2007/45

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The price of silence: tradeable noise permits and airports

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CORE DISCUSSION PAPER
2007/45

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July 2007

Abstract

This paper presents a market design for the management of noise disturbance created by aircraft traffic around large airports. A market for tradable noise permits allows noise generators to compensate harmed residents. We show that the noise permit markets allow the achievement of the planner's optimal allocation of flights provided that she/he does not over-weight the benefit of economic activity compared to the disutility of noise disturbances. The fact that zones are likely to be strategic players does not fundamentally alter this finding. Because of the market auctioneer's information constraints, noise permits are likely to redistribute windfall gains to residents located in non-critical zones. This entices landlords to increase their land/house rents there and to design smaller houses in the long run.

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T. Bréchet is also member of ECORE, the newly created association between CORE and ECARES. The authors thank participants to the Environmental Workshop at CORE, University of Lille 1, NARSC Toronto 2006 and Economic Seminar of MEDAD. They are also grateful to Paul Belleflamme, Jan Bruekner, Denny Ellerman, Paul Madden, Yann Ménière and Kurt Van Dender for fruitful comments on preliminary versions of the paper.

This paper presents research results of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by the authors.

1 Introduction

The air transport industry has become a key factor of economic activity. Air transport generates significant social and economic benefits related to trade, employment, investment, tourism and leisure opportunities. However, like most human activities, air transport also generates less welcome external effects, among which noise disturbances are probably the most salient ones[1]. Noise disturbances constitute a hot and topical societal problem for all major airports. Indeed, there seem to be no clear solutions to the issues about how to accommodate the residents suffering from noise disturbances and about how to accordingly determine the number and the distribution of aircraft movements around the airports.

Intriguingly, the economic literature has to a large extend neglected to discuss the design of policy instruments that could efficiently balance social cost of noise disturbances and economic benefits of airport economic activity. While many discussions have focused on technological improvements (quieter airplanes, alternative land-off and kick-off procedures) or on the definition of noise standards, they have not addressed the question of social optimality.[2] It is well-known that command-and-control policies do not lead to social optimality in the context of asymmetric information, which is relevant to airport regulation since information about residents' noise disutility is difficult to collect (we shall come back on that point). Even though, in some situations, regulators have been able to create noise abatement incentives by imposing different fees in function of the (theoretical) noise pressures of aircraft categories, they have been unable (or unwilling) to calibrate those fees to the actual noise disutility of the residents surrounding airports. In fact, the efficiency of such fees has not been established; the adequacy of residents' (absence of) compensation has many times remained an open question; and the optimality of the spatial distribution of aircraft movements around airports has not been demonstrated.

It is often argued that residents are already compensated for the social cost of noise disturbances by lower housing rents and prices. Yet, the fact that

[1]Another topical external impact is its contribution to climate change (air transport contributes to 3 percent of world greenhouse gases emissions). To cope with this problem, air transport sector will soon be included in the emission targets negotiated under the Kyoto protocol and in the European Emission Trading Scheme (EU-ETS) which regulates carbon dioxide emissions.

[2]See for instance Janic (1999) and Brueckner (2003). Brueckner and Girvin (2006) discuss the optimal taxation of aircrafts given a fixed global quota of noise emissions but do not consider the social cost of residents' noise exposure.

residents are compensated does not imply that social costs are internalized. Whereas such costs can be shifted from tenants to landlords who are bound to offer lower house rents, they are generally not shifted to the firms that generate the noise externality. Except in the rare situations where airports acquire the surrounding properties[3], the social costs of noise disturbances are not internalized and there exists a need to design economic instruments that organize such an internalization.

The debate about the internalization of social costs is well-known to economists. Noting the reciprocal nature of harmful effects, Coase (1960) suggested to define appropriate property rights over the source of those effects and showed that regulation can be accomplished effectively and efficiently by a market. The present paper applies this idea in the context of noise disturbances and airport activity. In our setting, the tradeable noise permits will constitute the means of the negotiation between residents and airline companies. And in order to compensate the residents for noise exposure the property rights will be assigned to them. By selling those rights on a market, the residents will express their willingness to accept noise.

The contribution of our paper is to show how a local market of tradeable noise permits allows to implement the socially optimal number and spatial distribution of flights among routes. The market for tradeable noise permits is organized as follows. Residents are organized by zones that sell noise permits to airline companies. Each noise permit consists of a right for one aircraft to fly over a specific zone during some time period. The supply of permits is set by the zones and bounded only by technical feasibility.[4] A (neutral) auctioneer collects the bids of zones' supplies and airline companies' demands, determines the price that clears the market and redistributes the permits according to the bids. In equilibrium, the price of a noise permit is equal to both the marginal profit of the additional flight from/to the airport and to the disutility of additional noise disturbance in the most disturbed area. Like any other market for tradeable permits, this market for noise permits offers a distinct advantage over traditional instruments like command-and-control, urban planning or fees. Once the market is adequately designed it reaches an equilibrium in which social benefit of the airlines' activity is balanced with its social costs. Because residents have full rights on the noise permits, they are

[3]To our knowledge, Flughafen Dusseldorf is one airport which has pursued a policy of house purchase in noisy areas. Some airports have been relocated in housing free areas (*e.g.* Oslo, Montreal-Mirabel).

[4]The maximal number of permits is equal to the maximal number of movements the airport can sustain during the specified time period.

never worse off. At the same time, the social costs of noise disturbances are internalized through the purchase of noise permits by airline companies. This makes void the debate about airport noise disturbances, and local governments need no longer be involved in cumbersome information collection/studies on noise disturbances and in political arbitrage between supporters of environmental quality and airport economic activity. Thus, a local market where airline companies and airport residential neighbors can trade noise permits provides governments with an efficient instrument to correct the aircraft noise externalities.

Designing a market for tradeable permits in the context of noise disturbances raises many theoretical challenges. The first one relates to the complementarity of zones over the routes. This complementarity results from the fact that, to fly over a route an aircraft needs to buy the permits of all the zones of that route. We show that zones always supply tradeable noise permits and no zone is willing to block the airplane traffic. The impact of a specific zone on the permit price is limited by the presence of competing zones located on different routes. The second challenge relates to the spatial dimension of our market. In particular, we analyze how the design of zones and routes shapes the market equilibrium. We show that there exists a large class of feasible designs and that, under some conditions, it is possible for the market designer to replicate the socially optimal number and spatial distribution of flights with an appropriate design. The third challenge relates to the possibility for planes to fly over different jurisdictions where local governments' objectives naturally differ. The fourth challenge relates to the possibility of zones' strategic behaviors on the market design. The last challenge is to capture the long run effects of the market for tradeable noise permit on the urban structure.

There exists a vast literature on the evaluation of the impacts of airport activities on property values in residential areas. This literature mainly focuses on the environmental impact of airports on neighboring residents and economic activity. McMillen (2004), Nelson (2004) or Schipper (2004) propose recent updates. On the one hand, Schipper (2004) shows that the medium value of environmental costs in a set of 35 European airport areas is 0.0201 Euro per passenger-km, the noise costs counting for 75 pc.[5] Numerous empirical studies have confirmed that aircraft noise influences property values around airports. Using the hedonic approach, Baranzini and Ramirez (2005) have recently used housing market data to infer the noise impact on housing rents in Geneva, Switzerland. They show that the impact of all sources of noise on

[5]This means an environmental cost of about 2000 Euro per 100-seat aircraft flight over 1000 km.

housing rents is about 0.7 per cent per acoustic decibels [6] and about 1 per cent when considering exclusively airplane noise in the airport area. Interestingly, this measure does not significantly change with noise measuring procedures and with the institutional structure of the housing market (private versus government ownership). On the other hand the local economic benefits of airports are generally significant, covering tax revenues and direct and indirect employment opportunities. As soon as expansions are to be discussed, these issues become much trickier. Opening of new runways typically exacerbate the dilemma between noise disturbances and economic benefits. According to the US Federal Aviation Administration [7], 18 of the 31 large hub airports in the US plan to add runways in the next decade. On the other hand, Brueckner (2003) estimates that the O'Hare expansion would raise service related employment in the Chicago area by 185,000 jobs. Yet, this literature on costs and benefits of airport activity and noise disturbances does not address the problem of internalization of the externality between aircrafts, airports and residents.

One may also wonder about the ability or the willingness of governments and relevant institutions to implement the optimal number and spatial distribution of aircraft movements. In 2001, the International Civil Aviation Organization (ICAO) Assembly endorsed the concept of a "balanced approach" to aircraft noise management.[8] This consists of identifying the noise problem at an airport and then analyzing the various measures available to reduce noise through the exploration of four elements, namely *(i)* reduction at source (quieter aircraft), *(ii)* land-use planning and management, *(iii)* noise abatement operational procedures and *(iv)* operating restrictions. This organization aims to address the noise problem in the most cost-effective manner. In a recent directive, the European Commission also advocates a similar "balanced approach" to aircraft noise management (Directive 2002/30/EC, European Commission, 2002). The European Commission's aims are however broader than those of ICAO as the Commission seeks to limit or reduce the number of people significantly affected by the harmful effects of noise, and to achieve maximum environmental benefit in the most cost-effective manner. The cost-effectiveness of the policy is clearly claimed but the choice of the policy instrument able to implement the policy remains open, and this is the very purpose of this article.

[6] A unit of acoustic decibel (dBA) is a (logarithmic) measure of the sound pressure and therefore of the intensity of a sound perceived by humans. Because a sound is perceived differently according to its frequency, sound pressures are corrected by a "A weighting filter".

[7] Source: US Department of transportation.

[8] See *www.icao.int*.

Finally, our discussion on a market for tradeable noise permits also relates to environmental economics. Although tradeable permits have been promoted as a policy instruments for environmental issues since Dales (1968), they were generally regarded as impractical despite their theoretically attractive properties (cost-efficiency). Yet, since the 1990s, many instances of markets of tradeable permits have been successfully implemented, most notably for sulfur dioxide pollutant in the US power industry and for carbon dioxide in the EU (see Ellerman (2005) for a comprehensive introductory note on tradeable permits). All these market experiences are inspired by Montgomery (1972) who formally showed that under a global emission constraint, competitive markets of tradeable permits yield cost efficient allocations of pollution abatement whatever the distribution of permits amongst polluters. In such a market, the emission cap coincides with the total number of emission permits, those being emitted by the regulator and trade occurring among polluters.

Clearly, a market *à la* Montgomery seems inappropriate for our subject of concern, for two reasons. First, we are interested in a design that makes the airport activity acceptable to neighboring residents. Since the latter are victims of aircraft noise disturbances, this implies that property rights on noise are granted to those residents. Second, in the previously cited cases, regulators impose arbitrary global emission ceilings. By contrast, we look for a market design that is able to implement the socially optimal allocation of air traffic amongst aircraft routes and that does not require the regulator's intervention after the creation of the market.

This article offers a contribution that is both policy-oriented and methodological. It presents an original and efficient policy instrument to regulate noise disturbances around airports. It proposes a new application of the concept of tradeable permits to the issue of noise exposure, with an emphasis on the spatial dimension of the problem.

The article is organized as follows. Section 2 presents the setting while Section 3 derives and discusses our proposal for a market for tradeable noise permits. Section 4 proposes the optimal market design in the standard case of price taking zones. In Sections 5 and 6, two extensions are considered, one when there are flights over independent jurisdictions, and the other when some zones are strategic. In all cases we will see that the optimal solution can be implemented, but under specific conditions. Section 7 develops the short run and long run equilibria and shows how the market for tradeable noise permits shapes the city structure. The conclusion follows.

2 The setting

2.1 The airport

Let us consider a civil airport located in the neighborhood of a large city. Air traffic is organized along several routes that airplanes may take when they land and take off. Landing and take-off routes are determined by exogenous technical characteristic, in particular the direction of wind. Yet, within the same set of technical parameters, there exist several route possibilities. For example, after the take-off, aircrafts may remain at low altitude, or go up, and they may go right or left. We denote $\overline{\mathcal{R}}$ and $\overline{R}$ the set and the number of all technically possible routes ($\overline{R} = \#\overline{\mathcal{R}} \geq 1$). We denote $\mathcal{R}$ ($\mathcal{R} \subseteq \overline{\mathcal{R}}$) and R ($R \leq \overline{R}$) the set and number of routes that are actually used. Along a route $r \in \mathcal{R}$, airplanes generate noise disturbances that vary according to their altitude and acceleration. Let t be the distance from the airport on a given route and let y_r be the total number of planes on route r.

2.2 The residents

The residents are homogenous with respect to their income and disutility from noise disturbance but they differ according to their distance t to the airport. We shall assume that aircrafts are homogeneous in term of their noise emission. Therefore, residents are concerned only about the number of flights over their location.

Thus, resident i located on route r is endowed with an individual utility function equal to $U_r^i(t, y_r) = I^i - d_r(t, y_r)$, where I^i is her income and $d_r(t, y_r)$ is her disutility from noise disturbances. This disutility firstly depends on the distance from the airport: in general, the closer to the airport, the worse are the noise disturbances. It secondly depends on the number of flights. For simplicity we assume that $d_r(t, y_r) = \delta_r(t) y_r^2/2$ where $\delta_r(t)$ is a location-noise disutility parameter on that route. The parameter $\delta(t)$ reflects the loss of utility suffered by a resident when an aircraft flights over the location t on route r. So, $\delta(t)$ is typically larger in locations where aircraft have lower altitude, boost engine power and/or use flaps. This parameter depends on the profile of the route, which we take as given.[9] Figure 1 depicts an example of an airport with several routes as well as an example of a profile of location-noise disutility parameter $\delta_r(t)$. The model generalizes to any increasing and convex

[9] The profile of a route includes the altitude, direction, speed and acceleration of aircraft that are prescribed in landing and taking-off procedures.

disutility function d_r.

The population size at location t on route r is $n_r(t)$. Let $\beta_r(t) = \delta_r(t)n_r(t)$ be the total disutility parameter at location t on route r and let T_r be the distance at which noise disturbance is no longer considered.[10] One then computes the aggregate disutility parameter on route r as $B_r(t) \equiv \int_0^t \beta_r(t)dt$. Figure 2 provides an example of the distribution of the total disutility parameter on a route. It is important to observe that whereas individuals located close to the airport have higher disutility $\delta(t)$, the total disutility parameter $\beta(t)$ may be higher elsewhere because a larger population density there.

Insert Figure 1 about here

2.3 Airline companies

Airplanes are supposed to belong to independent profit maximizing companies. Each flight's profitability varies with its characteristics such as travelers demand, flight distance, indivisibility, etc. There thus exists vertical differentiation of flights which we summarize by the following profit function $\pi(x) = \overline{\pi} - x$ where x is the index of the flight $x \in [0, \overline{\pi}]$, P_r the price of the route taken and $\overline{\pi} > 0$ the profit of the most profitable flight. Under this assumption, all flights are profitable. We assume for simplicity that x is uniformly distributed on the interval $[0, \overline{\pi}]$.

2.4 The first-best

It is instructive to derive the allocation of flights under a command and control policy where a planner determines the number and the distribution of flights over possible routes. The planner maximizes a weighted sum of profits and resident utility. Its problem is to find the set of routes $\mathcal{R}$ and the allocation of flights $\{y_r\}$ such that

$$\max_{\mathcal{R}, \{y_r\}} W = \gamma \int_0^y (\overline{\pi} - x)\, dx - \sum_{r \in \mathcal{R}} \int_0^T d_r(t, y_r) n_r(t) dt \quad \text{s.t.} \quad y = \sum_{r \in \mathcal{R}} y_r$$

[10]Residential areas with low or zero noise disturbance are not considered for noise contention issues. Typically, this applies to areas with an annual average noise level lower than 55 decibels (L_{dn}).

where γ is the weight the planner puts on profit. It is zero when profits do not accrue to local residents. It can be larger than one when the airport activity generates consumer surplus or additional peripheral activity. For example, suppose that demands for travel are quadratic functions and that flights incur only marginal costs. Then consumer surplus is equal to half of the profit generated by firms. In this case, a planner considering all profits and consumer surplus would set $\gamma = 1 + 1/2 = 3/2$. When some share of consumption and profits accrues to individuals that do not belong to the jurisdiction of the planner, the latter would set a smaller γ.

Insert Figure 2 about here

The first order conditions read $\gamma(\overline{\pi} - y) = y_r B_r, \forall r \in \mathcal{R}$. The planner's solution yields the following total number of flights and its distribution amongst routes:

$$y^o = \frac{\overline{\pi}}{1 + \left[\gamma \sum_{r \in R} B_r^{-1}\right]^{-1}} \quad \text{and} \quad \frac{y_r^o}{y^o} = \frac{B_r^{-1}}{\sum_{s \in R} B_s^{-1}} \tag{1}$$

The optimal number of flights y^o increases with the aircraft profitability and decreases with the aggregate noise pollution in routes. The number of flights on a route falls as the aggregate noise pollution on this route increases.

Plugging this solution into the planner's objective, we get $W = \overline{\pi} y^o / 2$ which increases in y^o. Since y^o rises with the number of routes R, the planner chooses to use all routes: $\mathcal{R}^o = \overline{\mathcal{R}}$. This result naturally follows from the assumption that disutility is convex and, importantly, it does not mean that the number of flights will be equal on all routes, as shown in equation (1).

This solution would correspond to a command-and-control solution where the regulator would fix the number of flights on each route. However, command-and-control policy requires full information on residents' preferences, airlines profitability and distribution of noise disturbances. Furthermore, even under full information such a policy would not provide the regulator with the funds necessary to compensate residents. Moreover, this solution can not be decentralized by a tax system. Indeed, whereas a tax equal to $\overline{\pi} - y^o$ allows the planner to implement the total number of flights, there exists no tax instrument that allows it to allocate flights amongst routes. Furthermore, the redistribution of the proceeds of the fees to the residents is impossible without knowing their preferences. So a tax system would leave unsolved major parts of the problem (flight allocation and compensation).

Thus, neither command-and-control nor fees seem adequate to optimally regulate noise disturbances around the airport. Alternative policy instruments are required. In the following we discuss the implementation of this optimal allocation of flights with a market for tradeable noise permits. Such a market would not only implement the optimal flight's allocation but would also compensate residents for noise disturbances.

3 The market for tradeable noise permits

We consider a market as in Coase's problem, that is, we define property rights (the right to benefit from a quiet environment), and we give these rights to the residents[11] and propose to create an instrument able to solve the bargaining problem between residents and airline companies, namely tradeable noise permits. Montgomery (1972) has proved that, under perfect competition such a market yields the optimal solution. We make Montgomery's (1972) assumption that firms and zone representatives are price takers. We also assume the presence of a 'neutral auctioneer' who collects bids until he finds an equilibrium.

The main feature of our market for tradeable noise permits is its spatial dimension. Residents are distributed over the space under aircraft routes and airline companies are required to buy the permits to take a route. A natural question arises about the possible organization of residents in this market. In this paper we assume that residents are organized in neighborhoods, which we here call 'zones'. Zones are heterogenous with respect to the noise exposure and to the size of the disturbed population. In each zone a representative agent acts in the market for tradeable noise permits on behalf of his/her co-residents. Such an agent may be a representative of a residents' association or of a municipality. The design of this market therefore includes the choice of routes, the definition of the zones and the role of the market auctioneer. Figure 1 and 2 depict the case of an airport with various routes that each includes three zones.

Regarding the spatial dimension (the definition of routes and zones) we have to introduce some notation. Let $\mathcal{T}_r$ denotes a partition of the interval $[0, T_r]$ of route $r \in \mathcal{R}$ into several zones, and let $Z_r = \#\mathcal{T}_r \geq 1$ be the corresponding number of zones. Hence $\mathcal{T}_r$ is equal to the set $\{[0, t_{r,1}], ..., (t_{r,z-1}, t_{r,z}], ..., (t_{r,Z_r-1}, T_r]\}$. We then denote by z the index of the zone $(t_{r,z-1}, t_{r,z}]$ and we

[11]It is well-known that, under the Coase Theorem, the optimal solution is reached whatever the allocation of property rights. Distributional effects will differ, however.

denote by $\mathcal{Z}_r = \{1, ..., Z_r\}$ the set of such indices for route r.

3.1 The supply and the demand of noise permits

In each zone the representative is allowed to sell noise permits at a price p_{rz}. To have one flight passing over the zone $z \in \mathcal{Z}_r$ #during a specified time period,[12]# an airline company must buy one permit at this price. Hence to take the route r, the airline company must purchase the bundle of zones' permits for a total the price of $P_r \equiv \sum_z p_{rz}$. We assume that the representative is utilitarian and considers the following aggregate utility function:

$$U_{rz}(y_{rz}) \equiv \int_{t_{r,z-1}}^{t_{r,z}} U_r^i(t, y_{rz}) n_r(t) dt$$

One can write

$$\int_{t_{r,z-1}}^{t_{r,z}} d_r(t, y_{rz}) n_r(t) dt = \frac{1}{2} \alpha_{rz} y_{rz}^2$$

where

$$\alpha_{rz} \equiv \int_{t_{r,z-1}}^{t_{r,z}} \beta_r(t) dt$$

is a parameter measuring the total disutility *over the zone.* Notice that aggregate disutility depends both on individual utility function and population size in that zone.[13] Given that the amount of money raised by the zone is equal to $p_{rz} y_{rz}$, the representative's utility is given by

$$U_{rz}(y_{rz}) = p_{rz} y_{rz} - \frac{1}{2} \alpha_{rz} y_{rz}^2$$

For the moment, we assume that representatives are price takers. So, each representative maximizes this function by choosing the amount of permits to sell and considering the price p_{rz} as a given. The first order condition is equal to $p_{rz} = \alpha_{rz} y_{rz}$, which yields the following supply of permits

$$y_{rz}^S(p_{rz}) = p_{rz}/\alpha_{rz} \tag{2}$$

[12]The attributes of the specified time period can be day/night, or smaller time periods, working days/week-ends. In case of several time periods, airlines redistribute their flight activity according the price of tradeable noise permit in each time period. For the sake of exposition, this paper focuses on a market defined over a single time period.

[13]We consider homogenous noise pressures amongst aircraft categories. When aircraft are heterogenous in noise pressures, the parameters $\delta_r(t)$, $\beta_r(t)$ and α_{rz} relate to the 'average' noise disutility at location t in zone z on route r. Incentives for noise abatement may easily be introduced by requiring airlines to purchase shares of noise permits that are proportional to the aircraft noise pressure.

The demand for permits comes from airline companies. We assume that flights are managed by independent profit maximizing companies. The latter have to purchase a bundle of permits to take a route. The profit function of the flight $x \in [0, \overline{\pi}]$ is now $\pi(x) = \overline{\pi} - x - P_r$ where P_r is the cost of taking route r. The demand for permits is therefore given by

$$y_r^D(P_1, ..., P_R) = \begin{cases} 0 & \text{if} \quad P_r > P_{r'} \\ [0, \overline{\pi} - P_r] & \text{if} \quad P_r = P_{r'} \\ \overline{\pi} - P_r & \text{if} \quad P_r < P_{r'} \end{cases}$$

3.2 The equilibrium

We now define the role of the auctioneer in the market for noise permits. The latter collects supplies and demands from residents and firms and is in charge to find an equilibrium price. As for any competitive market we assume that the only information the auctioneer gets is the amounts of permits supplied and demanded by residents and firms on each route. As a consequence, the auctioneer is not allowed to discriminate across zones. In our setting, this has a particular implication: the auctioneer must propose the same price for all zones located on a given route. Formally, $p_{rz} \equiv P_r/Z_r \; \forall r \in \mathcal{R}, \forall z \in \mathcal{Z}_r$.

The permits supplied by different zones over a same route are perfect complement for airline companies. At a given set of prices, the total supply is given by the most restrictive use of the route in every zone. That is,

$$y_r^S(P_r/Z_r) = \min_{z \in \mathcal{Z}_r} y_{rz}^S(P_r/Z_r)$$

Finally the instruments in that market design include the set of open routes $\mathcal{R} \subseteq \overline{\mathcal{R}}$, the definition of zones $\{\mathcal{T}_r\}_{r \in \mathcal{R}}$.

Definition 1 *A market equilibrium consists in a set of prices P_r^* and the numbers of flights y_r^* such that, given the value of the policy instruments $\{\mathcal{R}, \{\mathcal{T}_r\}\}$, the market clears, that is, $y_r^D(P_1^*, ..., P_R^*) = y_r^S(P_r^*/Z_r)$, $\forall r \in \mathcal{R}, \forall z \in \mathcal{Z}$.*

Such an equilibrium implies that there exists no allocation of routes that, at prevailing prices, would be preferred by any airline company or by any zones' representative. It also means that no resources are lost, given that the auctioneer makes no profit.

In this market equilibrium, the zone with the highest total disutility offers the smallest number of permits and thus it determines the number of flights over the route. We call this zone the *critical zone*. This can be seen by using the supply function defined above:

$$\min_{z\in\mathcal{Z}_r} y^S_{rz}(P_r/Z_r) = \min_{z\in\mathcal{Z}_r} \{P_r/(Z_r\alpha_{rz})\} = P_r/(\overline{\alpha}_r Z_r)$$

where

$$\overline{\alpha}_r \equiv \max_{z\in\mathcal{Z}_r} \alpha_{rz}$$

is a parameter measuring the total disutility over the critical zone. Equating this supply function with demand yields the following proposition.

Proposition 1 *The equilibrium in the market for tradeable noise permits exists and is unique. It is such that,*

$$P^*_r = P^* \equiv \frac{\overline{\pi}}{1+\sum_{r\in\mathcal{R}} (\overline{\alpha}_r Z_r)^{-1}} \quad \text{and} \quad y^* = \frac{\overline{\pi}}{1+\left(\sum_{r\in\mathcal{R}} (\overline{\alpha}_r Z_r)^{-1}\right)^{-1}} \tag{3}$$

Given that $y^S_r = P^*/(\overline{\alpha}_r Z_r)$ *we also have*

$$\frac{y^*_r}{y^*} = \frac{(\overline{\alpha}_r Z_r)^{-1}}{\sum_{s\in\mathcal{R}} (\overline{\alpha}_s Z_s)^{-1}} \tag{4}$$

Proof. See appendix. ■

Clearly, the design of the noise permits' market may have an impact on the equilibrium price and number of flights. It is easy to show that adding new routes reduces the prices of noise permits and raises the total aircraft activity ($\mathcal{R} \subset \mathcal{R}' \Rightarrow P^* > P^{*\prime}$ and $y^* < y^{*\prime}$). The impact of design of zones is manyfold: it does not only depend on the number of zones on each route but also on the definition of critical zones. Suppose indeed that the market designer adds a zone on route r, without altering any critical zones. Therefore, we have that Z_r increases while $\overline{\alpha}_s, \forall s \in \mathcal{R}$, remain constant. From Proposition 1, it follows that, in equilibrium, the price of routes increases and the total number of flights decreases. Also, flights are re-allocated away from route r. This needs not to be the case for different design of the routes. Suppose indeed that the market designer adds a new zone on route r while keeping the noise disturbances equal in every zone: $\alpha_{rz} \equiv \overline{\alpha}_r \equiv B_r/Z_r\ \forall z$. This implies that $\overline{\alpha}_r Z_r$ remains constant and that neither the price of routes nor the total number of flights change as Z_r increases. By the same token, the allocation of flights over routes remains unchanged. This is because a rise in the number of zones is associated (1) to a fall in the population of each zone and (2) to a reduction in its share of the noise permit revenue on the route, $P_r y_r / Z_r$. The first effect reduces the disutility of zone representatives who, at a given price, augment their supply of noise permits. The second effect reduces the revenues of zone representatives who restrict their supply. In equilibrium the two effects exactly balance.[14] Finally, it is easy to construct examples where the market designer adds new zones but reduces the size of critical zones so that the equilibrium permit price fall and the number of flights increases. Actually, in the design of zones ($\mathcal{T}_r$), the number of zones and the size of the critical zones are, to some extent, substitute instruments.[15]

The properties of the market for noise permits crucially depends on the complementarity of noise disturbances on each route. Indeed, noise disturbance is a complementary bad for all zones on the same route since aircrafts must fly over all those zones. Typically, this may lead to a "tragedy of the commons" where agents do not internalize the global effect of their decisions. In our context, each residents' representative owns the right to issue noise permits and does not internalize the benefits that other zones have. One may conjecture that the number of permits and flights is inefficiently low. Another way to see this is to observe that noise permits offered by different zones on a same route are complementary goods. Therefore noise permits is suspected to be subject to Cournot's conjecture about under-provision of complementary goods. In this conjecture, independent suppliers of complementary goods would set inefficiently high price levels because they would not internalize the effect of their decisions on others. In our setting, the tragedy of commons exists but is under the control of the market designer. By defining the zones, the latter is able to tune this effect and is also able to set the level of flight activity. Notice that the market designer is even able to neutralize the tragedy of commons by designing homogenous zones ($\alpha_{rz} \equiv \overline{\alpha}_r \equiv B_r/Z_r$). In this case the creation of new zones is balanced with a smaller disutility in the zones, which leaves prices and flight activity unchanged.

Because the division of routes into heterogenous zones allows to tune the total air traffic, it constitutes an important aspect of the design of a market for noise permits. In the following we show that this market allows the implement the social planner's solution.

[14] These effects may not balance if zone representative are not utilitarian. Yet, the assumption of utilitarian zone representatives constitutes an interesting benchmark.

[15] In practice, it may be unrealistic to consider the very large number of zones on a route of a given size.

4 An optimal market design

The market designer has a considerable degree of freedom in the divisions of routes into zones. It then is natural to ask whether and when an appropriate market design leads to the social planner's outcome in terms of route allocation and total number of flights.# In our market for tradeable permits the planner has the ability to choose two sets of instruments: the routes and the definition of zones. In this section we show the condition under which the market designer is able to implement the social planner's optimal solution by an adequate choice of these instruments. To this end, we compare the first best allocation with the competitive equilibrium. We do this, first, in the general setting. Then, we simplify the market design by imposing that all zones have equal length in all routes. Finally we consider the case where noise disturbances cover several jurisdictions. In all cases we will see that the optimal market design for the market of tradeable noise permits is achievable for some set of parameters.

4.1 The generic case

Can the market designer define the routes and the zones such that the market for noise permits replicates social planner's optimal solution? The instruments for the market design include the set of routes $\mathcal{R} \in \overline{\mathcal{R}}$ and the definition of zones $\{\mathcal{T}_r\}$. We say that the planner can implement the optimal number and allocation of flights with a market for noise permits if and only if there exists a set of instruments $(\mathcal{R},\{\mathcal{T}_r\})$ such that $y_r^* = y_r^o$ $\forall r \in \mathcal{R}^o$. The efficient route allocation will be obtained if and only if $\frac{y_r^o}{y^o} = \frac{y_r^*}{y^*}$, that is, by using (1) and (4)

$$\frac{\sum_{s\in\mathcal{R}} (\overline{\alpha}_s Z_s)^{-1}}{\sum_{s\in\mathcal{R}^o} B_s^{-1}} = \frac{(\overline{\alpha}_r Z_r)^{-1}}{B_r^{-1}} \quad \forall r \in \mathcal{R} \tag{5}$$

The efficient activity level will be reached if and only if $y^* = y^o$, that is, by using (1) and (3)

$$\frac{\sum_{s\in\mathcal{R}} (\overline{\alpha}_s Z_s)^{-1}}{\sum_{s\in\mathcal{R}^o} B_s^{-1}} = \gamma \tag{6}$$

These two equalities imply

$$\overline{\alpha}_r Z_r = \frac{1}{\gamma} B_r, \quad \forall r \in \mathcal{R} \tag{7}$$

Furthermore, plugging expression (7) in (5) yields the equality

$$\sum_{s\in\mathcal{R}} B_s^{-1} = \sum_{s\in\mathcal{R}^o} B_s^{-1}$$

which implies that $\mathcal{R} = \mathcal{R}^o$. Since $\mathcal{R}^o = \overline{\mathcal{R}}$, the planner allows residents located on any feasible routes to supply noise permits: $\mathcal{R} = \overline{\mathcal{R}}$.

Condition (7) determines the trade-off in the design of the market for noise permits. To fulfill this condition, the market designer can either adapt the size of the critical zone or the number of zones. Note that the value of $\overline{\alpha}_r Z_r$ used in expression (7) is a function of the design of zones. This function $\overline{\alpha}_r Z_r$ is bounded below by B_r when there is only one zone ($Z_r = 1$). As shown in the proof below, it also is easy to find a design of zones such that this function $\overline{\alpha}_r Z_r$ is made equal to any real number above B_r. As a result there always exists a design of zone such that the value $Z_r \overline{\alpha}_r$ lies above B_r. From this argument, it comes that the market designer will be able to find a design of zones that verifies condition (7) only if γ is not too larger, formally, if $\gamma \leq 1$. The following proposition states the necessary and sufficient condition for implementability.

Proposition 2 *The social planner's optimal allocation of flights can be implemented by a market for tradeable noise permits if and only if $\gamma \leq 1$. In this case, the market designer opens all feasible routes ($\mathcal{R} = \mathcal{R}^o = \overline{\mathcal{R}}$) and sets the design of zones $\{\mathcal{T}_r\}$ such that $\overline{\alpha}_r Z_r = B_r/\gamma, \quad \forall r \in \overline{\mathcal{R}}$.*

Proof. See Appendix. ■

Under Proposition 2 the market designer is able to implement its optimal allocation of flights by an appropriate design of zones only if he/she puts not too large weight on the economic activity of aircraft ($\gamma \leq 1$). In this case, he/she prefers a low aircraft activity and he/she can raise either the number of non-critical zones or the size of the critical zones so that the prices of routes increase. The number of zones and the size of the critical zones are therefore substitutable instruments, which gives the planner some freedom in market design. By contrast, the planner is not able to implement its optimal solution with a market for tradeable noise permits when he/she puts too high a weight on profits or economic activities ($\gamma > 1$). In this case, he/she desires a flight activity that conflicts too strongly with the residents' interest. Residents indeed set prices of tradeable noise permits that are too high and the aircraft activity remains too small compared to the optimal level even if the planner has reduced the price of flying over a route to its minimal value (when $Z_r = 1$).

It is interesting to discuss the situation where noise disturbances are uniformly distributed over each route. For instance, this can occur when aircrafts keep the same altitude on neighborhoods. In this case, the location-noise disutility parameter is constant on each route: $\delta_r(t) = \delta_r$. So, condition (7) writes as

$$\frac{\int_{t_{r,\overline{z}-1}}^{t_{r,\overline{z}}} n_r(t)dt}{\int_0^T n_r(t)dt} = \frac{1}{Z_r \gamma} \qquad \forall r$$

The market design requires only the information on population densities. The left hand side of this equation represents the proportion of the population of route r in the critical zone. Once this proportion is chosen, the optimal number of zones can be set. For example, suppose that $\gamma = 1/2$ and that the market design includes a critical zone covering 40 percent of the population on the route. Then the optimal number of zones is $Z_r = 5$.

It also is interesting to discuss the dual situation where the population density is uniform over each route. This case approximates the situation where one route passes over a highly populated city and another route passes over a less dense sprawl. In this case, the population density is constant on each route: $n_r(t) = n_r$. So, condition (7) writes as

$$\frac{\int_{t_{r,\overline{z}-1}}^{t_{r,\overline{z}}} \delta_r(t)dt}{\int_0^T \delta_r(t)dt} = \frac{1}{Z_r \gamma} \qquad \forall r$$

The market design requires only information on noise profiles and *individual* disutility of noise exposure. Assuming that noise levels are proportionally reflected in the individual disutility of noise exposure, the market designer can base his/her design of zones on the noise profiles.

In Proposition 2 the number of zones and the boundaries of critical zones are two substitutable instruments. Imposing a restriction on one of these instruments does not necessarily prevents the market designer from implementing the social optimum. For instance, if the number of zones Z_r cannot be freely chosen, the market designer is still able to choose the boundaries of the critical zone (i.e. choose $\overline{\alpha}_r$). Conversely, if the critical zones cannot be freely chosen, then he/she is still able to choose the number of zones such that condition (7) holds. However, one may wonder about the feasibility of the implementation of the social optimum allocation when the market designer faces some constraints on those two instruments. Of course, when the zones are exogenously defined for instance by administrative boundaries, the market designer has no degree of freedom in his/her design and the social optimum cannot be attained. In the following sections, we consider an intermediate situation in which the market design is still possible but for a smaller set of parameters.

4.2 An example of a restricted design

Let us consider for expositional purposes the case of an additional constraint that imposes that routes have equal lengths $T_r \equiv T$ and equal number of zones $Z_r \equiv Z$. As a result, routes are divided into equal intervals, $t_{z-1} - t_z = T/Z$. We here show that this restriction simultaneously fixes the number and the boundaries of zones and that it reduces the market designer's degree of freedom more than necessarily. By equation (7) we get that,

$$\overline{\alpha}_r Z = \frac{B_r}{\gamma}$$

which must be compatible with $Z \geq 1$. We remind that the function $\overline{\alpha}_r Z$ is bounded below by B_r $(Z = 1)$. This function also is bounded above by $T\beta_r^{\max}$ where $\beta_r^{\max} \equiv \max_{t\in[0,T]} \beta_r(t)$. Indeed, zone lengths are equal to T/Z and, when Z is large enough the disutility parameter of the critical zone, $\overline{\alpha}_r$, can be approximated by $\beta_r^{\max} * (T/Z)$. As a result, $\overline{\alpha}_r Z$ is approximated by $T\beta_r^{\max}$ and tends to this value when $Z \to \infty$. The market designer is then able to implement the optimal number and allocation of flights with a market for tradeable noise permits if B_r/γ takes any value between these two bounds. This argument yields the following proposition.

Proposition 3 *Suppose the market designer is constrained to design zones and routes with equal lengths. Then, the socially optimal allocation of flights can be implemented by a market for tradeable noise permits if and only if* $B_r/\left(T\beta_r^{\max}\right) < \gamma \leq 1$.

Hence, the constraint of zones of equal length does not eliminate the possibility of implementation of the social optimum via a market for tradeable noise permits but it reduces the set of parameters γ for which this is possible. The market designer is not able to implement the optimal allocation of flights when γ is too low because he/she is unable to design a set of zones that induces a high enough price of noise permits and thus a low enough aircraft activity. However, the restriction on equal lengths of zones obliges him/her to reduce at the same time the size and the total disutility of critical zones, which has the effect of decreasing the price.

In this section we have analyzed the design of markets for noise permits and we have established the conditions for which a market designer can implement the socially optimal allocation of flight under the three following conditions: all zones belong to the same jurisdiction, all zone representatives are price takers and the spatial distribution of residents is given. We relax each of those assumptions in the following three sections.

5 Flights over independent jurisdictions

In some situations airport disturbances may cover different jurisdictions with conflicting interests. The airport may for instance bring local benefits to only one jurisdiction (employment, accessibility, ...) whereas flights pass over zones that belong to several jurisdictions. One example is the case of the Brussels Airport that offers large job opportunities to the Flemish region and that dispatches flights over both the Flemish and Brussels regions. In many respects, the two regions' governments are independent institutions. Such a case can be handled in our setting. In this section, we establish the conditions under which a noise permits market can be an appropriate economic instrument for the jurisdiction that hosts the airport, receives the major part of its benefits and organizes the noise permits market.

We assume two jurisdictions A and B, with jurisdiction A hosting the airport. Hence, each route r includes at least a zone in jurisdiction A and maybe some zones in jurisdiction B. Let $\mathcal{T}_r^A \equiv \{...,(t_{r,z-1}^A, t_{r,z}^A],...\}$ be the set of all zones under route r in jurisdiction A. Aggregate noise disturbance measure for this jurisdiction is equal to $B_r^A \equiv \int_{\mathcal{T}_r^A} \beta_r(t)dt \leq B_r$.

We also assume that jurisdiction A is not altruistic with respect to jurisdiction B's residents and that it has a share of economic returns: $\gamma^A \in [0, \gamma]$ $(\gamma^A + \gamma^B = \gamma)$. When jurisdiction A plans its optimal airport activity, it maximizes

$$W^A = \gamma^A \int_0^y (\overline{\pi} - x)\, dx - \sum_{r \in \mathcal{R}} \int_0^{T_r^A} d_r(t, y_r)dt \quad \text{s.t.} \quad y = \sum_{r \in \mathcal{R}} y_r$$

which gives the same solution as in Section 2.4 except that γ and B_r must now be replaced by γ^A and B_r^A :

$$y^A = \frac{\overline{\pi}}{1 + \left[\gamma^A \sum_{r \in \mathcal{R}^A} (B_r^A)^{-1}\right]^{-1}} \quad \text{and} \quad \frac{y_r^A}{y^A} = \frac{\left(B_r^A\right)^{-1}}{\sum_{s \in \mathcal{R}^A} (B_s^A)^{-1}}$$

For the same reason as in Section 2, jurisdiction A chooses to open all routes: $\mathcal{R}^A = \overline{\mathcal{R}}$.

The first expression shows that if jurisdiction A has the full economic benefit of the airport activity $(\gamma^A = \gamma)$ then it plans a higher flight activity than the multi-jurisdictional planner discussed in Section 2. Indeed, jurisdiction A promotes higher flight activity because it accounts for a smaller number of disturbed individuals. Yet, this statement is not always true. One can indeed check that the necessary and sufficient condition for a higher number of flights,

$y^A \geq y^o$, is given by the condition: $\gamma^A/\gamma \geq [\sum_r (B_r^{-1})]/[\sum_r (B_r^A)^{-1}]$. Hence, if jurisdiction A gets a small enough share of economic benefit it will plan a flight activity smaller than the multi-jurisdictional planner's.

The second expression shows that flight allocation amongst a route decreases as the noise disturbances over jurisdiction A's territory increase. In particular, jurisdiction A will allocate a large proportion of flights on the routes that pass over few residents of its jurisdiction (small B_r^A).

We now ask whether jurisdiction A is able to implement its own social optimal allocation of flights while not making jurisdiction B worse off. Because a market for tradeable noise permits cannot make residents in any zone worse off, residents of jurisdiction B surely have an interest to accept the implementation of such a market. Therefore, the question simply becomes whether a market designer is able to choose the design of routes and zones that implements jurisdiction A's social optimal allocation of flights. A natural constraint in this design is that no zone can overlap the two jurisdictions. For a given design of routes and zones, the noise permits market satisfies the same conditions as in Proposition 1. As in Section 4.1, the airport activity depends on the number and the design of zones and routes. which can be chosen to satisfy the optimal flight allocation for jurisdiction A's planner. Comparing the above expression to Proposition 1, jurisdiction A must choose a design such that

$$Z_r \overline{\alpha}_r = \frac{1}{\gamma^A} B_r^A \quad \forall r \tag{8}$$

The argument is the same as in Section 4.1 except that there must be at least two zones, one in each jurisdiction. There are two types of routes to consider. First, consider the routes that do not pass over jurisdiction B. In that case we get that $B_r^A = B_r$ and that expression (8) is identical to the one in Proposition 1. Therefore, we need the condition $\gamma^A \leq 1$. Second consider the routes that pass over jurisdiction B. Note firstly that there exist two jurisdictions with at least one zone in each. So, the number of zones must be larger than or equal to 2. Secondly, observe that the largest value of the coefficient $\overline{\alpha}_r$ of critical zone is given by either B_r^A or $B_r - B_r^A$, depending on whether the critical zone is in jurisdiction A or B. Therefore the smallest value of $Z_r \overline{\alpha}_r$ is obtained when the market designer sets the smallest number of zones. That is, when $Z_r = 2$ and $Z_r \overline{\alpha}_r = 2 \max\{B_r^A, B_r - B_r^A\}$. Finally, it is possible to set zones so as to increase the value of $Z_r \overline{\alpha}_r$ and find a market design that satisfies expression (8). As a result, a market design is feasible if only if $2 \max\{B_r^A, B_r - B_r^A\} \leq \frac{1}{\gamma^A} B_r^A$. This argument is summarized in the following proposition.

Proposition 4 *Suppose the market designer of jurisdiction A is able to choose the routes, the number of zones and their boundaries. Then, the jurisdiction A's optimal allocation of flights can be implemented by a market for tradeable noise permits if and only if* $\gamma^A \leq \min_{r\in\mathcal{R}^B}\{1/2, B_r^A/\left[2(B_r - B_r^A)\right]\}$ *where* $\mathcal{R}^B$ *is the set of routes that pass over jurisdiction* B.

The model with two jurisdictions imposes two restrictions on the parameter γ^A in order to allow the market designer to implement its preferred allocation with a tradeable noise permits market. When those restrictions are binding, the noise permits market yields a high price and a low flight activity although jurisdiction A puts a high weight on its own economic benefit and promotes a higher flight activity.

The first restriction ($\gamma^A \leq 1/2$) stems from the existence of a second jurisdiction that obliges jurisdiction A to design more than two zones. Since the noise permit price increases with the numbers of zones, jurisdiction A cannot obtain a price as low as the one obtained by the multi-jurisdiction planner who is able to design a market with a unique zone.

The second restriction ($\gamma^A \leq B_r^A/\left[2(B_r - B_r^A)\right]$) stems from jurisdiction A' opportunity to shift noise disturbances to the neighboring jurisdiction B. Jurisdiction A may indeed be willing to concentrate the flight activity on the routes that host only a small share of its residents (small B_r^A) but that host a large share of the residents located in jurisdiction B (large $B_r - B_r^A$). Yet, this strategy cannot be achieved by a market for noise tradeable permits because jurisdiction B's residents do not allow the noise permit price to fall. This line of argument gives some insight into how jurisdiction A should design the residential zones to reach its optimal outcome. If the optimal number of zones Z_r is large (low γ^A), jurisdiction A may ask for bids from small associations of residents (e.g. at district level) in both jurisdictions. If the optimal number of zones Z_r is large, it may organize one zone per route in its territory and ask the other jurisdiction to bid on behalf of its residents. In any case, a multi-jurisdictional government may not be required in such a context.

Note finally that *jurisdiction A should keep the control of the market design over the design of routes and zones passing over jurisdiction B to be able to implement its optimal number and spatial distribution of flights*. Indeed, suppose this is not the case. That is, jurisdiction A aims at inducing its optimal aircraft activity $\{y_r^A\}$ by setting its market design $\left\{\mathcal{T}_r^A\right\}$ that satisfies condition (8) while jurisdiction B is able to choose its design $\left\{\mathcal{T}_r^B\right\}$. Because the aircraft activity is fixed to $\{y_r^A\}$, the noise disutility is fixed everywhere. For the same reason, the demand is fixed to y^A and the price of routes is given by a same value P. As a result, the profits of airline companies are also fixed. Therefore, two terms of jurisdiction B's objectives, noise disutility and profits, are fixed. Jurisdiction B is nevertheless able to alter the last term in its objective related to the proceeds from noise permits on each route: $Py_r^A \ Z_r^B / \left(Z_r^A + Z_r^B\right), \ r \in \mathcal{R}^B$. As a result, jurisdiction B's best strategy is to augment those proceeds by dividing some segments of its routes into infinitely, $Z_r^B \rightarrow \infty$. Jurisdiction A is obviously unable to respond to this strategy because it cannot diminish both Z_r^A and $\overline{\alpha}_r$ to zero in order to keep the condition (8) binding. As a result, a jurisdiction is able to implement its optimal solution only if it receives the full set of rights on the market design. The rights on the market design should be allocated in a way different from the residents' rights on issuing of noise permits.

6 Non competitive markets

In the above market design zone representatives and airlines are assumed to be price takers in the tradeable permits market. This assumption is questionable. Indeed, some airlines may be price makers as they demand a large share of noise permits. At the same time, some critical zones are likely to be price makers because the number of technically feasible routes is not expected to be large, and because non critical zones have no impact on the number of flights over routes. Because our paper focuses on the issue of granting noise permits to residents, it is natural to concentrate our discussion on residents' market power. We therefore elaborate a game theoretic foundation for the noise permits market where the zone representatives can be strategic.

We propose a market design in which each zone representative simultaneously decides on a finite number of noise permits to supply. The 'neutral auctioneer' allocates flights according to the minimum number of permits on each route. This corresponds to a Cournot Nash equilibrium where zone representatives fix numbers of flights. In this section we show the conditions under which there exists a design of zones $\{\mathcal{T}_r\}$ such the social optimal allocation of flights can be implemented by a market for tradeable noise permits under imperfect competition.

Let the utility of the zone representative be defined as $U_{rz}(y_{rz}) = p_{rz}y_{rz} - \alpha_{rz}y_{rz}^2/2$ as before. Each representative sets a number of noise permits y_{rz} over its zone. The market auctioneer sets the number of permits to its minimum over each route, $y_r = \min_z\{y_{rz}\}$; it allocates the same price $P = P_r$ for all routes and all zones $p_{rz} = P/Z_r$; and it balances noise permit supply $\sum_r y_r$ with demand $y \equiv \overline{\pi} - P$. Because the price on routes P depends on other zones, the utility in a zone will depend on other zones. Let y_{-rz} be the set of all supplies of zones different from rz: i.e. $y_{-rz} \equiv \{y_{ij}\}_{ij \neq rz}$. Utility of zone z on route r is given by

$$U_{rz}\left(y_{rz}, y_{-rz}\right) = \frac{1}{Z_r} P\left(y_{rz}, y_{-rz}\right) \min_j \{y_{rj}\} - \alpha_{rz} \left(\min_j \{y_{rj}\}\right)^2 / 2$$

where the market price is equal to

$$P\left(y_{rz}, y_{-rz}\right) \equiv \overline{\pi} - \sum_s \min_j \{y_{sj}\}$$

This utility depends on the number of flights supplied by the critical zone on route r, i.e. $\min_j\{y_{rj}\}$, and it depends on the revenues from the sales of permit which decrease with the number of zones Z_r and with the supplies of critical zones on all routes. A Cournot Nash equilibrium is defined as the number of flights y_{rz}^c such that

$$y_{rz}^c \in \arg\max_{y_{rz}} U_r\left(y_{rz}, y_{-rz}^c\right) \quad \text{for all } r, z$$

We formally derive the best response correspondences and the equilibrium in the Appendix. We here provide an informal proof. The main idea is that non-critical zones are never enticed to supply less than critical zones' supply. So, the supply of critical zones will bind at the equilibrium. As a result, the supply of a route is given by the supply of its critical zone $y_r \equiv \min_z\{y_{rz}\}$ and the utility of the critical zone can be re-written as function $U_r\left(y_r, y_{-r}\right)$ that depends on the supply of the critical zone and its rivals. The best responses then can be written as

$$y_r^{BR}\left(y_{-r}\right) = \arg\max_{y_r} U_r\left(y_r, y_{-r}\right) = \frac{\overline{\pi} - \sum_{s \neq r} y_s}{2 + \alpha_{rz} Z_r}$$

In equilibrium we must have that $y_r^c = y_r^{BR}\left(y_{-r}^c\right)$ for all r. Solving this equality for all routes r, we get

$$y^c = \frac{\overline{\pi}}{1 + \left[\sum_s \left(1 + \overline{\alpha}_s Z_s\right)^{-1}\right]^{-1}} \quad \text{and} \quad \frac{y_r^c}{y^c} = \frac{\left(1 + \overline{\alpha}_r Z_r\right)^{-1}}{\sum_s \left(1 + \overline{\alpha}_s Z_s\right)^{-1}} \tag{9}$$

One readily checks that the number of flights falls with the number of zones Z_r on a route r and with noise disturbances of the critical zones $\overline{\alpha}_r$. Also, the routes that are allocated fewer flights are those with higher aggregate noise disturbances in critical zones: $y_r^c \leq y_s^c \iff \overline{\alpha}_r \geq \overline{\alpha}_s$.

We may compare the Cournot equilibrium with the competitive equilibrium holding the design of routes and zones fixed. First, it is easy to check that the number of permits is smaller in the Cournot situation: $y^c < y^*$. Market power naturally decreases the number of permits. Yet, as the number of routes (not zones!) increases the Cournot number of permits converges to the competitive number because $y^c \to \overline{\pi}$ and $y^* \to \overline{\pi}$. Note that a planner can restore the equilibrium number of flights by reducing the number of zones Z. Second, comparing (4) and (9), one can show that

$$\frac{y_r^c}{y^c} \geq \frac{y_r^*}{y^*} \iff \sum_s \frac{(\overline{\alpha}_s Z_s)^{-1} - (\overline{\alpha}_r Z_r)^{-1}}{(1 + \overline{\alpha}_s Z_s)} \geq 0$$

which is true for $r = \arg\max \overline{\alpha}_s Z_s$ and false for $r = \arg\min \overline{\alpha}_s Z_s$. Hence there exists a subset of routes $\mathcal{R}_c \in \mathcal{R}$ that accept a larger proportion of flights under imperfect competition than under perfect competition and a complementary subset $\mathcal{R} \backslash \mathcal{R}_c$ that accept a smaller proportion of flights. Routes that accept a proportionally larger number of flights under imperfect competition have higher $\overline{\alpha}_r Z_r$. It instructive to study the case where the number of zones is equal: $Z_r = Z$. Then, it naturally comes that a route hosting a critical zone with higher noise disturbances accepts a higher proportion of flights under the Cournot equilibrium than under the competitive one. In other words, when shifting from price taking to price making behavior, such a route reduces proportionally less its supply of permits. This is consistent with the fact that the critical zone on this route displays a higher marginal disutility of noise and, as a result, produces a lower supply elasticity of noise permits. Similarly, one can study the case where critical zones have the same noise disturbance parameters: $\overline{\alpha}_r = \overline{\alpha}$. Then, a critical zone on a route with a larger number of zones will proportionally reduce less its offer of flights when it is able to use its market power. Indeed, the revenue increase caused by the contraction of supply must be shared amongst more zones, so that incentives to exert market power are lower.

Is it possible to replicate the optimal number and allocation of flights under this non competitive equilibrium? In particular, one may expect that representatives of critical zones use their market power to reduce the flight activity its the socially optimal level. Comparing the non competitive equilibrium and the first best allocation yields the following proposition.

Proposition 5 *The social planner's optimal allocation of flights can be implemented by a non competitive market for tradeable noise permits if and only if* $\gamma \leq \min_r \{B_r/(B_r+1)\} < 1$. *In this case, the market designer opens all* *feasible routes ($\mathcal{R}^c = \mathcal{R}^o = \overline{\mathcal{R}}$) and sets the design of zones $\{\mathcal{T}_r\}$ such that $\overline{\alpha}_r Z_r = B_r/\gamma - 1$, $\forall r \in \overline{\mathcal{R}}$.*

Proof. See Appendix. ■

As in the case of perfectly competitive market for noise permits, the optimal allocation of flights can be replicated by the market. Yet, the higher market power of zone representatives reduces the set of parameters γ for which the market for tradeable noise permits implements the first best. At a given design, critical zones offer fewer permits in the non competitive markets so that equilibrium prices are higher and flight activity is smaller. Therefore, the designs under non competitive markets imply smaller flight activity than the same designs under competitive market. As a consequence, when $\gamma \in [0, \min_r B_r/(B_r + 1)]$ so that the planner desires a intermediate level of flight activity, the market designer is unable to implement this flight activity under non competitive markets although it is able to implement it under competitive markets.

7 Residential mobility and city structure

We have shown in the previous sections that a market for tradeable noise permits allows the implementation of a central planner's solution. The main advantage of tradeable permits is their ability to automatically adjust for short term perturbations (*e.g.*, a demand increase) and to allow the revelation of residents' noise disutility. Thus, from a short term viewpoint, the flexibility of this instrument ranks high. The question of how such a market may impact on the economy in the medium and long terms however deserves some attention. To be more precise, our objective in this section is to analyze the effects of implementing a market for tradeable noise permits on the land market and, therefore, on the structure of residential areas. The rational is that, through the proceeds received by residents from the selling of noise permits, the housing and land markets may be affected.

The key feature in the following analysis is the time horizon in which the markets under analysis clear. One may reasonably assume that noise permits should be traded quite frequently, say on a quarterly or annual basis. Hence, this market clearing would occur with a higher frequency than that of the housing market in which search and legal procedures easily span a year, or even more. The land market is the one that clears in the longest time span, say some years. Hereafter, we will disentangle the following three time horizons depending on which market clears: short term (the market for tradeable noise permits clears), medium term (housing market clears) and long term (land market clears). Our current interested lies in the links between these three markets.

For this analysis, let us introduce a new kind of agent, landlords. We model the relationship between airport, residents and landlords as a sequential game that goes as follows. First, landlords choose the lot size which hosts residents. Second, residents choose a place to locate. Third, the land price adjusts. Finally, residents organize themselves and participate in the market for tradeable noise permits. This sequential game reflects the above mentioned idea that noise permits are traded more frequently than land or houses. As in Section 3 we assume that residents and their representatives are price takers.[16]

In this section we focus on the impact of tradeable noise permits on city structure. More specifically, we want to analyze how the population located in each zone is affected by the market for tradeable noise permits. For that reason we neglect any heterogeneity within each zone. As a consequence, we now assume that zones are populated by residents with homogenous noise disutility, that is, $\delta_r(t) = \delta_{rz}$ and $n_r(t) = n_{rz}$, $t \in [t_{r,z-1}, t_{r,z}]$, where n_{rz} is the population size within zone z on route r and where δ_{rz} is the location noise disutility parameter in that zone. Therefore the noise disutility parameter $\beta(t)$ is constant over each zone and equal to $n_{rz}\delta_{rz} = \alpha_{rz}$. Under this assumption, the objectives of residents and their zones representatives are perfectly congruent and the redistribution of the proceeds of noise permits is not an issue.

Each resident's preference for his/her residence lot size s is described by a concave utility function $v(s)$ where $v(0) = 0$ and $\infty > v' > 0 > v''$ for all $s \geq 0$. Note that the population density is given by $1/s$. So, resident i's utility is given by the revenue of noise permits, the disutility of noise, the utility and the rent for residential space:

$$V_{rz}^i = \frac{p_{rz} y_r}{n_{rz}} - \delta_{rz}\frac{y_r^2}{2} + v(s_{rz}) - s_{rz}R_{rz}$$

where R_{rz} is the land rent (per acre). Because residents are homogenous they have the same use of space s_{rz}. The zone's representative has an aggregate utility given by

$$V_{rz} = p_{rz}y_r - \alpha_{rz}\frac{y_r^2}{2} + n_{rz}v(s_{rz}) - n_{rz}s_{rz}R_{rz}$$

Finally, let us assume that the residential area of each disturbed zone consists of a land strip with unit width and with a length of $T_{rz} = t_{r,z-1} - t_{r,z}$.

[16] A similar analysis holds in the case of non competitive markets for tradeable noise permits.

So, T_{rz} is both the length and the residential area of zone z on route r. In the long run, lot sizes cover the residential area so that $T_{rz} = n_{rz}s_{rz}$.

We solve this game backwards, starting with the equilibrium on the noise permits market.

7.1 Short term

In the last period, the land rent R_{rz} and the lot size s_{rz} of each resident in zone z on route r are given. The representative asks for a financial compensation for the aircraft activity that maximizes his/her utility and the market for noise permits clears. The first order condition is the same as in Section 3 and yields the same supply of noise permits: $y_{rz}^S = p_{rz}/\alpha_{rz}$. Hence, the market for tradeable noise permits yields the same outcome; that is, the equilibrium price p^* and the flight allocations y_r^* are given by Proposition 1. The identity of critical zones is still given by $z = \arg\min_z \alpha_{rz}$. The property rights on silence given to residents provides them with some benefits from a so-called *windfall gain*. This corresponds to the additional utility each resident obtains from the sales of noise permits. Indeed using (2) the total utility of a representative over zone z can be computed as

$$U_{rz} = p_{rz}^2/(2\alpha_{rz}) > 0 \tag{10}$$

so that residents get a positive rent from the sales of noise permits. Therefore, in equilibrium, resident i's utility is given by

$$U_{rz}^i = \frac{U_{rz}}{n_{rz}} = \frac{p_{rz}^{*2}}{2n_{rz}\alpha_{rz}} = \frac{1}{2\delta_{rz}}\left(\frac{P^*}{Z_r}\frac{s_{rz}}{T_{rz}}\right)^2 \tag{11}$$

which is positive, increases with the equilibrium price of noise permits P^* and falls as lot size s_{rz} shrinks (and therefore population n_{rz} rises).

The existence of windfall gains is well-known in the literature on environmental economics and stems from the fact that a property right is given for free on productive inputs which acquire a price as soon as the market for tradeable permits is operational. As the compensation offered by noise permits is larger than the noise disturbance disutility in the zones, residents earns a windfall gain. So, as expected, the existence of this windfall gain will influence medium and long term equilibria.

7.2 Mid term

In the second period, residents move across locations while the land price R_{rz} adjusts. The lot sizes and critical zones are still fixed. Let V_o^i be residents' utility outside the disturbed zones. Reasonably, the city is supposed to be large enough so that residents' utility outside the disturbed zones, V_o^i, is independent of the land and permits market in the disturbed zones. Empirical evidence supports this assumption in many cases.[17] As soon as residents are mobile, they locate in disturbed zones only if, when doing so, they do not get less utility than outside, that is, only if $V^i \geq V_o^i$. At the mid-term equilibrium, this inequality binds and land rents absorb any utility difference. In equilibrium, this leads to a rent defined as

$$R_{rz}^* = \frac{v(s_{rz}) - V_o^i + U_{rz}^i}{s_{rz}} \tag{12}$$

Hence, windfall gains are transferred to landlords in the mid term through higher land rents. Land rents are higher in zones where residents earned a larger windfall gain. Resident utility is the same as in other parts of the city. For tractability we will assume in the sequel that the windfall gains do not outweigh residents' utility in non-disturbed areas ($V_o^i > U_{rz}^i$).

7.3 Long term

In the first period, landlords decide on the lot size s_{rz}. We assume a competitive land market where the landlords are numerous and price takers. They do not consider other landlords' behavior in their decision process. In particular, they do not anticipate the aggregate migration of residents that may result from their lot size choices. Hence, each landlord finds the optimal lot size s_{rz} that maximizes his/her land rent R_{rz}^* given by (12) and taking the zone's population n_{rz} as given. So, his/her optimal rent solves the following first order condition

$$v(s_{rz}) - s_{rz}v'(s_{rz}) = V_o^i - U_{rz}^i. \tag{13}$$

By our assumption on v, the left hand side of this equality strictly increases from and above zero as the lot size s_{rz} increases from zero. Therefore, landlords reduce lot sizes in zones earning larger windfall gains U_{rz}^i. Because the proceeds of noise permits are redistributed to residents, each landlord has an incentive to divide his/her lots and to offer a place to more residents, recouping a larger share of the permit proceeds. At a given price of noise permits, each resident's

[17]By using noise exposure maps for 35 major US airports, Morrison *et al.* (1999) conclude that noise disturbances typically affect less than 2 percent of the total number of housing units in each considered metropolitan area.

windfall gain U^i_{rz} decreases as lot sizes shrink. Yet, the windfall gain recouped in the long-term by landlords also diminishes, since

$$\frac{U^*_{rz}}{T_{rz}} = \frac{p^{*2}_{rz}}{2\alpha_{rz}T_{rz}} = \frac{1}{2\delta_{rz}}\left(\frac{P^*}{Z_r}\right)^2 \frac{s_{rz}}{T_{rz}}$$

also falls as s_{rz} decreases. This reflects the coordination failure amongst landlords.

On the long run path, as landlords reduce lot sizes, the number of residents in disturbed zones rises so that the price of noise permits (weakly) increases but so that the proceeds of each individual resident decrease. Windfall gains then decrease and an equilibrium can be reached with small lot sizes. In the Appendix we indeed show that the conditions (13) and $T_{rz} = n_{rz}s_{rz}$ yields a unique solution with finite lot size s^*_{rz}.

We can compare this value either to the lot size in the absence of both noise permits and aircraft activity or to the lot size in the absence of noise permits but in the presence of aircraft activity. First, in the absence of both noise permits and aircraft activity, residents get no windfall gain ($p^* = U^i_o = 0$) and the long-term lot space is equal to s^o which solves $v(s) - sv'(s) = V^i_o$. This is the lot size of the undisturbed areas. Comparing this to expression (13), we can readily infer that the long term lot space with noise permits s^*_{rz} is smaller than s^o_{rz}. Therefore, the introduction of the market for noise permits and aircraft activities decreases lot sizes and thus increases the population in each zone ($s^*_{rz} < s^o$ and $n^*_{rz} \equiv T_{rz}/s^*_{rz} > T_{rz}/s^o \equiv n^o_{rz}$).

Second, in the absence of noise permits but in the presence of aircraft activity, residents get the noise disturbances without any compensation. The noise disutility must be compensated by higher lot size. This implies a fall in each zone population. Indeed, at the long run equilibrium each resident gets a utility equal to $V^i_{rz} = -\delta_{rz}y^2_r/2 + v(s_{rz}) - s_{rz}v'(s_{rz})$ which must be equal to the utility in non disturbed zones V^i_o. Therefore, the long run equilibrium condition is equal to $v(s_{rz}) - s_{rz}v'(s_{rz}) = V^i_o + \delta_{rz}\frac{y^2_r}{2}$ which gives the solution s^{oo}_{rz}. It is easy to check that the lot size is even larger and that population is even smaller ($s^{oo}_{rz} > s^o_{rz} > s^*_{rz}$ and $n^{oo}_{rz} \equiv T_{rz}/s^{oo}_{rz} < n^o_{rz} < n^*_{rz}$). We summarize this discussion in the following proposition.

Proposition 6 *There exists a unique long term equilibrium with a market for noise permits. In this equilibrium, land rents fully capture the windfall gain that residents obtain in the market for noise permits. The introduction of the market for noise permits decreases lot sizes and increases the population in each zone* ($s^*_{rz} < s^o < s^{oo}_{rz}$ *and* $n^*_{rz} < n^o_{rz} < n^{oo}_{rz}$).

Proof. See appendix. ■

In the long run landlords entice the residents located outside the disturbed zones to come in the disturbed areas and to share the windfall gains offered to residents. Hence, the population increases in zones, raising in turn the price of permits and reducing the flight activity.

Whereas the previous proposition compares lot and population sizes with their level without noise permit market, we now compare lot and population sizes between zones of different routes. Using expression (13) we have already established that landlords reduce lot sizes in zones earning larger windfall gains U^i_{rz}. Therefore, zones with higher population density will offer larger (individual) windfall gains. Yet, population densities are determined by the long run equilibrium. The following proposition determines the relationship between those population densities and the characteristics of zones, namely their exposure to noise and their areas.

Proposition 7 *In the long term equilibrium, each resident benefits from a larger (individual) windfall gain in zones with higher population density. The population density is higher in zones with lower individual noise exposure (δ_{rz}) and with smaller land supply (T_{rz}). That is, $U^i_{rz} > U^i_{rz'} \iff 1/s^*_{rz} > 1/s^*_{rz'} \iff \delta_{rz}T^2_{rz} < \delta_{rz'}T^2_{rz'}\ \forall z \neq z'\ \forall r$.*

Proof. See appendix. ■

The intuition goes as it follows. First, landlords divide their lots in smaller parcels when residents benefit from high individual windfall gains. The population density is therefore larger in zones with larger windfall gains. Second, consider two zones of equal size. Then, the one with the lower noise exposure will have the higher population density. This is because lower noise exposures increase windfall gains and entice landlords to attract more residents.

Finally, consider two zones with equal individual noise exposure. Then, the zone with the smaller area will have the higher population density in the long run. Indeed, if the smaller area had the same population density then it would host a smaller population so that individual windfall gains would be larger. This would entice landlords to attract additional residents, which would increase the population density in that zone. As a result, the population density is necessarily higher in the zone with the smaller area.

The above Proposition also gives us an additional message: the identity of critical zones may be different in the short and in the long run because population densities are different.[18] This result suggests that, in order to

[18]To make thinks simple, let us concentrate on an example in which residents have initially

preserve the optimality of the market design, *the implementation of the market for noise permits must be accompanied with a land use policy that restricts changes in lot sizes.*

8 Conclusion

The main strand of literature on noise disturbances around airports has neglected the issue of policy design to make airport facilities internalize the negative externality to surrounding residents. In this paper we focus on Coase's (1960) idea to achieve this goal through noise permit markets. We suggest to organize residents in zones and to allow them to offer noise permits that must be bought by airline companies to fly over their zones. In such a design the noise externality is internalized and residents can then never be worse off with the airport traffic. Local governments need no longer be involved in cumbersome information collection/studies on noise disturbances and in political arbitrage between supporters of environmental quality and airport economic activity.

We show that the noise permit markets allow to achieve the planner's optimal allocation of flights provided that she/he does not put too much weight on the benefits of the economic activity compared to the disutility of noise disturbances. The possibility that some zones may be strategic players does not fundamentally alter this finding. In the long run, because the market auctioneer is not allowed to perfectly discriminate, noise permits offer a windfall gain to residents located on the flight routes. This entices landlords to increase their land/house rents and to design smaller houses in the long run.

This article proposes an original solution to the regulation of noise disturbance around airports. We acknowledge that further research must be undertaken on additional issues like e.g. the possible cooperative behaviors of zones, the dominance of some airline companies in the noise permit market, a finer

no noise disturbance and have same lot sizes (i.e. $s^o_{rz} = s^o$). Furthermore, suppose that the planner is able to set zones such that each zone offers the same windfall gain in the short run. This means that all zones are critical and have the same disutility parameter $\alpha^o_{rz} = \alpha^o_{rz'}$, which implies that $\delta_{rz}T_{rz} = \delta_{rz'}T_{rz'}$ since zones initially have the same short term population density ($1/s^o_{rz} = 1/s^o$). In the long run, we then get $\alpha^*_{rz} > \alpha^*_{rz'}$ which is equivalent to $\delta_{rz}T_{rz}/s^*_{rz} > \delta_{rz'}T_{rz'}/s^*_{rz'}$, or $s^*_{rz} < s^*_{rz'}$or, by virtue of Proposition 6, is true iff $T_{rz} < T_{rz'}$ or equivalently iff $\delta_{rz} > \delta_{rz'}$. That is, population density ($1/s_{rz}$) is larger in zones with higher individual noise disutility. In this case, only the zone with the highest individual noise disutility becomes critical in the long term although all zones are critical in the short run. As a result, in the long term changes in lot size and population density may alter the identity of critical zones.

organization of the market place (auctioneer's task/algorithm), the weather constraints, the heterogeneity of aircraft noise levels and the heterogeneity of residents.

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9 Appendices

Proof of Proposition 1 Demands for noise permits are non-increasing and supplies are non-decreasing functions of prices P_r. There thus exists a unique equilibrium. Because on every route r, supply is smaller than demand at $P_r = 0$, the equilibrium price is interior ($P_r > 0$). Therefore, at the equilibrium, routes have same prices: $P_r = P^*$. Hence, $\sum_r y_r^S(P^*/Z_r) = \sum_r y_r^D(P^*, ..., P^*)$ which gives $\sum_r P^*\overline{\alpha}_r^{-1}Z_r^{-1} = \overline{\pi} - P^*$. This yields equations (3) and (4).□**Proof of Proposition 2** Let $F(\mathcal{T}_r)$ be the function that returns the value of $\overline{\alpha}_r Z_r$ for any design of zones $\mathcal{T}_r$ on route r. We just need to prove that F is defined over $[B_r, \infty)$. We first prove that there exists a design of zones, $\mathcal{T}_r$, such that $\overline{\alpha}_r Z_r$ can be made equal to any real number above B_r. That is, $\forall x \geq B_r$, $\exists \mathcal{T}_r$ such that $F(\mathcal{T}_r) = x$. This is true for the following possible design $\mathcal{T}_r$. It is such that the number of zones Z_r is set to the integer strictly above $[x/B_r]$, the boundaries of the critical zone $\overline{z}$ are set so that $\overline{\alpha}_r = x/Z_r$ and the boundaries of other zones z are set so that $\overline{\alpha}_r > \alpha_{rz}$. Second we prove that $F(\mathcal{T}_r)$ is not bounded from above. Indeed, choose for instance a critical zone with small length εT_r ($\varepsilon > 0$) that includes the location $t_r^{\max} \equiv \max_{t\in[0,T_r]} \beta_r(t)$ and, choose non-critical zones with equal lenght $\varepsilon T_r/M$ ($M \geq 1$) which is smaller than the lenght of the critical zone. Then, the number of critical zones is equal to $Z_r = 1 + M(1-\varepsilon)/\varepsilon$. For small enough ε, Z_r can be approximated by M/ε and $\overline{\alpha}_r Z_r$ by $\beta_r(t_r^{\max})\varepsilon T_r Z = \beta_r(t_r^{\max}) T_r M$. It results from this design that $\overline{\alpha}_r Z_r$ can be set as large as wanted if M is set to a large enough value.□**Proof of Expression (9)** Let y_{-rz} be the set of all supplies of zones different from rz: i.e. $y_{-rz} \equiv \{y_{r'z'}\}_{r'z'\neq rz}$. Utility of zone z on route r is given by $U_{rz}(y_{rz}, y_{-rz}) = \frac{1}{Z_r}\left[\overline{\pi} - \sum_{r'} \min_{z'}\{y_{r'z'}\}\right] \min_{z'}\{y_{rz'}\} - \alpha_{rz}(\min_{z'}\{y_{rz'}\})^2/2$. A Cournot Nash equilibrium is defined as the number of flights y_{rz}^c such that $y_{rz}^c \in \arg\max_{y_{rz}} U_r\left(y_{rz}, y_{-rz}^c\right)$ for all r, z. The equilibrium is easily characterized. Suppose first that zone rz is the critical zone on route

r. Then, its best response to other zones' supplies is given by

$$\begin{aligned}
\widehat{y}_{rz}\left(y_{-rz}\right) &= \arg\max_{y_{rz}} U_r\left(y_{rz}, y_{-rz}\right) \\
&= \arg\max_{y_{rz}} \frac{1}{Z_r}\left[\overline{\pi} - y_{rz} - \sum_{r' \neq r} \min_{z'}\{y_{r'z'}\}\right] y_{rz} - \alpha_{rz}\left(y_{rz}\right)^2/2 \\
&= \frac{\overline{\pi} - \sum_{r' \neq r} \min_{z'}\{y_{r'z'}\}}{2 + Z_r \alpha_{rz}} \qquad (14)
\end{aligned}$$

This best response decreases with the aggregate noise disturbance α_{rz}. Second, suppose that zone zr is not the critical zone on route r but that zone rz' ($rz' \neq rz$) is critical. Therefore, we must have that $\alpha_{rz} < \alpha_{rz'}$ and $\widehat{y}_{rz}\left(y_{-rz}\right) > \widehat{y}_{rz'}\left(y_{-rz'}\right)$. In this case, zone rz is indifferent to any offer higher than $\widehat{y}_{rz'}\left(y_{-rz'}\right)$ since such offers will not change the number of flights on route r. Also, zone rz will not offer any number of flights below $\widehat{y}_{rz'}\left(y_{-rz'}\right)$. Indeed, this would reduce the number of flights further below its preferred level as a critical zone. Hence, the best reply of zone rz is given by the following correspondence:

$$y_{rz}^{BR}\left(y_{-rz}\right) = \begin{cases} \widehat{y}_{rz}\left(y_{-rz}\right) & \text{if } \widehat{y}_{rz}\left(y_{-rz}\right) \leq \min_{z' \neq z} y_{rz'} \\ \left[\min_{z' \neq z} y_{rz'}, \infty\right) & \text{otherwise} \end{cases} \qquad (15)$$

At the equilibrium, we must have that $y_{rz}^c = y_{rz}^{BR}\left(y_{-rz}^c\right)$ for all rz. Then, we successively get that

$$\begin{aligned}
y_r^c &= \min_z y_{rz}^{BR}\left(y_{-rz}^c\right) \\
&= \min_z \widehat{y}_{rz}\left(y_{-rz}^c\right) \\
&= \min_z \frac{\overline{\pi} - \sum_{r' \neq r} \min_{z'}\{y_{r'z'}^c\}}{2 + \alpha_{rz} Z_r} \\
&= \frac{\overline{\pi} - \sum_{r' \neq r} \min_{z'}\{y_{r'z'}^c\}}{2 + \overline{\alpha}_r Z_r} \\
&= \frac{\overline{\pi} - \sum_{r' \neq r} y_{r'}^c}{2 + \overline{\alpha}_r Z_r}
\end{aligned}$$

where the first and last equalities stems from the fact that the auctioneer takes the minimum offer of permits on each route, where the second and third equalities follow from (15) and (14)and , where the fourth equality uses $\overline{\alpha}_r = \max_z \alpha_{rz}$. Solving this equality for all routes r, we get expressions (9).□ **Proof**

of Proposition 5 By comparing the Cournot equilibrium (9) with the first best (1), we obtain

$$y^c = y^o \iff \gamma \sum_s B_s^{-1} = \sum_s (1 + \overline{\alpha}_s Z_s)$$

$$\frac{y_r^c}{y^c} = \frac{y_r^o}{y^o} \iff \frac{1 + \overline{\alpha}_r Z_r}{B_r} = \frac{\sum_s B_s^{-1}}{\sum_s (1 + \overline{\alpha}_s Z_s)^{-1}} \; \forall r$$

Combining both equalities we get $\frac{1+Z_r\overline{\alpha}_r}{B_r} = \frac{1}{\gamma} \; \forall r$ or equivalently,

$$Z_r \overline{\alpha}_r = \frac{1}{\gamma} B_r - 1 \;\; \forall r.$$

□**Proof of Proposition 6** We first prove the property $\delta_{rz} T_{rz}^2 > \delta_{rz'} T_{rz'}^2 \iff s_{rz}^* > s_{rz'}^*, \forall z \neq z' \; \forall r$. Indeed, suppose the contrary statement that, for a given price p^*, the inequalities $\delta_{rz} T_{rz}^2 \geq^*$ hold together. Then we get $\frac{1}{2\delta_{rz}} \left(\frac{p^* s_{rz}}{T_{rz}} \right)^2 < \frac{1}{2\delta_{rz'}} \left(\frac{p^* s_{rz'}}{T_{rz}} \right)^2$ which is equivalent to $v(s_{rz}) - s_{rz} v'(s_{rz}) > v(s_{rz'}) - s_{rz'} v'(s_{rz'})$. By (13), this implies that $s_{rz} > s_{rz'}$, a contradiction. We then prove the existence and uniqueness of the equilibrium. Using $\overline{\alpha}_r^{-1} = (\max_z \alpha_{rz})^{-1} = (\max_z n_{rz} \delta_{rz})^{-1} = (\max_z s_{rz}^{-1} T_{rz} \delta_{rz})^{-1} = \min_z (s_{rz} T_{rz}^{-1} \delta_{rz}^{-1})$, one can show that

$$\begin{aligned} P^* s_{rz} &= \frac{\overline{\pi} s_{rz}}{1 + \sum_s \overline{\alpha}_s^{-1} Z_s^{-1}} \\ &= \frac{\overline{\pi} s_{rz}}{1 + \sum_s \min_z (s_{sz} T_{sz}^{-1} \delta_{sz}^{-1}) Z_s^{-1}} \\ &= \frac{\overline{\pi} s_{rz}}{1 + \min_z (s_{rz} T_{rz}^{-1} \delta_{rz}^{-1}) Z_r^{-1} + \sum_{s \neq r} \min_z (s_{sz} T_{sz}^{-1} \delta_{sz}^{-1}) Z_s^{-1}} \end{aligned}$$

This is a strictly increasing and continuous function of s_{rz} that has a zero at $s_{rz} \to 0$ and that is linear for sufficiently high s_{rz}. Therefore, using expression (11) it is easy to check that the right hand side of equation (13) decreases from and below V_o^i and falls to $-\infty$ as the lot size s_{rz} rises from 0 to ∞. As a result, equation (13) has a unique a solution. So, the long-term equilibrium lot size exists and is unique.□

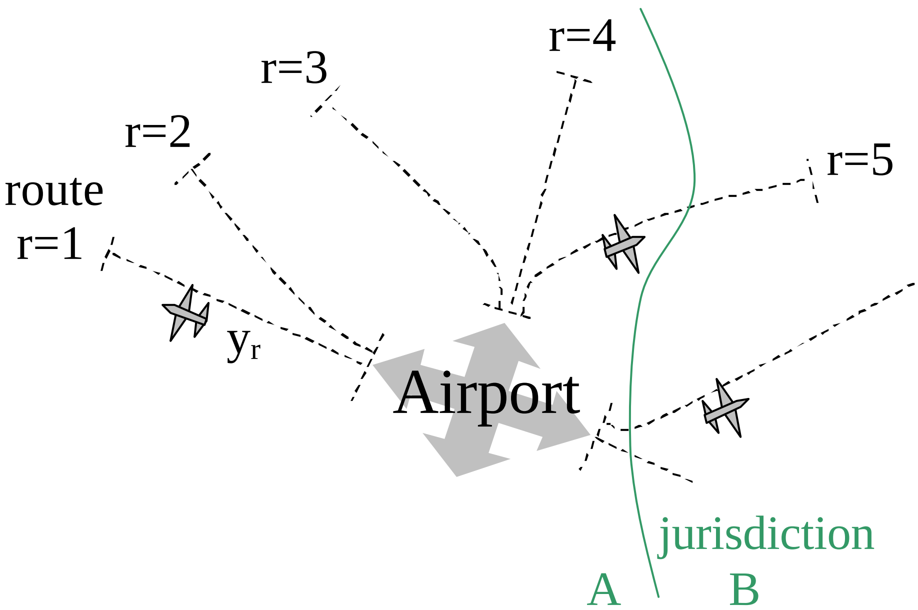

Figure 1 : Feasible routes from an airport.

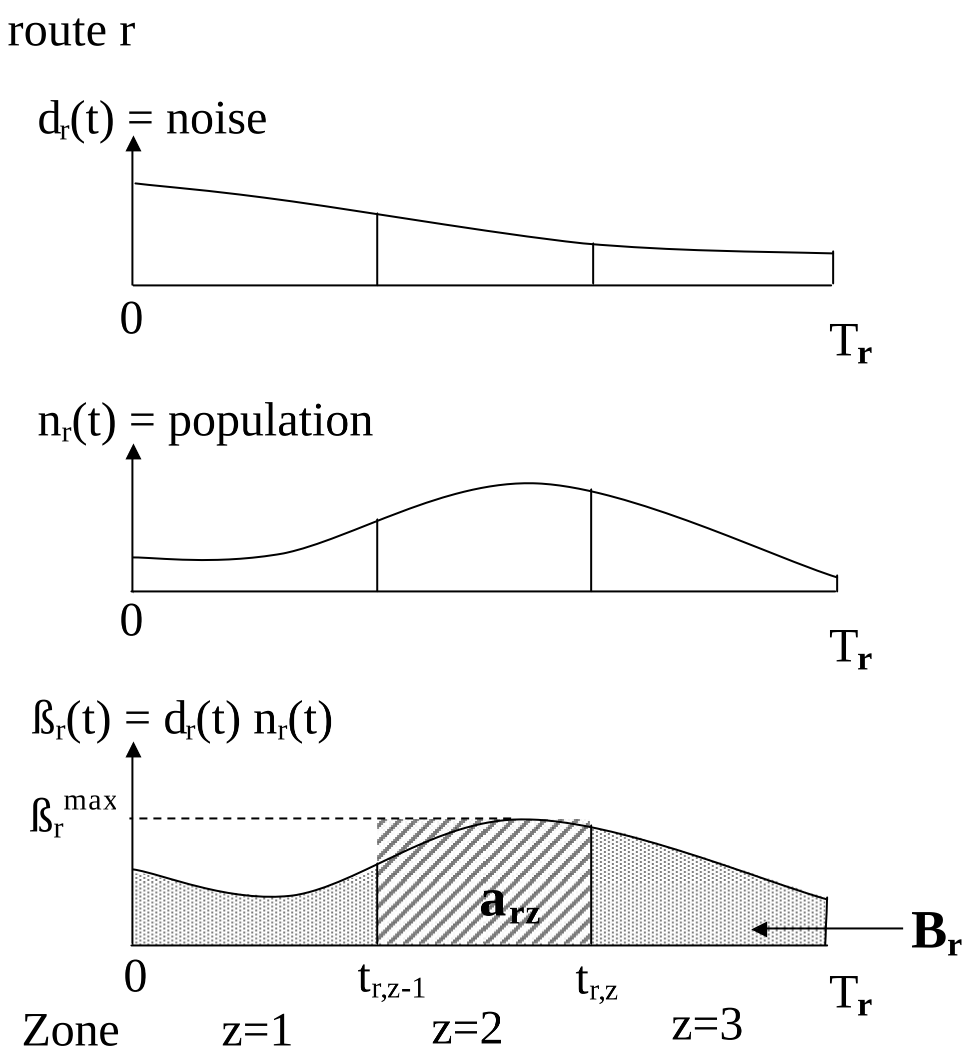

Figure 2 : Distribution of noise disutility parameter on route r and possible design of zones.