UK business investment: long-run elasticities and short-run dynamics

Colin Ellis

and

*Simon Price**

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*Monetary Analysis Division, Bank of England, Threadneedle Street, London, EC2R 8AH.
E-mail: colin.ellis@bankofengland.co.uk, simon.price@bankofengland.co.uk

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Abstract

Theory tells us that output, the capital stock and the user cost of capital are related. From the capital accumulation identity, it also follows that the capital stock and investment have a long-run proportional relationship. The dynamic structure thus implies a multi-cointegrating framework, in which separate cointegrating relationships are identifiable. This has been used to justify the estimation of investment equations embodying a reduced-form long-run relationship between investment and output (rather than between the capital stock and output). In this paper, a new investment equation is estimated in the full structural framework, exploiting a measure of the capital stock constructed by the Bank, and a long series for the cost of capital. A CES production function is assumed, and a well-determined estimate of the elasticity of substitution is obtained by a variety of measures. The robust result is that the elasticity of substitution is significantly different from unity (the Cobb-Douglas case), at about 0.45. Overidentifying restrictions on the long-run relationship are all accepted. Although the key long-run parameter (the elasticity of substitution) is highly robust to alternative specifications, single-equation investment relationships may obscure the dynamics. There is evidence that the Johansen method is oversized, but given this, a test for excluding the capital accumulation identity from the investment equation is much better than using a single-equation ECM.

Key words: Investment, capital stock, identification, multicointegration.

JEL classification: C32, E22.

Summary

Neoclassical theory tells us that a profit-maximising firm's desired capital/output ratio depends on the real user cost of capital: this is the long-run equilibrium relationship. On the steady-state growth path, firms remain at the optimal capital/output ratio by re-investing to offset depreciation and steady-state growth in the capital stock. With a stationary depreciation rate, this implies that in long-run equilibrium the investment/capital ratio is fixed. This is a second long-run equilibrium relationship.

In this paper we exploit a measure of the capital stock constructed at the Bank, and a real user cost of capital measure that explicitly incorporates relative prices. We relax the standard assumption of Cobb-Douglas technology that restricts the elasticity of substitution to unity, and instead use a constant elasticity of substitution (CES) production function that nests Cobb-Douglas as a special case. As described above, our theoretical framework implies two long-run equilibrium relationships: one between capital, output, and the real user cost; and the other between investment and capital. These theoretical long-run relationships imply restrictions on the model. They also imply a single reduced-form long-run relationship between investment, output and the real user cost.

We estimate this system as a vector error-correction mechanism (VECM) using the Johansen method. Our two long-run relationships form the basis for the two cointegrating vectors in the model. The model is statistically well specified and the overidentifying theoretical restrictions on the model are accepted. A key result is that the elasticity of substitution between labour and capital in production is significantly lower than unity at a little under 0.45. This estimate is obtained by a variety of measures and estimation techniques, and, as judged by external estimates, is plausible. This is a remarkable result, because most studies of aggregate investment have found it hard to find a significant relationship of the correct sign between investment and the user cost.

The model also tells us how investment and capital respond when the system is not in long-run equilibrium. Investment responds when the capital/output ratio is away from equilibrium, while capital responds when the investment/capital ratio is away from equilibrium. This last result is consistent with a log-linearisation of the capital accumulation identity. As with other aggregate investment models, the model takes a long time to reach the long-run equilibrium.

Despite the robust nature of our elasticity of substitution estimate, different estimation methods yield different results for the dynamics of investment. In particular, single-equation estimation results suggest that investment responds to disequilibrium in the investment/capital ratio, while our system estimation results suggest it does *not* respond to the investment/capital ratio.

We investigate this puzzle using simulations. We specify a model assuming the VECM results are correct, and use it to generate artificial data series for the four variables. Investment models are estimated on the artificial data using the single-equation and system techniques, and tested to see which technique correctly estimates the 'true' model. The system estimation is better at correctly estimating the dynamics than single-equation estimation, but rejects the restrictions

from the theoretical long-run relationships too often. The single-equation results find the investment/capital ratio to be significant because they implicitly estimate the reduced-form long-run relationship, rather than the two separate long-run relationships.

1. Introduction

This paper re-examines the aggregate business investment relationship within a neoclassical framework. This is a bold enterprise, as it is a commonplace that the effect of user cost is usually utterly insignificant on aggregate data. But over our sample, with new data and a new treatment of the investment decision in a cointegrating framework, we uncover a robust estimate of the user cost elasticity, consistent with a sensible estimate of the elasticity of substitution.

The empirical background is that researchers have found it difficult to find a role for the user cost of capital. Thus in research into UK investment published over the past decade,(1) the roles of variously debt, profits, capacity utilisation and uncertainty have been used to augment Q and other models of investment.(2) Moreover, researchers have often been uncertain how reliable estimates of the capital stock are. Since Bean (1981), it has been common to exploit the steady-state relationship between investment and the capital stock and model long-run investment, rather than capital. However, this approach conflates capital accumulation and investment dynamics into one equation.

In this paper, we take advantage of some new data and a new dynamic specification to return to the aggregate business investment equation. We use a real user cost of capital series, incorporating a weighted average cost of finance, that we push back to 1970. Moreover, although ONS publish a series for the stock of capital, it was suspended in the 2002 National Accounts *Blue Book*. Work at the Bank of England has been undertaken into the construction of new estimates, and it is an equivalent measure to the ONS' that we use here.(3) We then pay particular attention to the endogenous dynamics of the system. The reason why this is an issue is that the capital stock is determined by the capital accumulation identity (CAI). This identity is a difference equation explaining the growth of capital. It follows that there are steady-state implications for the relationship between investment and capital. Thus there are two relationships that will hold in steady state. One is a relationship between the capital stock and its drivers (output and the user cost); this follows from the first-order conditions of the profit-maximising firm (FOC). The other is the steady state of the accumulation identity. In a non-stationary environment it may be helpful to estimate both of these relationships. As investment and the capital stock are linked via accumulation, we have an example of multicointegration. There are also interesting questions about the loadings (error correction coefficients) of the variables in the system, which have implications for the dynamics. And to anticipate the results, we find that we can successfully estimate a user cost neoclassical model of investment where the key parameter, the elasticity of substitution, is both well determined and plausible.

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(1) For example, Bakhshi and Thompson (2002), Carruth, Dickerson and Henley (2000), Cuthbertson and Gasparro (1995) and Price (1995). Oliner *et al* (1995) estimate 'traditional' and Euler equation models for US investment, none of which do particularly well.
(2) Tobin's Q can be derived within the standard neoclassical framework.
(3) See Oulton (2001) and Oulton and Srinivasan (2003); the latter explains the methodology employed to create our series. This work has also involved the construction of a volume index of capital services (VICS), as an alternative to the conventional stock measure. Each measure has its own uses, as discussed in Oulton (2001) and Oulton and Srinivasan (2003), and VICS is especially useful in productivity and growth accounting. However, the appropriate measure of investment in the VICS approach is not the aggregate (National Accounts) measure, but one in which, like the VICS, the amount of investment in each asset is weighted together by its rental price. Moreover there is no simple analogue of the user cost.

In the next section we ask what the first-order conditions and accumulation identity imply for the long run. Then we spell out the implications for estimation within a vector error correction (VECM) framework, and estimate the model in Sections 4 and 5. Section 6 explores the finite sample properties of the methods with a Monte Carlo exercise, and Section 7 concludes.

2. Theory; first-order conditions and identities

The key long-run relationship explaining investment in the neoclassical model is the first-order condition (FOC) for capital from a profit-maximising firm. Given stringent conditions on the time paths of prices, one type of capital, a CES production function with constant returns to scale, quadratic costs of adjustment to growth in the capital stock and labour augmenting technical progress, an investment equation may be derived[(4)] that is driven by the static first-order condition,

$$k = y + \theta - \sigma r \qquad \textbf{(1)}$$

where lower case denotes natural logs, K is the capital stock, Y output, θ is a constant and R denotes the real user cost of capital (RCC). [(5)] σ is the elasticity of substitution between factors of production. Note that investment does not appear anywhere in the relationship: rather, the long run relates capital (the cumulated depreciated stock of past investment flows) to output and RCC.

The CAI states that the stock of capital in any period is equal to the depreciated stock from the previous period plus (gross) investment. It is shown in **(2)**, where I_t denotes investment during time t, K_t is the capital stock at end time t and δ the depreciation rate.[(6)]

$$K_{t+1} = (1-\delta)K_t + I_{t+1} \qquad \textbf{(2)}$$

Bean (1981) uses this to substitute for capital. Taking logs, **(2)** implies that in the steady state[(7)]

$$k = i - \ln(\delta + g^K) \qquad \textbf{(3)}$$

where g^K denotes the growth rate of capital.[(8)] By substituting **(3)** into equation **(1)**, we derive a long-run relationship **(4)** which links investment to output and the real user cost.[(9)]

$$i = y + \theta - \sigma r + \ln(\delta + g^K) \qquad \textbf{(4)}$$

(4) See, eg Chapter VI in Sargent (1979).
(5) The data used in this note are described in the data appendix.
(6) It is assumed investment is installed at the end of the period.
(7) Steady state here is to be interpreted as a constant growth path.
(8) This growth rate, assumed stationary, is driven by the growth in the exogenous variables driving the economy – for example, total factor productivity or population growth. In the context of a VECM conditioned on a limited number of endogenous variables, it is a function of the constants.
(9) As g^K and δ are stationary, in estimation they can be excluded from the long-run cointegrating vector. Note that in principle δ may be non-stationary, perhaps due to asset-level effects: see eg Tevlin and Whelan (2003) or Bakhshi, Oulton and Thompson (2003).

This procedure was used in Bakhshi and Thompson (2002). The authors argue the long run will not take account of past (accumulated) investment 'gaps', so they also included capacity utilisation as an integral control variable.(10)

It is helpful to think about this in a cointegrating framework. In what follows we assume that *k*, *i*, *y* and *r* are all I(1) variables, which is verified in practice. The FOC **(1)** defines a long-run relationship between {*k*, *y*, *r*}, and the CAI **(3)** also implies that {*k*, *i*} cointegrate. This is an example of multicointegration, a notion introduced by Granger and Lee (1991). Multicointegration may occur wherever there are stock-flow relationships: for example, between consumption, income and wealth (as wealth is cumulated savings); or between product sales, output and inventories. In some cases, such as the latter, it can be helpful to use techniques for analysing I(2) series (see Engsted and Haldrup (1999)). However, in our case as *k* is I(1), **(3)** implies that *i* is also I(1) (or *vice versa*). This follows from stationary non-zero depreciation.(11)

Thus there are two long-run relationships in the data, **(1)** and **(3)**, of which **(4)** is the reduced form. It will therefore also be a cointegrating relationship. Whether this necessarily enters a single ECM as a unique relationship depends on the variables included in the cointegrating set; we return to this shortly.

3. Modelling investment in a VECM framework

From Section 2, we have four endogenous variables, {*i, k, y, r*} and two cointegrating vectors (CVs) from equations **(1)** and **(3)**. To re-state, these are:

$$k = y + \theta - \sigma r \qquad \textbf{(1)}$$

$$i = k + \gamma \qquad \textbf{(3a)}$$

where θ and γ are constants. In VECM notation our two-CV model can be written as:

$$\Delta X_t = \Gamma(L)\Delta X_{t-1} + \Pi X_{t-1} + \Phi D \qquad \textbf{(5)}$$

where *L* is the lag operator, *X* is a matrix of I(1) variables, some of which may be weakly exogenous to the long-run relationship, and *D* is a set of I(0) variables both weakly exogenous to and insignificant in the long-run cointegration space. *D* may contain deterministic terms such as the constant and trend, and intervention dummies.

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(10) Bean (1981) argues that omitting such a variable may not lead to significant biases. Note that output itself is linked to capital via the production function and therefore incorporates a degree of integral control.
(11) As can be seen from the CAI (2), if δ were zero and *I* were I(1) then *K* would be I(2). The implicit depreciation rate is not trended, and passes ADF and Phillips-Perron tests for stationarity.

For a four-variable, two-vector VECM the long run can be decomposed as the reduced rank form

$$\Pi X_{t-1} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \\ \alpha_{41} & \alpha_{42} \end{bmatrix} \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \end{bmatrix} \begin{bmatrix} i \\ k \\ y \\ r \end{bmatrix}_{t-1} \quad \textbf{(6)}$$

In this notation the β_{ij} define the long-run relationships (the cointegrating vectors), and the α_{ij} the response of each variable to the two CVs (the 'loadings'). Although two vectors exist, it does not necessarily follow that all variables respond to all vectors. If a loading is zero, we have weak exogeneity: variable *i* is weakly exogenous to the parameters in vector *j* if $\alpha_{ij} = 0$. Note that this does not imply that the variable is exogenous in the economic sense. From an econometric point of view, unless *k*, *y* and *r* are all weakly exogenous to the long-run relationships, efficient estimation requires the VECM to be estimated. But note that if this condition did hold, single-equation estimation of the long-run parameters in an investment ECM would be unbiased. Moreover, there are sufficient restrictions to identify all the parameters. We would have

$$\Delta i_t = a_0 + dynamics + a_1(i_{t-1} - k_{t-1}) + a_2(k_{t-1} - y_{t-1} - \beta r_{t-1}) \quad \textbf{(7)}$$

4. VECM results

In this section we estimate the structure outlined above using the Johansen method. This is maximum likelihood, and assumes Gaussian errors, so it is important to ensure that the residuals in the underlying VAR are normal and white noise. In order to determine the lag structure, we began by estimating an unrestricted VAR.(12) Lag order selection criteria suggested two lags,(13) but on the serial correlation criteria, eight lags were required. There is still some evidence of serial correlation with a significant LM test at six lags, but the test is then short of degrees of freedom. The residual correlograms reveal no problems. This number of lags implies that the VAR is almost certainly overparameterised, which reduces the power of the tests. Thus we should err on the side of caution when determining the number of cointegrating vectors (use lower critical values). But the consequences of using too low a lag length are usually thought to be more severe.(14) On diagnostic failures in a cointegrating context, Hendry and Juselius (2000) conclude that '[s]imulation studies have demonstrated that statistical inference is sensitive to the validity of some of the assumptions, such as, parameter non-constancy, serially correlated residuals and residual skewness, while moderately robust to others, such as excess kurtosis (fat-tailed distributions) and residual heteroscedasticity.' On normality, each equation in the VAR comfortably passed tests for skewness, but there was evidence for excess kurtosis at the 1%

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(12) Results are reported using dummies for 1985 Q1 and Q2 for a spike in investment, which we attribute to the timing of a pre-announced change in tax allowances. All estimation is over the full sample (1970 Q2 – 2001 Q4), adjusted for lags.
(13) But information criteria are not helpful in determining lag lengths in cointegrating VECMs: Cheung and Lai (1993).
(14) There are many Monte Carlo studies of finite sample properties of the Johansen and other tests for cointegration, examining deviations form the maintained assumptions. Much of this literature is summarised in Maddala and Kim (1998). Results can be sensitive to the order in which assumptions about constants, trends and exogeneity are tested: see Greenslade *et al* (2002). We are guided by Hendry and Juselius (2000).

level in the *r* and *i* equations, and at the 10% level in the *y* equation. While the kurtosis result should lead to caution in interpreting results, there is no reason to suppose inference is fatally flawed.

4.1 Results for full variable set {i, k, y, r}

Cointegration test results for the four endogenous variables are shown in Table A, and we conclude that there are indeed 2 cointegrating vectors. With four variables and two cointegrating vectors, we need two restrictions per vector for exact identification. **(1)** and **(3)** allow us to overidentify both vectors, and these (overidentifying) restrictions are accepted by the data when we impose them (Table B).(15)

Only one parameter of interest is freely estimated in the cointegrating vectors, the elasticity of substitution, which is found to be 0.443. Remarkably, this is consistent with a wide variety of single-equation estimates we examined (ie Engle-Granger, DOLS) with varying dependent long-run variables (capital vs investment): these estimates were all in the range 0.40-0.49.(16) This is in the region of other estimates using labour demand relationships; see eg Barrell and Pain (1997) who report an estimate of 0.48 for the UK private sector, and Hubert and Pain (2001) who report well-determined estimates of around 0.5 for a panel of manufacturing industries.(17)

Table A: Cointegration tests: {*i, k, y, r*}

Null hypothesis: no. of cointegrating equations	Trace statistic	Max-Eigen statistic
None	62.6**	30.3*
At most 1	32.4*	21.6*
At most 2	10.7	10.1

*(**) indicates rejection of null at 5% (1%) significance level.

(15) Note that the fact that investment and capital are found to be cointegrated with a (-1, 1) vector implies that non-stationary depreciation is not a concern using our data over our sample: this potential concern was noted in footnote 6. However, if the share of 'new' investment goods in total investment continues to increase (eg computers), this issue is likely to be important in the future.
(16) These results are reported in the annex. In addition, both single-equation and VECM estimates (not reported) of the elasticity using alternative user cost measures, eg altering the expected change in the relative price term (see data appendix), were mainly in the range 0.3 – 0.6.
(17) NIGEM's estimate for the United Kingdom is slightly higher at 0.66 (see NIESR (2002)), while Chirinko *et al*'s (2002) panel-based estimate for the United States is slightly lower at 0.40. NIGEM's estimate for the United States is 0.57. But all these estimates are significantly different from one, as implied by Cobb-Douglas.

Table B: Estimated long run of VECM

Sample: 1972:3-2001:4
LR test for binding restrictions (rank = 2):

Chi-square(3)	3.585119
Probability	0.309889

Cointegrating Eq:	CointEq1 CAI	CointEq2 FOC		
r(-1)	0.000000	-0.443255 (0.03273) [-13.5408]		
i(-1)	-1.000000	0.000000		
y(-1)	0.000000	1.000000		
k(-1)	1.000000	-1.000000		
Constant	-4.032444	0.231877		
Error Correction:	**Δr**	**Δi**	**Δy**	**Δk**
CointEq1 loading (CAI)	1.043869 (0.32799) [3.18259]	0.083615 (0.29191) [0.28644]	0.020181 (0.08178) [0.24676]	-0.005900 (0.00547) [-1.07843]
CointEq2 loading (FOC)	0.462588 (0.11291) [4.09686]	0.104653 (0.10049) [1.04140]	0.055950 (0.02815) [1.98722]	0.000542 (0.00188) [0.28771]

Chart 1: Capital to output ratio

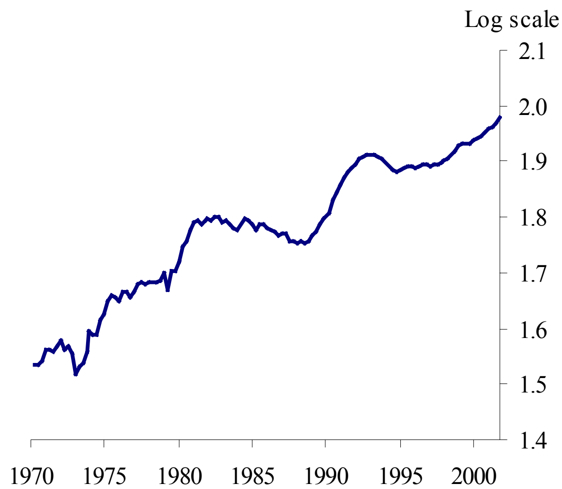
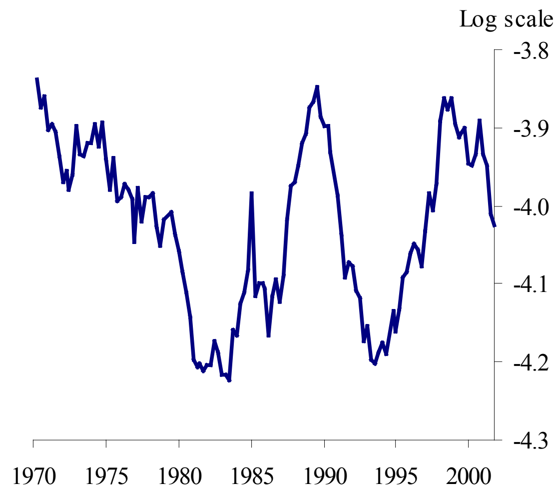


Chart 2: Investment to capital ratio



As an informal check, Charts 1 and 2 show the capital to output and investment to capital ratios. Evidently, the former is far from stationary, and while the latter is very persistent, it is untrended. These impressions are consistent with the formal statistical results.

Given we have identified cointegrating vectors, we can now examine possible restrictions on the loadings (error correction coefficients), which will have implications for weak exogeneity (WE) of the endogenous variables with respect to the two vectors. In VECM analysis, we often have no priors or interpretation of these restrictions, but in our case there may be something to say. From Table B, the unrestricted estimates suggest that investment is weakly exogenous with repect to the first vector (CAI), and capital to the second (FOC); the t-statistics in both cases are very close to zero, at just 0.3. Moreover, as we spell out shortly, these zero restrictions are precisely what we would expect. Although there is no prior reason to expect this, the Δy loading with respect to CAI is also very insignificant. The resulting estimates when we impose these restrictions (as well as the CV restrictions) are shown in Table C. The full set of restrictions is comfortably accepted, and all remaining loadings are now significant.(18)

Table C: VECM estimates with loading restrictions

Sample: 1972:3-2001:4
LR test for binding restrictions (rank = 2):

Chi-square(6)	3.770924
Probability	0.707644

Cointegrating Eq:	**CointEq1 CAI**	**CointEq2 FOC**		
r(-1)	0.000000	-0.442782 (0.03323) [-13.3241]		
i(-1)	-1.000000	0.000000		
y(-1)	0.000000	1.000000		
k(-1)	1.000000	-1.000000		
Constant	-4.032444	0.233540		
Error Correction:	Δr	Δi	Δy	Δk
CointEq1 loading CAI	1.048345 (0.32574) [3.21836]	0.000000 (0.00000) [NA]	0.000000 (0.00000) [NA]	-0.007430 (0.00191) [-3.89219]
CointEq2 loading FOC	0.464288 (0.11229) [4.13490]	0.075886 (0.03509) [2.16267]	0.050343 (0.01924) [2.61641]	0.000000 (0.00000) [NA]

The loading coefficient on capital to the first vector (CAI) is small numerically. But the estimate of 0.007 is plausible. To see why this is the case, note that the CAI can be re-written as

$$\Delta k_t \approx \frac{\Delta K_t}{K_{t-1}} = -\delta + \frac{I_t}{K_{t-1}} \qquad \textbf{(8)}$$

We estimate something similar, namely

$$\Delta k_t = \varphi_0 + dynamics + \varphi_1 (i_{t-1} - k_{t-1}) \qquad \textbf{(9)}$$

Thus the capital stock ecm is an approximation to the CAI. It is an approximation partly because δ is time varying, and partly because of the lag in the investment flow. We would expect $\varphi_1 \approx \frac{I}{K}$. Over the sample period the mean ratio of investment to capital is 0.0178. The long-run coefficient (φ_1 adjusted for dynamics) is 0.0176. So this seemingly small loading coefficient is about what we would expect, given the data.

(18) When we followed Greenslade *et al* (2002) and tested for cointegration imposing these weak exogeneity restrictions, our results were unaffected; ie we found two cointegrating vectors.

The other non-zero loading of the capital accumulation identity is in the user cost of capital equation. At first sight this looks odd. Why should 'disequilibrium' in an identity feed back into the user cost? But the long run is the steady-state investment to capital ratio, equal to depreciation and the steady-state growth rate of capital. This means that the relationship can yield information about what firms are expecting to occur. When the capital stock is increasing at a rate exceeding the steady state, firms' investment is high relative to the steady state. And firms will wish to do this when they expect future developments that are consistent with a higher capital stock. This interpretation of the VECM results recalls the present-value approach which has proved useful in several areas in economics: the current account; consumers' expenditure; and, most relevantly for our purposes, in the determination of stock prices. One relevant exposition on equity valuations is in Campbell and Shiller (1988). They show that the linearised present value of the firm implies that if there is a high dividend to price ratio, agents must be expecting either high returns on assets in the future, or low dividend growth rates. This follows purely from the identity and rational expectations. The interesting corollary for our purposes is that there are implications for q, and therefore investment. This was first explored by Abel and Blanchard (1986), and more recently by Lettau and Ludvigson (2002) and Robertson and Wright (2002). To understand what is going on, suppose that q exceeds unity. This implies (see, eg Hayashi (1982)) that the firm must be planning to increase the capital stock, and this is why q is a sufficient statistic for investment (or more strictly, for the growth in the capital stock). But while q is indeed increasing in future growth in the capital stock, it is also decreasing in the required rate of return (and increasing in future dividend payouts, although this is not particularly germane to our problem). The point is that above-average current growth in the capital stock (or equivalently, an above equilibrium investment to capital stock ratio) should be associated with lower future market returns (lower future cost of capital). And this is precisely what is happening in the VECM. i-k and the user cost are not cointegrated as the ratio does not depend on the user cost, but the former nevertheless contains some information about future changes in the latter. When i-k exceeds the long-run value future returns must be expected to fall, which implies a run of negative rates of change in the user cost.

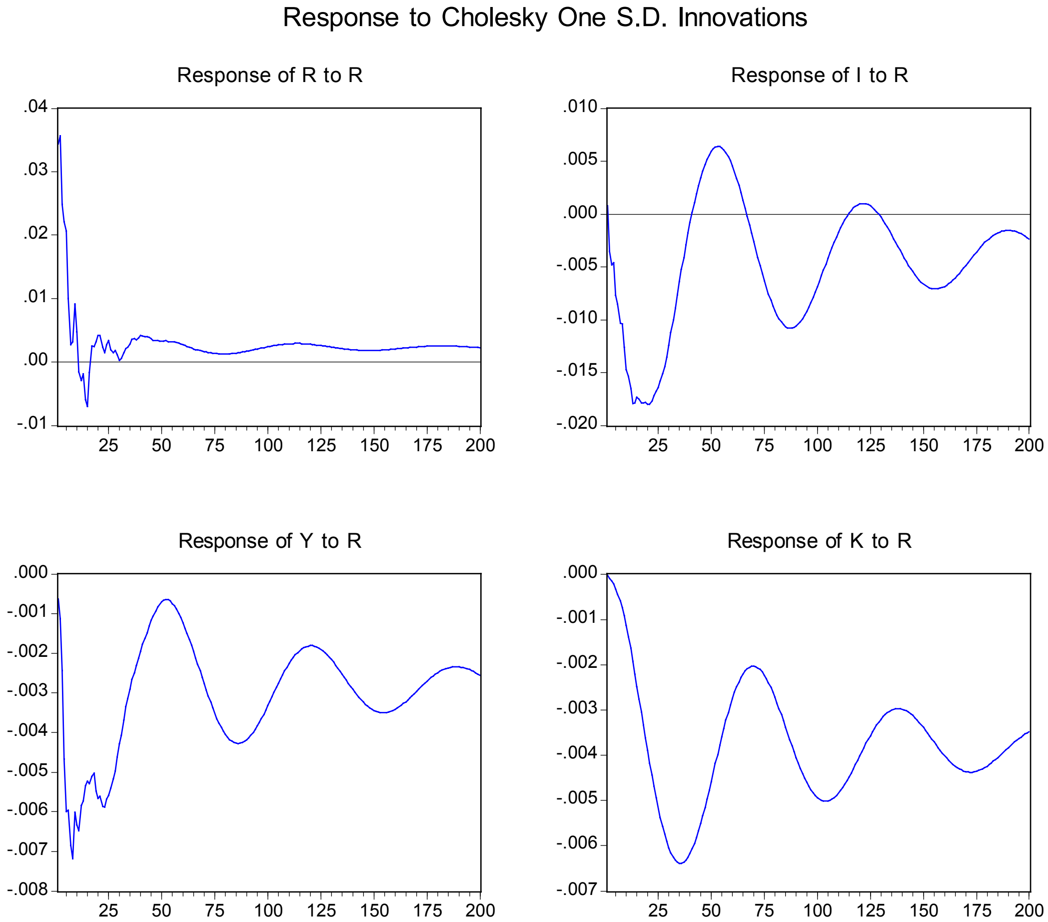
One interesting way to test this mechanism would be to see if the i-k ratio has predictive power for excess returns to equities. Table D reports the results of regressing quarterly excess (over the three-month T-bill rate) returns from the UK FT-Actuaries All-Share Total Return Index at different horizons on the lagged ratio. To account for autocorrelation induced by overlapping horizons the Newey-West correction is applied. The table reveals that the ratio does have predictive power at horizons greater than a year, peaking at around 16 quarters.

Table D: Predictive power of *i-k* ratio for FTSE excess returns

Forecast Horizon (quarters)	**1**	**4**	**8**	**12**	**16**	**20**	**24**
Coefficient	-0.08	-0.27	-0.61	-1.00	-1.14	-1.15	-0.95
t-statistic	-0.93	-1.64	-3.07	-3.96	-4.12	-4.01	-3.06
R^2	0.01	0.05	0.19	0.29	0.30	0.26	0.18

The loading on the FOC in the user cost equation is explicable in terms of the long-run relationship; in the steady state, the marginal revenue product is equated with the user cost. Equilibration takes place via investment or the return on capital. The positive output loading on the FOC is less easy to explain. But it is hard to judge details of the dynamic process from the single-equation loadings, as all the dynamics in the system are important.[19] Thus Chart 3 reports the impulse responses of the endogenous variables to a one standard deviation shock to the real user cost (r).[20] The vertical scale is log difference (ie approximately % difference/100), and the horizontal scale is time periods in quarters. Investment, output and capital all overshoot the long-run effect and then oscillate around the eventual long-run response. This protracted oscillation is driven by the loading on CAI in the dynamic user cost equation being greater than unity. In the next section we exclude insignificant dynamic terms and the loading falls below one: impulse responses from this more parsimonious system exhibited less oscillation. The response of capital is fairly slow, with the maximum (overshooting) effect occurring after six years with the further unwind persisting beyond this. This drawn-out response is mirrored in the investment and output responses. Part of this is likely to be linked to the fact that the real user cost does not stabilise until at least 30 quarters after the initial shock.

Chart 3: System impulse responses



(19) Full results are available on request.
(20) We use a Cholesky ordering where the user cost is placed first. Clearly, this is an arbitrary identification scheme. An alternative approach would be to seek to identify structural disturbances.

4.2 SUR VECM results

We have established the existence of two cointegrating vectors and tested the overidentifying restrictions implied by theory. Nevertheless, with eight lags the dynamics of the system are grossly overparameterised. Thus we looked for a parsimonious restriction of the system, estimated by seemingly unrelated regression (SUR). When we tested down without imposing the weak exogeneity restrictions, the dynamic capital equation responded to both cointegrating vectors. However, investment remained weakly exogenous to the capital accumulation identity.[21]

4.3 Results for restricted cointegrating variable sets: irreducible cointegration

Our model predicts that two cointegrating vectors exist among the four variables, and we have successfully identified them using the overidentifying restrictions from theory. It should be clear, however, that if we restrict attention to limited sets of variables then it should be possible to estimate unique vectors, albeit with a possible loss of efficiency. This idea was formalised and explored by Davidson (1994, 1998) as the notion of 'irreducible cointegration'. An irreducible cointegrating relation is one from which no variable can be omitted without loss of the cointegration property. Such relations may be structural or reduced form. The advantage of the procedure he suggests is that, under certain circumstances, when the model is overidentified, it enables the researcher to obtain information about the underlying structure directly from the data. However, as in our case, this will not always be true: theory is still required. If our approach is correct, we should find evidence for unique structural and irreducible cointegrating vectors in each of the sets $\{k, y, r\}$ and $\{i, k\}$, from **(1)** and **(5)**. Tables E and F reveal support for this.[22] Moreover, the restrictions on the two vectors are accepted (p values of 0.44 and 0.63 for FOC and CAI respectively). The pattern, significance and rough magnitudes of the loadings are consistent with those from the full system. The reduced form **(4)** also implies that $\{i, y, r\}$ should constitute an irreducible cointegrating set. Table G gives the test results. The trace statistic indicates cointegration at the 5% significance level, and the failure of the max-Eigen statistic to reject the null of no cointegration is very marginal. So this last result suggests that the Bean reduced-form specification will work. Nevertheless, the fact that we are estimating a reduced-form specification means that the adjustment dynamics conflate the capital adjustment and accumulation.

Table E: Cointegration tests: $\{k, y, r\}$

Null hypothesis: no. of cointegrating equations	Trace statistic	Max-Eigen statistic
None	37.1**	21.9*
At most 1	15.2	14.0

*(**) indicates rejection of null at 5% (1%) level.

(21) Full results are available on request.

(22) The coefficient restrictions are essential for the irreducibility argument, as without them there is evidence that $\{y, r\}$ and $\{k, r\}$ cointegrate, which have no sensible economic interpretations. Thus we effectively argue that $\{i\text{-}y, r\}$, $\{i\text{-}k\}$ and $\{k\text{-}y, r\}$ form conditional irreducible cointegrating sets. This is consistent, as mentioned in Section 4.1 above, with the time series for the capital to output and investment to capital ratios.

Table F: Cointegration tests: {*k, i*}

Null hypothesis: no. of cointegrating equations	Trace statistic	Max-Eigen statistic
None	18.5*	18.4*

*(**) indicates rejection of null at 5% (1%) level.

Table G: Cointegration tests: {*i, y, r*}

Null hypothesis: no. of cointegrating equations	Trace statistic	Max-Eigen statistic
None	31.0*	18.9
At most 1	12.1	10.9

*(**) indicates rejection of null at 5% (1%) level.

5. Impulse responses

The results we have obtained thus far imply that investment equilibrates the capital stock equilibrium condition. This automatically embodies an integral control mechanism through the capital accumulation identity. The cointegrating relationship implied by the capital accumulation identity does not equilibrate through the investment equation. However, no variables are weakly exogenous with respect to either of the cointegrating relationships so efficient estimation requires full system estimation. But given the theory and results in Table G if the cointegrated set excludes capital a single equation might be estimated.(23) Table H reports the restricted equation. The loading is very similar to that obtained in the estimation underlying Table G (-0.13 with t ratio –2.58). Table I reports the comparable results of using the long-run VECM results, re-estimated parsimoniously by OLS.

(23) The specification in Table H differs from that in the Annex A1 as there is no capacity utilisation term, included for comparability with previous Bank work, but the long-run parameter and adjustment terms are very close.

Table H: Reduced form

Dependent Variable: Δi
Sample: 1972:3-2001:4
Included observations: 118

Variable	Coefficient	Std. Error	t-Statistic	Prob.
Constant	-0.497205	0.117819	-4.220088	0.0001
Δi(-2)	0.140725	0.078126	1.801246	0.0744
Δi(-3)	0.243819	0.079059	3.083996	0.0026
Δi(-4)	0.260874	0.079953	3.262853	0.0015
Δr(-1)	-0.094995	0.059426	-1.598555	0.1128
i(-1)-y(-1)	-0.131323	0.030064	-4.368117	0.0000
r(-1)*	**0.445334**	0.086550	5.145397	0.0000
D85_1	0.082034	0.029182	2.811107	0.0059
D85_1(-1)	-0.145333	0.029200	-4.977103	0.0000
R-squared	0.381631	Mean dependent var		0.008487
Adjusted R-squared	0.336246	S.D. dependent var		0.035217
S.E. of regression	0.028691	Akaike info criterion		-4.191229
Sum squared resid	0.089729	Schwarz criterion		-3.979905
Log likelihood	256.2825	Durbin-Watson stat		1.999177
Durbin-Watson stat	1.983020	Prob(F-statistic)		0.000000

* Long-run value.

Table I: Single equation from VECM long-run results

Dependent Variable: Δi
Sample: 1972:3-2001:4
Included observations: 118

	Coefficient	Std. Error	t-Statistic	Prob.
FOC*	**0.102084**	0.038902	2.624121	0.0099
Δi(-7)	0.281172	0.076393	3.680612	0.0004
Δy(-5)	0.247692	0.278022	0.890909	0.3749
Δk(-7)	-6.339864	1.514348	-4.186532	0.0001
Constant	0.062747	0.014437	4.346145	0.0000
1985:1 dummy	0.076325	0.028761	2.653763	0.0091
1985:1 dummy lagged	-0.153061	0.028776	-5.319031	0.0000
R-squared	0.393568	Mean dependent var		0.008487
Adjusted R-squared	0.360788	S.D. dependent var		0.035217
S.E. of regression	0.028156	Akaike info criterion		-4.244620
Sum squared resid	0.087997	Schwarz criterion		-4.080257
Log likelihood	257.4326	Durbin-Watson stat		2.163611

* Defined as in Table C. Normalisation on -*k*.

In order to calculate the different responses of investment and capital under these approaches, we constructed two models, each of which comprised the dynamic investment equation and the capital accumulation identity. In specifying these models, we treated y and r as exogenous, as we are concerned here purely with the investment and capital stock dynamics. Thus the responses differ from those in Chart 3. Our impulse was a permanent +1% shock to the level of the real user cost (R).

The estimated response of capital under the two methods is shown below in Chart 4, with particular periods singled out in Table J. In each case the long-run adjustment of capital reflects the estimated elasticity of substitution in each model, as it must, which are similar in each model. Due to the small size of investment relative to the capital stock (investment represents just under 2% of the capital stock), in both cases it takes a considerable length of time to reach the long run.

Table J: Impulse responses at forecast horizons

	Quarters after shock		
Response of:	4	8	12
Capital			
Reduced form	-0.015	-0.043	-0.076
VECM	-0.007	-0.027	-0.055
Investment			
Reduced form	-0.315	-0.512	-0.566
VECM	-0.18	-0.363	-0.512

Chart 4: Capital impulse responses

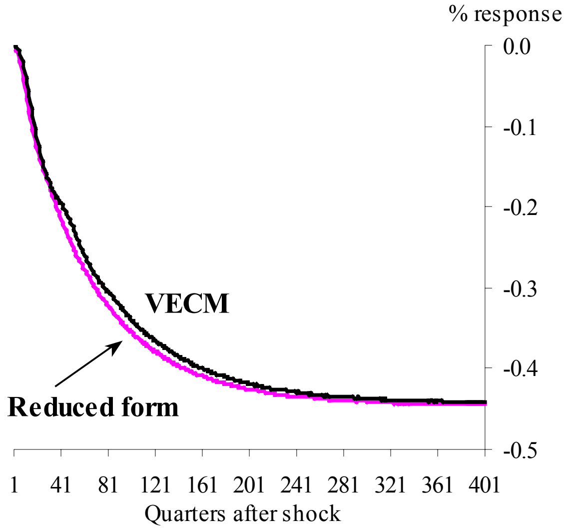


Chart 5: Investment impulse responses

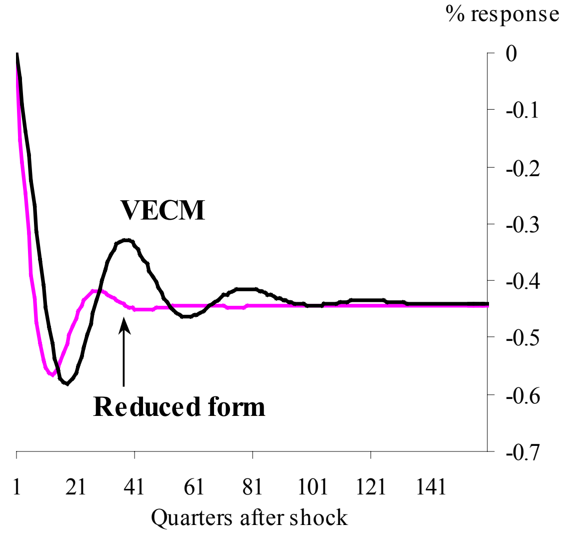


Chart 5 shows impulse responses for investment. These converge to the long run rather more quickly than the capital stock, but near-complete convergence is still protracted. Investment overshoots in both models. In principle there is no special reason why the two should give different answers, but as a matter of algebra, the reduced form is quicker partly because the long run is defined directly in terms of investment.

6. Monte Carlo simulations

We are advocating system estimation. But this may come at the expense of low power or incorrectly sized test statistics. To investigate this, we carried out a Monte Carlo exercise, maintaining the restricted structure in the VECM.[24] As *a priori* nothing rules out the possibility that both cointegrating vectors enter the investment ECM, we conducted three experiments. The first (Method 1) is to estimate by OLS a single equation embodying both CVs; the second (Method 2), the single reduced-form CV, again in an OLS regression; the third (Method 3), the correctly specified VECM, estimated by the Johansen method.

(24) In general we need to ensure that the model is estimated consistently under both the null and the alternative, and in non-stationary environments this may not always be true. But here we maintain the reduced rank hypothesis throughout, so this is not an issue.

We used the parsimonious SUR system[25] as the underlying data generating process, although we replace the capital ECM with the capital accumulation identity, where depreciation is stochastic, calibrating the normal additive error on δ to match the variance of capital stock growth. Shocks to the other three variables (investment, output and the user cost) were calibrated using the variances of the errors from the SUR system. Having generated 200 observations for each variable, we then estimated using each of the three methods outlined above, using the same sample size as for our model based on UK data (120 observations). We then imposed restrictions on each of the three estimation methods and tested for whether these were accepted. The results reported below are based on 10,000 repetitions.

Table K: Monte Carlo results

	Method 1		Method 2	Method 3		
Significance level	FOC significant	CAI significant	RF significant	OIR rejected	CAI significant, conditional on OIR accepted	CAI WE and OIR jointly rejected
	power	**size**	power	**size**	**size**	**size**
10%	89.53	73.67	96.18	62.48	0.08	52.18
5%	80.27	62.17	91.15	51.82	0.02	42.02
Estimated elasticity of substitution (actual = 0.443)						
Mean	0.432		0.396	0.430		
Median	0.410		0.406	0.443		
5th Percentile	0.183		0.201	0.382		
95th Percentile	0.593		0.579	0.518		

In Table K the first column reports the proportions with which the true hypothesis that the FOC is significant in a dynamic OLS regression; the power of the test. For a notional size of 5%, it is 80.1%. But the striking result is that the regression fails to reject the true hypothesis that the CAI should be excluded in the majority of cases.[26] That is, the actual size of the test is much greater than the notional, and inference would be profoundly misleading. By contrast, the reduced-form approach is highly robust, with a power of 93.1% at a notional size of 5%. In other words, the single reduced-form cointegrating vector is highly significant in single equations.

The VECM is oversized on the set of long-run overidentifying restrictions (OIR), rejecting the (true) restrictions just over 50% of the time at the notional 5% significance level. But, conditional on these restrictions being accepted, the size of the test on the (true) hypothesis that the CAI should be excluded from the investment ECM is very small at just 0.1%. Thus single-equation techniques will lead one to falsely include the CAI in the estimated investment equation in the majority of cases, but this is much less likely to happen in a system context.

(25) Results available on request.

(26) This follows from the fact that the two normalised long-run relationships both contain a term in the capital stock with unit coefficients (positive or negative, depending on the normalisation): *i-k* and *k-y*. There is also the reduced-form relationship, *i-y*, and this is a restriction on the loadings of the two long-run coefficients (the restriction being that they are equal and opposite in sign). This restriction is easily accepted, in the actual data as well as in the Monte Carlo experiments.

Indeed, it will almost never occur. However, the true overidentifying restrictions will be falsely rejected roughly half the time (at a notional size of 5%).

The other question of interest is the estimate of the elasticity of substitution, σ. The table gives the mean and median estimates. The distributions are highly skewed, and the VECM mean is substantially biased upwards, but the median estimate is very close to the true value (0.44). The 95% confidence interval is also reported, and it is clear that the VECM will estimate the parameter with far more precision than the other methods.

Finally, we bootstrapped the residuals to avoid making possibly incorrect distributional assumptions. We drew six period blocks to capture possible lumpiness in the depreciation series. The main difference to the Monte Carlo results is that the simulations now reveal a much reduced skew (Table L).

Table L: Bootstrapped results

	Method 1		Method 2	Method 3		
Significance level	FOC significant	CAI significant	RF significant	OIR rejected	CAI significant, conditional on OIR accepted	CAI WE and OIR jointly rejected
	power	**size**	power	**size**	**size**	**size**
10%	93	85.14	98.72	61.7	0.08	51.71
5%	85.84	76.22	96.35	51.00	0.10	41.37
Estimated elasticity of substitution (actual = 0.443)						
Mean	0.402		0.413	0.379		
Median	0.417		0.418	0.443		
5th Percentile	0.282		0.262	0.395		
95th Percentile	0.531		0.555	0.500		

7. Conclusions

Economic theory tells us that output, the capital stock and the user cost of capital are cointegrated. But from the capital accumulation identity, it also follows that the capital stock and investment share a long-run relationship. This has been used to justify the estimation of long-run investment, rather than capital stock, equations. Econometrically, the dynamic structure suggests that a multi-cointegrating framework should exist, in which separate cointegrating relationships are identifiable. This turns out to be the case for UK business sector investment. A new investment equation is estimated, exploiting a measure of the capital stock constructed by the Bank, and a long series for the weighted cost of capital. A CES production function is assumed, and a well-determined estimate of the elasticity of substitution is obtained for a variety of measures. The robust result is that the elasticity of substitution is significantly different from unity (the Cobb-Douglas case), at about 0.45. Overidentifying restrictions on the long-run relationship are all accepted. Although the key long-run parameter (the elasticity of substitution) is highly robust to alternative specifications, single-equation investment relationships may obscure the dynamics. Thus we finally conclude that if we are interested only in knowing the elasticity of substitution, estimation using an OLS long-run investment relationship will return an unbiased estimate, although the Monte Carlo results show that the VECM is considerably more accurate. In a macroeconometric modelling context, however, we may prefer to model investment via the long-run capital stock relationship, with the capital accumulation identity as a separate relationship.

Data appendix

Business investment (I) and GDP (Y) are available from the Office for National Statistics (ONS) National Accounts data, with quarterly backruns to 1965 and 1955 respectively. We use capital stock and associated depreciation series that have been constructed in-house following Oulton and Srinivasan (2003). In particular, we use a four-asset wealth measure of the capital stock, assuming that the asset split of business investment is the same as whole-economy investment (excluding dwellings), which is available from National Accounts data.(27)

The other key variable is the real user cost of capital (*RCC*). The full Hall-Jorgensen *RCC* is defined as:

$$RCC = \frac{P_K}{P_Y}\{c + \delta - E(\frac{\dot{P}_K}{P_K} - \frac{\dot{P}_Y}{P_Y})\}\frac{1 - PVIC}{1 - corptax}$$

where

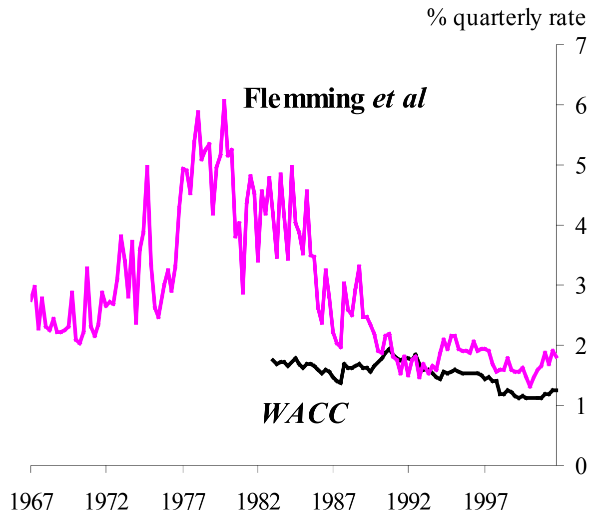
c	denotes	real cost of finance
P_k	denotes	price of capital goods (the IBUS deflator)
P_y	denotes	price of all goods (the GDP deflator)
PVIC	denotes	present value of investment allowances
corptax	denotes	corporation tax rate

The price variables are available from National Accounts data, as is the effective corporation tax rate. *PVIC* is based on Bank calculations, following Mayes and Young (1993). The expected relative price inflation term is an unobserved variable, assumed to be zero. We also experimented using a variety of assumptions about this variable including ARMA forecasts, filtered *ex-post* expectations and backward-looking averages, but excluding this term improved the fit of singly-estimated investment equations.

A more problematic issue is construction of a 'real cost of finance' variable. From 1982 Q1, a weighted average cost of capital (*WACC*) can be calculated, following Brealey and Myers (2000). But wanting to conduct estimation prior to 1982, previous work(28) has used an alternative cost of finance described in Flemming *et al* (1976). Essentially this measure is the ratio of current earnings to the financial valuation of companies. While in principle measuring a similar concept, over the same sample the Flemming measure is more volatile than *WACC* (see Chart A1) and the correlation is not particularly strong (0.48 over the whole sample).

(27) We are grateful to Jamie Thompson for his help in constructing the series. ONS suspended their estimates of the capital stock in the latest *Blue Book*: see National Statistics (2002).
(28) See Bakhshi and Thompson (2002).

Chart A1: *WACC* and Flemming cost of finance

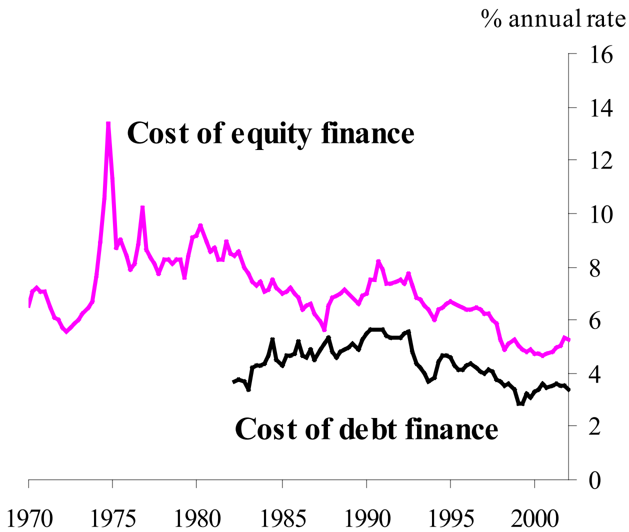


As an alternative we decided to construct a back series for *WACC* to 1970. *WACC* is calculated as the weighted average of the cost of debt finance (R^D) and the cost of equity finance (R^E):

$$WACC_t = \lambda R_t^D + (1-\lambda)R_t^E$$

The weight (λ) is taken from company balance sheet data on the relative use of debt and equity finance: a weight of 15% is placed on debt, and 85% on equity. We used fixed weights for debt and equity finance: using time-varying weights, to capture changes in the relative importance of debt and equity in company balance sheets, had only a negligible impact on the aggregate *WACC* series.

Chart A2: *WACC* components



The real cost of debt finance is calculated as the risk-free real interest rate, measured using the ten-year spot rate for the index-linked yield curve, plus a measure of spreads. This measure of spreads is for investment-grade bonds, spliced together from the FTSE Debentures and Loans Index and the Merrill Lynch Index. The real cost of equity finance is calculated using a simple dividend discount model. Using these calculations, the cost of equity finance is readily available back to 1965, but the cost of debt finance only from 1982 (Chart A2). Thus to construct a *WACC* we need to backcast the cost of debt finance.

We constructed three different models to backcast the cost of debt finance using macro variables eg GDP, base rates, and inflation. Two of the three models included leads of the dependent variable (DV). The first model estimated the real risk-free rate and the spreads measure separately, including leading DVs (a 'bottom-up' approach); the second estimated the cost of debt finance directly including leading DVs (a 'top-down' approach); and the third was a 'simple' equation for R^D with no DVs included. The three different models produced very different backcasts for the cost of debt finance, shown in Chart A3. However, given the small weight of debt finance, the resulting backcast *WACC* series were almost indistinguishable (Chart A4). Given this, we picked the simple approach, due to concerns about the stability of the models including leading DVs.

Note that when we estimated the VECM over the non-backcast *WACC* sample (1982 onwards), the estimate of the elasticity of substitution was broadly the same (0.43, compared to 0.44 over the whole sample), but the overidentifying restrictions were not accepted. Given the simulation results in Section 6, this is perhaps unsurprising.

Chart A3: Backcast cost of debt finance

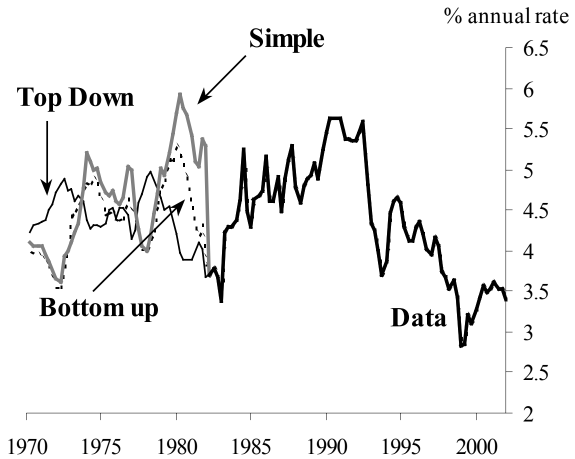
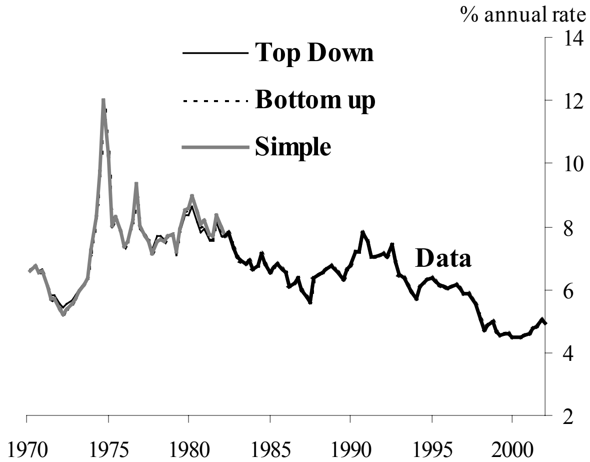


Chart A4: Backcast *WACC*



Having constructed our cost of finance term, we now have our four endogenous variables.(29) These are shown below in Charts A5 to A8 (note log scales). Unit root tests for all four variables are shown in Tables A1 and A2.(30) The tests indicate that all variables are I(1) at standard significance levels. The source of the non-stationarity in the user cost is primarily the fall in the relative price of capital, although the real interest rate is also somewhat trended over this sample. The result certainly holds in sample, although some readers may be sceptical that the fall will continue. However, it is quite feasible for relative prices to be non-stationary indefinitely if there are sources of differential productivity growth. This may be true in the production of capital versus other types of good.

(29) All data are consistent with the *2002 Blue Book*.
(30) For ADF tests, we included as many lagged differences in the auxiliary regressions as were significant.

Chart A5: Investment (i)

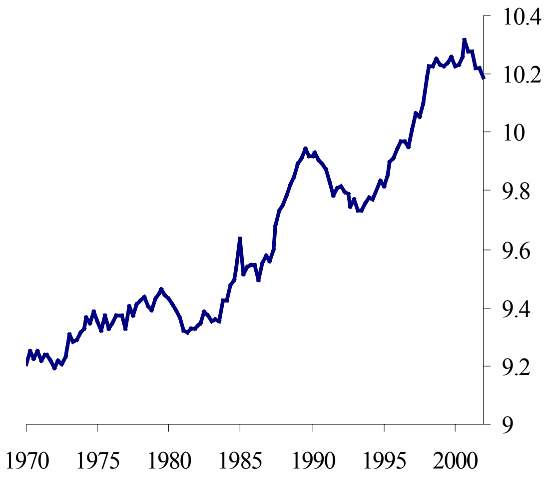


Chart A6: Capital stock (k)

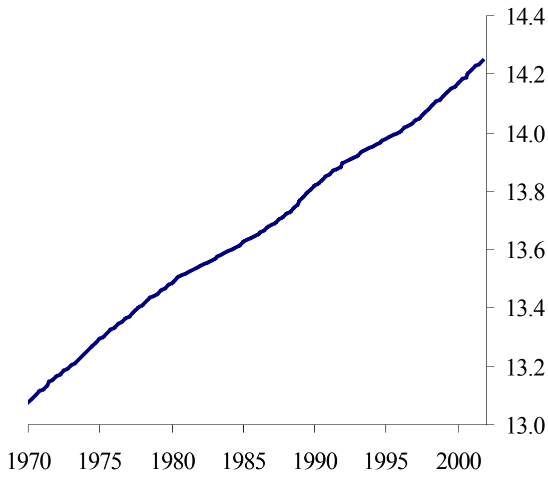


Chart A7: Real user cost of capital (r)

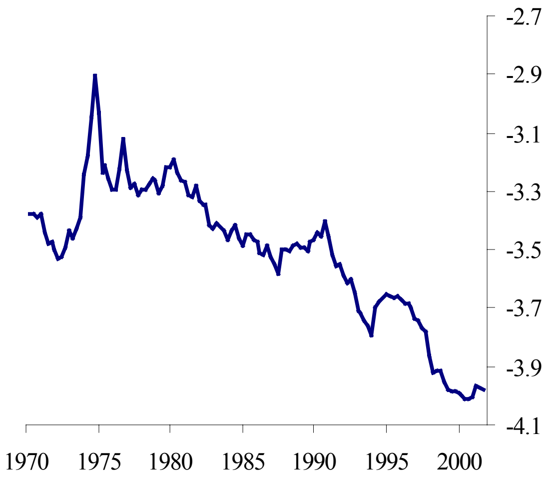


Chart A8: GDP (y)

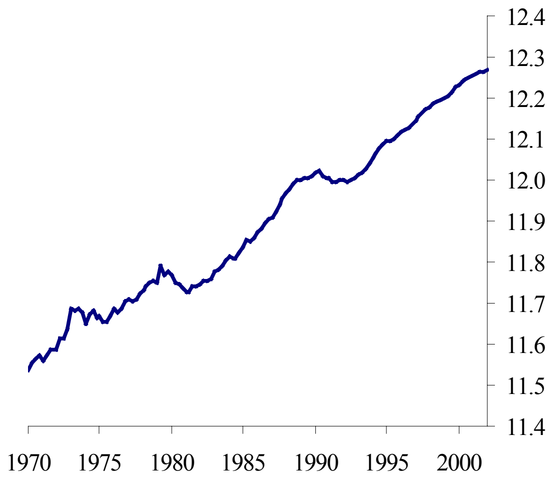


Table A1: ADF tests

Variable	Level	First difference
i	-0.734	-3.569***
k	-0.179	-2.666*
r	-0.380	-5.746***
y	0.029	-4.786***

* (**, ***) indicates rejection of null at the 10% (5%, 1%) significance level.

Table A2: KPSS and Phillips-Perron tests

Variable	KPSS $H_0 = I(0)$	Phillips-Perron $H_0 = I(1)$
i	2.395***	-0.150
k	2.626***	-0.434
r	2.050***	-0.542
y	2.597***	-0.194

* (**, ***) indicates rejection of null at the 10% (5%, 1%) significance level.

Annex: Baseline single-equation results

Variables in single-equation estimation are as listed in the main text and the data appendix, namely:

I	Business investment
K	Capital stock
Y	Gross domestic product
R	Real user cost of capital
FKU	Capital utilisation (taken from Larsen, Neiss and Shortall (2002))
D85_1	Dummy variable for 1985 Q1 (tax allowance change)

All estimation was carried out over the full data sample (1970 Q2 – 2001 Q4), adjusted for lags. FKU is an 'integral control variable', referred to in the main body of the text; (unreported) results were similar with other measures, eg the CBI capacity utilisation balance. A baseline least squares equation is shown in Equation A1, estimated using a 'general to specific' approach with four lags of all variables. It differs in dynamic detail from Table H in the main text, for comparability with Bakhshi and Thompson (2002).

Equation A1: Single-equation estimates with investment in the long run

Dependent Variable: Δi
Sample: 1971:2-2001:4

	Coefficient	Std. Error	t-Statistic	Prob.
Constant	-1.176709	0.244373	-4.815219	0.0000
Δi(-4)	0.153563	0.079923	1.921385	0.0571
FKU(-1)	0.633699	0.187134	3.386332	0.0010
1985:1 dummy	0.106353	0.028699	3.705791	0.0003
1985:1 dummy lagged	-0.124903	0.028905	-4.321193	0.0000
i(-1)-y(-1)	-0.138680	0.029959	-4.628988	0.0000
r(-1)*	**0.489132**	0.080269	6.093684	0.0000
R-squared	0.362020	Mean dependent var		0.008108
Adjusted R-squared	0.329021	S.D. dependent var		0.034759
S.E. of regression	0.028473	Akaike info criterion		-4.224528
Sum squared resid	0.094039	Schwarz criterion		-4.064485
Log likelihood	266.8085	Durbin-Watson stat		2.100567

* Long-run value.

The estimated elasticity of substitution is 0.49, significantly different from unity (as would be implied under Cobb-Douglas technology). The equation is well specified in terms of the usual residual test criteria (normality, homoscedasticity, serial correlation and ARCH test). In addition we estimated a dynamic ordinary least squares (DOLS) regression with two leads to compare the estimated elasticity of substitution in the long-run relationship. Results are shown below. The DOLS estimate of 0.41 is a little lower than that in the Equation A1 above (0.49), but our baseline estimate is not significantly different from the lower DOLS estimate.

Equation A2: DOLS equation estimates (two leads): investment in the long run

Dependent Variable: i-y
Sample: 1971:1-2001:2

	Coefficient	Std. Error	t-Statistic	Prob.
Constant	-6.629227	0.532656	-12.44560	0.0000
r	**-0.411598**	0.032129	-12.81099	0.0000
Δr(-1)	-0.032689	0.167817	-0.194790	0.8459
Δr(-2)	0.067514	0.165997	0.406716	0.6850
Δr(+1)	-0.366995	0.162094	-2.264084	0.0255
Δr(+2)	-0.480255	0.160652	-2.989412	0.0034
Δ(i(-1) – y (-1))	0.387622	0.234414	1.653578	0.1010
Δ(i(-2) – y (-2))	0.275979	0.230675	1.196396	0.2341
FKU(-1)	2.929605	0.533884	5.487349	0.0000
1985:1 dummy	0.103962	0.080356	1.293777	0.1984
1985:1 dummy lagged	-0.084816	0.082443	-1.028776	0.3058

R-squared	0.697812	Mean dependent var	-2.249935
Adjusted R-squared	0.670588	S.D. dependent var	0.136100
S.E. of regression	0.078114	Akaike info criterion	-2.175460
Sum squared resid	0.677298	Schwarz criterion	-1.922638
Log likelihood	143.7030	Durbin-Watson stat	0.334799

We also replicated the single-equation results using the capital stock rather than investment in the long run. The corresponding results are shown below.

Equation A3: Single-equation estimates with capital in the long run

Dependent Variable: Δi
Sample: 1971:2-2002:1

	Coefficient	Std. Error	t-Statistic	Prob.
Constant	0.064972	0.046968	1.383310	0.1692
Δi(-3)	0.206489	0.079180	2.607845	0.0103
Δi(-4)	0.239706	0.080228	2.987801	0.0034
Δk(-1)	-6.035088	1.653706	-3.649433	0.0004
1985:1 dummy	0.087218	0.029647	2.941914	0.0039
1985:1 dummy lagged	-0.141157	0.029565	-4.774447	0.0000
(k(-1) – y(-1))	-0.106458	0.037287	-2.855120	0.0051
r(-1)*	**0.492888**	0.111973	4.401848	0.0000

R-squared	0.332974	Mean dependent var	0.008108
Adjusted R-squared	0.292372	S.D. dependent var	0.034759
S.E. of regression	0.029240	Akaike info criterion	-4.163746
Sum squared resid	0.098321	Schwarz criterion	-3.980839
Log likelihood	264.0704	Durbin-Watson stat	2.033090

* Long-run value.

Once again the estimated elasticity of substitution is close to 0.5. And yet again DOLS estimation (see below) suggested a slightly smaller coefficient, but not radically different.

Equation A4: DOLS equation estimates (two leads): capital in the long run

Dependent Variable: k-y
Sample: 1971:1-2001:2

	Coefficient	Std. Error	t-Statistic	Prob.
Constant	0.378849	0.109244	3.467919	0.0007
r	**-0.395811**	0.030943	-12.79168	0.0000
Δr(-1)	0.011360	0.154672	0.073448	0.9416
Δr(-2)	0.030776	0.153128	0.200980	0.8411
Δr(+1)	-0.339067	0.153246	-2.212560	0.0290
Δr(+2)	-0.497018	0.153367	-3.240703	0.0016
Δ(k(-1) – y (-1))	1.183177	0.677267	1.746989	0.0834
Δ(k(-2) – y (-2))	0.912514	0.672837	1.356218	0.1778
1985:1 dummy	0.037677	0.075759	0.497334	0.6199
1985:1 dummy lagged	0.035428	0.075806	0.467358	0.6412
R-squared	0.633111	Mean dependent var		1.778145
Adjusted R-squared	0.603629	S.D. dependent var		0.118681
S.E. of regression	0.074719	Akaike info criterion		-2.271751
Sum squared resid	0.625289	Schwarz criterion		-2.041913
Log likelihood	148.5768	Durbin-Watson stat		0.134395

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