$$

From now on, foreign variables (those with an asterisk) are taken as constant, and their steady state deviations or growth rates are, therefore, set to zero. Hence, consumption and the terms-of-trade in (48) can be eliminated by (49) and (45), yielding

$$\frac{l-\overline{l}}{\overline{l}} = \varphi\frac{x-\overline{x}}{\overline{x}}, \quad \text{where } \varphi \equiv \left[\frac{1-\vartheta}{\sigma}\frac{\overline{c}}{\overline{y}} + \left\{\frac{\overline{c}}{\overline{y}}\eta\vartheta + \left[\frac{1}{1-\vartheta} - \frac{\overline{c}}{\overline{y}}\right]\eta^*\right\}\right] > 0. \tag{50}$$

Note that the sign of φ follows from the maintained assumption $\overline{c} \leq \overline{y}$. Combination of the linearized first order conditions for labor (6) and consumption (5) yields

$$\frac{w-\overline{w}}{\overline{w}} = (\gamma-1)\frac{l-\overline{l}}{\overline{l}} + \sigma\frac{c-\overline{c}}{\overline{c}},$$

which together with the linearized labor demand function (18) and (43), allows to eliminate real marginal cost by

$$mc_H - \overline{mc}_H = \frac{\epsilon-1}{\epsilon}\frac{\psi}{\xi(\xi+\rho)}\frac{x-\overline{x}}{\overline{x}}, \qquad \psi \equiv \xi(\xi+\rho)\frac{1}{1-\vartheta} + (\gamma-1)\,\varphi > 0,$$

where we used that $\overline{mc}_H = \frac{\epsilon-1}{\epsilon}$ holds, in the inflation equation (17), yielding

$$\dot{\pi}_H = \rho(\pi_H - \overline{\pi}_H) - \frac{\psi}{\overline{x}}(x-\overline{x}). \tag{51}$$

6 References

Benhabib, J., Schmitt-Grohe, S., and M. Uribe (2001), Monetary policy and multiple equilibria, American Economic Review 91, 167-185.

Blanchard, O.J., and C.M. Kahn (1980), The solution of linear diffrence models under rational expectations, Econometrica, 48, 1305-1313.

Calvo, G. (1983), Staggered prices in a utility-maximizing framework, Journal of Monetary Economics 12, 383-398.

Cavallo, M., and F. Ghironi (2002), Net foreign assets and the exchange rate: redux revived, Journal of Monetary Economics 49 (2002), 1057-1097.

Clarida, R., J. Galí, and M. Gertler (1999), The science of monetary policy: a new Keynesian perspective, Journal of Economic Literature 37, 1661-1707.

Clarida, R., J. Galí, and M. Gertler (2000), Monetary policy rules and macroeconomic stability: evidence and some theory, Quarterly Journal of Economics 115, 147-180.

Corsetti, G., and P. Pesenti (2000), Welfare and macroeconomic interdependence, Quarterly Journal of Economics 116, 412-445.

Edwards, S. (2002), Does the current account matter?, NBER working paper 8275.

Galí, J., and T. Monacelli (2002), Optimal monetary policy and exchange rate volatility in a small open economy, mimeo, Universitat Pompeu Fabra.

Kollmann, R. (2001), The exchange rate in a dynamic-optimizing business cycle model with nominal rigidities: a quantitative investigation, Journal of International Economics 55, 243-262.

Lane, P. (2001), The new open economy macroeconomics: a survey, Journal of International Economics, 54, 235-266.

Lane, P., and G. Milesi-Ferretti (2001), Long-term capital movements, NBER Macroeconomics Annual.

Linnemann, L., and A. Schabert (2001), Monetary policy, exchange rates, and real indeterminacy, mimeo, University of Cologne.

Obstfeld, M., and K. Rogoff (1996), Foundations of international macroeconomics, MIT Press, Cambridge, Ma.

Parrado, E., and A. Velasco (2002), Optimal interest rate policy in a small open economy, NBER Working Paper 8721.

Schmitt-Grohé, S., and M. Uribe (2002), Closing small open economy models, NBER working paper 9270.

Taylor, J.B. (1993), Discretion versus policy rules in practice, Carnegie-Rochester Conference Series on Public Policy 39, 195-214.

Turnovsky, S.B. (1997), International macroeconomic dynamics, MIT Press, Cambridge,

Ma.
Woodford, M. (2001), The Taylor rule and optimal monetary policy, American Economic Review 91 P. & P., 232-237.
Yun, T (1996)., Nominal price rigidity, money supply endogeneity, and business cycles, Journal of Monetary Economics 37, 345-370.