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Abstract

In a number of countries one observes a steady decline in defined benefits pensions schemes, public or private, funded or unfunded, and a simultaneous expansion of defined contributions plans. One of the consequences of this trend is to deprive individuals at the time of their retirement from the benefit of collective annuitization. Collective annuities can be distinguished from individual ones in two ways. First, they tend to be cheaper because of their scale and because of inefficiencies in private annuity markets. Second they redistribute resources from short-lived to long-lived individuals. Our paper studies the role of collective annuities. Both their redistributive incidence and efficiency aspects are accounted for. We assume that lifetime is uncertain and that there is a positive correlation between longevity and earnings. Collective annuitization (in part or in total) can be imposed on private savings or it can be "bundled" with a redistributive pension scheme. We show that the case for applying collective annuitization to private savings is weak. The case is stronger when collective annuities are associated with redistributive pensions. However, even in that case, collective annuitization may mitigate the redistributive benefits associated with the pension system.

Keywords: annuities, public pensions, differential longevity.

JEL Classification: G23, H55, J18

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1 Introduction

In a number of countries we observe today a puzzling trend regarding retirement benefits. Pension plans, funded or not, public or private, are progressively shifting from being defined benefits to becoming defined contributions. This trend implies that retirees are subject to new risks: financial market risk for the funded schemes and longevity risk for all schemes.

This paper is concerned by this second type of risk. An increasing number of workers are now offered a lump-sum amount instead of an annuity income at time of retirement. Admittedly, nobody stops them from purchasing annuities. These would not only provide insurance against the longevity risk but also offer (at least in principle) a higher return than traditional investments. However, in reality many retirees are reluctant to buy an annuity, so much that this behavioral pattern is often referred to as "annuity puzzle". Even though a number of papers have recently dealt with this issue, it appears fair to say that the jury is still out when it comes to the explanation of this puzzle. Among the usual factors one finds bequest motives, precautionary saving, family solidarity and irrationality; see Brown *et al.* (2005). Part of the explanation may also lie in the imperfections of private annuity markets. In reality the rates of return of individual annuities are much below actuarially fair levels and often significantly less attractive than the implicit return of collective annuities; see Finkelstein and Poterba (2000, 2002).

Collective annuities tend to offer much better returns than private annuities for reasons of scale and of risk pooling. They are however open to the objection that they can be regressive. It is well known that average longevity tends to increase with income. The life expectancy gap between high and low income people or between unskilled manual workers and executives can amount to over 5 years. This has led several people to show that when taking into account this correlation between longevity and income public pension scheme are not as progressive as they appear at first sight; see, e.g., Coronado *et al.* (2000).

In this paper, we study the redistributive role of collective annuities. We first assume that there is no public pension scheme, but just a mandatory scheme of collective annuities (Section 2). We show that the case for social annuitization is weak except when private annuities have a very large loading factor or when the regressive effect of collective annuities is dominated by their insurance effect. We then look at a more realistic setting wherein collective annuities are linked to a (linear) pension scheme (Section 3). We contrast two schemes: a pure contributory (Bismarckian) pension system and a flat rate (Beveridgean) pension system. Finally, we introduce nonlinear pension schemes (Section 4).

2 Mandatory collective annuitization

We start by assuming that some fraction of otherwise freely chosen private saving has to be invested in a collective annuity. The rest is invested in private annuities. The collective annuity offers the same return to everyone; private annuities, on the other hand, are responsive to longevity differentials. Private annuities are subject to a loading factor. Consequently, the collective annuity has, on average, a higher net return than private annuities. However, for individuals with low longevity this cost wedge is not sufficient to make the collective annuity preferable to private annuities.

Individuals live at most for two period. Each individual works in the first period and retires in the second. An individual of type i is characterized by a pair (w_i, π_i) where w_i is labor productivity in the first period and π_i denotes the probability to be alive in the second period. We assume that w_i and π_i are positively correlated. Let c_i, d_i, s_i and ℓ_i denote individual i's first and second period consumption, saving and labor supply. Throughout the paper we assume away liquidity constraints so that s_i can be positive as well as negative.[1] An individual's (expected) lifetime utility is given by

$$u(c_i - h(\ell_i)) + \pi_i u(d_i) = u(x_i) + \pi_i u(d_i), \tag{1}$$

where $x_i = c_i - h(\ell_i)$ is first period consumption net of the (monetary) disutility of labor.

[1] This is of no relevance in the current section. It will, however, become important in the subsequent sections where a pension system is introduced.

The interest rate is zero. Individuals can purchase annuities in the private market with a loading factor β ($\beta \geq 1$). As a result, if all saving is invested in private annuities, we have $d_i = s_i/\beta\pi_i$. If instead it is invested in the collective annuity second period consumption is given by $d_i = s_i/\hat{\pi}$, where

$$1/\hat{\pi} = \frac{\sum n_i s_i}{\sum n_i \pi_i s_i} \tag{2}$$

is the rate of return of collective annuity.[2] More generally, if there is a mandatory share of saving α to be invested in collective annuities, the second period consumption is given by $d_i = R_i s_i$ where:

$$R_i = \frac{\alpha}{\hat{\pi}} + \frac{1-\alpha}{\beta\pi_i}. \tag{3}$$

For future reference, it is important to stress that $\hat{\pi}$ is larger than the average survival probability, $\bar{\pi}$. Let us define $\gamma_i = s_i/\bar{s}$ and rewrite (2) as $\hat{\pi} = \sum n_i \gamma_i \pi_i$. We have $\sum n_i \gamma_i = 1$ so that

$$\hat{\pi} - \bar{\pi} = \sum n_i \pi_i (\gamma_i - 1) = \text{cov}(\gamma, \pi) > 0.$$

Given that s_i is a normal good, $\text{cov}(w, \pi) > 0$ (positive correlation between productivity and longevity) implies $\text{cov}(\gamma, \pi) > 0$ and thus $\hat{\pi} > \bar{\pi}$.

2.1 Individual choice

The problem of an individual of type i is given by:

$$\max_{\ell_i s_i} u[w_i \ell_i - h(\ell_i) - s_i] + \pi_i u\left[s_i\left(\frac{\alpha}{\hat{\pi}} + \frac{1-\alpha}{\beta\pi_i}\right)\right].$$

The first order conditions are given by

$$h'(\ell_i) = w_i$$

$$u'(x_i) = u'(d_i)\,\pi_i R_i.$$

The solutions to this problem are denoted $\ell_i^* = \ell_i^*(w_i)$ and $s_i^* = s_i^*(w_i, \pi_i, R_i)$.

[2] This expression follows from the budget constraint of the collective annuity: $\hat{\pi}\sum n_i s_i = \sum n_i \pi_i s_i$. As already mentioned, it is possible that the rate of return of collective annuities is smaller than the one of private annuities for individuals with a low survival probability (so that $1/\beta\pi_i > 1/\hat{\pi}$ or $\beta\pi_i < \hat{\pi}$)

2.2 First-best optimum

The first-best optimum is obtained by maximizing the following Lagrangian:

$$\mathcal{L}_{FB} = \sum n_i \left[\varphi_i \left(u\left(c_i - h\left(\ell_i\right)\right) + \pi_i u\left(d_i\right)\right) - \mu\left(c_i + \pi_i d_i - w_i \ell_i\right)\right]$$

where φ_i are social weights. Note that with $\varphi_i = 1$, the objective follows the pure utilitarian approach.

The first order conditions of this problem yield:

$$\varphi_i u'\left(x_i\right) = \varphi_i u'\left(d_i\right) = \mu$$

$$h'\left(\ell_i\right) = w_i.$$

With a pure utilitarian objective we would transfer resources not only from high productivity individuals to low productivity ones, but also from short-lived individuals to long-lived ones. The first redistribution is widely accepted; the second one is more questionable. Let us illustrate this second type of redistribution with a simple example. Assume that $w_i = w$ and that π_i only takes two values with $\pi_2 > \pi_1$. With uniform weights, individuals 1 consume the same amount as individuals 2 ($d_1 = d_2 = d$) but over a shorter (expected) lifetime. To be more precise, there is a larger proportion of type 2 individuals than of type 1 individuals who effectively consume d. Consequently, on average, individuals of type 1 pay for individuals of type 2. Now, when wages are also different and under positive correlation ($w_2 > w_1$) utilitarism also calls for redistribution from high- to low- wage individuals and the overall direction of redistribution is ambiguous. The impact of the redistribution according to life expectancies can be mitigated (or even eliminated) by setting $\varphi_1 > \varphi_2$.[3]

In the remainder of the paper we concentrate on the utilitarian case with $\varphi_i = 1$ keeping in mind that it implies a questionable redistribution from short-lived to long-

[3]For example, consider the following parameters: $u(\cdot) = \log$, $h(\ell) = \ell^2/2$, $w = 1$, $\pi_1 = 0$, $\pi_2 = 1$. We need $\varphi_1 = 2$, $\varphi_2 = 1$ to have both $\varphi_1 u'(c_1) = \varphi_2 u'(c_2) = \varphi_2 u'(d_2)$ with $c_1 = 1/2$, $c_2 = d_2 = 1/4$ which are also the *laissez-faire* levels of consumption.

An other approach is to concavify the flow of utility. Then the social criterion is

$$\sum n_i \left[\pi_i V\left(u\left(c_i\right) + u\left(d_i\right)\right) + \left(1 - \pi_i\right) V\left(u\left(c_i\right)\right)\right]$$

where V is a strictly convave transformation. This is the approach of Bommier *et al.* (2007).

lived individuals.[4] With a positive correlation between longevity and productivity the overall direction of (first-best) redistribution is thus ambiguous.

To decentralize the optimum, one needs individualized lump-sum taxes. If $\beta = 1$, the choice of saving will be efficient; so is also the choice of labor. If $\beta > 1$, lump-sum transfers in both periods are required, which amount to controlling saving.

2.3 Optimal degree of mandatory collective annuitization

We now assume that the only instrument available is the parameter α that determines what share of savings is to be invested in the collective annuity. Utilitarian social welfare can now be expressed as:

$$\mathcal{L}_{CA} = \sum n_i \left[u\left(w_i \ell_i^* - s_i^* - h\left(\ell_i^*\right)\right) + \pi_i u\left(s_i^* R_i\right)\right] \tag{4}$$

Recall that R_i is defined by (3) and is a function of α. Without further restriction on u, the relationship between α and R_i is extremely complex. This is because $\widehat{\pi}$ that appears on the RHS of (3) is itself a function of R_i (through s_i). To bypass this difficulty assume for the time being that utility is logarithmic ($u = \ln$) so that $\partial s_i^* / \partial R_i = 0$.[5] This, in turn, implies that $\widehat{\pi}$ does not depend on α. Consequently, we have by differentiating $\mathcal{L}_{CA}$ with respect to α:

$$\frac{\partial \mathcal{L}_{CA}}{\partial \alpha} = \sum n_i \pi_i \frac{s_i}{d_i} \frac{dR_i}{d\alpha} = \sum n_i \frac{\pi_i \left(\beta \pi_i - \hat{\pi}\right)}{\left(\beta \pi_i - \hat{\pi}\right)\alpha + \hat{\pi}} \tag{5}$$

Focusing on the two polar values of α, we obtain:

$$\frac{\partial \mathcal{L}_{CA}}{\partial \alpha} = \frac{\beta \mathrm{var} \pi}{\hat{\pi}} - \frac{\left(\hat{\pi} - \beta \bar{\pi}\right)\bar{\pi}}{\hat{\pi}} \text{ for } \alpha = 0 \tag{6}$$

$$= \bar{\pi} - \frac{\hat{\pi}}{\beta} \text{ for } \alpha = 1. \tag{7}$$

To interpret these formulas, first assume that $\beta = 1$ and that there is no heterogeneity in income. In (6) we have two terms. The first one is positive and represents the

[4] We shall, however, briefly return to weighted welfare function in Section 4.

[5] With a logarihtmic specification the second term on the RHS of (4) is given by

$$\pi_i \ln(s_i^* R_i) = \pi_i \ln s_i^* + \pi_i \ln R_i,$$

so that R_i does not affect the solution of the maximization problem.

insurance role of collective annuities. Recall that in the first-best, one wants to equalize consumptions levels across individuals with different mortality profiles. Thus, for *uniform* level of savings invested in annuities, the collective annuitization redistributes from the short-lived to the long-lived persons. The higher is the variance of π the higher is the concern for this type of redistribution.

However, for any given return on annuities, savings are higher for long-lived individuals. Specifically, with the logarithmic utility function we have

$$s_i = \frac{\pi_i z_i}{1+\pi_i},$$

where $z_i = w_i \ell_i^* - h(\ell_i^*)$ is independent of π_i. Consequently, part of the redistribution induced by collective annuitization is offset by lower contributions in collective annuities from the short-lived individuals. The second term in (6) represents the welfare loss associated with this effect; it is proportional to $\text{cov}(\gamma, \pi) = (\bar{\pi} - \hat{\pi})$ (which reflects the link between savings and survival probability).[6] To sum up, collective annuitization is desirable if the first term measuring the redistributive concern dominates the second one measuring the counter redistributive effect of collective annuities.

These arguments are all for the case where wages are equal. Unequal earnings further strengthen the odds for a corner solution at $\alpha = 0$ (and thus the case *against* collective annuities). To see this, assume that with equal wages ($w_i = \overline{w}$) the right hand side of (6) is positive. Now, a more unequal distribution of w_i. leads to an *increase* in $\text{cov}(\gamma, \pi) = \hat{\pi} - \bar{\pi}$ because γ_i (the share of saving of type i's individuals) is a positive function of π_i and w_i. The formal proof of this property is somewhat tedious and given in the Appendix.[7] Intuitively, it is not surprising. The point is that since γ increases both in π and in w, which in turn are positively correlated, the variability of w tends to strengthen the relationship between π and γ. Put differently, an increase in π now affects γ in two ways: directly (because γ increases with π) and indirectly because a higher π goes hand in hand with a higher w (and γ is increasing in w). With a higher

[6] Recall that γ is proportional to s.

[7] It requires an assumption that is stronger than what is needed for the rest of the paper. Specifically, we assume that π and w are both increasing in i (rather than simply assuming positive correlation between π and w).

covariance term, we will go from an optimal positive value of α to a lower level of α or to a corner solution at $\alpha = 0$.

Observe that even when the RHS of (6) is positive, it is *never* optimal to have full collective annuitization ($\alpha = 1$) when there is no loading factor associated with private annuities ($\beta = 1$). The right hand side of (7) is then equal to $-\operatorname{cov}(\gamma, \pi)$ and is negative. To understand this, let us start again with the case where there is no income heterogeneity. If there is full collective annuitization, the rate of return on savings is the same for every type of individuals while savings are higher for long-lived individuals. In other words, second period consumption will be higher for long-lived individuals. However, in the first best, consumption levels should be equalized between the different types of individuals. Thus starting from full collective annuitization, it is always desirable to introduce individualized annuitization (reduce α) so as to reduce the redistribution from short to long-lived individuals.[8]. Introducing income heterogeneity reinforces this result because highly productive individuals are also the ones who live long and thus those who save more.

Now let us introduce $\beta > 1$. Expression (6) can be rewritten as

$$\frac{\partial \mathcal{L}_{CA}}{\partial \alpha} = \frac{\operatorname{var}\pi\,(\bar{\pi} - \hat{\pi})\,\bar{\pi}}{\hat{\pi}} + (\beta - 1)\,\frac{E\pi^2}{\hat{\pi}} \text{ for } \alpha = 0,$$

which shows that a higher level of β makes collective annuities more desirable. This does not come as a surprise. When the loading factor on private annuities is sufficiently important, even people who live shorter than the average individuals can find collective annuities more attractive than individual annuities.

3 Collective annuitization and public pension

In the previous section, there was no public pension scheme. Collective annuities were just seen as a mandatory scheme in which some fraction of otherwise private savings had to be invested. We have seen that under these circumstance there are little grounds for social annuitization as long as longevity and productivity are positively correlated

[8]This reduction in α does not affect first-period consumption levels (saving and labor supply are indepednant of α).

and as long as the loading factor of private annuities is not prohibitive. In reality, social annuities typically come along with public pensions schemes that are more or less redistributive. We now consider the case where earnings are subject to a payroll tax at rate τ which finances annuitized pension benefits that are either earnings related or uniform.

3.1 Contributive schemes

We now drop the assumption of a loading factor ($\beta > 1$) and return to the general class of (quasi-linear) utility functions as defined by (1).[9] We start with the case where the pension system is contributive (Bismarckian) in the sense that an individual's annuitized pension benefits are proportional to his contributions (payroll taxes). When the pension system is purely contributive, one might be tempted to think that it is "neutral". In other words, the payroll tax would be indeterminate. However, the pension system we consider relies on collective annuitization (with individuals differing not only in productivity but also in survival probability). Consequently, it is not "completely contributive" in the sense that benefits are not related to survival probabilities. In other words, while there is no (explicit) redistribution according to income levels, there is redistribution according to mortality levels. The setting is thus not that different from the one considered in the previous section. The main modification is that individual contributions are now a fraction of income rather than of savings.

Pension benefit for a worker with productivity w_i is

$$p_i = \frac{\tau_i y_i}{\hat{\pi}} \tag{8}$$

where $y_i = w_i \ell_i$ and where the return of the collective annuity is redefined as

$$\hat{\pi} = \frac{\sum n_i \pi_i y_i}{\bar{y}}.$$

Observe that

$$\hat{\pi} - \bar{\pi} = \frac{\text{cov}\,(y, \pi)}{\bar{y}}. \tag{9}$$

[9] A loading factor associated with private annuities would have a similar impact as in the previous sections. Specifically, it would strengthen the case for collective annuities (which have a higher return) and thus for public pensions.

Individuals choose labor supply and savings given the pension scheme. Consequently, they maximize

$$U_i = u\left(w_i\left(1-\tau\right)\ell_i - h\left(\ell_i\right) - s_i\right) + \pi_i u\left(\frac{s_i}{\pi_i} + p_i\right), \tag{10}$$

where p_i is defined by (8), with respect to ℓ_i and s_i.[10] The FOCs are given by

$$u'\left(c_i\right) = u'\left(d_i\right) \quad \text{and} \quad h'\left(\ell_i\right) = w_i\left(1-\tau+\frac{\pi_i}{\hat{\pi}}\tau\right). \tag{11}$$

Observe that the term $(1-\tau+\pi_i\tau/\hat{\pi})$ in (11) measures the annuitized pension system's net rate of return for individual i (characterized by w_i and π_i). With a Bismarckian system this rate does not (directly) depend on w_i, but it does increase with π_i. The net return is positive for $\pi_i > \hat{\pi} > \bar{\pi}$ (and for these individuals we have $h'\left(\ell_i\right) > w_i$). Under positive correlation between productivity and longevity, the individuals with positive return are also those with the highest productivities. In other words, high productivity individuals benefit from collective annuitization.

With a utilitarian objective, the optimal pension system is then obtained by maximizing

$$\mathcal{L}_{CP} = \sum n_i\left[u\left(w_i\left(1-\tau\right)\ell_i^* - h\left(\ell_i^*\right) - s_i^*\right) + \pi_i u\left(\frac{s_i^*}{\pi_i} + \frac{\tau w_i \ell_i^*}{\hat{\pi}}\right)\right],$$

with respect to τ, where the stars are used to denote the induced solution to the individual problem. The FOC is given by

$$\frac{\partial \mathcal{L}_{CP}}{\partial \tau} = \sum n_i u'\left(c_i\right)\left[\left(\frac{\pi_i}{\hat{\pi}} - 1\right)y_i - \tau y_i \frac{\pi_i}{\hat{\pi}}\frac{\partial \hat{\pi}}{\partial \tau}\right] = 0,$$

which can be rearranged to yield

$$\tau = \frac{\sum n_i u'\left(c_i\right) y_i\left(\pi_i - \hat{\pi}\right)}{\sum n_i u'\left(c_i\right) w_i \pi_i\left(\dfrac{1}{\hat{\pi}}\dfrac{\partial \hat{\pi}}{\partial \tau}\right)}. \tag{12}$$

To interpret this formula, we start by an extreme example in which labor supply is inelastic (and set at $\ell_i = 1$ with $h(1) = 0$) so that the problem of the consumer is:

$$\max_{s_i} \quad u\left(w_i\left(1-\tau\right) - s_i\right) + \pi_i u\left(\frac{s_i}{\pi_i} + \frac{\tau w_i}{\bar{\pi}}\right).$$

[10] Recall that there is no liquidity constraint.

Without liquidity constraint the solution implies $c_i = d_i$ so that

$$c_i = \frac{w_i \bar{\pi}}{1 + \pi_i} \left((1 - \tau) \bar{\pi} + \tau \pi_i \right), \tag{13}$$

which implies that $\partial c_i / \partial \pi_i < 0$ as long as $\tau < \bar{\pi}/(1 + \bar{\pi})$.

Utilitarian welfare is now given by

$$\mathcal{L}_{FL} = \sum n_i \left[u \left(w_i (1 - \tau) - s_i^* \right) + \pi_i u \left(\frac{s_i^*}{\pi_i} + \frac{\tau w_i}{\bar{\pi}} \right) \right],$$

so that

$$\frac{\partial \mathcal{L}_{FL}}{\partial \tau} = \sum n_i u' (c_i) \left[\frac{\pi_i}{\bar{\pi}} - 1 \right] w_i. \tag{14}$$

This can be rewritten as:

$$\frac{\partial \mathcal{L}_{FL}}{\partial \tau} = \frac{1}{\bar{\pi}} \operatorname{cov} \left(u' (c) \, w, \pi \right) - \operatorname{cov} (w, \pi) \frac{E u' (c) \, w}{\bar{w}}. \tag{15}$$

To interpret (14) and (15) we first assume that all individuals have the same wage. In that case a contributive pension system is desirable because it redistributes income from short- to long-lived individuals. To see this note that with $w_i = \bar{w}$, (15) becomes:

$$\frac{\partial \mathcal{L}_{FL}}{\partial \tau} = \operatorname{cov} \left(u' (c) \, , \pi \right)$$

which is positive as long as $\tau < \bar{\pi}/(1+\bar{\pi})$ and $s^* \neq 0$. When $\tau = \bar{\pi}/(1+\bar{\pi})$ we have $s^* = 0$ and $\operatorname{cov} (u' (c) \, \pi) = 0$. The reason for this result is that we use a utilitarian objective which is invariant to longevity differences. It implies a first-best allocation with identical yearly consumption for everyone (so that the short-lived pay for the long-lived). With equal wages, the contributive system is effectively first-best optimal.

When wages are different and positively correlated with π_i the second term of (15) is negative and tends to reduce the tax rate. As to the first term which is positive for uniform wages, it can turn negative if the utility function is sufficiently concave and the variance of w_i dominates that of π_i. In the special case where the utility is logarithmic, one easily checks from (13) that this covariance is positive (as long as $\tau < \bar{\pi}/(1 + \bar{\pi})$). We thus may have a positive tax rate that is a compromise between the components of (15).

Let us once again stress that a positive first term in (15) (for low payroll taxes and when the variance of productivities is small) is due to our utilitarian criterion. With social weights decreasing with π, the case for a positive tax rate would be weaker.

Going back to expression (12) with endogenous labor supply, its numerator receives the same interpretation as that just given for (14) and (15). As to the denominator, it reflects the impact of the tax on the returns of the collective annuity (via its impact on labor supply and incomes). In the vicinity of $\tau = 0$, one shows that it is positive.[11]

To conclude this section, we have seen that the only reason why a contributive system with collective annuitization might be desirable is due to a utilitarian bias towards long-lived individuals. This result is not surprising. The regressive incidence of social security system when longevity and earnings are positively correlated has been underlined by several studies; see e.g. Coronado *et al.* (2000) and Bommier *et al.* (2006).

As we show in the next subsection, the case for collective annuitization is strengthened when it is combined with a redistributive rather than a contributive pension system.

3.2 Flat rate pensions

We now consider the case of a flat rate annual pension. In other words, everyone receives the same annual benefit but on a lifetime basis those who live longer receive more than those who die early. The annual pension is now given by

$$p = \frac{\tau \sum w_i \ell_i}{\bar{\pi}}. \tag{16}$$

The individual optimization problem continues to be given by (10) but with $p_i = p$ defined by (16). The individual FOCs are now given by

$$u'(c_i) = u'(d_i) \quad \text{and} \quad h'(\ell_i) = w_i(1-\tau), \tag{17}$$

where the condition with respect to ℓ has changed because with flat pensions an increase in (first-period) labor supply does not entitle an individual to extra pension benefits.

[11] Take the case where $h(\ell) = \ell^2/2$. Then we have

$$\frac{d\hat{\pi}}{d\tau} = \frac{Ew^2(\hat{\pi} - \pi)}{Ew^2\left(\hat{\pi} - \tau\left(\hat{\pi}^2 - \pi\right)\right)}.$$

.

Utilitarian welfare is given by

$$\begin{aligned}\mathcal{L}_{FP} &= \sum n_i u\left(w_i \ell_i^* \left(1-\tau\right) - h\left(\ell_i^*\right) - s_i^*\right) + \pi_i u\left(\frac{s_i^*}{\pi_i} + p\right) \\ &\quad -\mu \sum n_i \left(\pi_i p - \tau w_i \ell_i^*\right),\end{aligned}$$

where ℓ_i^* and s_i^* come from individual optimization conditions (and are suitable redefined).

The FOC for an interior maximum are the following:

$$\begin{aligned}\frac{\partial \mathcal{L}_{FP}}{\partial \tau} &= \sum n_i u'\left(x_i\right)\left(-w_i \ell_i^*\right) + \mu \sum n_i \left(w_i \ell_i^* + \tau w_i \frac{\partial \ell_i^*}{\partial \tau}\right) = 0 \\ \frac{\partial \mathcal{L}_{FP}}{\partial p} &= \sum n_i \pi_i u'\left(d_i\right) - \mu\ \bar{\pi} = 0.\end{aligned}$$

Combining these two conditions, we obtain:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \tau}\bar{\pi} + \frac{\partial \mathcal{L}}{\partial p}\sum n_i w_i \ell_i^* &= -\bar{\pi} \sum n_i u'\left(x_i\right) w_i \ell_i^* \\ &\quad + \sum n_i \pi_i u'\left(d_i\right) \sum n_i w_i \ell_i^* \\ &\quad + \mu \tau \sum n_i w_i \frac{\partial \ell_i^*}{\partial \tau} = 0,\end{aligned}$$

which can be rewritten as

$$-\bar{\pi}\ \operatorname{cov}\left(u'\left(x\right), w\ell^*\right) + \operatorname{cov}\left(u'\left(x\right), \pi\right) + \mu \tau E w \frac{\partial \ell^*}{\partial \tau} = 0,$$

which in turn yields

$$\tau = \frac{-\bar{\pi}\ \operatorname{cov}\left(u'\left(x\right), w\ell^*\right) + \operatorname{cov}\left(u'\left(x\right), \pi\right)}{-\mu E w \dfrac{\partial \ell^*}{\partial \tau}}. \tag{18}$$

The denominator of (18) is the usual efficiency term in linear income tax formulas. The numerator on the other hand measures the redistributive impact. The second term of the denominator is expected to be negative with a positive correlation between wages and longevity. It measures the "reverse" redistribution that is brought about by the collective annuity (that favors the productive who also tend to live longer). This effect was already present in the previous sections. The new feature brought about by the flat pension is the first term. Its structure and interpretation is standard. A flat pension that is financed by a proportional payroll tax redistributes from high to low productivity individuals. The magnitude of this term depends on the inequality of earnings and on the concavity of the utility function. The optimal payroll tax rate is then determined by trading off these two effects.

To sum up, collective annuities continue to have a potentially adverse redistributive impact in this setting. However, this effect is mitigated by combining the collective annuities with a redistributive pension scheme. In other words, when bundled with a suitably designed pension scheme collective annuities may be desirable (even when they are not more efficient than private annuities).

4 Non linear scheme

So far we have concentrated on linear policies. First we have assumed that some fraction of private saving (which is otherwise freely chosen) is allocated to collective annuities. Then we have introduced a linear pension scheme under which a proportional payroll tax was used to finance annuitized pension benefits. We have seen that there is a conflict between the redistributive pension scheme and the regressivity of the underlying collective annuitization. A natural question to ask at this stage is whether the results reflect fundamental properties of collective annuities or instead are mere artifacts of the (artificial) restrictions imposed on the different policies.

To study the robustness of this result we shall now take a different view on this problem. Rather than specifying arbitrary (linearity) restrictions on the policy we now characterize the "best" (utilitarian) allocation that can be achieved given the available information and study how is can be implemented through a (possibly nonlinear) pension system that may or may not rely on annuitization. For reasons that will become clear below, we adopt a slightly different specification of preferences in this section and assume that utility is given by

$$u\left(c_i\right)-h\left(\ell_i\right)+\pi_i u\left(d_i\right). \tag{19}$$

This specification differs from (1) in that labor supply is now separable from the other goods. Productivity w_i, labor supply, ℓ_i and longevity π_i cannot be observed. There are two types of individuals, indexed $i = 1, 2$, characterized each by a vector (w_i, π_i). Type 2 is the more productive one so that we have $w_2 \geq w_1$. Regarding life expectancy we continue to concentrate on the case of positive correlation $(\pi_2 \geq \pi_1)$ in which the more productive also have a higher survival probability. This is the empirically most relevant case on which we have concentrated in the previous sections. The second case with $\pi_2 \leq \pi_1$ is empirically less appealing but constitutes an interesting benchmark. Another departure from the analysis in the previous sections is that we allow for the introduction of social weighs that may be negatively correlated to longevity. To keep the notation simple, we take $\varphi_1 = \varphi$ and $\varphi_2 = 1$. In other words, when $\pi_1 < \pi_2$ we set $\varphi_1 \geq 1$ and when $\pi_1 > \pi_2$ we set $\varphi_1 \leq 1$.

The Lagrangian of this problem can be expressed as:

$$\begin{aligned}
\mathcal{L}_{NL} &= \sum_i^2 n_i \left[\varphi_i \left(u\left(c_i\right) - h\left(\ell_i\right) + \pi_i u\left(d_i\right)\right) - \mu\left(c_i + \pi_i d_i - w_i \ell_i\right)\right] \\
&\quad + \lambda_1 \left[u\left(c_1\right) - h\left(\frac{y_1}{w_1}\right) + \pi_1 u\left(d_1\right) - u\left(c_1\right) h\left(\frac{y_1}{w_2}\right) - \pi_1 u\left(d_2\right)\right] \\
&\quad + \lambda_2 \left[u\left(c_2\right) - h\left(\frac{y_2}{w_2}\right) + \pi_2 u\left(d_2\right) - u\left(c_2\right) + h\left(\frac{y_2}{w_1}\right) - \pi_2 u\left(d_1\right)\right], \quad (20)
\end{aligned}$$

where λ_1, λ_2 and μ are the Lagrange multipliers associated with the incentive constraints and the resource constraint. This yields the following FOCs:

$$\frac{\partial \mathcal{L}_{NL}}{\partial c_1} = n_1 \left[\varphi u'\left(c_1\right) - \mu\right] + \left(\lambda_1 - \lambda_2\right) u'\left(c_1\right) = 0, \quad (21)$$

$$\frac{\partial \mathcal{L}_{NL}}{\partial c_2} = n_2 \left[u'\left(c_2\right) - \mu\right] + \left(\lambda_2 - \lambda_1\right) u'\left(c_2\right) = 0, \quad (22)$$

$$\frac{\partial \mathcal{L}_{NL}}{\partial d_1} = n_1 \pi_1 \left[\varphi u'\left(d_1\right) - \mu\right] + \left(\lambda_1 \pi_1 - \lambda_2 \pi_2\right) u'\left(d_1\right) = 0, \quad (23)$$

$$\frac{\partial \mathcal{L}_{NL}}{\partial d_2} = n_2 \pi_2 \left[u'\left(d_2\right) - \mu\right] + \left(\lambda_2 \pi_2 - \lambda_1 \pi_1\right) u'\left(d_2\right) = 0, \quad (24)$$

$$\frac{\partial \mathcal{L}_{NL}}{\partial y_1} = -n_1 \left[\varphi h'\left(\frac{y_1}{w_1}\right) \frac{1}{w_1} - \mu\right] - \lambda_1 h'\left(\frac{y_1}{w_1}\right) \frac{1}{w_1} + \lambda_2 h'\left(\frac{y_1}{w_2}\right) \frac{1}{w_2} = 0, \quad (25)$$

$$\frac{\partial \mathcal{L}_{NL}}{\partial y_2} = -n_2 \left[h'\left(\frac{y_2}{w_2}\right) \frac{1}{w_2} - \mu\right] - \lambda_2 h'\left(\frac{y_2}{w_2}\right) \frac{1}{w_2} + \lambda_1 h'\left(\frac{y_2}{w_1}\right) \frac{1}{w_1} = 0. \quad (26)$$

4.1 First-best

As a benchmark, we study the first-best allocation that can be achieved when all variables are publicly observable. To derive this solution we can use (21)–(26) and set $\lambda_1 = \lambda_2 = 0$ (i.e., ignore the incentive constraints). This yields

$$\varphi u'(c_1) = u'(c_2) \quad ; \quad \frac{u'(c_2)}{u'(d_2)} = \frac{u'(c_1)}{u'(d_1)} = 1 \quad ; \quad h'(\ell_i) = u'(c_i)\, w_i, \tag{27}$$

which implies $\ell_2 \geq \ell_1$ (with a strict inequality when $w_2 > w_1$). It also implies $c_1 = d_1 \gtreqless c_2 = d_2$ for $\varphi \gtreqless 1$. In words, consumption levels are equalized between periods and adjusted for differences in longevity. Observe that longevity affects the solution only through the social weights. In the utilitarian case when $\varphi = 1$, consumption levels do not depend on $\pi' s$. Either way, the more productive individuals work more than the less productive.

To decentralize this solution one needs (personalized) lump-sum taxes and transfers. Earnings would not be taxed and individuals could invest their savings in fair annuities.

We now turn to the second-best problem. First, we look at the utilitarian case and then we introduce social weights.

4.2 Second-best in a utilitarian case

The first-best solution is not feasible when individual types (productivities and survival probabilities) are not observable. More precisely, it follows from (27) that the incentive constraint of individual 2 is violated as long as $w_2 > w_1$. This is because consumption levels are equalized but the productive have to work more. Note that this is true irrespective of the correlation between w and π.

Let us now turn to the second-best solution. Assume that the incentive constraint that is violated at the first-best is also the one that is binding in the second-best so that we have $\lambda_2 > 0$ and $\lambda_1 = 0$. The FOC (21)–(26) can then be rearranged as follows

$$\frac{u'(c_2)}{u'(d_2)} = 1 \quad ; \quad h'\left(\frac{y_2}{w_2}\right) = u'(c_2)\, w_2, \tag{28}$$

$$\frac{u'(c_1)}{u'(d_1)} = \frac{n_1 - \lambda_2 \dfrac{\pi_2}{\pi_1}}{n_1 - \lambda_2}, \tag{29}$$

$$\frac{h'\left(\dfrac{y_1}{w_1}\right)}{u'(c_1)} = w_1 \left[1 + \frac{\lambda_2}{\mu n_1}\left(h'\left(\frac{y_1}{w_2}\right)\frac{1}{w_2} - h'\left(\frac{y_1}{w_1}\right)\frac{1}{w_1}\right)\right] < w_1. \tag{30}$$

Condition (28) yields the traditional no distortion at the top property. Equation (30) is also standard and implies a positive marginal (payroll) tax rate on the low productivity type's labor income. The interesting expression from the perspective of this paper is (29) which shows how the trade-off between c_1 and d_1 is distorted. When $\pi_1 = \pi_2$ we have a single dimension of heterogeneity and (under separable preferences) the Atkinson and Stiglitz property applies.[12] This means that all savings are invested in private (fair) annuities which are not taxed. There is nothing which resembles a collective annuity in this solution. When $\pi_1 < \pi_2$ (positive correlation between w and π) we have $u'(c_1)/u'(d_1) < 1$ so that there is a downward distortion on d_1 (compared to c_1). The way this distortion translates into the tax policy depends on the specific implementation. One possibility is to have a tax on type 1's savings which otherwise are invested in a fair annuity. Alternatively, one can directly provide an annuity to type 1 which offers an implicit return that is smaller than $1/\pi_1$. Since type 2 faces an undistorted trade-off the implicit return of his annuity is $1/\pi_2$. Consequently, the second-best implies a certain equalization of the return of annuities between individuals. However, this equalization is not in general going to be complete so that the solution cannot be implemented by a simple collective annuity.

4.3 Second-best solution with social weights

We now introduce φ as a social weight for individuals 1. As long as $\lambda_2 > 0$, $\lambda_1 = 0$, qualitative results are unchanged. Focusing on the trade-off between c and d, there is

[12] And it is to have this as a benchmark that we have adopted the separable specification in this section.

no distortion for individuals 2 and for individuals 1, we have

$$\frac{u'(c_1)}{u'(c_2)} = \frac{\varphi n_1 - \lambda_2 \frac{\pi_2}{\pi_1}}{\varphi n_1 - \lambda_2} < 1. \tag{31}$$

Consequently, the sign of the distortion does not change with φ. However, one easily verifies that the middle term in (31) increases with φ. Consequently, the magnitude of the distortion decreases as the weight put on the short-lived individual increases. In the previous subsection we have argued that this distortion reduces the wedge between the returns of individual annuities. A higher social weight on type 1 individuals reverses this effect and increases the wedge so that we move further away from collective annuities. In Sections 2 and 3 we have not explicitly considered social weights, but we have argued that such weights would further weaken the case for collective annuities. The results we have just discussed show that this remains true with nonlinear instruments.

Finally, let us have a look at the case of negative correlation $\pi_1 > \pi_2$. Assume further that the social weights given on the long-lived individuals is very low. It is then not impossible to have $\lambda_2 = 0$ and $\lambda_1 > 0$. In that case, there is no distortion on the choice of c_1 and d_1 but for 2 we have:

$$\frac{u'(c_2)}{u'(d_2)} = \frac{n_2 - \lambda_1 \frac{\pi_1}{\pi_2}}{n_2 - \lambda_1} > 1.$$

In words, we have a subsidy on saving by individuals 2. This brings us even further away from collective annuities because the net returns to individual annuities are further apart than in an actuarially fair world.

5 Conclusion

One of the properties of most public pension schemes is not only to force people to save but also to provide them with a constant flow of income after retirement. Such a collective and mandatory annuitization is known to be more cost-efficient than market based annuities. Their drawback is to penalize workers with short lifetime as benefits are longevity invariant.

In this paper we have studied the desirability of collective annuities (from a utilitarian perspective) in a number of different settings. First, we have considered the case where collective annuitization is introduced on a stand alone basis (i.e., not combined with a public pension system). Specifically, we have assumed that some fraction of private savings (which is otherwise freely chosen) is allocated to collective annuities. In that setting, the case for collective annuities is weak, except if the loading factor on private annuities is very large. The reason is that by redistributing from the short-lived to the long-lived, collective annuities also redistribute from low to high productivity individuals (under the plausible assumption of positive correlation between productivity and longevity).

Next, we have considered collective annuities that are bundled with a public pension system. We have shown that a purely contributive system is welfare improving only if the redistribution from short-lived to long-lived individuals dominates the redistribution from high income to low income individuals. A pension system that provides flat rate benefits is more likely to be welfare improving. However, its generosity is mitigated by the regressive effect of collective annuitization. Finally, in a non linear taxation setting, collective annuities as such are not needed to implement the second-best allocation. However, we show that the wedge between the returns of individual annuities ought to smaller then in an actuarially fair world. This can be seen as a step towards collective annuities but an incomplete step because the rates of return are never equalized across type (as they are for collective annuities).

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Appendix

We want to establish that a more unequal distribution of w leads to an *increase* in $\text{cov}(\gamma,\pi)$. The is done in the following lemma where we compare the covariance with two distributions of w (with the same average wage). The first distribution implies equal wages for everyone. The second distribution implies wages that are increasing with π. We establish this property for the case where w and π are increasing function of i and where the type space is continuous.

Lemma 1 *Consider two distributions $(w_1(i),\pi(i))$ and $(w_2(i),\pi(i))$ each characterized by the same density $n(i)$ defined over the interval $[i_-,i_+]$ where $w_1(i)=w$ for every $i\in[i_-,i_+]$ and $\bar{w}_2=\int_{i_-}^{i_+}w(i)\,n(i)\,di=w$. Further denote*

$$\gamma_j(i)=\frac{\pi(i)\,z(w_j(i))/(1+\pi(i))}{\int_{i_-}^{i_+}[\pi(i)\,z(w_j(i))/(1+\pi(i))]\,n(i)\,di}$$

where $z(w_j(i))=w_j(i)\,l(w_j(i))-h(l(w_j(i)))$ is an increasing function of $w_j(i)$. Then if $w_2(i)$ and $\pi(i)$ are both increasing with i, $\text{cov}(\gamma_1,\pi)<\text{cov}(\gamma_2,\pi)$.

Proof. Since $\bar{w}_2=\bar{w}_1=w$ and $E(\gamma_j)=1$ for $j=1,2$, one has:

$$\begin{aligned}
&\text{cov}(\gamma_1,\pi)-\text{cov}(\gamma_2,\pi)=E(\pi\gamma_1)-E(\pi\gamma_2)\\
&=\int_{i_-}^{i_+}\pi(i)\left[\frac{\pi(i)\,z(w_1(i))/(1+\pi(i))}{\int_{i_-}^{i_+}[\pi(i)\,z(w_1(i))/(1+\pi(i))]\,n(i)\,di}\right.\\
&\qquad\left.-\frac{\pi(i)\,z(w_2(i))/(1+\pi(i))}{\int_{i_-}^{i_+}[\pi(i)\,z(w_2(i))/(1+\pi(i))]\,n(i)\,di}\right]n(i)\,di
\end{aligned}$$

Denoting $K(i)=\pi(i)/(1+\pi(i))$, and using $w_1(i)=w$ for every $i\in[i_-,i_+]$, this can be rewritten as

$$\begin{aligned}
&\text{cov}(\gamma_1,\pi)-\text{cov}(\gamma_2,\pi)\\
&=\frac{\Delta(i_+)\,n(i)}{\left(\int_{i_-}^{i_+}\pi(i)/(1+\pi(i))\,n(i)\,di\right)\left(\int_{i_-}^{i_+}\pi(i)\,z(w_2(i))/(1+\pi(i))\,n(i)\,di\right)}di \qquad \text{(A1)}
\end{aligned}$$

where

$$\begin{aligned}
\Delta(i_+) &= \left(\int_{i_-}^{i_+}\pi(i)\,K(i)\,n(i)\,di\right)\left(\int_{i_-}^{i_+}z(w_2(i))\,K(i)\,n(i)\,di\right)\\
&\quad-\left(\int_{i_-}^{i_+}\pi(i)\,z(w_2(i))\,K(i)\,n(i)\,di\right)\left(\int_{i_-}^{i_+}K(i)\,n(i)\,di\right)
\end{aligned}$$

Since the denominator of $(A1)$ is positive, the sign of $\text{cov}(\gamma_1, \pi) - \text{cov}(\gamma_2, \pi)$ thus only depends upon the sign of the numerator of $(A1)$.

Denote

$$\begin{aligned}\Delta(i_*) &= \left(\int_{i_-}^{i_*} \pi(i) K(i) n(i) di\right)\left(\int_{i_-}^{i_*} z(w_2(i)) K(i) n(i) di\right) \\ &\quad - \left(\int_{i_-}^{i_*} \pi(i) z(w_2(i)) K(i) n(i) di\right)\left(\int_{i_-}^{i_*} K(i) n(i) di\right).\end{aligned}$$

It is straightforward to see that $\Delta(i_-) = 0$. Moreover, for any $i_* \in [i_-, i_+]$,

$$\frac{d(\Delta(i_*))}{di_*} =$$

$$K(i_*) n(i_*) \left[\pi(i_*) \left(\int_{i_-}^{i_*} z(w_2(i)) K(i) n(i) di\right) + z(w_2(i_*)) \left(\int_{i_-}^{i_*} \pi(i) K(i) n(i) di\right)\right]$$

$$-K(i_*) n(i_*) \left[\pi(i_*) z(w_2(i_*)) \left(\int_{i_-}^{i_*} K(i) n(i) di\right) + \left(\int_{i_-}^{i_*} \pi(i) z(w_2(i)) K(i) n(i) di\right)\right]$$

Factorizing by $K(i_*) \pi(i_*) z(w_2(i_*)) n(i_*)$, this yields

$$\frac{d(\Delta(i_*))}{di_*} = K(i_*) \pi(i_*) z(w_2(i_*)) n(i_*) \left[\int_{i_-}^{i_*} K(i) \left(\frac{z(w_2(i))}{z(w_2(i_*))} - 1\right)\left(1 - \frac{\pi(i)}{\pi(i_*)}\right) n(i) di\right]$$

Since $\pi(i)$ and $z(w_2(i))$ are both increasing with i, one always have

$$\left(\frac{z(w_2(i))}{z(w_2(i_*))} - 1\right)\left(1 - \frac{\pi(i)}{\pi(i_*)}\right) < 0$$

so that $d(\Delta(i_*))/di_* < 0$. Together with $\Delta(i_-) = 0$, this shows that $\Delta(i_*)$ is negative for every i_* and in particular for $i_* = i_+$. The numerator in $(A1)$ is thus negative while the denominator is positive so that $\text{cov}(\gamma_1, \pi) - \text{cov}(\gamma_2, \pi) < 0$ which proves the lemma.

■