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Berliant, Marcus and Yu, Chia-Ming
Washington University in St. Louis

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Marcus Berliant† and Chia-Ming Yu‡

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Abstract: Agglomeration can be caused by asymmetric information and a locational signaling effect: The location choice of workers signals their productivity to potential employers. The cost of a signal is the cost of housing at a location. When workers' price elasticity of demand for housing is negatively correlated with their productivity, skill-biased technological change causes a core-periphery bifurcation where the agglomeration of high-skill workers eventually constitutes a unique stable equilibrium. When workers' price elasticity of demand for housing and their productivity are positively correlated, skill-biased technological improvements will never result in a core-periphery equilibrium. This paper claims that location can at best be an approximate rather than a precise sieve for high-skill workers. (*JEL Classifications*: D51; D82; R13)

Keywords: Agglomeration; Adverse Selection; Asymmetric Information; Locational Signaling

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†Department of Economics, Washington University, Campus Box 1208, 1 Brookings Drive, St. Louis, MO 63130-4899. Phone: (1-314) 935-8486, Fax: (1-314) 935-4156, e-mail: berliant@artsci.wustl.edu; and Division of the Humanities and Social Sciences, California Institute of Technology.

‡Department of Economics, Washington University, Campus Box 1208, 1 Brookings Drive, St. Louis, MO 63130-4899. Phone: (1-314) 552-1773, Fax: (1-314) 935-4156, e-mail: cyua@artsci.wustl.edu.

1 Introduction

As shown in Baum-Snow and Pavan [2009], US wages were more than 30 percent higher in metropolitan areas with over 1.5 million inhabitants than in rural areas in the year 2000. Furthermore, their model indicates that ability sorting and returns to experience across locations are crucial elements in explaining the wage premium in large cities. Glaeser and Mare [2001] show that sorting on human capital accounts for about one-third of the city-size wage gap in the US. Moreover, Gould [2007] demonstrates that migration of high-skill workers is important in justifying the urban productivity premium which is amplified by steeper experience profiles in urban areas. These analyses suggest that workers signal their skill and experience using their locations. That is, the location choice of workers can signal their productivity to potential employers. The signaling cost is the price of housing at a location. Locational signaling is also consistent with a constant product of city rankings and growth rates in population for top-ranking cities: The U.S. Census Bureau data show that, from 1990 to 2000, this constant is around 0.11 for the top three cities.[1] That is, though housing rents are high and population is dense in top-ranking cities, these costs do not inhibit new migrants from moving in. This paper analyzes the effects of locational signaling behavior, in particular how and whether locational signaling effects can generate agglomeration.

One natural question is: How can we empirically distinguish locational signaling effects from agglomeration externalities? Agglomeration externalities and spillovers are widely analyzed in the literature, for example, Henderson [1986], Henderson et al. [1995], Glaeser et al. [1992], and Feldman and Audretsch [1999]. Under the framework of agglomeration externalities,

[1] For the top 50 cities in U.S. from 1990 to 2000, we can get (city growth rate)= $0.12 + 0.0001*$(city ranking). The coefficient on city rankings is small; that is, for the top 50 cities, the growth rate is almost constant.

an increase in the ratio of high-skill labor in one region causes more than a proportional increase in the average real wage (or an increase in labor's marginal product). In a locational signaling model, an increase in the ratio of high-skill labor in a region yields a proportional increase in the average real wage. The predictions of a signaling model are consistent with the findings in Aghion et al. [1999, p. 1644]: "The main argument put forward against the skill-biased technical change hypothesis is that we have not observed an increase in the rate of productivity growth since the early 1980s."

Households' private information includes their productivity, which varies among individuals. When locations can possibly reveal workers' productivities, it is natural to ask why in practice some locations are attached to a signal for high productivity of workers, while others are not. For example, fashion designers in Milan, software programmers in Seattle, entertainers in Hollywood, financiers on Wall Street, or high-tech workers in Silicon Valley can be viewed as having a higher productivity than do workers in the same field in other locations. These observations could be due to learning from other workers, or interaction with R&D in these locations; however, they could also due to a locational signaling effect. Many tools are used to signal workers' abilities since information about workers' skill is very important to firms and workers, for example: college diplomas, professional certificates, and academic alliance memberships.[2] It is interesting to examine how high-skill workers can use locational agglomeration to distinguish themselves from other workers, and how effective location can be as a reference for workers' productivity.

In the literature, Starrett [1978] proves a spatial impossibility theorem: If there is no relocation cost, space is homogeneous (consumers' preferences and firms' technologies are independent of location), the economy is closed, and there are perfect and complete markets everywhere, there is no compet-

[2] In urban economics, for example, there is the UEA.

itive equilibrium involving costly transportation of any commodity. Fujita and Thisse [2002] interpret this theorem further and show that either there is no agglomeration of agents in equilibrium or there is no equilibrium at all. However, Starrett's theorem offers only sufficient conditions for no agglomeration in equilibrium; these conditions are not necessary. Therefore, when one of the conditions is violated, it is not clear whether firms or workers will agglomerate or not. Berliant and Kung [2008] is the first paper analyzing how asymmetric information causes agglomeration. Using a screening model, they show that workers can agglomerate and be sorted by skill in equilibrium due to asymmetric information in the labor market. This paper focuses on a complementary question: When there is asymmetric information, does an agglomeration emerge in equilibrium due to the signaling value of the choice of location? The shadow cost of location, and thus of the signal, is the price of housing in the region.

Krugman [1991a] and New Economic Geography models adopt increasing returns to scale to explain the agglomeration of manufacturing firms in one region. When transportation cost is decreased as transportation technology is improved, a core-periphery pattern is more likely in equilibrium. It is natural to ask: Is a core-periphery configuration more likely to constitute an equilibrium when there are no increasing returns to scale in production, but rather asymmetric information?

Many economic agglomeration phenomena in reality cannot be satisfactorily explained by increasing returns to scale. As expounded in Krugman [2009], "the history of such classic localizations as that of the car industry seemed to suggest that concentrations due to increasing returns peaked before World War II." Thus, "there is good reason to believe that the world economy has, over time, actually become less characterized by the kinds of increasing-returns effects emphasized by new trade and new geography." That is, there is a need to offer economic explanations other than increasing returns to scale in explaining the agglomeration of industries without increasing returns. A signaling incentive potentially fills this need.

In contrast to considering aggregate uncertainty in Berliant and Yu [2009], idiosyncratic uncertainty (individual specific information) is the source of asymmetric information in this paper. We consider a model with two homogeneous regions and two types of workers, with high and low productivity, respectively. Workers are mobile across regions while differences in regional wages and housing rents determine their migration incentives. We first analyze the case when workers' price elasticity of demand for housing is negatively correlated with their productivity. In this case, as shown in Figure 10, there are at least three equilibria: a completely symmetric equilibrium where every type of worker is evenly distributed over both regions, and two partially segregated equilibria (or say core-periphery equilibria) where high-productivity workers are agglomerated in one region. The partially segregated equilibria are always stable. When the difference in workers' productivities is small, the completely symmetric equilibrium is stable; when the difference in workers' productivity is large enough, the completely symmetric equilibrium becomes unstable. When the difference in workers' productivity is very large, in addition to the unstable completely symmetric equilibrium and two stable core-periphery equilibria, there are two unstable asymmetrically integrated equilibria. On the other hand, when workers' price elasticity of demand for housing is positively correlated with workers' productivity, as shown in Figure 9, there always exists a completely symmetric equilibrium but there is no core-periphery equilibrium. The completely symmetric equilibrium is stable when the difference in workers' productivities is not large. When the difference in productivities is very large, there are three unstable equilibria where one of them is completely symmetric and two of them are asymmetrically integrated equilibria.

For example, though a higher wage for workers in the fashion industry in Milan attracts workers in an alternative region to migrate to Milan, due to a larger aggregate housing demand, there will be a higher housing rent in Milan to offset workers' migration incentives. As shown in Figure 1,[3] when high-productivity workers have a lower price elasticity of demand for housing than low-productivity workers, the utility cost of signaling for high-productivity workers is lower than the utility cost of signaling for low-productivity workers at the core-periphery equilibrium. Therefore, for a given wage premium in Milan, there is a long-run segregated equilibrium such that all the high-productivity workers agglomerate in Milan while the low-productivity workers reside in both Milan and the alternative region. When high-productivity workers have a higher price elasticity of demand for housing than low-productivity workers, as shown in Figure 2, the signaling cost for high-productivity workers is higher than that for low-productivity workers under any core-periphery configuration. This intuition is verified in this paper, which suggests a potentially testable implication of our model, namely the prevalence of agglomeration of high-skill workers as a function of the correlation of skill and demand elasticity for housing.

Notice that, in either a segregated or an integrated equilibrium, no region is fully occupied by high-productivity workers alone. That is, there is no completely segregated equilibrium, but a semi-pooled equilibrium may exist.[4] On the other hand, there is always a completely pooled equilibrium in our model. Therefore, it is only possible to ensure that any worker who does not reside in Milan is a low-productivity worker. For every worker in Milan, it is impossible to guarantee that his/her productivity is high in any equilibrium. *This observation indicates that location at best is an approximate instead of a precise sieve for high-productivity workers.*

[3]We shall explain the figures introduced here in detail later in the paper. This is a preview.

[4]The core-periphery equilibrium in this paper corresponds to a semi-pooling equilibrium where some types of senders choose the same message (location) and other types choose different messages (locations).

Furthermore, if we consider a continuous increase in high-skill workers' productivity relative to that of low-skill workers, a core-periphery fork bifurcation is present (Figure 10), even if there are no increasing returns to scale in production and knowledge spillovers. In other words, the agglomeration of high-productivity industries can be attributed to the existence of a locational signaling effect. Since, intuitively, increasing returns to scale in fashion design seems bizarre, the agglomeration of fashion industries in Milan can be explained from a signaling viewpoint.

Signaling cost in our model is determined by housing prices, and housing prices are different for different distributions of workers. In contrast with most signaling models where the marginal signaling cost is exogenous, i.e., Spence [1973], Wilson [1977], Grossman [1981], and Rothschild and Stiglitz [1976], the marginal signaling cost is endogenous in our paper. That is, signaling cost affects workers' migration incentives, and after their migration, the distribution of workers' types further influences the signaling cost. We explore the question: Does the interaction between migration and marginal signaling cost yield a separated equilibrium? The same type of endogeneity also holds in cheap-talk models like Crawford and Sobel [1982] and Austen-Smith and Banks [2000].

In what follows, our model is introduced in Section 2. Additionally, necessary and sufficient conditions for the existence of stable core-periphery equilibria and for the stability of integrated equilibria are presented. Several numerical examples and related welfare analyses are offered in Section 3. Conclusions are in Section 4.

2 Model

There are two regions $k \in K \equiv \{x, y\}$ with the same land endowment $\bar{s}$. There are two types of mobile workers $i \in N \equiv \{H, L\}$ with population

$n^H, n^L \in \mathbb{R}_{++}$, respectively, where the productivity of H-type workers is higher than that of L-type workers. H-type (L-type) workers can be interpreted as high-skill (low-skill) workers, or can be interpreted as experienced (novice) workers. With the second interpretation, the appearance of a segregated equilibrium implies that returns to experience are important in explaining city size wage premium.

Throughout this paper, workers' type is indexed by a superscript and location is indexed by a subscript. The (endogenous) population of i-type workers living in k is denoted by n_k^i, and the (exogenous) aggregate population in the model is $n = n^H + n^L$. Firms cannot recognize any worker's type directly; however, firms know the (equilibrium) distribution of workers' types over the two regions and can infer the probability of a worker's type using his/her location. Utility is CES. Let s_k^i, z_k^i be each i-type worker's house size and the consumption of composite goods in region k, $i \in N$, $k \in K$, respectively. Let p_k denote the rent per unit of housing and w_k denote the worker's wage in k, $k \in K$. Each worker is endowed with one unit of labor. The rents are collected and consumed by an absentee landlord, denoted by A, who is endowed with all the housing. Let $\varphi_k^i \equiv (s_k^i, z_k^i)$, $i \in N \cup \{A\}$, $k \in K$. The absentee landlord has an inelastic supply of housing $\bar{s}$ in each region and maximizes the rent that he can collect, i.e., $\max_{z_k^A} \sum_{k \in K} z_k^A$, subject to $z_k^A \leq p_k \, \bar{s}$, $\forall k \in K$, and has an inelastic supply of housing in all cities.[5] The optimization problem for H-type workers in region k, $k \in K$, is

$$\begin{aligned} \max \;\; & u_k^H(\varphi_k^H) = [(s_k^H)^{\frac{\alpha-1}{\alpha}} + (z_k^H)^{\frac{\alpha-1}{\alpha}}]^{\frac{\alpha}{\alpha-1}} \\ \text{s.t.} \;\; & p_k \, s_k^H + z_k^H \leq w_k, \\ & s_k^H, z_k^H \in \mathbb{R}_+; \end{aligned} \tag{1}$$

[5]Except for asymmetric information, our model satisfies all the assumptions in Starrett's theorem. That is, asymmetric information is the only source of agglomeration in this model.

whereas the optimization problem for L-type workers in k is

$$\begin{aligned} \max \;\; & u_k^L(\varphi_k^L) = [(s_k^L)^{\frac{\beta-1}{\beta}} + (z_k^L)^{\frac{\beta-1}{\beta}}]^{\frac{\beta}{\beta-1}} \\ \text{s.t.} \;\; & p_k\, s_k^L + z_k^L \leq w_k, \\ & s_k^L, z_k^L \in \mathbb{R}_+. \end{aligned} \tag{2}$$

Assume that $\alpha, \beta > 1$. Either $\alpha > \beta$ holds, which implies that workers' price elasticity of demand for housing is positively correlated with productivity, or $\alpha < \beta$ holds, implying that workers' price elasticity of demand for housing and productivity are negatively correlated.[6]

To simplify the analysis, assume that each worker inelastically supplies one unit of labor, so we need not be concerned about monitoring and voluntary participation constraints. Every firm hires one worker at most. Each firm can adopt a high type technology together with a H-type labor to produce Y^H, or adopt a low type technology together with a L-type labor to produce Y^L, where $0 < Y^L < Y^H$. The corresponding profit in k is $Y^H - w_k$ and $Y^L - w_k$, respectively, $k \in K$. When any firm adopts a high type technology with a L-type worker, the output is zero. On the other hand, when a firm adopts a low type technology and a H-type worker, the output is Y^L, which is lower than Y^H. That is, no firm would prefer to adopt a technology that is incompatible with the type of the hired worker. Firms maximize their expected profit, and their actual behavior in choosing technology will be explained later. Every firm or worker is so small that he/she cannot influence competitive market prices. Furthermore, assume that there is free entry of firms, and thus, every firm earns zero expected profit in equilibrium. Finally, workers choose locations to maximize their utilities, including the consideration that firms can possibly learn about workers' types only from observing their locations.

[6]When $\alpha = \beta$, either there are an infinite number of equilibria or there is no long-run equilibrium, which is not a case of interest.

To extract the influence of signaling effects, assume that there is no commuting; that is, workers can work only in the place where they live. In other words, this is a regional, not city, model. However, H-type and L-type workers are allowed to migrate to earn a higher utility.[7] Denote ρ^H (ρ^L) as the ratio of H-type (L-type) workers in the world living in x, and thus $1-\rho^H$ ($1-\rho^L$) is the ratio of all H-type (L-type) workers living in y. The population in x and y, given (ρ^H, ρ^L), can be expressed as $n_x = \rho^H n^H + \rho^L n^L$ and $n_y = (1-\rho^H)n^H + (1-\rho^L)n^L$, respectively.

To characterize locational signaling effects, the market process is given as follows. First, each firm hires a worker without knowing his/her productivity. Though firms do not know each worker's type, suppose that firms do not misperceive; that is, they know the actual equilibrium proportion of H-type workers in each region and thus have a common distribution over a worker's type conditional on his/her equilibrium location. Then, since there is a free entry of firms, each firm in a region pays its worker a wage according to the expected profit in the region. After learning the type of worker that the firm hires, the firm chooses its production technology to maximize ex post profit or minimize ex post loss. A mixed adoption of technology is assumed not available for firms.[8]

Note that given (ρ^H, ρ^L), since there is free entry of firms, each firm earns

[7] When H-type workers are mobile but L-type workers are immobile, there are similar bifurcations.

[8] Surely, changing the specified market process can change the results of our model. For example, when firms are assumed to choose their technology before knowing workers' type, the chosen technology must be the same for all firms in one region (since there is no difference between firms in the same region). Moreover, given workers' distribution is not completely symmetric, when the high technology is chosen in one region in equilibrium, the other region will choose the low technology. Since the H-type (L-type) workers can be hired only in the region adopting the high (low) technology, a core-periphery equilibrium is immediate for any not-completely symmetric initial distribution of workers. Actually, this setting is more like a screening model as analyzed in Berliant and Kung [2008], instead of a signaling model. In addition, when firms pay the wage after they know workers' type, there is no need for signaling. Therefore, the market process specified here is more appropriate in presenting a story for signaling effects than alternative assumptions.

zero expected profit. Thus, the wages for every worker in region x and y are[9]

$$w_x(\rho^H, \rho^L) = \frac{1}{n_x}(\rho^H n^H Y^H + \rho^L n^L Y^L), \tag{3}$$

$$w_y(\rho^H, \rho^L) = \frac{1}{n_y}[(1-\rho^H) n^H Y^H + (1-\rho^L) n^L Y^L]. \tag{4}$$

Let us temporarily leave workers' mobility aside. Short-run equilibrium is defined as a competitive market equilibrium, given a population distribution over the two regions.

Definition 1 (Short-Run Equilibrium)
$(\varphi_k^{H*}, \varphi_k^{L*}, \varphi_k^{A*}, w_k^*, p_k^*)_{k\in K}$ *constitutes a short-run equilibrium if, given an arbitrary* (ρ^H, ρ^L), *workers choose optimal consumptions, firms make competitive wage offers for the distribution of workers, and the housing and the composite good markets in each region clear. That is:*

(a) $u_k^i(\varphi_k^{i*}) \geq u_k^i(\varphi_k^i)$, *for all* $\varphi_k^i \in \mathbb{R}_+^2$ *satisfying* $p_k\, s_k^i + z_k^i \leq w_k$, $\forall i \in N$, $k \in K$;

(b) $w_x^* = \frac{1}{n_x}(\rho^{H*} n^H Y^H + \rho^{L*} n^L Y^L)$, *and*
$w_y^* = \frac{1}{n_y}[(1-\rho^{H*}) n^H Y^H + (1-\rho^{L*}) n^L Y^L]$;

(c) $\rho^{H*}\, n^H\, s_x^{H*} + \rho^{L*}\, n^L\, s_x^{L*} = \bar{s}$,
$(1-\rho^{H*})\, n^H\, s_y^{H*} + (1-\rho^{L*})\, n^L\, s_y^{L*} = \bar{s}$,
$(\rho^{H*}\, z_x^{H*} + (1-\rho^{H*})\, z_y^{H*})\, n^H + (\rho^{L*}\, z_x^{L*} + (1-\rho^{L*})\, z_y^{L*})\, n^L + z_x^{A*} + z_y^{A*}$
$= n^H\, Y^H + n^L\, Y^L$, *where* $z_k^{A*} = p_k\, \bar{s}$, $k \in K$.[10]

The short-run equilibrium, by Walras' law, is determined by conditions (a), (b), and the first two (or the last two) equalities in (c). Theorem 1 shows that the short-run equilibrium exists and is unique.

Theorem 1 *For each* $(\rho^H, \rho^L) \in [0,1]\times[0,1]$, *there exists a unique short-run equilibrium.*

[9] The main purpose of this paper is to characterize agglomeration across regions, instead of migration within one region; therefore, wage inequality within the same region is not considered here.

[10] Recall that land in all regions is owned by one absentee landlord.

Proof. It is obvious from (b) that $w_k^* = w_k(\rho^H, \rho^L)$, $k \in K$, can not be empty or multiple-valued. Substituting w_k^* into workers' utility maximization problems (1) and (2), we have workers' optimal consumptions as functions of p_k and (ρ^H, ρ^L); that is, $\varphi_k^H(p_k, w_k(\rho^H, \rho^L)) = [w_k(\rho^H, \rho^L)/(p_k + p_k^\alpha), w_k(\rho^H, \rho^L)/(1 + p_k^{1-\alpha})]$ and $\varphi_k^L(p_k, w_k(\rho^H, \rho^L)) = [w_k(\rho^H, \rho^L)/(p_k + p_k^\beta), w_k(\rho^H, \rho^L)/(1 + p_k^{1-\beta})]$, all are well-defined demand functions. Finally, the equilibrium housing prices can be solved from substituting demands into market clearing conditions, i.e., $\rho^H\, n^H\, s_x^H(p_x, w_x^*) + \rho^L\, n^L\, s_x^L(p_x, w_x^*) = \bar{s}$ and $(1-\rho^H)\, n^H\, s_y^H(p_y, w_y^*) + (1-\rho^L)\, n^L\, s_y^L(p_y, w_y^*) = \bar{s}$. Though there are no explicit solutions for these two equalities, we can solve housing prices for each given (ρ^H, ρ^L), and since the excess demand function for housing in k, $k \in K$, is continuous and monotonically decreasing in p_x and p_y, for all p_x, $p_y \in \mathbb{R}_{++}$, $p_k^* = p_k(\rho^H, \rho^L)$ is uniquely determined for each (ρ^H, ρ^L). Accordingly, we have well-defined equilibrium consumptions, $\varphi_k^{i*} = \varphi_k^i(p_k^*, w_k^*) = \varphi_k^i(\rho^H, \rho^L)$, $i \in N$, $k \in K$. *Q.E.D.*

When workers' mobility is considered, workers have to choose their optimal locations according to the utilities from living in the two regions. Since i-type workers' indirect utility from living in region k is $u_k^i(\varphi_k^{i*})$, $i \in N$, $k \in K$, the equilibrium condition for no further migration is

$$u_x^i(\varphi_x^{i*}) = u_y^i(\varphi_y^{i*}), \text{ if } \rho^{i*} \in (0,1),\ \forall\, i \in N. \tag{5}$$

However, when all i-type workers are agglomerated in region k, $i \in N$, $k \in K$, i-type workers' utility in the other region k', $k' \in K$ where $k' \neq k$, is not defined. Following the literature, the potential wage and housing rent for i-type workers in k' is defined as the limit of the equilibrium wage and equilibrium rent in k' when the ratio of i-type workers in k' approaches zero. Then, the potential utility for i-type workers in k' is defined according to their potential wage and potential housing rent in k'. Given this setting, the signaling equilibrium concept is in fact defined by a pair $(\rho^{H*}, \rho^{L*}) \in$

$[0,1] \times [0,1]$, and the corresponding $(\varphi_k^{H*}, \varphi_k^{L*}, \varphi_k^{A*}, w_k^*, p_k^*)_{k \in K}$ that satisfies following conditions.

Definition 2 (Signaling Equilibrium)
$((\varphi_k^{H*}, \varphi_k^{L*}, \varphi_k^{A*}, w_k^*, p_k^*)_{k \in K}, \rho^{H*}, \rho^{L*})$ constitutes a signaling equilibrium when $(\varphi_k^{H*}, \varphi_k^{L*}, \varphi_k^{A*}, w_k^*, p_k^*)_{k \in K}$ constitutes a short-run equilibrium for (ρ^{H*}, ρ^{L*}), and, in addition, no worker in any region has an incentive to migrate to the other region. That is, in addition to conditions (a)-(c) in Definition 1, it is required that
(d) $u_x^i(\varphi_x^{i*}) = u_y^i(\varphi_y^{i*})$ *if* $\rho^{i*} \in (0,1)$, $\forall i \in N$, $k \in K$;
$u_x^H(\varphi_x^{H*}) \geq \lim_{\rho^H \to 1} u_y^H(\varphi_y^H[p_y(\rho^H, \rho^{L*}), w_y(\rho^H, \rho^{L*})])$, *if* $\rho^{H*} = 1$;
$u_x^L(\varphi_x^{L*}) \geq \lim_{\rho^L \to 1} u_y^L(\varphi_y^L[p_y(\rho^{H*}, \rho^L), w_y(\rho^{H*}, \rho^L)])$, *if* $\rho^{L*} = 1$;
$u_y^H(\varphi_y^{H*}) \geq \lim_{\rho^H \to 0} u_y^H(\varphi_y^H[p_y(\rho^H, \rho^{L*}), w_y(\rho^H, \rho^{L*})])$, *if* $\rho^{H*} = 0$;
$u_y^L(\varphi_y^{L*}) \geq \lim_{\rho^L \to 0} u_y^L(\varphi_y^L[p_y(\rho^{H*}, \rho^L), w_y(\rho^{H*}, \rho^L)])$, *if* $\rho^{L*} = 0$.

The long-run signaling equilibrium can be solved by a system of equations including (a), (b), (d), and, by Walras' Law, the first two (or the last two) equations of condition (c) in Definition 1. That is, the equilibrium housing rents are determined by

$$h_x \equiv \rho^H n^H (p_x + p_x^\alpha)^{-1} + \rho^L n^L (p_x + p_x^\beta)^{-1} - \frac{\bar{s}}{w_x} = 0, \tag{6}$$

$$h_y \equiv (1-\rho^H) n^H (p_y + p_y^\alpha)^{-1} + (1-\rho^L) n^L (p_y + p_y^\beta)^{-1} - \frac{\bar{s}}{w_y} = 0. \tag{7}$$

Substituting equilibrium consumption and equilibrium prices into the utility functions, we have workers' difference in indirect utilities from living in the regions. Letting $u_k^{i*} = u_k^i(\varphi_k^{i*})$, in order to have an easy decomposition of signaling gains and signaling costs, we take a natural logarithm of indirect utility functions.[11]

$$\log u_x^{H*} - \log u_y^{H*}$$
$$= (\log w_x - \log w_y) - \left(\log(1+p_x^{1-\alpha})^{\frac{-1}{\alpha-1}} - \log(1+p_y^{1-\alpha})^{\frac{-1}{\alpha-1}}\right), \quad (8)$$
$$\log u_x^{L*} - \log u_y^{L*}$$
$$= (\log w_x - \log w_y) - \left(\log(1+p_x^{1-\beta})^{\frac{-1}{\beta-1}} - \log(1+p_y^{1-\beta})^{\frac{-1}{\beta-1}}\right). \quad (9)$$

Notice that $\log w_x - \log w_y$ is interpreted as a signaling gain (if it is positive), or signaling loss (if it is negative) from living in x comparing to living in y, which is the same for both types of workers. On the other hand, the signaling cost of living in x relative to living in y is $\log(1+p_x^{1-\alpha})^{\frac{-1}{\alpha-1}} - \log(1+p_y^{1-\alpha})^{\frac{-1}{\alpha-1}}$ and $\log(1+p_x^{1-\beta})^{\frac{-1}{\beta-1}} - \log(1+p_y^{1-\beta})^{\frac{-1}{\beta-1}}$ for H-type and L-type workers, respectively.

Equilibrium is a solution to a system of four nonlinear simultaneous equations (6), (7), (8), and (9). It is interesting to notice that if $(\rho^{H*}, \rho^{L*}) = (\frac{1}{2}, \frac{1}{2})$ constitutes an equilibrium, the result is exactly the case where both types of workers are equally distributed over the two regions, which is called a completely symmetric equilibrium; whereas if either $(\rho^{H*}, \rho^{L*}) = (1, 0)$ or $(\rho^{H*}, \rho^{L*}) = (0, 1)$ in equilibrium, there is a segregated equilibrium. Letting $f \equiv \log u_x^{H*} - \log u_y^{H*}$ and $g \equiv \log u_x^{L*} - \log u_y^{L*}$, the following lemma ensures the existence of an interior equilibrium.

Lemma 1 *Equal-dispersion* $(\rho^{H*}, \rho^{L*}) = (1/2, 1/2)$ *always constitutes a signaling equilibrium.*

Proof. Given $(\rho^H, \rho^L) = (1/2, 1/2)$, it is known that $w_x = w_y$, and from (6) and (7), it can be checked that $p_x = p_y$. Since $w_x = w_y$ and $p_x = p_y$ imply $f = 0$ and $g = 0$, we have that $(\rho^H, \rho^L) = (1/2, 1/2)$ is always one of the solutions to (6), (7) and $\log u_x^{H*} = \log u_y^{H*}$, $\log u_x^{L*} = \log u_y^{L*}$. *Q.E.D.*

In addition to the existence of a signaling equilibrium, the stability of a

[11] Since $Y^L > 0$ implies $w_x, w_y > 0$, the CES indirect utilities are greater than 0, so it is safe to use a logarithmic transformation.

long-run equilibrium should also be examined. The definition of stability for an equilibrium is given as follows.

Definition 3 (Stability of Equilibrium)
For any small deviation of one type of workers from the equilibrium worker distribution, given that firms can only recognize a worker's type according to their beliefs generated by the worker's equilibrium location, if the utility difference from living in different locations drives the perturbed workers back to their equilibrium locations, the equilibrium is stable; otherwise, the equilibrium is called unstable.

Note that, given condition (d) in Definition 2, a core-periphery configuration (i.e, $\rho^{H*} = 0$ or $\rho^{H*} = 1$) is always a stable equilibrium when it constitutes an equilibrium. However, a completely symmetric equilibrium can be stable or unstable.

For a given $(\log u_x^{i*},\ \log u_y^{i*})$, $i \in N$, we consider standard dynamics with multiple types of workers. When $\log u_x^{i*} > \log u_y^{i*}$ $(\log u_x^{i*} < \log u_y^{i*})$, $i \in N$, i-type workers in y (x) surely have incentive to move to x (y). In order to explore the stability of signaling equilibria, following Krugman [1991b], Fukao and Benabou [1993], and Forslid and Ottaviano [2003], for $i \in N$, let $\dot{\rho}^i$ describe the ad hoc dynamics:

$$\dot{\rho}^i \equiv \frac{d\rho^i}{dt} = \begin{cases} \max\{0, \gamma\,(\log u_x^{i*} - \log u_y^{i*})\} & \text{if } \rho^i = 0, \\ \gamma\,(\log u_x^{i*} - \log u_y^{i*}) & \text{if } \rho^i \in (0,1), \\ \min\{0, \gamma\,(\log u_x^{i*} - \log u_y^{i*})\} & \text{if } \rho^i = 1. \end{cases} \tag{10}$$

Notice that $\gamma > 0$ represents a measure of the speed of adjustment in the ratio of i-type workers across regions, $i \in N$ (as emphasized in Krugman [1991b], "γ is an inverse index of the cost of adjustment"). That is, when $\log u_x^{i*} > \log u_y^{i*}$ $(\log u_x^{i*} < \log u_y^{i*})$, i-type workers in y (x) migrate to x (y) with a speed of $|\dot{\rho}^i|$. From the specified ad hoc dynamics, two curves corresponding to $\dot{\rho}^H = 0$ and $\dot{\rho}^L = 0$ can be drawn on the (ρ^H, ρ^L) plane as shown in Figures 3 to 8.

Intuitively, when ρ^H increases, fixing ρ^L and all parameters, since the population in x (y) increases (decreases), the demand for and the equilibrium price of houses in x (y) increase (decrease) and at the same time, the average productivity or wage of workers in x (y) increases (decreases). Therefore, $\log u_x^{i*} - \log u_y^{i*}$, $i \in N$, may not be a monotonic function of ρ^H. On the other hand, given ρ^H and parameters, when ρ^L increases, the demand for housing in x increases and the average productivity of workers in x decreases. That is, there is no benefit but only damage for any resident in x when there are low-skill migrants coming from y, so $\log u_x^{i*} - \log u_y^{i*}$, $i \in N$, is monotonically decreasing in ρ^L. Notice that the signaling gain is the same for both types of workers in the same region. As illustrated in Figure 1, when the price elasticity of demand for housing for H-type workers is smaller than that for L-type workers, the signaling cost for H-type workers is less than the signaling cost for L-type workers at the core-periphery equilibrium, and thus, H-type workers have a stronger incentive to migrate to the region with a higher wage, which causes an agglomeration of H-type workers in the *ex post* core region. By contrast, in Figure 2, when the price elasticity of demand for housing for H-type workers is larger than that for L-type workers, the signaling cost for H-type workers is higher than the signaling cost for L-type workers. In this case, there is no equilibrium with an agglomeration of any type of worker. In the interesting cases with $n^H < n^L$, these intuitions are verified by the following numerical simulations.

3 Numerical Examples

Since there is no closed-form solution for the simultaneous equations (6)-(9), some numerical examples are analyzed here. Given $n^H = 1$, $n^L = 2$, $Y^L = 1$, $\bar{s} = 1$, $\alpha = 2$, $\beta = 4$, and $Y^H = 1.25$, a corresponding phase diagram is shown in Figure 3. Here, productivity and the elasticity of demand for housing are negatively correlated. In the phase diagram, from $f \equiv \log u_x^{H*} - \log u_y^{H*}$ and $g \equiv \log u_x^{L*} - \log u_y^{L*}$, it can be checked that $\dot{\rho}^H < 0$ ($\dot{\rho}^H > 0$) for all (ρ^H, ρ^L)-points above (below) the curve of $\dot{\rho}^H = 0$. In addition, $\dot{\rho}^L < 0$ ($\dot{\rho}^L > 0$) for all (ρ^H, ρ^L)-points above (below) the curve of $\dot{\rho}^L = 0$. Letting $\phi^i(\rho^H) = \{\rho^L | \log u_x^{i*}(\rho^H, \rho^L) = \log u_y^{i*}(\rho^H, \rho^L)\}$, $i \in N$, the phase diagram shows that $\phi^i(\rho^H)$, $i \in N$, is single valued and non-empty for $\rho^H \in [0, 1]$. The phase diagram also shows that a necessary and sufficient condition for a stable completely symmetric equilibrium is $\phi^{H'}(\rho^H) \leq 0$ at $\rho^H = 1/2$. A sufficient condition for the existence of a core-periphery equilibrium is $\phi^L(\rho^H) < \phi^H(\rho^H)$ at $\rho^H = 1$ or $\phi^L(\rho^H) > \phi^H(\rho^H)$ at $\rho^H = 0$. All these conditions are satisfied in Figure 3 where there exist three equilibria: one stable completely symmetric equilibrium and two stable core-periphery equilibria at $(\rho^{H*}, \rho^{L*}) = (0, 0.61)$ and $(\rho^{H*}, \rho^{L*}) = (1, 0.39)$. At $(\rho^{H*}, \rho^{L*}) = (1/2, 1/2)$, $u_x^{H*} = u_y^{H*} = 2.28$, $u_x^{L*} = u_y^{L*} = 1.44$, and $u^{A*} = 1.81$. Since firms' expected profit is zero in all equilibria, *ex ante* social welfare function is defined as the sum of workers' and the landlord's utilities. The social welfare equals 6.97 in the completely symmetric equilibrium, which is higher than the social welfare of 6.94 at any core-periphery equilibrium.[12] The reason is that locational signaling is unproductive, and thus it is not socially optimal to agglomerate high-skill workers in our model.

Given the same parameters, when Y^H increases to 2, as shown in Figure 4, the completely symmetric equilibrium becomes unstable, though there are still two stable core-periphery equilibria, $(\rho^{H*}, \rho^{L*}) = (0, 0.41)$ and $(\rho^{H*}, \rho^{L*}) = (1, 0.59)$. When $Y^H = 4$, besides the two core-periphery equilibria at $(\rho^{H*}, \rho^{L*}) = (0, 0.23)$ and $(\rho^{H*}, \rho^{L*}) = (1, 0.77)$, there are three unstable integrated equilibria at $(\rho^{H*}, \rho^{L*}) = (0.09, 0.27)$, $(\rho^{H*}, \rho^{L*}) = (1/2, 1/2)$, and $(\rho^{H*}, \rho^{L*}) = (0.91, 0.73)$. In both the cases of $Y^H = 2$ and 4, *ex ante* social welfare in

[12] Since at $(\rho^{H*}, \rho^{L*}) = (1, 0.39)$, $u_x^{H*} = 2.27$, $u_y^{H*} = 2.25$, $u_x^{L*} = u_y^{L*} = 1.43$, and $u^{A*} = 1.81$, the core-periphery equilibrium is in fact Pareto-dominated by the completely symmetric equilibrium.

the completely symmetric equilibrium is higher than that in other equilibria. Letting W^{CS}, W^{AI}, and W^{CP} denote social welfare in the completely symmetric, asymmetrically integrated, and core-periphery equilibrium, respectively, it can be checked that $W^{CS} > W^{AI} > W^{CP}$ for all $Y^H > 2$.

A core-periphery bifurcation is present when a high-skill biased technological improvement is considered as a continuous process. Denote $Y^H(S)$ as the sustain point where a given core-periphery pattern can be sustained, i.e., $Y^H(S) = \min\{Y^H | \phi^H(1) \geq \phi^L(1)\}$, and $Y^H(B)$ to be a break point where the symmetric equilibrium starts to become unstable, i.e., $Y^H(B) = \{Y^H | \phi^{H\prime}(\frac{1}{2}) = 0\}$. As shown in Figure 10, given the above parameters and when $\alpha = 2$, $\beta = 4$, the sustain point is at $Y^H(S) = 1$ while the break point is at $Y^H(B) = 1.56$. It can be checked that in all core-periphery equilibria, population in the core region is larger than population in the periphery region. Moreover, the difference in population of different regions increases with the difference between Y^H and Y^L. The divergent trends in urban and rural population are confirmed by data in U.S. Census Bureau [1990] (Table 1) which shows that in addition to the increasing difference in urban and rural population, the percentage of US urban population in total population is increasing over time, and the percentage of US rural population is decreasing from 1950 to 1990.

On the other hand, when productivity and the elasticity of demand for land are positively correlated, given $n^H = 1$, $n^L = 2$, $Y^L = 1$, $\bar{s} = 1$, $\alpha = 4$, $\beta = 2$, and $Y^H = 1.25$, the unique equilibrium is completely symmetric which is also stable. When $Y^H = 2$, the unique completely symmetric equilibrium is unstable. When Y^H further increases to be 4, there are three integrated equilibria at $(\rho^{H*}, \rho^{L*}) = (0.09, 0.31)$, $(\rho^{H*}, \rho^{L*}) = (1/2, 1/2)$, and $(\rho^{H*}, \rho^{L*}) = (0.91, 0.69)$. None of these integrated equilibria is stable and there is no core-periphery equilibrium. Moreover, social welfare in the completely symmetric equilibrium is higher than that in other equilibria.

As shown in Figure 9, when $\alpha = 4$, $\beta = 2$, the break point is at $Y^H(B) = 1.78$ and there is no core-periphery equilibrium for all $Y^H > Y^L$. Finally, denote $Y^H(F)$ as a bifurcation point where the number of interior equilibria starts to be greater than 1. As shown in Figures 5 and 8, $Y^H(F)$ is the Y^H value such that when the relative positive steepness of curves with $\dot{\rho}^H = 0$ and $\dot{\rho}^L = 0$ at $(\rho^H, \rho^L) = (1/2, 1/2)$ starts to switch, i.e., $Y^H(F) = \{Y^H | \phi^{H\prime}(\frac{1}{2}) = \phi^{L\prime}(\frac{1}{2})\}$, it can be checked that $Y^H(F) = 2$ for both cases with $\alpha = 2$, $\beta = 4$, and $\alpha = 4$, $\beta = 2$. The dotted curves in Figures 10 and 9 represent unstable equilibria and solid lines represent stable equilibria.

Beginning from a uniform distribution of both types of worker over the two regions, when a skill-biased technological change is considered (that is, Y^H increases over time while Y^L is a constant), when $\alpha < \beta$, we can have a core-periphery bifurcation as shown in Figure 10. As the productivity of high-skill workers increases, since the signaling cost is lower for high-skill workers than low-skill workers around $(\rho^H, \rho^L) = (1/2, 1/2)$, high-skill workers have a stronger incentive to deviate to another region than low-skill workers once the distribution of workers is slightly perturbed. The breakdown of the uniform distribution of workers leads to a migration of some high-skill workers from one region (*ex post* periphery) to another region (*ex post* core), namely the "first migration wave." After the migration of these high-skill workers, firms start to notice the difference between average productivities in the two regions, and thus, a positive signaling effect is attached to the region with a higher ratio of high-skill workers. That is, firms start to pay workers different wages according to their locations. Though short-run equilibrium housing cost in the region with a higher ratio of high-skill workers increases (and housing cost in the other region decreases), both high-skill and low-skill workers are attracted to the region where the initial high-skill migration led, namely the "second migration wave." In the long-run equilibrium, high-skill workers are agglomerated in the core region, and low-skill workers are non-degenerately distributed in both regions. Low-skill workers have the same utility level in both regions, and they have no incentive to move in equilibrium. Since, in this case, the realized core-region is determined by the region with an initially higher ratio of high-skill labor than the other region, this paper implies that any event or policy that attracts high-skill labor plays a crucial role in the beginning of the development of a region.

4 Conclusions

Even without any increasing returns to scale in production, this paper illustrates that the agglomeration of high-skill labor, and thus the agglomeration of high-technology firms, can be caused by asymmetric information and locational signaling effects, even if the regional housing cost (endogenous signaling cost) is increasing in the high-skill population residing there.

When workers' price elasticity of demand for housing is negatively correlated with their productivity, there exist stable core-periphery equilibria. In this case, sorting on skill occurs, which accounts for the city size wage premium. Furthermore, since the agglomeration of high-skill labor is unproductive under locational signaling, social welfare in any core-periphery equilibrium is less than that in the completely symmetric equilibrium. On the other hand, when workers' price elasticity for housing is positively correlated with their productivity, no core-periphery equilibrium can be sustained. Though there always exists a completely symmetric equilibrium, it is stable only if the difference between high-skill and low-skill workers' productivity is not too large. When the difference in workers' productivity is very large, there are, additionally, two unstable symmetrically integrated equilibria. Therefore, when a skill-biased technological change is considered, a core-periphery fork bifurcation occurs under locational signaling effects.

In summary, though the appearance of a core region is not socially optimal, the conclusions of this paper shed light on the importance of path-dependence or policies that attract high-skill labor for the development of a region, even when there are no increasing returns to scale, knowledge spillovers, or externalities. Moreover, in any segregated equilibrium, the agglomeration of high-skill labor in one region is mixed with a portion of low-skill labor. This suggests that when location signals workers' productivity and the signaling cost is determined by the housing market at a location, location can at best be a reference for rather than a guarantee of workers' high productivity.

Many extensions of the ideas presented here come to mind, for example, adding further heterogeneity to workers and firms, or adding firm investment in physical capital. Moreover, the techniques introduced here can be extended to models where firms have private information, or to models where both firms and workers have private information.

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Year	Urban population (percent of total)	Rural population (percent of total)	The difference in urban and rural population
1950	96846817 (64.0%)	54478981 (36.0%)	42367836
1960	125268750 (69.9%)	54045425 (30.1%)	71223325
1970	149646617 (73.6%)	53565309 (26.4%)	96081308
1980	167050992 (73.7%)	59494813 (26.3%)	107556179
1990	187053487 (75.2%)	61656386 (24.8%)	125397101

Table 1: Source: U.S. Census Bureau [1990], (CPH-2).

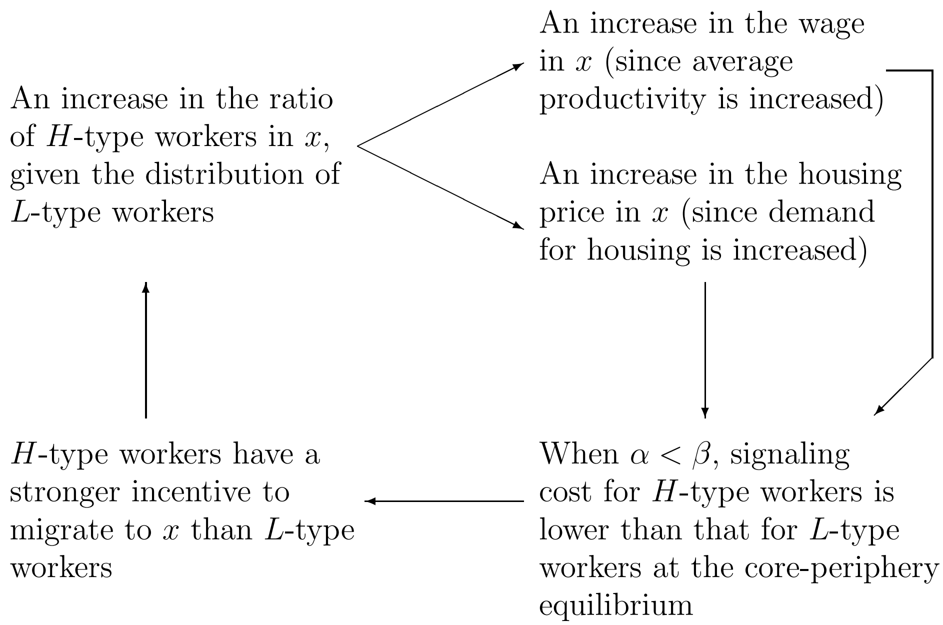

Figure 1: The logic and intuition for the existence of a core-periphery equilibrium when $\alpha < \beta$.

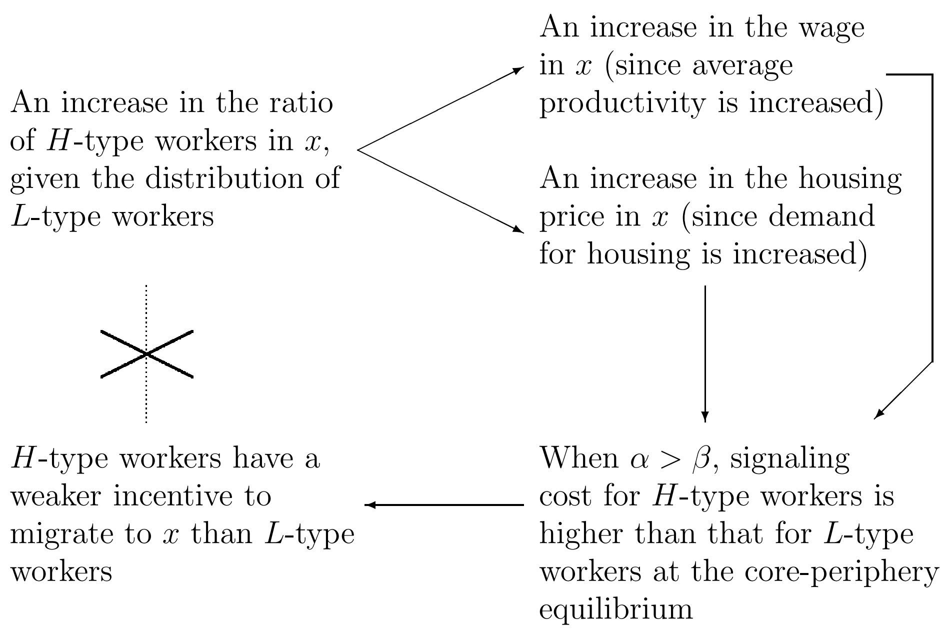

Figure 2: The logic and intuition for the non-existence of a core-periphery equilibrium when $\alpha > \beta$.

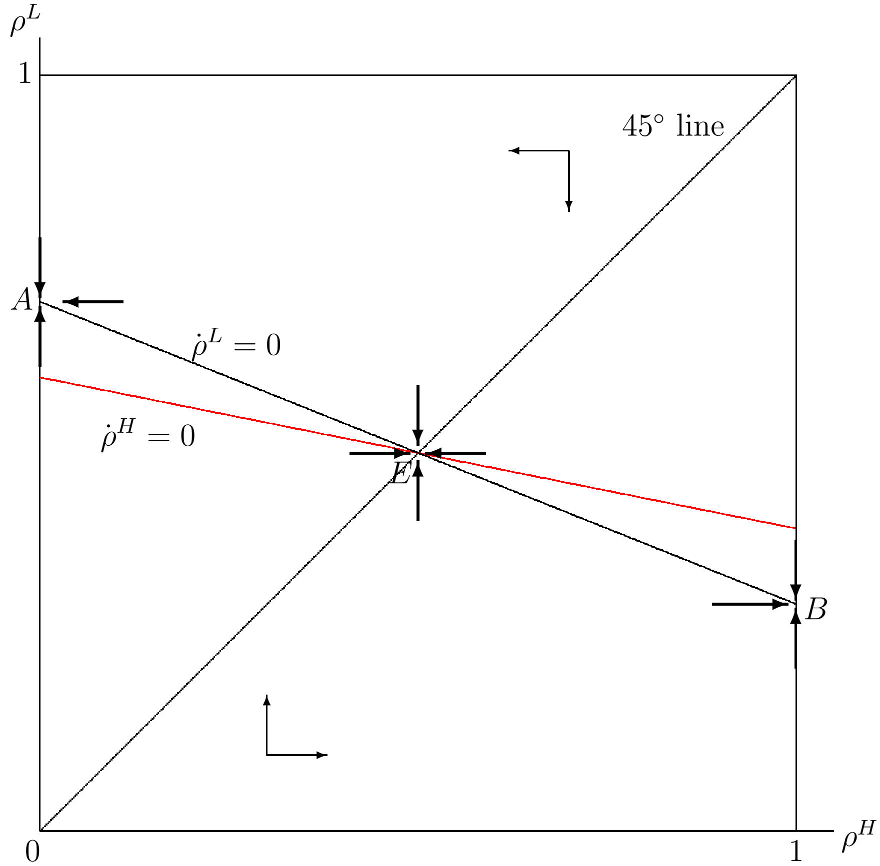

Figure 3: When $\alpha = 2, \beta = 4$, given $n^H = 1, n^L = 2, \bar{s} = 1, Y^L = 1$, and $Y^H = 1.25$, there exist two stable core-periphery equilibria, points $A = (0, 0.61)$ and $B = (1, 0.39)$. In addition, since $\phi^{H\prime}(\rho^H) < 0$ at $\rho^H = \frac{1}{2}$, the completely symmetric equilibrium at point $E = (\frac{1}{2}, \frac{1}{2})$ is stable.

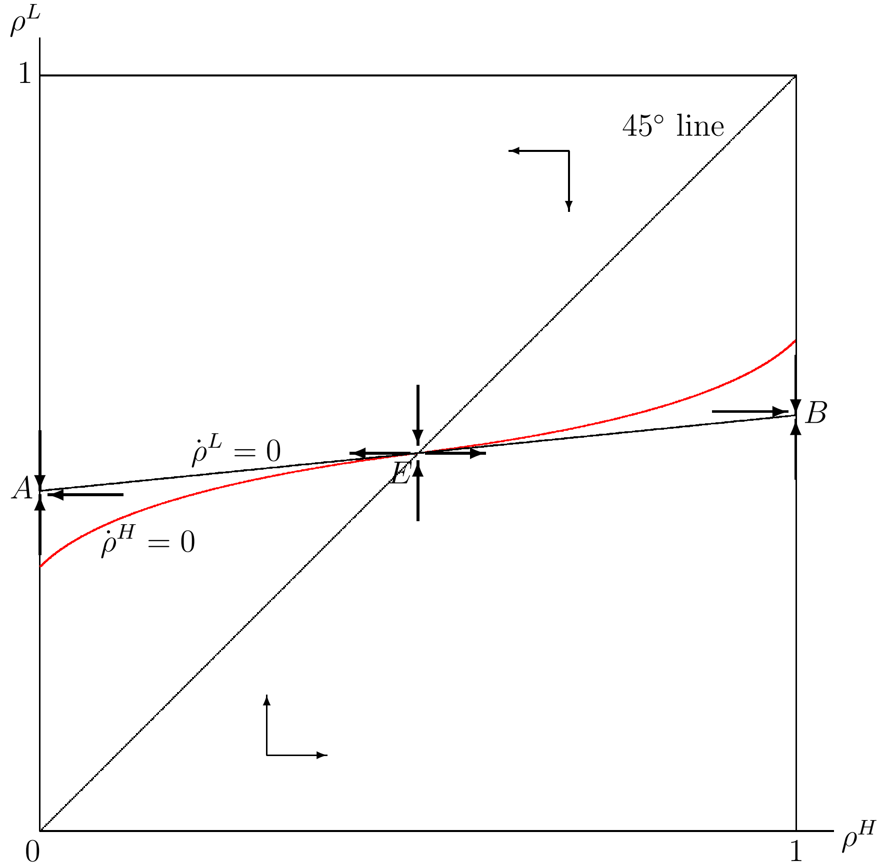

Figure 4: When $\alpha = 2, \beta = 4$, given $n^H = 1, n^L = 2, \bar{s} = 1, Y^L = 1$, and $Y^H = 2$, there exist stable core-periphery equilibria at points $A = (0, 0.41)$ and $B = (1, 0.59)$, and the completely symmetric equilibrium is unstable.

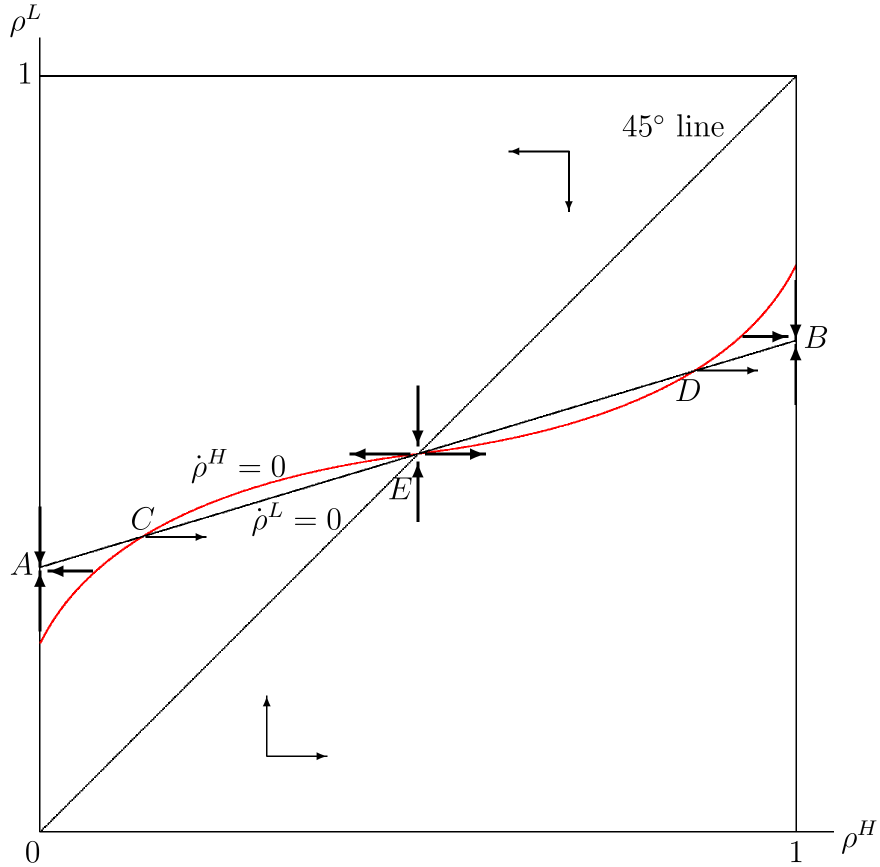

Figure 5: When $\alpha = 2, \beta = 4$, given $n^H = 1, n^L = 2, \bar{s} = 1, Y^L = 1$, and $Y^H = 4$, there exist three unstable integrated equilibria, points $C = (0.09, 0.27)$, $D = (0.91, 0.73)$, and $E = (\frac{1}{2}, \frac{1}{2})$, and there are two stable core-periphery equilibria at points $A = (0, 0.23)$ and $B = (1, 0.77)$.

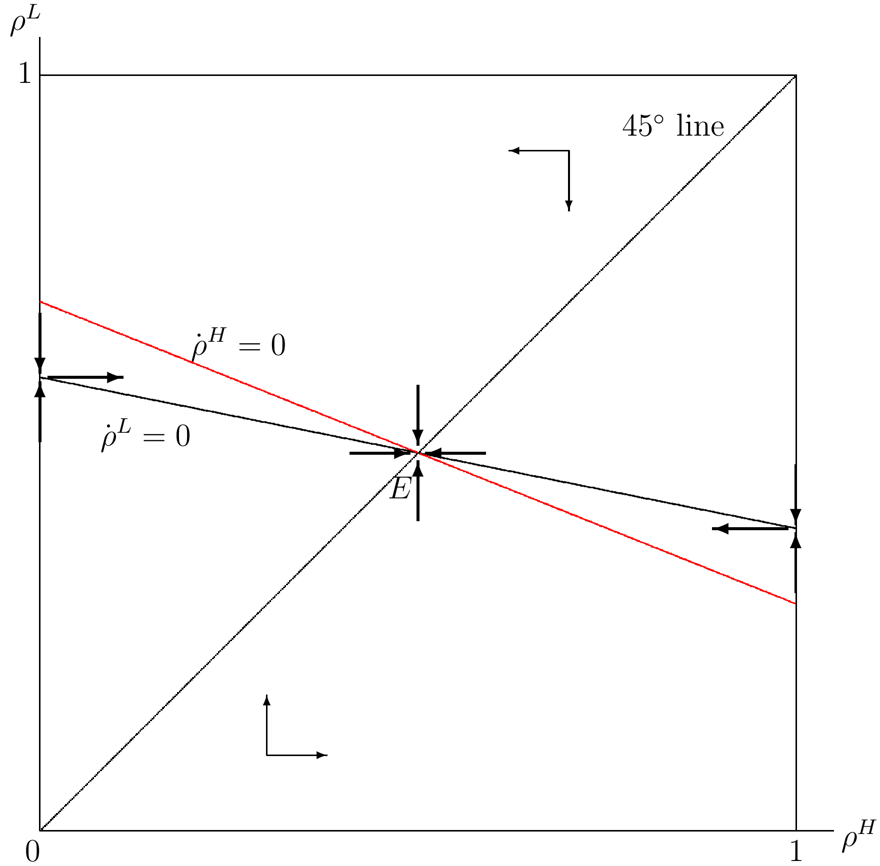

Figure 6: When $\alpha = 4, \beta = 2, n^H = 1, n^L = 2, \bar{s} = 1, Y^L = 1$ and $Y^H = 1.25$, there exists a unique stable completely symmetric equilibrium; however, there does not exist any core-periphery equilibrium.

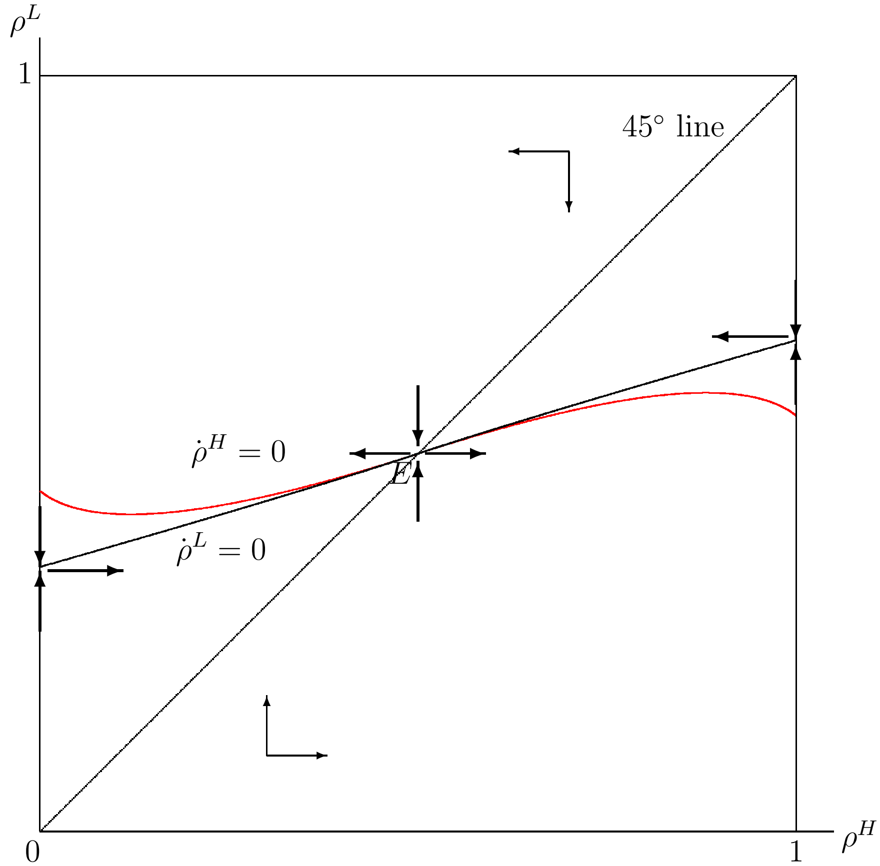

Figure 7: When $\alpha = 4, \beta = 2, n^H = 1, n^L = 2, \bar{s} = 1, Y^L = 1$ and $Y^H = 2$, there exists a unique equilibrium which is completely symmetric and unstable.

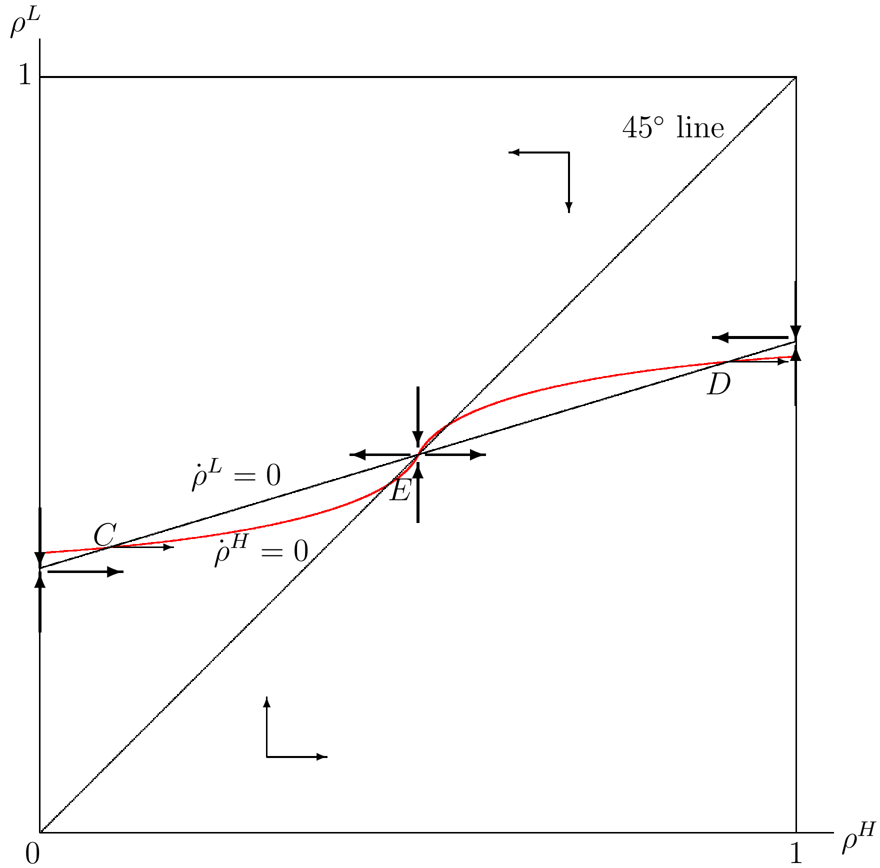

Figure 8: When $\alpha = 4, \beta = 2, n^H = 1, n^L = 2, \bar{s} = 1, Y^L = 1$ and $Y^H = 4$, there exist three unstable integrated equilibria at $C = (0.09, 0.31)$, $D = (0.91, 0.69)$, and $E = (\frac{1}{2}, \frac{1}{2})$, respectively. In addition, there is no core-periphery equilibrium.

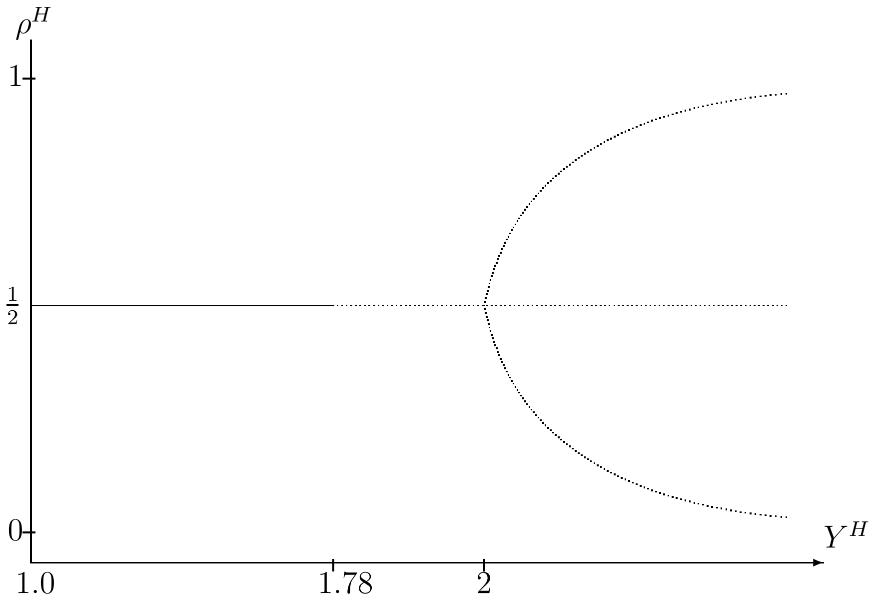

Figure 9: The fork bifurcation when productivity and the elasticity of demand for land are positively correlated, given $n^H = 1$, $n^L = 2$, $\bar{s} = 1$, $Y^L = 1$, $\alpha = 4$, and $\beta = 2$.

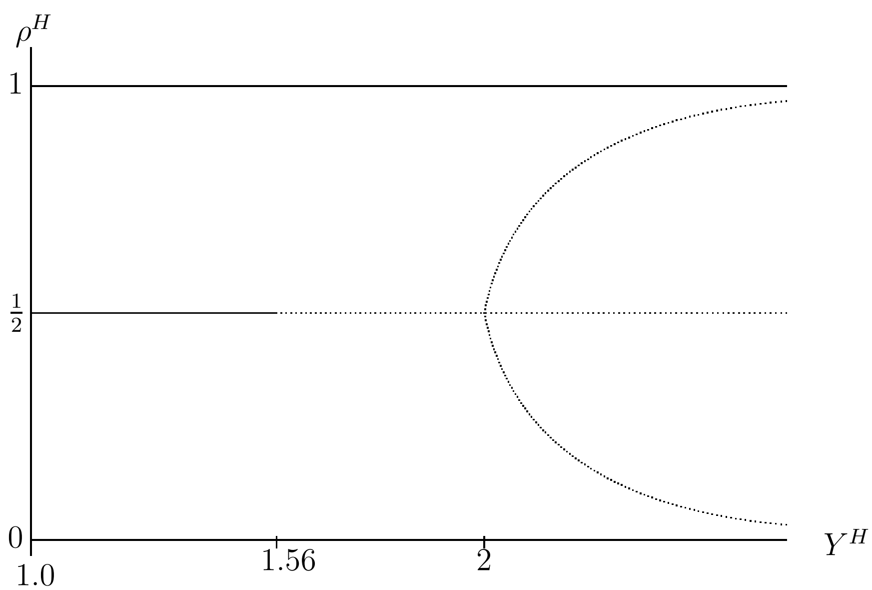

Figure 10: The fork bifurcation when productivity and the elasticity of demand for land are negatively correlated, given $n^H = 1$, $n^L = 2$, $\bar{s} = 1$, $Y^L = 1$, $\alpha = 2$, and $\beta = 4$.