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Thematic clubs and the supremacy  
of network externalities

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**Abstract**

We raise the problem of the minorities' survival in the presence of positive network externalities. We rely on the example of thematic clubs to illustrate why and in which circumstances such survival problems might appear, first considering the case of simple network externalities and then the case of cross network externalities.

**Keywords:** thematic clubs, network externalities, cross network externalities

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## 1. INTRODUCTION

Consider two thematic clubs offering each a differentiated service to its members, as it would be the case, for instance, of a jazz club or a coffee proposing philosophical forums. Such clubs create opportunities for their members to listen to jazz music (or discuss philosophy). They also serve to generate social networks, facilitating interactions among club members. Generally, both these services are positively valued by their members.

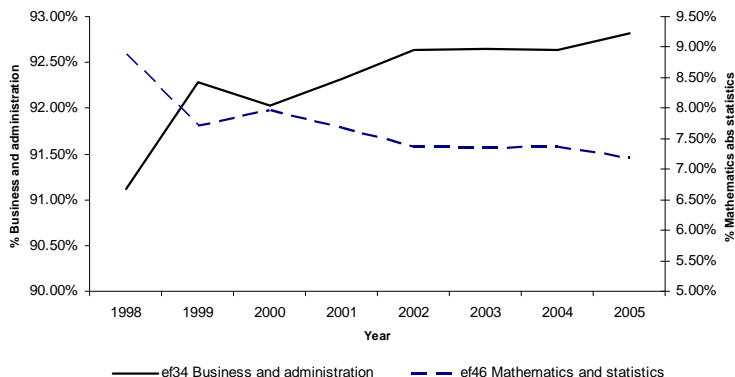
In this paper, we wish to raise the following question. Assume that the size of the population of jazz music fans is very large compared with the philosophers' population size. Are there any circumstances in which philosophical forums would purely and simply disappear because of the supremacy of social network externalities over the individual preferences for the themes patronized by the clubs?

Similar questions might arise in many other contexts. For instance, this is often the case when choosing a specific career among the different diplomas offered by a university. In some circumstances, even when individuals are particularly interested in a given scientific or professional area, they might be diverted from it when the number of people working in this area is not sufficiently large to generate social recognition of the field. When many individuals share these expectations and all of them sufficiently value social recognition, that diploma might purely and simply disappear from academic programs in favour of alternative and more fashionable fields.

Figure 1 shows the evolution from 1998 to 2005 of % of European (EU-27) 3rd cycle students choosing between the diploma in Business and Administration *versus* Mathematics and Statistics. This figure, even if focusing on two diplomas only, points out a huge asymmetry between the number of students who choose Mathematics and Statistics vs Business and Administration (e.g., in 1998, only 8.9% of the students have chosen the former diploma, while 91.1% have preferred the latter). Moreover, it also illustrates that this asymmetry is progressively growing over time (e.g., in 2005, only 7.2% of the students have chosen Mathematics and Statistics, which means that in 7 years, the relative importance of Mathematics and Statistics Diploma has decreased 19 percentage points!). As we argued before, the existence of positive network externalities might explain (at least partially) this evolution.

At a smaller scale, a similar phenomenon has been observed in economics departments, particularly, with courses in general equilibrium theory which were so fashionable in the seventies. Nowadays they have disappeared from most departments' programs where they have been replaced by more fashionable courses, like courses in game theory or neuroeconomics.

These and many other similar examples point out the problem of the minorities' survival in the presence of positive network externalities. In this paper, we rely on the example of thematic clubs to illustrate why and



**FIG. 1** :% of 3rd cycle students in ef34 and ef46 (Source: Eurostat)

in which circumstances such survival problems might appear, first considering the case of simple network externalities and then the case of cross network externalities. *Simple network externalities* take place when individuals' utility for a given activity depends not only on its intrinsic nature, but also on the number of individuals sharing the same activity. This is typically the case with thematic clubs, whose members are generally interested in the theme of the club but also in the social interactions that the club generates. Similarly, the notion of *cross network externalities* arises when the utility of individuals of a given category depends not only on its intrinsic nature, but also on the number of individuals of another category who share the same activity, and *vice-versa*. For instance, in the case of clubs, these categories could be men and women. While individuals in both categories value the specific activity offered by the club, they only value social interactions when those take place with the opposite sex.

The example of clubs has been chosen with an illustrative purpose only. We suggest in this paper that the supremacy of network externalities over individual preferences can be viewed as a broader phenomenon, arising in a wide range of human activities. Therefore, when the supremacy effect takes place, some of these human activities might purely and simply disappear in a quasi-irreversible way. In this case, we suggest that there might be an intertemporal inconsistency between the interest of today's agents and the diversity of choice made available to future generations.

In the following sections, we provide a formal model where it is possible to identify the necessary and sufficient conditions guaranteeing the survival of the club offering the service in line with the preferences of the minority group. In section 2, we start by considering the case of simple network externalities, while cross network externalities are examined in section 3.

Finally, section 4 sets up the main conclusions.

## 2. SIMPLE NETWORK EXTERNALITIES

We start by setting the basic model in which simple network externalities are analyzed. To this end, consider two differentiated clubs (club 1 and 2), each of them generating simple network externalities. The universe of individuals is divided into two distinct populations. A fraction  $\lambda_1$  of the total population (population 1) prefers the activity offered by club 1, while the other fraction,  $\lambda_2$  (population 2) prefers the activity offered by club 2, with  $\lambda_1 + \lambda_2 = 1$ .

Individuals in each population do not value equally the specific activity proposed by their intrinsically preferred club, while on the contrary, all of them are identical in terms of the benefit they get from social interactions. More precisely, we denote by  $U_{1j,t}(m)$  the utility obtained by an individual  $m$  who belongs to population 1 and attends club  $j$ ,  $j = 1, 2$ , at period  $t$ :

$$\begin{aligned} U_{11,t}(m) &= m + \alpha N_{1,t-1}; \\ U_{12,t}(m) &= \alpha N_{2,t-1}, \end{aligned}$$

where  $m$  is uniformly distributed on the interval  $[0, 1]$  and corresponds to the benefit obtained from the activity in club  $j$ . As for the term  $\alpha N_{j,t-1}$ , it corresponds to the benefit obtained from social interactions among the members in club  $j$  ( $\alpha > 0$  represents the *intensity* of the network externalities). Analogously, the utility of an individual  $m$  in population 2 is given by

$$\begin{aligned} U_{21,t}(m) &= \alpha N_{1,t-1}; \\ U_{22,t}(m) &= m + \alpha N_{2,t-1}. \end{aligned}$$

Network externalities are assumed to be perceived by individuals with a time lag of one period. Accordingly, at period 0, each individual chooses his/her club only according to his/her preferences over the services rendered by each club. Therefore,  $N_{1,0} = \lambda_1$  and  $N_{2,0} = \lambda_2$ . Assume also that, at period 0, the two populations have asymmetric sizes - say, without loss of generality,  $\lambda_1 > \lambda_2$ . Then, at period 0, club 1 constitutes a larger network than club 2 ( $N_{1,0} > N_{2,0}$ ) and, everything else being the same, club 1 becomes more attractive for individuals in population 1, but as well for individuals in population 2. Consider the individual  $m_0$  in population 2 for whom the equality

$$\alpha N_{1,0} = m_0 + \alpha N_{2,0}$$

holds. For this individual, the network benefit from attending club 1 instead of club 2,  $\alpha(N_{1,0} - N_{2,0})$ , exactly offsets the opportunity cost of not participating in his/her preferred activity ( $m_0$ ). Therefore, at period 1,

some individuals of population 2 decide to switch to club 1 and the size of the network of this club increases from  $N_{1,0}$  to  $N_{1,1}$  as given by

$$\begin{aligned} N_{1,1} &= N_{1,0} + m_0 N_{2,0}; \\ &= \lambda_1 + \alpha(N_{1,0} - N_{2,0})\lambda_2. \end{aligned}$$

At the end of period 1, the network of club 1 is reinforced, and this club becomes even more attractive to individuals in both populations. Once more, club 1 attracts new individuals from population 2 and reinforces the size of its network again, and so on. More generally, at period  $t+1$ , network sizes are given by

$$N_{1,t+1} = \lambda_1 + \alpha(N_{1,t} - N_{2,t})\lambda_2 \quad (1)$$

and

$$N_{2,t+1} = 1 - N_{1,t+1}. \quad (2)$$

Incorporating equation (2) into equation (1), the network size of club 1 at period  $t+1$  may be re-written as

$$N_{1,t+1} = (1 - \lambda_2) - \alpha\lambda_2 + 2\alpha\lambda_2 N_{1,t},$$

which corresponds to a first order linear difference equation whose general solution is given by

$$N_{1,t} = \frac{1 - (\alpha + 1)\lambda_2}{1 - 2\alpha\lambda_2} - \frac{\alpha\lambda_2(1 - 2\lambda_2)}{1 - 2\alpha\lambda_2} (2\alpha\lambda_2)^t. \quad (3)$$

As for the network size of club 2 at period  $t$ , it is simply given by

$$N_{2,t} = 1 - N_{1,t}. \quad (4)$$

As shown in the next proposition, the survival of both clubs in the long run occurs if and only if the valuation of social interactions by the potential clubs' members is not too significant.

**PROPOSITION 1.** *Both clubs survive in the long run with strictly positive memberships if and only if*

$$\alpha < 1. \quad (5)$$

*Proof.* From equation (3), convergence occurs whenever  $2\alpha\lambda_2 < 1$ . Whenever this is the case, in order to guarantee that both clubs have strictly positive memberships, the steady state market share of club 1 ( $N_1^* = \lim_{t \rightarrow \infty} N_{1,t}$ ) must be strictly smaller than 1. This is equivalent to

$$\frac{1 - (\alpha + 1)\lambda_2}{1 - 2\alpha\lambda_2} < 1 \iff \alpha < 1.$$

■

When the survival condition is not met, the activity proposed by the club which is intrinsically the most preferred by the minority irremediably disappears due to the supremacy of network externalities. As shown in the following proposition, the date of elimination depends not only on the intensity of network externalities but also on the relative sizes of the initial populations.

PROPOSITION 2. *When the "survival condition" (5) is violated, the minority club 2 is eliminated at date  $T^*$ , with*

$$T^* = \frac{1}{\ln(2\alpha\lambda_2)} \ln\left(\frac{(\alpha - 1)}{\alpha(1 - 2\lambda_2)}\right).$$

In the next section, we move a step further and consider the case of cross network externalities.

### 3. CROSS NETWORK EXTERNALITIES

Assume the same situation as before, but now the members of a club value social interactions only when they take place with members of the opposite sex. This is often the case because individuals frequently go to clubs not only to share common interests with other members, but also to get some "*amorous rendez-vous*"! In such circumstances, each club offers a specific service but also serves as a platform embarking two categories of members: men and women.

With cross network externalities, the situation is accordingly more complex than under simple network externalities. In the latter case, there are only two categories of agents : those who prefer philosophy to jazz and those who prefer jazz to philosophy (regardless of the sex of the membership). In the former case, there are in fact four categories: women (men) who prefer jazz to philosophy and women (men) who prefer philosophy to jazz.

We consider two differentiated clubs (platforms), say club 1 and 2, interacting simultaneously with two categories of members: women ( $W$ ) and men ( $M$ ). Each of those categories can be subdivided according to their preferences over the activity developed in each club. Accordingly, we must consider two sub-populations of women (population 1 and population 2) and two sub-populations of men. In the side of women,  $\mu_1$  and  $\mu_2$  ( $\mu_1 + \mu_2 = 1$ ) represent the mass of women in sub-populations 1 and 2, respectively. Analogously, in the men' side,  $\eta_1$  and  $\eta_2$  ( $\eta_1 + \eta_2 = 1$ ) represent the mass of men in sub-populations 1 and 2, respectively.

In the women' side, the utility obtained by the woman  $v$  that belongs

to sub population 1 and attends club  $j$  ( $j = 1, 2$ ) at period  $t$  is given by

$$\begin{aligned} U_{11,t}^W(v) &= v + \beta N_{1,t-1}^M \\ U_{12,t}^W(v) &= \beta N_{2,t-1}^M, \end{aligned} \quad (6)$$

where  $v$  is uniformly distributed on the interval  $[0, 1]$  and corresponds to the benefit obtained by this woman from the activity in club  $j$ . As for the term  $\beta N_{j,t-1}^M$ , it corresponds to the benefit from social interaction with men attending the same club ( $\beta > 0$  represents the *intensity* of the network externalities). Analogously, the utilities of the woman  $v$  that belongs to population 2 are

$$\begin{aligned} U_{21,t}^W(v) &= \beta N_{1,t-1}^M \\ U_{22,t}^W(v) &= v + \beta N_{2,t-1}^M. \end{aligned} \quad (7)$$

By symmetry, in the men' side, the utility obtained by the man  $k$  that belongs to sub population 1 and attends club  $j$  ( $j = 1, 2$ ) at period  $t$  is given by

$$\begin{aligned} U_{11,t}^M(k) &= k + \theta N_{1,t-1}^W \\ U_{12,t}^M(k) &= \theta N_{2,t-1}^W, \end{aligned} \quad (8)$$

where  $k$  is uniformly distributed on the interval  $[0, 1]$  and corresponds to the benefit obtained by this man from the activity in club  $j$ . As for the term  $\theta N_{j,t-1}^W$ , it corresponds to the benefit from social interaction with women attending the same club ( $\theta > 0$  represents the *intensity* of the network externalities).

Analogously, the utilities of the man  $k$  that belongs to population 2 are

$$\begin{aligned} U_{21,t}^M(k) &= \theta N_{1,t-1}^W \\ U_{22,t}^M(k) &= k + \theta N_{2,t-1}^W. \end{aligned} \quad (9)$$

At period 0, all individuals in both categories choose to attend their most intrinsically preferred club, i.e.,  $N_{1,0}^W = \mu_1$ ;  $N_{2,0}^W = \mu_2$ ;  $N_{1,0}^M = \eta_1$  and  $N_{2,0}^M = \eta_2$ . We assume :  $\mu_1 > \mu_2$  and  $\eta_1 > \eta_2$ .

At period 0, club 1 attracts a larger male membership (given  $\eta_1 > \eta_2$ ) which, everything else the same, makes this club more attractive to women in population 1 but also to women in population 2. As a result, the woman  $v_0$  in the "natural membership" of club 2 becomes indifferent between club 1 and 2:

$$\begin{aligned} v_0 + \beta N_{2,0}^M &= \beta N_{1,0}^M \\ v_0 &= \beta(N_{1,0}^M - N_{2,0}^M). \end{aligned} \quad (10)$$



Consequently, at period 1, the number of women attending club 1 and 2 are now given by:

$$N_{1,1}^W = \mu_1 + \beta(N_{1,0}^M - N_{2,0}^M)\mu_2 \quad (11)$$

and

$$\begin{aligned} N_{2,1}^W &= (1 - \beta(N_{1,0}^M - N_{2,0}^M))\mu_2 \\ &= 1 - N_{1,1}^W, \end{aligned}$$

respectively.

More generally, at period  $t$ , the female membership' sizes corresponding to club 1 and 2 are given by

$$N_{1,t}^W = \mu_1 + \beta(N_{1,t-1}^M - N_{2,t-1}^M)\mu_2 \quad (12)$$

and

$$N_{2,t}^W = 1 - N_{1,t}^W. \quad (13)$$

By analogy, it is easy to derive the male membership' sizes corresponding to club 1 and 2 at period  $t + 1$ , as

$$N_{1,t}^M = \eta_1 + \theta(N_{1,t-1}^W - N_{2,t-1}^W)\eta_2 \quad (14)$$

and

$$N_{2,t}^M = 1 - N_{1,t}^M. \quad (15)$$

Applying equations (14) and (15), with  $t = t - 1$ , we compute  $N_{1,t-1}^M$  and  $N_{2,t-1}^M$ , respectively. Then, we substitute these values in equation (12) and we obtain the female membership' size of club 1 conditional on the female membership two periods before, namely

$$N_{1,t}^W = (\mu_1 + 2\beta\eta_1\mu_2 - 2\beta\theta\eta_2\mu_2 - \beta\mu_2) + (4\beta\theta\eta_2\mu_2) N_{1,t-2}^W, \quad (16)$$

which corresponds to a second order linear difference equation and determines the trajectory of the female memberships of club 1 :

$$N_{1,\frac{t}{2}}^W = \frac{B}{1-A} + \left( N_{1,0}^W - \frac{B}{1-A} \right) A^{\frac{t}{2}} \quad (\text{when } t \text{ is even}) \quad (17)$$

$$N_{1,\frac{t-1}{2}}^W = \frac{B}{1-A} + \left( N_{1,1}^W - \frac{B}{1-A} \right) (A)^{-\frac{1}{2}} A^{\frac{t}{2}} \quad (\text{when } t \text{ is odd}), \quad (18)$$

with

$$B = \mu_1 + 2\beta\eta_1\mu_2 - 2\beta\theta\eta_2\mu_2 - \beta\mu_2 \quad (19)$$

$$A = 4\beta\theta\eta_2\mu_2. \quad (20)$$

The same analysis can be performed in the men' side, where the size of the male membership of each club can be rewritten as a second order linear difference equation, namely

$$N_{1,t}^M = (\eta_1 + 2\theta\mu_1\eta_2 - 2\theta\beta\mu_2\eta_2 - \theta\eta_2) + (4\theta\beta\mu_2\eta_2) N_{1,t-2}^M. \quad (21)$$

Equation (21) determines the trajectory of the male memberships of club 1 :

$$N_{1,\frac{t}{2}}^M = \frac{C}{1-A} + \left( N_{1,0}^M - \frac{C}{1-A} \right) A^{\frac{t}{2}} \quad (\text{when } t \text{ is even}) \quad (22)$$

$$N_{1,\frac{t-1}{2}}^M = \frac{C}{1-A} + \left( N_{1,1}^M - \frac{C}{1-A} \right) (A)^{-\frac{1}{2}} A^{\frac{t}{2}} \quad (\text{when } t \text{ is odd}), \quad (23)$$

with  $A$  given by equation (20) and

$$C = \eta_1 + 2\theta\mu_1\eta_2 - 2\theta\beta\mu_2\eta_2 - \theta\eta_2. \quad (24)$$

To determine the trajectories of the memberships in the case of simple network externalities, we had simply to specify the initial membership' sizes at period 0. Since, in equations (16) and (21), the effectiveness of network externalities takes a time lag of two periods, in the presence of cross network externalities, initial conditions should now be specified over period 0 and 1. However, whenever trajectories converge, the dynamics of membership' sizes do not depend on the initial conditions. The next proposition identifies the necessary and sufficient condition for convergence of female and male membership' sizes.

**PROPOSITION 3.** *The sequences of female and male membership' sizes of each club,  $\{N_{1,t}^W\}$ ;  $\{N_{2,t}^W\}$ ;  $\{N_{1,t}^M\}$  and  $\{N_{2,t}^M\}$ , converge if, and only if*

$$4\beta\theta\eta_2\mu_2 < 1. \quad (25)$$

*Proof.* It follows directly from equations (17)-(18) and (22)-(23). ■

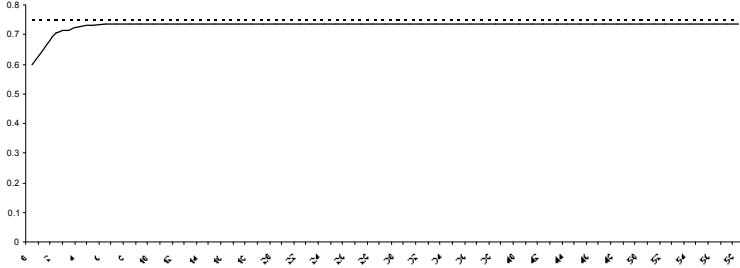
However, the convergence condition identified in proposition 3 is not sufficient to guarantee the survival of both clubs in the long run, since the steady state membership' sizes of both categories must be strictly smaller than 1. From equations (17)-(18) and (22)-(23), we compute the steady state membership' sizes for each category as

$$(N_1^W)^* = \frac{B}{1-A} = \frac{\mu_1 + 2\beta\eta_1\mu_2 - 2\beta\theta\eta_2\mu_2 - \beta\mu_2}{1 - 4\beta\theta\eta_2\mu_2}; \quad (26)$$

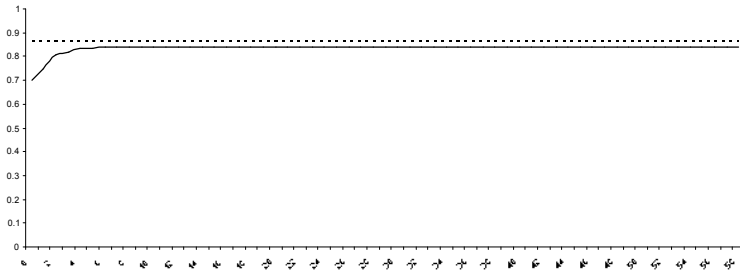
$$(N_2^W)^* = 1 - (N_1^W)^*;$$

$$(N_1^M)^* = \frac{C}{1-A} = \frac{\eta_1 + 2\theta\mu_1\eta_2 - 2\theta\beta\mu_2\eta_2 - \theta\eta_2}{1 - 4\beta\theta\eta_2\mu_2}; \quad (27)$$

$$(N_2^M)^* = 1 - (N_1^M)^*.$$



**FIG. 2** :Trajectory of club 1's female membership



**FIG. 3** :Trajectory of club 1's male membership

From equations (26) and (27), it follows that

$$(N_1^W)^* < 1 \Leftrightarrow \theta < \frac{1}{2\beta\eta_2} - \frac{1-2\eta_2}{2\eta_2}; \quad (28)$$

$$(N_1^M)^* < 1 \Leftrightarrow \theta < \frac{1}{2\beta\mu_2 + (1-2\mu_2)}. \quad (29)$$

Figures 1 and 2 illustrate the trajectories of female and male memberships, assuming  $\beta = 0.5$ ;  $\theta = 1$ ;  $\mu_1 = 0.6$  and  $\eta_1 = 0.7$ , which means that both conditions (28) and (29) hold.

Notice that, in the presence of cross network externalities, the survival analysis becomes more complex and four possible outcomes can be *a priori* expected. First, both clubs could survive in the long run, attracting

members of both categories (men and women). However, there are other possibilities with survival of both clubs: a club could survive in the long run with a single category of members (men or women). Finally, it is also possible that the club in line with the preferences of the minority is eliminated, and only one club survives.

The following propositions identify the necessary and sufficient conditions for each of these alternatives to realize.

PROPOSITION 4. *Both thematic clubs (club 1 and 2) survive in the long run with members of both categories if and only if both men and women are not too much attracted by social interactions, namely when one of the following conditions holds:*

$$(i) \quad 0 < \beta < 1 \text{ and } \theta < \frac{1}{2\beta\mu_2 + (1 - 2\mu_2)}$$

$$(ii) \quad 1 < \beta < \frac{1}{(1 - 2\eta_2)} \text{ and } \theta < \frac{1}{2\beta\eta_2} - \frac{(1 - 2\eta_2)}{2\eta_2}.$$

*Proof.* In appendix. ■

PROPOSITION 5. *The thematic club specialized in the activity preferred by the minority (club 2) survives in the long run with a single category of members (men or women) if and only if that category is not too much attracted by social interaction with the opposite sex, while the other category is. Namely, the steady state membership of club 2 is restricted to men if and only if*

$$\beta > 1 \text{ and } \frac{1}{2\beta\eta_2} - \frac{(1 - 2\eta_2)}{2\eta_2} < \theta < \frac{1}{2\beta\mu_2 + (1 - 2\mu_2)}.$$

*Conversely, the steady state membership of club 2 is restricted to women if and only if one of the following condition holds:*

$$0 < \beta < 1 \text{ and } \frac{1}{2\beta\mu_2 + (1 - 2\mu_2)} < \theta < \frac{1}{2\beta\eta_2} - \frac{(1 - 2\eta_2)}{2\eta_2}.$$

*Proof.* In appendix. ■

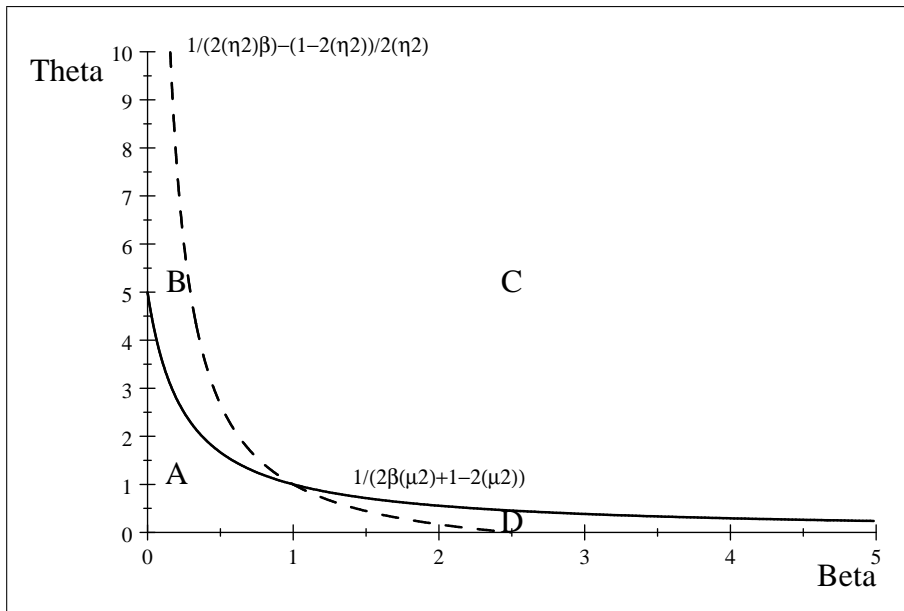
PROPOSITION 6. *The thematic club specialized in the activity preferred by the majority (club 1) survives alone in the long run if and only if both categories of members value social interactions with the opposite sex sufficiently high; more precisely, when one of the following conditions holds:*

$$(i) \quad 0 < \beta < 1 \text{ and } \theta > \frac{1}{2\beta\eta_2} - \frac{(1 - 2\eta_2)}{2\eta_2}$$

$$(ii) \quad \beta > 1 \text{ and } \theta > \frac{1}{2\beta\mu_2 + (1 - 2\mu_2)}.$$

*Proof.* In appendix. ■

The following figure illustrates the above propositions, assuming  $\mu_2 = 0.4$  and  $\eta_2 = 0.3$  and representing the survival regions in the space  $(\beta, \theta)$  of the preferences intensities for network externalities .



**FIG 4:** Survival analysis ( $\mu_2 = 0.4$  and  $\eta_2 = 0.3$ ).

In region *A*, proposition 4 holds and both women and men exhibit preferences with relatively weak network intensities so that club 2 succeeds to survive, even if it proposes a theme in line with the taste of the minority. In regions *B* and *D*, proposition 5 holds so that club 2 is able to attract a single category of members only. More precisely, in region *B* (*D*), men (women) highly value the social relations with the opposite sex while, conversely, women (men) do not; as a result, in region *B* (*D*), the steady state membership of the club proposing the activity preferred by the minority (club 2) is exclusively made of women (men). Finally, in region *C*, club 2 does not survive at all, because both categories of members value very highly (too highly?) the social interactions with the opposite sex.

Additionally, as a consequence of the above propositions, survival conditions not only depend on the intensity of preferences for network externalities ( $\beta$  and  $\theta$ ), but also on the initial asymmetry resulting from intrinsic preferences for the themes proposed by each club ( $\mu_1$  and  $\mu_2$ , in the case of women, and  $\eta_1$  and  $\eta_2$  in the case of men). In general, for given network intensities, the higher the asymmetry in the initial dimension of the populations (i.e., the higher  $\mu_1$  and  $\eta_1$ , or equivalently, the lower  $\mu_2$  and  $\eta_2$ ),

the harder it is for club 2 to survive with a "mixte" membership. One significant exception to this statement is when both categories of members have the same preference intensities with respect to network externalities ( $\beta = \theta$ ). In this case, it is easy to check that the outcomes identified in proposition 5 cannot occur. More precisely, when  $\beta = \theta$ , either both clubs survive with mixte memberships ( $0 < \theta < 1$ ), or only the thematic club preferred by the majority succeeds to survive ( $\theta > 1$ ).

#### 4. CONCLUSION

The example of clubs studied in the above sections emphasizes the existence of an endogenous mechanism possibly generating the permanent disappearance of certain human activities. This phenomenon has also been stressed in the literature on "evolution stability and societal outcomes" (see Bowles, (2004)), exhibiting some stable equilibrium where, in the long run, an *a priori* heterogeneous population specializes on a single type. This approach however is based on the effect of mismatching between members of different types and the resulting updates of their traits. Differently, in our approach, the basic mechanism lies on the supremacy of network externalities: the disappearance of certain human activities relies on the fact that individual preferences for the intrinsic values obtained from participating to these activities are not sufficiently strong to overcome the intensity of the network externalities they generate from period to period.

Of course, within our framework, the individuals who were initially participating to these activities do not object against their disappearance, since none of them is forced to choose an alternative which would differ from his/her optimal individual choice. Consequently, it cannot be argued that the mechanism evoked above would be harmful for those who were initially concerned.

It is important however to stress the quasi-irreversibility character related to the disappearance of such activities. When the network on which their very existence is based, comes to vanish, only some external and voluntary intervention could possibly reinstate the interest of individuals for this activity. In the absence of such a voluntary intervention, younger and future generations could be definitely deprived from the opportunity of choosing themselves whether these activities are, or are not, meaningful to them. Furthermore, the diversity of their choice would be inexorably reduced due to their disappearance. The reduction of diversity due to the supremacy of network effects could eventually create a social damage whenever focusing social values on a restricted number of alternatives only, transforms the set of social activities and opinions into a uniform, and perhaps boring, landscape.

The evolution of the daily press industry provides a significant example of the possible consequences of this endogenous mechanism, known under the name of the *circulation spiral*. This phenomenon has been identified by

Furhoff (1973) and formally analyzed in Gabszewicz *et al* (2007). According to the circulation spiral theory, the interaction between the newspapers' and advertising markets drives the newspaper with the smaller readership into a vicious circle finally leading to its disappearance. The essence of this theory rests on the positive network externalities existing between the two markets : a larger readership attracts more advertising and again more readers. The smaller newspaper loses readers, aggravating the problem of selling advertising, and has finally to close down. The main consequence of the disappearance of the minority newspaper is not that some readers have lost their preferred outlet: all of them have finally agreed to switch to the rival one, due to the larger benefit obtained from more informative advertising in it. The true damage comes from the fact that, while disappearing, the smaller newspaper leaves a single outlet only available to the readership in the future. The pluralism of opinions, so important for the spreading and survival of democracies, might have henceforth disappeared for ever from the journalistic landscape.

The last example points out the fact that the theory developed above about the persistence, or disappearance, of activities through time can be as well applied to the case of economic commodities. The most spectacular examples of such disappearances of goods are provided by goods submitted to fashion, like clothes, perfumes, touristic services, a.s.o. Fashionable goods generally survive (have a positive demand) as long as the fashion survives. When a different fashion rises, it can take the place of the existing one which has to disappear.

One should not be too pessimistic however. First, individuals may participate simultaneously to several activities (multihoming)<sup>1</sup>. This opportunity has not been taken into account in our analysis since it is implicitly supposed here that individuals adhere to a club at the exclusion of the other. The possibility of double membership should strengthen the probability of survival of the minorities since participation in the minorities' interests does not preclude to share as well the interests for the activities preferred by the majority. Second, firms or institutions offering economic goods or services generating positive network externalities can also develop several strategies to countervail the superior size of the rivals' networks. In a context of price competition between firms, one example of such strategies could be the introduction of price discounts, or even negative prices, as it has been the case for instance in the European press industry, with a significant number of newspapers offering special gifts to attract readers.

Our model does not take into account the existence of such strategies since it deals with individual choices operating in a context from which price competition is absent. Should the clubs impose entry fees to their members, it would open the door to practises analysed in a static context

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<sup>1</sup>Recent studies have stressed the importance of multihoming in the static context of price competition with two-sided markets, see Armstrong and Wright (2007) and Gabszewicz and Wauthy (2004).

by industrial economists interested in networks and two-sided markets (for an excellent recent survey, see Farrell, J. and P. Klemperer (2007); on two sided markets in particular, see Rochet and Tirole (2003) and Armstrong (2006)).<sup>2</sup>

Finally, the emergence of new fashions does not always entail that the old ones have to vanish. Their simultaneous presence then enriches the choice possibility set of the individuals. Such a cohabitation arises when network externalities which have brought the fashionable objects in light are not sufficiently strong to prevent the birth and the development of parallel objects or activities, creating and keeping alive their own networks of fans.

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<sup>2</sup>To our knowledge, the question of minorities' survival in a context of product differentiation, price interaction and lagged network externalities (in both cases of simple network externalities and cross network externalities) has not been so far addressed in a dynamic context. Doganoglu (2003) provides a dynamic framework for the analysis of price competition in the presence of simple network externalities. Unfortunately, this study rules out the possibility of the smaller network's eviction and, consequently, is not able to provide us any guidance on the questions related to the survival of the minority's variants in the presence of networks.



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## Appendix

### Proof of proposition 4

Club 2 survives with a mixte membership if and only if

$$\begin{cases} \theta < \frac{1}{2\beta\eta_2} - \frac{1-2\eta_2}{2\eta_2} \\ \theta < \frac{1}{1-2\mu_2+2\beta\mu_2} \end{cases} \quad (30)$$

or equivalently,

$$\theta < \min \left\{ \frac{1}{2\beta\eta_2} - \frac{1-2\eta_2}{2\eta_2}; \frac{1}{1-2\mu_2+2\beta\mu_2} \right\}$$

with

$$\begin{aligned}
& \frac{1}{2\beta\eta_2} - \frac{1-2\eta_2}{2\eta_2} < \frac{1}{1-2\mu_2+2\beta\mu_2} \Leftrightarrow \\
& \Leftrightarrow 4\beta\mu_2 - 2\mu_2 - \beta - 2\beta^2\mu_2 + 4\beta^2\mu_2\eta_2 - 4\beta\mu_2\eta_2 + 1 < 0 \\
& \Leftrightarrow \left( \frac{2\mu_2-1}{2\mu_2(1-2\mu_2)} > \beta \vee \beta > 1 \right) \text{ (since } 2\beta^2\mu_2(-1+2\eta_2) < 0) \Leftrightarrow \\
& \Leftrightarrow \beta > 1 \left( \text{since, by definition } \beta > 0 > \frac{2\mu_2-1}{2\mu_2(1-2\eta_2)} \right)
\end{aligned}$$

Therefore

$$0 < \beta < 1 \Rightarrow \frac{1}{2\eta_2\beta} - \frac{1-2\eta_2}{2\eta_2} > \frac{1}{1-2\mu_2+2\beta\mu_2} \quad (31)$$

$$\beta > 1 \Rightarrow \frac{1}{2\eta_2\beta} - \frac{1-2\eta_2}{2\eta_2} < \frac{1}{1-2\mu_2+2\beta\mu_2} \quad (32)$$

Accordingly, both clubs survive with a mixte membership if and only if

$$0 < \beta < 1 \text{ and } \theta < \frac{1}{2\beta\mu_2+1-2\mu_2} \quad (33)$$

or

$$\beta > 1 \text{ and } \theta < \frac{1}{2\eta_2\beta} - \frac{1-2\eta_2}{2\eta_2} \quad (34)$$

Finally, since we are restricting ourselves to the case of positive externalities, the non-negativity constraint over  $\theta$  must be taken into account. In condition (33),

$$\begin{aligned}
& \theta > 0 \Leftrightarrow \\
& \Leftrightarrow \frac{1}{1-2\mu_2+2\beta\mu_2} > 0 \\
& \text{Always true } (\beta > 0)
\end{aligned}$$

And condition (33) corresponds to condition (i) in proposition 4. In condition (34),

$$\begin{aligned}
& \theta > 0 \Leftrightarrow \\
& \Leftrightarrow \frac{1}{2\eta_2\beta} - \frac{1-2\eta_2}{2\eta_2} > 0 \\
& \beta < \frac{1}{(1-2\eta_2)}, \text{ with } \frac{1}{(1-2\eta_2)} > 1 \forall 0 < \eta_2 < \frac{1}{2} \quad (35)
\end{aligned}$$

This means that condition (34) has to be re-written as

$$1 < \beta < \frac{1}{(1-2\eta_2)} \text{ and } \theta < \frac{1}{2\eta_2\beta} - \frac{1-2\eta_2}{2\eta_2}$$

,which corresponds to condition (ii) in proposition 4.■

**Proof of proposition 5**

Club survives with a female membership only if and only if

$$\begin{cases} \theta < \frac{1}{2\eta_2\beta} - \frac{1-2\eta_2}{2\eta_2} \\ \theta > \frac{1}{1-2\mu_2+2\beta\mu_2} \end{cases} \Leftrightarrow \quad (36)$$

$$\Leftrightarrow \frac{1}{1-2\mu_2+2\beta\mu_2} < \theta < \frac{1}{2\eta_2\beta} - \frac{1-2\eta_2}{2\eta_2}$$

which is possible if and only if the values of the parameters are such that

$$\frac{1}{1-2\mu_2+2\beta\mu_2} < \frac{1}{2\eta_2\beta} - \frac{1-2\eta_2}{2\eta_2}$$

which, according to (31) and (32) only happens when

$$0 < \beta < 1$$

Therefore, club 2 attracts a female membership only if and only if  $0 < \beta < 1$  and  $\frac{1}{2\beta\mu_2+1-2\mu_2} < \theta < \frac{1}{2\eta_2\beta} - \frac{1-2\eta_2}{2\eta_2}$ , as stated in proposition 5.

Similarly, the membership of club 2 is restricted to men if and only if

$$\begin{cases} \theta > \frac{1}{2\eta_2\beta} - \frac{1-2\eta_2}{2\eta_2} \\ \theta < \frac{1}{1-2\mu_2+2\beta\mu_2} \end{cases} \Leftrightarrow \quad (37)$$

$$\Leftrightarrow \frac{1}{2\eta_2\beta} - \frac{1-2\eta_2}{2\eta_2} < \theta < \frac{1}{1-2\mu_2+2\beta\mu_2}$$

The previous condition requires

$$\frac{1}{2\eta_2\beta} - \frac{1-2\eta_2}{2\eta_2} < \frac{1}{1-2\mu_2+2\beta\mu_2}$$

which according to (31) and (32) only happens when

$$\beta > 1$$

Moreover, since the first condition in (37) now imposes a lower threshold on the parameter  $\theta$ , the non-negativity condition on this parameter (condition (35)) is not binding. Therefore, as stated in proposition 5, club 2 survives with a male membership only when  $\beta > 1$  and  $\frac{1}{2\eta_2\beta} - \frac{1-2\eta_2}{2\eta_2} < \theta < \frac{1}{1-2\mu_2+2\beta\mu_2}$ . ■

**Proof of proposition 6**

Club 2 doesn't get a positive membership if and only if

$$\begin{cases} \theta > \frac{1}{2\eta_2\beta} - \frac{1-2\eta_2}{2\eta_2} \\ \theta > \frac{1}{1-2\mu_2+2\beta\mu_2} \end{cases} \quad (38)$$

,which is equivalent to require

$$\theta > \max \left\{ \frac{1}{2\eta_2\beta} - \frac{1-2\eta_2}{2\eta_2}, \frac{1}{1-2\mu_2+2\beta\mu_2} \right\} \quad (39)$$

Recalling equations (31) and (32), the solutions to inequation (39) are:

$$0 < \beta < 1 \text{ and } \theta > \frac{1}{2\eta_2\beta} - \frac{1-2\eta_2}{2\eta_2} \quad (40)$$

$$\beta > 1 \text{ and } \theta > \frac{1}{1-2\mu_2+2\beta\mu_2} \quad (41)$$

again, since equation (41) imposes a lower threshold on  $\theta$ , the non-negativity condition (35) is not binding. Condition (40) is condition (i) in proposition 6 and condition (41) is condition (ii) in the same proposition. ■