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Summary

This article demonstrates that when finite lifetime is introduced in a Lucas (1988) growth model, the environmental policy may enhance growth both in the short- and the long-run, while pollution does not influence educational activities, labor supply is not elastic and human capital does not enter the utility function. This is because finite lifetime and the appearance of newborns at each date creates a turnover of generations which disconnects the aggregate consumption growth to the interest rate. We show that the shorter is the horizon, the greater the effect of the environmental policy on growth, because the higher the "generational turnover effect". We also demonstrate that when time is not the single production factor in education, the environmental policy promotes growth only if time remains the predominant factor. Otherwise, the crowding-out effect of the tighter environmental policy dominates the "generational turnover effect" and growth diminishes. Finally, when the source of pollution is final output rather than physical capital and time is the single factor in education, the environmental does not affect growth in the steady-state, despite the "generational turnover effect". Nevertheless, if the education good is introduced, the positive influence of the environmental policy appears again.

Keywords: Growth, Environment, Overlapping generations, Human capital

JEL Classification: C, O

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1 INTRODUCTION

While the link between environment and growth has already been extendedly investigated since the 80s (see Xepapadeas, 2005; Brock and Taylor, 2005), conclusions about the influence of the environmental policy on economic growth remain open.

The purpose of this article is to contribute to the debate, by re-examining the influence of the environmental policy on growth when finite lifetime is taken into account, in an economy where growth is driven by human capital accumulation. It demonstrates that finite lifetime introduces a new channel of transmission between the environment and economic performances based on the turnover of generations. Conversely to previous works in the field, a win-win environmental policy may be implemented while pollution does not influence educational activities, labor supply is inelastic and human capital does not enter the utility function.

Most of the industrialized countries are now becoming knowledge- and education-based economies using more and more human capital instead of physical capital to produce. And education played a major role in the industrialization of the South-East Asian countries during 7os and 80s decades.[1] Nevertheless, few theoretical works investigate environmental issues in a framework where human capital is the engine of growth and economic prosperities. A noteworthy exception is the seminal article by Gradus and Smulders (1993) where authors demonstrate that, in a model à la Lucas (1988) where pollution originates from physical capital, the environmental concerns do not influence the steady-state growth rate of human capital accumulation, but when pollution is assumed to influence education activities, the engine of growth.[2] This result comes from the fact that the growth rate of consumption relies on the after-tax interest rate and the rate of time preferences and that the tax-rate is invariant with pollution tax in the steady-state when labor supply is inelastic. When the environmental

[1]See World Bank (1993) for empirical evidence of this role. Grimaud and Tournemaine (2007) point out this role to justify the need to investigate the link between environment and growth through the channel of education.

[2]Specifically, pollution may influence education in two ways. Either by depreciating the stock of human capital (the case of Gradus and Smulders, 1993), or by reducing the ability to train (as shown by van Ewijk and van Wijnbergen, 1995).

taxation increases, the after-tax interest rate reduces and becomes lower than the returns to human capital. Consequently, the investment in physical capital drops in favor to human capital accumulation. Final production becomes more intensive in human capital and the allocation of human capital in production diminishes. This mechanism perpetuates until the after-tax interest rate backs to its initial value equal to the rate of returns in human capital accumulation. Because the aggregate consumption growth in the steady-state relies only on the after-tax interest rate, it is not modified by the higher pollution tax.

Assuming that labor supply is elastic and pollution originates from the stock of physical capital, Hettich (1998) finds a positive influence of the environmental policy on human capital accumulation, in a Lucas' setting. The increase in the environmental tax compels firms to increase their abatement activities at the expense of the household's consumption. To counteract this negative effect, households substitute leisure to education and the growth rate rises. Nevertheless, the author demonstrates that his result is very sensitive to the assumption about the source of pollution. When the source of pollution is final output rather than physical capital, the link between the environment and growth does not longer exist. By taxing output, a higher tax reduces both the returns to physical capital and the wage rate which contributes to the returns to education. The incentives for agents to invest more in education vanish.

More recently, in a model combining Research & Development activities and human capital accumulation à la Lucas, where education directly enters the utility function as a consumption good and knowledge from R&D reduces the flow of pollution emissions, Grimaud and Tournemaine (2007) demonstrate that a tighter environmental policy promotes growth. By increasing the price of the good whose production pollutes the higher tax rate reduces the relative cost of education and therefore incites agents to invest in human capital accumulation. Because education is the engine of growth, the growth rises in the steady-state. As highlighted by the authors, the key assumption is the introduction of the education as a consumption good in utility which lets the returns to education dependent from the environmental policy. When education does not influence utility, the returns to education is exogenous and therefore is not affected by the policy.

In the present article, we re-examine the link between the environmental policy and growth in a Lucas' setting, assuming just that lifetime is finite. We use a Yaari (1965) - Blanchard (1985) overlapping generations model where growth is driven by human capital accumulation à la Lucas (1988) and pollution arises from physical capital. We study both the long-run balanced growth path and the transition.

Our results are threefold. First, we demonstrate that when lifetime is finite and physical capital is the source of pollution, a tighter environmental policy enhances growth even if pollution does not affect educational activities, labor supply is inelastic and human capital does not enter the utility function. Indeed, finite lifetime introduces a turnover of generations which disconnects aggregate consumption growth to the interest rate. This "*generational turnover effect*" incites individuals to invest in human capital (the non-polluting factor) and offsets the crowding-out effect tied to the reduction of the returns to physical capital, not only during the transition but also in the steady-state. Second, when we relax the assumption that only time is required to educate and we assume that an education good is used as input in education activities, the "*generational turnover effect*" continues to operate and the environmental policy enhances growth, only if the part of the education good in human capital accumulation remains small. Otherwise, the crowding-out effect leads the environmental policy to be harmful to growth. Finally, when the source of pollution is final output rather than physical capital, the environmental policy does not influence growth in the Lucas' setting, for the reasons emphasized by Hettich (1998). Nevertheless, when the education good is introduced, the positive influence of the environmental policy appears again.

In section 2, we expose the model. Section 3 investigates the steady-state equilibrium. Section 4 examines the transitional dynamics of the model and compare the transitional effects of a tighter environmental policy on the economy when lifetime is finite and infinite, using numerical simulations. Section 5 discusses results, assuming alternative assumptions about production factors in education and the source of pollution. Section 6 concludes.

2 The model

Let's consider a Blanchard (1985) overlapping generations model with human capital accumulation and environmental concerns. Time is continuous. Each individual born at time s faces a constant probability of death per unit of time $\lambda \geq 0$. Consequently his life expectancy is $1/\lambda$. When λ increases, the life span decreases. At time s, a cohort of size λ is born. At time t, this cohort has a size equal to $\lambda e^{-\lambda(t-s)}$ and the constant population is equal to $\int_{-\infty}^{t} \lambda e^{-\lambda(t-s)} ds$. There are insurance companies and there is no bequest motive.

The expected utility function of an agent born at $s \leq t$ is:[3]

$$\int_{s}^{\infty} [\log c_{s,t} - \zeta \log \mathcal{P}_t] \, e^{-(\varrho+\lambda)(t-s)} dt \tag{1}$$

where $c_{s,t}$ denotes consumption in period t of an agent born at time s, $\varrho \geq 0$ is the rate of time preference and ζ measures the weight in utility attached to the environment, that is environmental care.

The representative agent can increase his stock of human capital by devoting time to schooling, according to Lucas (1988):

$$\dot{h}_{s,t} = B \, [1 - u_{s,t}] \, h_{s,t} \tag{2}$$

where B is the efficiency of schooling activities, $u_{s,t} \in]0,1[$ is the part of human capital allocated to productive activities at time t for the generation born at s and $h_{s,t}$ is the stock of human capital at time t of an individual born at time s. Note that we make no assumption about the influence of pollution on individual human capital accumulation.

Households face the following budget constraint:

$$\dot{a}_{s,t} = [r_t + \lambda] \, a_{s,t} + u_{s,t} h_{s,t} w_t - c_{s,t} \tag{3}$$

where $a_{s,t}$ is the financial wealth in period t and ω_t represents the wage rate per effective unit of human capital $u_{s,t}h_{s,t}$. The representative agent chooses the time path for $c_{s,t}$ and his working

[3] We use logarithmic utility to simplify the study of the transitional dynamics. In appendix D, we demonstrate that our results remain valid when the intertemporal elasticity of substitution of the consumption is different from unity.

time $u_{s,t}$ by maximizing (1) subject to (2) and (3). It yields

$$\dot{c}_{s,t} = [r - \varrho]\, c_{s,t} \tag{4}$$

Integrating (3) and (4) and combining the results gives the consumption at time t of an agent born at time s:

$$c_{s,t} = (\varrho + \lambda)\,[a_{s,t} + \omega_{s,t}]$$

where $\omega_{s,t} \equiv \int_t^\infty [u_{s,\nu} h_{s,\nu} w_\nu]\, e^{-\int_t^\nu [r_\varsigma + \lambda] d\varsigma} d\nu$ is the present value of lifetime earning.It also gives the equality between the rate of return to human capital and the effective rate of interest:

$$\frac{\dot{w}_t}{w_t} + B = r_t + \lambda \tag{5}$$

Due to the simple demographic structure, all individual variables are additive across individuals. Consequently, the aggregate consumption equals

$$C_t = \int_{-\infty}^t c_{s,t} \lambda e^{-\lambda(t-s)} ds = (\varrho + \lambda)\,[K_t + \Omega_t] \tag{6}$$

where $\Omega_t \equiv \int_{-\infty}^t \omega_{s,t} \lambda e^{-\lambda(t-s)} ds$. The aggregate stock of physical capital is defined by

$$K_t = \int_{-\infty}^t a_{s,t} \lambda e^{-\lambda(t-s)} ds$$

and the aggregate human capital is

$$H_t = \int_{-\infty}^t h_{s,t} \lambda e^{-\lambda(t-s)} ds, \tag{7}$$

We assume that the human capital of an agent born at current date, $h_{t,t}$, is inherited from the dying generation.[4] Because the mechanism of intergenerational transmission of knowledge is complex, we make the simplifying assumption that the human capital inherited from the dying generation is a constant part of the aggregate level of human capital such that $h_{t,t} = \eta H_t$ with $\eta \in \,]0, 1[$ (see Song, 2002).

[4]This assumption is not crucial for our results. Assuming that $h_{t,t}$ is exogenous would give the same qualitative results.

The productive sector is competitive. Firms produce the final good Y with the following technology:

$$Y_t = K_t^{\alpha}\left[\int_{-\infty}^{t} u_{s,t}h_{s,t}\lambda e^{-\lambda(t-s)}ds\right]^{1-\alpha}, \qquad 0<\alpha<1$$

with Y_t being the aggregate final output. $\int_{-\infty}^{t} u_{s,t}h_{s,t}\lambda e^{-\lambda(t-s)}ds$ is the amount of the aggregate stock of human capital used in production.

Following Gradus and Smulders (1993), pollution flow is assumed to increase with the stock of physical capital K and reduces with abatement activities D (which uses final output one for one):

$$\mathcal{P}_t = \left[\frac{K_t}{D_t}\right]^{\gamma}, \qquad \gamma > 0 \tag{8}$$

We assume that the government implements an environmental policy to incite firms to reduce their net flow of pollution. To do so, the government taxes the net flow of pollution by firms and transfers to them the fruit of the taxes to fund their abatement activities. Consequently, firms under perfect competition pay a pollution tax on their net pollution $\mathcal{P}_t$ and they choose their abatement activities D_t (whose cost equals D_t) and the amount of factors which maximize their profits $\pi_t = Y_t - r_tK_t - w_t\left[\int_{-\infty}^{t} u_{s,t}h_{s,t}\lambda e^{-\lambda(t-s)}ds\right] - \vartheta_t\mathcal{P}_t - D_t + T_t^p$ where ϑ_t is the pollution tax rate and T_t^p denotes transfers from the public sector with $T_t^p = \vartheta_t\mathcal{P}_t$. Firms take as given these transfers and pay each production factor at its marginal productivity to maximize profit:

$$r_t = \alpha\frac{Y_t}{K_t} - \vartheta_t\gamma\frac{\mathcal{P}_t}{K_t}$$

$$w_t = (1-\alpha)K_t^{\alpha}\left[\int_{-\infty}^{t} u_{s,t}h_{s,t}\lambda e^{-\lambda(t-s)}ds\right]^{-\alpha} \tag{9}$$

$$D_t = \vartheta_t\gamma\mathcal{P}_t \tag{10}$$

From equations (8) and (10), we obtain $\mathcal{P}_t = \left[\gamma\frac{\vartheta_t}{K_t}\right]^{-\gamma/(1+\gamma)}$. Because in the long-run the net flow of pollution must be constant to have a constant environmental quality, the environmental tax is assumed to evolve at the same rate of growth than the stock of physical capital.[5]

[5]It is quite easy to demonstrate that this assumption is verified in the centralized economy.

Intuitively, ϑ_t increases over time to encourage firms to increase abatement activities to limit pollution which rises with the physical capital stock. Consequently, we define $\tau \equiv \vartheta_t/K_t$, the environmental tax normalized by the physical capital and we obtain:

$$\mathcal{P} = \Phi(\tau)^{-\gamma}$$

$$D_t = \Phi(\tau)K_t$$

with $\Phi(\tau) \equiv (\gamma\tau)^{1/(1+\gamma)}$. Because τ is fixed by the government and therefore has no transitional dynamics, $\mathcal{P}$ is independent of time.

3 The general equilibrium and the balanced growth path

The final market clearing condition is:

$$Y_t = C_t + \dot{K}_t + D_t,$$

Differentiating (6) with respect to time and using the expression of dK_t/dt and $d\Omega_t/dt$ gives:

$$\dot{C}_t/C_t = r_t - \varrho - \lambda(\varrho + \lambda)K_t/C_t \tag{11}$$

Aggregate consumption growth differs from individual consumption growth (given by equation 4) by the term $\lambda K_t/C_t$ which represents what Heijdra and Ligthart (2000) called the "*generational turnover effect*". This effect appears because at each date a new generation is born and a cross-section of the existing population dies. Because new agents born without financial assets, their consumption is lower than the average consumption and therefore the "*generational turnover effect*" reduces the growth rate of the aggregate consumption. This *generational effect* rises with the probability to die λ: on one hand, agents die at a higher frequency (that increases the generational turnover) and on the other hand the propensity to consume out of wealth $\varrho + \lambda$ increases due to the shorter horizon.

Differentiating (7) with respect to time and using the fact that $u_{s,t} = u_t$,[6] the aggregate accumulation of human capital is:

$$\dot{H}_t = B\left[1 - u_t\right] H_t - (1-\eta)\lambda H_t$$

The first term in the right-hand side of the equation represents the increase in the aggregate human capital due to the investment of each alive generation in education at time t. The second term represents the loss of human capital due to the vanishing of dying generation net from the intergenerational transmission of human capital. Indeed, on the one hand, a part λ of the living cohort born at s with a stock of human capital equal to $h_{s,t}\lambda e^{-\lambda(t-s)}$ vanishes reducing growth by $\lambda \int_{-\infty}^{t} h_{s,t}\lambda e^{-\lambda(t-s)} ds = \lambda H_t$ when all generations are aggregated. On the other hand, at the same time, a new cohort of size λ appears, adding $\lambda h_{t,t}$ to growth, with $h_{t,t} = \eta H_t$ and $\eta \in]0, 1[$ (see above). This net loss reduces the aggregate accumulation of human capital.

Using previous results,[7] we can write the dynamics of the model as:

$$\begin{aligned} \dot{x}_t =& \ \left\{\left[\alpha - 1\right](b_t u_t)^{1-\alpha} - \varrho - \lambda(\varrho + \lambda)x_t^{-1} + x_t\right\} x_t \\ \dot{b}_t =& \ \left\{B\left[1 - u_t\right] - (1-\eta)\lambda - (b_t u_t)^{1-\alpha} + x_t + \Phi(\tau)\right\} b_t \\ \dot{u}_t =& \ \left\{(\alpha^{-1} - 1)B + (\alpha^{-1} - 1)\Phi(\tau) + B u_t - (\alpha^{-1} + \eta - 1)\lambda - x_t\right\} u_t \end{aligned} \tag{12}$$

where $x_t \equiv C_t/K_t$ and $b_t \equiv H_t/K_t$.

Along the balanced growth path, C, K, H, D and Y evolve at a common positive rate and the allocation of human capital across sectors is constant. Consequently, from (6) $\dot{x} = \dot{b} = \dot{u} = 0$. Combining the equations of (12) enables us to express the implicit value of $u^\star$ (see appendix A):

$$\Gamma(u^\star) \equiv \left[Bu^\star - \eta\lambda - \varrho\right]\left\{\left[u^\star + \mathcal{A}\right]B - \lambda\left[\mathcal{A} + \eta\right] + \mathcal{A}\Phi(\tau)\right\} - \lambda(\varrho + \lambda) = 0 \tag{13}$$

with $\mathcal{A} \equiv \alpha^{-1} - 1 > 0$.

[6] Using (9), the equalization of the rates of return given by equation (5) implies that the rate of return to human capital is independent of s, therefore all individuals allocate the same effort to schooling: $u_{s,t} = u_t$.

[7] From (5), (9) and the fact that $\int_{-\infty}^{t} u_{s,t}h_{s,t}\lambda e^{-\lambda(t-s)} ds = u_t H_t$ because $u_{s,t} = u_t$, we obtain $\dot{u}_t/u_t = \dot{K}_t/K_t - \dot{H}_t/H_t - \alpha^{-1}\left[r_t + \lambda - B\right]$.

This equality defines a unique $u^\star \in]0,1[$ which decreases with τ, for $\tau \in]\underline{\tau}, \bar{\tau}[$ (see appendix A). This last condition means that the pollution tax must be higher than a minimal value in order to encourage agents to invest in human capital accumulation. Note that when lifetime is infinite, $\lambda = 0$, the allocation of human capital into the production $u^\star$ is independent from τ along the balanced growth path.[8] When the horizon shortens (λ increases), the allocation of human capital into final production $u^\star$ rises: agents invest less in human capital. Finally, using (12), we can express the ratio H/K along the balanced growth path as:

$$b^\star = \left[\alpha^{-1}(B-\lambda) + \alpha^{-1}\Phi(\tau)\right]^{1/(1-\alpha)} u(\tau)^{\star-1}$$

The increase in the environmental tax leads the final production to be less intensive in physical capital and more intensive in human capital because it increases the cost of physical capital: $b^\star$ rises. The ratio consumption to physical capital along the balanced growth path is given by:

$$x^\star = \left[\alpha^{-1} + u^\star(\tau) - 1\right] B - (\alpha^{-1} + \eta - 1)\lambda + \left(\alpha^{-1} - 1\right)\Phi(\tau)$$

and is an increasing function of the environmental tax rate.[9]

Finally, the growth of the market economy along the BGP is:

$$g^\star = B\left[1 - u^\star(\tau)\right] - (1-\eta)\,\lambda.$$

The BGP rate of growth increases with τ: environmental policy has a positive impact because it encourages agents to invest more in the no-pollutant factor of production: the human capital. Consequently, by assuming finite lifetime, it is possible to implement a win-win environmental policy in a Lucas (1988) growth model. Nevertheless, the environmental tax rate must be higher than $\hat{\tau} > \underline{\tau}$ to have $g^\star > 0$: a minimum environmental policy is required to obtain a sustainable equilibrium in the market economy.

Note that if the horizon is short (λ is high), the effect of the environmental policy on growth is greater, as demonstrated in appendix A.

[8] When $\lambda = 0$, $u^\star$ is defined (from equation 13) by $\Gamma(u^\star) \equiv [Bu^\star - \varrho]\left\{[u^\star + \mathcal{A}]\,B + \mathcal{A}\Phi(\tau)\right\} = 0$. It is straightforward that the positive solution of this equality is $u^\star = \varrho/B > 0$ like in Lucas (1988).

[9] Note that, when lifetime is infinite ($\lambda = 0$), $b^\star$ and $x^\star$ remain increasing functions of τ.

4 The transitional dynamics

In this section we examine the features of the equilibrium and we investigate the trajectory of the economy out of the steady-state using the time-elimination method. We compare the influence of the environmental policy during the transition both when lifetime is finite and when lifetime is infinite.

The dynamical system (12) may be linearized around the steady-state and becomes:

$$\begin{pmatrix} \dot{x}_t \\ \dot{b}_t \\ \dot{u}_t \end{pmatrix} = \mathcal{J} \times \begin{pmatrix} x_t - x^\star \\ b_t - b^\star \\ u_t - u^\star \end{pmatrix}$$

where $\mathcal{J}$ is the Jacobain matrix evaluated at the neighbourhood of the steady-state:

$$\mathcal{J} \equiv \begin{pmatrix} x^\star + \lambda(\varrho+\lambda)x^{\star-1} & (\alpha-1)b^{\star-\alpha}u^{\star 1-\alpha}x^\star & (\alpha-1)u^{\star-\alpha}b^{\star 1-\alpha}x^\star \\ b^\star & -(1-\alpha)(b^\star u^\star)^{1-\alpha} & -b^\star(B+(1-\alpha)u^{\star-\alpha}b^{\star 1-\alpha}) \\ -u^\star & 0 & Bu^\star \end{pmatrix}$$

with

$$\det(\mathcal{J}) = -(\varrho+\lambda)B(1-\alpha)u^\star(b^\star u^\star)^{1-\alpha}\left[x^\star + \lambda(\varrho+\lambda)x^{\star-1}\right] < 0$$

and (from $\dot{x}_t = 0$ in the steady-state)

$$\text{Trace}(\mathcal{J}) = 2\lambda(\varrho+\lambda)x^{\star-1} + \varrho + bu^\star > 0$$

Because there are two control variables (u and x) and one state-variable (b), the negative determinant and the positive trace of the Jacobian matrix imply that there are two positive eigenvalues and one negative eigenvalue. Therefore, the equilibrium is saddle-path stable.

Due to the complexity of the model, we use numerical simulations to look at the transitional dynamics, and especially the transitional impact of the environmental tax. We calibrate the model to obtain realistic values of the growth rate of GDP and the probability of death for the US economy. From the *World Development Indicators 2005* by the World Bank, life expectancy was 77.4 years in 2003, the growth rate was 3.3% during the period 1990-2002 and a public health expenditures as percentage of GDP was 6.55%. Since the expected lifetime is the reverse of the probability of death per unit of time λ, we want λ to be close to $1/77.4 = 0.0128$. We adjust other variables to obtain such values for our benchmark case.

Table 1 summarizes the benchmark value of parameters and Table 2 summarizes the exercise of comparative statics for log utility.

α	η	ϱ	B	γ	$\lambda^{(1)}$	$\lambda^{(2)}$
0.3	0.85	0.025	0.0735	0.3	0.0128	0

(1) finite lifetime (2) infinite lifetime

Table 1. Benchmark value of parameters

	Finite lifetime				*Infinite lifetime*	
	$\lambda = 0.0128$		$\lambda = 0.0200$			
	$\tau = 0.01$	$\tau = 0.1$	$\tau = 0.01$	$\tau = 0.1$	$\tau = 0.01$	$\tau = 0.1$
$g^\star$	3.32%	3.42%	2.35%	2.55%	4.85%	4.85%
$\mathcal{P}^\star$	3.82	2.24	3.82	2.24	3.82	2.24
$u^\star$	0.5218	0.5084	0.6389	0.6109	0.3401	0.3401
$r^{\star(1)}$	0.0607	0.0607	0.0535	0.0535	0.0735	0.0735
$x^{\star(2)}$	0.1959	0.3254	0.1815	0.3100	0.2233	0.3537
$b^{\star(3)}$	0.2503	0.5831	0.1760	0.4468	0.4849	0.9986

$^{(1)}\alpha Y/K - \Phi(\tau)$ $^{(2)}C/K$ $^{(3)}H/K$

Table 2. The increase in the environmental tax along the BGP

Graph 1 and Graph 2 draw the temporal evolution of the main variables towards the new steady-state when an unanticipated increase in the environmental tax is implemented by the government, respectively for finite and infinite lifetime.

What differs between finite and infinite lifetime is mainly the size of the variations and the fact that all variables tend towards a new steady-state value in the finite lifetime case. Let consider this case to look at the out-of steady-state evolution of variables (Graph 2).

First of all, an increase in the environmental taxation lowers instantaneously the interest rate (Figure 4 in Graph 2) and drops the growth rate of physical capital which becomes negative (Figure 6 in Graph 2). Because the returns to education becomes higher, agents allocate their resource towards human capital accumulation: the allocation of human capital into the manufacturing sector falls (Figure 1 in Graph 2) while the growth rate of human capital jumps (Figure 5 in Graph 2). The manufacturing sector becomes more intensive in human capital and the ratio human capital to physical capital (b) rises (Figure 2 in Graph 2). The fall in u and the increase in b contributes to rise the interest rate and reduces the incentives of agents to allocate human capital into the educational sector: u rises while the increase in the human capital physical capital ratio decelerates. This continues up to the interest rate comes to its initial value (which equals $B + \lambda$). While the allocation of human capital into production and the growth rates of the physical capital, the human capital, consumption and final output back to their initial steady-state value when the lifetime of agents is infinite, this is not the case when the lifetime is finite (see Graph 1): agents allocate more human capital into the educational sector ($u\star$ is lower) and that promotes human capital accumulation, aggregate consumption growth and aggregate output growth. It comes from the *generational turnover effect* on the aggregate consumption growth whose size relies on the aggregate consumption to physical capital ratio.[10] Because during the transition, this ratio increases, in the same time the *generational turnover effect* reduces (the difference between the consumption of the newborn and the average consumption lowers), and the aggregate consumption growth is fostered (with respect to the infinite case).

5 Discussion

In the previous section, we demonstrated that the "*generational turnover effect*" , due to the appearance of newborn at each date and the death of a part of the population, leads to a positive impact of the environmental policy in the Lucas (1988) model without assuming that a better environmental quality enhances ability to learn, that labor supply is elastic or that education enters the utility function as a consumption good.

[10]Remark that the externality in the aggregate human capital accumulation due to the generational turnover $(1 - \eta)\lambda H_t$ is not required to obtain our result. Letting $\eta = 1$ in equation (13), that is assuming that all the human capital of the dying generation is inherited by newborn, the allocation of human capital into production $u^\star$ remains negatively influenced by the environmental tax. The only effect is a higher growth rate in the steady-state.

Nevertheless, according to empirical evidence, education is not only the fruit of a private time investment. It requires either physical capital (see for example King and Rebelo, 1990) or public expenditures (see for example Glomm and Ravikumar, 2001). Furthermore, as highlighted by Hettich (1998), the assumption about the source of pollution may completely change the impact of the environmental policy on growth. Do our results continue to hold when the technology in the educational sector is modified in such a way and/or when an alternative source of pollution is introduced?

5.1 EDUCATION GOOD IN HUMAN CAPITAL ACCUMULATION

Following Rebelo (1991), we consider that, besides time, education requires an educational input produced with physical and human capital. To simplify things we suppose that it is produced in the same way than the final output. The fact that households buy an educational input will modify their decisions to invest their time to educational activities and the influence of the environmental policy on human capital accumulation and growth.

At time t, each agent born at s buy $z_{s,t}$ units of final output which increase the productivity of the time that they invest in education, such as:

$$\dot{h}_{s,t} = B\left[(1-u_{s,t})h_{s,t}\right]^{1-\delta} z_{s,t}^{\delta} \tag{14}$$

and

$$\dot{a}_{s,t} = \left[r_t + \lambda\right] a_{s,t} + u_{s,t}h_{s,t}w_t - c_{s,t} - z_{s,t} \tag{15}$$

Utility maximization implies that $u_{s,t}$ and the ratio $\dfrac{z_{s,t}}{h_{s,t}}$ are independent from s (conveniently, we denote $\tilde{z}_t \equiv \dfrac{z_{s,t}}{(1-u_{s,t})h_{s,t}}$). Furthermore, it gives (see appendix B)

$$\tilde{z}_t = \frac{\delta}{1-\delta} w_t \tag{16}$$

and the equalization of the returns to education

$$(1-\delta)\frac{\dot{w}_t}{w_t} + B(1-\delta)^{1-\delta}\delta^{\delta}w_t^{\delta} = r_t + \lambda \tag{17}$$

The aggregate accumulation of human capital is then written as

$$\dot{H}_t = B(1-u_t)\tilde{z}_t^{\delta} - (1-\eta)\lambda \tag{18}$$

The amount of final output used as educational good is $Z_t \equiv \int_{-\infty}^{t} z_{s,t}\lambda e^{-\lambda(t-s)}ds = \tilde{z}_t(1-u_t)H_t$ from previous results. Consequently, from (16) it can be expressed as a function of final output:

$$Z_t = \frac{\delta(1-\alpha)}{1-\delta}\left(\frac{1-u_t}{u_t}\right)Y_t \tag{19}$$

and the market clearing condition becomes:

$$\left(1+\frac{(1-\alpha)\delta}{1-\delta}\left(1-\frac{1}{u_t}\right)\right)Y_t = \dot{K}_t + C_t + \Phi(\tau) \tag{20}$$

Consequently, the dynamical system is summarized by

$$\dot{x}_t = \left\{\left[\alpha - \left(1+\frac{(1-\alpha)\delta}{1-\delta}\left(1-\frac{1}{u_t}\right)\right)\right](b_t u_t)^{1-\alpha} - \varrho - \lambda(\varrho+\lambda)x_t^{-1} + x_t\right\}x_t$$

$$\begin{aligned}\dot{b}_t = \Big\{&B(1-u_t)[\delta(1-\delta)^{-1}(1-\alpha)(b_t u_t)^{-\alpha}]^{\delta} - (1-\eta)\lambda \\ &- \left(1+\frac{(1-\alpha)\delta}{1-\delta}\left(1-\frac{1}{u_t}\right)\right)(b_t u_t)^{1-\alpha} + x_t + \Phi(\tau)\Big\}b_t\end{aligned}$$

$$\dot{u}_t = \left\{\frac{1}{\alpha(1-\delta)}\left[B(1-\delta)^{1-\delta}\delta^{\delta}(1-\alpha)^{\delta}(b_t u_t)^{-\alpha\delta} - \alpha(b_t u_t)^{1-\alpha} + \Phi(\tau) - \lambda\right] - \dot{b}/b\right\}u_t$$

Along the steady-state, from the last equation of the dynamical system, we obtain

$$\alpha(b^{\star}u^{\star})^{1-\alpha} - \Phi(\tau) + \lambda - B(1-\delta)^{1-\delta}\delta^{\delta}(1-\alpha)^{\delta}(b^{\star}u^{\star})^{-\alpha\delta} = 0 \tag{21}$$

This relation states $b^{\star}u^{\star}$ as an increasing function of $\Phi(\tau)$. We define $\mathcal{R}(\tau) \equiv b^{\star}u^{\star}$ with $d\mathcal{R}(\tau)/d\tau > 0$. The two remaining equations of the dynamical system evaluated at the steady-state ($\dot{x}_t = 0$ and $\dot{b}_t = 0$) enable to write $u^{\star}$ with respect to τ:

$$\begin{aligned}\Upsilon(u^{\star}) \equiv\ &\frac{\lambda(\varrho+\lambda)}{B\left[u^{\star}-\delta\right](1-\delta)^{-\delta}\delta^{\delta}(1-\alpha)^{\delta}\mathcal{R}(\tau)^{-\alpha\delta} - \varrho - \eta\lambda} + (\mathcal{A}_1(u^{\star})+\eta)\lambda \\ &- \mathcal{A}_1(u^{\star})\Phi(\tau) - \left[(1-\delta)\mathcal{A}_1(u^{\star}) - \delta + u^{\star}\right]B\left(\frac{\delta}{1-\delta}\right)^{\delta}(1-\alpha)^{\delta}\mathcal{R}(\tau)^{-\alpha\delta} = 0\end{aligned} \tag{22}$$

with $u^\star - \delta > 0$ to obtain $x^\star > 0$ and $\mathcal{A}_1(u^\star) \equiv \left(\frac{1-\alpha}{1-\delta}\right)\left(1 - \frac{\delta}{u^\star}\right) > 0$. Since $\Upsilon(u^\star)$ is a decreasing function of $u^\star$, it defines a unique allocation of human capital to production in the steady-state. Nevertheless, it also imposes that the part of the education good in the accumulation of human capital does not exceed $u^\star$. Finally, the impact of the environmental taxation on $u^\star$ is no clear-cut. Consequently, conversely to the case where only time is used as input in education, here the influence of the environmental taxation on the allocation of time in education is not obvious.

When we consider the case of infinite lifetime ($\lambda = 0$), $u^\star$ is independent from τ and because the aggregate accumulation of human capital then only depends negatively on the human capital to physical capital ratio, growth in the economy decreases with the environmental taxation due to the crowding-out effect of the environmental taxation. When lifetime is finite ($\lambda > 0$), the previous effect exists, through the human capital to physical capital ratio, but there is also the "*generational turnover effect*" which operates in the opposite way. Is it high enough to compensate or to offset the crowding-out effect? That is the question we investigate in the following.

The aggregate growth rate in the economy is given by:

$$g^\star = B(1 - u^\star)[\delta(1-\delta)^{-1}(1-\alpha)\mathcal{R}(\tau)^{-\alpha}]^\delta - (1-\eta)\lambda \tag{23}$$

Because the impact of the environmental taxation on $u^\star$ is ambiguous and because $\mathcal{R}(\tau)$ is positively influenced by the environmental tax, the global impact on the steady-state growth rate is not clear-cut and very cumbersome to derive analytically. That the reason why we use numerical simulations to investigate this impact. Because we demonstrated that the environmental policy enhances growth when only time is used as input for education (that is $\delta = 0$) and it is straightforward from our previous results that environmental policy is harmful for growth when only final output is used to increase human capital (that is $\delta = 1$), we look at the impact of an increase in environmental taxation for different values of δ the part of human capital accumulation made by the education good. We report results in table 3.

The first insight of our simulations is that the positive effect of the environmental policy due to finite lifetime exists, for the parameters values chosen. The second insight is that this positive effect offsets the crowding-out effect only if the part of the education good in human capital accumulation (δ) is small enough.

Consequently, the mechanism highlighted in the previous sections, based on the "*generational turnover effect*" due to finite lifetime still operates when an education good (produced with human and physical capital) is introduced, but it is reduced and may be offset if the part of time that individuals invest in education becomes too small.

Table 3. Impact of the environmental policy according to δ.

	$\delta = 0.01$		$\delta = 0.08$		$\delta = 0.09$		$\delta = 0.5$	
	$\tau = 0.01$	$\tau = 0.1$	$\tau = 0.01$	$\tau = 0.1$	$\tau = 0.01$	$\tau = 0.1$	$\tau = 0.01$	$\tau = 0.1$
$g^\star$	3.05%	3.15%	1.907%	1.914%	1.785%	1.778%	0.195%	-0.099%
$\mathcal{P}^\star$	3.82	2.24	3.82	2.24	3.82	2.24	3.82	2.24
$u^\star$	0.5567	0.5429	0.7070	0.6991	0.7232	0.7167	0.9299	0.9804
$r^\star$	0.0569	0.0567	0.0445	0.0431	0.0434	0.0419	0.0334	0.0266
$x^\star$ (C/K)	0.1858	0.3136	0.1539	0.2750	0.1513	0.2718	0.1257	0.2419
$b^\star$ (H/K)	0.2171	0.5219	0.1283	0.3435	0.1220	0.3297	0.0712	0.1944

Positive effect on growth *Negative effect on growth*

5.2 Alternative specification of pollution

As demonstrated by Hettich (1998), the specification of pollution may modify the impact of the environmental taxation on growth. In a Uzawa-Lucas (1988) model with elastic labor supply, he finds that the environmental taxation enhances growth when the source of pollution is the stock of physical capital. Nevertheless, assuming that the source of pollution is rather final output, he obtains no effect of the environmental taxation. Indeed, when final production is the source of pollution, the environmental tax not only impacts the interest rate, but also the wage rate, erasing the positive effect which transits through the elastic labor supply. In this section, we re-examine our previous results assuming that the source of pollution is final output rather than physical capital.

In this case, we have

$$\mathcal{P}_t = \left[\frac{Y_t}{D_t}\right]^{\gamma}$$

Therefore, profit maximization in the final output production leads to

$$r_t = \left(1 - \vartheta_t \gamma \frac{\mathcal{P}_t}{Y_t}\right) \alpha \frac{Y_t}{K_t}$$

$$w_t = \left(1 - \vartheta_t \gamma \frac{\mathcal{P}_t}{Y_t}\right) (1-\alpha) K_t^{\alpha} \left[\int_{-\infty}^{t} u_{s,t} h_{s,t} \lambda e^{-\lambda(t-s)} ds\right]^{-\alpha}$$

and

$$D_t = \vartheta \gamma \mathcal{P}_t$$

Defining $\hat{\tau} \equiv \vartheta_t / Y_t$, the environmental tax normalized by the final output production we obtain:

$$\mathcal{P} = \Phi(\hat{\tau})^{-\gamma}$$

$$D_t = \Phi(\hat{\tau}) Y_t$$

with $\Phi(\hat{\tau}) \equiv (\gamma \hat{\tau})^{1/(1+\gamma)}$.

The final market clearing condition becomes:

$$\left[1 - \Phi(\hat{\tau})\right] Y_t = C_t + \dot{K}_t$$

and the dynamical system gets

$$\dot{x}_t = \left\{[1 - \Phi(\hat{\tau})](\alpha - 1)(b_t u_t)^{1-\alpha} - \varrho - \lambda(\varrho + \lambda) x_t^{-1} + x_t\right\} x_t$$

$$\dot{b}_t = \left\{B(1 - u_t) - (1 - \eta)\lambda - [1 - \Phi(\hat{\tau})](b_t u_t)^{1-\alpha} + x_t\right\} b_t$$

$$\dot{u}_t = \left\{(\alpha^{-1} - 1)B + B u_t - (\alpha^{-1} + \eta - 1)\lambda - x_t\right\} u_t$$

Solving it in the steady-state, the last equation of the system gives

$$x^{\star} = (\alpha^{-1} - 1)B + B u^{\star} - (\alpha^{-1} + \eta - 1)\lambda$$

and from the two first equations evaluated at the steady-state, we obtain

$$\varrho + \lambda(\varrho + \lambda)x^{\star -1} = -(1-\alpha)B(1-u^\star) + (1-\alpha)(1-\eta)\lambda + \alpha x^\star$$

The two previous expressions enable to state the steady-state value $u^\star$. It is straightforward that the time allocation to production is independent from the environmental taxation, like in Hettich (1998) and consequently, despite finite lifetime, the environmental policy does not affect the growth rate in the steady-state.

Nevertheless, let's consider the case of the previous section where final output is used in the educational activity. With production flow arising from the final output rather physical capital, the dynamical system becomes:

$$\dot{x}_t = \left\{ (1-\Phi(\hat{\tau}))\left[\alpha - 1\right](b_t u_t)^{1-\alpha} - \frac{(1-\alpha)\delta}{1-\delta}\left(1 - \frac{1}{u_t}\right)(b_t u_t)^{1-\alpha} - \varrho - \lambda(\varrho+\lambda)x_t^{-1} + x_t \right\} x_t$$

$$\begin{aligned}\dot{b}_t = \big\{ & B(1-u_t)[\delta(1-\delta)^{-1}(1-\alpha)(b_t u_t)^{-\alpha}]^\delta - (1-\eta)\lambda \\ & - \left(1 - \Phi(\hat{\tau}) + \frac{(1-\alpha)\delta}{1-\delta}\left(1 - \frac{1}{u_t}\right)\right)(b_t u_t)^{1-\alpha} + x_t \Big\} b_t \end{aligned} \quad (24)$$

$$\dot{u}_t = \left\{ \frac{1}{\alpha(1-\delta)}\left[B(1-\delta)^{1-\delta}\delta^\delta(1-\alpha)^\delta(b_t u_t)^{-\alpha\delta} - \alpha(1-\Phi(\hat{\tau}))(b_t u_t)^{1-\alpha} - \lambda\right] - \dot{b}/b \right\} u_t$$

Along the steady-state, from the last equation of the previous system

$$(1-\Phi(\hat{\tau}))\alpha(b^\star u^\star)^{1-\alpha} + \lambda - B(1-\delta)^{1-\delta}\delta^\delta(1-\alpha)^\delta(b^\star u^\star)^{-\alpha\delta} = 0 \quad (25)$$

This relation states $b^\star u^\star$ as an increasing function of $\Phi(\tau)$. We define $\hat{\mathcal{R}}(\tau) \equiv b^\star u^\star$ with $d\hat{\mathcal{R}}(\tau)/d\tau > 0$. The allocation of human capital into the output sector in the steady-state $u^\star$ is given by the equality:

$$\frac{\lambda(\varrho+\lambda)}{B(u^\star - \delta)[\delta(1-\delta)^{-1}(1-\alpha)\mathcal{R}(\tau)^{-\alpha}]^\delta - \varrho - \eta\lambda} =$$
$$(1-\alpha)\left(1 + \delta - \frac{\delta}{u^\star}\right)\mathcal{R}(\tau)^{1-\alpha} - \left[(1-u^\star) - (1-\delta)\delta^\delta\right]B(1-\alpha)^\delta\mathcal{R}(\tau)^{-\alpha\delta} - \eta\lambda$$

with $1+\delta-\frac{\delta}{u^\star}>0$ and $(1-u^\star)-(1-\delta)\delta^\delta>0$ (see Appendix C). This equality defines a unique $u^\star$ which decreases in τ. Because the growth rate in the steady-state remains $g^\star = B(1-u^\star)-(1-\eta)\lambda$, the environmental policy has a positive impact on the growth rate in the steady-state when the flow of pollution arises from the stock of physical capital and final output is used as education good in human capital accumulation.

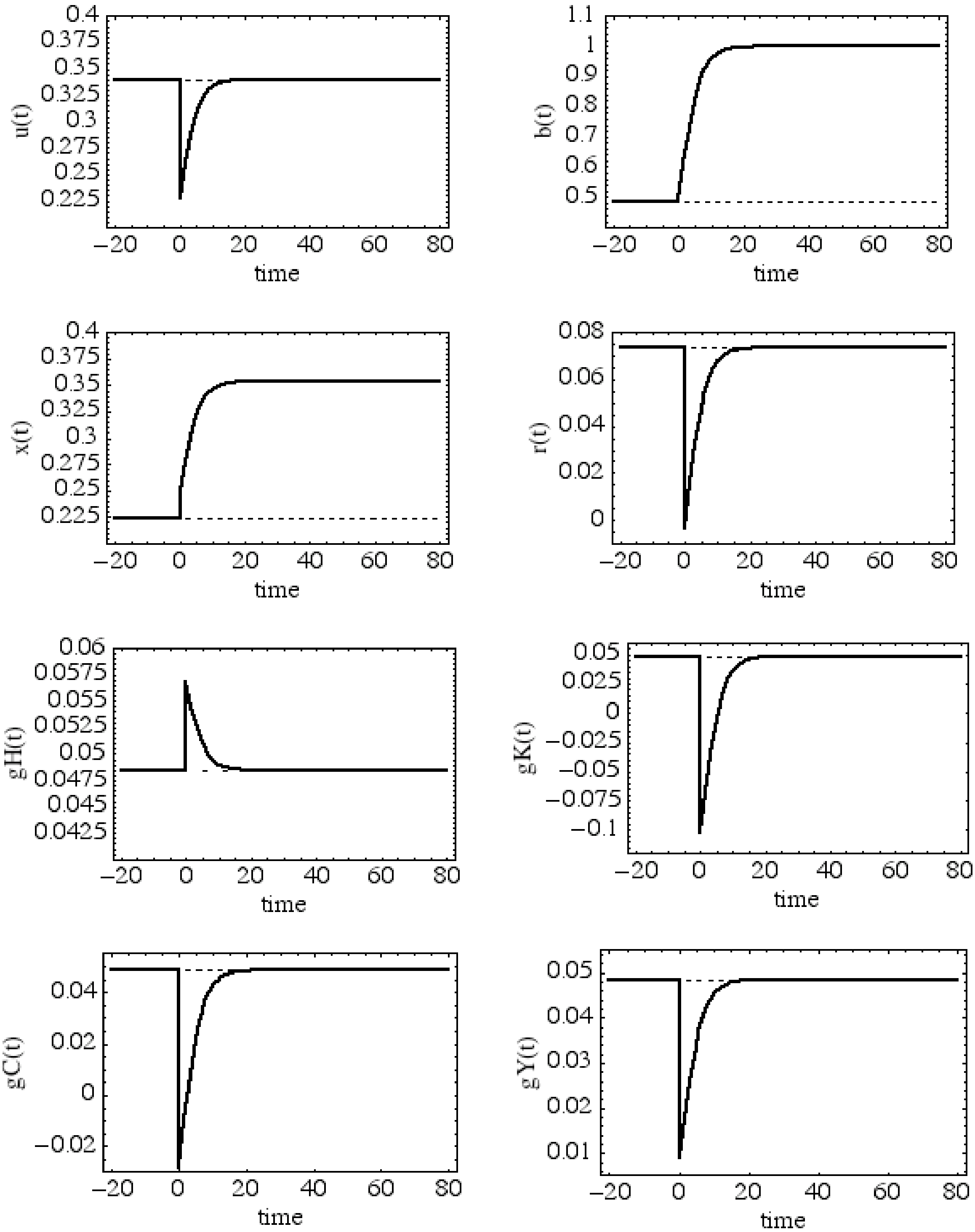
6 Conclusion

This article demonstrates that, if finite lifetime is taken into account, a win-win environmental policy may be implemented in an economy where growth is driven by human capital accumulation à la Lucas (1988), while pollution does not influence educational activities, labor supply is not elastic and human capital does not enter the utility function. This is because finite lifetime and the appearance of newborns at each date creates a turnover of generations which disconnects the aggregate consumption growth to the interest rate. We show that the shorter is the horizon, the greater the effect of the environmental policy on growth, because the higher the "*generational turnover effect*" .

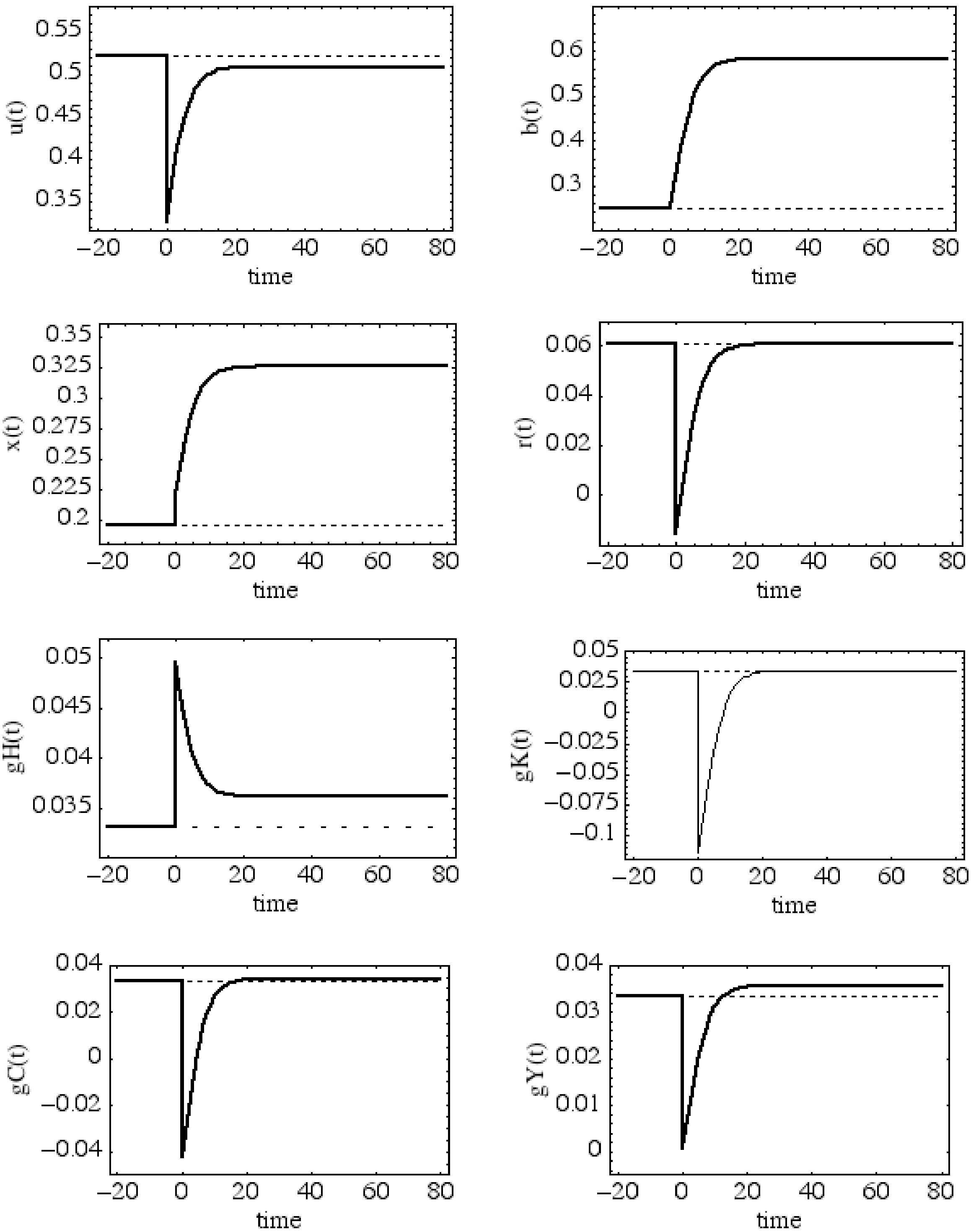
We also demonstrate that when time is not the single production factor in education, the environmental policy promotes growth only if time remains the predominant factor. Otherwise, the crowding-out effect of the tighter environmental policy dominates the "*generational turnover effect*" and growth diminishes.

Finally, when the source of pollution is final output rather than physical capital and time is the single factor in education, the environmental does not affect growth in the steady-state, despite the "*generational turnover effect*" . Nevertheless, if the education good is introduced, the positive influence of the environmental policy appears again.

Graph 1. Transitional effects of an increase in the environmental tax with INFINITE lifetime



Graph 2. Transitional effects of an increase in the environmental tax with FINITE lifetime



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APPENDIX A

On the one hand , combining the equations of (12), we obtain $x^\star = [u^\star + \mathcal{A}]\, B - \lambda\, [\mathcal{A} + \eta] + \mathcal{A}\Phi(\tau) > 0$ and on the other hand we find $x^\star = \frac{\lambda(\varrho+\lambda)}{Bu^\star - \eta\lambda - \varrho} > 0$. Equalizing these two expressions gives $\Gamma(u^\star)$, which is the difference between two positive terms:

$$\Gamma(u^\star) \equiv \{Bu^\star - \eta\lambda - \varrho\}\, \{[u^\star + \mathcal{A}]\, B - \lambda\, [\mathcal{A} + \eta] + \mathcal{A}\Phi(\tau)\} - \lambda(\varrho + \lambda) = 0$$

with $\mathcal{A} \equiv \alpha^{-1} - 1 > 0$.

From the implicit function theorem, the influence of τ on $u^\star$ is given by $u^{\star\prime} \equiv -\frac{\partial\Gamma(u^\star)/\partial\tau}{\partial\Gamma(u^\star)/\partial u^\star}$. If we note $\Gamma(u) = \Gamma_1(u) \times \Gamma_2(u) - \lambda(\varrho + \lambda)$, from the previous footnote it comes that $\Gamma_1(u^\star)$ and $\Gamma_2(u^\star)$ are both positive. Therefore $u^{\star\prime} = \frac{-\mathcal{A}\chi'(\tau)\Gamma_1(u^\star)}{B[\Gamma_1(u^\star)+\Gamma_2(u^\star)]}$ is negative and $u^\star$ is a decreasing function of τ. $\Gamma(u^\star) = 0$ defines a unique value for $u^\star \in]0, 1[$ if $\Gamma(0) < 0$ and $\Gamma(1) > 0$. These two conditions define $\underline{\tau}$ and $\bar{\tau}$ such that $\Gamma(0)|_{\tau=\bar{\tau}} = 0$ and $\Gamma(1)|_{\tau=\underline{\tau}} = 0$.

If we note $g^{\star\prime} = dg^\star/d\tau = -Bu^{\star\prime}$, the effect of the environmental policy on growth with respect to the horizon is given by $dg^{\star\prime}/d\lambda$. Because $g^{\star\prime} = \mathcal{A}\chi'(\tau)\left[1 + \frac{\Gamma_2(u^\star)}{\Gamma_1(u^\star)}\right]^{-1} =$

$\mathcal{A}\chi'(\tau)\left[1+\frac{\lambda(\varrho+\lambda)}{\Gamma_1(u^\star)^2}\right]^{-1}$, the sign if its derivative with respect to λ is the same than the sign of the derivative of $\Gamma_1(u^\star)$ with respect to λ. Remembering that $\frac{\partial u^\star}{\partial\lambda} \equiv -\frac{\partial\Gamma(u^\star)/\partial\lambda}{\partial\Gamma(u^\star)/\partial u^\star} = \frac{\eta}{B} + \frac{\mathcal{A}}{B[\Gamma_1(u^\star)+\Gamma_2(u^\star)]} > 0$, we obtain $\frac{\partial\Gamma_1(u^\star)}{\partial\lambda} = B\frac{\partial u^\star}{\partial\lambda} - \eta = \frac{\mathcal{A}}{B[\Gamma_1(u^\star)+\Gamma_2(u^\star)]} > 0$. Consequently $dg^{\star\prime}/d\lambda > 0$.

APPENDIX B

At time t, each agent born at s buy $z_{s,t}$ units of final output which increase the productivity of the time that they invest in education, such as:

$$\dot{h}_{s,t} = B(1-u_{s,t})^{1-\delta}h_{s,t}^{1-\delta}z_{s,t}^{\delta}$$

The program of the households becomes:

$$\begin{array}{rl} \max\limits_{c_{s,t},z_{s,t},a_{s,t},h_{s,t},u_{s,t}} & \int_s^\infty \left[\log c_{s,t} - \zeta\log\mathcal{P}_t\right]e^{-(\varrho+\lambda)(t-s)}dt \\ s.t. & \dot{a}_{s,t} = [r_t+\lambda]\,a_{s,t} + u_{s,t}h_{s,t}w_t - c_{s,t} - z_{s,t} \\ & \dot{h}_{s,t} = B(1-u_{s,t})^{1-\delta}h_{s,t}^{1-\delta}z_{s,t}^{\delta} \end{array}$$

The Hamiltonian of the program may be written as:

$$\mathcal{H} = [\log c_{s,t} - \zeta\log\mathcal{P}_t] + \pi_{1,t}\left[(r_t+\lambda)a_{s,t} + u_{s,t}h_{s,t}w_t - c_{s,t} - z_{s,t}\right] + \pi_{2,t}B(1-u_{s,t})^{1-\delta}h_{s,t}^{1-\delta}z_{s,t}^{\delta}$$

The F.O.C. are

$$\frac{\partial\mathcal{H}}{\partial c_{s,t}} = 0 \quad\Rightarrow\quad \frac{1}{c_{s,t}} = \pi_{1,t} \tag{26}$$

$$\frac{\partial\mathcal{H}}{\partial z_{s,t}} = 0 \quad\Rightarrow\quad \pi_{1,t} = \pi_{2,t}\delta B(1-u_{s,t})^{1-\delta}\left(\frac{z_{s,t}}{h_{s,t}}\right)^{\delta-1} \tag{27}$$

$$\frac{\partial\mathcal{H}}{\partial u_{s,t}} = 0 \quad\Rightarrow\quad \pi_{1,t}w_t = \pi_{2,t}B(1-\delta)(1-u_{s,t})^{-\delta}\left(\frac{z_{s,t}}{h_{s,t}}\right)^{\delta} \tag{28}$$

$$\frac{\partial\mathcal{H}}{\partial a_{s,t}} = -\dot{\pi}_{1,t} + (\varrho+\lambda)\pi_{1,t} \quad\Rightarrow\quad \pi_{1,t}(r_t+\lambda) = -\dot{\pi}_{1,t} + (\varrho+\lambda)\pi_{1,t} \tag{29}$$

$$\frac{\partial\mathcal{H}}{\partial h_{s,t}} = -\dot{\pi}_{2,t} + (\varrho+\lambda)\pi_{2,t} \quad\Rightarrow$$

$$\pi_{1,t}w_t u_{s,t} + \pi_{2,t}B(1-\delta)(1-u_{s,t})^{1-\delta}\left(\frac{z_{s,t}}{h_{s,t}}\right)^{\delta} = -\dot{\pi}_{2,t} + (\varrho+\lambda)\pi_{2,t} \tag{30}$$

$$\frac{\partial \mathcal{H}}{\partial h_{s,t}} = -\dot{\pi}_{2,t} + (\varrho + \lambda)\pi_{2,t} \qquad \Rightarrow$$

$$\pi_{1,t} w_t u_{s,t} + \pi_{2,t} B(1-\delta)(1-u_{s,t})^{1-\delta}\left(\frac{z_{s,t}}{h_{s,t}}\right)^{\delta} = -\dot{\pi}_{2,t} + (\varrho+\lambda)\pi_{2,t} \tag{31}$$

First of all, equation (28) implies that the ratio $\frac{z_{s,t}}{(1-u_{s,t})h_{s,t}}$ is independent from s and equation (31) implies that $u_{s,t}$ is independent from s. Consequently, the ratio $\frac{z_{s,t}}{h_{s,t}}$ is independent from s. Conveniently, we denote $\tilde{z}_t \equiv \frac{z_{s,t}}{(1-u_{s,t})h_{s,t}}$.

From (26) and (29), we obtain

$$\dot{c}_{s,t} = (r_t - \varrho)c_{s,t}$$

Equations (27) and (28) give:

$$\tilde{z}_t = \frac{\delta}{1-\delta} w_t \tag{32}$$

It implies that $u_{s,t} = u_t$ independent from s. Furthermore (27) and (31) give:

$$\frac{\dot{\pi_{2,t}}}{\pi_{2,t}} = \varrho + \lambda - B\left[1-\delta\right]\tilde{z}_t^{\delta}$$

Differentiating (28) with respect to time, we obtain:

$$\frac{\dot{\pi_{1,t}}}{\pi_{1,t}} + \frac{\dot{w}_t}{w_t} = \frac{\dot{\pi_{2,t}}}{\pi_{2,t}} + \delta\frac{\dot{\tilde{z}_t}}{\tilde{z}_t}$$

Replacing by the expressions of $\frac{\dot{\pi_{1,t}}}{\pi_{1,t}}$ and $\frac{\dot{\pi_{2,t}}}{\pi_{2,t}}$, it gives

$$\frac{\dot{w}_t}{w_t} - \delta\frac{\dot{\tilde{z}_t}}{\tilde{z}_t} + B(1-\delta)\tilde{z}_t^{\delta} = r_t + \lambda$$

which means that the returns to education must be equal to the returns to physical capital.

We can re-write this relation in terms of w_t and r_t:

$$(1-\delta)\frac{\dot{w}_t}{w_t} + B(1-\delta)^{1-\delta}\delta^{\delta}w_t^{\delta} = r_t + \lambda$$

From equation (32), it is possible to express $\tilde{z}_t$ in terms of Y_t:

$$\tilde{z}_t = \frac{\delta}{1-\delta}(1-\alpha)\frac{Y_t}{u_t H_t}$$

Now, we can write that the amount of final output used as educational good is $Z_t \equiv \int_{-\infty}^{t} z_{s,t}\lambda e^{-\lambda(t-s)}ds =$ $\int_{-\infty}^{t}\left(\frac{z_{s,t}}{(1-u_{s,t})h_{s,t}}\right)(1-u_{s,t})h_{s,t}\lambda e^{-\lambda(t-s)}ds = \tilde{z}_t(1-u_t)\int_{-\infty}^{t} h_{s,t}\lambda e^{-\lambda(t-s)}ds = \tilde{z}_t(1-u_t)H_t$.

Consequently:

$$Z_t = \frac{\delta(1-\alpha)}{1-\delta}\left(\frac{1-u_t}{u_t}\right)Y_t,$$

the market clearing condition is written as:

$$\left(1+\frac{(1-\alpha)\delta}{1-\delta}\left(1-\frac{1}{u_t}\right)\right)Y_t = \dot{K}_t + C_t + \Phi(\tau)$$

and the aggregate accumulation of human capital is:

$$\dot{H}_t = B(1-u_t)\tilde{z}_t^{\delta} - (1-\eta)\lambda$$

Therefore, the dynamical system is summarized by

$$\begin{aligned}
\dot{x}_t &= \left\{\left[\alpha - \left(1+\frac{(1-\alpha)\delta}{1-\delta}\left(1-\frac{1}{u_t}\right)\right)\right](b_t u_t)^{1-\alpha} - \varrho - \lambda(\varrho+\lambda)x_t^{-1} + x_t\right\}x_t \\
\dot{b}_t &= \Big\{B(1-u_t)[\delta(1-\delta)^{-1}(1-\alpha)(b_t u_t)^{-\alpha}]^{\delta} - (1-\eta)\lambda \\
&\quad - \left(1+\frac{(1-\alpha)\delta}{1-\delta}\left(1-\frac{1}{u_t}\right)\right)(b_t u_t)^{1-\alpha} + x_t + \Phi(\tau)\Big\}b_t \\
\dot{u}_t &= \left\{\frac{1}{\alpha(1-\delta)}\left[B(1-\delta)^{1-\delta}\delta^{\delta}(1-\alpha)^{\delta}(b_t u_t)^{-\alpha\delta} - \alpha(b_t u_t)^{1-\alpha} + \Phi(\tau) - \lambda\right] - \dot{b}/b\right\}u_t
\end{aligned}$$

APPENDIX C

The two first equations of the dynamical system (24) evaluated in the steady-state ($\dot{x}_t = 0$ and $\dot{b}_t = 0$) enable to define $x^\star$ as a function $\mathcal{X}_1(\underset{-}{u^\star}, \underset{-}{\tau})$:

$$x^\star = \frac{\lambda(\varrho+\lambda)}{B(u^\star-\delta)[\delta(1-\delta)^{-1}(1-\alpha)\mathcal{R}(\tau)^{-\alpha}]^{\delta} - \varrho - \eta\lambda}$$

with $u^\star > \delta$ to obtain $x^\star > 0$. From $\dot{b}_t = 0$ and (25) we can also define $x^\star$ as a function $\mathcal{X}_2(\underset{+}{u^\star}, \underset{+}{\tau})$:

$$x^\star = (1-\alpha)\left(1+\delta-\frac{\delta}{u^\star}\right)\mathcal{R}(\tau)^{1-\alpha} - \left[(1-u^\star)-(1-\delta)\delta^{\delta}\right]B(1-\alpha)^{\delta}\mathcal{R}(\tau)^{-\alpha\delta} - \eta\lambda$$

with $1+\delta-\frac{\delta}{u^\star}>0$ and $(1-u^\star)-(1-\delta)\delta^\delta>0$.

The allocation of human capital into the output sector in the steady-state $u^\star$ is given by the equality:

$$\mathcal{X}_1(\underset{-}{u^\star},\underset{-}{\tau})=\mathcal{X}_2(\underset{+}{u^\star},\underset{+}{\tau}) \tag{33}$$

APPENDIX D

In this appendix, we relax the assumption of logarithmic utility to show that this simplifying assumption does not the qualitative results.

We just give the main equations of model. The expected utility of an agent born at $s\leq t$ becomes

$$\int_s^\infty \frac{[c_{s,t}\mathcal{P}_t^{-\zeta}]^{1-1/\sigma}-1}{1-1/\sigma}e^{-(\varrho+\lambda_s)(t-s)}dt \tag{34}$$

with $\sigma\neq 1$ the elasticity of intertemporal substitution.

The individual consumption of an agent born at s becomes

$$c_{s,t}=\Delta_t^{-1}[a_{s,t}+\omega_{s,t}] \tag{35}$$

where $\Delta_t\equiv\int_t^\infty e^{-(\sigma\varrho+\lambda)(\nu-t)-(1-\sigma)\int_t^\nu r_\mu d\mu}d\nu>0$ is the propensity to consume out of wealth.[11] Similarly, we can express the aggregate consumption at time t:

$$C_t=\int_{-\infty}^t c_{s,t}\lambda e^{-\lambda(t-s)}ds=\Delta_t^{-1}\left[K_t+\Omega_t\right] \tag{36}$$

The aggregate consumption growth rate (equation 11) becomes

$$\dot{C}_t=\sigma\left[r_t-\varrho\right]C_t-\lambda\Delta_t^{-1}K_t$$

Finally, the dynamics of the economy is summarized by the following system (with respect to the case where $\sigma=1$, the intertemporal evolution of the propensity to consume out of wealth

[11] $\Delta_t>0$ insures that individual utility is bounded.

is added):

$$\begin{aligned}
\dot{x}_t &= \left\{[\alpha\sigma - 1]\,(b_t u_t)^{1-\alpha} + (1-\sigma)\Phi(\tau) - \sigma\varrho - \lambda\Delta_t^{-1}x_t^{-1} + x_t\right\} x_t \\
\dot{b}_t &= \{B\,(1-u_t) - (1-\eta)\lambda - (b_t u_t)^{1-\alpha} + x_t + \Phi(\tau)\}\, b_t \\
\dot{u}_t &= \{(\alpha^{-1}-1)\,B + (\alpha^{-1}-1)\Phi(\tau) + Bu_t - (\alpha^{-1}+1-\eta)\,\lambda - x_t\}\, u_t \\
\dot{\Delta}_t &= -1 + [(1-\sigma)\alpha(b_t u_t)^{1-\alpha} - (1-\sigma)\Phi(\tau) + \sigma\varrho + \lambda]\,\Delta_t
\end{aligned} \tag{37}$$

where $x_t \equiv C_t/K_t$ and $b_t \equiv H_t/K_t$.

Along the balanced growth path, the allocation of human capital into the manufacturing sector is defined by:

$$\begin{aligned}
\Gamma_s(u^\star) \equiv [B\,(\sigma - 1 + u^\star) + (1-\sigma-\eta)\lambda - \sigma\varrho]\,\{[u^\star + \mathcal{A}]\,B - \lambda\,[\mathcal{A} + \eta] + \mathcal{A}\Phi(\tau)\} \\
- \lambda\Delta^{\star -1} = 0
\end{aligned} \tag{38}$$

with $\mathcal{A} \equiv \alpha^{-1} - 1 > 0$ and $\Delta^{\star -1} = \sigma\varrho + \sigma\lambda + (1-\sigma)B > 0$. It is straightforward that $\Gamma_s(u^\star) = 0$ defines a unique $u^\star$ decreasing with τ.

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