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over competing divisions

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Abstract

This paper examines a three-stage model of divisionalization where, first, two parent firms create independent units, second, the parent firms allocate cost reduction levels over these units, and third, the resulting units compete in a Cournot market given their current costs of production. The introduction of the cost reduction phase is shown to reduce the incentives toward divisionalization severely, relative to other existing models. Namely, the scope for divisionalization in equilibrium reduces as the marginal cost of the cost reducing investment decreases, and eventually vanishes. A second-best welfare analysis shows that, for any given market structure, the equilibrium investment decisions of the parent firms are socially optimal. In addition, to no divisionalization outcome is sustainable in equilibrium only if it is socially optimal.

I. INTRODUCTION. This paper deals with divisionalization, *i.e.* the practice pursued by a parent firm of dividing its production among a number of competing units. Many recent studies argue that a major incentive for divisionalization is the commitment effect it generates toward a relatively large output on the part of the parent firm. In this regard, Baye *et al.* (1996) show that if the cost of creating a new division is small, then each firm forms autonomous units in equilibrium. This fact seems to work against the conventional rationale that increasing competition is detrimental to firms, while provides a strategic ground for thinking about the diffusion of such procedures as divisionalization, franchising and divestiture. A comparable issue relates to the point made by Salant *et al.* (1983) that exogenous mergers in a Cournot market may not be profitable to the merging firms to the extent that the outsiders react to the merger by expanding too much their production. It is comparable since horizontal mergers can be seen as the inverse action for divisionalization. In a close spirit, firms bringing to the marketplace several divisions, aim to mimic a Stackelberg-leader behavior. Even so, this reduces equilibrium profits and increases social welfare relative to the non divisionalization scenario. In addition, as the cost of divisionalization gets smaller, the equilibrium aggregate output converges to that of perfect competition.[2]

The main finding that firms may choose to enhance product market competition through divisionalization has proved to be robust to the inclusion of further assumptions like product or spatial differentiation.[3] Here I consider divisionalization in a context where two parent firms also decide on the allocation of cost reducing investments over their divisions. This specification traces back to a traditional look on the

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[2] A similar result is in Corchón and González-Maestre (2000).
[3] See, for instance, Ziss (1998) and González-Maestre (2001).

subject at hand according to which the fundamental effect of divisionalization is in separating strategic from operational issues.[4] In the analysis to follow the strategic issue is about allocating resources to divisions and lies within the competence of a parent firm, while the operational decision pertains to product market competition and is up to the firm's units. Formally, I examine the following three-stage game. In the *divisionalization* stage, two parent firms simultaneously decide how many divisions to form. The resulting divisions are ordained to operate in the market as independent units, while each parent firm acts by the same criterion as a general office which collects the sum of its divisions' profits net of the cost of divisionalization. In the second or *investment* stage, the parent firms simultaneously undertake separate actions, *i.e.* one for each of their divisions. An action here is an investment lowering the constant marginal cost of the respective division. The parent firms allocate investments over units in order to maximize their divisions' aggregate (non cooperative) profit net of the investment cost. Finally, in the *market* stage, all the divisions created in the first stage engage in Cournot competition, given their actual marginal costs.

The investment problem faced by a parent firm in stage two is clearly reminiscent of that addressed by a precompetitive cartel allocating investments over its members, and extensively analyzed by d'Aspremont and Jacquemin (1988), Kamien *et al.* (1992), Amir *et al.* (2003), and many others in the literature on R&D joint ventures. In this case however, and unlike the standard framework for the mentioned line of research, the strategic interaction in the investment phase is not completely collusive, postulating collusive behavior only for those divisions headed by the same parent company, while displaying competition between those others relating to different firms. A second, relevant difference is that while precompetitive cartelization is usually meant

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[4] See, for instance, Tirole (1988) and the references therein.

3

to involve firms already active in an industry, the first stage divisionalization process considered here is fundamentally one of firm creation. More specifically, the present setting should not be rephrased in terms of two-joint venture oligopolistic competition with endogenous number of members, since the more appropriate way to model the latter would probably describe a problem of coalition formation between a given number of *existing* firms.

The introduction of the investment phase into a standard model of divisionalization alters the incentives for firms to form divisions. As summarized in the central result of this paper, the ultimate action of it is to discourage divisionalization severely. The first and immediate effect of embarking in the cost reducing investment is that the overall cost of creating independent units gets a rise equal to the total expenditure borne by the parent firm in stage two. The above, however, turns out not to limit firms' divisionalization decisions. On the contrary, the scope for divisionalization is shown to increase the larger the marginal cost of the investment. This seemingly counterintuitive statement is entirely in line with a second, decisive effect of investment in this model. By lowering the marginal cost of each firm's independent units, investing in stage two makes the *competitive* effect of starting a new division stronger, either because this division will operate at a *low* marginal cost, or because it will engage in market competition with very fierce rivals. When the cost of investment is relatively small and hence at least some of a firm's divisions achieve a large cost reduction level, this inhibits the process of division creation. The analysis is separated in two parts according as to whether the best response of a parent firm in stage two involves a symmetric or asymmetric investment allocation over units. For the symmetric case, I show that there exists a unique symmetric equilibrium (and possibly other asymmetric ones) and that the latter sometimes involves no divisionalization, even when the

4

cost of creating a new division is equal to zero. For the asymmetric case the result is more striking. Given an upper bound on the number of competing divisions, there exists a unique equilibrium, always involving no divisionalization, whatever the cost of divisionalization is.

This study also conducts a welfare evaluation of the industry performance for the symmetric case mentioned above in terms of second-best criterion, *i.e.* conditional on the third-stage Cournot equilibrium. I first consider a given market structure, that is a given number of divisions, and show that the equilibrium level of cost reducing investment in stage two is socially optimal. This does not imply that the industry selects the socially optimal amount of investment when the number of divisions is endogenously determined at the equilibrium of the first stage. As a second step, however, I focus on the no divisionalization scenario and argue that the latter is sustainable as equilibrium of the whole game only if it is socially optimal.

The paper proceeds as follows. Section II presents the model, sections III.a and III.b deal with the cases of symmetric and asymmetric allocation of cost reductions over divisions, respectively. Section IV explores welfare issues, and section V briefly concludes, with a connection with the mentioned literature on collusive joint ventures and horizontal mergers. The appendix contains the proofs of the propositions.

II. THE MODEL. I consider the effect of cost reducing investments on the incentives for two parent firms, indexed by $i = 1, 2$, to separate their production over independent units. Accordingly, the remainder of this study examines a three-stage oligopoly game with the following timing structure. In the first stage, parent firm i decides on the number δ_i of own divisions to bring to the market. Creating a new division has a constant cost of $c > 0$, which is common to both firms. The result-

ing divisions operate as independent profit maximizer units, selling an homogeneous product in a market with linear demand $P(Q) = a - Q$, where $Q = \sum_{i=1}^{2} \sum_{j=1}^{\delta_i} q_{ij}$ is total output and q_{ij} denotes the output of division ij, *i.e.* the *jth* division of firm i. Divisions are *ex ante* identical in that they share the same technology with *initial* constant marginal cost m. I assume both $a > 2m$ and $(a - m)^2 > c$.[5] In the second stage however, parent firm i selects a vector $\mathbf{x}_i \in [0, m]^{\delta_i}$ of cost reductions, one for each of its divisions, so that the effective marginal cost of division ij becomes $m_{ij} = m - x_{ij}$. The cost of obtaining x_{ij} is $\gamma \frac{x_{ij}^2}{2}$, $\gamma > 0$, reflecting decreasing returns in the cost reducing activity. In the third stage, the divisions created in stage one engage in Cournot competition, given their effective production costs. Let $\theta_{ij} = (a - m_{ij})$. The profit of division ij is written $\pi_{ij} = (\theta_{ij} - Q) q_{ij}$. The profit of parent firm i is equal to the sum of its divisions' profits net of the costs of both divisionalization and cost reducing investments. Namely, $\pi_i = \sum_{j=1}^{\delta_i} \left(\pi_{ij} - \gamma \frac{x_{i,j}^2}{2} \right) - c \delta_i$.[6] Since I examine subgame perfect Nash equilibria (SPNE), I solve the game by going backward from the third to the first stage.

Stage three is a k-division Cournot oligopoly, with $k = \delta_1 + \delta_2$. Division ij solves the first order condition $\theta_{ij} - Q - q_{ij} = 0$. Summing the latter over divisions gives $Q = \sum_{i=1}^{2} \sum_{j=1}^{\delta_i} \theta_{ij} / (k + 1)$, along with the equilibrium quantity and profit of division ij,

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[5] These assumptions are standard. The former ensures that in the case where the two parent firms decide to form only one division each, both divisions find it profitalbe to operate in the equilibrium of the market subgame. The latter is sufficient to ensure that the equilibrium in the divisionalization stage is interior.

[6] The game at hand extends the divisionalization game proposed by Baye *et al.* to encompass cost reducing investments. In this respect, the cost reducing technology in stage two is the same as in the model of d'Aspremont and Jacquemin, with the exception of a zero spillover rate.

equal to

$$q_{ij}^c\left(\mathbf{x}_i,\,\mathbf{x}_z\right) = \max\left\{\left((k+1)\,\theta_{ij} - \sum_{i=1}^{2}\sum_{j=1}^{\delta_i}\theta_{ij}\right) / (k+1)\;,\,0\right\} \text{ and } \pi_{ij}^c\left(\mathbf{x}_i,\,\mathbf{x}_z\right) = q_{ij}^c{}^2,$$

$$(1)$$

respectively, with $i = 1, 2$, $i \neq z$.

In stage two, parent firm i chooses $\mathbf{x}_i$ in order to maximize the sum of its divisions' profits conditional on the Cournot equilibrium in the market phase. As specified in recent studies on R&D joint ventures, a crucial issue here is whether the optimal allocation involves identical or different cost reduction levels over divisions. As a preliminary step, following Salant and Shaffer (1998),[7] I address this point by looking at the case where for any pair ij and ih of firm i's divisions, the respective cost reduction levels x_{ij} and x_{ih} vary so that their sum remains constant, equal to w, provided the level of cost reduction of all the other divisions is left unchanged. Let $R_i\left(\mathbf{x}_i,\,\mathbf{x}_z\right) = \sum_{j=1}^{\delta_i}\left(\pi_{ij}^c\left(\mathbf{x}_i,\,\mathbf{x}_z\right) - \gamma\frac{x_{ij}^2}{2}\right)$ denote the revenue of firm i, net of the investment cost but gross of the cost of divisionalization. For given w and $\mathbf{x}_z$, then R_i can be seen as a function of x_{ij} only, $i.e.$

$$\overline{R}_i\left(x_{ij}\right) = \left(\frac{k(a-m+x_{ij})-(w-x_{ij})-F}{k+1}\right)^2 + \left(\frac{k(a-m+(w-x_{ij}))-x_{ij}-F}{k+1}\right)^2 - \frac{\gamma\left((w-x_{ij})^2+x_{ij}^2\right)}{2} + H,$$

$$(2)$$

where F and H are constant terms. Since divisions are $ex\ ante$ identical, R_i is symmetric along the path of identical cost reductions. In addition, since R_i is smooth, any interior cost reduction profile with $x_{ij} = x_{ih}$ is a local extremal point of $\overline{R}_i\left(x_{ij}\right)$.[8]

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[7] Also see Amir and Tesoriere (2007 a,b). Van Long and Soubeyran (2001) propose to apply their results on this regard to a model where the sole incumbent in an industry decides first how many divisions to create and then on the allocation of a cost reducing capital to them. Finally, a potential entrant decides whether to enter or not and with how many divisions. This setup is close enough to the present one, but it does not consider duopoly and rather focuses on the investment stage, leaving open the question of determining the equilibrium number of divisions.

[8] To see that, notice that by symmetry of R_i, any feasible reallocation of cost reductions on a

So, the profile of interest is a maximum if $\overline{R}_i$, evaluated at it, is strictly concave. From

(2), it follows that $\frac{d_i^2 \overline{R}_i(x_{ij})}{dx_{ij}^2} < 0$ if and only if $\gamma > 2$. When the latter is satisfied, R_i

is strictly concave along any feasible locus of sum preserving cost reductions. This

is a sufficient condition for the optimal allocation to be symmetric, since it means

that, for any pair of firm i's divisions, any feasible asymmetric cost reduction profile

is strictly dominated by the symmetric one lying on the sum preserving locus passing

through it. Analogously, when $\gamma < 2$, $\overline{R}_i$ is strictly convex.[9] This gives the following

result.

Lemma 1: (a) If $\gamma > 2$, then the best response of parent firm i to a cost reduc-

tion allocation $\mathbf{x}_z$ of parent firm z involves the same level of cost reduction for each

division. (b) If $\gamma < 2$, then the best response of parent firm i involves a vector of cost

reductions lying on the boundary of the feasible set $[0, m]^{\delta_i}$.

Proof: (a) The result follows from the discussion above. *(b)* Since $\overline{R}_i$ is strictly

convex, any interior pair of cost reductions x_{ij} and x_{ih} is dominated by the two

boundary profiles lying on the sum preserving locus passing through them. Since the

best response is well defined, it must involve a boundary profile of cost reductions. ∎

The previous lemma allows to separate the analysis of stage two in two cases. This

is done in the following section.

III.a. SYMMETRIC COST REDUCTIONS. Assume $\gamma > 2$. Then, from

Lemma 1, in stage two, firm 1 maximizes the sum of its divisions' profits net of

the investment costs under the constraint of identical cost reductions, taking the

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w-isosum locus of the form $x_{ij} + \varepsilon$ and $x_{ij} - \varepsilon$, would give parent firm i the same profit, for any ε.

[9]Hence $\gamma < 2$ is necessary for the optimal allocation to involve asymmetric cost reductions. I do not consider the zero measure case $\gamma = 2$, where R_i is flat along any sum preserving locus.

investment decision of firm 2 as given. The latter amounts to solving

$$\max_{x_1 \,\le\, m} \left\{ R_1\left(x_1, X_2\right) = \delta_1 \left(\frac{(\delta_2+1)(a-m+x_1) - \sum\limits_{j=1}^{\delta_2} \theta_{2j}}{\delta_1+\delta_2+1} \right)^2 - \delta_1 \frac{\gamma(x_1)^2}{2} \right\}, \qquad (3)$$

where $X_2 = \sum\limits_{j=1}^{\delta_2} x_{2j}$. Let $x_1^*\left(X_2\right) = \frac{2(a-m-X_2)(\delta_2+1)}{\gamma(1+\delta_1+\delta_2)^2 - 2(1+\delta_2)^2}$ denote the unique root of $\frac{\partial R_1(x_1,\, X_2)}{\partial x_1}$, and let $\widehat{x_1}\left(X_2\right) = \max\left\{0\ ,\ x_1^*\left(X_2\right)\right\}$. Since R_1 is strictly concave,[10] the reaction function of firm 1 in the investment stage is continuous and single valued and is given by

$$r_1\left(X_2\right) = \arg\max\left\{R_1\left(x_1, X_2\right): \ x_1 \le m\right\} = \min\left\{\widehat{x_1}\left(X_2\right)\ ,\ m\right\}, \qquad (4)$$

and analogously for firm 2. From (4) one obtains that stage two has a unique and symmetric equilibrium $x_i^N\left(\delta_i, \delta_z\right) = \frac{2(a-m)(\delta_z+1)}{\gamma(1+\delta_i+\delta_z)^2 - 2(1+\delta_z)}$, $i = 1, 2,\ i \ne z$.[11]

I can finally consider the divisionalization decision of firm 1 which, in stage 1, selects δ_1 conditional on the stage-two equilibrium $\left(x_1^N, x_2^N\right)$, in order to maximize its profit, taking δ_2 as given. Substituting x^N in R_i defined in (3) allows to write the maximization problem at hand as

$$\max_{\delta_1} \left\{ \pi_1\left(\delta_1, \delta_2\right) = \sum_{j=1}^{\delta_1} R_1\left(\delta_1, \delta_2\right) - c\delta_1 \right\}, \qquad (5)$$

which has an analogous counterpart for firm 2. Since the Cournot equilibrium of the market stage and the Nash equilibrium of the investment stage are both unique, every

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[10] In fact $\frac{\partial^2 R_1}{\partial x_1^2} < 0$ if and only if $\gamma > 2\frac{\delta_2^2+2\delta_2+1}{(\delta_1+\delta_2+1)^2}$. Concavity follows from $\frac{\delta_2^2+2\delta_2+1}{(\delta_1+\delta_2+1)^2} < 1$.

[11] Uniqueness follows in the usual way from a contraction argument, given the linearity of $r\left(.\right)$ and the fact that $r_1' r_2' < 1$ (globally) can be checked to hold whenever $R\left(x\right)$ is concave. Further, it is interesting to compare (1) and (4). This shows that at the equilibrium both parent firms are active in the product market. Finally, it can be shown that a sufficient condition for x^N to be interior, *i.e.* for $x^N \le m$, is $\gamma > \frac{4}{9}\frac{a}{m}$, which is assumed in the remainder of this section- see Proposition 1 below.

Nash equilibrium of the game with the above payoff leads to a SPNE of the whole three-stage game. So, for the sake of brevity, in the remainder of the section I will mention the Nash equilibrium of the divisionalization stage only. For the framework discussed here, the main result is summarized by the following proposition.

Proposition 1: Assume $\gamma > \max\left\{2, \frac{4}{9}\frac{a}{m}\right\}$. (a) The divisionalization stage has a unique symmetric Nash equilibrium where each parent firm creates the same number of divisions δ^e. (b) There exist $\gamma^* > 2$, and a decreasing function $w(\gamma)$, such that if and only if either $\gamma < \gamma^*$ or $\gamma \geq \gamma^*$ and $\frac{(a-m)^2}{c} < w(\gamma)$, then $\delta^e = 1$.

A proof is given in the appendix, together with the definition of both γ^* and $w(\gamma)$. Here no divisionalization means that in equilibrium each parent firm brings to the product market just one unit.[12] The latter occurs if both the extent of the market and the marginal cost of the investment are small relative to the cost of divisionalization, i.e. $\frac{(a-m)^2}{c} < w(\gamma)$. In fact a very small γ, i.e. $\gamma < \gamma^*$, suffices independently of c. This is of some interest since it departs from the analogous result for the standard case in the mentioned paper by Baye *et al.,* according to which firms create multiple divisions in equilibrium if divisionalization costs are low enough. In that case, when c tends to zero, the equilibrium number of divisions increases without bound and the equilibrium output converges to that of perfect competition. Now, note that for the model here Cournot aggregate output increases in δ[13] and that x_i^N tends to zero

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[12] In this section I am treating the number of divisions as a continuous variable. As shown in the proof, the result depends on the first derivative of a parent firm's payoff with respect to the number of divisions, evaluated along the diagonal, being negative at $\delta = 1$. See Baye *et al.* This is an equilibrium since staying out of the market would give the parent firms zero profit, while overall Cournot profit with optimal investment choices and two firms is positive.

[13] Notice that unlike a standard one-shot Cournot game as analyzed by Amir and Lambson (2000), here increasing the number of divisions affects the equilibrium investment choices of parent companies and in turn their units' marginal costs. Total output as a function of k is written $kq^c(k,x) = k\frac{a-m+x}{k+1}$. Substituting x^N for x and taking derivatives with respect to k yields $\frac{dkq^c(k)}{dn} = \frac{\gamma(a-m)\left(n^2+(\gamma-2)(n+1)^2\right)}{\left(\gamma(n+1)^2-(n+2)\right)^2} > 0$.

as δ tends to infinity. Thus convergence to perfect competition, that is to marginal cost pricing, is an issue even in the presence of fixed (investment) costs. However, following the same argument reported in the proof of Proposition 1, one establishes that with zero divisionalization costs the equilibrium number of divisions remains finite, provided γ is not too large.[14] So the scope for divisionalization reduces with the ease of obtaining a cost reduction in stage two and vanishes when γ is sufficiently close to 2, which is the lower bound for the present analysis.

III.b. ASYMMETRIC COST REDUCTIONS. Assume $\gamma < 2$. Thus, from Lemma 1, the best response of parent firm i in stage two involves a cost reduction profile lying on the boundary of the feasible set $[0, m]^{\delta_i}$. The latter is consistent with a symmetric allocation only when each division is given the maximal cost reduction m, otherwise it implies an asymmetric distribution of cost reductions over units. If one looks at γ as an index of decreasing returns to investment, an explanation is that, when this index is low, the inefficiency due to undertaking different levels of the investment across units is offset by the gain from increasing the market share going to the more efficient division under Cournot competition.

Now, for the case at hand, the best response in the investment phase is conditional on (δ_1, δ_2). This gives room for a number of possible solutions, one for each candidate equilibrium pair of divisions. The analysis of this case is interesting in itself and would be relevant at least for the literature on asymmetric joint ventures, but is not immediate[15] and far beyond the scope of this paper. Here, for simplicity, I impose

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[14] One can show that the first derivative of a parent firm's payoff (relative to δ) evaluated along the diagonal tends to zero from below when c tends to zero and δ to infinite, provided $(\gamma^2 - 3\gamma - 1) < 0$, *i.e.* $\gamma < 3.3028$. This is sufficient for forming an infinite number of divisions never being a best response. As an instance, setting $c = 0$, the Nash equilibrium involves no more than 5 divisions per firm if $\gamma < 5.7044$, than 4 if $\gamma < 4.7443$, than 3 if $\gamma < 3.8038$, and than 2 if $\gamma < 2.9023$.

[15] Consider for instance the spectum of solutions for the two firm case in the cartel problem addressed by d'Aspremont and Jacquemin, as studied in Amir and Tesoriere (2007 b).

an upper bound of two on the number of divisions available to a parent company. Moreover, I restrict the study to a region of parameters within which the stage-two best response calls for asymmetric cost reductions over divisions and, in particular, for a zero cost reduction level for one of them. The result in this subsection is the following proposition, where use is made of the definitions: $w_1 \equiv 3.6667$, and $w_2(\gamma) \equiv \frac{4}{3}(2\gamma - 1)$.

Proposition 2: Assume $\frac{a}{m} < \min\{w_1, w_2(\gamma)\}$ and $\gamma \in (1.6, 2)$. In addition, assume that each parent firm can create at most two divisions. (a) The best response of parent firm i in stage two to a cost reduction allocation $\mathbf{x}_z$ of parent firm z, conditional on $\delta_i = 2$, involves a zero cost reduction for one and only one division, independently of δ_z. (b) The divisionalization stage has a unique and symmetric Nash equilibrium where each parent firm creates just one division.

A proof is given in the appendix. While Proposition 1 leaves open the possibility of other (possibly asymmetric) equilibria, in the case of interest no divisionalization is the only observable outcome, whatever the value of c is. This might not be surprising, given that γ is taken to be smaller than in Section III.a. However it is interesting, since it emphasizes the effect of investment in this model. Unlike the case of symmetric cost reductions, under the assumptions of Proposition 2, deviating from the symmetric equilibrium, *i.e.* creating a new unit, reduces the overall investment of the parent firm.[16] This is mainly because the latter internalizes the action of investment on the new division's profit, that is negative, so that starting a new division induces the parent firm to rise the marginal cost of the existing one. This reduces efficiency relative to the other company and inhibits divisionalization.

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[16] Namely, with symmetric cost reductions, $2x^N(2,1) - x^N(1,1) = 2\frac{(a-m)(\gamma-2)}{(9\gamma-4)(4\gamma-1)}$, while with asymmetric allocations, $x^N(1,1) - x^N(2,1) = 2\frac{(a-m)(7\gamma-10)}{(9\gamma-4)(8\gamma-7)}$. See the proof of Proposition 2 for details.

IV. SECOND-BEST WELFARE ANALYSIS. First-best welfare maximization mandates that price should be equal to marginal cost. Since in this framework there are not external effects from the investment activity, the efficient way to obtain a given cost reduction requires that the necessary investment is undertaken by one division only. Thus, divisionalization is wasteful, since it would saddle society with the charge of duplicating the costs of both investment and divisionalization itself. There should be only one firm active in the market, doing the optimal amount of investment $x^w = \min\left\{\frac{a-m}{\gamma-1}, m\right\}$, and producing output $Q^w = a - m - x^w$.[17] As widely recognized, putting this into practice presents the major problems of restricting access to the market to just one firm and imposing the marginal-cost rule. A second-best criterion that considers welfare conditional on the Cournot equilibrium in the product market is usually accepted as a more workable standard. This section develops a second-best welfare analysis of the equilibrium for the case $\gamma > 2$ discussed in Section III.a, under the additional restriction of having at least two firms active in the market. The latter represents the appropriate benchmark for the no divisionalization scenario.

As argued above, when $\gamma > 2$, deviating from an interior symmetric profile of investments along the sum preserving locus of cost reductions is not privately profitable.[18] Since along this locus the sum of firms' marginal costs is kept constant, given Cournot competition in the market stage, so is aggregate quantity[19] and hence consumer surplus. It follows that deviating from the symmetric profile is socially undesirable too. So, in line with Lemma 1, second-best welfare maximization requires

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[17] The cost of x^w and c are recuperated by means of transfers.

[18] From (2) is Section II it is clear that such deviation would reduce industry profits also if the two divisions of interest were headed by two different parent companies.

[19] See Bergstrom and Varian (1985).

identical cost reductions over divisions. Welfare is then written

$$W\left(k,x\right) = \int_{0}^{kq^{c}(k,x)} \left(P\left(s\right) - m + x\right)ds - k\left(\frac{\gamma}{2}x^{2} + c\right), \tag{6}$$

where k is the total number of divisions, $q^{c}\left(k,x\right)$ is the symmetric $k-$division Cournot equilibrium output produced by each unit, and x is the amount of cost reduction going to each division. I first consider a given market structure, that is a given number of divisions k, and address the issue of determining the socially optimal amount of cost reducing investment $x_{w}\left(k\right)$. The first point in this section is that, given Cournot competition in the market stage, the social value of an extra unit of cost reduction is proportional to that assigned to it by a parent company when both firms form the same number of divisions and each division is given the same level of cost reduction. This means that for any k, the investment undertaken by the industry at the equilibrium of the second stage is socially optimal in terms of second-best welfare criterion. However, the previous statement does not imply that the industry performance is socially optimal when the number of divisions is endogenously determined at the equilibrium of the three-stage game, in view of the fact that x_{w} and x^{N} depend on the socially optimal number of divisions k_{w} and on the equilibrium number of divisions δ^{e} active in the product market, respectively. Since there is no reason for $k_{w} = 2\delta^{e}$ to hold, a complete welfare analysis would be in order. Here I limit the study to assessing the welfare properties of the no divisionalization scenario described in Proposition 1 above. Unlike under the fist-best standard, here society might benefit from divisionalization since the equilibrium output $kq^{c}\left(k,x\right)$ is increasing in k. This gain must be offset against the action of k on total profits through both the reduction of $x_{w}\left(k\right)$ and the increase of the total cost of divisionalization c it generates. Let

14

(k_w, x_w) denote the (unique) socially optimal pair of number of divisions and cost reduction level per division, in terms of the mentioned second-best criterion. This pair is determined jointly in order to maximize $W(k, x)$. Recall, for clarity, that $x^N(k)$ is the second-stage equilibrium cost reduction level when the total number of divisions is k, and δ^e is the number of divisions established by each parent firm at the equilibrium of the whole game. The second point in the present analysis is that the no divisionalization scenario is sustainable in equilibrium only if it is socially optimal. The following proposition summarizes these results.

Proposition 3: (a) *Assume* $\gamma > 2$. *For any given* k, $x^N(k) = x_w(k)$. (b) *Assume* $\gamma > \max\left\{2, \frac{4}{9}\frac{a}{m}\right\}$. $\delta^e = 1$ *only if* $k_w = 2$.

A proof is given in the appendix. Notwithstanding the relevance of part (b) of the proposition, social and private incentives towards divisionalization generally are not the same. Specifically, as underlined in the proof, there are parameter regions within which $k_w = 2$ but there is divisionalization in equilibrium. Since $x^N(k)$ is decreasing, at least in these cases, there should be less units operating in the product market, at a lower marginal cost each. However, part (a) portrays a suitable scenario for policy issues, suggesting that regulating the number of divisions for each company in an industry is sufficient to induce the socially optimal outcome independently of any concern about firms investment decisions. I will briefly return on this in the next section.

V. CONCLUDING REMARKS. This section concludes the paper with two remarks. The first relates to R&D Joint Ventures (RJV) by means of a comparison between the welfare analysis outlined by Proposition 3 and that provided in the literature on that subject. A RJV is a group of firms which coordinate their cost

reducing R&D decisions before competing in the product market, in order to maximize joint non cooperative profit. A primary interest for the mentioned studies on RJV is the action of precompetitive collusion on social welfare. So these studies have usually compared the scenario where the n firms active in an industry form a RJV with that involving noncooperative behavior for the whole of the game. The comparison is very often concerned with the case of external effects flowing from private R&D (spillovers). In this regard, Suzumura (1992) argues that if spillovers are sufficiently large, the fully noncooperative equilibrium R&D level is socially insufficient in terms of second-best criterion, while the RJV equilibrium R&D level is socially insufficient for any spillover rate, and that, with zero spillover, the non cooperative R&D equilibrium level is socially excessive if there is a sufficiently large number of firms in the industry.

A minor comment is that, for the linear case discussed in this article, non cooperative R&D in a duopoly with zero spillover is second-best efficient by Proposition 3.[20] A more significant annotation is that while full R&D cartelization of the industry performs poorly independently of the environment, for any given market structure, competing divisions undertake the optimal investment level. This candidates divisionalization as a preferable system for organizing R&D. A practicable extension of this research would consider technological disclosure in the form of voluntary or involuntary spillovers in order to appraise R&D divisionalization as a way to internalize the efficiency effect of sharing knowledge.

The second remark relates to the mentioned link between divisionalization and horizontal mergers. In light of this connection, and given Propositions 1 and 2, one would expect that the perspective of undertaking cost reducing investment should

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[20] This encompasses the case depicted by d'Aspremont and Jacquemin. As far as I know this property is unnoticed in the existing literature.

alter firms' incentives in favor of mergers. This would not rely on efficiency effects as far as marginal costs are constant and there are decreasing returns to investment. Consider the example made by Salant *et al.* to illustrate the so called and well known *80% rule*, according to which mergers not involving more than this share of industry output are not profitable: there is an industry where k Cournot competitors share the same technology and face the same demand as in the framework analyzed throughout this paper. If h of these firms were to merge, the number of firms would become $k - h + 1$. Prior to merging the aggregate profit of the insiders would be $h\pi(k) = h\frac{(a-m)^2}{(k+1)^2}$, while post merger per firm profit would be $\pi(k - h + 1) = \frac{(a-m)^2}{(k-h+2)^2}$. Merger is profitable for the insiders if and only if $\pi(k - h + 1) > h\pi(k)$, which is equivalent to $k < \frac{\sqrt{h}(2-h)-1}{1-\sqrt{h}}$. The latter says that for the merger to happen the number of firms active prior to the merger must be small relative to the number of insiders. For instance, if $k = 5$, then no merger involving less than four firms is profitable. Consider now the three-stage game where, in stage one, an exogenous number of firms h decides whether or not to merge, in stage two, the resultant number of firms decide upon a cost reduction investment in a non cooperative way and in the same manner as a parent firm in this paper, and, in stage three, these firms compete à la Cournot with their actual marginal costs. Leaving the details apart,[21] if the merger takes place, given equilibrium investment decisions, then the insider obtain $\pi_*(k - h + 1) = \frac{(a-m)^2\gamma\left(\gamma(k-h+2)^2-2(k-h+1)^2\right)}{\left(\gamma(k-h+2)^2-2(k-h+1)\right)^2}$, while if they do not merge their aggregate profit is $h\pi_*(k) = h\frac{(a-m)^2\gamma\left(\gamma(1+k)^2-2k^2\right)}{\left(\gamma(1+k)^2-2k\right)^2}$. They decide to merge if and only if $\pi_*(k - h + 1) > h\pi_*(k)$. Now, the latter is a much weaker requirement than the analogous condition for the case without investment, provided γ is small. The explanation is analogous to

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[21]I assume $\gamma > \frac{2k^2}{(k+1)^2}$.

that used for Propositions 1 and 2 above. When the marginal cost of the investment reduces, the anticompetitive effect of a merger increases. For instance, if $k = 5$ and $\gamma = 2$ then the merger is profitable if $h \geq 3$, while if $\gamma = 1.5$ it suffices $h \geq 2$. Along the same line, if $k = 10$ and $\gamma = 1.7$, then any merger is profitable.

Appendix

Proof of Proposition 1.

Assume $\gamma > \max\left\{2, \frac{4}{9}\frac{a}{m}\right\}$. Let

$$\gamma_1 = \frac{2\delta^3 + 3\delta^2 + \delta\sqrt{(4\delta^4 + 12\delta^3 + 21\delta^2 + 10\delta + 1)} + 5\delta + 2}{1 + 4\delta + 4\delta^2},$$

$$\gamma_2 = 2\frac{3\delta^3 + 4\delta^2 + \delta\sqrt{(5 + 26\delta + 48\delta^2 + 34\delta^3 + 13\delta^4)} + 6\delta + 2}{4\delta^3 + 12\delta^2 + 9\delta + 2},$$

and $\gamma_3 = \frac{4\delta^3 + 6\delta^2 + 17\delta + \delta\sqrt{(13 + 60\delta + 124\delta^2 + 48\delta^3 + 16\delta^4)} + 6}{3(1 + 2\delta)^2}.$

(a) I first show that there exists a symmetric Nash equilibrium (SNE) (δ^e, δ^e) of the game with payoff $\pi_i(\delta_i, \delta_z)$, $i = 1, 2$, $i \neq z$, as defined in (5). Treating δ as a real number, let $\sigma(\delta) = \left.\frac{\partial \pi_i(\delta_i, \delta_z)}{\partial \delta_i}\right|_{\delta_i = \delta_z = \delta}$. Since $\sigma(0) = \frac{\gamma(a-m)^2 + c(2-\gamma)}{\gamma - 2} > 0$, if a SNE exists then it must satisfy the first order condition $\sigma(\delta^e) = 0$. Since $\sigma(\delta)$ is continuous, and $\lim_{\delta \to \infty} \sigma(\delta) = -c$, there exists at least one δ^* such that $\sigma(\delta^*) = 0$. The latter condition is written

$$(a - m)^2\gamma\,\Theta(\gamma, \delta) = c\left(\gamma(1 + 2\delta)^2 - 2(\delta + 1)\right)^3, \tag{A1}$$

where

$$\Theta(\gamma, \delta) = \gamma^2\left(1 + 6\delta + 12\delta^2 + 8\delta^3\right) - \gamma\left(8\delta^4 + 16\delta^3 + 26\delta^2 + 18\delta + 4\right) +$$
$$4\left(1 + \delta^3 + 3\delta^2 + 3\delta\right).$$

18

$(A1)$ implies $\Theta\left(\gamma,\delta\right) > 0$ which is equivalent to $\gamma > \gamma_1$. Let now $\Delta\left(\delta\right) = \left.\frac{\partial^2 \pi_i(\delta_i,\delta_z)}{\partial \delta_i^2}\right|_{\delta_i=\delta_z=\delta}$. More specifically $\Delta\left(\delta\right) = -2\frac{(a-m)^2\gamma^2\Phi(\gamma,\delta)}{(4\gamma\delta^2+4\gamma\delta+\gamma-2\delta-2)^4}$, where

$$\Phi\left(\gamma,\delta\right) = \gamma^2\left(2+16\delta^5+64\delta^4+88\delta^3+56\delta^2+17\delta\right)$$

$$-\gamma\left(8+48\delta^5+112\delta^4+172\delta^3+144\delta^2+56\delta\right)+8+44\delta+72\delta^2+32\delta^3-8\delta^4-16\delta^5.$$

Since $\Delta\left(\delta\right) < 0$ if and only if $\gamma > \gamma_2$, since $\gamma_1 > \gamma_2$ if $\delta > 1$, and $\gamma_2 < 2$ if $\delta < 1$, it holds that under the first order condition $(A1)$, $\gamma > \max\left\{\gamma_1,\gamma_2\right\}$, and the second order condition is satisfied. So $\left(\delta^*,\delta^*\right)$ is a SNE.

To show uniqueness, I demonstrate that $\sigma\left(\delta\right)$ has negative slope whenever $\sigma\left(\delta\right) = 0$, so that the latter can occur at most once. Let $\Xi\left(\delta\right) = \frac{\partial\sigma(\delta)}{\partial\delta}$. Since

$$\Xi\left(\delta\right) =$$

$$\frac{2(a-m)^2\gamma\left(\gamma^2\left(12\delta^2+12\delta+3\right)-\gamma\left(16\delta^3+24\delta^2+26\delta+9\right)+6\delta^2+12\delta+6\right)-3c\left(\gamma(1+2\delta)^2-2\delta-2\right)^2(4\gamma(1+2\delta)-2)}{\left(\gamma(1+2\delta)^2-2\delta-2\right)^3}$$

$$-3\sigma\left(\delta\right)\frac{4\gamma(1+2\delta)-2}{\gamma(1+2\delta)^2-2\delta-2},$$

and since $\sigma\left(\delta\right) = 0$ implies

$$3c\left(\gamma\left(1+2\delta\right)^2-2\delta-2\right)^2\left(4\gamma\left(1+2\delta\right)-2\right) =$$

$$6\left(2\gamma\left(1+2\delta\right)-1\right)\left(a-m\right)^2\gamma\frac{\left(6\delta+8\delta^3+12\delta^2+1\right)\gamma^2-\left(26\delta^2+4+16\delta^3+8\delta^4+18\delta\right)\gamma+12\delta+4\delta^3+12\delta^2+4}{\left(\gamma(1+2\delta)^2-2(\delta+1)\right)},$$

it follows that

$$\Xi\left(\delta\right)|_{\sigma(\delta)=0} = -2\left(a-m\right)^2\gamma^2\times$$

$$\frac{\gamma^2\left(48\delta^4+96\delta^3+72\delta^2+24\delta+3\right)-\gamma\left(32\delta^5+80\delta^4+192\delta^3+196\delta^2+82\delta+12\right)+\left(16\delta^4+64\delta^3+116\delta^2+68\delta+12\right)}{\left(\gamma(1+2\delta)^2-2(\delta+1)\right)^4}.$$

Since the numerator of the right hand side of the previous identity is positive if and only if $\gamma > \gamma_3$, since $\gamma_1 > \gamma_3$ if $\delta > 1$, and $\gamma_3 < 2$ if $\delta < 1$, it holds that under the first order condition $(A1)$, $\gamma > \max\left\{\gamma_1,\gamma_3\right\}$ and $\Xi\left(\delta\right)|_{\sigma(\delta)=0} < 0$. So there exists a

19

unique δ^* such that $\sigma(\delta^*) = 0$, so that there exists a unique SNE for the game with payoff as in (5).

(b) Let $\gamma^* = 2.1031$, and $w(\gamma) = \frac{(9\gamma-4)^3}{\gamma(3\gamma(9\gamma-4)+2(16-30\gamma))}$. $w(\gamma)$ is decreasing for $\gamma > \gamma^*$. Consider $\sigma(\delta)$ defined in (a) above and note that

$$\sigma(1) = \frac{(a-m)^2\gamma(3\gamma(9\gamma-4)+2(16-30\gamma))-c(9\gamma-4)^3}{(9\gamma-4)^3}.$$

The result follows from the uniqueness of the SNE, the fact that $\sigma(0) > 0$, and the fact that $\sigma(1) < 0$ if and only if either $\gamma < \gamma^*$ or $\gamma \geq \gamma^*$ and $\frac{(a-m)^2}{c} < w(\gamma)$. ∎

Proof of Proposition 2.

Let $w_1 = 3.6667$ and $w_2(\gamma) = \frac{4}{3}(2\gamma-1)$. Assume $\gamma \in (1.6, 2)$, $\frac{a}{m} < \min\{w_1, w_2(\gamma)\}$, and that each parent firm can create at most two divisions.

(a) Consider parent firm 1 and focus on the stage-two investment problem for the case $\delta_1 = 2$. From Lemma 1, the best response to a cost reduction allocation $\mathbf{x}_2$ involves a cost reduction profile $\mathbf{x}_1$ lying on the boundary of the feasible set $[0, m]^2$. Let now $\widehat{\pi}_1(x, X_2)$ denote the second stage payoff for parent firm 1 when its first division is given the maximal cost reduction m and the second one is given the cost reduction level x. Specifically:

$$\widehat{\pi}_1(x, X_2) =$$

$$\left(\frac{ka-(a-m+x)-\delta_2(a-m)-X_2}{k+1}\right)^2 + \left(\frac{k(a-m+x)-a-\delta_2(a-m)(a-m)-X_2}{k+1}\right)^2 - \frac{\gamma}{2}\left(m^2 + x^2\right),$$

where $X_2 = \sum_{j=1}^{\delta_2} x_{2j}$. Note that $\frac{\partial\widehat{\pi}_1(x,X_2)}{\partial x}\Big|_{x=0} = 2\frac{(\delta_2(a-m)+X_2)(1-k)-2ka+(a-m)\left(1+k^2\right)}{(k+1)^2} < 0$, and $\frac{\partial^2\widehat{\pi}_1(x,X_2)}{\partial x^2} = 2\frac{1+k^2}{(k+1)^2} - \gamma < 0$, for the parameters of interest. So giving the second division a positive cost reduction is never a best response for parent firm 1 (and analogously for parent firm 2).

20

(b) When making a divisionalization decision at the first stage, either parent company considers its own payoff conditional on the equilibrium in stage two for the divisionalization scenario that would obtain, taking the first-stage decision of the rival as given. I first consider the case where parent firm i has one division and parent firm z has two divisions. At the equilibrium of stage two, $x_i(1, 2) = 3\frac{a-m}{8\gamma-7}$, $x_{z1}(1, 2) = 2\frac{a-m}{8\gamma-7}$, and $x_{z2}(2, 1) = 0$, the latter due to part (a) above. Equilibrium profits net of divisionalization costs are $\pi_i(1, 2) = \frac{\gamma(a-m)^2(8\gamma-9)}{2(8\gamma-7)^2} - c$, and $\pi_z(2, 1) = 2\frac{(5+4\gamma^2-9\gamma)(a-m)^2}{(8\gamma-7)^2} - 2c$. Second, I consider the case where both parent firms bring to the market one division only. The equilibrium of stage two is symmetric with each division obtaining $x_i(1, 1) = 4\frac{a-m}{9\gamma-4}$. Profit net of the cost of divisionalization is $\pi_i(1, 1) = \gamma\frac{(a-m)^2(9\gamma-8)}{(9\gamma-4)^2} - c$. I finally consider the case where both parent firms bring to the market two divisions. The equilibrium of stage two is symmetric with the two divisions of firm i, obtaining $x_{i1}(2, 2) = 6\frac{a-m}{25\gamma-28}$, and $x_{i2}(2, 2) = 0$, respectively, the latter due to part (a) above. Profit, net of the cost of divisionalization for parent firm i is $\pi_i(2, 2) = 2\left(\frac{(25\gamma^2-59\gamma+34)(a-m)^2}{(25\gamma-28)^2} - c\right)$. All the above holds for $i = 1, 2$, $i \neq z$. In addition, for the parameters of interest, all the equilibria above are stable under best reply dynamics, unique, and interior. For the parameters at hand, it holds that $\pi_i(1, 2) > \pi_i(2, 2)$, $\pi_i(2, 1) > \pi_i(2, 2)$, $\pi_i(1, 1) > \pi_i(1, 2)$, and $\pi_i(1, 1) > \pi_i(2, 1)$. So the divisionalization stage has a unique and symmetric Nash equilibrium where each parent company sets $\delta^e = 1$ ∎

Proof of Proposition 3.

(a) Assume $\gamma > 2$. Let $x^N(k) = \min\left\{\frac{(a-m)(k+2)}{\gamma(1+k)^2-(k+2)}, m\right\}$ define the cost reduction obtained by one unit at the Nash equilibrium of the investment stage when the total number of divisions is k. Also let $x_w(k)$ be the socially optimal cost reduction

level given a fixed number of divisions k, in terms of second-best welfare criterion. Finally, let other subscripts denote partial derivatives. $x_w(k)$ solves $\max\limits_{x \leq m} W(k,x) = \int_0^{k\frac{a-m+x}{k+1}} (a-t-m+x)\, dt - k\left(c + \frac{\gamma}{2}x^2\right)$. Since $W_{xx} = k\frac{(k+2)-\gamma(1+k)^2}{(k+1)^2} < 0$, $x_w(k)$ $= \min\{x(n), m\}$, where $x(n)$ is the unique root of W_x. Since $x(n) = \frac{(a-m)(k+2)}{\gamma(1+k)^2-(k+2)}$, $x_w(k) = x^N(k)$.

(b) Assume $\gamma > \max\left\{2, \frac{4}{9}\frac{a}{m}\right\}$. Let

$$g(\gamma) = \left(12\gamma(1-2\gamma) + 8\left(1+\gamma^3\right) + 4\sqrt{(4\gamma+1)}\,(\gamma-2)\right)^{\frac{1}{3}}.$$

Also let $\gamma^* = 2.1031$ and $w(\gamma) = \frac{(9\gamma-4)^3}{\gamma(3\gamma(9\gamma-4)+2(16-30\gamma))}$. Finally, let (k_w, x_w) denote the socially optimal number of divisions and the socially optimal level of cost reduction for each division, respectively, in terms of second-best welfare criterion, that is, given Cournot competition in the product market and $k \geq 2$. (k_w, x_w) solves $\max\limits_{x \leq m,\, k \geq 2}$ $W(k,x) = \int_0^{k\frac{a-m+x}{k+1}} (a-t-m+x)\, dt - k\left(c + \frac{\gamma}{2}x^2\right)$. For the parameters of interest, $x_w(k) = x(n)$ defined in part (a) above, and the constraint on x is not binding. Treating k as a real number, let $\overline{W}_k(k)$ denote $W_k(k, x_w(k)) = \frac{\gamma(a-m)^2\left(2(\gamma-2)(k+1)-k^2\right)}{2\left(\gamma(k+1)^2-(k+2)\right)^2} - c$. Since $\overline{W}_k(k)$ is continuous, since $\overline{W}_k(0) = \frac{\gamma(a-m)^2-c(\gamma-2)}{\gamma-2} > 0$, and $\lim\limits_{k \to \infty}$ $\overline{W}_k(k) = -c$, there exists at least one $k^* > 0$ such that $\overline{W}_k(k^*) = 0$. Consider now $\frac{d\overline{W}_k(k)}{dk} = \gamma^2(a-m)^2\frac{3(2-\gamma)(k+1)^2+k^3}{\left(\gamma(k+1)^2-(k+2)\right)^3}$. The latter has a unique root k^{**}, with $\frac{d\overline{W}_k(k)}{dk} \lesseqgtr 0 \Leftrightarrow k \lesseqgtr k^{**}$. So $\overline{W}_k(k) \uparrow c$ as $k \to \infty$ and there is only one such k^*. Let now $|H| = n\gamma^2(a-m)^2\frac{3\left(\gamma(k+1)^2-k-2\right)-k(k+3)^2}{(k+1)^2\left(\gamma(k+1)^2-k-2\right)^2}$ denote the determinant of the Hessian of $W(k,x)$, evaluated at $x = x_w(k)$ defined above. $|H|$ has unique root $k^{***}(\gamma) = \frac{1}{2}g(x) + \frac{2(\gamma^2-2\gamma)}{g(x)} + \gamma - 2$. In addition, $k^{***}(\gamma) > 0$ for $\gamma > 2$ and $|H|$ $\gtreqless 0 \Leftrightarrow k \lesseqgtr k^{***}$. Note now that at k^* it must be $\left(2(\gamma-2)(k+1) - k^2\right) > 0$. The latter is equivalent to $k < \left(\gamma - 2 + \sqrt{(-2\gamma+\gamma^2)}\right)$. Since $\left(\gamma - 2 + \sqrt{(-2\gamma+\gamma^2)}\right) - k^{***}(\gamma) = \frac{g(\gamma)\left(2\sqrt{(\gamma^2-2\gamma)}-g(\gamma)\right)+4\gamma(2-\gamma)}{2g(\gamma)}$, and since the latter is negative for $\gamma > 2$,

$|H| > 0$ when $k = k^*$. Finally, $W_{kk} = -3\frac{(a-m+x)^2}{(n+1)^4} < 0$. So the Hessian of $W(k, x)$ is negative definite at (k^*, x^*) and the latter is the unique solution to the maximization problem at hand. In the case of no divisionalization, $k = 2$. Now, $\operatorname{sign}\overline{W}_k(2) = \operatorname{sign}\left(\frac{(a-m)^2}{c}\frac{(3\gamma-8)\gamma}{(9\gamma-4)^2} - 1\right)$. The right hand side of the previous identity is negative if and only if either $\gamma < 2.6667$, or $\frac{(a-m)^2}{c} < \frac{(9\gamma-4)^2}{(3\gamma-8)\gamma}$. Recall from part (b) of the proof of Proposition 1 that there is not divisionalization in equilibrium if and only if $\gamma < \gamma^*$ or $\gamma \geq \gamma^*$ and $\frac{(a-m)^2}{c} < w(\gamma)$. Since $\gamma^* < 2.66667$ and $w(\gamma) - \frac{(9\gamma-4)^2}{(3\gamma-8)\gamma}$ < 0 for $\gamma > 2.66667$, whenever each firm forms just one division in equilibrium, it holds $\overline{W}_k(2) < 0$. Since the solution of the second-best welfare maximization problem is unique and interior, and $\overline{W}_k(0) > 0$, in these cases, it must be $k^* = 2$. ∎

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