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Abstract

This paper provides a real option methodology for evaluating R&D investment opportunities assuming that potential competitors can enter in the market. As it is well known, R&D investments are made often in a phased manner and so each stage creates an opportunity (option) for subsequent investment. Therefore, R&D projects can be considered as 'Compound Exchange Options' in which investments present uncertainty both in the gross project value and in costs.
According to Majd and Pindyck (1987), in a real options context, "dividends" are the opportunity costs inherent in the decision to defer an investment project and so deferment implies the loss of project's cash flows. Moreover, Trigeorgis (1996) incorporates the preemption effect through the "competitive dividends" which are the cash flows that can be eroded by anticipated competitive arrivals.
In this paper we propose to value, using Montecarlo simulation, the R&D investments of a pioneer firm assuming that the Development cost can be spent in two moments: t_2 or t_3. If the Develpment cost is realized in t_2 no firms enters in the market since the rivals' R&D plan is not yet concluded otherwise, if the investment D is delayed at time t_3 waiting better market conditions, other rivals can enter in the market and so the opportunity costs ("dividends") increase.

Keywords: Real options; R&D; Monte Carlo methods; Competitive dividends.
JEL Codes: G13; O32; C15; D40.

1 Introduction

In recent years, academic studies have argued that traditional valuation model, NPV (net present value), can not adequately capture the value of an investment project. Therefore, the real options approach becomes very important to give explicit valutations of managerial flexibility to grow, delay, scale down or abandon projects (Smith 2003). The main difference between the NPV rule and real options approach is that while the first considers uncertainty as a risk to be minimized, the second takes uncertainty as an opportunity for maximizing the value of a project.
In real options, the options involve real assets as opposed to financial ones. So the project value can be valued quantitatively using the analogue option pricing theory as developed in the modern finance. In case of financial options, optimal exercise strategies can be derived without consideration of strategic interaction across option holders. However, it is widely acknowledged that a competitive environment may have a considerable impact on the valuation of real options.
Under the threat of competition, the exercise of options strategically depends on the tradeoff between the benefits and costs of going ahead with an investment against waiting for more information. Waiting can have an informational benefit (McDonald and Siegel 1986). However, if a firm chooses to defer exercising its option until better information is received (thus resolving uncertainty), it runs the risk that other firms may preempt it by exercising first (Zhu and Weyant 1999). Such an early exercise by a competitor can erode the profits or even force the option to expire prematurely.
If a real options is held in isolation by a single firm, no competitive interations should be considered and so, the decision to invest is not influenced by any rival's actions (Dixit and Pindyck 1994). But, most of the real investment opportunity are shared with other firms. Then, when this is the case, the models should incorporate this competitive dimension, since it could have a considerable impact on the decision to invest. Game theory is a regular tool that come down of industrial organization and model in imperfect competition as is presented in Grenadier (1996), Weeds (2002), Tsekrekos (2003), Huisman (2001) and so on. In these papers the problem is, basically, the following: two competing firms have the option to invest and enter in a duopoy market; the first company to do so (the Leader or pioneer) may benefit, after investing, from a temporary or permanent competitive advantage over the other firm (the Follwer), by securing, for example, a higher market share.
In addition, during the last two decades, numerous studies have attempted to apply option pricing formula to R&D investment. As it is well know, R&D investments are made often in a phased manner, with the commencement of subsequent phase being dependent on the successuful completation of the preceding phase. Each stage provides information about the R&D success and creates an opportunity (option) for subsequent investment. Therefore, R&D projects can be considered as 'Compound Exchange Options' in which investments present uncertainty both in the gross project value and in costs. The most important valuation models of exchange options are given in Margrabe (1978), McDonald and Siegel (1985), Carr (1988), Carr (1995) and Armada et al. (2007). In particular way, McDonald and Siegel

(1985) and Margrbe (1978) value a simple European exchange option (SEEO), Carr (1988) develops a model to price a compound European exchange option (CEEO) and Carr(1995) and Armada et al. (2007) present the pricing of a simple American exchange option (SAEO). However, if we consider the possibility that the development investment can be realized in two moments before the maturity time, we have a pseudo compound American exchange option. In this case numerical approximation is an important task as withnessed by the contribution of Tilley (1993), Barraquand and Martineau (1995), Broadie and Glasserman (1997), Longstaff and Schwartz (2001).
Aim of this paper is to propose a new way to value the R&D investment opportunity of a pioneer firm assuming that potential rivals can enter in the market. The sudden entry of a competitor may significantly diminish the current cash flow deriving from an existing asset and so alter its value. In real market terms, dividends may represent several types of opportunity costs caused by holding the real option unexercised, namely the loss of project's cash flows when an investment opportunity is postponed as it shown in Majd and Pindyck (1987) and Trigeorgis (1996). Specifically, we consider that R&D project deferment may contribute to the early entrance of competitors in a competitive environment that produces a negative impact on the R&D project value.
The structure of this paper is organized as follows. In Section 2, we present the option model to value the R&D investment opportunity of a pioneer firm assuming that other rivals can enter in the market. First of all, we analyses the case with a single and two potential rivals and, after that, we propose a general case with the threat of k potential competitors. We use a Monte Carlo approach to value the pioneer R&D opportunity. In Section 3 we propose a numerical application showing as the pioneer's R&D opportunity value depend on the number of potential rivals that can erode cash-flows and on the information revalation. Finally, Section 4 concludes.

2 The Basic Model

In this section, we propose to value the R&D investment opportunity of a pioneer firm assuming that potential competitors can enter in the market. The sudden entry of a competitor may significantly diminish the current cash flow deriving from an existing asset and so alter its value. In this connection, we consider that the Development cost can be spent in two moments: t_2 or t_3. If the Development investment is realized in t_2 no firm enters in the market, as the rivals' R&D plan is not yet concluded. Otherwise, if the Development cost is delayed at time t_3 waiting better market conditions, other rivals can enter in the market and so the opportunity costs, namely dividend-yields, increase.
About the frame of an R&D investment, we denote by:

- R the Research Investment spent at initial time $t_0 = 0$;
- IT the Investment Technology payed at time t_1. We suppose that $IT = \varphi D$ is a proportion of asset D;

- D the Development Investment that can be realized at time t_2 or at time t_3. Obviously, we suppose that $t_1 < t_2 < t_3$;
- V the Gross Project value obtained spending the Develpment Investment D.

We assume that V and D follow a geometric brownian motion process given by:

$$\frac{dV}{V} = (\mu_v - \delta_v)dt + \sigma_v dZ_v \tag{1}$$

$$\frac{dD}{D} = (\mu_d - \delta_d)dt + \sigma_d dZ_d \tag{2}$$

$$cov\left(\frac{dV}{V}, \frac{dD}{D}\right) = \rho_{vd}\sigma_v\sigma_d\, dt. \tag{3}$$

where μ_v and μ_d are the expected rates of return on asset V and D respectively, δ_d and δ_v are the corresponding dividend yields, σ_v^2 and σ_d^2 are the respective variance rates, Z_v and Z_d are the brownian standard motions of assets V and D with correlation ρ_{vd}.
In addition, assuming by q and p the R&D success probability of pioneer firm and potential competitors respectively, we introduce two Bernoulli random variates X and Y:

$$X : \begin{cases} 1 & q \\ 0 & 1-q \end{cases} \qquad Y : \begin{cases} 1 & p \\ 0 & 1-p \end{cases}$$

The R&D Investment of pioneer firm generates an information revelation that influences the investment decision of other firms. So, if pioneer R&D investment is successful, the rivals probability p changes in positive information revelation p^+. Using Dias (2004) model, it results that:

$$p^+ = Prob[Y = 1/X = 1] = p + \sqrt{\frac{1-q}{q}} \cdot \sqrt{p(1-p)} \cdot \rho(X, Y) \tag{4}$$

where the correlation $\rho(X, Y)$ is measure of information revelation from pioneer to rivals.
About the evolution of asset V, we can distinguish two situations:

- If the pioneer firm spends the Development Investment D at time t_2, it obtains the monopolistic Gross Project value V at time t_2. In this case no rival is entered in the market and so no cash flow is lost. We can state that $\delta_v = 0$ in Eq.(1) and consequently the evolution of asset V evolution becomes:

$$\frac{dV}{V} = \mu_v dt + \sigma_v dZ_v. \tag{5}$$

- If the pioneer firm postpones the Development Investment D at time t_3 waiting better market conditions, other firms can enter in the market and so to reduce the pioneer cash flows. In this case δ_v assumes positive values depending on the number of rivals that invest with success in R&D. We denote by k the number of potential rivals and by h the number of competitors that realize successfully the R&D project, with $h \leq k$. The Gross project value for the pioneer firm is the asset value V at time t_3 assuming positive values of δ_v, that we denote by $V^{h\delta}$. The evolution of asset $V^{h\delta}$ is given by Eq.(1) considering positive dividends:

$$\frac{dV^{h\delta}}{V^{h\delta}} = (\mu_v - h\delta_v)dt + \sigma_v dZ_v. \tag{6}$$

In particular way, in case of all rivals failure ($h = 0$), the Gross project value for the pioneer is the asset V at time t_3 without dividends. In this situation, the evolution of asset V is given by Eq.(5).

2.1 First Case: $k = 1$.

First of all, we analyse the pioneer R&D investment opportunity at time t_2 assuming that only a single firm can enter in the market and therefore $k = 1$. This situation may characterized an oligopoly market. The investment D will be realized at time t_2 if the exercised value $V_{t_2} - D_{t_2}$ is bigger than waiting value and so the option value:

$$V_{t_2} - D_{t_2} > (1 - p^+)s(V_{t_2}, D_{t_2}, \tau_2) + p^+ s(V^\delta_{t_2}, D_{t_2}, \tau_2) \tag{7}$$

where:

- $s(V_{t_2}, D_{t_2}, \tau_2)$ is the value at time t_2 of a Simple European Exchange Option (SEEO) without dividends δ_v and with maturity $\tau_2 = t_3 - t_2$. This option is obtained in case of rival's R&D failure ($h = 0$) with a probability $1 - p^+$. Its final payoff is $\max[V_{t_3} - D_{t_3}, 0]$;
- $s(V^\delta_{t_2}, D_{t_2}, \tau_2)$ is the value at time t_2 of a SEEO with positive dividends δ_v and with maturity τ_2. This option is obtained in case of rival's R&D success ($h = 1$) with a probability p^+. Its final payoff is $\max[V^\delta_{t_3} - D_{t_3}, 0]$.

Using Margrabe (1978) and McDonald & Siegel (1984) models, we have that:

$$s(V_{t_2}, D_{t_2}, \tau_2) = V_{t_2} N(d_1(P_{t_2}, \tau_2)) - D_{t_2} e^{-\delta_d \tau_2} N(d_2(P_{t_2}, \tau_2)) \tag{8}$$

and

$$s(V^\delta_{t_2}, D_{t_2}, \tau_2) = V_{t_2} e^{-\delta_v \tau_2} N(d_1(P^\delta_{t_2}, \tau_2)) - D_{t_2} e^{-\delta_d \tau_2} N(d_2(P^\delta_{t_2}, \tau_2)) \tag{9}$$

where:

- $P_{t_2} = V_{t_2}/D_{t_2}$; $P^\delta_{t_2} = P_{t_2} e^{-\delta_v \tau_2}$; $\sigma = \sqrt{\sigma_v^2 - 2\rho_{vd}\sigma_v\sigma_d + \sigma_d^2}$;
- $d_1(a, b) = \frac{\log(a) + \left(\frac{\sigma^2}{2} + \delta_d\right) b}{\sigma\sqrt{b}}$; $d_2(a, b) = d_1(a, b) - \sigma\sqrt{b}$
- $N(d)$ is the cumulative standard normal distribution.

Assuming the asset D as numeraire, we can reduce the by-dimensionality of inequality (7) to one variable $P = \frac{V}{D}$:

$$\begin{aligned} P_{t_2} - 1 &> (1-p^+)\left[P_{t_2} N(d_1(P_{t_2}, \tau_2)) - e^{-\delta_d \tau_2} N(d_2(P_{t_2}, \tau_2))\right] \\ &+ p^+ \left[P_{t_2} e^{-\delta_v \tau_2} N(d_1(P^{\delta}_{t_2}, \tau_2)) - e^{-\delta_d \tau_2} N(d_2(P^{\delta}_{t_2}, \tau_2))\right] \end{aligned}$$

namely:

$$P_{t_2} - 1 > (1-p^+) s(P_{t_2}, 1, \tau_2) + p^+ s(P^{\delta}_{t_2}, 1, \tau_2) \tag{10}$$

where:

- $s(P_{t_2}, 1, \tau_2)$ is the value of a Simple European Option (SEO) whose underlying asset is P_{t_2} without dividends ($\delta_v = 0$), the exercise price is equal to 1 and the risk-free rate is $r = \delta_d$;
- $s(P^{\delta}_{t_2}, 1, \tau_2)$ is the value of a SEO whose underlying asset is P_{t_2} with dividends δ_v, the exercise price is equal to 1 and the risk-free rate is $r = \delta_d$.

We determine the critical ratio price P_1^* that makes indifferent to invest or not at time t_2 solving the following equation:

$$\begin{aligned} P_1^* - 1 &= (1-p^+)\left[P_1^* N(d_1(P_1^*, \tau_2)) - e^{-\delta_d \tau_2} N(d_2(P_1^*, \tau_2))\right] \\ &+ p^+ \left[P_1^* e^{-\delta_v \tau_2} N(d_1(P_1^* e^{-\delta_v \tau_2}, \tau_2)) - e^{-\delta_d \tau_2} N(d_2(P_1^* e^{-\delta_v \tau_2}, \tau_2))\right] \end{aligned} \tag{11}$$

So, the value of this investment opportunity at time t_1 is a particular Pseudo Simple American Exchange option (PSAEO) that can be exercised at time t_2 and t_3, whose maturity time is $\tau = t_3 - t_1$. Denoting by $S_1(V_{t_1}, D_{t_1}, \tau)$ this PSAEO with $k = 1$, its payoff will be:

- $(V_{t_2} - D_{t_2}) > 0 \quad$ if $P_{t_2} \geq P_1^*$;
- $p^+ \max[V^{\delta}_{t_3} - D_{t_3}, 0] + (1-p^+) \max[V_{t_3} - D_{t_3}, 0] \quad$ if $P_{t_2} < P_1^*$.

So, if $P_{t_2} \geq P_1^*$, the investment D is realized at time t_2 otherwise, if $P_{t_2} < P_1^*$, the investment D is postponed at time t_3 and it will be carried out if the option is in the money. The pioneer firm does not know at time t_1 if the rival's R&D investment will be successfull or not. Therefore, we must consider the expectation value between two options. For instance, if $p^+ = 1$ (we are sure that the rival's R&D investment is successful), then the option payoff at time t_3 is $\max[V^{\delta}_{t_3} - D_{t_3}, 0]$; if $p^+ = 0$ (we are sure that the rival's R&D investment has failed), then the option payoff at time t_3 is $\max[V_{t_3} - D_{t_3}, 0]$.

In order to determine the value of $S_1(V_{t_1}, D_{t_1}, \tau)$ with the threat of a single competitor ($k = 1$), we implement a Monte Carlo simulation assuming the asset D as numeraire. So we price te PSAEO as the expectation value of discounted cash-flows under the risk-neutral probability $\mathbb{Q}$:

$$\begin{aligned} S_1(V_{t_1}, D_{t_1}, \tau) &= e^{-r\tau_1} E_{\mathbb{Q}} \left[(V_{t_2} - D_{t_2}) \mathbf{1}_{\{P_{t_2} \geq P_1^*\}} \right] \\ &+ e^{-r\tau} E_{\mathbb{Q}} \left[p^+ \max(V_{t_3}^{\delta} - D_{t_3}, 0) \mathbf{1}_{\{P_{t_2} < P_1^*\}} \right] \\ &+ e^{-r\tau} E_{\mathbb{Q}} \left[(1 - p^+) \max(V_{t_3}^{\delta} - D_{t_3}, 0) \mathbf{1}_{\{P_{t_2} < P_1^*\}} \right] \end{aligned} \tag{12}$$

where $\tau_1 = t_2 - t_1$. Denoting by:

- $A_1 \equiv e^{-r\tau_1} E_{\mathbb{Q}} \left[(V_{t_2} - D_{t_2}) \mathbf{1}_{\{P_{t_2} \geq P_1^*\}} \right]$;
- $A_2 \equiv e^{-r\tau} E_{\mathbb{Q}} \left[p^+ \max(V_{t_3}^{\delta} - D_{t_3}, 0) \mathbf{1}_{\{P_{t_2} < P_1^*\}} \right]$;
- $A_3 \equiv e^{-r\tau} E_{\mathbb{Q}} \left[(1 - p^+) \max(V_{t_3} - D_{t_3}, 0) \mathbf{1}_{\{P_{t_2} < P_1^*\}} \right]$;

and using the asset D as numeraire given by Eq.(46) (see Appendix A), we have that:

$$\begin{aligned} A_1 &= e^{-r\tau_1} D_{t_2} E_{\mathbb{Q}} \left[(P_{t_2} - 1) \mathbf{1}_{\{P_{t_2} \geq P_1^*\}} \right] \\ &= e^{-r\tau_1} D_{t_1} e^{(r-\delta_d)\tau_1} E_{\mathbb{Q}} \left[(P_{t_2} - 1) \mathbf{1}_{\{P_{t_2} \geq P_1^*\}} \frac{d\,\widetilde{\mathbb{Q}}}{d\,\mathbb{Q}} \right] \\ &= D_{t_1} e^{-\delta_d \tau_1} E_{\widetilde{\mathbb{Q}}} \left[(P_{t_2} - 1) \mathbf{1}_{\{P_{t_2} \geq P_1^*\}} \right]; \end{aligned} \tag{13}$$

$$\begin{aligned} A_2 &= e^{-r\tau} D_{t_3} E_{\mathbb{Q}} \left[p^+ \max(P_{t_3}^{\delta} - 1, 0) \mathbf{1}_{\{P_{t_2} < P_1^*\}} \right] \\ &= e^{-r\tau} D_{t_1} e^{(r-\delta_d)\tau} E_{\mathbb{Q}} \left[p^+ \max(P_{t_3}^{\delta} - 1, 0) \mathbf{1}_{\{P_{t_2} < P_1^*\}} \frac{d\,\widetilde{\mathbb{Q}}}{d\,\mathbb{Q}} \right] \\ &= D_{t_1} e^{-\delta_d \tau} E_{\widetilde{\mathbb{Q}}} \left[p^+ \max(P_{t_3}^{\delta} - 1, 0) \mathbf{1}_{\{P_{t_2} \geq P_1^*\}} \right]; \end{aligned} \tag{14}$$

$$\begin{aligned} A_3 &= e^{-r\tau} D_{t_3} E_{\mathbb{Q}} \left[(1 - p^+) \max(P_{t_3} - 1, 0)) \mathbf{1}_{\{P_{t_2} < P_1^*\}} \right] \\ &= e^{-r\tau} D_{t_1} e^{(r-\delta_d)\tau} E_{\mathbb{Q}} \left[(1 - p^+) \max(P_{t_3} - 1, 0) \mathbf{1}_{\{P_{t_2} < P_1^*\}} \frac{d\,\widetilde{\mathbb{Q}}}{d\,\mathbb{Q}} \right] \\ &= D_{t_1} e^{-\delta_d \tau} E_{\widetilde{\mathbb{Q}}} \left[(1 - p^+) \max(P_{t_3} - 1, 0) \mathbf{1}_{\{P_{t_2} \geq P_1^*\}} \right]. \end{aligned} \tag{15}$$

where $\widetilde{\mathbb{Q}}$ is the new risk-neutral probability equivalent to $\mathbb{Q}$, as illustrated in Appendix A. Using Eqs. (13), (14) and (15), we can write that:

$$S_1(V_{t_1}, D_{t_1}, \tau) = D_{t_1} S_1(P_{t_1}, 1, \tau) \tag{16}$$

where $S_1(P_{t_1}, 1, \tau)$ is the value of Pseudo Simple American option (PSAO) whose underlying asset is P_{t_1}, the exercise price is equal to 1 and the risk-free rate $r = \delta_d$. Therefore we can write, using the Monte Carlo approximation with a single stochastic factor P, the PSAO $S_1(P_{t_1}, 1, \tau)$ that can be exercised at time t_2 or t_3 as:

$$\begin{aligned} S_1(P_{t_1}, 1, \tau) &\approx \frac{\sum_{j\in H_1} e^{-\delta_d \tau_1}(\hat{P}_{t_2,j} - 1)}{m} \\ &+ \frac{\sum_{j\in \bar{H}_1} e^{-\delta_d \tau}\left(p^+ \max[\hat{P}^{\delta}_{t_3,j} - 1, 0]\right)}{m} \\ &+ \frac{\sum_{j\in \bar{H}_1} e^{-\delta_d \tau}\left((1-p^+) \max[\hat{P}_{t_3,j} - 1, 0]\right)}{m} \end{aligned} \tag{17}$$

where $H_1 = \{j = 1...m \,\text{s.t.}\, \hat{P}_{t_2,j} \geq P_1^*\}$, $\hat{P}_{t_2,j}$ are the simulated price of asset P and m is the number of simulations to determine the PSAO.
Finally, the investment IT will be realized at time t_1 if the underlying value $S_1(V_{t_1}, D_{t_1}, \tau)$ is bigger than exercise price $IT = \varphi D_{t_1}$. This investment opportunity can be valued at time t_0 as a Compound Exchange option (CEO) $c(S_1, IT, t_1)$ whose underlying asset is $S_1(V_{t_1}, D_{t_1}, \tau)$, the exercise price is $IT = \varphi D_{t_1}$ and the maturity date is t_1. The final payoff is:

$$\begin{aligned} c(S_1, IT, 0) &= \max[S_1(V_{t_1}, D_{t_1}, \tau) - IT, 0] \\ &= D_{t_1}\left(\max[S_1(P_{t_1}, 1, \tau) - \varphi, 0]\right) \end{aligned} \tag{18}$$

So, using the asset D as numeraire, we can write the value of pionner R&D investment opportunity $c(S_2, IT, t_1)$ at intial time $t_0 = 0$ as:

$$\begin{aligned} c(S_1, IT, t_1) &= e^{-rt_1} D_{t_1} E_{\mathbb{Q}}\left[\max(S_1(P_{t_1}, 1, \tau) - \varphi, 0)\right] \\ &= e^{-rt_1} D_0 e^{(r-\delta_d)t_1} E_{\mathbb{Q}}\left[\max(S_1(P_{t_1}, 1, \tau) - \varphi, 0)\frac{d\,\widetilde{\mathbb{Q}}}{d\,\mathbb{Q}}\right] \\ &= D_0 e^{-\delta_d t_1} E_{\widetilde{\mathbb{Q}}}\left[\max(S_1(P_{t_1}, 1, \tau) - \varphi, 0)\right] \end{aligned} \tag{19}$$

Using Monte Carlo simulation, it is possible to approximate:

$$c(S_1, IT, t_1) \approx D_0 e^{-\delta_d t_1}\left(\frac{\sum_{i=1}^{n} \max[S_1(\hat{P}^i_{t_1}, 1, \tau) - \varphi, 0]}{n}\right) \tag{20}$$

where n is the number of simulation paths to compute the CEO. So for each path simulation i^{th}, we have the price $\hat{P}^i_{t_1}$ at time t_1 and starting from this value, we implement a new Monte Carlo simulation to determine the i^{th} option $S_2(\hat{P}^i_{t_1}, 1, \tau)$.
Finally, investing R at time t_0, the pioneer firm obtains, in case of success with a probability q, the investment opportunity which value is given by CEO $c(S_1, IT, t_1)$. So the Research investment R will be realized at time t_0 if the profit Π given by:

$$\Pi = -R + q\, c(S_1, IT, t_1) \tag{21}$$

is positive. Fig. 1 summarizes the pioneer R&D investment with the threat of a single competitor.

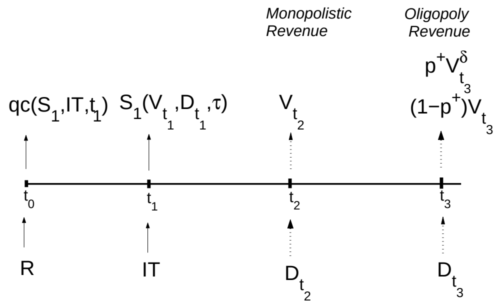

Figure 1: The frame of a pioneer R&D investment with the threat of a single competitor.

Moreover, the appendix B illustrates the Matlab algorithm to simulate the pioneer R&D investment opportunity with $k = 1$.

2.2 Second Case: $k = 2$.

Now we analyse the situation in which there are $k = 2$ potential firms that can enter in the market. So, the pioneer will be realize the investment D at time t_2 if the exercised value $V_{t_2} - D_{t_2}$ is bigger than waiting value, namely the option value:

$$V_{t_2} - D_{t_2} > (1-p^+)^2 s(V_{t_2}, D_{t_2}, \tau_2) + 2p^+(1-p^+) s(V_{t_2}^{\delta}, D_{t_2}, \tau_2) + (p^+)^2 s(V_{t_2}^{2\delta}, D_{t_2}, \tau_2) \tag{22}$$

where $s(V_{t_2}^{2\delta}, D_{t_2}, \tau_2)$ is the value at time t_2 of a SEEO with positive divedends $2\,\delta_v$ whose final payoff is $\max[V_{t_3}^{2\delta} - D_{t_3}, 0]$. So we can remark that pioneer will obtain at time t_2 the monopolistic revenue $s(V_{t_2}, D_{t_2}, \tau_2)$ without to lose dividends in case of both rivals' failure ($h = 0$) with probability $(1 - p^+)^2$, the oligopolistic revenue $s(V_{t_2}^{\delta}, D_{t_2}, \tau_2)$ losing the cash flow δ_v in case of R&D success of only one rival ($h = 1$) with probability $2(1 - p^+)p^+$ and finally, the revenue $s(V_{t_2}^{2\delta}, D_{t_2}, \tau_2)$ in case of both competitors' success ($h = 2$) with probability $(p^+)^2$.
Using Margrabe (1978) and McDonald & Siegel (1984) models we have that:

$$s(V_{t_2}^{2\delta}, D_{t_2}, \tau_2) = V_{t_2} e^{-2\delta_v \tau_2} N(d_1(P_{t_2}^{2\delta}, \tau_2)) - D_{t_2} e^{-\delta_d \tau_2} N(d_2(P_{t_2}^{2\delta}, \tau_2)) \tag{23}$$

where $P_{t_2}^{2\delta} = P_{t_2} e^{-2\delta_v \tau_2}$. Assuming the asset D as numeraire, we can reduce the by-dimensionality to one variable $P = \frac{V}{D}$. So we rewrite Eq. (22) as:

$$\begin{aligned} P_{t_2} - 1 \quad > \quad & (1 - p^+)^2 \left[P_{t_2} N(d_1(P_{t_2}, \tau_2)) - e^{-\delta_d \tau_2} N(d_2(P_{t_2}, \tau_2))\right] \\ + \quad & 2(1 - p^+)p^+ \left[P_{t_2} e^{-\delta_v \tau_2} N(d_1(P_{t_2}^{\delta}, \tau_2)) - e^{-\delta_d \tau_2} N(d_2(P_{t_2}^{\delta}, \tau_2))\right] \\ + \quad & (p^+)^2 \left[P_{t_2} e^{-2\delta_v \tau_2} N(d_1(P_{t_2}^{2\delta}, \tau_2)) - e^{-\delta_d \tau_2} N(d_2(P_{t_2}^{2\delta}, \tau_2))\right] \end{aligned} \tag{24}$$

namely:

$$P_{t_2} - 1 > (1-p^+)^2 s(P_{t_2}, 1, \tau_2) + 2p^+(1-p^+) s(P^{\delta}_{t_2}, 1, \tau_2) + (p^+)^2 s(P^{2\delta}_{t_2}, 1, \tau_2) \quad (25)$$

where $s(P^{2\delta}_{t_2}, 1, \tau_2)$ is the value of a SEO whose underlying asset is P_{t_2} with dividends $2\delta_v$. We determine the critical ration price P_2^* that makes indifferent to invest or not at time t_2 solving the following equation:

$$\begin{aligned} P_2^* - 1 &= (1-p^+)^2 \left[P_2^* N(d_1(P_2^*, \tau_2)) - e^{-\delta_d \tau_2} N(d_2(P_2^*, \tau_2)) \right] \\ &+ 2(1-p^+)p^+ \left[P_2^* e^{-\delta_v \tau_2} N(d_1(P_2^* e^{-\delta_v \tau_2}, \tau_2)) - e^{-\delta_d \tau_2} N(d_2(P_2^* e^{-\delta_v \tau_2}, \tau_2)) \right] \\ &+ (p^+)^2 \left[P_2^* e^{-2\delta_v \tau_2} N(d_1(P_2^* e^{-2\delta_v \tau_2}, \tau_2)) - e^{-\delta_d \tau_2} N(d_2(P_2^* e^{-2\delta_v \tau_2}, \tau_2)) \right] \quad (26) \end{aligned}$$

The value of this investment opportunity at time t_1 with the threat of $k = 2$ competitors entry is a PSAEO that can be exercised at time t_1 and t_2, whose maturity time is $\tau \equiv t_3 - t_1$ and that we denote by $S_2(V_{t_1}, D_{t_1}, \tau)$. Its payoff will be $(V_{t_2} - D_{t_2})$ if $P_{t_2} \geq P_2^*$ and so the pioneer realizes the investment D at time t_2, otherwise it will be:

$$(p^+)^2 \max[V^{2\delta}_{t_3} - D_{t_3}, 0] + 2p^+(1-p^+) \max[V^{\delta}_{t_3} - D_{t_3}, 0] + (1-p^+) \max[V_{t_3} - D_{t_3}, 0]$$

if $P_{t_2} < P_2^*$, and so the investment D is postponed at time t_3. As it is shown previously, in order to determine the value of $S_2(V_{t_1}, D_{t_1}, \tau)$ we use the Monte Carlo approach. So we price the PSAEO as the expectation value of discounted cash-flows under the risk-neutral probability $\mathbb{Q}$:

$$\begin{aligned} S_2(V_{t_1}, D_{t_1}, \tau) &= e^{-r\tau_1} E_{\mathbb{Q}} \left[(V_{t_2} - D_{t_2}) \mathbf{1}_{\{P_{t_2} \geq P_2^*\}} \right] \\ &+ e^{-r\tau} E_{\mathbb{Q}} \left[(p^+)^2 \max(V^{2\delta}_{t_3} - D_{t_3}, 0) \mathbf{1}_{\{P_{t_2} < P_2^*\}} \right] \\ &+ e^{-r\tau} E_{\mathbb{Q}} \left[2p^+(1-p^+) \max(V^{\delta}_{t_3} - D_{t_3}, 0) \mathbf{1}_{\{P_{t_2} < P_2^*\}} \right] \\ &+ e^{-r\tau} E_{\mathbb{Q}} \left[(1-p^+)^2 \max(V_{t_3} - D_{t_3}, 0) \mathbf{1}_{\{P_{t_2} < P_2^*\}} \right] \end{aligned} \quad (27)$$

Also in this case, using the asset D as numeraire given by Eq.(46) illustrated in Appendix A, we have that:

$$S_2(V_{t_1}, D_{t_1}, \tau) = D_{t_1} S_2(P_{t_1}, 1, \tau) \quad (28)$$

Therefore, using the Monte Carlo simulation, we can price the PSAO $S_2(P_{t_1}, 1, \tau)$ using the stochastic factor P as:

$$\begin{aligned} S_2(P_{t_1}, 1, \tau) &\approx \frac{\sum_{j \in H_2} e^{-\delta_d \tau_1} (\hat{P}_{t_2,j} - 1)}{m} \\ &+ \frac{\sum_{j \in \bar{H}_2} e^{-\delta_d \tau} (p^+)^2 (\max[\hat{P}^{2\delta}_{t_3,j} - 1, 0])}{m} \\ &+ \frac{\sum_{j \in \bar{H}_2} e^{-\delta_d \tau} 2 (p^+)^2 (1 - p^+) \max[\hat{P}^{\delta}_{t_3,j} - 1, 0]}{m} \\ &+ \frac{\sum_{j \in \bar{H}_2} e^{-\delta_d \tau} (1 - p^+)^2 \max[\hat{P}_{t_3,j} - 1, 0]}{m} \end{aligned} \tag{29}$$

where $H_2 = \{j = 1...m \,\text{s.t.}\, P_{t_2,j} \geq P_2^*\}$ and m is the number of simulations.
Also in this case, the investment IT will be realized at time t_1 if the underlying value $S_2(V_{t_1}, D_{t_1}, \tau)$ is bigger than exercise price $IT = \varphi D_{t_1}$, otherwise the firm prefers to abandon the project. So this investment opportunity can be valued at time t_0 as a Compound Exchange option $c(S_2, IT, t_1)$ whose underlying asset is $S_2(V_{t_1}, D_{t_1}, \tau)$, the exercise price is $IT = \varphi D_{t_1}$ and the maturity date is t_1 as $t_0 = 0$. The final payoff is:

$$\begin{aligned} c(S_2, IT, 0) &= \max[S_2(V_{t_1}, D_{t_1}, \tau) - IT, 0] \\ &= D_{t_1} (\max[S_2(P_{t_1}, 1, \tau) - \varphi, 0]) \end{aligned} \tag{30}$$

So, using the asset D given by Eq.(46), we can write the value at time t_0 of pioneer R&D investment opportunity $c(S_2, IT, t_1)$ with the threat of $k = 2$ rivals:

$$\begin{aligned} c(S_2, IT, t_1) &= e^{-rt_1} D_{t_1} E_{\mathbb{Q}} [\max(S_2(P_{t_1}, 1, \tau) - \varphi, 0)] \\ &= e^{-rt_1} D_0 e^{(r-\delta_d)t_1} E_{\mathbb{Q}} \left[\max(S_2(P_{t_1}, 1, \tau) - \varphi, 0) \frac{d\,\widetilde{\mathbb{Q}}}{d\,\mathbb{Q}} \right] \\ &= D_0 e^{-\delta_d t_1} E_{\widetilde{\mathbb{Q}}} [\max(S_2(P_{t_1}, 1, \tau) - \varphi, 0)] \end{aligned} \tag{31}$$

Using Monte Carlo simulation, it is possible to approximate:

$$c(S_2, IT, t_1) \approx D_0 e^{-\delta_d t_1} \left(\frac{\sum_{i=1}^{n} \max[S_2(P^i_{t_1}, 1, \tau) - \varphi, 0]}{n} \right) \tag{32}$$

where n is the number of simulation paths to determine the CEO.

2.3 General Case with k potential competitors entry.

Finally, we analyse the general case with k rivals that can enter in the market. So, the investment D will be realized at time t_2 if the exercised value $V_{t_2} - D_{t_2}$ is greater than waiting value:

$$V_{t_2} - D_{t_2} > \sum_{h=0}^{k} \left(\begin{array}{c} k \\ h \end{array} \right) (p^+)^h (1-p^+)^{k-h} s(V_{t_2}^{h\delta}, D_{t_2}, \tau_2) \tag{33}$$

Using Margrabe (1978) and McDonald & Siegel (1984) models we have that:

$$s(V_{t_2}^{h\delta}, D_{t_2}, \tau_2) = V_{t_2} e^{-h\delta_v \tau_2} N(d_1(P_{t_2}^{h\delta}, \tau_2)) - D_{t_2} e^{-\delta_d \tau_2} N(d_2(P_{t_2}^{h\delta}, \tau_2)) \tag{34}$$

where $P_{t_2}^{h\delta} = P_{t_2} e^{-h\delta_v \tau_2}$. Assuming the asset D as numeraire we reduce the by-dimensionality of Eq.(33) to one variable $P = \frac{V}{D}$:

$$P_{t_2} - 1 > \sum_{h=0}^{k} \left(\begin{array}{c} k \\ h \end{array} \right) (p^+)^h (1-p^+)^{k-h} s(P_{t_2}^{h\delta}, 1, \tau_2) \tag{35}$$

After that, we determine the critical ratio price P_k^* that makes indifferent to invest or not at time t_2 solving the following equation:

$$P_k^* - 1 = \sum_{h=0}^{k} \left(\begin{array}{c} k \\ h \end{array} \right) (p^+)^h (1-p^+)^{k-h} s((P_k^*)^{h\delta}, 1, \tau_2) \tag{36}$$

where:

$$s((P_k^*)^{h\delta}, 1, \tau_2) = P_k^* e^{-h\delta_v \tau_2} N(d_1(P_k^* e^{-h\delta_v \tau_2}, \tau_2)) - D_{t_2} e^{-\delta_d \tau_2} N(d_2(P_k^* e^{-h\delta_v \tau_2}, \tau_2)).$$

We price the PSAEO $S_k(V_{t_1}, D_{t_1}, \tau)$ with the threat of k competitors entry as the expectation value of discounted cash-flows under the risk-neutral probability $\mathbb{Q}$:

$$\begin{aligned} S_k(V_{t_1}, D_{t_1}, \tau) &= e^{-r\tau_1} E_{\mathbb{Q}} \left[(V_{t_2} - D_{t_2}) \mathbf{1}_{\{P_{t_2} \geq P_k^*\}} \right] \\ &+ e^{-r\tau} E_{\mathbb{Q}} \left[\sum_{h=0}^{k} \left(\begin{array}{c} k \\ h \end{array} \right) (p^+)^h (1-p^+)^{k-h} \max(V_{t_3}^{h\delta} - D_{t_3}, 0) \mathbf{1}_{\{P_{t_2} < P_k^*\}} \right] \end{aligned} \tag{37}$$

Using the same procedure, it results that:

$$S_k(V_{t_1}, D_{t_1}, \tau) = D_{t_1} S_k(P_{t_1}, 1, \tau) \tag{38}$$

By Monte Carlo simulation, we can approximate the value of PSAO $S_k(P_{t_1}, 1, \tau)$ as:

$$\begin{aligned} S_k(P_{t_1}, 1, \tau) &\approx \frac{\sum_{j \in H_k} e^{-\delta_d \tau_1}(\hat{P}_{t_2,j} - 1)}{m} \\ &+ \frac{\sum_{j \in \bar{H}_k} e^{-\delta_d \tau}(1-p^+)^n \max[\hat{P}_{t_3,j} - 1, 0]}{m} \\ &+ \frac{\sum_{j \in \bar{H}_k} e^{-\delta_d \tau} k(p^+)(1-p^+)^{k-1} \max[\hat{P}^{\delta}_{t_3,j} - 1, 0]}{m} \\ &+ \cdots \\ &+ \frac{\sum_{j \in \bar{H}_k} e^{-\delta_d \tau} k(p^+)^{k-1}(1-p^+) \max[\hat{P}^{(k-1)\delta}_{t_3,j} - 1, 0]}{m} \\ &+ \frac{\sum_{j \in \bar{H}_k} e^{-\delta_d \tau} (p^+)^k \max[\hat{P}^{k\delta}_{t_3,j} - 1, 0]}{m} \end{aligned} \tag{39}$$

where $H_k = \{j = 1...m \, \text{s.t.} \, P_{t_2,j} \geq P_k^*\}$ and m is the number of simulations to compute the PSAO.
Finally, using Monte Carlo simulation again, it is possible to approximate:

$$c(S_k, IT, t_1) \approx D_0 e^{-\delta_d t_1} \left(\frac{\sum_{i=1}^{n} \max[S_k(\hat{P}^i_{t_1}, 1, \tau) - \varphi, 0]}{n} \right) \tag{40}$$

where n is the number of simulation paths in order to have the CEO.

3 Real Application

To illustrate the concepts and equations presented, we develop a numerical example to value a pioneer R&D investment opportunity assuming the threat of competitors that can erode the cash-flows. In particular we consider that:

- Gross Project Value: V_0=220 000 $;
- Research Investment: R= 36 500 $;
- Investment Technology: IT_0=30 000 $;
- Proportion of D required for IT: $\varphi = \frac{IT}{D} = 0.12$
- Development Investment: D_0= 250 000 $;
- Market and Costs Volatility: $\sigma_v = 0.55$; $\sigma_d = 0.23$;
- Correlation between V and D: $\rho_{vd} = 0.10$;
- Dividend-Yelds of V: $\delta_v = 0.30$;
- Dividend-Yelds of D: $\delta_d = 0.05$;

- Initial time: $t_0 = 0$;
- Expiration Time of CEO (Deadline Investment IT): $t_1 = 2$ years;
- First time to exercise the PSAEO (First time to realize D): $t_2 = 3$ years;
- Expiration time of PSAEO (Second time to realize D): $t_3 = 4$ years;
- Pioneer's and Rivals' success probabilities: $q = 0.60$; $p = 0.55$.

The Gross Project Value V_0 is the present value of the R&D project's discounted expected cash flows. V is the underlying asset that the pioneer receives realizing investment D. We assume that the evolution of asset V follows Eq. (5) if no rival invests with success in R&D, or Eq. (6) if h rivals realize successfully the R&D program.
The Development investment D_0 is the current amount of capital that the pioneer needs to invest today to receive the R&D project's value. We assume that D follows the geometric Brownian motion defined in Eq. (2) and it can be spent at time $t_2 = 3$ or at time $t_3 = 4$.
As we consider a two-stage R&D plan, we considering two different investments denoted by R and IT. The first is the Reserch investment spent a time initial time $t_0 = 0$ that allows to obtain, in case of success, the investment opportunity, while IT_0 represents the current amount of Investment Technology to research innovations. We suppose that IT is a proportion φ of asset D; therefore it assumes the identical stochastic process of D, except that it occurs at time $t_1 = 2$.
Conveniently, we assume that the quoted shares and trade options of similar company are adequate proxies in order to value the volatility of assets V and D.
In our application, $\delta_v = 0.30$ is the competitive dividend, namely the loss of cash-flows deferring the R&D project owing to each competitor entrance in the market. Moreover, as pointed out by McDonald and Siegel (1986), δ_d is negative when there are carrying costs associated with a project's capital costs, while δ_d is positive when there are technological befenits from the capital cost's level from deferring the project. We suppose $\delta_d = 0.05$.
In addition, we consider that pioneer has an higher and more efficient Know-How than potential rivals and so, pioneer success probability is $q = 0.60$ while rivals' one is $p = 0.55$.
Evaluation of R&D investment opportunity is generally a complex and computationally intensive problem. To determine the simulated R&D investment opportunity values, we have assumed the same number of path-simulations to compute the CEO and PSAO, and so $n = m = 10\,000$. Moreover, Appendix B shows the Matlab algorithm about the Stratified Sample method to simulate the asset P that allows to reduce the variance. We assume that $a = 0.80$.
Table 1 illustrates four simulated Monte Carlo values about the pioneer R&D investment. In particular way, we have considered three situation: in the first case there is a single potential competitor and so $k = 1$, in the second case we assume two potential rivals and so $k = 2$, and finally we have three potential competitors and therefore $k = 3$. For each situation we suppose three value about the information revelation from pioneer to rivals: $\rho(X, Y) = 0$, 0.40 and 0.80. Moreover,

we have considered the average among the four simulations in order to determine a single simulated value of CEO. The last column summarizes the pioneer profit Π that invests in R&D given by Eq.(21).

Table 1: Monte Carlo Simulations

k	$\rho(X,Y)$	P_k^*	1^{st} MC	2^{nd} MC	3^{rd} MC	4^{th} MC	CEO	Π
1	0	1.3836	64 826	64 978	64 513	64 876	64 798	**+2 379**
1	0.40	1.2818	63 540	63 692	63 229	63 594	63 514	**+1 608**
1	0.80	1.2081	62 426	62 578	62 117	62 486	62 402	**+941**
2	0	1.1790	61 146	61 296	60 841	61 211	61 123	**+174**
2	0.40	1.1128	60 275	60 425	59 973	60 345	60 254	−348
2	0.80	1.0679	59 760	59 911	59 461	59 833	59 741	−655
3	0	1.0982	60 712	60 862	60 403	60 780	60 689	−87
3	0.40	1.0522	59 812	59 963	59 513	59 884	59 793	−624
3	0.80	1.0245	59 255	59 405	58 957	59 329	59 236	−958

Using the real option methodology, we are able to determine the value of pioneer R&D investment opportunity assuming the threat of rivals entry. In particular way we can observe that, when the information revelation $\rho(X,Y)$ and the number of potential competitors k increase, then the value of CEO goes down. By the competitive dividends, the option value takes into account the reduction of cash-flows that potential competitors can erode when the pioneer delays the realization of R&D waiting better market conditions. In addition, the increasing information revelation benefits the rivals R&D success and so the pioneer's option reduces its value. We can state that, if $\Pi < 0$, the pioneer firm preferes to abandon the R&D investment project at time t_0 otherwise, if $\Pi > 0$, the pioneer realizes the investment R at time t_0. For our adapted numbers, it results that pioneer invests R at time t_0 when $k = 1$ for each information revelation value and when $k = 2$ without information revelation. In this case the profit Π is positive, as it is denoted by bold value in Table 1. In the other situations, the pioneer prefers abandon the R&D program. Obviously, if $R \neq 36\,500\,\$$, the pioneer's decision about the realization of R&D project investment changes.

Finally, if the pioneer realizes the investments R at time t_0 and IT at time t_1, it must decide whether to invest D at time t_2 or to delay the decision at final date t_3, running the risk that potential rivals can erode its cash-flows. In particular, we can interpret the asset P as a profitability index since it is the ratio between the market value V and the development investment D. So, if the profitability index P_{t_2} at time t_2 is bigger than critical market value P_k^*, the pioneer realizes the development investment D at time t_2, otherwise it delays the decision at time t_3. We can observe that, when the information revelation $\rho(X,Y)$ and the number of potential rivals k increase, the critical ratio P_k^* decreases. This means the reduction of critical profitability for which is indifferent to realize the investment D or to wait to invest. For instance, we assume that $\rho(X,Y) = 0.40$ and the profitability index of pioneer R&D project $P_{t_2} = 1.07$ at time t_2. If the number of potential rivals $k = 2$, the pioneer prefers delay the investment decision at time t_3 waiting better market conditions since $P_{t_2} < 1.1128$, while if the threat increases and so $k = 3$, the pioneer prefers invests at time t_2 in order to avoid the loss of cash-flows (preempt the rivals) since $P_{t_2} > 1.0522$. In conclusion, the increase of threat arising from potential rivals' number and the information revelation reduces the critical level P_k^* under which the pioneer postpones the investment D at final time t_3, as it is illustrated in Table 2.

Table 2: Critical Ratios P_k^*

$\rho(X,Y)$	P_4^*	P_5^*	P_6^*	P_7^*	P_8^*	P_9^*	P_{10}^*
0	1.0571	1.03411	1.0205	1.0124	1.0075	1.0045	1.0027
0.40	1.0251	1.0121	1.0058	1.0027	1.0012	1.0005	1.0002
0.80	1.0087	1.0029	1.0009	1.0002	1.00008	1.00002	1.000006

4 Concluding Remark

In this paper we have shown as the information revelation and the threat of potential competitors can affect the pioneer decision about the realization of R&D investment. As it well known, the evaluation of R&D investment opportunity is generally complex and computationally intensive problem. In fact, an R&D investment is made in a phased manner, in which each stage provides information about the R&D success and creates an opportunity (option) for subsequent investment. Therefore, we have considered a two-stage R&D plan valuated as a compound option using the Monte Carlo approach. To reduce the variance of simulation we have used the Stratified Sample method.

In particular way, we have assumed that the threat of rivals' entrance in the market can diminish the cash-flows deriving from pioneer R&D project. To reach this objective, we have used the dividends as the cash-flows eroded by other rivals when an investment opportunity is postponed. We have analyzed two particular cases, $k = 1, 2$ and then a general case k. Finally, we have illustrated numerically that both the pioneer's R&D opportunity value (CEO) and the critical profitability index P_k^* decrease when the information revelation $\rho(X,Y)$ and the number of potential competitors k increase. This means the reduction of pioneer's opportunity to realize the investment R at time t_0 and to delay the investment D at time t_3 waiting better market conditions.

A General Computation

The typical simulation approach is to price the PSAEO as the expectation value of discounted cash-flows under the risk-neutral probability $\mathbb{Q}$. So, for the risk-neutral version of the Eqs. (1) and (2), it's enough replace the expected rates of return μ_i by the risk-free interest rate r plus the premium-risk, namely $\mu_i = r + \lambda_i \sigma_i$, where λ_i is the asset's market price of risk, for $i = V, D$. So, we obtain the risk-neutral stochastic equations:

$$\frac{dV}{V} = (r - \delta_v)dt + \sigma_v(dZ_v + \lambda_v dt) = (r - \delta_v)dt + \sigma_v dZ_v^* \tag{41}$$

$$\frac{dD}{D} = (r - \delta_d)dt + \sigma_d(dZ_d + \lambda_d dt) = (r - \delta_d)dt + \sigma_d dZ_d^* \tag{42}$$

The Brownian processes $dZ_v^* \equiv dZ_v + \lambda_v dt$ and $dZ_d^* \equiv dZ_d + \lambda_d dt$ are the new Brownian motions under the risk-neutral probability $\mathbb{Q}$ and $Cov(dZ_v^*, dZ_d^*) = \rho_{vd} dt$. Applying the Ito's lemma, we can reach the equation for the ratio-price simulation $P = \frac{V}{D}$ under the risk-neutral measure $\mathbb{Q}$:

$$\frac{dP}{P} = \left(\delta_d - \delta_v + \sigma_d^2 - \sigma_v \sigma_d \rho_{vd}\right) dt + \sigma_v dZ_v^* - \sigma_d dZ_d^* \tag{43}$$

Applying the log-transformation for D_t, under the probability $\mathbb{Q}$, it results:

$$D_t = D_0 \exp\left\{(r - \delta_d)t\right\} \cdot \exp\left(-\frac{\sigma_d^2}{2}\, t + \sigma_d Z_d^*(t)\right). \tag{44}$$

We have that $U \equiv \left(-\frac{\sigma_d^2}{2}\, t + \sigma_d Z_d^*(t)\right) \sim \mathcal{N}\left(-\frac{\sigma_d^2}{2} t, \sigma_d \sqrt{t}\right)$ and therefore $\exp(U)$ is a log-normal which expectation value is $E_{\mathbb{Q}}\left[\exp(U)\right] = \exp\left(-\frac{\sigma_d^2}{2} t + \frac{\sigma_d^2}{2} t\right) = 1$. So, by Girsanov's theorem, we can define the new probability measure $\widetilde{\mathbb{Q}}$ equivalent to $\mathbb{Q}$ and the Radon-Nikodym derivative is:

$$\frac{d\,\widetilde{\mathbb{Q}}}{d\,\mathbb{Q}} = \exp\left(-\frac{\sigma_d^2}{2}\, t + \sigma_d Z_d^*(t)\right). \tag{45}$$

Hence, using the Eq. (44), we can write:

$$D_t = D_0\, e^{(r - \delta_d)t} \cdot \frac{d\,\widetilde{\mathbb{Q}}}{d\,\mathbb{Q}} \tag{46}$$

By the Girsanov theorem, the processes:

$$d\hat{Z}_d = dZ_d^* - \sigma_d dt \tag{47}$$

$$d\hat{Z}_v = \rho_{vd} d\hat{Z}_d + \sqrt{1 - \rho_{vd}^2}\, dZ' \tag{48}$$

are two Brownian motions under the risk-neutral probability measure $\widetilde{\mathbb{Q}}$ and Z' is a Brownian motion under $\widetilde{\mathbb{Q}}$ independent of $\hat{Z}_d$. By the Brownian motions defined in the Eqs. (47) and (48), we can rewrite the Eq. (43) for the asset P under the risk-neutral probability $\widetilde{\mathbb{Q}}$. So it results that:

$$\frac{dP}{P} = (\delta_d - \delta_v)\, dt + \sigma_v\, d\hat{Z}_v - \sigma_d\, d\hat{Z}_d \tag{49}$$

Using the Eq. (48), it results that:

$$\sigma_v d\hat{Z}_v - \sigma_d d\hat{Z}_d = (\sigma_v \rho_{vd} - \sigma_d)\; d\hat{Z}_d + \sigma_v \left(\sqrt{1-\rho_{vd}^2}\right) dZ' \tag{50}$$

where $\hat{Z}_v$ and Z' are independent under $\widetilde{\mathbb{Q}}$. Therefore, as $(\sigma_v d\hat{Z}_v - \sigma_d d\hat{Z}_d) \sim \mathcal{N}(0, \sigma\sqrt{dt})$, we can rewrite the Eq. (49):

$$\frac{dP}{P} = (\delta_d - \delta_v)\, dt + \sigma dZ_p \tag{51}$$

where $\sigma = \sqrt{\sigma_v^2 + \sigma_d^2 - 2\sigma_v\sigma_d\rho_{vd}}$ and Z_p is a Brownian motion under $\widetilde{\mathbb{Q}}$. Using the log-transformation, we obtain the equation for the risk-neutral price simulation P with dividends:

$$P_t^{h\delta} = P_0 \exp\left\{ \left(\delta_d - h\delta_v - \frac{\sigma^2}{2}\right) t + \sigma Z_p(t) \right\} \tag{52}$$

and without dividends:

$$P_t = P_0 \exp\left\{ \left(\delta_d - \frac{\sigma^2}{2}\right) t + \sigma Z_p(t) \right\} \tag{53}$$

Using Eqs. (52) and (53), it is plain that $P_t^{h\delta} = P_t e^{-h\delta_v\, t}$.

B Matlab Algorithm with $k = 1$.

In this section we present the Matlab algorithm to determine the pioneer R&D investment opportunity value assuming that $k = 1$. Using Eq. (52), we can observe that $Y = \ln(\frac{P_t^{h\delta}}{P_0})$ follows a normal distribution with mean $(\delta_d - h\delta_v - \frac{\sigma^2}{2})t$ and variance $\sigma^2 t$. So, the random variable Y can be generate by inverse of the normal cumulative distribution function $Y = F^{-1}(u; (\delta_d - h\delta_v - \frac{\sigma^2}{2})t, \sigma^2 t)$ where u is a function of a uniform random variable $U[0, 1]$. Using Matlab algorithm, we can generate the n simulated prices $\hat{P}_t^i$, for $i = 1...n$, as:

```
Pt=P0*exp(norminv(u,(dD-h*dV)*t-0.5*sig^2*t,sig*sqrt(t)))
```

where $u = rand(1, n)$ are the n random uniform values between 0 and 1. To reduce the variance of results, we propose the Stratified Sample with two intervas. So we concentrate the sample in the region where the function payoff g is more variable. So we consider the piecewise $a\, g(u_1) + (1-a)\, g(u_2)$ where $u_1 \sim U[0\, a]$ and $u_2 \sim U[a\, 1]$ as an individual sample. The Matlab algorith is the following:

```
function R&DValue = CONCK1(V0,D0,R,IT,T1,T2,T3,sigV,...
sigD,rhoVD,dV,dD,q,p,riv,m,n,a); %Inputs;
sig=sqrt(sigV.^2+sigD.^2-2*rhoVD*sigV*sigD); %Variance of P;
fi=(IT/D0); %Proportion of IT;
dd=(dV-dD);
P0=V0/D0;
%Rival success probability after Information Revelation;
pp=p+sqrt((1-q)/(q))*sqrt(p*(1-p))*riv;
t1=T2-T1;
t2=T3-T2;
t=T3-T1;
u=rand(1,n); %Random uniform values between 0 and 1;
%Value of Asset P in T1 with dV=0;
PT1=P0*exp(norminv(a*u,dD*T1-sig^2*T1/2,sig*sqrt(T1)));
PaT1=P0*exp(norminv(a+(1-a)*u,dD*T1-sig^2*T1/2,sig*sqrt(T1)));
for i=1:n;
h=rand(1,m);
g=rand(1,m);
%Asset Values P at time T2 and T3 with and without dividends;
PT2=PT1(i).*exp(norminv(h,dD*(t1)-0.5*(sig^2)*(t1),sig*sqrt(t1)));
PT3=PT2.*exp(norminv(g,dD*(t2)-0.5*(sig^2)*(t2),sig*sqrt(t2)));
PT3e=PT2.*exp(norminv(g,-dd*(t2)-0.5*(sig^2)*(t2),sig*sqrt(t2)));
PaT2=PaT1(i).*exp(norminv(h,dD*(t1)-0.5*(sig^2)*(t1),sig*sqrt(t1)));
PaT3=PaT2.*exp(norminv(g,dD*(t2)-0.5*(sig^2)*(t2),sig*sqrt(t2)));
PaT3e=PaT2.*exp(norminv(g,-dd*(t2)-0.5*(sig^2)*(t2),sig*sqrt(t2)));
d1=(log(PT2*exp(dD*(t2)))+0.5*(sig.^2)*(t2))/(sig*sqrt(t2));
d2=(log(PT2*exp(dD*(t2)))-0.5*(sig.^2)*(t2))/(sig*sqrt(t2));
dd1=(log(PT2*exp(-dd*(t2)))+0.5*(sig.^2)*(t2))/(sig*sqrt(t2));
dd2=(log(PT2*exp(-dd*(t2)))-0.5*(sig.^2)*(t2))/(sig*sqrt(t2));
da1=(log(PaT2*exp(dD*(t2)))+0.5*(sig.^2)*(t2))/(sig*sqrt(t2));
da2=(log(PaT2*exp(dD*(t2)))-0.5*(sig.^2)*(t2))/(sig*sqrt(t2));
dda1=(log(PaT2*exp(-dd*(t2)))+0.5*(sig.^2)*(t2))/(sig*sqrt(t2));
dda2=(log(PaT2*exp(-dd*(t2)))-0.5*(sig.^2)*(t2))/(sig*sqrt(t2));
%Conditions that makes indifferent the investment D in T2;
cond1=PT2-1-(1-pp).*PT2.*normcdf(d1)+(1-pp).*exp(-dD*t2).*normcdf(d2)-...
pp.*PT2.*exp(-(dV)*t2).*normcdf(dd1)+pp.*exp(-dD*t2).*normcdf(dd2);
conda1=PaT2-1-(1-pp).*PaT2.*normcdf(da1)+(1-pp).*exp(-dD*t2).*normcdf(da2)-...
pp.*PaT2.*exp(-(dV)*t2).*normcdf(dda1)+pp.*exp(-dD*t2).*normcdf(dda2);
%Simulations;
```

```
for j=1:m;
    if cond1(j)>=0;
        r(j)=0;
        v(j)=exp(-dD*t1)*(max(PT2(j)-1,0));
    else
        r(j)=exp(-dD*t)*((1-pp)*(max(PT3(j)-1,0))+pp*(max(PT3e(j)-1,0)));
        v(j)=0;
    end
    if conda1(j)>=0;
        ra(j)=0;
        va(j)=exp(-dD*t1)*(max(PaT2(j)-1,0));
    else
        ra(j)=exp(-dD*t)*((1-pp)*(max(PaT2(j)-1,0))+pp*(max(PaT3e(j)-1,0)));
        va(j)=0;
    end
end
    w=a*(r+v)+(1-a)*(va+ra);
%Value of i^th PSAEO
    PSAO(i)=mean(w);
end
S=max(PSAO-fi,0);
Y=mean(S)
CEO=D0*exp(-dD*T1)*Y
Profit=q*CEO-R
```

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