2008/76

Equilibrium models for the carbon leakage problem

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CORE DISCUSSION PAPER
2008/76

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December 2008

Abstract

Carbon leakage in this pape ris the phenomenon whereby Electricity Intensive Industries subject to harsh environmental standards move their activity or part of it to more environmentally lenient regions. Carbon leakage has been mentioned as a possible outcome of the EU Emission Trading Scheme. Different studies are underway to assess the reality of the phenomenon and to devise policies to mitigate its possible impact. One remedy, proposed by the Energy Intensive Industries is to combine free emission allowances with a pricing of electricity whereby energy émissions and transmission costs are bundled and sold on an average cost basis. The paper attempts to model this proposal.

We cast the problem in a spatial model of the power sector where generators can develop new capacities, the transmission system is organized on a flowgate basis, emission allowances are auctioned, except possibly for industries, and traded. The consumer market is decomposed in two segments. Industries purchase electricity according to some form of average cost price, the rest of the market is supplied at marginal cost. These equilibrium models are non convex. We present the models and discuss their properties. Companion papers report policy implications.

Keywords: carbon leakage, emission trading scheme, electricity, energy policies, equilibrum, complementarity.

JEL Classification: C61, L23, O14, O33

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This work has been financially supported by the Chair Lhoist Berghmans in Environmental Economics and Management and by the Italian project PRIN 2006, *Generalized monotonicity: models and applications*, whose national responsible is Elisabetta Allevi. The authors gratefully acknowledge Thierry Bréchet, Andreas Ehrenmann and Chefi Triki for their in depth comments and helpful advices.

This paper presents research results of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by the authors.

1 Introduction

This paper addresses a relatively unexplored subject in the literature of electricity restructuring. We consider the question of investing in new generation capacities in a market where sales are subject to two different price regimes. A fraction of the consumer market procures electricity through long-term contracts at average cost based price; the other fraction is supplied at marginal cost price. Physical arbitrage between the two market segments is impossible. Generators invest to satisfy both segments of the market. The problem is inspired by a persistent problem in the European restructured electricity market, which has recently become more acute with the introduction of carbon trade (Emission Trading Scheme or ETS). Large industrial consumers, in particular energy intensive industries (EIIs), desire long-term contracts at prices that are independent of short-term movements of the spot market (Businesseurope [1]). They object to procuring the bulk of their electricity at prices driven by power exchanges (PX). At the same time the whole history of the restructuring of the power sector has revealed the importance of the short-term market to send correct price signals and guarantee the good functioning of the market. It is thus essential to retain a significant fraction of the market operating on the basis of short-term prices. The introduction of carbon trading in Europe gave a new impetus to EIIs' demand. Carbon is an additional risk factor that affects both the price of allowances and of electricity. EIIs, which did not like the price volatility created by restructuring, find additional reason to complain because of this additional carbon driven price volatility.

The quest of the EIIs suggests an easy response. Financial instruments allow one to hedge electricity and carbon price risks, at least in principle. There is thus no need for long-term contracts directly negotiated with generators. But EIIs do not trust prices derived from power exchanges by virtue of the sole financial arbitrage. They do their own computations and come with costs for base load supplies that they compare to the prices that they get on the market. They conclude that they would be better off with a long term cost based contract. This raises the interesting methodological question as to why averages of short-term prices, such as determined on power exchanges, would differ from the average generation cost computed by EIIs. The old theory of peak load pricing in optimally adjusted generation systems suggests that they should not. But even removing this argument would not help; an investor cannot today secure a long-term financial contract to guarantee the price of its power supply because the market does not trade long-term risks. Even if possibly justifiable in theory because of equality between short and long term marginal costs, the sole recourse to the power exchange would not work in practice.

We treat the question with techniques directly inspired from standard electricity modelling that we adapt to accommodate the average cost prices demanded by EIIs. We extend existing models to tackle these dual markets and cast the extended model in an investment context. Investment models of restructured systems are still unusual and we believe that the one presented here is novel. Our treatment attempts to reflect some important features of the European situation: notwithstanding the claim for an internal electricity market, energy policies remain quite different among Member States in Europe, especially when it comes to nuclear investments. We thus cast the problem in a spatial set up where different zones are subject to different restrictions on generation. This raises the question of the geographic coverage of the average cost contracts demanded by EIIs: should they be zonal or regional (a single price for all zones)? Power Exchanges prices differ by the location of the PX, but EIIs have sometimes demanded a single electricity price throughout Europe. Also average cost demands a careful definition and verifiability of costs and these also depend on the location. We believe that these questions have not been addressed yet in the modeling of restructured electricity systems.

The paper is organized as follows. Section 2 provides a general description of the power market studied and the model implemented. Section 3 presents in details a reference model whose mathematical features are discussed in Section 4. Sections 5 and 6 are respectively devoted to the presentation of the average cost based models and their mathematical characteristics. Both Sections 3 and 5 model only the additional electricity cost induced by the ETS. A more general framework is presented in Section 7. In Section 8 we describe our results from a mathematical point of view and, finally, we report our conclusions in Section 9.

2 Market Description and Modeling Steps

We consider a spatial power market segmented into different zones linked by flowgates. Generators operate a set of technologies in each zone. There are two types of consumers in each zone, namely Energy Intensive Industries (EIIs) and other consumers (N-EIIs). They differ by the flexibility of their demand. EIIs can more or less easily relocate their activities in other parts of the world in response to changes of electricity and carbon prices: their electricity demand is thus more flexible than the one of other consumers. The grid is represented as a set of flowgates under the control of a single system operator (the current organization of the European grid is more primitive than this flowgate model). We adopt a standard technological representation of generation plants. Each technology is described by its investment and operations costs as well as by its conversion efficiency and emission factors. There are existing capacities, but generators can also invest in new capacities. Because of lack of an adequate technological information and relevant data, the demand sectors are represented by linear demand curves. Apart from some recent studies (see Hidalgo et al. [3], Reinaud [7], [8] and Szabò et al. [9]), little information is indeed available on the reaction of these sectors to carbon and electricity prices. We suppose throughout that agents are price takers. All these are standard assumptions in electricity systems modeling.

EIIs incur direct and indirect ETS driven costs. Electricity prices increase because they cumulate allowance and fuel costs: this is the indirect cost. EIIs also incur direct costs because of their emissions; they depend on the allowances that they receive free. We want to analyze both these indirect electricity and direct carbon costs. We begin with a reference model presented in Section 3. This model describes a perfectly competitive market where all agents maximize their benefits. Specifically, generators maximize the profits accruing from supplying the two consumer groups. Investors optimize the profits derived from investing in new plants and leasing existing and new capacities to generators. The TSO maximize the profits from selling flowgate services. Finally, consumers maximize their surpluses. We then modify this model by segmenting the market and introducing different pricing schemes for the two consumer groups. This new model is described in Section 5. It supposes that EIIs buy electricity at average cost thanks to special long-term contracts while N-EIIs still pay a marginal cost based electricity price. This implies a change of generators' behavior, which still maximize the profits from supplying N-EIIs, but minimize the cost of producing electricity for EIIs. We consider two average cost pricing systems: one applies a single regional price to all EII customers. The other applies a different average cost price to the EIIs in each zone. These two models focus on the sole power sector and assume an emission market restricted to the electricity sector without trade with the EIIs. We relax this simplifying assumption in the third part of the analysis presented in Section 7. We there assume that industries participate to the ETS and are also directly affected by the carbon cost due to their emissions. This makes the model more realistic, but implies some changes. We first slightly modify the emission constraint by including the EIIs. The most significant change concerns the EIIs' demand function. We assume that the industrial electricity consumption depends on two prices: the electricity price that can be either marginal or average cost based, and the carbon cost expressing the EIIs' position with respect to the emission market. The update of the ETS legislation indeed leaves it for later to decide whether EIIs would receive free allowances in the period 2013-2020 and hence what their position will be on that market. We model both the cases with and without free allowances for EIIs. In the first case, we suppose a benchmarking method where free allowances are granted proportionally to output (but not to emissions) to EIIs.

3 The Reference Model

The reference model concentrates on the sole indirect ETS induced cost that results from higher electricity prices. The model involves three commodity or service markets: energy, allowances and transmission. Generators, the TSO and two consumer groups operate on some or all of these markets. In order to simplify the discussion, the electricity demand is only decomposed into two time segments, peak and off peak. We assume that EIIs' consumption is constant over the year, that is in these two time segments. Prices are equal to the marginal cost. Investors can build new capacities in stage 0; they lease both new and old capacities to generators in stage 1. Because the model is formulated and solved by mathematical programming techniques it is rather flexible and can be extended easily.

3.1 Notation of the Reference Model

Following Stoft [6], we conduct the whole discussion on an hourly basis or in MWh. We thus express capacities in MWh and not in MW.

3.1.1 Notation

- **Sets**

$i \in I$	Zones;
$f \in F$	Generators;

$l \in L$	Flowgates;
$t \in T$	Time segments in stage 1;
$k \in K$	Technology;

- **Parameters**

$X_{f,i,k}$	Existing generation capacity of technology k run by generator f located in zone i (MWh);
$I_{f,i,k}$	Unitary investment/capacity costs of technology k run by generator f, in zone i (€/MWh);
$c_{f,i,k}$	Production costs of technology k owned by generator f in zone i (€/MWh);
τ_t	Duration in % of time segment t;
a_i^1	Intercept of EIIs' demand function in zone i (€/MWh);
b_i^1	Slope of EIIs' demand function in zone i (€/MWh^2);
$a_{i,t}^2$	Intercept of N-EIIs' demand function in zone i, time segment t (€/MWh);
$b_{i,t}^2$	Slope of N-EIIs' demand function in zone i, time segment t (€/MWh^2);
$Linecap_l$	Capacity of flowgate l (MWh);
$PTDF_{i,l}$	Flow in flowgate l due to a unit injection in zone i with corresponding withdrawal at the hub;
e_k	Emission factor by technology k (ton/MWh);
E	Emission cap in ton per hour.

- **Variables**:

$x_{f,i,k}$	New capacity of technology k used by generator f located in zone i (MWh);
$y_{f,i,k,t}$	Production of generator f by technology k, located in zone i, time segment t (MWh);
$P^1(d_i^1, i)$	EIIs' annual inverse demand function in zone i (€/MWh);
$P^2(d_{i,t}^2, i, t)$	N-EIIs' annual inverse demand function in zone i, time segment t (€/MWh);
d_i^1	EIIs' demand in zone i (MWh);

$d^2_{i,t}$	N-EIIs' demand in zone i, time segment t (MWh);
π_t	Electricity price at the hub in time segment t (€/MWh);
$p_{i,t}$	Marginal electricity price in zone i, time segment t (€/MWh);
$\mu^{+,-}_{l,t}$	Congestion rent of flowgate l, depending on flow direction $(+,-)$ and time segment t (€/MWh);
$\nu_{f,i,k,t}$	Marginal hourly value of capacity (scarcity rent) of technology k used by generator f in zone i, time segment t (€/MWh).

Note that some variables depend on time t and others do not. As argued before, we assume that EIIs' electricity consumption is constant over the year and hence their demand does not depend on time segment t. We express the inverted demand functions (price as function of quantities) in the form $P^2_{i,t}(d^2_{i,t}) = a^2_{i,t} - b^2_{i,t}d^2_{i,t}$ for N-EIIs and $P^1_i(d^1_i) = a^1_i - b^1_i d^1_i$ for EIIs.

3.2 Agents' Models

3.2.1 Transmission market and TSO's profit maximization.

We adopt a flowgate representation of the transmission system of the whole region and assume that the TSO maximizes the gains accruing from selling transmission services compatible with the network capacity. The $PTDF$ matrix determines the flow through flowgate l resulting from a unit injection in zone i and withdrawal at the hub. Network constraints impose that the flow in a flowgate does not exceed its capacity ($Linecap_l$). This must hold for every time segment. Assuming no losses, $\sum_k y^t_{i,k,t}$ and $(d^1_i + d^2_{i,t})$ are respectively the energy injection and withdrawal services. The TSO's maximization problem in each time segment t is then as follows:

$$\textbf{Max} \quad \sum_i p_{i,t} \cdot (d^1_i + d^2_{i,t} - \sum_{f,k} y_{f,i,k,t}) \qquad s.t. \tag{1}$$

$$-Linecap_l \leq \sum_i PTDF_{i,l}(\sum_{f,k} y_{f,i,k,t} - d^1_i - d^2_{i,t}) \leq Linecap_l \qquad (\mu^{\pm}_{l,t}) \quad \forall\, l,t \tag{2}$$

Note that $\mu^+_{l,t}$ and $\mu^-_{l,t}$ are the dual variables of the upper and lower transmission constraints respectively, while $p_{i,t}$ is the zonal electricity price. The complementarity conditions of this problem are stated as:

$$0 \leq Linecap_l \mp \sum_i PTDF_{i,l}(\sum_{f,k} y_{f,i,k,t} - d^1_i - d^2_{i,t}) \perp \mu^{\pm}_{l,t} \geq 0 \quad \forall\, l,t \tag{3}$$

Condition (3) states that one of the dual variables $\mu^+_{l,t}$ and $\mu^-_{l,t}$ is positive when the flowgate l is congested in time segment t.

3.2.2 Generators maximize profits.

Generators maximize hourly profits within their generation capacity constraints in each time segment t:

$$\textbf{Max} \quad \sum_{i,k} p_{i,t} \cdot y_{f,i,k,t} - \sum_{i,k} (c_{f,i,k} + e_k \cdot \lambda) \cdot y_{f,i,k,t} \qquad s.t. \tag{4}$$

$$0 \leq X_{f,i,k} + x_{f,i,k} - y_{f,i,k,t} \qquad (\nu_{f,i,k,t}) \quad \forall\, f, i, k, t \tag{5}$$

Generators operate both existing $X_{f,i,k}$ and new $x_{f,i,k}$ capacities to produce a non-negative quantity of electricity $y_{f,i,k,t}$ as indicated by constraint (5). This condition is matched with the dual variable $\nu_{f,i,k,t}$ representing the scarcity rent, i.e. the marginal capacity value, of both new and old power plants. This optimization problem can be transformed in complementarity conditions as indicated below:

$$0 \leq X_{f,i,k} + x_{f,i,k} - y_{f,i,k,t} \perp \nu_{f,i,k,t} \geq 0 \quad \forall\, f, i, k, t \tag{6}$$

$$0 \leq c_{f,i,k} + e_k \cdot \lambda + \nu_{f,i,k,t} - p_{i,t} \perp y_{f,i,k,t} \geq 0 \quad \forall\, f, i, k, t \tag{7}$$

Condition (6) restates constraint (5) and indicates that the scarcity rent $\nu_{f,i,k,t}$ is positive when the total available capacity $(X_{f,i,k} + x_{f,i,k})$ is fully utilized. Condition (7) summarizes the generator's optimal behaviour and expresses production efficiency: the electricity price $p_{i,t}$ recovers the fuel $(c_{f,i,k})$ and the emission opportunity $(e_k \cdot \lambda)$ costs as well as the scarcity rent $(\nu_{f,i,k,t})$ whenever generation $(y_{f,i,k,t})$ is positive.

3.2.3 Investors maximize profits.

As already explained in Section 2, we suppose that investors maximize profits by building new generation capacities $x_{f,i,k}$ and leasing them to generators at price $\nu_{f,i,k,t}$ in each time segment t. This is expressed as follows:

$$\textbf{Max} \quad \sum_{i,k} (\sum_{t} \tau_t \cdot \nu_{i,k,t} - I_{f,i,k}) \cdot x_{f,i,k} \tag{8}$$

Revenues accrue from the difference between the unitary marginal value of the new capacity $\nu_{f,i,k,t}$ and the investment cost $I_{f,i,k}$. Variable $x_{f,i,k}$ is non-negative. Condition (9) states the complementary version of this problem.

$$0 \leq I_{f,i,k} - \sum_{t} \tau_t \cdot \nu_{f,i,k,t} \perp x_{f,i,k} \geq 0 \quad \forall\, f, i, k \tag{9}$$

3.2.4 N-EIIs and EIIs maximize surpluses.

N-EIIs' hourly surplus maximization problem in time segment t and zone i is defined as:

$$\textbf{Max.} \quad \int_0^{d^2_{i,t}} P^2_{i,t}(\xi) d\xi - p_{i,t} \cdot d^2_{i,t} \tag{10}$$

Taking into account that $P^2_{i,t}(d^2_{i,t}) = p_{i,t}$ and after replacing $P^2_{i,t}(d^2_{i,t})$ by its affine expression, it holds that:

$$p_{i,t} = a^2_{i,t} - b^2_{i,t} \cdot d^2_{i,t} \qquad \forall\, i, t \tag{11}$$

Recall that we assume a constant industrial electricity consumption over time. This implies a slight modification of the EIIs' model compared to the N-EIIs.

$$\textbf{Max} \qquad \int_0^{d^1_i} P^1_i(\xi)d\xi - \sum_t \tau_t \cdot p_{i,t} \cdot d^1_i \tag{12}$$

or after expressing the optimality condition and replacing the functions $P^1_i(\xi)$ by its affine expression:

$$\sum_t \tau_t \cdot p_{i,t} = a^1_i - b^1_i \cdot d^1_i \quad \forall\, i \tag{13}$$

The quantities of electricity consumed by N-EIIs ($d^2_{i,t}$) and by EIIs (d^1_i) are non-negative. Moreover, both consumer groups pay identical electricity prices $p_{i.t}$.

3.2.5 Clearing of the energy market.

We assume an hourly balance of the energy market. According to the flowgate representation of the network, this market clears at π_t, the price set at the hub. This price depends on time t. Equality (14) expresses this idea.

$$\sum_{f,i,k} y_{f,i,k,t} - \sum_i d^1_t - \sum_i d^2_{i,t} = 0 \qquad (\pi_t) \quad \forall\, t \tag{14}$$

$$p_{i,t} = \pi_t + \sum_l PTDF_{i,l}(-\mu^+_{l,t} + \mu^-_{l,t}) \quad \forall\, i, t \tag{15}$$

The hub price π_t can be considered as a floor price that after adding the transmission costs, $(-\mu^+_{l,t} + \mu^-_{l,t}) \cdot PTDF_{i,l}$, gives the zonal marginal electricity prices $p_{i,t}$ paid by consumers as indicated in condition (15). Note that zonal electricity prices are equal to the hub price when there is no congestion.

3.2.6 Clearing of the emission market.

This reference model only considers the emissions of power plants. Since National Allocation Plans (NAPs) refer to annual targets, we impose one emission constraint, with its associated dual variable λ, representing the allowance price. The hourly emission cap E results from the division of the annual power sectors NAPs by 8760, the yearly number of hours. The emission constraint is defined as follows:

$$0 \leq E - \sum_{f,i,k,t} \tau_t \cdot e_k \cdot y_{f,i,k,t} \qquad (\lambda) \tag{16}$$

Total hourly emissions are computed by multiplying the technology emission factor e_k by the hourly electricity production $y_{f,i,k,t}$. Condition (17) is the complementarity of constraint (16). It means that the allowance price λ is positive when the emission constraint is binding.

$$0 \leq E - \sum_{f,i,k,t} \tau_t \cdot e_k \cdot y_{f,i,k,t} \perp \lambda \geq 0 \tag{17}$$

4 Properties of the Perfect Competition Model

The reference model extends the standard capacity expansion model and the associate peak load pricing theory developed in the early days of electricity economics by including transmission and emission markets. It is a convex problem as stated in Proposition 1.

4.1 Existence of Equilibrium

Proposition 1 *The set of conditions (3), (6), (7), (9), (11), (13), (14), (15) and (17) are the KKT conditions of a convex optimization problem. They have a convex solution set.*

Proof: See Appendix 1. □

4.2 Windfall Profits and Losses

The model supposes a fixed amount of allowances, but purposely avoids discussing property rights on these allowances. It is well known that free allowances allocated once for all irrespectively of agents' decisions (as in the US acid rain program, but not in the EU-ETS) do not distort these decisions. We assume that this is the case here and bypass questions of distortion of incentives to invest or operate. Another issue is whether the allocation of free allowances creates windfall profits or losses. The application of a new environmental regulation changes the value of a firm's assets therefore possibly justifying taxes and subsidies (free allowances). We quantify this change by comparing the value of the firm's assets before and after the inception of the EU-ETS. For instance, a nuclear plant benefits from a CCGT plant setting the electricity price in the ETS, while a coal plant loses because of its high emission charges. This may justify taxing windfall profits in the former case and granting free allowances in the latter. Propositions 2 and 3 shed some light on this issue. We look at the question by referring to optimally adjusted generation systems as introduced in the early theories of peak load pricing. Consider first a generation system developed without carbon legislation. Introduce then an emission trading scheme and assume a certain fraction of free allowances. The question is whether this creates profits or induces losses for the power companies, either before or after they adjust capacity, to adapt to the new situation. We state the following propositions.

Proposition 2 *Suppose an optimally adjusted generation system when there is no carbon legislation. The profits of investors are zero at the equilibrium solution. Suppose an optimally adjusted generation system under an emission trading scheme. The profits of investors are zero at the equilibrium solution if the operators of these generations capacities do not receive free allowances.*

Proof: See Appendix 2. □

The introduction of a carbon legislation changes the value of its capacities leading to gains or losses. These can be measured before or after the company adjust to this new situation by investing in new equipments. We assess these changes in the following proposition.

Proposition 3 *Suppose a generation system optimally adjusted in the absence of carbon legislation. Introduce an emission trading scheme and assume that the generation system adapts to this new legislation taking the generation structure developed without the ETS as existing capacities. Let $\nu^{CO_2}_{f,i,k,t}$ be the marginal value in time segment t of capacity k, owned by generator f located in zone i obtained under these conditions. The change of value of plant (k, f, i) after the introduction of the ETS is:*

$$\sum_t \tau_t \cdot \nu^{CO_2}_{f,i,k,t} - I_{f,i,k}$$

Proof: Appendix 3. □

The same formula applies to the change of value of the asset before adjustment of capacities, but with the $\nu^{CO_2}_{f,i,k,t}$ computed with existing capacities and no investments.

5 Electricity Cost and Investments: Average Cost Pricing Models

We now turn to models where electricity is sold to EIIs at average cost. Recall that we distinguish two types of contracts that differ by their geographic expanse. A zonal contract is one where the EIIs are charged the average cost of their zone: the price excludes transmission costs between zones. A regional contract supposes a single average cost for all zones; the price includes cross-border transmission costs. In this section, average cost models only deal with the (indirect) electricity ETS induced costs, which EIIs complain most about. Because average costs need to be auditable or at least credible for EIIs, we assume that their are based on capacities dedicated by generators to EIIs, the rest of the capacity supplying N-EIIs. Generators then sell at marginal cost to N-EIIs and at average cost to EIIs. We assume that generators maximize the profit of selling electricity to N-EIIs (without exercising market power) and minimize the cost of supplying EIIs. The structure of the transmission and allowance markets remains unchanged. Also the investors' problem is not modified.

5.1 Notation of the Average Cost Models

The parameters of the models are identical to those of Section 3.2. We list the variables. Recall that EIIs' variables do not depend on time, in contrast with those relative to N-EIIs. We identify EIIs and N-EIIs' variables respectively with "*1*" and "*2*".

- **Variables**

$y^1_{f,i,k}$; $y^2_{f,i,k,t}$	Hourly generation by technology k run by generator f in zone i to supply EIIs and N-EIIs (MWh);
$X^1_{f,i,k}$; $X^2_{f,i,k}$	Existing capacity of technology k that generator f located in i dedicates to EIIs and N-EIIs (MWh);
$x^1_{f,i,k}$; $x^2_{f,i,k}$	New capacity of technology k that generator f located in i dedicates to EIIs and N-EIIs (MWh);
$\nu^1_{f,i,k}$; $\nu^2_{f,i,k,t}$	Marginal hourly value of capacity (scarcity rent) of technology k used by generator f in zone i allocated to EIIs and N-EIIs (€/MWh);
$\nu_{f,i,k}$	Marginal hourly value of capacity of technology k used by generator f in zone i (€/MWh). See below for the relation with $\nu^1_{f,i,k}$ and $\nu^2_{f,i,k,t}$;
d^1_i; $d^{t,2}_i$	Hourly power consumption respectively by EIIs and N-EIIs located in zone i (in MWh). N-EIIs' consumption differs by season t;
p^1	Regional average cost price of electricity paid by EIIs (€/MWh). This price is the sum of the regional production and allowance costs $pprod^1$ and of the average regional transmission cost $ptrans^1$;
p^1_i	Zonal average cost price of electricity paid by EIIs (€/MWh). This price in the sum of the regional production and allowance costs in the zone i;
$p^{t,2}_i$	Zonal marginal price paid by N-EIIs in each season t (€/MWh);
π^2_t	Electricity price at the hub in each time segment t for N-EIIs (€/MWh);
θ^1	Marginal cost at the hub of the electricity generated by capacities dedicated to EIIs (€/MWh). This variable intervenes in the regional average cost price model;
θ^1_i	Marginal cost at the zone i of the electricity generated by capacities dedicated to EIIs (€/MWh). These variables intervene in the zonal average cost price model.

5.2 Regional Average Cost Pricing Model

The regional average cost pricing arrangement assumes that EIIs form a power purchase consortium that buys electricity produced by dedicated plants located in different zones of the network through long-term contracts. This is akin to the French Exeltium consortium. There is no change in the organization of the N-EIIs' problem with respect to the reference model.

5.2.1 Transmission market and TSO's profit maximization.

The TSO's model is similar to the one presented in Section 3.2. We state the complementarity conditions:

$$0 \leq Linecap_l \mp \sum_i PTDF_{i,l}(\sum_{f,k} y^1_{f,i,k} + \sum_{f,k} y^2_{f,i,k,t} - d^1_i - d^2_{i,t}) \perp \mu^{\pm}_{l,t} \geq 0 \quad \forall\, l,t \tag{18}$$

obtained from condition (3) after replacing the variable $y_{f,i,k,t}$ respectively by $y^1_{f,i,k}$ and $y^2_{f,i,k,t}$ to account for the segmentation of the market into EIIs and N-EIIs.

5.2.2 Generators maximize profits from supplying the N-EIIs.

The N-EII sector is still supplied at marginal cost. As in Section 3.2, generators maximize the profits accruing from supplying N-EIIs in each time segment t at the prevailing prices $p^2_{i,t}$.

$$\textbf{Max} \quad \sum_{i,k} p^2_{i,t} \cdot y^2_{f,i,k,t} - \sum_{i,k}(c_{f,i,k} + e_k \cdot \lambda) \cdot y^2_{f,i,k,t} \qquad s.t. \tag{19}$$

$$0 \leq X^2_{f,i,k} + x^2_{f,i,k} - y^2_{f,i,k,t} \qquad (\nu^2_{f,i,k,t}) \quad \forall\, f,i,k,t \tag{20}$$

Relation (20) states that generators operate dedicated existing ($X^2_{f,i,k}$) and new ($x^2_{f,i,k}$) capacities to supply N-EIIs. Again, $y^2_{f,i,k,t}$, $X^2_{f,i,k}$ and $x^2_{f,i,k}$ are non-negative variables. The complementarity conditions become:

$$0 \leq c_{f,i,k} + e_k \cdot \lambda + \nu^2_{f,i,k,t} - p^2_{i,t} \perp y^2_{f,i,k,t} \geq 0 \quad \forall\, f,i,k,t \tag{21}$$

$$0 \leq X^2_{f,i,k} + x^2_{f,i,k} - y^2_{f,i,k,t} \perp \nu^2_{f,i,k,t} \geq 0 \quad \forall\, f,i,k,t \tag{22}$$

As condition (7), condition (21) states that, in perfect competition, the marginal electricity price $p^2_{i,t}$ equals the sum of the marginal production ($c_{f,i,k}$), emission ($e_k \cdot \lambda$) and capacity ($\nu^2_{f,i,k,t}$) costs when generation ($y^2_{f,i,k,t}$) is positive.

5.2.3 Generators minimize the cost of supplying EIIs.

A different modeling applies for generation and supply to the EIIs' segment. Average cost pricing allows one to retain production efficiency, but sacrifices allocation efficiency. Generators no longer maximize profits from selling at marginal cost, but they still minimize generation and supply (here transmission) costs. Their problem is stated as follows:

$$\textbf{Min} \quad \sum_{f,i}(c_{f,i,k} + e_k \cdot \lambda) \cdot y^1_{f,i,k} - (\sum_{i,l,t} \tau_t \cdot PTDF_{i,l}(-\mu^{t,+}_l + \mu^{t,-}_l)) \cdot (\sum_k y^1_{f,i,k} - d^1_i) \tag{23}$$

$s.t.$

$$0 \leq X^1_{f,i,k} + x^1_{f,i,k} - y^1_{f,i,k} \qquad (\nu^1_{f,i,k}) \quad \forall\, f,i,k \tag{24}$$

$$\sum_{f,i} y^1_{f,i} - \sum_i d^1_i = 0 \quad (\theta^1) \tag{25}$$

The first term $(c_{f,i,k} + e_k \cdot \lambda)$ of this objective function is the sum of the production and emission costs; the second term, $(\sum_{i,l,t} \tau_t \cdot PTDF_{i,l}(-\mu_l^{t,+} + \mu_l^{t,-}))$, collects the congestion costs caused by sales from generators to EIIs. Conditions (24) and (25) state the capacity constraints and the energy balance of the consortium on the EIIs' market. As before, variables $y^1_{f,i,k}$, $X^1_{f,i,k}$ and $x^1_{f,i,k}$ are non-negative.

The KKT conditions of this problem are:

$$0 \leq c_{f,i,k} + e_k \cdot \lambda + \nu^1_{f,i,k} - \theta^1 - (\sum_{t,l} \tau_t \cdot PTDF_{i,l}(-\mu_l^{t,+} + \mu_l^{t,-})) \perp y^1_{f,i,k} \geq 0 \quad \forall\, f, i, k \tag{26}$$

$$0 \leq X^1_{f,i,k} + x^1_{f,i,k} - y^1_{f,i,k} \perp \nu^1_{f,i,k} \geq 0 \quad \forall\, f, i, k \tag{27}$$

When condition (26) is binding, generator f supplies EIIs with a positive quantity of electricity $y^1_{f,i,k}$. This implies that the sum of the marginal production $(c_{f,i,k})$, emission $(e_k \cdot \lambda)$ and capacity $(\nu^1_{f,i,k})$ costs equals the value $(\theta^1 + \sum_{t,l} \tau_t \cdot PTDF_{i,l}(-\mu_l^{t,+} + \mu_l^{t,-}))$. The variable θ^1 is the marginal electricity price that EIIs would pay at the hub in a hypothetical perfectly competitive regime. It is the analogue of the π^2_t in the N-EIIs' problem and appears as the complementarity variable of the energy balance in the EIIs' market (see condition (25)). In fact, the EIIs' model embeds both an "accounting" and an "economic" electricity prices. The former is the average cost price p^1, which EIIs effectively pay to generators and the TSO. The second is the transfer price θ^1 that guarantees the efficiency of electricity production and capacity allocation between the two consumer groups. One indeed observes in condition (26) that the marginal price θ^1 adds to the weighted congestion costs $(\sum_{i,l,t} \tau_t \cdot PTDF_{i,l}(-\mu_l^{t,+} + \mu_l^{t,-}))$ to arrive at a marginal cost price similar to $p^2_{i,t}$ in the N-EIIs' model.

Finally, condition (27) is the capacity constraint. The quantity of electricity that generators produce for EIIs $(y^1_{f,i,k})$ cannot exceed the dedicated capacity $(X^1_{f,i,k} + x^1_{f,i,m})$. The scarcity rent $(\nu^1_{f,i,k})$ paired with this condition is positive only when power plant (f, i, k) is at capacity.

5.2.4 Efficiency of the capacity allocation between N-EIIs and EIIs.

Capacities are endogenously allocated between EIIs and N-EIIs. We make this allocation efficient by equalizing the marginal values of the capacities dedicated to the two consumer groups. This is stated in the following complementarity conditions that we further analyze in the next section:

$$0 \leq X_{f,i,k} - X^1_{f,i,k} - X^2_{f,i,k} \perp \nu_{f,i,m} \geq 0 \quad \forall\, f, i, k \tag{28}$$

$$0 \leq X^2_{f,i,k} + x^2_{f,i,k} - y^2_{f,i,k,t} \perp \nu^2_{f,i,k,t} \geq 0 \quad \forall\, f, i, t, k \tag{29}$$

$$0 \leq X^1_{f,i,k} + x^1_{f,i,k} - y^1_{f,i,k} \perp \nu^1_{f,i,k} \geq 0 \quad \forall\, f, i, k \tag{30}$$

$$0 \leq \nu_{f,i,k} - \sum_t \tau_t \cdot \nu^2_{f,i,k,t} \perp X^2_{f,i,k} \geq 0 \quad \forall\, f, i, k \tag{31}$$

$$0 \leq \nu_{f,i,k} - \nu^1_{f,i,k} \perp X^1_{f,i,k} \geq 0 \quad \forall\, f, i, k \tag{32}$$

Condition (28) is the global constraint on existing capacities. It imposes that the sum of capacities $X^2_{f,i,k}$ and $X^1_{f,i,k}$, respectively reserved for N-EIIs and EIIs does not exceed the existing capacity $X_{f,i,k}$ ($X_{f,i,k}$ is a parameter). The variable $\nu_{f,i,m}$, pairing (28), is the global scarcity rent. Constraints (29) and (30) set the marginal values of the capacities in the two market segments. They were stated before as (22) and (27) and are recalled here as we gather all relations relevant to the capacity allocation mechanism. Conditions (31) and (32) equalize $\nu_{f,i,k}$ to $\nu^1_{f,i,k}$ and the weighted sum of $\nu^2_{f,i,k,t}$. As will be stated in the next section, this guarantees production efficiency by forcing the equality of the marginal values of capacity of the consumer segments, irrespectively of the fact that the different electricity pricing schemes distort allocative efficiency.

5.2.5 Investors maximize profits.

Investors maximize their profits by building new plants and leasing existing and new capacities to generators in each time segment t. Their problem reflects the split of capacity between the two consumer groups:

$$\textbf{Max} \quad \sum_{i,k}(\sum_t \tau_t \cdot \nu^2_{i,k,t} - I_{f,i,k}) \cdot x^2_{f,i,k} + \sum_{i,k}(\nu^1_{i,k,t} - I_{f,i,k}) \cdot x^1_{f,i,k} \tag{33}$$

The complementarity conditions are as follows:

$$0 \leq I_{f,i,k} - \sum_t \tau_t \cdot \nu^2_{f,i,k,t} \perp x^2_{f,i,k} \geq 0 \quad \forall\ f, i, k \tag{34}$$

$$0 \leq I_{f,i,k} - \nu^1_{f,i,k} \perp x^1_{f,i,k} \geq 0 \quad \forall\ f, i, k \tag{35}$$

5.2.6 N-EIIs and EIIs maximize surpluses.

As in the reference case, both N-EIIs and EIIs maximize their surpluses. The formulation of the N-EIIs' problem is slightly adapted from Section 3.2.4 to new situation. The EIIs' problem becomes:

$$\textbf{Max} \quad \int_0^{d^1_i} P^1_i(\xi)d\xi - p^1 \cdot d^1_i \qquad or \qquad p^1 = a^1_i - b^1_i \cdot d^1_i \quad \forall\ i \tag{36}$$

where the weighted sum of the marginal electricity prices $p_{t,i}$ in condition (13) of Section 3.2.4 is now substituted with p^1, the regional average cost price that represents the average production, emission and transmission costs of the EIIs' dedicated power units (see equality (41)).

5.2.7 Clearing of the energy market.

The separation of the market into its EIIs and N-EIIs' segments requires two market clearings. Conditions (37) and (38) respectively state the energy balance and the formation of the zonal marginal cost electricity prices on the N-EIIs segment. The mechanism is as in Section 3.2.5, but is here restricted to the sole N-EIIs' sector.

$$\sum_{f,i,k} y^2_{f,i,k,t} - \sum_i d^2_{i,t} = 0 \quad (\pi^2_t) \quad \forall\ t \tag{37}$$

$$p^2_{i,t} = \pi^2_t + \sum_l PTDF_{i,l}(-\mu^{t,+}_l + \mu^{t,-}_l) \quad \forall\ i, l \tag{38}$$

As announced in Section 5.2.3 the EIIs' market involves an "economic" transfer price θ^1 and an "accounting" average cost price p^1 that we now define. Condition (39) computes the average production cost $pprod^1$: it accounts for the annual fuel, emission and capacity costs of the existing ($X^1_{f,i,k}$) and new ($x^1_{f,i,k}$) power plants assigned to EIIs. The average congestion cost is computed in equality (40). The sum of the average production and transmission costs results in the total average price p^1 paid by EIIs (41).

$$pprod^1 = \frac{\sum_{f,i,k} y^1_{f,i,k} \cdot (c_{f,i,k} + e_k \cdot \lambda) + \sum_{f,i,k} I_{f,i,k} \cdot (X^1_{f,i,k} + x^1_{f,i,k})}{\sum_i d^1_i} \tag{39}$$

$$ptrans^1 = \frac{\sum_{i,l,t}(\tau_t \cdot PTDF_{i,l}(\mu^{t,+}_l - \mu^{t,-}_l) \cdot (\sum_{f,k} y^1_{f,i,k} - d^1_i))}{\sum_i d^1_i} \tag{40}$$

$$p^1 = pprod^1 + ptrans^1 \tag{41}$$

The "economic" transfer price θ^1 only plays a technical role that will appear in the proof of some propositions in Section 6. It suffices to note here that it is paired with condition (42) that is identical to (25) of Section 5.2.3.

$$\sum_{f,i,k} y^1_{f,i,k} - \sum_i d^1_i = 0 \quad (\theta^1) \tag{42}$$

5.2.8 Clearing of the emission market.

As condition (17) in Section 3.2.6, the emission constraint only accounts for the CO_2 of the power sector, here expressed as the global contribution for supplying both EIIs ($\sum_{f,i,k} e_k \cdot y^1_{f,i,k}$) and N-EIIs ($\sum_{t,f,i,k} e_k \cdot y^2_{f,i,k,t}$). Recall that, as the rest of the model, the emission cap E is expressed in hour. We directly report the complementarity form.

$$0 \leq E - (\sum_{f,i,k} e_k \cdot y^1_{f,i,k} + \sum_{t,f,i,k} \tau_t \cdot e_k \cdot y^2_{f,i,k,t}) \perp \lambda \geq 0 \tag{43}$$

5.3 Zonal Average Cost Pricing Model

We now turn to an alternative model where one assumes that EIIs pay a zonal average cost price. The structure of the model is similar to the one of the previous section. We therefore limit the presentation to the required small changes of the EIIs' model.

5.3.1 Transmission market and TSO's profit maximization.

Because EIIs conclude contracts with local electricity producers, their demand no longer contributes to the network congestion (this is the official fiction of the flowgate model discussed in Europe) and therefore disappears from the TSO's optimization problem. Complementarity condition (44) replaces (18) of Section 5.2.1.

$$0 \leq Linecap_l \mp \sum_i PTDF_{i,l}(\sum_{f,k} y^2_{f,i,k,t} - d^2_{i,t}) \perp \mu^{\pm}_{l,t} \geq 0 \quad \forall\, l, t \tag{44}$$

5.3.2 Generators minimize the cost of supplying EIIs.

In contrast with the regional average cost pricing model where industries constitute a single purchasing consortium, we here assume a purchasing consortium in each zone (the current insufficient development of cross border trade in Europe makes this view quite realistic). Generators in each zone i therefore solve the following cost minimization problem:

$$\mathbf{Min} \quad \sum_{i,k}(c_{f,i,k} + e_k \cdot \lambda) \cdot y^1_{f,i,k} \qquad s.t. \tag{45}$$

$$0 \leq X^1_{f,i,k} + x^1_{f,i,k} - y^1_{f,i,k} \qquad (\nu^1_{f,i,k}) \quad \forall\, f, i, k \tag{46}$$

$$\sum_f y^1_{f,i} - d^1_i = 0 \quad (\theta^1_i) \qquad \forall\, i \tag{47}$$

As in the regional average cost model, the variable θ^1_i represents an internal transfer price among generators. In contrast with the regional model, it here depends on zone i. The corresponding KKT condition becomes:

$$0 \leq c_{f,i,k} + e_k \cdot \lambda + \nu^1_{f,i,k} - \theta^1_i \perp y^1_{f,i,k} \geq 0 \quad \forall\, f, i, k \tag{48}$$

$$0 \leq x^1_{f,i,k} - y^1_{f,i,k} \perp \nu^1_{f,i,k} \geq 0 \quad \forall\, f, i, k \tag{49}$$

to be added to the market balance (47).

5.3.3 N-EIIs and EIIs maximize surpluses.

The N-EIIs' maximization problem obtains from Section 5.2.6 after substituting the regional (p^1) with the zonal (p^1_i) average cost prices:

$$\mathbf{Max} \quad \int_0^{d^1_i} P^1_i(\xi)d\xi - p^1_i \cdot d^1_i \quad or \quad p^1_i = a^1_i - b^1_i \cdot d^1_i \quad \forall\, i \tag{50}$$

5.3.4 Clearing of the energy market.

The energy market clears for both market segments in each zone. The N-EIIs' model remains unchanged with respect to that presented in Section 5.2.8 and leads to the formation of the marginal prices p_i^2. EIIs now buy at a zonal average cost price computed as follows:

$$p_i^1 = \frac{\sum_{f,k} y^1_{f,i,k} \cdot (c_{f,i,k} + e_k \cdot \lambda) + \sum_{f,k} I_{f,i,k} \cdot (X^1_{f,i,k} + x^1_{f,i,k})}{d_i^1} \tag{51}$$

Finally, the clearing of the emission market, the endogenous allocation of existing and new capacity to N-EIIs and EIIs, the investors as well as the generators maximization problems are identical to those described in Section 5.2.

6 Properties of the Average Cost Pricing Models

The above models involve average cost prices and hence can no longer be formulated as optimization problems. The relations (39), (40), (41) and (51), equating prices to average costs respectively in the regional and the zonal average cost models, introduce non convexities. The trivial, one plant and one consumer segment equilibrium model, exhibited in Figure 1 shows that the problem can have a multiplicity of isolated solutions or be infeasible. We try to shed some light on this existence and multiplicity problem by resorting to arguments that do not involve convexity. The discussion is conducted on the regional model, but also applies to the zonal model.

While the overall model is non convex, it contains a useful convex sub-model. Consider the relations obtained from the overall model by dropping the definition of the average cost prices and the demand system of the EIIs, namely conditions (36), (39), (40) and (41). Taking the demand of the EIIs as fixed and leaving the demand of the N-EIIs responsive to price, we obtain a sub-model that we refer to as the partial average cost model. We first state the following proposition.

Proposition 4 *The partial average cost model is formed by the KKT conditions of a convex optimization problem. It has a convex set of solutions.*

Proof: See Appendix 5. □

6.1 Production Efficiency

A by-product of this proposition is that the allocation of generation capacity between EIIs and N-EIIs is economically efficient and that the price θ^1 and the time weighted average of π_t^2 at the hub are equal. Before proving it, we introduce the following Lemma:

Lemma 1 *Suppose the regional average cost model has an equilibrium. The EIIs' scarcity rent $\nu^1_{f,i,k}$ equals the time weighted sum of the N-EIIs' scarcity rent $\nu^2_{f,i,k,t}$.*

Proof: This equality directly derives from (31) and (32). □

Proposition 4 and the Lemma imply the following corollaries:

Corollary 1 *Suppose the regional average cost model has an equilibrium. The allocation of capacities between EIIs and N-EIIs is production efficient at equilibrium. Generation is also efficient in the sense that* θ^1 *and the time weighted average of* π_t^2 *are equal.*

Proof: See Appendix 6. □

Corollary 2 *Suppose the zonal average cost model has an equilibrium. The allocation of capacities between EIIs and N-EIIs is production efficient at equilibrium. Generation is also efficient in the sense that* θ_i^1 *and the time weighted average of* $p_{i,t}^2$ *are equal in each zone* i*.*

Proof: See Appendix 7. □

6.2 Existence of Equilibrium

In order to proceed further in the analysis of existence and multiplicity of equilibrium, we slightly modify the current set up to endow it with additional continuity properties. We first assume that the solution of the partial average cost model is unique. This is easily done by making the short term cost functions of the generators strictly convex. This guarantees that the solution of the partial average cost model is a continuous mapping of the vector of EIIs' demands. Because total production and transmission costs are computed on the basis of this solution using algebric operations, the partial average cost model also defines a continuous mapping from EIIs' demand into EIIs induced production and transmission costs. We also slightly modify the definition of the average cost price by capping it at a level that is higher than the largest intercept of the demand functions of the EIIs (see Figure 2 in Appendix 4). This is done by restating the average cost price as $p^{1'}$ and assuming that its value is equal to $min[p^1, cap]$, namely the minimum between the actual p^1 and the imposed cap. Note that this modification is not innocuous; it implies that we accept that generators do not cover their fixed costs when demand is too low (that is when $p^{1'}$ is effectively smaller than p^1). We argue next that this adaptation of the price is quite realistic, but first state an existence result. We refer to the models with modified average cost price as the modified models (see Appendix 4).

Consider the set $\prod_i [0, a_i^1/b_i^1]$ of feasible demand vectors of EIIs. As just discussed, the solution of the partial average cost model is a continuous mapping of the vector of EIIs' demands into the production and transmission costs incurred by EIIs. The combination of the capped average cost price, defined above, and of the demand relation (36) constitutes a mapping from the production and transmission costs of EIIs into the set of EIIs' demand vectors. The combination of these two mappings is a continuous mapping from the set of feasible EIIs' demand vectors into itself. We can then state the following proposition.

Proposition 5 *The modified average cost model has an equilibrium.*

Proof: The proof is a direct application of Brower's fixed point theorem with a continuous mapping from $\prod_i [0, a_i^1/b_i^1]$ into itself. □

It remains to explain the role of the modification of the pricing mapping. As can be seen from Figure 2, an average cost pricing model can be infeasible when the average cost price is too high for the demand. In practice, this means that the demand vanishes (since there is no feasible demand) and that the generator is left with stranded assets (assets for which it cannot recover the cost). This is exactly what the modified average pricing scheme represents.

6.3 Numerical Considerations

The two average cost models are non-convex and hence typically more difficult to solve than convex problems. Particular difficulties arise here because of the possibility that there is no positive solution for some zonal EIIs. This is so when capacity charges contribute too much to the average cost price, an occurrence that we cannot identify with certainty in a non-convex equilibrium model. We try to mitigate these numerical difficulties by solving the average cost based models as a sequence of two different sub-problems. We first solve a preliminary model by simulating a perfectly competitive power market where, like in the average cost pricing models, EIIs and N-EIIs are supplied by dedicated capacities, but both buy electricity at the marginal cost price. This preliminary problem is convex and has always a solution that is used as starting point for solving the average cost pricing problem. This procedure is applied to the two average cost pricing models. These non-convex models may have either no or multiple positive solutions. The empirical results of our simulations (see Oggioni and Smeers [4], [5]) show that, apart from one case, all models have positive solutions, even though possibly multiple. These disjoint solutions are detected by changing the starting point of the algorithm. We shall return to this point in Section 8.

Another important observation is that the regional average cost pricing model is more complex than the zonal average cost problem. Specifically, the regional average cost price model involves the sum of the average production (variables and capacity) costs of the dedicated units and the average transmission charges paid by EIIs for their use of the congested transmission grid. Production costs include only primal variables. The computation of the transmission cost involves the product of primal (injection and withdrawals in $\sum_{f,k} y^1_{f,i,k} - d^1_i$), and dual variables ($\mu_l^{t,+}$, $\mu_l^{t,-}$), corresponding to the marginal congestion costs (see, for instance, condition (40)). The absence of the average transmission charges should simplify the zonal average cost problem. Surprisingly we did not notice any difference in the solution of the two types of problems.

6.4 Windfall Profits and Losses

As for the reference model we do not discuss the possibility of distorted incentives due to decision dependent allocations of free allowances. Our concern remains focused on the identification of windfall profits or losses on existing assets. We do not repeat the justification of this concern here and simply state some propositions that extend the results already found in the reference model.

Proposition 6 *Suppose an optimally adjusted generation system under an emission trading scheme where EIIs are priced at average generation, transmission and generation costs. The profits of investors are zero at the equilibrium solution if the operators of these generation capacities do not receive free allowances.*

Proof: The proof is identical to the one of Proposition 2 for the profits of N-EIIs. The absence of profit on investments for EIIs results from the average cost pricing. □

Suppose now a system optimally adjusted under the assumption of no carbon legislation (zonal marginal price). The introduction of a carbon legislation together with average cost based contracts changes the value of generation capacities as stated in the following proposition.

Proposition 7 *Suppose an optimally adjusted generation system when there is no carbon legislation and one applies zonal marginal prices. Let $X^0_{f,i,k}$ be the capital stock obtained as the solution of that equilibrium model. Introduce an emission trading scheme and assume that the generation system adapts to this new legislation taking $X^0_{f,i,k}$ as existing capacities. Let $\sum_t \tau_t \nu^{CO_2}_{f,i,k,t}$ be the weighted marginal values of capacity k, located in i, obtained by solving the average cost model (regional or zonal). The change of value of this plant (k, f, i) due to the introduction of the ETS is:*

$$\sum_t \tau_t \cdot \nu^{CO_2}_{f,i,k,t} - I_{f,i,k}$$

Proof: The proof is identical to the one of Proposition 3. □

7 Electricity and Carbon Costs

We now assume that EIIs participate to the ETS and model both their carbon (direct) and electricity (indirect) costs. Allowances intervene in EIIs' profit and loss as a cost or subsidy depending on the (still to be decided) European policy on free allowances. Full auctioning imposes costs, free allowances provide subsidies. We model both policies and let the carbon component appear in the new formulation of the EIIs' demand either as an additional cost or a subsidy. A first scenario supposes full auctioning and hence considers carbon as a cost that we obtain by multiplying the endogenous allowance price by the EIIs' emission factors. A second scenario assumes some free allowances and represents the subsidy accruing from them as the product of the allowance price and an allowance factor (to be distinguished from the emission factor) that reflects the gap between the EIIs' emissions and the amount of free allowances. We calibrate this coefficient on the basis of the Community Independent Transaction Log (for the emission), Eurostat [2] and UCTE [10] (for electricity consumption) websites. We chose 2005 data since, in this year, emissions have been recognized to be independently and consistently verified.

We model the ETS induced carbon and electricity costs on the basis of the equilibrium problems presented in Sections 3 and 5 that we modify as described below. Perfect competition constitutes again the reference counterfactual for assessing the regional and zonal average costs pricing models. We introduce two features with respect to sections 3 and 5. One is a straightforward modification of the emission constraint that accounts for EIIs emissions. The second change is less trivial and described in the following subsection. Additional information and results of these models are provided in a companion paper (Oggioni and Smeers [5]).

7.1 Modeling the Electricity and Carbon Costs

This section offers a method for embedding the impact of both the electricity price and carbon costs in the electricity demand of the EIIs. It relies on simplifying assumptions that substitute the lack of information on the industrial processes of these consumers. We assume that emissions are proportional to production, which is itself proportional to power consumption (linear technology). Global emissions are then determined by multiplying electricity demand by an emission factor that, because of the lack of information, we compute on an aggregated basis. Specifically, we distinguish emission factors by country, but not by production sector. Sectoral data would relieve this restriction. These emission factors are obtained by dividing the EIIs' 2005 emissions (provided by the Community Independent Transaction Log) by the EIIs' annual reference demand that we compute on the basis of data available on the Eurostat [2] and UCTE [10] websites.

Let π denote the profit function of the aggregate industrial sector. It is defined as:

$$\pi = p_y \cdot y - p_e \cdot e - p_o \cdot o - p_{co_2} \cdot co_2 \tag{52}$$

where y is the total output sold at price p_y. To produce y, industries consume non electricity inputs o and electricity e that they buy at prices p_o and p_e respectively. Industries also face a price p_{co_2} for each allowance. The total environmental cost is the product of that price and the net purchase of allowances, let co_2, that is, the difference between the emissions and the free allowances.

We make the important policy assumption that free allowances are granted proportionally to production (according to a benchmarking method) and the technological assumption that the amount of electricity e, allowances co_2 and input o depend on production as follows: *(1)* $e = \alpha \cdot y$; *(2)* $co_2 = (\beta - \gamma) \cdot y$ and *(3)* $o = \varphi(y)$. Note that α and β are two technical parameters that respectively represent the electricity consumption and the emission factor per unit of output y; γ is a policy parameter defining the number of free allowances received per unit of output and, finally, $\varphi(y)$ is the consumption of non electricity input as a function of the output y. By allowing for a slightly more general treatment of the consumption of non electricity input $\varphi(y)$, we can write (52) as a function of the sole output y and obtain:

$$\pi(y) = p_y \cdot y - p_e \cdot \alpha \cdot y - p_{co_2} \cdot (\beta - \gamma) \cdot y - p_o \cdot \varphi(y) \tag{53}$$

A first step towards the modified industrial demand of electricity obtains by first computing the first order condition of (53) with respect to y:

$$\frac{\partial \pi}{\partial y} = p_y - p_e \cdot \alpha - p_{co_2} \cdot (\beta - \gamma) - p_o \cdot \varphi'(y) = 0 \tag{54}$$

With the objective to separately identifying the electricity price (p_e), which measures the indirect ETS cost, and the allowance price (p_{co_2}) which determines the direct carbon cost, we re-write condition (54) as follows:

$$p_e \cdot \alpha + p_{co_2} \cdot (\beta - \gamma) = p_y - p_o \cdot \varphi^{'}(y) \tag{55}$$

Dividing by α on the left and right hand side of equation (55), one gets:

$$p_e + p_{co_2} \cdot (\frac{\beta - \gamma}{\alpha}) = \frac{1}{\alpha}\left[p_y - p_o \cdot \varphi'(y)\right] \tag{56}$$

which is akin to a demand function: the left hand side is a combination in fixed proportion of the electricity and allowance prices (€/MWh), while the right hand side is only a function of the output of the firm (the price of the output is fixed in this partial equilibrium model). Because we are assuming linear demand functions throughout, we now impose the particular functional form $\varphi^{'}(y) = a + b \cdot y$ and write:

$$p_e + p_{co_2} \cdot (\frac{\beta - \gamma}{\alpha}) = \frac{1}{\alpha}\left[p_y - p_o \cdot (a + b \cdot y)\right] \tag{57}$$

Using the relation $e = \alpha \cdot y$, we substitute y in (57) and obtain:

$$p_e + p_{co_2} \cdot (\frac{\beta - \gamma}{\alpha}) = p_y\frac{1}{\alpha} - p_o\frac{a}{\alpha} - p_o\frac{b}{\alpha}e \tag{58}$$

By setting $A = p_y\frac{1}{\alpha} - p_o\frac{a}{\alpha}$ and $B = p_o\frac{b}{\alpha}$, we have:

$$p_e + p_{co_2} \cdot (\frac{\beta - \gamma}{\alpha}) = A - Be \tag{59}$$

Equation (59) represents the inverted industrial demand of electricity, where the electricity consumption e depends on electricity price p_e and carbon component $p_{co_2} \cdot (\frac{\beta-\gamma}{\alpha})$. This representation of the demand function allows for the modeling of different allowance allocation methods. In the full grandfathering case $\beta = \gamma$ and the carbon component does not affect EIIs' power consumption. In contrast, when allowances are fully auctioned, the factor $\gamma = 0$ and the carbon component becomes $\frac{\beta}{\alpha}$, which is also taken as the emission factor of EIIs. All possible intermediate allowance allocation scenarios are easily defined by imposing $\beta \neq \gamma$. The EIIs' demand for allowances is:

$$allowance \quad demand = \frac{\beta - \gamma}{\alpha} \cdot e$$

At this stage of the decision process, both full auctioning and partial free allowances to EIIs are compatible with the proposed ETS Directive for the period 2013-2020. Note that we assume full auctioning in the power sector.

7.2 Implied Modifications of the Model

The inclusion of the carbon component in the industrial electricity demand modifies EIIs' behavior and requires a slight change of formulation in the models in Sections 3 and 5. Adopting the notation of the previous sections where the allowance price p_{co_2} is equal to λ, and the industrial electricity demand e is d_i^1, the EIIs' surplus maximization problem becomes:

- **EIIs maximize their surplus**

$$\textbf{Max} \qquad \int_0^{d_i^1} P_i^1(\xi)d\xi - \sum_t \tau_t \cdot p_{i,t} \cdot d_i^1 - \lambda \cdot \frac{\beta - \gamma}{\alpha} \cdot d_i^1 \tag{60}$$

or:

$$\sum_t \tau_t \cdot p_{i,t} + \lambda \cdot \frac{\beta - \gamma}{\alpha} = a_i^1 - b_i^1 \cdot d_i^1 \quad (d_i^1) \quad \forall\, i \tag{61}$$

This optimization problem applies to the reference model, but can be easily adapted to the regional and the zonal average cost models by replacing the marginal electricity price $p_{i,t}$ respectively with p^1 and p_i^1.

- **Clearing of the emission market**

We also modify the emission constraint as follows:

$$0 \leq E - (\sum_{t,f,i,k} \tau_t \cdot e_k \cdot y_{f,i,k,t} + \frac{\beta}{\alpha} \cdot d_i^1) \perp \lambda \geq 0 \tag{62}$$

where the hourly cap E now also includes industrial emissions and the term $\frac{\beta}{\alpha} \cdot d_i^1$ represents the hourly emissions of the industrial sector. This emission constraint extends condition (17) of the reference model in Section 3. It is readily adapted to the average cost pricing models by substituting the variable $y_{f,i,k,t}$ with $y^1_{f,i,k,t}$ and $y^2_{f,i,k,t}$ appearing in the capacity splitting of those models.

8 Mathematical Analysis of the Results

Oggioni and Smeers [4] and [5] offer an extensive discussion of the type of policy results obtained with these models. We here comment on some numerical aspects. As already mentioned the average cost pricing models are non-convex and can have multiple solutions or be infeasible. We explained in Section 6 how one can in principle remove infeasibility by closing some industrial activities and possibly stranding generation capacities. Except in one case, our simulations always found feasible models. We first discuss these feasible cases.

Most case studies conducted in Oggioni and Smeers [4]and [5] conclude with feasible models, but reveal some non-convexity effect. As already mentioned, it is sometimes necessary to resort to a two stages approach where one first solves the perfect competition model in order to get a starting point for the average cost model. PATH would not find a solution without this good starting point. One also finds that different starting points can lead to different capacity allocation between EIIs and N-EIIs. Notwithstanding these problems, the results are stable in the sense that perturbations of the model lead to globally similar policy effects. For instance and of particular relevance, modifying the assumption of EIIs' price elasticity smoothly changes the total capacity allocated to EIIs (this capacity determines the electricity price to EIIs) even though the details of this allocation can change (shift between new and existing capacities in the allocation). Finally, although the regional average cost price model is in principle more complex because of the combination of primal and dual variables in the transmission cost component, we do not encounter particular problems in case studies conducted with that model.

We failed to solve one problem among the many reported in Oggioni and Smeers [5]. It is impossible with today's knowledge to algorithmically conclude to the infeasibility of a non-convex equilibrium model as it can also be a failure of the algorithm. This infeasible case is a zonal average cost model where EIIs have to bear both the electricity and the full CO_2 costs (full auctioning). Sensitivity analysis however revealed that a slight relaxation of full auctioning by the granting of some free allowances to EIIs made the model feasible. This suggests that full auctioning effectively made the model infeasible and EIIs could not manage both the electricity and carbon costs.

9 Conclusion

The restructuring of the electricity industry has generated an extensive literature on market simulation models. A lot of attention was devoted to studying the impact of different market architectures and structures with particular emphasis devoted to the exercise of market power. The problem taken up in this paper is different and we believe new: it requires a shift of paradigm away from a perfect competition system to an organization of the market where a different pricing regime is introduced for a certain class of consumers, in this case, the Electricity Intensive Industries. These industries almost universally complain about their being offered PX or PX driven prices that they find both too high and too volatile. They ask for average cost based contracts that they see as more stable and less expensive because not based on marginal costs. We model this claim by constructing a model of the electricity market with the demand sector segmented in two parts. EIIs are offered average cost based contracts while the rest of the market remains supplied at PX based prices (or time averages of PX based prices). We consider two types of average cost contracts that differ by their geographic expanse. One assumes a single regional price; the other supposes several zonal prices; the rest of the market is supplied at marginal cost. While these models are non-convex, they perform surprisingly smoothly at least for the cases studied in this project. The results of these models are presented in companion papers (Oggioni and Smeers [4], [5]); they are also interesting in terms of policy: average cost based contracts can indeed mitigate the impact of the ETS, but they remain far from removing it. The implementation of this remedy is also complicated by the differences of energy policies among EU Member States. Models need to rely on reliable data in order to be credible. This is where shortcomings are most significant. While there is a lot of discussion on the problem of carbon leakage, one finds very little empirical basis to construct a truly reliable prospective study of the problem (see Reinaud [7], [8] as studies of the current situation).

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Appendix 1: Proof of Proposition 1

Proof of Proposition 1: Consider the following maximization problem:

$$\textbf{Max} \quad \int_0^{d_i^1} P_i^1(\xi)d\xi + \sum_t \tau_t \left[\int_0^{d_{i,t}^2} P_{i,t}^2(\xi)d\xi - \sum_{f,i,k} c_{f,i,k} \cdot y_{f,i,k,t} \right] - \sum_{f,i,k} I_{f,i,k} \cdot x_{f,i,k}$$

s.t.

$$0 \leq X_{f,i,k} + x_{f,i,k} - y_{f,i,k,t} \qquad (\tau_t \nu_{f,i,k,t}) \quad \forall\ f,i,k,t$$

$$\sum_{f,i,k} y_{f,i,k,t} - \sum_i d_i^1 - \sum_i d_{i,t}^2 = 0 \qquad (\tau_t \pi_t) \quad \forall\ t$$

$$-Linecap_l \leq \sum_i PTDF_{i,l}(\sum_{f,k} y_{f,i,k,t} - d_i^1 - d_{i,t}^2) \leq Linecap_l \qquad (\tau_t \mu_{l,t}^{\pm}) \quad \forall l, t$$

$$0 \leq E - \sum_{f,i,k,t} \tau_t \cdot e_k \cdot y_{f,i,k,t} \qquad (\lambda)$$

$$0 \leq y_{f,i,k,t}, \quad x_{f,i,k}, \quad d_{i,t}^2, \quad d_i^1 \quad \forall\ f, i, k, t$$

The objective function is the difference between a concave function (the consumers' willingness to pay $\sum_t \tau_t \int_0^{d_{i,t}^2} P_{i,t}^2(\xi) d\xi$ and $\int_0^{d_i^1} P_i^1(\xi) d\xi$)) and linear functions representing the generators' operating and investment costs ($\sum_t \tau_t \sum_{f,i,k} c_{f,i,k} \cdot y_{f,i,k,t}$ and $\sum_{f,i,k} I_{f,i,k} \cdot x_{f,i,k}$). All constraints are linear. This means that the problem is convex. The solution set is then also convex. The KKT conditions of a convex problem suffice to characterize its global optimal solutions. We derive these conditions taking into account the non-negativity of these variables and show that they reproduce the set of complementarity conditions.

- Derivative w.r.t. variable $y_{f,i,k,t}$:

$$0 \leq c_{f,i,k} + e_k \cdot \lambda + \nu_{f,i,k,t} - p_{i,t} \perp y_{f,i,k,t} \geq 0 \quad \forall\ f, i, k, t$$

- Derivative w.r.t. variable $x_{f,i,k}$:

$$0 \leq I_{f,i,k} - \sum_t \tau_t \cdot \nu_{f,i,k,t} \perp x_{f,i,k} \geq 0 \quad \forall\ i, k$$

- Derivative w.r.t. variable $d_{i,t}^2$:

$$p_{i,t} = a_{i,t}^2 - b_{i,t}^2 \cdot d_{i,t}^2 \qquad \forall\ i, t$$

- Derivative w.r.t. variable d_i^1:

$$\sum_t \tau_t \cdot p_{i,t} = a_i^1 - b_i^1 \cdot d_i^1 \quad \forall\ i$$

where $p_{i,t}$ is defined as $\pi_t + \sum_l PTDF_{i,l}(-\mu_{l,t}^+ + \mu_{l,t}^-)$. In addition, we consider the complementarity conditions of the inequality constraints (generation and transmission capacity and the emission constraints) and the clearing of the energy market:

$$0 \leq X_{f,i,k} + x_{f,i,k} - y_{f,i,k,t} \perp \nu_{f,i,k,t} \geq 0 \quad \forall\ f, i, k, t$$

$$0 \leq Linecap_l \mp \sum_i PTDF_{i,l}(\sum_{f,k} y_{f,i,k,t} - d_i^1 - d_{i,t}^2) \perp \mu_{l,t}^{\pm} \geq 0 \quad \forall\ l, t$$

$$0 \leq E - \sum_{f,i,k,t} \tau_t \cdot e_k \cdot y_{f,i,k,t} \perp \lambda \geq 0$$

$$\sum_{f,i,k} y_{f,i,k,t} - \sum_i d_i^1 - \sum_i d_{i,t}^2 = 0 \qquad \forall\, t$$

□

Appendix 2: Proof of Proposition 2

Proof of Proposition 2: Relation (9) states that an investor (with positive variable $x_{f,i,k}$) makes zero profit on its investment after receiving the capacity rent $\nu_{f,i,k,t}$ from the plant's operator.

Relation (7) states that the operator of the plant, which produces a positive quantity of electricity $y_{f,i,k,t}$ and receives the electricity price $p_{i,t}$, makes zero profit after deducting the fuel cost (parameter $c_{f,i,k}$), the allowance cost (variable $e_k \cdot \lambda$) and paying the rent ($\nu_{f,i,k,t}$) to the owner of the plant.

In conclusion, neither the investor nor the generator make any profit respectively from investing in capacity and operating it after paying for its costs.

□

Appendix 3: Proof of Proposition 3

Proof of Proposition 3: Let $X^0_{f,i,k}$ be the capacity of plant f, i, k obtained as part of the solution of the equilibrium problem without ETS. Insert these $X^0_{f,i,k}$ as existing capacities $X_{f,i,k}$ in relation (5) describing the maximization of the profit of the generator. Let $\nu^{CO_2}_{f,i,k,t}$ be obtained in the solution of the ETS problem with these existing capacities.

Relation (6) states that $\nu^{CO_2}_{f,i,k,t}$ is the value received by the owner of capacity $X^0_{f,i,k}$ from the operator of the plant. Because the value of this capacity was $I_{f,i,k}$ (parameter) when it was invested before the inception of the ETS, the change of value of a unit capacity is $(\sum_t \tau_t \cdot \nu^{CO_2}_{i,k,t} - I_{f,i,k})$. Summing up over all capacities, the change of value of the assets of firm f is equal to:

$$\sum_{k,i} X^0_{f,i,t} \sum_{i,k} (\sum_t \tau_t \cdot \nu^{CO_2}_{i,k,t} - I_{f,i,k})$$

□

Appendix 4: Observations on Average Cost Pricing System

Figure 1 illustrates that the non-convexity induced by the average cost price may lead either to a multiplicity of solutions or to no solution. The downward sloping linear curve represents the demand function. The hyperbolic curve represents an average cost price of the form $\frac{K}{Q}+C$ where K is the fixed cost and C is the proportional cost. The latter curve represents the price p^1 in an average cost pricing system. The average cost price curve goes to infinity when

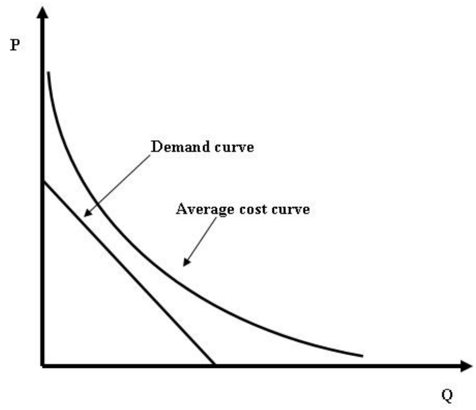

Figure 1: Average Cost Price Curve p^1

the quantity of electricity demanded is close to zero. By capping the average cost pricing curve, we obtain a new average cost price curve that we denote as $p^{1'}$ as depicted in Figure 2. The price is now $min[p^1, cap]$, which guarantees a solution $(0, cap)$ to the average cost based model, where 0 is the quantity of electricity demanded and cap is the price limit imposed.

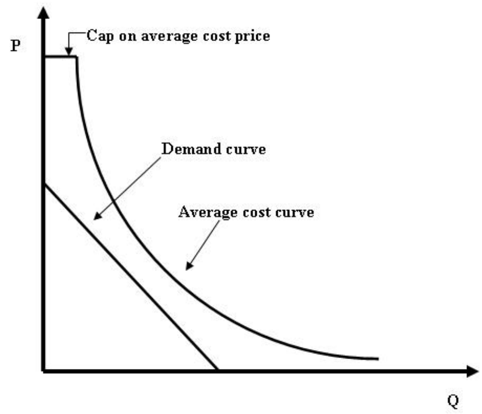

Figure 2: Capped Average Cost Price Curve $p^{1'}$

Appendix 5: Proof of Proposition 4

Proof of Proposition 4: The optimization problem we are looking for is as follows:

$$\textbf{Max} \quad \sum_t \tau_t \left[\int_0^{d_{i,t}^2} P_{i,t}^2(\xi)d\xi - \sum_{i,k} c_{f,i,k} \cdot y_{f,i,k,t}^2 \right] - \sum_{f,i,k} c_{f,i,k} \cdot y_{f,i,k}^1 - \sum_{f,i,k} I_{f,i,k} \cdot (x_{f,i,k}^2 + x_{f,i,k}^1)$$

s.t.

$$0 \le X_{f,i,k} - X^2_{f,i,k} - X^1_{f,i,k} \qquad (\nu_{f,i,k}) \quad \forall\ f,i,k$$

$$0 \le X^2_{f,i,k} + x^2_{f,i,k} - y^2_{f,i,k,t} \qquad (\tau_t \nu^2_{f,i,k,t}) \quad \forall\ f,i,k,t$$

$$0 \le X^1_{f,i,k} + x^1_{f,i,k} - y^1_{f,i,k} \qquad (\nu^1_{f,i,k}) \quad \forall\ f,i,k$$

$$\sum_{f,i,k} y^2_{f,i,k,t} - \sum_i d^2_{i,t} = 0 \qquad (\tau_t \pi^2_t) \quad \forall\ t$$

$$\sum_{f,i,k} y^1_{f,i,k} - \sum_i d^1_i = 0 \qquad (\theta^1) \quad \forall\ t$$

$$-Linecap_l \le \sum_i PTDF_{i,l}(\sum_{f,k} y^2_{f,i,k,t} - y^1_{f,i,k} - d^2_{i,t} - d^1_i) \le Linecap_l \qquad (\tau_t \mu^{\pm}_{l,t}) \quad \forall\ l,t$$

where d^1_i is now a parameter.

$$0 \le E - \sum_{f,i,k,t} \tau_t \cdot e_k \cdot y^2_{f,i,k,t} - \sum_{f,i,k} e_k \cdot y^1_{f,i,k} \qquad (\lambda)$$

$$0 \le y^2_{f,i,k,t},\quad x^2_{f,i,k},\quad X^2_{f,i,k},\quad d^2_{i,t} \quad \forall\ f,i,k,t$$

$$0 \le y^1_{f,i,k},\quad x^1_{f,i,k},\quad X^1_{f,i,k} \quad \forall\ f,i,k$$

The objective function is the difference between a concave function (the N-EIIs' willingness to pay $\sum_t \tau_t \int_0^{d^2_{i,t}} P^2_{i,t}(\xi)d\xi)$), and the sum of linear functions representing the generators' operating cost ($\sum_{t,f,i,k} \tau_t \cdot c_{f,i,k} \cdot y^2_{f,i,k,t}$ and $\sum_{f,i,k} c_{f,i,k} \cdot y^1_{f,i,k}$) and the investments cost ($\sum_{f,i,k} I_{f,i,k} \cdot (x^2_{f,i,k} + x^1_{f,i,k})$). Again, all constraints are linear. This implies that the problem is convex and hence also its solution set. Thanks to convexity, the KKT conditions are necessary and sufficient to characterize a global optimal solution. They are indicated in the following, noting that variables are non-negative:

- Derivative w.r.t. variable $y^2_{f,i,k,t}$:

$$0 \le c_{f,i,k} + e_k \cdot \lambda + \nu^2_{f,i,k,t} - p^2_{i,t} \perp y^2_{f,i,k,t} \ge 0 \quad \forall\ f,i,k,t$$

- Derivative w.r.t. variable $y^1_{f,i,k}$:

$$0 \le c_{f,i,k} + e_k \cdot \lambda + \nu^1_{f,i,k} - \theta^1 - \sum_t \tau_t \cdot \sum_l PTDF_{i,l}(-\mu^+_{l,t} + \mu^-_{l,t}) \perp y^1_{f,i,k} \ge 0 \quad \forall\ f,i,k$$

- Derivative w.r.t. variable $x^2_{f,i,k}$:

$$0 \le I_{f,i,k} - \sum_t \tau_t \cdot \nu^2_{f,i,k,t} \perp x^2_{f,i,k} \ge 0 \quad \forall\ i,k$$

- Derivative w.r.t. variable $x^1_{f,i,k}$:

$$0 \leq I_{f,i,k} - \nu^1_{f,i,k} \perp x^1_{f,i,k} \geq 0 \quad \forall\ i,k$$

- Derivative w.r.t. variable $X^2_{f,i,k}$:

$$0 \leq \nu_{f,i,k} - \sum_t \tau_t \nu^2_{f,i,k,t} \perp X^2_{f,i,k} \geq 0 \quad \forall\ f,i,k$$

- Derivative w.r.t. variable $X^1_{f,i,k}$:

$$0 \leq \nu_{f,i,k} - \nu^1_{f,i,k} \perp X^1_{f,i,k} \geq 0 \quad \forall\ f,i,k$$

- Derivative w.r.t. variable $d^2_{i,t}$:

$$p^2_{i,t} = a^2_{i,t} - b^2_{i,t} \cdot d^2_{i,t} \qquad \forall\ t,i$$

where $p^2_{i,t}$ is defined as $\pi^2_t + \sum_l PTDF_{i,l}(-\mu^+_{l,t} + \mu^-_{l,t})$. Like in proof 9, we consider the complementarity conditions of the inequality constraints (generation and transmission capacity and emission constraints) and the equality constraints defining the energy balance of the two markets:

$$0 \leq X_{f,i,k} - X^2_{f,i,k} - X^1_{f,i,k} \perp \nu_{f,i,k} \geq 0 \quad \forall\ f,i,k$$

$$0 \leq X^2_{f,i,k} + x^2_{f,i,k} - y^2_{f,i,k,t} \perp \nu^2_{f,i,k,t} \geq 0 \quad \forall\ f,i,k,t$$

$$0 \leq X^1_{f,i,k} + x^1_{f,i,k} - y^1_{f,i,k} \perp \nu^1_{f,i,k} \geq 0 \quad \forall\ f,i,k$$

$$0 \leq Linecap_l \mp \sum_i PTDF_{i,l}(\sum_{f,k} y^2_{f,i,k,t} - y^1_{f,i,k} - d^2_{i,t} - d^1_i) \perp \mu^\pm_{l,t} \geq 0 \quad \forall\ l,t$$

$$0 \leq E - \sum_{f,i,k,t} \tau_t \cdot e_k \cdot y^2_{f,i,k,t} - \sum_{f,i,k} e_k \cdot y^1_{f,i,k} \perp \lambda \geq 0$$

$$\sum_{f,i,k} y^2_{f,i,k,t} - \sum_i d^2_{i,t} = 0 \quad (p^2_{i,t}) \quad \forall\ t$$

$$\sum_{f,i,k} y^1_{f,i,k} - \sum_i d^1_i = 0 \quad (\theta^1) \quad \forall\ t$$

□

Appendix 6: Proof of Corollary 1

Proof of Corollary 1: The marginal electricity prices θ^1 and $p^2_{i,t}$ respectively match the EIIs' and the NEIIs' energy balance constraints (25) and (37). Constraint (26) shows that θ^1, increased by the transmission cost ($\sum_{t,l} \tau_t \cdot PTDF_{i,l}(-\mu_l^{t,+} + \mu_l^{t,-})$), equals the fuel cost (parameter $c_{f,j,k}$), the allowance cost (variable $e_k \cdot \lambda$) and the capacity cost (variable $\nu^1_{f,i,k}$). Constraint (21) shows that $p^2_{i,t}$ equals the sum of the fuel cost (parameter $c_{f,i,k}$), the allowance cost (variable $e_k \cdot \lambda$) and the capacity cost (variable $\nu^2_{f,i,k,t}$). Note that thanks to constraints (31) and (32), variables $\nu^1_{f,i,k}$ and $\nu^2_{f,i,k,t}$ (weighted by hours) are equal (see Lemma 1). This implies that $\theta^1 + \sum_{t,l} \tau_t \cdot PTDF_{i,l}(-\mu_l^{t,+} + \mu_l^{t,-})$ equals $p^2_{i,t}$. Since $p^2_{i,t}$ results from the sum of π^2_t and the transmission costs $\sum_l PTDF_{i,l}(-\mu_l^{t,+} + \mu_l^{t,-})$, this implies that θ^1 is equal to the time average sum of π^2_t. $\square$

Appendix 7: Proof of Corollary 2

Proof of Corollary 2: The reasoning of this proof is as in Corollary 9, after comparing (47) and (48) respectively with (37) and (21). Due the EIIs' zonal electricity balance, θ^1_i in condition (48) is equal to $p^2_{i,t}$ in condition (21). $\square$

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