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On production and abatement time scales  
in sustainable development.  
Can we loosen the *sustainability screw* ?

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CORE DISCUSSION PAPER  
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**On production and abatement time scales  
in sustainable development.  
Can we loosen the *sustainability screw* ?**

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**Abstract**

In this paper we carry out a preliminary exploration of a time scales' conjecture, which postulates that "reasonable" notions of sustainability must include a suitable synchronisation of time scales of both the processes of human development and those of the natural environment. We perform our analysis within a coarse, five variable, model of man-nature interactions expressed as a system of differential equations where production and human capital are coupled with both renewable and non-renewable natural resource. We demonstrate a phenomenon that we name the "sustainability screw" that describes a spiral like trajectory of the three key variables: non-renewable and renewable resources as well as the production capital. Under many plausible scenarios, this spiral tends unacceptably fast to an undesirable equilibrium. However, we also show that by adjusting the ratio of "intensity of production effort" and "intensity of abatement effort", parameters of the relative time scales of production and natural recovery processes can be altered in a manner that produces, arguably, more sustainable trajectories.

**Keywords:** sustainable optimization systems, viability, multiple time scale.

**JEL Classification:** C61, Q01 - **MSC Classification:** 37N40, 34E99

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# 1 Introduction

According to P. Hawken, the author of “The Ecology of Commerce: A Declaration of Sustainability” [8], *sustainability is an economic state where the demands placed upon the environment by people and commerce can be met without reducing the capacity of the environment to provide for future generations*. While, nowadays, most would agree with the underlying principle embedded in the preceding statement many still regard it as utopian. Furthermore, whether intentionally or unintentionally, this notion of sustainability appears to focus more on the ultimate “system equilibrium”<sup>1</sup> of human and environmental interactions rather than on the dynamic nature of these interactions. Concerning the latter, the 1972 “The Limits to Growth” controversial publication [10] prepared for the so-called Club of Rome, predicted a rather bleak future for humanity driven by the dynamics of population growth and environmental exploitation leading to exhaustion. Despite many criticisms of that report, the incontestable fact remains that - in the regions where both population and production continue to grow - pressure on natural resources increases.

Nevertheless, there is also a significant school of thought - sometimes referred to as “free-market environmentalism” - that places a lot of confidence in economic development, globalisation and human capital as drivers that eliminate poverty and ultimately lead to increased environmental protection and reduced population growth. Although there are many examples where technological advances averted negative environmental impacts<sup>2</sup> there is clearly a vast array of, potentially, adverse consequences of the existing trends in man-nature interactions.

Thus an obvious prerequisite for a meaningful discussion on “sustainability” is that we first understand, at least qualitatively, the underlying dynamics of man-nature interactions. It is in this context that we set our goal in this paper: we want to carry out a, preliminary, exploration of the following “time scales”<sup>3</sup> conjecture that, in conceptual terms, postulates that *reasonable notions of sustainability must include a suitable synchronisation of time scales of both the processes of human development and those of the natural*

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<sup>1</sup>Also, the notion of *providing for future generations* leaves scope for interpretation of what “providing” means.

<sup>2</sup>For example, the New York city councillors feared at the beginning of the 20th century that, given the rapid increase of horse driven street cars, the amount of the horse manure would be unbearable for the city by the early twenties.

<sup>3</sup>We have borrowed the *time scale* notion from optimal control, see e.g., [11]. In broad terms it signifies that, in a multi-variable dynamical system, some variables change significantly slower than others.

*environment.*

A full-scale modelling and exploration of the above conjecture would need to involve detailed modelling of the biological, chemical and physical processes and coupling these with economic growth dynamics. The latter is beyond the scope of this preliminary study. However, we begin by considering the two aspects that are relatively easy to capture and that, indirectly, affect the time scales of both the production and natural resource processes. We name these two aspects “*intensity of production effort*” and “*intensity of abatement effort*”, respectively. We believe that when these are appropriately quantified and calibrated their ratio strongly, albeit indirectly, reflects the underlying relative time scales of production and natural recovery processes.

We perform our analysis within a coarse, five variable, dynamical system where the variables are intended to capture the trends in *non-renewable (mineral) resources* ( $M$ ), *renewable natural resources* ( $R$ ), *production capital* ( $K$ ), and *human capital* ( $H$ ). As to the notion of sustainability we will use it to refer to a multi-variable dynamic process, in which no variable diminishes below some socially acceptable level. This is in the spirit of Aubin’s viability theory (see [2]) and obviates the requirement to be overly prescriptive in a formal definition of this fundamental and yet frequently controversial concept. That formulation obviates the need for an *optimisation* criterion, under which the above trends are generated. We note here an enlightening discussion in [1] on whether *optimal* solutions can be sustainable. Our approach can be viewed as, perhaps, more robust than optimisation under the *sustainability criterion* proposed there. This is because we allow for social welfare to decrease. Indeed, in our setting a substantial decrease in the production capital variable  $K$  will be responsible for social welfare decline.

We demonstrate a phenomenon that we name the “*sustainability screw*” that describes a spiral like trajectory in the  $M - R - K$  phase space of the three key variables: non-renewable and renewable resources as well as the production capital. Under many plausible scenarios, this spiral tends unacceptably fast to an undesirable equilibrium (low-resource, low-production steady state). This behaviour is illustrated in Figure 1 that is explained in more detail in Section 3.2.

However, we show that there are configurations of the intensity of production and abatement parameters that substantially alter the “thread of the screw” to arrive at a more desirable equilibrium. Of course, the time taken by the key variables to pass whatever thresholds we might be interested in is intimately related to the thread mentioned in the preceding sentence. Arguably, controlling that thread by a judicious choice of parameters could “buy time” for new technologies to become viable in dealing with environmental problems.

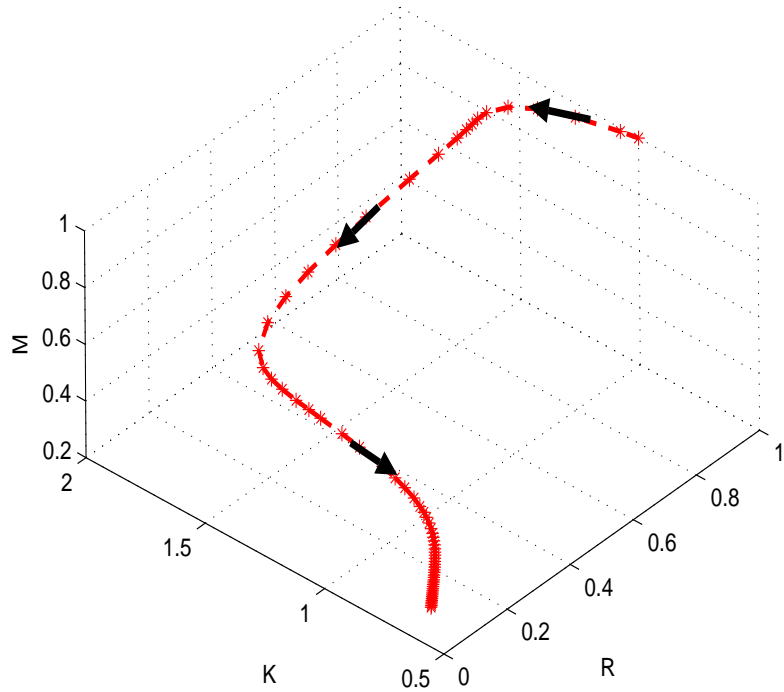


Figure 1: Sustainability screw.

The differential equations in the postulated coupled model have been calibrated to produce trajectories that qualitatively behave like a number of, historically measured, natural surrogates for the variables in question. In particular, the shape of the trajectory generated by the homogeneous human capital equation resembles the global working-age population<sup>4</sup> growth. Similarly, the shape of the trajectory generated by the renewable resources equation resembles the trend of the planet's biocapacity as modelled by the Global Footprint Network [12]. See Section 3 for details of these calibrations.

We hope that this preliminary discussion of the times scales' conjecture and the sustainability screw will stimulate further research aimed at identifying,

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<sup>4</sup>We proxy human capital by working-age population.

perhaps, more sustainable development pathways than those practiced in the recent decades.

What follows is a brief outline of what this paper contains. The core dynamic model is formulated and calibrated in Section 2. We analyse the business-as-usual time profiles of the variables of interest in Section 3. In Section 4 we generate alternative time profiles that correspond to different time scales of the production and abatement processes. We end the paper with the Concluding Remarks aimed at stimulating a further discussion around the *sustainability screw*.

## 2 The model

### 2.1 Systems dynamics

We begin by considering the following variables that relate to a country, region or otherwise a “closed” system:

- Time:  $t \in [0, \infty)$ .
- Non-renewable resource:  $M(t) \in \mathcal{R}_+, \forall t$ .
- Renewable resource:  $R(t) \in \mathcal{R}_+, \forall t$ .
- Production capacities:  $K(t) \in \mathcal{R}_+, \forall t$ .
- Abatement capital:  $Q(t) \in \mathcal{R}_+, \forall t$ .
- Human capital:  $H(t) \in \mathcal{R}_+, \forall t$ .

In Section 3, we will suggest which information gathered statistically can *proxy* the above variables’ historical time profiles. This will also enable us to make their meaning more precise.

The nature-man system that we purpose to consider is portrayed by five, coupled, ordinary differential equations.

The first equation describes the non-renewable resources  $M$ . By its very nature  $M$  is a diminishing variable. However, due to discovery, it may temporarily grow. We will use the following mathematical expression for  $M(t)$ ,  $t \in [0, \infty)$ :

$$\dot{M} = -\varphi_M M + C e^{-\delta t}, \quad (1)$$

where coefficients  $\varphi_M > 0, C > 0, \delta > 0$ . The amount  $\varphi_M M$  represents the non-renewable resource contribution to the growth of production capacities



$K$ , see equation (3). If no new resources are discovered then  $M(t)$  decays exponentially, at a constant rate  $\varphi_M$ . The exogenous term  $Ce^{-\delta t}$  represents the yet to be discovered resource; its long run value is zero as a consequence of the non-renewable nature of these resources.

The second equation describes the renewable resources  $R$ ; we can also think that  $\frac{1}{R}$  is a measure for aggregated environmental pollution. The dynamics of this variable is such that, if unperturbed and not *too much* depleted, it should converge to some pristine value  $\bar{R}$  (probably once attained in the remote past). On the other hand, the resources availability will worsen if certain level of “pollution” is exceeded and the resources lose the capacity to renew. In that case we expect  $R$  to converge to some low value  $\underline{R}$ . This implies that the homogenous solution to an equation for  $R$  needs to bifurcate at some critical level  $R_c > 0$ . We propose the following mathematical expression for  $R(t)$ ,  $t \in [0, \infty)$  that contains a logistic growth term:

$$\dot{R} = \sigma \left( -R + \frac{\gamma}{1 + be^{-\alpha R}} \right) - \varphi_R R + \gamma_Q Q. \quad (2)$$

Coefficient  $\sigma > 0$  weighs the strength, with which  $R$  follows the convergence paths to  $\bar{R}$  or  $\underline{R}$ , relative to the other (*i.e.*, “intra-systemic”) effects. Positive constants  $\gamma, b, \alpha$  describe the logistic growth function for  $R$ . The amount  $\varphi_R R$ ,  $\varphi_R > 0$  represents the renewable resource contribution to growth of production capital  $K$ , see equation (3). On the other hand,  $\gamma_Q Q$ , ( $\gamma_Q > 0$ ) represents the positive impact of abatement capital on the resource renewal.

The third equation describes the production capital  $K$ . When it is fully utilised, then  $K$  will determine consumption. We assume the dynamics of this variable is linear

$$\dot{K} = \varphi_M M + \varphi_R R - (\varphi + \psi_K + \kappa_K)K + \varphi_H H \quad (3)$$

All coefficients  $\varphi, \psi_K, \kappa_K, \varphi_H$  are non negative; in particular,  $\varphi > 0$  characterises the depreciation rate of  $K$ . Quantities  $\psi_K K$  and  $\kappa_K K$  represent the capital sector’s contributions to the growth of abatement capital (see (3)) and of human capital (5), respectively<sup>5</sup>. The amount  $\varphi_H H$  captures the abatement intensification attributable to human capital.

The fourth equation describes abatement capital  $Q$ . We assume the dynamics of this capital to be linear:

$$\dot{Q} = \psi_K K - \gamma_Q Q. \quad (4)$$

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<sup>5</sup>However, as explained after equation (5), the medium-term human capital growth is assumed autonomous in this model.

For simplicity, we assume that  $Q$  grows at a rate that is proportional to the production capital  $K$  and is depleted at a rate proportional to its own size. As its sole purpose is to alleviate pressures placed on the renewable resources by the production processes,  $Q$  does not depreciate of its own accord because the production sector takes full care of its maintenance.

The fifth equation describes formation of human capital  $H$ . We assume the dynamics of this process is nonlinear:

$$\dot{H} = a \left( 1 - \frac{H}{c(R, K)} \right) H, \quad (5)$$

where  $a > 0$ . We further assume that  $c(R, K) = \text{const} > 0$ . This reflects our view that most of people who constitute (or will constitute) human capital in the period that corresponds to a “medium-term” planning horizon (30-50 years, say) have already been born. However, ultimately, it will be natural to make explicit the dependence of  $c(R, K)$  on availability of the natural resource  $R$  (or its reciprocal that carries information about levels of pollution) *and* the consumption levels as represented by production capacities  $K$ . A possible functional representation of such a “well-being” dependence of on  $R$  and  $K$  might be of the form:

$$c(R, K) = c_0 \left( 1 + \frac{R - R_0}{R_0} + \frac{K - K_0}{K_0} \right), \quad c_0 \geq 0 \quad (6)$$

where the terms inside the bracket could be positively weighted; in particular the second term’s weight could depend on  $\kappa_K$ .

While, at first sight, the *human capital* expression (5) may appear radically different from the classical so-called “Lucas-Uzawa” formulation (e.g., see [9]), in fact it is quite consistent with the latter, if we interpret the factor  $1 - \frac{H}{c(R, K)}$  as an “implicit control” of the carrying capacity of the environment on the growth of the human capital. Certainly, the potentially limiting impact of the carrying capacity of the natural environment on human civilisation is well documented in history (e.g., Easter Island in 16th and 17th centuries)

Furthermore, we suppose that - after a suitable normalisation - the state variables assume known values  $M_0 = 1, R_0 = 1, K_0 = 1, Q_0 = 1$  and  $H_0 = 1$  at some initial time  $t = 0$ . This is sufficient to capture our interest in the variables’ *variation* rather than in their absolute levels.

## 2.2 Calibration and uses of the model

We need to calibrate the model (1)-(5) of Section 2.1 so that it is, qualitatively, consistent with historical trends and/or widely accepted projections for

appropriate surrogates of the variables of interest. Towards that goal we collected statistical data on historical profiles of *real* variables that can proxy our hypothetical variables  $M, R, K, Q$  and  $H$ . The profiles should match, at least qualitatively, the model's trajectories for certain, default, parameter values.

Projections from the so calibrated model (1)-(5) will constitute a status-quo or a *business-as-usual*, benchmark, scenario. It will be seen that this benchmark scenario demonstrates the highly undesirable behaviour whereby the essential variables  $M, R, K$  all spiral down to unacceptably low levels.

We will also produce time profiles obtained for a modified parameter set where the modifications will correspond to policies of *time synchronisation* between human efforts and the natural processes. A joint presentation of evolutions of the  $M, R, K$  variables in the three dimensional space – *the sustainability screw* – will be used for visualisation of sustainability analysis. In particular, the status-quo projections will be compared with those obtained for a (better) synchronised policy.

### 3 Model calibration

#### 3.1 Historical time profiles

##### 3.1.1 $M$ - Non renewable resources

We consider oil reserves as a proxy for  $M$ . We believe oil will condition the other variables behaviour in a short-to-medium term (30-50 years).

Figure 2 is a snapshot of the oil production history, as well as includes possible future production curve, see [5]. Indeed, depending on how we steer our oil consumption in the coming years, there can be a number of scenarios.

As the ultimate reserves are finite, the looming fact is that we are depleting these non renewable resources, faster or slower, sooner or later. Left panel of Figure 3 is taken from [6]. When we observe what happens after (approximately) 2005, we can see that an exponential decrease in the oil production is imminent. In particular, one should expect that the oil stock would drop to 20-30% of its original value, after about 50 years of exploitation. This is the kind of drop captured by our model, see the right panel of this figure. This particular profile is a result of running the calibrated model for 50 years, see Section 3.2 (and Figure 7).

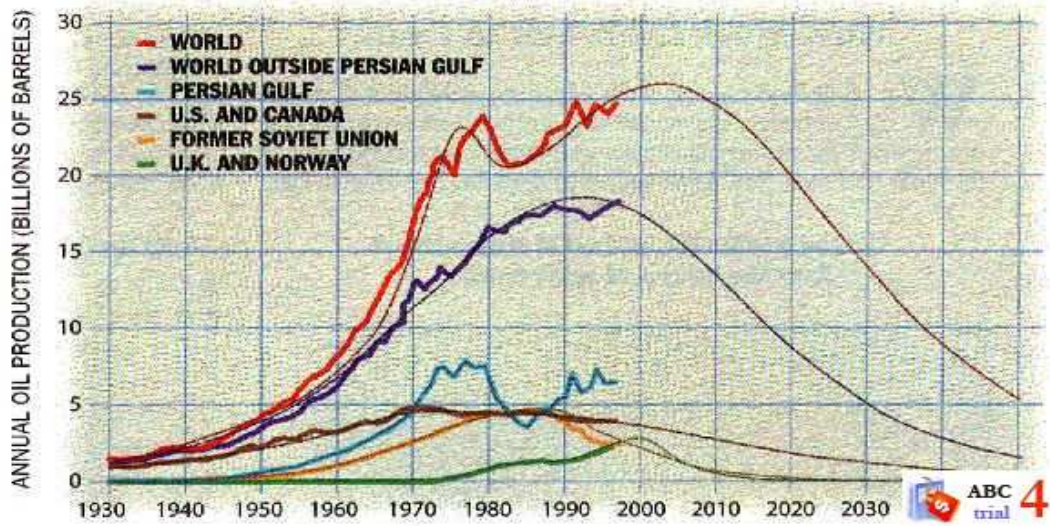


Figure 2: World Oil production at a glance.

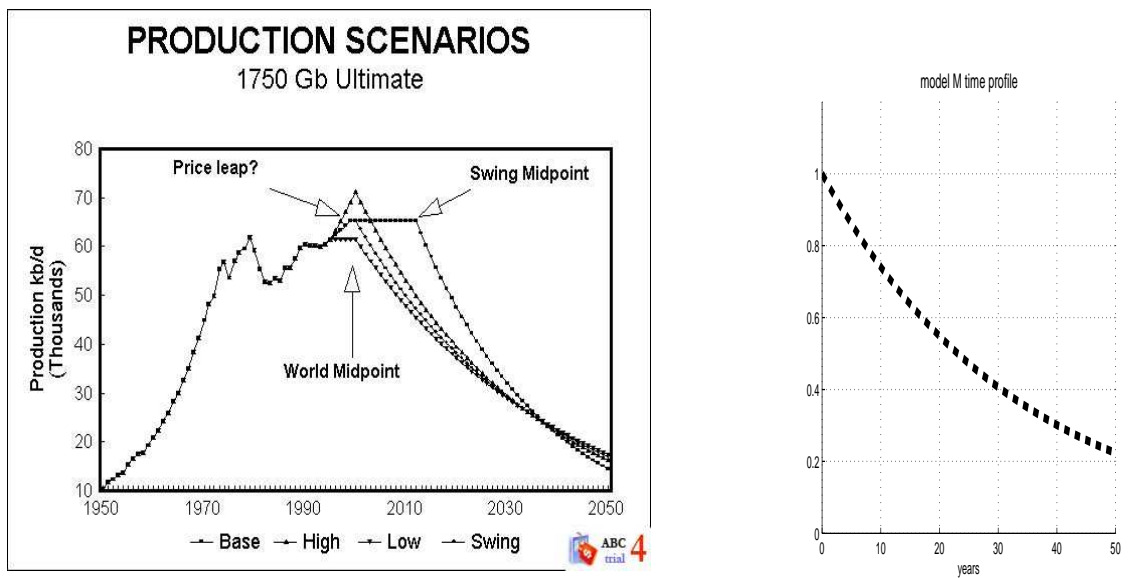


Figure 3: World oil production: future scenarios

### 3.1.2 $R$ - Renewable resources or $R$ - state of environment

We consider *biocapacity*<sup>6</sup> of the planet earth (see, [7]) as a proxy for  $R$ .

The Living Planet Report 2006 [7] confirms that we are using the planet's resources faster than they can be renewed - the latest data available indicate that humanity's *Ecological Footprint*, our impact upon the planet (see [7]), has more than tripled since 1961. Our footprint now exceeds the world's ability to regenerate by about 25 per cent.

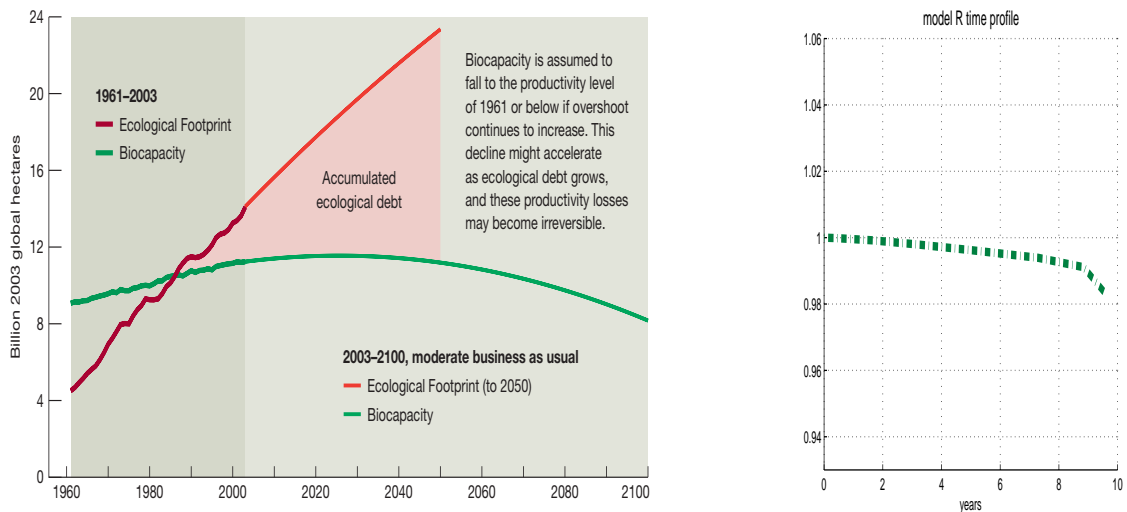


Figure 4: Ecological Footprint and Biocapacity

The left panel of Figure 4 is taken from [7]. The historical part of the biocapacity and footprint data is shown in darker grey shaded region and it is almost constant. We observe that *biocapacity* is almost constant in the considered period. The line shown in the right panel of this figure is obtained from our calibrated model. While it is almost flat at the beginning, it starts falling

<sup>6</sup>Total biocapacity is measured in global hectares - defined as the total biocapacity divided by the total physical area generating it. In 2003, the earth's total biocapacity was stated to be 11.2 billion gha (Ggha). However, a more useful measure is the biocapacity per head of population in units of global hectares per capita (gha/cap). This describes the average land area available to sustain each person. In 2003, since there was a population of 6.3 billion humans sharing the earth's 11.2 Ggha, the biocapacity was therefore 1.784 global hectares per person.

sharper than the Living Planet predictions. However, notice that in the comment placed on their figure Living Planet expects the “decline to accelerate”. This is what might happen due to a sharp increase of the production capital that we observe in Figure 5. (For the full time profiles of the calibrated model for 50 years, see Section 3.2 and Figure 7 in particular).

### 3.1.3 $K$ - Production capital

We will consider GDP (Gross Domestic Product) as proxy for production capital  $K$ . Figure 5 left panel depicts World GDP time profile. We also show the time profile of GNI, which grows in a similar pace but more smoothly (both are in Current US dollars per capita, PPP<sup>7</sup>).

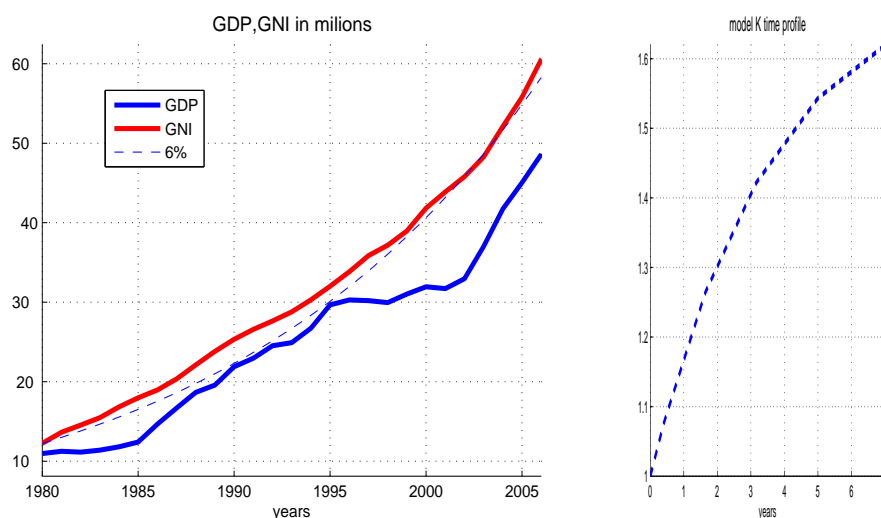


Figure 5: World’s GDP and GNI profiles.

Gross Domestic Product (GDP) per capita, is the total annual output of a country’s economy, given here in current international dollars. As is customary, GDP per capita is the total market value of all final goods and services

<sup>7</sup>PPP is *purchasing power parity*. An international dollar adjusted for PPP has the same purchasing power over GDP (or GNI) as a U.S. dollar in the United States and buys an equivalent amount of goods or services irrespective of the country. PPP rates provide a standard measure allowing comparisons of real price levels between countries, just as conventional price indexes allow comparison of real values over time. Values are in current dollars and are not adjusted for inflation.

produced in a country in a given year, equal to total consumer, investment, and government spending, divided by the mid-year population.

On the other hand, Gross National Income (GNI) measures the total domestic and foreign value added claimed by residents. GNI comprises gross domestic product (GDP) plus net receipts of primary income (compensation of employees and property income) from nonresident sources.

As we see in Figure 5 left panel, each of these indices' growth averages about 6% p.a. Data for Figure 5 are taken from World Bank [14].<sup>8</sup>

We observe that the data on GDP (and GNI) follow a convex graph in the historical period. Basic intuition suggests that the process of *growth* corresponding to these profiles is unsustainable. Furthermore, the data for the last decade must have been “contaminated” by the financial bubble (which burst in 2008). All this and also because our model variable is production capital  $K$ , and GDP (or GNI) serves as a “proxy” for the former, compel us to propose a model for  $K$  that can produce *concave* profiles. The right panel of Figure 5 is a fragment of the calibrated model profile (run for 50 years, see Section 3.2 and Figure 7 in particular). We notice that the (concave) model time profile replicates the fast growth in the historical data for the last eight years<sup>9</sup>.

### 3.1.4 $Q$ - Abatement capital

We did not find reliable data on *global* abatement capital. Some countries publish their data on proportions of GDP spent on abatement. A rather obvious conclusion from the proportions analysis is that  $Q$  grows when  $K$  grows, albeit not necessarily at the same velocity. See Figure 7 in Section 3.2 for the calibrated model profile of abatement capital.

### 3.1.5 $H$ - Human capital

We consider world's *working-age* population as a proxy for this variable. The solid (blue) line in left panel of Figure 6 shows world population of 15 to 65 years old from 1950 till current and the projection until 2050. Data source is [13].

The dashed line (black) represents the working age population numbers obtained from equation (5) calibrated for the “natural” units *i.e.*, billions. The right panel of Figure 6 presents the time profile of  $H$ , which is the same profile as in the left panel but re-scaled and presented in relative terms. We

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<sup>8</sup>The same data are also available at World Resources Institute's data site, [13].

<sup>9</sup>  $\frac{\text{GNI}(2007)}{\text{GNI}(2000)} = 1.67$ ;  $\frac{K(8)}{K(0)} = 1.68$ .

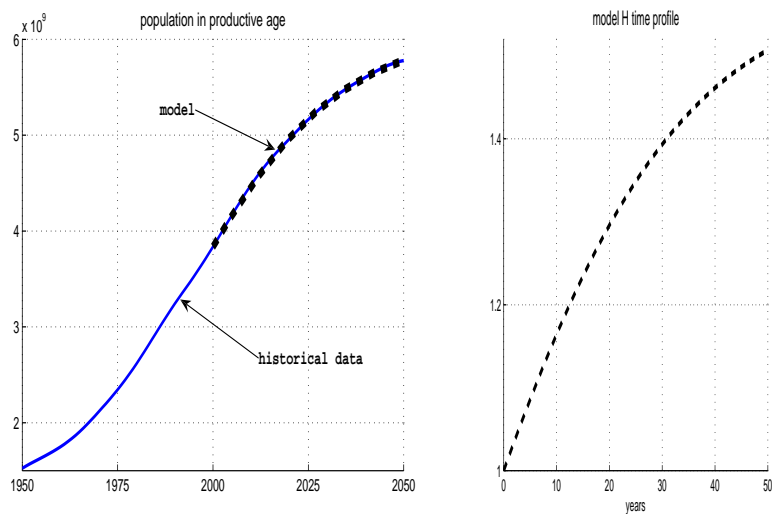


Figure 6: World’s working age population.

notice a high degree of coincidence between both the model and historical data after 2000.

Typically, it is very difficult to obtain historical data such as these or to find them in the open literature. Notable exceptions are [3] and [4]<sup>10</sup>. We believe the concordance of the calibration run with the 2000-2006 panel data is a reasonable test that our model has passed. We also notice that these data (*i.e.*, [13]) were used by other institutions for “sustainability” projections; in particular, World Bank uses data on *working-age population* to compute *dependency ratio*. Furthermore, our forecast (until 2050) matches that of [13].

### 3.2 Calibration runs

The calibrated parameter set used in the runs generated below is as follows:  $\varphi_M = 0.03, C = 0, \sigma = 3.4852943, \gamma = 2, b = 35.0625, \alpha = 3.718, \varphi_R = 0.35, \gamma_Q = 0.07, \varphi = 0.1054, \psi_K = 0.075, \kappa_K = 0.1, \phi_H = 0.1, a = 0.0479114, c = 6.091 \cdot 10^9$ .

Figure 7 shows the calibrated model runs for 50 years. The initial time is identified as 2000. The results for the first 5-8 years of each variable’s run

<sup>10</sup>Gathering historical data enabled those authors to develop a quantitative theory to link population density to human capital formation.



have been commented in Section 3.1. We have concluded that, in broad terms, the opening fragments of the calibrated runs correspond to their respective historical data.

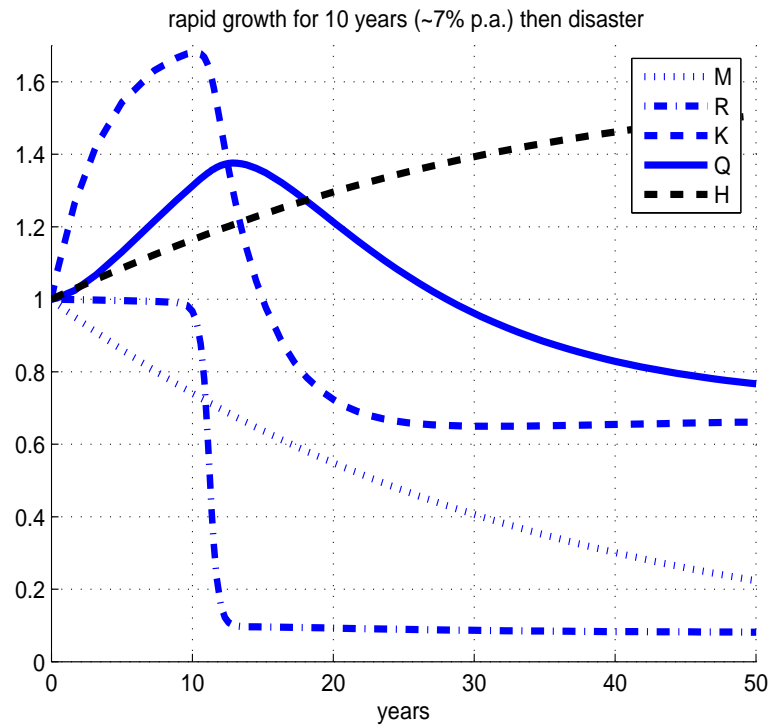


Figure 7: Calibration runs.

The continuations of the historical runs, according to our model, predict a rather gloomy future: the fragile balance of renewable resources is tipped down and they become in low supply. This, together with gradual exhaustion of nonrenewable resources, causes production capital to decline, which eventuates in less abatement capital contributing to resource renewal. This situation is captured by the “sustainability screw” already shown in Figure 1 in the Introduction. We see that as nonrenewable resources  $M$  become scarce, spiraling “down” are both the renewable resources  $R$  and the production capital  $K$ .

Consider the following ratio index

$$\Delta = \frac{\phi_M + \phi_R + \phi_H}{\gamma_Q}. \quad (7)$$

The numerator is a sum of coefficients that describe the amounts of *resources* (non-renewable, renewable and human) utilised for growth of production capital  $K$ . Coefficient  $\gamma_Q$  is the denominator and describes how much production capital is devoted to help regrow renewable resources. The larger the numerator, the faster  $K$  increases; the larger the denominator, the faster the renewable resources recover.

In view of the above, we can view  $\Delta$  as the ratio of the “intensity of production effort” over the “intensity of abatement effort”. We conjecture that  $\Delta$  conveys information on how human and natural processes are *synchronised* and influences the relative time scales of production and natural recovery processes. In particular, larger values of  $\Delta$  will signify that  $K$  growth dominates that of  $R$ .

The value of this index computed for the “screw” in Figure 1 in the Introduction is  $\Delta = 6.8571$ .

## 4 Policy runs

Suppose that there is a possibility of diminishing the value of  $\varphi_R$  (previously set at 0.35) by a factor of  $\frac{2}{3}$  to the new value of  $\varphi_R = 0.2333$ . This could be a political decision or a result of market forces.

A smaller  $\varphi_R$  would correspond to a slower exploitation of renewable resources. Presumably, the variable  $R$  would thus be able to re-grow and leave the dangerous zone of where a bifurcation<sup>11</sup> to the low steady state may occur. The value of  $\Delta$  for  $\varphi_R = 0.2333$  (and when the other parameters remain unchanged) is  $\Delta = 5.1905$ .

Figure 8 shows the time profiles of the model variables after  $\phi_R$  was diminished in year 9.

We observe that dedicating less renewable resources to  $K$  has a negative effect on growth. However, more resources  $R$  become available (dash-dotted line) and they feed into the growth process of capital that quickly recovers and overtakes the path corresponding to the business-as-usual scenario.

While we cannot claim that the pace of those changes is reproduced by Figure 8 exactly, we are certain to have captured a reaction of the man-nature

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<sup>11</sup>We remind the reader that the growth function in equation (2) is logistic and that the corresponding time profile of  $R$  can converge (bifurcate) to a “low” or “high” steady state.

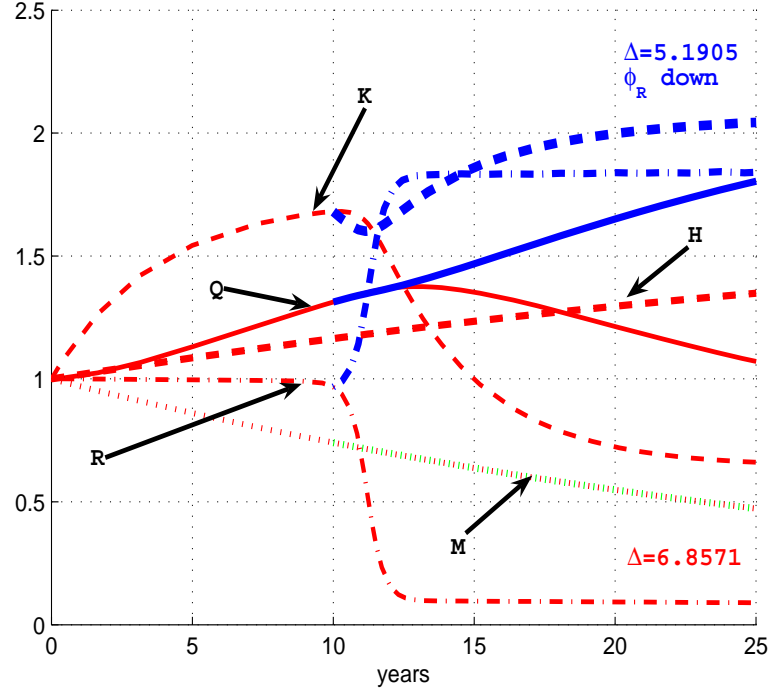


Figure 8: Bifurcation in  $R$  ( $\varphi_R$  down).

system, represented by equations (1)-(5), to a change in the *time-scale* of the component processes .

Figure 9 shows the bifurcated sustainability screws for  $\varphi_R$  diminished in year 9. We observe that after  $R$  stabilised at the “high” steady state, production capital grew substantially which resulted in high abatement capital that helped nonrenewable resources to remain at the high steady state.

Diminishing  $\varphi_R$  might be perceived by some (politicians) as a “negative” modification because it depresses (albeit temporarily only) the production capital growth. Increasing  $\gamma_Q$  should not have that connotation yet it should induce a similar result in terms of a long-term improvement of the system sustainability.

Assume  $\gamma_Q = .1202$ , which corresponds to an increase in abatement capital

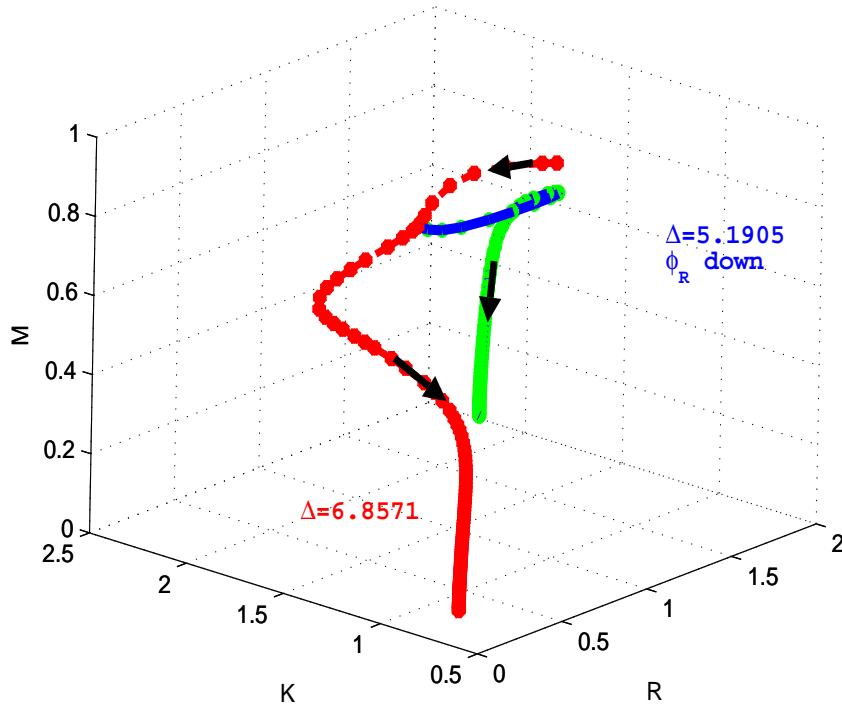


Figure 9: Bifurcated sustainability screws.

use of about 72%. Figure 10 shows the time profiles of the model variables after  $\gamma_Q$  was increased in year 9.

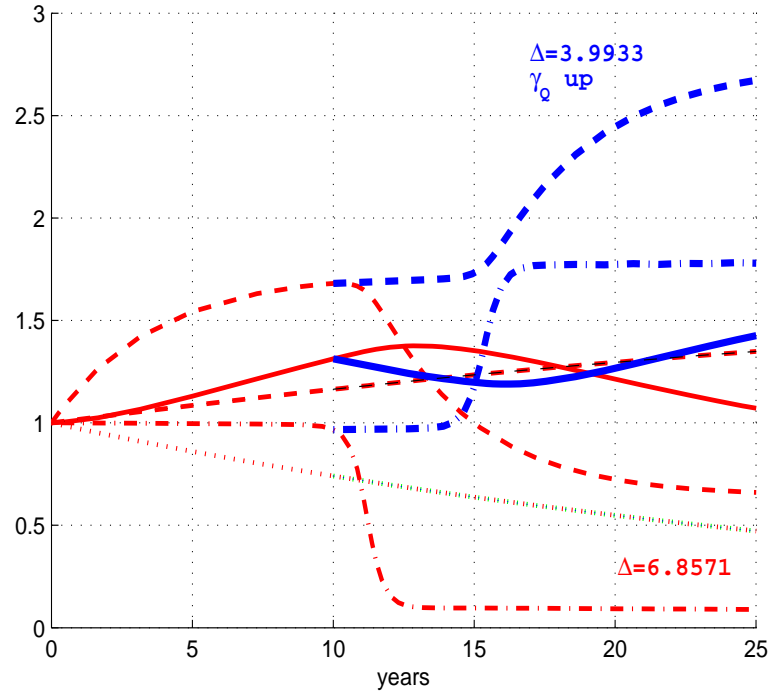


Figure 10: Bifurcation in  $R$  ( $\gamma_Q$  up).

Expectantly dedicating more abatement capital to  $R$  has a temporary diminishing effect on the abatement capital stock. The production capital stabilises for some time mainly because the renewable resources do not grow. On the other hand, the drop of  $R$  is averted and, after some period of zero growth, this resource converges to the “high” steady state. This enables production capital to grow as well. In turn, the abatement capital grows too that further stabilises  $R$  at the high steady state.

Again, we cannot claim that the pace of those changes and the levels of the variables are reproduced by Figure 10 exactly. However, we can see a possible reaction of the man-nature system, represented by equations (1)-(5), to a change in the *time-scale* of the component processes.

Figure 11 shows the bifurcated sustainability screws for  $\gamma_Q$  increased in year 9. The processes depicted in this figure behave qualitatively similar to those presented in Figure 9.

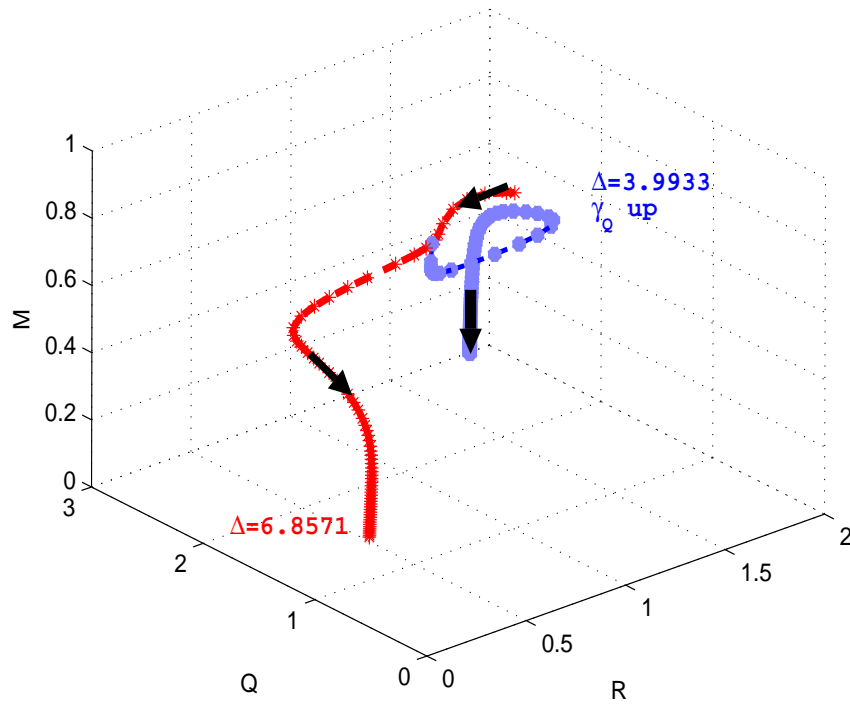


Figure 11: Bifurcated sustainability screws.

## 5 Concluding remarks

As mentioned earlier, we hope that this very preliminary discussion of the times scales' conjecture and the sustainability screw will stimulate further research aimed at identifying more sustainable development pathways than those practiced in the recent decades. Perhaps, one of the essential aspects of modelling human-nature interactions is that it, inevitably, involves two sets of

time scales: the physical, chemical and biological scales of our planet and its biosphere and the human generated time scales of population and economic growth. We would like to argue that when these two sets of time scales are sufficiently "out of synch", the trajectories of many key variables in the global human-nature system will naturally follow pathways that most people would deem "unsustainable" according to any reasonable notion of sustainability.

Since, the parameters affecting the time scales of natural processes are largely beyond human control<sup>12</sup>, it is natural to focus on adjusting parameters that affect the time scales of human development processes, such as the "intensity of production effort" and "intensity of abatement effort" parameters experimented with in this paper.

We conclude by pointing out that this kind of parametric adjustment of the "relative velocities" of equations in a dynamical systems links closely to the rapidly developing subject of *singularly perturbed* systems. These are systems whose behaviour with nonzero values of certain parameters (no matter how small they are) is essentially different from that when the parameters are equal to zero. In analysing such systems we must consider what would happen if our estimate of a given parameter were given a slightly different value? Will the systems evolve in the same way with only slight changes to the outcomes, or will they evolve in a radically different way? An analogy with environmental systems is natural: *Which human induced "perturbations" dissipate harmlessly, and which fundamentally alter the system?*

Frequently, a singularly perturbed system refers to a dynamical system containing *slow* and *fast* components which change their values with rates proportional to zero powers (slow) and negative powers (fast) of a small parameter. A general approach to singularly perturbed optimal control problems was proposed in [11]. It is based on the capability (in many cases) to approximate the slow motions by the solutions of a certain averaged control system. In this averaging process the fast dynamics are not neglected. Indeed, the fast variables are controlled in such a way as to attain a desirable steady state distribution with respect to the slow variables. An analogy with the more sustainable land use practices in many traditional societies (such as periodic resting of paddocks, rotation of crops or selective harvesting of timber such as in the Menominee Indian Forest) seems irresistible.

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<sup>12</sup>Of course, some would argue that technologies such as genetically modified foods are a counter-example to this rather sweeping statement.

## References

- [1] Arrow K., Dasgupta P., Goulder L., Daily G., Ehrlich P., Heal G., Levin S., Maler K., Schneider S. Starrett, and Walker B., 2004, "Are we consuming too much?" *Journal of Economic Perspectives*, 18(3), pp. 147-172.
- [2] Aubin, J.-P., 1997, *Dynamical Economic Theory - A Viability Approach*. Springer.
- [3] Boucekkine, R., de la Croix, D., Peeters, D. 2007, "Early literacy achievements, population density, and the transition to modern growth", *Journal of the European Economic Association* 5, pp. 183-226.
- [4] Boucekkine, R., de la Croix, D., Peeters, D. 2007, "Demographic, Economic, and Institutional Factors in the Transition to Modern Growth in England: 1530-1860", *Population and Development Review Supplement* 2008, v. 34, pp. 126-48.
- [5] Campbell, C. and Laherrere, J., (1998), "The End of Cheap Oil", *Scientific American* 278(3), pp. 60-65.
- [6] Campbell, C. J., (1996), *The Twenty First Century The World's Endowment of Conventional Oil and its Depletion*. Version available at <http://www.hubbertypeak.com/campbell/camfutur.htm>
- [7] *The Living Planet Report*.(2006) Published by WWF, Switzerland available at [http://www.panda.org/news\\_facts/publications/living\\_planet\\_report](http://www.panda.org/news_facts/publications/living_planet_report)
- [8] Hawken, P., (1994), *The Ecology of Commerce: A Declaration of Sustainability*. HarperBusiness, New York.
- [9] Lucas, R.E., 1988, "On the mechanics of economic development." *Journal of Monetary Economics*, 22, pp. 3-42.



- [10] Meadows, D.H, D.L. Meadows, J. Randers and W.H. Beyrens, 1972, “The Limits to Growth”. A report for the Club of Rome. Earth Island Ltd.
- [11] Gaitsgory, V. and M.-T. Nguyen, 2002, “Multiscale singularly perturbed control systems: Limit occupational measures sets and averaging”. *SIAM J. Control Optim.*, 41, pp. 954-974.
- [12] Humanity's Footprint 1961-2003, *available at*  
[http://www.footprintnetwork.org/gfn\\_sub.php?content=global\\_footprint](http://www.footprintnetwork.org/gfn_sub.php?content=global_footprint)  
*on 21 October 2008.*
- [13] World Resource Institute. *data available at*  
<http://earthtrends.wri.org/text/economics-business/variable-638.html>
- [14] <http://ddp-ext.worldbank.org/ext/ddpqq/member.do?method=getMembers&userid=1&queryId=135>
-

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