Economic Growth: Theory and numerical solution methods

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List of EXCEL files and MATLAB programs
(ordered by sections)

Chapter 2. The neoclassical growth model under a constant savings rate

- ***Solow_deterministic.xls***. Section 2.5.3
  A single realization for the constant savings rate, deterministic growth model, is analyzed.
  - *Discrete spreadsheet:* The solution to the model is computed from the exact solution as well as from linear and quadratic approximations. A numerical comparison between them is performed.
  - *Increasing time path* spreadsheet: the case of an economy converging to steady state from below
  - *Decreasing time path* spreadsheet: the case of an economy converging to steady state from above

- ***Change_steadystate.m***. Section 2.5.3
  Characterization of steady-state effects from changes in structural parameters other than the savings rate

- ***Change_savings.xls***. Section 2.5.4
  Short- and long-run effects of a permanent increase in the value of the savings rate. A large and a small increase in savings rate are considered in different spreadsheets.
  - *C-increases* (1) and (2) spreadsheets: it presents the case of an economy where steady-state consumption is higher after the increase in savings.
  - *C-decreases* (1) and (2) spreadsheets: it presents the case of an economy where steady-state consumption is lower after the increase in savings

- ***Change_savings.m***. Section 2.5.4
  It performs the same exercise as ***Change_savings.xls***

- ***Dynamic_inefficiency.xls***. Section 2.5.5
  Characterization of dynamically efficient and inefficient steady-states

- ***Solow_dynamic.m***. Section 2.5.5
  The program performs the same exercise as ***Dynamic_inefficiency.xls***.

- ***Solow_stochastic.xls***. Section 2.6.1
  An approximate and an exact solution to the stochastic version of the constant savings rate economy. Four different realizations are displayed in as many spreadsheets

- ***Solow_stochastic.m***. Sections 2.5.3 and 2.6.1
  The program performs the same exercise as ***Solow_stochastic.xls***. This program can also be used to reproduce the exercise in *Solow_deterministic.x*ls file by setting a variance parameter to zero.

Chapter 3. Optimal Growth in continuous time

- ***CK_continuous time.xls***. Section 3.1.6
    - ***Steady-state*** spreadsheet: steady-state levels for different parameterizations
    - ***Speed of convergence*** spreadsheet: Speed of convergence to steady state, as a function of structural parameter values
    - ***Convergence. Risk aversion*** and ***Risk aversion*** spreadsheets: Convergence trajectories for different values of the intertemporal elasticity of substitution of consumption. It shows the potential pitfalls in using discrete time observations from the solution to the continuous time economy
    - ***Change in output share of k*** spreadsheet: Steady-state effects of a change in the value of the output elasticity of capital

- ***CK_c_steady state.m.*** Section 3.1.6
  A program to analyze the sensitivity of the steady state levels of endogenous variables in the continuous time version of the Cass Koopmans model with respect to values of structural parameters.

- ***CK_c_transition.m.*** Section 3.1.6
  Impulse response functions in the continuous time version of the Cass Koopmans model. This program calls function ***ss_ck_c.m.***, which lays out first order conditions for the planner's problem and computes the transition matrix in the log-linear approximation to the model. Impulse response functions are obtained by taking an initial condition on the stock of capital different form the steady-state level.

- ***CK_c_change structural parameters.m.*** Section 3.1.6
  Short- and long-run analysis of effects of a change in a structural parameter in the continuous time Cass-Koopmans economy. The program is written to consider changes in the output elasticity of capital, but changes in any other parameter can be considered alternatively. Like ***CK_impulse responses .m***, this program calls function ***ss_ck_c.m***.

- ***CK_Taxes_deterministic.xls***. Section 3.8.2
  Solution to the deterministic planner's problem under consumption and income taxes. It shows the need to appropriately use stability conditions

Chapter 4. Optimal Growth in discrete time

- ***CK_d_transition.m.*** Section 3.8.2
  This program computes the transition in the discrete time version of the Cass-Koopmans model without uncertainty using different treatments of stability. This program calls function ***ss_ck_d.m,*** which lays out first order conditions for the discrete-time version of the Cass-Koopmans economy. These conditions can be used either to compute steady-state levels of the endogenous variables, as well as the transition matrix for the linear approximation to the model.

- ***CK_solution_changes in tax.xls.*** Section 3.8.3
    - ***Steady state*** spreadsheet: Long-run effects of different fiscal policy experiments
    - ***Transition*** spreadsheet: Short- and long-run effects of some fiscal policy experiments

- ***CK_d_long run tax changes.m.*** Section 3.8.3
  A program to analyze long-run (steady-state) effects of changes in either the consumption tax rate, the income tax rate or both in the deterministic, discrete time

Cass-Koopmans economy. The analysis can be done either keeping revenues constant, or maintaining a constant ratio of revenues to output. This program calls the following functions: *i*) ***ss_ck_d.m***, which solves for the steady-state, *ii*) ***ss_ck_d1.m***, which solves for the steady-state keeping revenues constant (in this case, the consumption tax becomes an endogenous variable, which is solved for so that revenues remain constant for any income tax rate), *iii*) ***ss_ck_d2.m***, which solves for the steady-state keeping constant the ratio of revenues to total income (in this case, the consumption tax is an endogenous variable which is solved for so that the revenues to output ratio is the same for any income tax rate).

- ***CK_d_long_short run tax changes.m.*** Section 3.8.3
  A program to analyze short- and long-run effects in the deterministic, discrete-time Cass-Koopmans economy of changes in either the consumption tax rate, the income tax rate, or both, while keeping constant either total revenues, or the ratio of revenues to income. This program calls the same functions as ***CK_d_long_run tax changes.m***.

Chapter 5. Numerical solution methods

- ***Simple_models.xls.*** Section 5.2.3
  A sample realization form the analytical solution for each of the two economies considered in this section

- ***Simple_planner_problemxls.*** Sections 5.3.1 and 5.3.4
  A single realization from the planner's problem without taxes, using the linear quadratic approximation as well as Uhlig's undetermined coefficients approach.

- ***lq.m.*** Section 5.3.1
  This program computes a single realization from the discrete-time version of the Cass-Koopmans growth model under uncertainty, using the linear-quadratic approximation solution method. The program calls function ***util.m***, which lays out the objective function so that the numerical derivatives needed to implement the solution method can be computed

- ***CK_solution_BK.xls.*** Section 5.3.3 and 5.4.2
  A single realization from the solution to the planner's problem with taxes using Blanchard and Kahn's approach

- ***Blanchard Kahn.m.*** Section 5.3.3
  The Blanchard-Kahn method is used to compute a numerical solution to the discrete-time version of the Cass-Koopmans economy subject to productivity shocks. A single realization is obtained and graphs for the simulated series for the main variables in the model are displayed.

- ***uhlig.m.*** Section 5.3.4
  This program computes a single realization from the discrete-time version of the Cass-Koopmans growth model under uncertainty using Uhlig's solution method, based on a log-linear approximation. The program calls function *ss_lsims.m*, as in ***LSIMS.m***.

- ***CK_Stochastic.xls.*** Section 5.3.5
  Numerical solutions for the planner's problem without taxes using the eigenvalue-eigenvector decomposition approach

- ***CK_Stochastic_taxes.xls.*** Section 5.3.5

Numerical solutions for the planner problem under consumption and income taxes, using the eigenvalue-eigenvector decomposition approach. A single realization from the stochastic economy is obtained for alternative values of the consumption and income tax rates, maintaining the same values for structural parameters.

- ***CK_Stochastic_taxes_structural_parameters.xls.*** Section 5.3.5
  Numerical solutions for the planner's problem using the eigenvalue-eigenvector decomposition approach. A single realization from the stochastic economy is obtained for alternative values of the vector of structural parameters, maintaining the same consumption and income tax rates.

- ***SIMS.m.*** Section 5.3.5
  This program computes a single realization of the discrete-time version of the Cass-Koopmans growth model under uncertainty, using Sims's eigenvalue-eigenvector decomposition method and a linear approximation. This program calls function ***ss_sims.m***, to solve for the steady-state as well as to compute the transition matrix in the linear approximation to the first order conditions for the optimization problem.

- ***LSIMS.m.*** Section 5.3.5
  This program computes a single realization from the discrete-time version of the Cass-Koopmans growth model under uncertainty, using Sims's eigenvalue-eigenvector decomposition method and a log-linear approximation. This program calls function ***ss_lsims.m*** to solve for the steady-state as well as to compute the transition matrix in the log-linear approximation to the first order conditions for the optimization problem.

- ***marcet.m.*** Section 5.3.6
  This program solves for the polynomial used in the den Hann-Marcet solution method to approximate the expectations term in the Euler equation in the discrete-time neoclassical growth model under uncertainty. The program calls ***srmarcet.m*** to compute this sum of squared residuals

- ***marcet1.m*** *and* ***marcet2.m.*** Section 5.3.6
  These two programs solve the discrete-time version of the Cass-Koopmans growth model under uncertainty, using the parameterized expectations method of den Hann and Marcet. *marcet1.m* uses command "fminunc to minimize the sum of squared residuals, while *marcet2.m* uses a Gauss-Newton algorithm. In both cases, *marcet.m* needs to be used in advance to compute initial values for the parameters in the polynomial used to represent the conditional expectations.

- ***BK impulse response.m*** Section 5.4.3
  This program computes responses to an impulse in the innovation in the autoregressive process for productivity in the stochastic, discrete-time version of the Cass-Koopmans economy. The Blanchard and Kahn method is used times series for the endogenous variables along the transition.

- ***methods.m*** Section 5.5
  This program solves solve the discrete-time version of the Cass-Koopmans growth model under uncertainty, using different solution methods: linear-quadratic approximation, Sims' method with either a linear or a log-linear approximation, Blanchard-Kahn method, Uhlig method, and the parameterized expectations method by den Hann-Marcet. The user must select the specific solution method by activating the corresponding line in the program, and deactivating the alternative lines. N simple realizations are obtained and the characteristics of the solution are summarized through sample statistics: mean, standard deviation, coefficient of variation, volatility relative to that of output, correlation coefficients with

contemporaneous, lagged or leaded output,…). We compute these statistics for each realization to then obtain the average and standard deviation for each one of them across the set of simulations (sample realizations). These basic statistics needed for a discussion along the lines of the Real Business Cycle literature. Depending on the solution method chosen, the program calls functions: ***MLQ.m, MSIMS.m, MLSIMS.m, MBK.m, MUHLIG.m, MMARCET.m.***

- ***coll_cheb.m.*** Section 5.5
  This program solves the deterministic, Cass-Koopmans optimal growth model using projection methods and Chebychev polynomials. It calls functions ***res.m*** and ***Cd.m***.

- ***g_cheb_s_3.m.*** Section 5.5
  This program solves the stochastic version of the optimal Cass-Koopmans growth model using projection methods and Chebychev polynomials of order 3. It calls functions ***res_s_3g.m, psi_3.m, Cds_3.m, root_h.m, herm.m***.

- ***g_cheb_s_4.m.*** Section 5.5
  This program solves the stochastic version of the optimal Cass-Koopmans growth model using projection methods and Chebychev polynomials of order 4. It calls functions ***res_s_4g.m, psi_4.m, Cds_4.m, root_h.m, herm.m***.

- ***g_cheb_s_5.m.*** Section 5.5
  This program solves the stochastic version of the optimal Cass-Koopmans growth model using projection methods and Chebychev polynomials of order 5. It calls functions ***res_s_5g.m, psi_5.m, Cds_5.m, root_h.m, herm.m***.

Chapters 6. Basic models of endogenous growth

- ***DynamicLaffer.xls***. Section 5.5.1
  The possibility of dynamic Laffer effects in the AK endogenous growth economy is shown in this file.

- ***AK_Stochastic.m***. Section 5.6.2
  This program computes a single realization of the numerical solution to the stochastic version of the AK economy.

- ***mAK_Stochastic.m***. Section 5.6.2
  This program computes an arbitrary number (chosen by the user) of realizations of the numerical solution to the stochastic version of the AK economy, from which the sample distribution for any statistic of interest can then be obtained.

- ***Dfase.m***. Section 5.9.3
  This program computes the phase diagram for the Jones and Manuelli endogenous growth economy.

- ***AKModel.xls***. Section 5.9.4
  A single realization from the numerical solution to the stochastic version of the AK economy.

- ***AK_JMs.m***. Section 5.9.4
  This program computes a single realization of the numerical solution to the stochastic version of the Jones and Manuelli endogenous growth economy, from which the sample

distribution for any statistic of interest can be obtained. Realizations from the AK economy can be obtained as a special case.

- ***mAK_JMs.m***. Section 5.9.4
  This program computes a number of realizations (chosen by the user) of the numerical solution to the stochastic version of the Jones and Manuelli endogenous growth economy, from which the sample distribution for any statistic of interest can be obtained. Realizations from the AK economy can be obtained as a special case.

Chapter 7. Additional models of endogenous growth

- ***Simul_diffus.m.*** Section 7.2.3
  This program computes the transmission of a shock in the leader country to the follower country. The program can also be used to generate a realization of the numerical solution to the problems of the leader and the follower countries, given a realization of the stochastic shocks for both countries.

- ***Lucas_ss_w.m.*** Section 7.4.5
  These three programs compute steady-state effects of changes in tax rates on labour income, in the endogenous growth model with human capital accumulation.

- ***Lucas_ss_r.m.*** Section 7.4.5
  These three programs compute steady-state effects of changes in tax rates on capital income in the endogenous growth model with human capital accumulation.

- ***Lucas_ss_c.m.*** Section 7.4.5
  These three programs compute steady-state effects of changes in tax rates on consumption expenditures in the endogenous growth model with human capital accumulation.

- ***Lucas_sim1.m.*** Section 7.4.7
  This program computes a single realization of the endogenous growth model with human capital accumulation, which may include an externality in the production of the final good through the average stock of human capital. It can also be used to solve this model in case of equilibrium indeterminacy. The program uses the full Blanchard-Kahn approach by representing the model in state-space form, described as '*First simulation approach*' in the section devoted to describing the solution approach for this model.

- ***Human capital.xls.*** Section 7.4.7
  This EXCEL spreadsheet performs the same exercise as the ***Lucas_sim1.m*** program file, but without the externality produced by the possible presence of the average stock of human capital in the production of the final good.

- ***Lucas_sim2.m.*** Section 7.4.7
  This program computes a single realization of the endogenous growth model with human capital accumulation, without externalities in the production of the final good. The Blanchard-Kahn approach is used to solve for control variables, but the original nonlinear model is used to solve for the variables in levels, described as '*Second simulation approach*' in the section devoted to describing the solution approach for this model**.**

- ***Mlucas_sim.m.*** Section 7.4.7
  This program computes multiple realizations of the endogenous growth model with human capital accumulation. The user may include or not externalities in the production of the final good. The program can also be used for the case of indeterminacy of equilibrium, and two

alternative benchmark parameterizations leading to either situation are again provided. The multiple realizations are used to present average values of a wide variety of statistics, together with their standard deviations across the set of simulations.

Chapters 8 and 9. Growth in monetary economies

- ***Timing real balances.xls***. Section 8.4.3
  Steady-state welfare cost of inflation in two economies differing in the timing of real balances in the utility function.

- ***Steady-state fiscal policy.xls***. Section 8.5.2
  Steady-state values under alternative monetary policy choices, the economy being under an income tax.
  - *Case* 1 spreadsheet: the government chooses the income tax rate as well as the steady-state level of outstanding debt and the rate of inflation, while the steady-state levels of real balances and lump-sum transfers to consumers become endogenous
  - *Case* 2 spreadsheet: the government chooses the income tax rate and the steady-state size of the transfer, while leaving the steady-state stock of bonds to be endogenously determined
  - *Case* 3 spreadsheet: the government chooses the level of the steady-state transfer and the steady-state stock of per-capita debt

- ***SS inflation endogenous leisure.xls***. Section 8.6.2
  - *Case* 1 spreadsheet: the government realizes a lump-sum transfer to consumers, which is fully financed by increasing the money supply, and we compute the welfare cost of inflation
  - *Case* 2 spreadsheet: examines the welfare cost of inflation under a different type of separability in the utility function
  - *Case* 3 spreadsheet: examines the welfare cost of inflation under a still different type of separability in the utility function
  - *Case* 4 spreadsheet: the government realizes a lump-sum transfer to consumers, which is financed by increasing the money supply and by a tax on consumption. The government has an inflation target and a constant tax rate on consumption
  - *Case* 5 spreadsheet: the government realizes a lump-sum transfer to consumers, which is financed by increasing the money supply and by a tax on consumption. The government has an inflation target and it keeps constant the level of public debt

- ***Money_M_d.m***. Section 9.4
  This program solves the deterministic monetary economy when the monetary authority chooses the rate of money growth. The program computes steady-state values and a single sample realization for the solution

- ***Money_i_d.m***. Section 9.5
  This program solves the deterministic monetary economy when the monetary authority chooses the nominal rate of interest. The program computes steady-state values and a single sample realization for the solution

- ***Short-run nonneutrality.xls***. Section 9.6
  - ***Change nominal rates*** spreadsheet: government finances a transfer to consumers by printing money, issuing debt and raising proportional taxes on output, while using nominal rates as a control variable for monetary policy
  - ***Once and-for-all money change*** spreadsheet: the government makes a lump-sum transfer to the representative agent, which is financed through seignoriage and bond

issuing. In this case, the government uses the rate of growth of money supply as policy variable and we consider the effects of an experiment by which the growth rate experiences a permanent increase.
    - ***Gradual money change*** spreadsheet: we consider a gradual, but permanent change in money growth.

- ***S_i_npi_s.m***. Section 9.7.1
  A single realization of the solution to the stochastic monetary economy, under the assumption that the government chooses the nominal rate of interest according to a Taylor's rule without the presence of the rate of inflation.

- ***S_i_pi_s.m***. Section 9.7.1
  A single realization of the solution to the stochastic monetary economy, under the assumption that the government chooses the nominal rate of interest according to a Taylor's rule that includes the rate of inflation together with the output gap.

- ***mS_i.m***. Section 9.7.1
  This program computes the solution to the stochastic monetary economy, under the assumption that the government chooses the nominal rate of interest according to a Taylor's rule that includes the rate of inflation together with the output gap. A large number of sample realizations (chosen by the user) is computed, and the sample distribution for any statistic of interest can be obtained.

- ***S_M_s.m.*** Section 9.7.2
  This program computes a single sample realization of the solution for the stochastic monetary economy, under the assumption that the government controls the rate of growth of the money supply.

- ***mS_M.m***. Section 9.7.2
  This program solves the stochastic monetary economy, under the assumption that the government controls the rate of growth of the money supply. A large number of sample realizations (chosen by the user) are computed, and the sample distribution for any statistic of interest can be obtained.

- ***Neokeyn.m***. Section 9.8
  This program computes a single realization of the Neo Keynesian economy described in the text. Monetary policy is implemented by the monetary authority choosing the nominal rate of interest according to a Taylor's rule. It calls function ***nkeyn.m***.

- ***Nkeyprg.m.*** Section 9.8
  This program computes multiple realizations of the same economy considered in the previous program. It computes the first two moments of the empirical distribution for a number of statistics, and it stores the simulated data.