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Bidimensional Inequalities with an Ordinal Variable

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Inégalités Bidimensionnelles avec une Variable Ordinale

Résumé

Nous étudions les fondements normatifs de deux critères de dominance faciles à mettre en oeuvre et permettant de comparer des distributions de deux attributs, l'un étant cardinal et l'autre ordinal. Les critères que nous examinons sont, d'une part le premier quasi-ordre d'Atkinson et Bourguignon (1982), d'autre part une généralisation du déficit de pauvreté de Bourguignon (1989). Nous spécifions dans chaque cas les propriétés des fonctions d'utilité individuelles, qui garantissent que tous les observateurs éthiques bien-êtristes ayant de l'aversion pour l'inégalité classeront les distributions de la même manière que le critère de dominance. Nous identifions aussi les transformations élémentaires réductrices d'inégalité, qui permettent d'obtenir la distribution dominante à partir de la distribution dominée.

Mots-clés : Analyse Normative, Utilitarisme, Bien-Êtrisme, Dominance Stochastique Bidimensionnelle, Transformations Egalisantes

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Abstract

We investigate the normative foundations of two empirically implementable dominance criteria for comparing distributions of two attributes, where the first one is cardinal while the second is ordinal. The criteria we consider are Atkinson and Bourguignon's (1982) first quasi-ordering and a generalization of Bourguignon's (1989) ordered poverty gap criterion. In each case we specify the restrictions to be placed on the individual utility functions, which guarantee that all utility-inequality averse welfarist ethical observers will rank the distributions under comparison in the same way as the dominance criterion. We also identify the elementary inequality reducing transformations successive applications of which permit to derive the dominating distribution from the dominated one.

Keywords: Normative Analysis, Utilitarianism, Welfarism, Bidimensional Stochastic Dominance, Inequality Reducing Transformations

JEL : D30 ; D63 ; I32

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Bidimensional Inequalities with an Ordinal Variable♯

1. Introduction and Motivation

The normative foundations of the comparison of distributions of a single attribute between a given number of individuals are by now well-established. They originate in the equivalence between three statements that are considered relevant answers to the question of when a distribution $\mathbf{x}$ can be considered normatively better than a distribution $\mathbf{y}$. These statements, the equivalence of which was first established by Hardy, Littlewood and Polya (1952) and popularized later on among economists by Kolm (1969), Dasgupta, Sen and Starrett (1973), Sen (1997), Fields and Fei (1978) among others, are the following:

(i) Distribution $\mathbf{x}$ can be obtained from distribution $\mathbf{y}$ by means of a finite sequence of progressive – or equivalently Pigou-Dalton – transfers.

(ii) All utilitarian ethical observers who assume that individuals convert the attribute into well-being by means of the same concave utility function rank distribution $\mathbf{x}$ above distribution $\mathbf{y}$.

(iii) The poverty gap is lower in distribution $\mathbf{x}$ than in distribution $\mathbf{y}$ whatever the poverty line, or equivalently the Lorenz curve of distribution $\mathbf{x}$ lies everywhere above that of $\mathbf{y}$.

This remarkable result, which can be generalised in a number of ways, points to three different dimensions of the inequality measurement process.[1] The first statement aims at capturing the very *notion of inequality* reduction by associating it with elementary transformations of the distributions. The second statement is fundamentally normative and it assumes that society has an *aversion to inequality* which, in the utilitarian framework, is captured by the concavity of the utility function. To some extent the first statement helps in clarifying the meaning of the restriction imposed on the utility function in the second statement. While these two conditions shed light on two different facets of the inequality concept, they do not prove very useful for deciding in practice when one distribution is more unequal than another. This is particularly true for the second statement which requires an infinite number of comparisons to be made before a distribution can be declared less unequal than another. The third statement resolves this problem by providing easily *implementable criteria* that allow one to recover the ranking of distributions implied by the first and second statements.

The last condition can also been interpreted as a means of identifying those pairs of distributions for which a consensus prevails among all inequality averse utilitarian ethical

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[1] As shown by Dasgupta, Sen and Starrett (1973) the equivalence continues to hold when more flexible social welfare functionals are substituted for the utilitarian one. It is also possible to extend this result to the case where the distributions under comparison have differing means by adapting appropriately each of these three conditions (see Kolm (1969) and Shorrocks (1983)).

observers. If the Lorenz curves – or equivalently the poverty gap profiles – of two distributions do not intersect, then all the utilitarian ethical observers who have some aversion to inequality will rank these distributions in the same way. If, on the contrary, the Lorenz curves of two distributions cross, then it is always possible to find two such ethical observers who will rank these distributions in the opposite way. This points to what may be considered an inherent limitation of this approach: it is very unlikely that the above conditions will allow one to rank conclusively *all* the distributions under consideration. Actually this pessimistic view is exaggerated and there is empirical evidence that the comparison of the Lorenz curves permits conclusive verdicts to be obtained in a significant number of cases.[2] Nevertheless, the Lorenz criterion is often considered to be a first round approach which must be supplemented by the use of ethically more demanding indices in a second stage. It is traditional practice to require that these summary indices be compatible with any of the conditions of the Hardy-Littlewood-Polya result (see e.g. Foster (1985)).

Remarkable as they are, these foundations strictly concern distributions of a single attribute which is typically identified with individual income. However the ability of income alone to measure a person's well-being has been seriously challenged during the last thirty years and there has been an increasing concern for a more comprehensive approach. The focus on income is to a large extent justified on the assumption that it provides a good measure of the level of well-being achieved by an individual who behaves rationally in a comprehensive and fully competitive market environment. This neglects the fact that a number of commodities that contribute to a person's well-being cannot be given market values. Typical instances are amenities like recreational areas or publicly provided services like education or health care, for which markets are imperfect or even do not exist. Also, attributes such as family circumstances or health status affect a person's well-being but cannot be related to income. The recognition that a person's well-being cannot be fully summarized by income alone calls for a multidimensional approach to welfare and inequality measurement. The multidimensional nature of well-being is also at the heart of the capability approach developed by Sen (1985), where the functionings are generally associated with attributes such as health, educational level, but in no way with income alone. A practical instance of the capability approach is given by the *human development index* (HDI) of the United Nations, which aggregates in a single measure information about a country's literacy rate and attendance rates at primary, secondary and higher education, life expectancy at birth and GDP per capita. While the HDI represents an improvement upon existing measures of national well-being, it must be recognized that the aggregation procedure used in its definition is rather specific. A further difficulty is that the HDI is not sensitive to the inequality within any of the dimensions

[2]Sophisticated statistical inference techniques have been designed in order to test the robustness of the ranking of distributions based on the comparisons of the Lorenz curves. For instance, application of such techniques to the comparisons of ten countries from the Luxembourg Income Study database allowed Bishop, Formby and Smith (1993) to obtain conclusive verdicts in 34 out of 45 possible cases.

considered.[3]

While the last thirty years have witnessed a number of contributions that have proposed dominance criteria for comparing distributions of several attributes, one has to admit that the theory has not attained a degree of achievement comparable to that of the unidimensional approach. The Hardy-Littlewood-Polya result suggests three possible routes to address the question of the measurement of multidimensional inequality. The first of these – illustrated by Atkinson and Bourguignon (1982) – is to impose particular conditions on the utility function that are assumed to capture the aversion to multidimensional inequality and to explore their implications for the ranking of the situations under comparison. The main purpose is to find implementable dominance tests that permit one to check if it is possible to reach a consensus among these well-defined classes of social welfare functionals. Building on the results of Hadar and Russell (1974) in the multidimensional risk literature, Atkinson and Bourguignon (1982) have shown that first and second-order multidimensional stochastic dominance imply unanimity of judgements among all utilitarians for specific classes of utility functions. They have also suggested that an equivalence between their multidimensional stochastic dominance criteria and utilitarian unanimity might hold. Atkinson and Bourguignon (1987) have proposed a nice interpretation of their stochastic dominance criteria in the specific case where there are only two attributes, one of which is income and the other one an ordinal index of needs such as the household size. Their criteria and equivalence results, developed originally for distributions of attributes with an identical marginal distribution of needs, have been subsequently extended by Jenkins and Lambert (1993) and Bazen and Moyes (2003) to more general situations. Bourguignon (1989) has proposed an interesting empirically implementable criterion, that lies in between the first and second order criteria of Atkinson and Bourguignon (1982). He further identified the restrictions that have to be placed on the utility functions in order that the ranking of distributions agreed by all utilitarians coincides with that implied by his criterion.[4] However it is in general unclear what the meaning of the conditions to be satisfied by the utility functions is, unless one is able to uncover the underlying transformations of the distributions.

The second route consists in introducing particular transformations of the situations which aim at capturing the very idea of inequality reduction in a multidimensional context, and in deriving the properties of the utility function that guarantee that welfare increases as the result of such transformations. That was the strategy chosen by Kolm (1977), who considered the multiplication by a bistochastic matrix as the appropriate generalization of the idea of inequality reduction in a multidimensional framework. He proved that, if a situation is

[3]It follows that the HDI may record an improvement in a country's well-being even though inequality in the distribution of health, education or income has increased. Foster, Lopez-Calva and Székely (2005) propose a refinement of the HDI that is distributive-sensitive and show that substitution of this new measure to the original HDI considerably changes the way societies have ranked.

[4]Fleurbaey, Hagneré and Trannoy (2003) have shown that it is possible to refine Bourguignon's (1989) criterion by introducing equivalence scales and allowing these to vary within predefined intervals.

obtained from another one by means of such a transformation, then the former situation is ranked above the latter by all the utilitarian ethical observers who evaluate the individuals' well-being by means of a concave utility function, and conversely. Kolm's (1977) approach avoids the difficulty inherent in Atkinson and Bourguignon (1982) by making clear what is meant by inequality reduction and then by identifying the normative judgements consistent with it. On the other hand the elementary transformation he considers is rather specific: it consists in transferring an identical fraction of each attribute from one individual to another. This imposes very particular restrictions on the equalizing – in Kolm's terminology *rectifying* – process which rules out very natural extensions of the idea of a progressive transfer in a multidimensional setting. More importantly he did not produce a criterion that would implement the utilitarian unanimity over the class of concave utility functions.

The third route was followed by Koshevoy (1995) who suggested the use of the Lorenz zonotope as a generalization of the Lorenz curve in the multidimensional framework. Among other things, he proved that a sufficient condition for a distribution to dominate another one according to his criterion – the Lorenz zonotope of the former distribution is included in the Lorenz zonotope of the latter – is that it can be obtained from the latter through multiplication by a bistochastic matrix. However, this does not tell us a lot about the implicit equalization process embedded in the Lorenz zonotope criterion since the converse statement does not hold. It can actually be shown that there exist transformations – different from those obtained by multiplication by a bistochastic matrix – that are compatible with the Lorenz zonotope criterion. Furthermore Koshevoy (1995) did not provide any indication about the properties of the utility function that would guarantee that the ranking of distributions generated by the unanimity among all utilitarian ethical observers coincide with that implied by the Lorenz zonotope criterion. Nor did he give evidence that the welfarist approach to inequality measurement is compatible with the Lorenz zonotope quasi-ordering.[5] While the welfare criterion considered by Kolm implies Koshevoy's quasi-ordering, the converse implication does not hold, and it is therefore difficult to justify the use of the Lorenz zonotope from a normative standpoint.

This brief review of the literature indicates that the best that can be achieved is to provide equivalences between the unanimity among those utilitarian ethical observers who subscribe to some common values and, *either* an implementable criterion (Atkinson and Bourguignon (1982)), *or* a specific equalizing transformation (Kolm (1977)). In particular, none of the above contributions hint at the *elementary* transformations, which if performed a finite number of times, would be equivalent to the proposed dominance criteria. It is fair to recognize that the elementary transformations that were believed to lie behind the above-mentioned criteria have been discussed by various authors, including Atkinson and Bourguignon (1982) themselves, Ebert (2000) and Moyes (1999). However none of these

[5]Rigorously speaking the fact that the Lorenz zonotope cannot be given a utilitarian – and possibly a welfarist – justification does not mean that there exists no social welfare functions consistent with it.

papers has made a decisive step by proving that, if a distribution is ranked above another distribution by a criterion, then it is possible to obtain the dominating distribution from the dominated one by successive applications of appropriate transformations. In short, despite the relative wealth of attempts made in this direction during the last thirty years, there seems to be no multidimensional analogue to the Hardy-Littlewood-Polya equivalence result. In this paper, we aim at providing a step towards establishing such an equivalence for the case where there are only two attributes to be distributed, one of which has a cardinal nature while the other is only ordinally measurable.

Focusing on bidimensional distributions might look restrictive but it provides a simple enough structure in which the basic problem we are interested can be addressed. Most results extend to the general case where there are more than two attributes but this introduces additional complexity at the risk of making the main points less accessible. Our asymmetric treatment of the two attributes is certainly more disputable but there are a number of instances where only ordinal information is conveyed by the value of the attribute. For instance one may be interested in the evaluation of income distributions for households, who differ in size and composition as in Atkinson and Bourguignon (1987) or Bourguignon (1989). Even though precise information is provided concerning household composition such as the number and ages of the family's members, one may be reluctant to assign a cardinal meaning to the available figures. A similar situation arises when one has to compare the well-being of different populations on the basis of the joint distributions of income and health status of their members. Indicators like infant mortality or life expectancy are routinely used for measuring a person's health status and here again one might be willing to assume that the only reliable information is of an ordinal nature (see e.g. Allison and Foster (2004)).[6] The consumption of local public goods such as the quality of schooling (see Gravel, Moyes and Tarroux (2008)) is an other example of an attribute to which one might hesitate to give a cardinal meaning. On the other hand there is a wide agreement among economists on the fact that individual income per year – at least when used in a specific price configuration – is a cardinally meaningful attribute. It is therefore not unrealistic to consider two attributes that cannot be defined with the same degree of precision and which require therefore a different treatment in normative analysis.

As we emphasized earlier, the Hardy-Littlewood-Polya result provides a natural research agenda pointing at three different but actually complementary facets of inequality measurement. Our approach is much in line with Kolm's (1977) contribution in that we start with those elementary transformations which we believe capture the very notion of inequality reduction in our bidimensional setting and then look for the restrictions they imply for the utility function used by the ethical observer to evaluate the distributions. However we depart from Kolm's suggestion in a number of respects that will prove to be crucial for overcoming

[6] An important issue, which is particularly stressed by Allison and Foster (2004), concerns the implication for the ranking of the situations under comparisons of a change in the measurement scale of an attribute.

the limitations of previous studies. There is a large agreement in the profession for assimilating inequality reduction with progressive transfers when there is a single attribute. Things appear to be more complicated in the multidimensional framework and no consensus has been reached up to now considering the question of what is meant by inequality reduction. In this paper we will examine three types of elementary transformations, each of which is supposed to capture a specific feature of the equalizing process. To have a better view of the way one conceives of inequality reduction in a multidimensional context, it is helpful to return to the initial situation involving a single attribute. In this case there is a one-to-one correspondence between the ranking of individuals in terms of income and their ranking in terms of well-being. If an individual is richer than another, then her well-being as measured by *any* increasing utility function will be higher than the well-being achieved by the other individual. This correspondence breaks down when there is more than one attribute because of the multidimensionality of the attribute space. It is no longer possible to define a complete ordering of the individual situations in the attribute space that coincides with the ranking of these situations generated by *any* utility function. However it is possible to define a non-ambiguous ranking of the individual situations in terms of well-being: one needs simply to declare that one individual is better-off than another if the well-being of the former is higher than that of the latter for *all* non-decreasing utility functions. One then obtains a partial ranking that reduces to the standard dominance quasi-ordering of vectors: the individual who gets more of each attribute will be ranked above the other one by all monotone increasing utility functions. This is our first departure from Kolm (1977) who imposes no such restrictions on the pairs of individuals whose situations are *averaged* through the application of a bistochastic matrix. While there is no requirement in Kolm's approach that the individuals taking part in the equalization process be unambiguously ordered in terms of their attribute endowment, this ordering is crucial for the definition of the equalizing transformations we will consider.[7]

It seems quite natural to assume that, other things equal, a progressive transfer in the cardinal attribute between two individuals who have the same endowment of the ordinal attribute reduces inequality. For the sake of consistency with the unidimensional approach a *within-type progressive transfer* – as we will call it henceforth – is the first equalizing transformation we consider. The second transformation – which we refer to as a *favorable permutation* – is implicitly contained in Atkinson and Bourguignon (1982). It consists in giving the cardinal attribute endowment of the better-off individual in both attributes to the

[7]In Kolm's (1977) setting it is always possible to find two distributions and two individuals such that the first individual is better-off than the second according to an increasing utility function while the opposite situation occurs for another increasing utility function. The ranking of the individuals in terms of well-being is therefore conditional upon the choice of the utility function. A good example of the conditional approach in a framework similar to ours is Ebert (2006), who considers explicitly a two-stage process. In the first stage a better-off individual and a worse-off individual are specified on the basis of a predefined *social norm*. In the second stage one identifies the properties of the utility function which guarantee that social welfare increases when transfers in the cardinal attribute are performed between these two individuals in such a way that the inequality in their well-being is reduced.

worse-off individual in both attributes and *vice versa*. This permutation of the endowments in the cardinal attribute actually amounts to reducing the pairwise correlation – or positive association – existing between the two attributes. The favorable permutation is probably the most controversial elementary transformation we consider because it is not clear how inequality in the distribution of the two attributes is reduced as a result. Actually it is the inequalities of well-being between the two individuals involved in the favorable permutation that are reduced. More precisely, it can be established that, for any monotone utility function, the well-being of the poorer individual increases while that of the richer individual decreases bringing them closer on the utility scale. This consequence of a favorable permutation is reminiscent of Hammond's (1979) *equity condition* in an ordinal context and it is in this respect that it may be considered to be an equalizing transformation. The *between-type progressive transfer*, which is a natural extension of a progressive transfer in our particular framework, is our third transformation. It consists in transferring an amount of the cardinal attribute from a better-off individual in both attributes to a worse-off individual in both attributes in such a way that the beneficiary of the transfer is not made richer than the donor in the cardinal attribute. Contrary to a bistochastic transformation where the same equalizing process is applied to both attributes, a between-type progressive transfer entails redistribution in only one dimension, and this is our second point of departure from Kolm (1977). These three transformations are elementary in the sense that it seems difficult to conceive of simpler inequality-reducing operations into which they could be decomposed. Actually this is not perfectly true as far as between-type progressive transfers are concerned and one may argue they are not as elementary as they might look at first glance. As will be seen below, it is always possible to decompose a between-type progressive transfer into a within-type progressive transfer coupled with a favorable permutation. But an important qualification is needed for such a decomposition to be made: one has to add a dummy individual with appropriate endowments in both attributes. As it will become clear later this dummy individual is only instrumental and it will be suppressed at the end of the decomposition.

As far as the normative evaluation is concerned we take a slightly more restrictive approach than the traditional one which consists in endowing each ethical observer with a utilitarian social welfare functional and requiring unanimity of judgements among them. Indeed the utilitarian rule is a particular method for making distributive judgements that shows strickly-speaking no direct consideration of distributive justice. What matters for a utilitarian ethical observer is the sum of the individuals' utilities and not the way these utilities are distributed among the population. As argued by Sen (1997) it might be preferable to use more flexible aggregation rules that pay attention to the inequality in the distribution of individual utilities. We follow Sen and adopt the view that unanimity should be looked for over the larger family of all utility-inequality averse welfarist social welfare functionals. The utilitarian rule clearly belongs to this family of social welfare functionals, as do the Maximin and the Leximin ones.

As we will show in the paper our approach is not as restrictive as it might seem at first sight. Indeed, for a large class of utility functions, the application of unanimity among all utility-inequality averse welfarist ethical observers provides the same ranking of distributions as unanimity among the utilitarian ones.

Turning finally to the question of the implementation of the unanimity of value judgements we focus on two dominance criteria that have aroused contrasted interests in the profession. The first criterion, proposed by Atkinson and Bourguignon (1982), is the one that has received the greater attention in the literature. It declares that a situation dominates another situation if the graph of the joint distribution function of the former situation lies nowhere above that of the latter. This condition is actually the bidimensional version of the poverty measurement approach based on the headcount ratio: poverty as measured by the percentage of individuals who fall below predetermined levels of the two attributes is less in the first situation than in the second, for all possible values of the attributes' poverty lines. Bourguignon's (1989) ordered poverty gap quasi-ordering is the second implementable criterion that we examine in this paper. The poverty gap is computed for the cardinal attribute by assigning to each individual a poverty line that depends negatively on her endowment of the ordinal attribute. Other things equal, it is more difficult for someone in good health or living in an area with good public facilities to be considered deprived in income than for someone handicapped or living in the slums.

The organization of the paper is as follows. We introduce in Section 2 our bidimensional model along with our notation and preliminary definitions. In Section 3 we present the normative criteria, which will be used by the ethical observer in order to rank the distributions under comparison. We show that the unanimity of value judgements among all utilitarian ethical observers and all welfarist ethical observers coincide provided that the individual utility functions possess an appropriate *closedness* property, which is actually satisfied by all the classes of utility functions we consider throughout. Section 4 is concerned with the first model of Atkinson and Bourguignon (1982) and it provides the analogue to the Hardy-Littlewood-Polya result. More precisely we show that the domination of one situation over another is equivalent to requiring unanimity of value judgements among all welfarist ethical observers whose marginal utility of the cardinal attribute is non-negative and decreasing in the level of the ordinal attribute. This implies in turn that the dominating distribution can be obtained from the dominated one by means of a finite sequence of favorable permutations of the cardinal attribute. Section 5 examines the normative foundations of Bourguignon's (1989) ordered poverty gap dominance criterion. Introducing the additional requirement that the utility function is concave in the cardinal attribute for fixed levels of the ordinal attribute is enough to guarantee that the rankings of situations implied by the ordered poverty gap criterion and unanimity among all welfarist ethical observers be identical. Exploiting the possibility of adding dummy individuals to the original population allows us to show that it is possible to derive the dominating situation from the dominated one by means of a finite sequence of within-type progressive transfers followed by a finite sequence of favorable permutations, and conversely. Section 6 concludes the paper by summarizing the main results and hinting at possible avenues for future research.

2. Notation and Preliminary Definitions

We consider finite *societies*, where each individual is endowed with two attributes: the first attribute is cardinal and transferable between individuals while the second is ordinal and cannot be transferred between individuals. To make things simple we find it convenient to assimilate the first attribute with *income* and the second with *ability*. The latter term, however, should not be taken too literally and it may represent the ordering of households in terms of decreasing size (Atkinson and Bourguignon (1987)), or of individuals in terms of health (Allison and Foster (2004)), or in terms of consumption of local public goods (Gravel, Moyes and Tarroux (2008)). A *bidimensional distribution* or more compactly a *situation* for a population of n individuals is a $n \times 2$ matrix

$$\mathbf{s} \equiv (\mathbf{x}; \mathbf{a}) := \begin{bmatrix} x_1 & , & a_1 \\ \vdots & & \vdots \\ x_i & , & a_i \\ \vdots & & \vdots \\ x_n & , & a_n \end{bmatrix} = \begin{bmatrix} \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_i \\ \vdots \\ \mathbf{s}_n \end{bmatrix}, \tag{2.1}$$

such that $\mathbf{s}_i = (x_i, a_i)$ fully describes individual i, where $x_i \in D := [\underline{v}, \overline{v}] \subset \mathbb{R}$ and $a_i \in A := [\underline{h}, \overline{h}] \subset \mathbb{R}$ are respectively the income and the ability of individual i. The marginal *distributions of income* and *ability* in situation $\mathbf{s} \equiv (\mathbf{x}; \mathbf{a})$ are indicated respectively by $\mathbf{x} := (x_1, \ldots, x_n)^T \in D^n$ and $\mathbf{a} := (a_1, \ldots, a_n)^T \in A^n$, where the superscript T denotes the transposed vector. The general set of situations for a population of n individuals is denoted as

$$\mathbb{S}_n := \{\mathbf{s} \equiv (\mathbf{x}; \mathbf{a}) \mid (x_i, a_i) \in D \times A, \forall\, i = 1, 2, \ldots, n\}, \tag{2.2}$$

and we let $\mu(\mathbf{x})$ represent the *mean income* in situation $\mathbf{s} \equiv (\mathbf{x}; \mathbf{a})$.

We will make extensive use in what follows of the representation of the bidimensional distributions by means of their associated joint, marginal and conditional cumulative distribution functions. Given the situation $\mathbf{s} \equiv (\mathbf{x}; \mathbf{a}) \in \mathbb{S}_n$ we denote as $N(v, h) = \{i \mid x_i = v \text{ and } a_i = h\}$ the set of individuals who receive an income equal to v and who have an ability equal to h. The *joint density function* of $\mathbf{s} \equiv (\mathbf{x}; \mathbf{a})$ is given by

$$f(v, h) = n(v, h)/n, \ \forall\, v \in D, \ \forall\, h \in A, \tag{2.3}$$

where $n(v, h) = \#N(v, h)$. Similarly the set of individuals who receive an income no greater than v and whose ability is no greater than h is indicated by $Q(v, h) = \{i \mid x_i \leq v \text{ and } a_i \leq h\}$.

Letting $q(v,h) = \#Q(v,h)$, the *joint cumulative distribution function* is defined by

$$F(v,h) = q(v,h)/n, \ \forall\ v \in D, \ \forall\ h \in A. \tag{2.4}$$

The set of individuals whose income is equal to v is denoted as $N_1(v) = \{i \mid x_i = v\}$, while $N_2(h) = \{i \mid a_i = h\}$ indicates the set of individuals whose ability is equal to h. The *marginal density functions of income* and *ability* are respectively defined by

$$f_1(z) = n_1(v)/\,n \ \text{ and } \ f_2(h) = n_2(h)/\,n, \ \forall\ v \in D, \ \forall\ h \in A, \tag{2.5}$$

where $n_1(v) = \#N_1(v)$ and $n_2(h) = \#N_2(h)$. The set of individuals whose income in situation $\mathbf{s} \equiv (\mathbf{x};\mathbf{a})$ is no greater than v is indicated by $Q_1(v) = \{i \mid x_i \leq v\}$, while the set of individuals whose ability is no greater than h is denoted as $Q_2(h) = \{i \mid a_i \leq h\}$. The *marginal distribution functions of income* and *ability* are respectively defined by

$$F_1(z) = q_1(v)/\,n \ \text{ and } \ F_2(h) = q_2(h)/\,n, \ \forall\ v \in D, \ \forall\ h \in A, \tag{2.6}$$

where $q_1(v) = \#Q_1(v)$ and $q_2(h) = \#Q_2(h)$. The *conditional density functions of income* and *ability* are indicated by

$$f_1(v \,|\, h) = n(v,h)/\,n_2(h) \ \text{ and } \ f_2(h \,|\, v) = n(v,h)/\,n_1(v), \ \forall\ v \in D, \ \forall\ h \in A, \tag{2.7}$$

respectively. Let $Q_1(v \,|\, h) = \{i \mid x_i \leq v \text{ and } a_i = h\}$ represent the set of individuals, whose ability is equal to h and income no greater than v. Similarly we denote as $Q_2(h \,|\, v) = \{i \mid a_i \leq h \text{ and } x_i = v\}$ the set of individuals, whose income is equal to v and ability no greater than h. Then the *conditional distribution functions of income* and *ability* are defined by

$$F_1(v \,|\, h) = q_1(v \,|\, h)/\,n_2(h) \ \text{ and } \ F_2(h \,|\, v) = q_2(h \,|\, v)/\,n_1(v), \ \forall\ v \in D, \ \forall\ h \in A, \tag{2.8}$$

respectively, where $q_1(v \,|\, h) = \#Q_1(v \,|\, h)$ and $q_2(h \,|\, v) = \#Q_2(h \,|\, v)$.

Throughout this paper we are interested in the comparison of situations $\mathbf{s}^\circ \equiv (\mathbf{x}^\circ;\mathbf{a}^\circ), \mathbf{s}^* \equiv (\mathbf{x}^*;\mathbf{a}^*) \in \mathbb{S}_n$ $(n \geq 2)$. The associated joint, marginal and conditional density functions and distribution functions of $\mathbf{s}^*$ and $\mathbf{s}^\circ$ will be identified by means of the same superscripts. In practice it is not necessary to consider the range of all possible values for our two attributes: it is sufficient to make computations using the values of income and ability that actually occur in the situations under comparison. To this aim we define:

(2.9.a) $$M_1(\mathbf{s}) = \{v_j \in D \mid \exists\, i : x_i = v_j\} \subset D,$$

(2.9.b) $$M_2(\mathbf{s}) = \{h_j \in A \mid \exists\, i : a_i = h_j\} \subset A,$$

(2.9.c) $$M_1(\mathbf{s}^*, \mathbf{s}^\circ) = \{v_j \in D \mid \exists\, i : x_i^* = v_j \text{ or } x_i^\circ = v_j\} \subset D,$$

(2.9.d) $$M_2(\mathbf{s}^*, \mathbf{s}^\circ) = \{h_j \in A \mid \exists\, i : a_i^* = h_j \text{ or } a_i^\circ = h_j\} \subset A,$$

(2.9.e) $$m_1(\mathbf{s}) := \#M_1(\mathbf{s}),$$

(2.9.f) $$m_2(\mathbf{s}) := \#M_2(\mathbf{s}),$$

(2.9.g) $$m_1(\mathbf{s}^*, \mathbf{s}^\circ) := \#M_1(\mathbf{s}^*, \mathbf{s}^\circ),$$

(2.9.h) $$m_2(\mathbf{s}^*, \mathbf{s}^\circ) := \#M_2(\mathbf{s}^*, \mathbf{s}^\circ),$$

and we label the distinct elements in $M_1(\mathbf{s})$, $M_2(\mathbf{s})$, $M_1(\mathbf{s}^*, \mathbf{s}^\circ)$ and $M_2(\mathbf{s}^*, \mathbf{s}^\circ)$ so that $v_1 < v_2 < \cdots < v_{m_1(\mathbf{s})}$, $h_1 < h_2 < \cdots < h_{m_2(\mathbf{s})}$, $v_1 < v_2 < \cdots < v_{m_1(\mathbf{s}^*, \mathbf{s}^\circ)}$ and $h_1 < h_2 < \cdots < h_{m_2(\mathbf{s}^*, \mathbf{s}^\circ)}$, respectively.

Example 2.1. For the sake of illustration we introduce the four following situations where for graphical convenience we permit ability to take only two possible values

$$\mathbf{s}^1 = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 4 & 1 \\ 5 & 2 \\ 5 & 2 \end{bmatrix}, \ \mathbf{s}^2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 5 & 1 \\ 4 & 2 \\ 5 & 2 \end{bmatrix}, \ \mathbf{s}^3 = \begin{bmatrix} 3 & 1 \\ 1 & 2 \\ 5 & 1 \\ 4 & 2 \\ 4 & 2 \end{bmatrix} \ \text{ and } \ \mathbf{s}^4 = \begin{bmatrix} 4 & 1 \\ 1 & 2 \\ 4 & 1 \\ 3 & 2 \\ 5 & 2 \end{bmatrix}.$$

For later reference we note that all four situations have the same marginal distribution of ability.

On some occasion we will restrict attention to situations with the same mean income. In the particular case where the joint distribution function of one situation lies nowhere above the joint distribution function of the other situation, the equal mean income assumption implies that the situations under comparison have identical marginal distribution functions of income. This is precisely stated in the following lemma proved – as well as all formal results in this paper – in Gravel and Moyes (2006).

Lemma 2.1. Let $\mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n$ $(n \geq 2)$ and suppose that $\mu(\mathbf{x}^*) = \mu(\mathbf{x}^\circ)$. Then:

(2.10) $$\big[F^*(v,h) \leq F^\circ(v,h), \ \forall\, v \in D, \ \forall\, h \in A\big] \Longrightarrow \big[F_1^*(v) = F_1^\circ(v), \ \forall\, v \in D\big].$$

Before we examine the normative criteria which we will rely on for passing welfare judgements, we would like to insist on the implications of our informational constraints for these assessments. Let the situations $\mathbf{s}^\circ \equiv (\mathbf{x}^\circ; \mathbf{a}^\circ)$, $\mathbf{s}^* \equiv (\mathbf{x}^*; \mathbf{a}^*)$, $\tilde{\mathbf{s}}^\circ \equiv (\tilde{\mathbf{x}}^\circ; \tilde{\mathbf{a}}^\circ)$ and $\tilde{\mathbf{s}}^* \equiv (\tilde{\mathbf{x}}^*; \tilde{\mathbf{a}}^*)$ be such that

$$\tilde{x}_i^* = \alpha + \beta x_i^*, \ \ \tilde{x}_i^\circ = \alpha + \beta x_i^\circ, \ \ \tilde{a}_i^* = \phi(a_i^*) \ \text{and} \ \tilde{a}_i^\circ = \phi(a_i^\circ), \tag{2.11}$$

where $\alpha \in \mathbb{R}$, $\beta > 0$ and ϕ is increasing. Assuming that income is cardinally measurable and ability ordinally measurable amounts to considering that the pairs $\{\mathbf{s}^*, \mathbf{s}^\circ\}$ and $\{\tilde{\mathbf{s}}^*, \tilde{\mathbf{s}}^\circ\}$ convey the same information. Then all the normative criteria $\geq_J$ we will consider must have the property that

$$\forall\ \mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n : \mathbf{s}^* \geq_J \mathbf{s}^\circ \Longleftrightarrow \tilde{\mathbf{s}}^* \geq_J \tilde{\mathbf{s}}^\circ. \tag{2.12}$$

In other words the normative criteria $\geq_J$ are invariant with respect to particular modifications of the measurement scales of the attributes. This informational constraint is not innocuous: for instance, the well-known relative Lorenz quasi-ordering, which is widely used for making inequality comparisons in the single attribute case, is not invariant with respect to affine transformations of the variable.

3. The Welfare Criteria

Following the usual practice in the dominance approach, we assume (i) that all individuals transform their endowments of two attributes into well-being by means of the same utility function, and (ii) that the distribution of the individual utilities in a situation provides all the relevant information for appraising this situation from a normative point of view. It is convenient to think of an ethical observer who is in charge of evaluating the different situations under comparison on the basis of the distributions of utilities they generate. There are at least two interpretations that can be given to this way of proceeding. The first one is the *welfarist* approach according to which the utility function is the one actually used by the individuals in order to convert their endowments of the attributes into well-being.[8] It is assumed that well-being is a cardinally measurable and interpersonnally comparable variable that summarizes all the aspects of an individual's situation that are deemed relevant for normative evaluation. The other interpretation – the *non-welfarist* approach – is to view the utility function as reflecting the assessment of the individual's situation by the ethical observer. There is no presumption in the latter approach that utility is connected to the individual's actual well-being and one must rather interpret the utility function as a predefined *social norm*. While the dominance approach is compatible with both interpretations, the *symmetry* assumption according to which all individuals have the same utility function may look rather specific. Indeed, does a welfarist claim that all individuals convert their endowments of the attributes into well-being by using the same utility function? Or, in the non-welfarist model, does it

[8] We refer the reader to Blackorby, Bossert and Donaldson (2005) and Griffin (1986) for discussions of the welfarist approach in economics and philosophy, respectively.

mean that a bundle of attributes assigned to distinct individuals should be given the same evaluation by the social planner? The symmetry assumption is rather easy to defend in the non-welfarist context, provided that all the relevant individual attributes have been included. If an individual's situation can be comprehensively described, for the purpose of normative evaluation, by a bundle of attributes, then why should two individuals with the same bundle of attributes be treated differently? The Laplacian principle of *insufficient reason* would recommend that they be treated symmetrically in absence of a convincing reason for not doing so. The symmetry assumption is admittedly more difficult to justify in a welfarist setting since there is no reason a priori to assume that two individuals with the same bundle of attributes will achieve the same level of well-being. Yet, it is possible to defend symmetry by arguing, again, that since individuals are assumed to differ only by their endowment of the attributes it is natural to suppose that they convert the two attributes into well-being by means of the same utility function. For, if they were different in their ability to convert attributes into well-being, then the variables underlying these differences should be considered themselves as attributes and included in the list of arguments of the *fundamental utility function*, which is by definition the same for all individuals.[9] A similar idea has been applied by Kolm (1972) to preference orderings rather than to cardinally meaningful and interpersonally comparable utility functions. While this argument is not convincing when applied to preferences that are not assumed to be interpersonally comparable as shown by Broome (1993), we believe it makes sense when applied to cardinally measurable and interpersonnally comparable utility functions. These justifications of the symmetrical treatment of the individuals, in either the welfarist or non-welfarist interpretation, necessitate that the list of attributes that describe an individual's situation is sufficiently comprehensive. Admittedly, it is very unlikely that this requirement will be met in the two attributes case considered herein.

The utility achieved by individual i in situation $\mathbf{s} \equiv (\mathbf{x}; \mathbf{a})$ as envisaged by the ethical observer – be it welfarist or not – is indicated by $U(\mathbf{s}_i) = U(x_i, a_i)$. To simplify the exposition we assume throughout that the *utility function* $U : D \times A \to \mathbb{R}$ is twice differentiable in income and we denote as $\mathbb{U}$ the set of such functions.[10] We use $U(\mathbf{s}) \equiv U(\mathbf{x}; \mathbf{a}) := (U(x_1, a_1), \ldots, U(x_n, a_n))$ to indicate the *distribution of utility* generated by the situation $\mathbf{s} \equiv (\mathbf{x}; \mathbf{a}) \in \mathbb{S}_n$ when the utility function is $U \in \mathbb{U}$. The *utilitarian rule* ranks the situations under comparison on the basis of the sum of the utilities they generate. More precisely from the point of view of a utilitarian ethical observer endowed with the utility function $U \in \mathbb{U}$ the social welfare in situation $\mathbf{s} \equiv (\mathbf{x}; \mathbf{a}) \in \mathbb{S}_n$ is equal to $\sum_{i=1}^{n} U(x_i, a_i)$, and the situation

[9] A way of justifying further this argument would be to appeal to Howe's (1987) theorem according to which each member of a collection of n distinct concave and monotonic functions of m variables can be expressed as a projection of one concave and monotonic function of $n + m$ variables. A difficulty with this justification here is that the utility functions we consider need not be concave as we will see later on.

[10] The differentiability assumption is not restrictive since it is always possible in our framework to approximate the discontinuous functions we might face in the proofs by suitable continuous and differentiable functions (see Fishburn and Vickson (1978)).

$\mathbf{s}^*$ is considered to be no worse than situation $\mathbf{s}^\circ$, if and only if

$$\sum_{i=1}^{n} U\left(x_i^*, a_i^*\right) \geq \sum_{i=1}^{n} U\left(x_i^\circ, a_i^\circ\right). \tag{3.1}$$

The utility function U captures the utilitarian ethical observer's normative judgement and it is the only parameter by which such ethical observers can be distinguished. In order to rule out as much arbitrariness as possible we require all utilitarian ethical observers whose utility functions U belong to a given class $\mathbb{U}^* \subset \mathbb{U}$ to agree on the ranking of the situations under comparison.

Utilitarian Unanimity Rule. We will say that situation $\mathbf{s}^*$ is no worse than situation $\mathbf{s}^\circ$ for the utilitarian unanimity rule over the class $\mathbb{U}^* \subset \mathbb{U}$, if and only if

$$\sum_{i=1}^{n} U\left(x_i^*, a_i^*\right) \geq \sum_{i=1}^{n} U\left(x_i^\circ, a_i^\circ\right), \ \forall\, U \in \mathbb{U}^*. \tag{3.2}$$

The following obvious result, which originates in the additive separability of the utilitarian social welfare function, expresses the independence of the utilitarian rule with respect to the unconcerned individuals.

Lemma 3.1. Let $U \in \mathbb{U}$ and consider arbitrary situations $\mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n$ $(n \geq 2)$, $\mathbf{s} \in \mathbb{S}_q$, where $q \geq 1$. Then the following two statements are equivalent:

(a) $\sum_{i=1}^{n} U\left(x_i^*, a_i^*\right) + \sum_{i=1}^{q} U\left(x_i, a_i\right) \geq \sum_{i=1}^{n} U\left(x_i^\circ, a_i^\circ\right) + \sum_{i=1}^{q} U\left(x_i, a_i\right)$.

(b) $\sum_{i=1}^{n} U\left(x_i^*, a_i^*\right) \geq \sum_{i=1}^{n} U\left(x_i^\circ, a_i^\circ\right)$.

The utilitarian unanimity rule allows us to encompass different value judgements whose spectrum is fully captured by the subset $\mathbb{U}^*$ of admissible utility functions. Much of the discussion in this paper will address the question of what constitutes the relevant set of utility functions over which one is looking for unanimity.

However the utilitarian rule is just one among many other possible principles that can be used for aggregating individual utility levels. It has been criticized on the grounds that it pays no attention to the way utilities are distributed among the individuals. One might prefer to appeal to principles that express some aversion to the inequality in the distribution of individual utilities such as the Maximin or the Leximin. More precisely we are interested in those social welfare functionals that incorporate a concern for both efficiency and equity considerations. Formally the social welfare in situation $\mathbf{s} \in \mathbb{S}$ is given by $G(U(\mathbf{x};\mathbf{a})) := G\left(U\left(x_1, a_1\right), \ldots, U\left(x_n, a_n\right)\right)$, where $G : \mathbb{R}^n \longrightarrow \mathbb{R}$ is the social welfare functional. We will say that situation $\mathbf{s}^*$ is no worse than situation $\mathbf{s}^\circ$ for an ethical observer endowed with the utility function $U \in \mathbb{U}$ and the social welfare functional G, if and only if

$$G\left(U\left(\mathbf{x}^*; \mathbf{a}^*\right)\right) \geq G\left(U\left(\mathbf{x}^\circ; \mathbf{a}^\circ\right)\right). \tag{3.3}$$

The aversion to inequality is typically captured by the property of Schur-concavity and we indicate by $\mathbb{G}_{SC}$ the set of social welfare functionals that are monotone and Schur-concave (see Kolm (1969), Dasgupta, Sen and Starrett (1973)).[11] A particularly cautious – but on the other hand definitively uncontroversial – position consists in declaring that social welfare improves if and only if condition (3.3) holds whatever the social welfare functional and the utility function provided the former expresses a concern for efficiency and equity considerations.

Welfarist Unanimity Rule. We will say that situation $\mathbf{s}^*$ is no worse than situation $\mathbf{s}^\circ$ for the welfarist unanimity rule over the class of utility functions $\mathbb{U}^* \subset \mathbb{U}$ and the class of social welfare functionals $\mathbb{G}^* \subset \mathbb{G}$, if and only if

$$G\left(U\left(\mathbf{x}^*;\mathbf{a}^*\right)\right) \geq G\left(U\left(\mathbf{x}^\circ;\mathbf{a}^\circ\right)\right),\ \forall\, U \in \mathbb{U}^*,\ \forall\, G \in \mathbb{G}^*. \tag{3.4}$$

Clearly the welfarist unanimity rule implies the utilitarian unanimity rule whatever the class of utility functions. Actually since the converse statement does not generally hold it is more demanding to require unanimous agreement among all welfarist ethical observers than among all utilitarian ones. Interestingly is the fact that the utilitarian and welfarist unanimity rules coincide over a particular subset of the utility functions. Before we proceed to a precise statement of this result we first need to introduce some additional notation and definition. Let $\mathbf{u}^* := (u_1^*, \ldots, u_n^*)$ and $\mathbf{u}^\circ := (u_1^\circ, \ldots, u_n^\circ)$ be two distributions of individual utilities. Denote respectively as $\mathbf{u}^*_{(\ )}$ and $\mathbf{u}^\circ_{(\ )}$ the non-decreasing rearrangements of $\mathbf{u}^*$ and $\mathbf{u}^\circ$ defined by $u^*_{(1)} \leq u^*_{(2)} \leq \cdots u^*_{(n)}$ and $u^\circ_{(1)} \leq u^\circ_{(2)} \leq \cdots u^\circ_{(n)}$. Then we will say that $\mathbf{u}^*$ *generalised Lorenz dominates* $\mathbf{u}^\circ$, which we write as $\mathbf{u}^* \geq_{GL} \mathbf{u}^\circ$, if and only if

$$\frac{1}{n}\sum_{i=1}^{k} u^*_{(i)} \geq \frac{1}{n}\sum_{i=1}^{k} u^\circ_{(i)},\ \forall\, k = 1, 2, \ldots, n \tag{3.5}$$

(see Kolm (1969) and Shorrocks (1983)). An interesting result for our purpose is the fact that the rankings of the distributions of utilities by the welfarist unanimity rule and the generalised Lorenz quasi-ordering prove to be identical under certain conditions.

Lemma 3.2 (see e.g. Marshall and Olkin (1979)). Let $\mathbf{u}^*, \mathbf{u}^\circ \in \mathbb{R}^n$. Then the following two statements are equivalent:

(a) $\mathbf{u}^* \geq_{GL} \mathbf{u}^\circ$.

(b) $G\left(\mathbf{u}^*\right) \geq G\left(\mathbf{u}^\circ\right)$, for all $G \in \mathbb{G}_{SC}$.

Given the above result it is now possible to identify the restrictions to be imposed on the class of utility functions that guarantee that the rankings of situations implied by the utilitarian and welfarist unanimities are identical.

[11] The mapping $G : \mathbb{R}^n \longrightarrow \mathbb{R}$ is *Schur-concave* if $G(B\mathbf{u}) \geq G(\mathbf{u})$, for all $\mathbf{u} = (u_1, \ldots, u_n)$ and all bistochastic matrices B (see Marshall and Olkin (1979)). It is *monotone* if $G(\mathbf{u}) \geq G(\mathbf{v})$, for all $\mathbf{u} = (u_1, \ldots, u_n)$ and $\mathbf{v} = (v_1, \ldots, v_n)$ such that $u_i \geq v_i$, for all $i = 1, 2, \ldots, n$.

Proposition 3.1. Let $\mathbb{U}^* \subseteq \mathbb{U}$ and $\mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n$ $(n \geq 2)$. Then a sufficient condition for (3.2) and (3.4) to be equivalent is that

$$\forall\, U \in \mathbb{U}^* : [\Psi : \mathbb{R} \longrightarrow \mathbb{R} \text{ non-decreasing and concave }] \Longrightarrow \Psi \circ U \in \mathbb{U}^*. \tag{3.6}$$

The result above indicates that in order to obtain an equivalence between utilitarian and welfarist unanimities, it is sufficient that the class of utility functions under consideration be closed under the operation of applying monotonic and concave – but still identical for each individual – transformations. Since the classes of utility functions we will consider in the next two sections possess this property, Proposition 3.1 will make things simpler by allowing us to focus on the utilitarian unanimity rule without loss of generality.

4. Bidimensional Headcount Poverty Dominance

As we emphasized above, the influential contribution of Atkinson and Bourguignon (1982) is silent about the elementary equalizing transformations of situations that result in welfare improvements in terms of the dominance criteria they introduced. Because we are primarily interested in the way the attributes are distributed among the individuals, we will restrict our attention in a first stage to the comparisons of situations whose marginal distribution functions of income and ability are identical. We will say that situation $\mathbf{s}^*$ is obtained from situation $\mathbf{s}^\circ$ by means of an income *favorable permutation* – or equivalently that $\mathbf{s}^\circ$ is obtained from situation $\mathbf{s}^*$ by means of an income *defavorable permutation* – if there exist two individuals i and j such that:

$$x_j^* = x_i^\circ < x_j^\circ = x_i^*;\ a_i^* = a_i^\circ < a_j^\circ = a_j^*; \text{ and} \tag{4.1.a}$$

$$s_g^* = s_g^\circ,\ \forall\, g \neq i, j. \tag{4.1.b}$$

A favorable permutation consists in exchanging the cardinal attribute endowment of the better-off individual in both attributes with that of the worse-off individual in both attributes. Such a transformation involving two individuals i and j whose initial situations are respectively $\mathbf{s}_i^\circ = (x_i^\circ, a_i^\circ) = (u, h)$ and $\mathbf{s}_j^\circ = \left(x_j^\circ, a_j^\circ\right) = (v, k)$ with $u < v$ and $h < k$ is represented in Figure 4.1. The permutation of the endowments in the cardinal attribute actually reduces the correlation – or equivalently the positive association – existing between the two attributes. The resulting situation may be considered less unequal than the original situation in the sense that

$$U\left(x_i^\circ, a_i^\circ\right) < U\left(x_i^*, a_i^*\right) \leq U\left(x_j^*, a_j^*\right) < U\left(x_j^\circ, a_j^\circ\right), \text{ or} \tag{4.2.a}$$

$$U\left(x_i^\circ, a_i^\circ\right) < U\left(x_j^*, a_j^*\right) < U\left(x_i^*, a_i^*\right) < U\left(x_j^\circ, a_j^\circ\right), \tag{4.2.b}$$

for all monotone increasing utility functions U. On the other hand a favorable permutation has no impact on the distribution of each attribute in the population: the marginal distribution functions of income and ability are left unchanged and so is mean income. The next

Figure 4.1. A Favorable Permutation

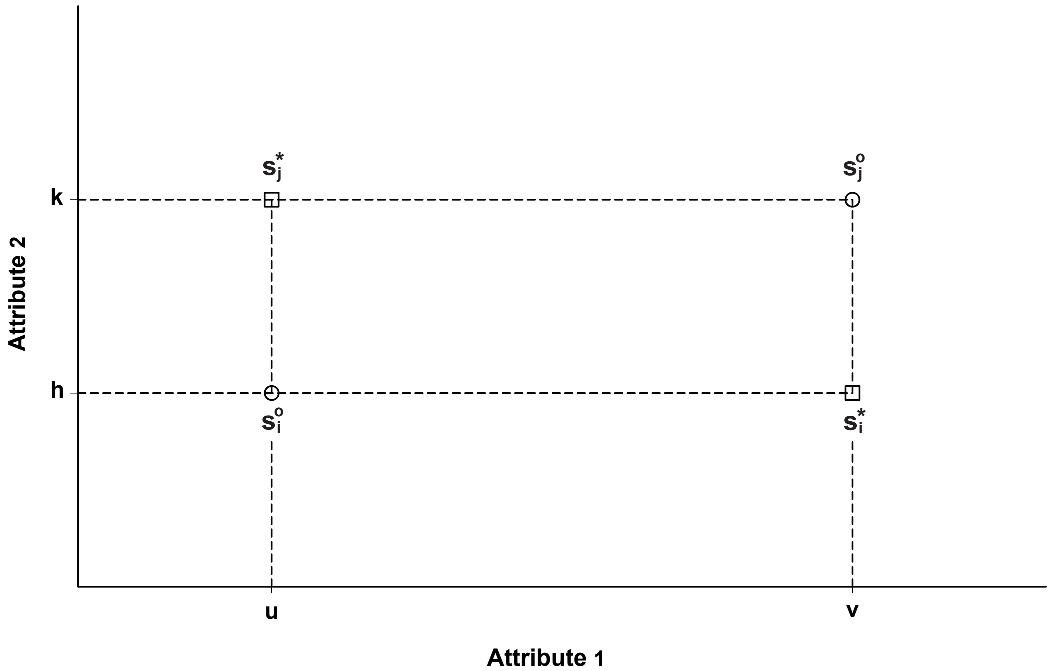
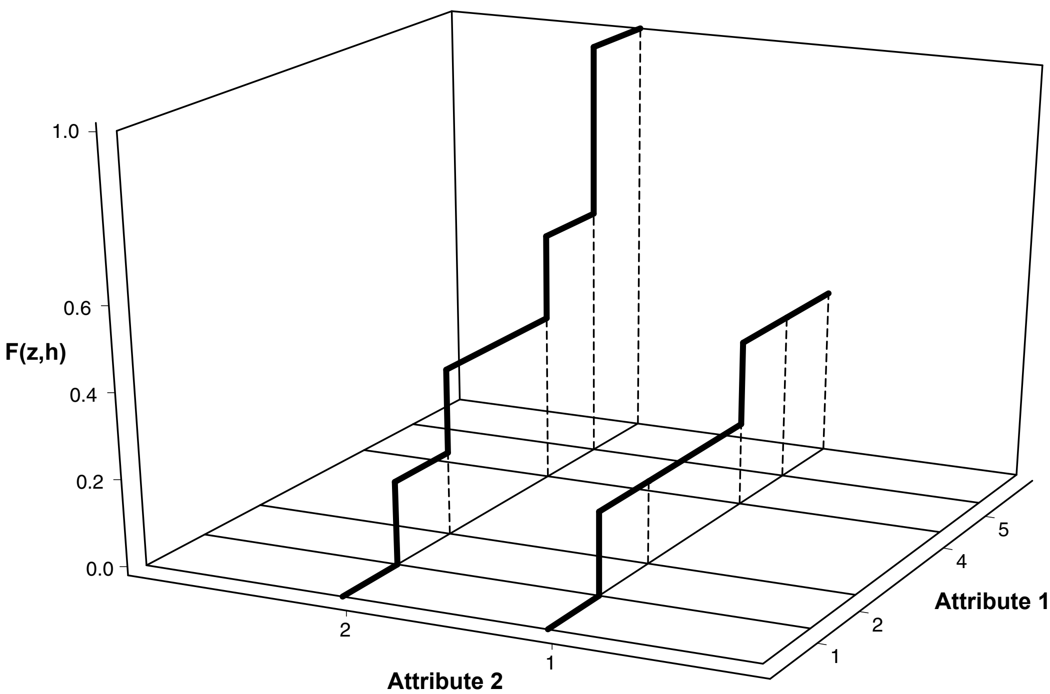


Figure 4.2. Joint Cumulative Distribution Function



result, the proof of which is obvious, identifies the conditions to be imposed on the utility function for social welfare as measured by the utilitarian rule to improve as the result of a favorable permutation.

LEMMA 4.1. For all $\mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n$ $(n \geq 2)$: $\sum_{i=1}^n U(x_i^*, a_i^*) \geq \sum_{i=1}^n U(x_i^\circ, a_i^\circ)$ whenever $\mathbf{s}^*$ is obtained from $\mathbf{s}^\circ$ by means of a favorable permutation, if and only if

$$U_v(v,h) \geq U_v(v,k),\ \forall\ v \in D,\ \forall\ h,k \in A\ (h < k), \tag{C1}$$

where $U_v(v,h)$ indicates the first derivative of $U(v,h)$ with respect to income.

According to Lemma 4.1 a non-increasing marginal utility of income is necessary and sufficient for a utilitarian ethical observer to consider that a favorable permutation results in a weak welfare improvement. We denote as

$$\mathbb{U}_1 := \{U \in \mathbb{U} \mid U \text{ satisfies (C1)}\} \tag{4.3}$$

the class of utility functions with non-increasing marginal utility of income. Atkinson and Bourguignon (1982) have shown that the ranking of situations by the utilitarian unanimity rule over the class $\mathbb{U}_1$ is actually implied by the ranking obtained by comparing the graphs of their joint cumulative distribution functions. Precisely, we will say that situation $\mathbf{s}^*$ *first order stochastic dominates* situation $\mathbf{s}^\circ$, which we write as $\mathbf{s}^* \geq_{FSD} \mathbf{s}^\circ$, if and only if:

$$F^*(v,h) \leq F^\circ(v,h),\ \forall\ v \in D,\ \forall\ h \in A. \tag{4.4}$$

Using (2.4), (2.7) and (2.8) and upon substitution, condition (4.4) can be equivalently rewritten as

$$\sum_{g=1}^{h} f_2^*(g)\, H^*(z \mid g) \leq \sum_{g=1}^{h} f_2^\circ(g)\, H^\circ(z \mid g),\ \forall\ z \in D,\ \forall\ h \in M_2(\mathbf{s}^*, \mathbf{s}^\circ), \tag{4.5}$$

where

$$H(z \mid h) := \frac{q_1(z \mid g)}{n_2(g)} \tag{4.6}$$

is the *headcount poverty* of the subpopulation of individuals having ability h in situation $\mathbf{s}$. We have represented in Figure 4.2 the joint cumulative distribution function – or equivalently the headcount poverty curves – of situation $\mathbf{s}^1$ in Example 2.1. Condition (4.5) expresses the fact that there is less poverty in situation $\mathbf{s}^*$ than in situation $\mathbf{s}^\circ$ for all possible values of the bidimensional poverty line (z,h), where poverty is measured by the percentage of individuals whose income and ability fall strictly below z and h, respectively. More precisely, we will say that situation $\mathbf{s}^*$ *headcount poverty dominates* situation $\mathbf{s}^\circ$, which we write as $\mathbf{s}^* \geq_{HP} \mathbf{s}^\circ$, when condition (4.5) – equivalently condition (4.4) – holds.

Lemma 4.1, in conjunction with Atkinson and Bourguignon's (1982) result, suggests that favorable permutations might be the elementary transformations that lie behind first order stochastic – equivalently headcount poverty – dominance. The following result shows that this intuition is correct under certain qualifications.

Lemma 4.2. Let $\mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n$ $(n \geq 2)$ such that $\mathbf{a}^* = \mathbf{a}^\circ$ and $\mu(\mathbf{x}^*) = \mu(\mathbf{x}^\circ)$. Then:

(a) $F^*(v,h) \leq F^\circ(v,h)$, for all $v \in D$ $\left(v \neq v_{m_1(\mathbf{s}^*,\mathbf{s}^\circ)}\right)$ and all $h \in A$ $\left(h \neq h_{m_2(\mathbf{s}^*,\mathbf{s}^\circ)}\right)$

implies that

(b) $\mathbf{s}^*$ is obtained from $\mathbf{s}^\circ$ by means of a finite sequence of favorable permutations of income.

Making use of Proposition 3.1 and Lemmas 4.1 and 4.2, we obtain the following result, which constitutes the analogue to the Hardy-Littlewood-Polya result for first order stochastic – equivalently headcount poverty – dominance.

Theorem 4.1. Let $\mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n$ $(n \geq 2)$ such that $\mathbf{a}^* = \mathbf{a}^\circ$ and $\mu(\mathbf{x}^*) = \mu(\mathbf{x}^\circ)$. Then statements (a), (b), (c) and (d) below are equivalent:

(a) $\mathbf{s}^*$ is obtained from s° by means of a finite sequence of individuals' permutations and/or favorable permutations.

(b) $\sum_{i=1}^n U(x_i^*, a_i^*) \geq \sum_{i=1}^n U(x_i^\circ, a_i^\circ)$, for all $U \in \mathbb{U}_1$.

(c) $U(\mathbf{x}^*; \mathbf{a}^*) \geq_{GL} U(\mathbf{x}^\circ; \mathbf{a}^\circ)$, for all $U \in \mathbb{U}_1$.

(d-1) $F^*(v,h) \leq F^\circ(v,h)$, for all $v \in D$ $\left(v \neq v_{m_1(\mathbf{s}^*,\mathbf{s}^\circ)}\right)$ and all $h \in M_2(\mathbf{s}^*, \mathbf{s}^\circ)$ $\left(h \neq h_{m_2(\mathbf{s}^*,\mathbf{s}^\circ)}\right)$;

(d-2) $F_1^*(v) = F_1^\circ(v)$, for all $v \in D$;

(d-3) $F_2^*(h) = F_2^\circ(h)$, for all $h \in M_2(\mathbf{s}^*, \mathbf{s}^\circ)$.

Sketch proof. The fact that statement (a) implies statement (b) follows from Lemma 4.1. Proposition 3.1 and the fact that the class of utility functions $\mathbb{U}_1$ is closed under composition by a non-decreasing and concave function guarantee the equivalence between statements (b) and (c). The fact that (d) is necessary for (b) to hold can be easily shown by suitably choosing degenerate utility functions in the class $\mathbb{U}_1$. Finally Lemma 4.2 establishes that statement (d) implies statement (a). □

Theorem 4.1 confirms the intuition that what we call favorable permutations are the elementary transformations that lie behind the bidimensional headcount poverty criterion. It is fair to note that Epstein and Tanny (1980) have proven a similar result but using a different argument. This, in conjunction with the fact that headcount poverty dominance is a necessary condition for a situation to be ranked above another one by all the utilitarian ethical observers

Figure 4.3.A. Joint Cumulative Distribution Functions of s^1 and s^2

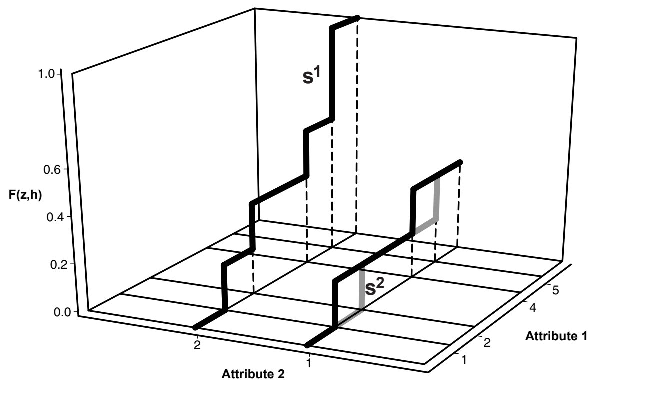


Figure 4.3.B. Joint Cumulative Distribution Functions of s^1 and s^3

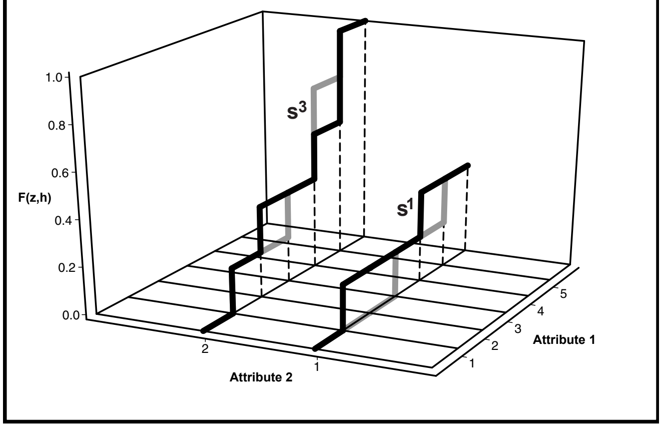


Figure 4.3.C. Joint Cumulative Distribution Functions of s^1 and s^4

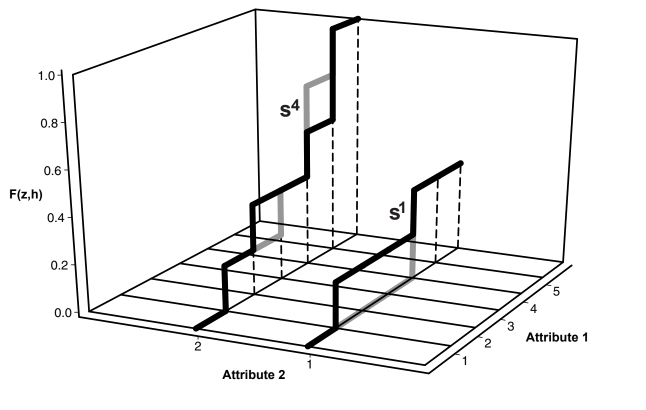


Figure 4.3.D. Joint Cumulative Distribution Functions of s^2 and s^3

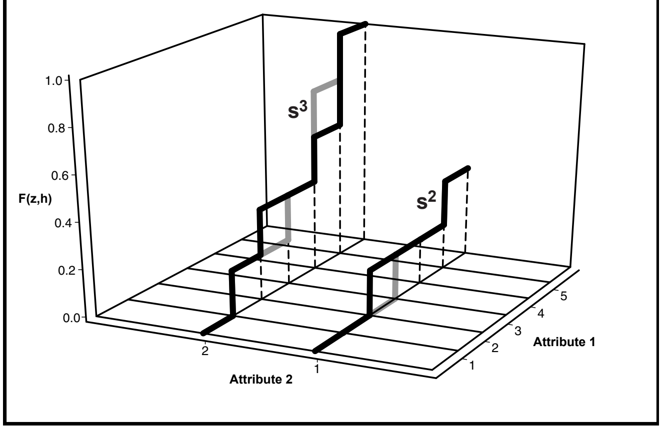


Figure 4.3.E. Joint Cumulative Distribution Functions of s^2 and s^4

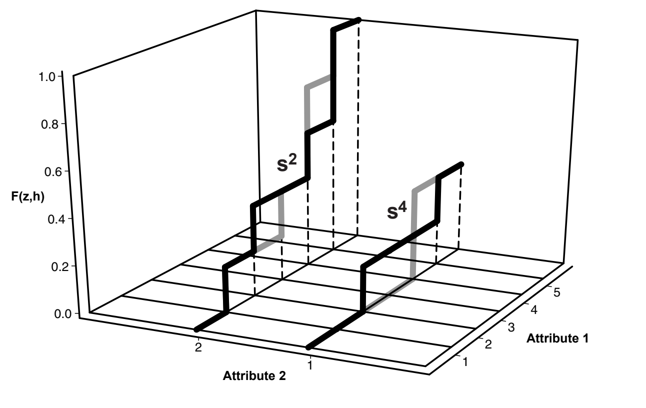
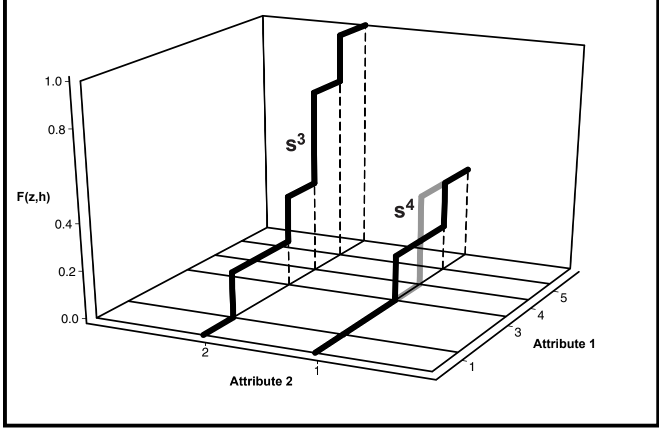


Figure 4.3.F. Joint Cumulative Distribution Functions of s^3 and s^4



whose marginal utility of income is non-increasing with ability, fills the gaps in Atkinson and Bourguignon's (1987) Proposition 2. A byproduct of Theorem 4.1 is the recognition that the aforementioned result of Atkinson and Bourguignon (1987) goes far beyond utilitarianism and actually holds for all monotone social welfare functionals exhibiting inequality aversion towards the distribution of individual utilities. We have represented in Figure 4.3 the joint cumulative distribution functions of the situations of Example 2.1. Application of the head-count poverty criterion indicates that $\mathbf{s}^2 \geq_{HP} \mathbf{s}^1$, which does not come as a surprise since situation $\mathbf{s}^2$ is derived from situation $\mathbf{s}^1$ by means of two favorable permutations. These are actually the only situations that can be ranked by the headcount poverty criterion, which means that for no other pair of situations it is possible to find a general agreement among the welfarist ethical observers whose marginal utility of income is non-increasing in ability.

So far we have restricted our attention to the case where the situations under comparison have identical marginal distributions of income and ability. While this allows us to focus on equity considerations, the practical relevance of the results we have obtained is limited. Theorem 4.1 can be easily extended to the general case where the marginal distributions are no longer fixed provided one is willing to introduce further restrictions on the utility functions. We will say that situation $\mathbf{s}^*$ is obtained from situation $\mathbf{s}^\circ$ by means of an *income increment* if there exists one individual i such that:

(4.7.a) $$x_i^* > x_i^\circ;\ a_i^* = a_i^\circ;\ \text{and}$$

(4.7.b) $$s_g^* = s_g^\circ,\ \forall\ g \neq i.$$

We will equally say that situation $\mathbf{s}^\circ$ is obtained from situation $\mathbf{s}^*$ by means of an *income decrement*. Contrary to a favorable permutation, which leaves the marginal distribution of income unchanged, an income increment moves the graph of the marginal distribution function of income downwards. Although it is immediately obvious, we find it convenient for later purposes to state the following result.

LEMMA 4.3. For all $\mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n$ $(n \geq 2)$: $\sum_{i=1}^n U(x_i^*, a_i^*) \geq \sum_{i=1}^n U(x_i^\circ, a_i^\circ)$ whenever $\mathbf{s}^*$ is obtained from $\mathbf{s}^\circ$ by means of an income increment, if and only if

(C2) $$U_v(v, h) \geq 0,\ \forall\ v \in D,\ \forall\ h \in A.$$

Similarly we will say that situation $\mathbf{s}^*$ is obtained from situation $\mathbf{s}^\circ$ by means of an *ability increment* if there exists one individual i such that:

(4.8.a) $$x_i^* = x_i^\circ;\ a_i^* > a_i^\circ;\ \text{and}$$

(4.8.b) $$s_g^* = s_g^\circ,\ \forall\ g \neq i.$$

Clearly the graph of the marginal distribution of ability will go down as the result of an ability increment and for later use we state the following:

Lemma 4.4. For all $\mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n$ $(n \geq 2)$: $\sum_{i=1}^n U(x_i^*, a_i^*) \geq \sum_{i=1}^n U(x_i^\circ, a_i^\circ)$ whenever $\mathbf{s}^*$ is obtained from $\mathbf{s}^\circ$ by means of an ability increment, if and only if

$$\text{(C3)} \qquad U(v,h) \leq U(v,k), \ \forall\, v \in D, \ \forall\, h,k \in A (h < k).$$

According to Lemmas 4.3 and 4.4, the monotonicity in both arguments of the utility function is necessary and sufficient for a utilitarian ethical observer to record income and ability increments as welfare-improving. We use

$$\text{(4.9)} \qquad \mathbb{U}_1^* := \{U \in \mathbb{U} \mid U \text{ satisfies (C1), (C2) and (C3)}\}$$

to indicate the class of monotone utility functions whose marginal utility of income is non-increasing in ability, and we obtain:

Theorem 4.2. Let $\mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n$ $(n \geq 2)$. Then statements (a), (b), (c) and (d) below are equivalent:

(a) $\mathbf{s}^*$ is obtained from s° by means of a finite sequence of individuals' permutations, income increments, ability increments and/or favorable permutations.

(b) $\sum_{i=1}^n U(x_i^*, a_i^*) \geq \sum_{i=1}^n U(x_i^\circ, a_i^\circ)$, for all $U \in \mathbb{U}_1^*$.

(c) $U(\mathbf{x}^*; \mathbf{a}^*) \geq_{GL} U(\mathbf{x}^\circ; \mathbf{a}^\circ)$, for all $U \in \mathbb{U}_1^*$.

(d) $F^*(v,h) \leq F^\circ(v,h)$, for all $v \in D$ and all $h \in M_2(\mathbf{s}^*, \mathbf{s}^\circ)$.

5. Bidimensional Ordered Poverty Gap Dominance

Here again we proceed in two steps and we first examine the case where the situations to be compared have identical marginal distributions of income and ability. We will say that situation $\mathbf{s}^*$ is obtained from situation $\mathbf{s}^\circ$ by means of a *within-type progressive transfer* of income if there exist two individuals i and j such that:

$$\text{(5.1.a)} \qquad x_i^\circ < x_i^* \leq x_j^* < x_j^\circ; \ a_i^\circ = a_i^* = a_j^* = a_j^\circ;$$

$$\text{(5.1.b)} \qquad x_i^* - x_i^\circ = x_j^\circ - x_j^*; \ \text{and}$$

$$\text{(5.1.c)} \qquad s_g^* = s_g^\circ, \ \forall\, g \neq i,j.$$

The marginal distribution function of ability is left unchanged by a within-type progressive transfer while the distribution of income conditional on ability is made more equal. One can easily identify the property of the utility function that guarantees that social welfare as measured by the utilitarian rule improves as the result of a within-type progressive transfer.

Lemma 5.1. For all $\mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n$ $(n \geq 2)$: $\sum_{i=1}^n U(x_i^*, a_i^*) \geq \sum_{i=1}^n U(x_i^\circ, a_i^\circ)$ whenever $\mathbf{s}^*$ is obtained from $\mathbf{s}^\circ$ by means of a within-type progressive transfer, if and only if

$$\text{(C4)} \qquad U_{vv}(v,h) \leq 0, \ \forall\, v \in D, \ \forall\, h \in A,$$

where $U_{vv}(v,h)$ indicates the second derivative of $U(v,h)$ with respect to income.

The multidimensional nature of the situations under comparison plays actually no role in the notion of a within-type progressive transfer, which is merely a restatement of the usual Pigou-Dalton transfer in our framework. The next transformation, which fully exploits the bidimensionality of a situation, constitutes in our model a very natural generalisation of a unidimensional progressive transfer. We will say that situation $\mathbf{s}^*$ is obtained from situation $\mathbf{s}^\circ$ by means of a *between-type progressive transfer* of income if there exist two individuals i and j such that:

$$\text{(5.2.a)} \qquad x_i^\circ < x_i^* \leq x_j^* < x_j^\circ; \ a_i^\circ = a_i^* < a_j^* = a_j^\circ;$$

$$\text{(5.2.b)} \qquad x_i^* - x_i^\circ = x_j^\circ - x_j^*; \text{ and}$$

$$\text{(5.2.c)} \qquad s_g^* = s_g^\circ, \ \forall\, g \neq i, j.$$

A between-type progressive transfer resembles a Pigou-Dalton transfer but there is a major difference: the beneficiary of the transfer must be poorer than the donor and she must also have a lower ability. Put differently, the transfer recipient must be deprived in both dimensions – income and ability – compared to the donor. This is illustrated in Figure 5.1 where individual i receives an additional income of $v - u$ while individual j gives away an amount of income equal to $t - u$, where $u < v < w < t$, $v - u = t - w$ and $h < k$. Both within-type and between-type progressive transfers leave the marginal distribution of ability unchanged. But contrary to a within-type progressive transfer, which has an unambiguous effect on the conditional distribution of income, here the graph of the conditional distribution function of income of the ability group to which the receiver belongs moves downwards, while the opposite situation arises for the ability group of the donor. The overall effect on the joint distribution function of income and ability is ambiguous and the bidimensional headcount poverty criterion does not allow one to sign the overall welfare impact of a between-type progressive transfer. Somewhat surprisingly, no condition in addition to the ones we have considered up to now need to be imposed on the utility function for a between-type progressive transfer to imply in a welfare improvement.

Lemma 5.2. For all $\mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n$ $(n \geq 2)$: $\sum_{i=1}^n U(x_i^*, a_i^*) \geq \sum_{i=1}^n U(x_i^\circ, a_i^\circ)$ whenever $\mathbf{s}^*$ is obtained from $\mathbf{s}^\circ$ by means of a between-type progressive transfer, if conditions (C1) and (C4) are satisfied.

According to Lemma 5.2 it is sufficient for welfare as conceived by a utilitarian ethical observer to increase as the result of a between-type progressive transfer that the marginal utility of

Figure 5.1. A Between-Type Progressive Transfer

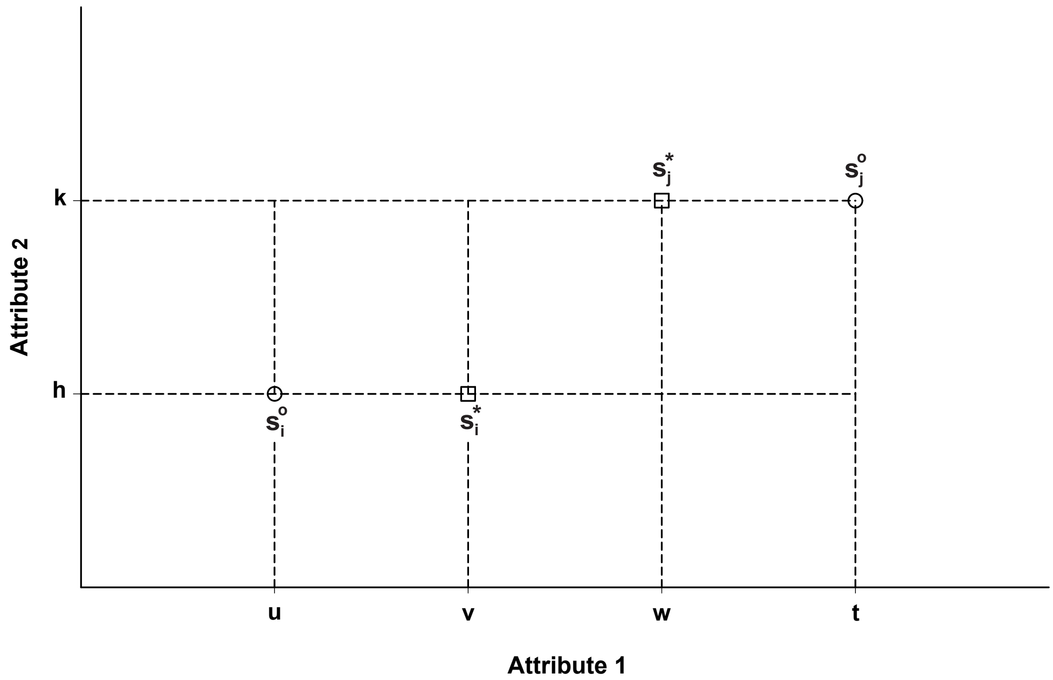
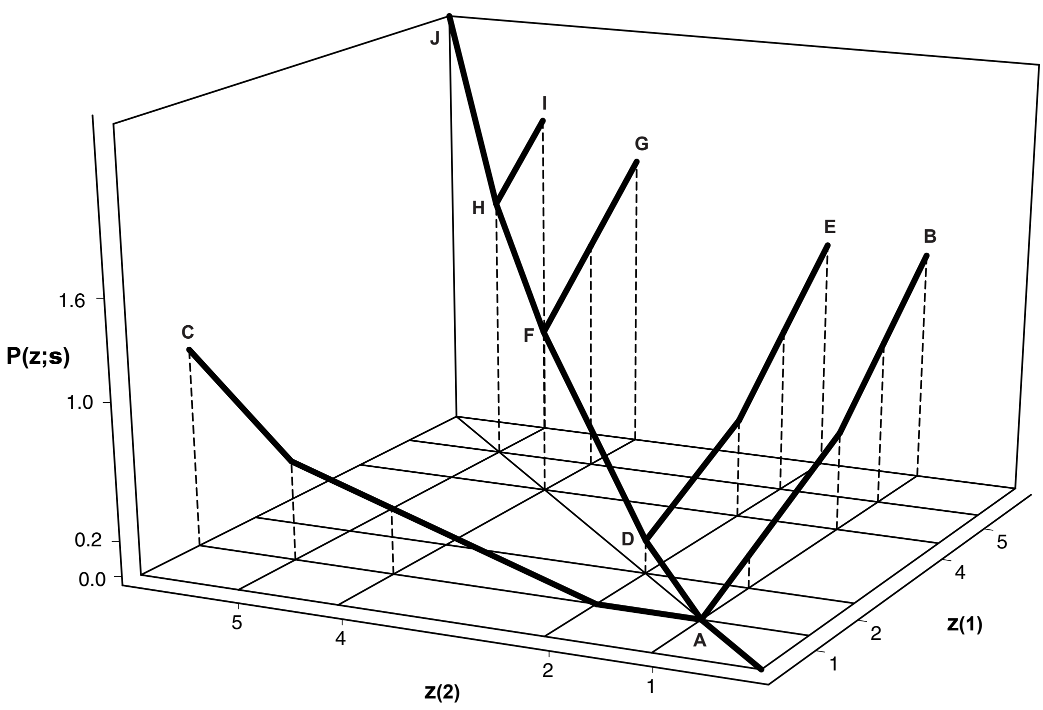


Figure 5.2. Ordered Poverty Gap Curves



income be non-increasing in income and in ability. The corresponding class of utility functions is indicated by

$$\mathbb{U}_2 := \{U \in \mathbb{U} \mid U \text{ satisfies (C1) and (C4)}\}\,. \tag{5.3}$$

Contrary to what happens with the elementary transformations we introduced previously – in particular the favorable permutations of income – it is not clear whether the conditions we obtain here are also necessary for a between-type progressive transfer to result in a welfare improvement.[12]

Bourguignon (1989) introduced a dominance criterion that coincides with the one which commands unanimity over all utilitarian ethical observers who use a utility function that is non-decreasing and concave in income and whose marginal utility in income is non-increasing in ability. In order to define Bourguignon's (1989) criterion, we first need to introduce additional notation and technicalities. An *ordered poverty line* is a non-decreasing mapping $\mathbf{z} : A \longrightarrow D$ such that $\mathbf{z}(h) \in D$ is the poverty line assigned to all the individuals whose abilities are equal to h. The poverty line faced by an individual is no longer exogenously given as in the standard unidimensional framework but it depends now on her personal situation. The *ordered poverty gap* in situation $\mathbf{s} \equiv (\mathbf{x}; \mathbf{a}) \in \mathbb{S}_n$, when the ordered poverty line is $\mathbf{z}$, is given by

$$\mathbb{P}(\mathbf{z}; \mathbf{s}) = \sum_{h \in M_2(\mathbf{s})} f_2(h) \int_{\underline{v}}^{\mathbf{z}(h)} F_1(t \mid h)\, dt = \sum_{h \in M_2(\mathbf{s})} f_2(h)\, P\left(\mathbf{z}(h) \mid h\right), \tag{5.4}$$

where

$$P\left(\mathbf{z}(h) \mid h\right) = \frac{1}{n_2(h)} \sum_{i \in Q_1(\mathbf{z}(h) \mid h)} (\mathbf{z}(h) - x_i) \tag{5.5}$$

is the *conditional poverty gap* of the group of individuals with ability equal to h when the poverty line is set to $\mathbf{z}(h)$. The ordered poverty gap inherits all the properties of the usual poverty gap: $\mathbb{P}(\mathbf{z}; \mathbf{s})$ is non-decreasing and convex over the income range $\left[v_1, v_{m_2(\mathbf{s})}\right]$, and linear over the intervals $[v_j, v_{j+1})$, with $j = 1, 2, \ldots, m_2(\mathbf{s}) - 1$, and $\left[v_{m_2(\mathbf{s})}, +\infty\right)$. In order to illustrate the definition above we have represented in Figure 5.2 the ordered poverty gap curves of situation $\mathbf{s}^1$ in Example 2.1. There are two ability groups – say the handicapped ($h = 1$) and the healthy ($h = 2$) – and thus two poverty lines $\mathbf{z}(1)$ and $\mathbf{z}(2)$. Given the properties of the poverty gap curves and since $\mathbf{z}(1) \geq \mathbf{z}(2)$ there are four ordered poverty gap curves corresponding to the convex piecewise linear lines AB, DE, EG and HI. Actually the curve AB is nothing but the conditional poverty gap curve of the group of handicapped

[12] Ebert (2000) makes use of a continuity argument to show that a decreasing in ability marginal utility of income and concavity with respect to income are also necessary for the sum of utilities to increase as the result of a between-type progressive transfer.

individuals weighted by their corresponding population share. Similarly the piecewise linear line AC represents the weighted conditional poverty gap curve of the group of healthy individuals. Summing the curves AB and AC we obtain the curve AJ which is the integral of the marginal distribution of income or equivalently the poverty gap for the whole population of individuals irrespective of their type. We will say that situation $\mathbf{s}^*$ *ordered poverty gap dominates* situation $\mathbf{s}^\circ$, which we write as $\mathbf{s}^* \geq_{OPG} \mathbf{s}^\circ$, if and only if:

$$\mathbb{P}\left(\mathbf{z};(\mathbf{x}^*;\mathbf{a}^*)\right) \leq \mathbb{P}\left(\mathbf{z};(\mathbf{x}^\circ;\mathbf{a}^\circ)\right), \ \forall\ \mathbf{z} \text{ such that } \mathbf{z}(h) \text{ is non-increasing in } h. \tag{5.6}$$

Definition (5.4) makes clear that the ordered poverty gap is the weighted sum of the conditional poverty gaps, where the weights are equal to the marginal densities of ability. For later reference we will say that situation $\mathbf{s}^*$ *conditional poverty gap dominates* situation $\mathbf{s}^\circ$, which we write as $\mathbf{s}^* \geq_{CPG} \mathbf{s}^\circ$, if and only if:

$$P^*\left(z \,|\, h\right) \leq P^\circ\left(z \,|\, h\right), \ \forall\ z \in D, \ \forall\ h \in M_2\left(\mathbf{s}^*, \mathbf{s}^\circ\right). \tag{5.7}$$

When the situations under comparison have identical marginal distributions of ability, it is immediately clear that conditional poverty gap dominance implies ordered poverty gap dominance, while the converse implication does not hold. Straightforward computations indicate that

$$\mathbb{P}\big(\mathbf{z};(\mathbf{s}^*;\mathbf{s}^\circ)^T\big) = \mathbb{P}\left(\mathbf{z};\mathbf{s}^*\right) + \mathbb{P}\left(\mathbf{z};\mathbf{s}^\circ\right), \ \forall\ \mathbf{s}^* \in \mathbb{S}_n, \ \forall\ \mathbf{s}^\circ \in \mathbb{S}_q \ (n, q \geq 1). \tag{5.8}$$

The next result, which is a direct consequence of the additive separability of the ordered poverty gap, will play a crucial role in subsequent developments.

Lemma 5.3. For all $\mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n$ $(n \geq 2)$ and all $\mathbf{s} \in \mathbb{S}_q$ $(q \geq 1)$, we have:

$$\mathbf{s}^* \geq_{OPG} \mathbf{s}^\circ \Longleftrightarrow \left(\mathbf{s}^*;\mathbf{s}\right)^T \geq_{OPG} \left(\mathbf{s}^\circ;\mathbf{s}\right)^T. \tag{5.9}$$

The following is nothing but a restatement of well-know results in the unidimensional case (see Berge (1963), Hardy, Littlewood and Polya (1952), Marshall and Olkin (1979)).

Lemma 5.4. Let $\mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n$ $(n \geq 2)$ such that $\mathbf{a}^* = \mathbf{a}^\circ$ and $\mu\left(\mathbf{x}^*\right) = \mu\left(\mathbf{x}^\circ\right)$. Then $\mathbf{s}^* \geq_{CPG} \mathbf{s}^\circ$ implies that $\mathbf{s}^*$ is obtained from $\mathbf{s}^\circ$ by means of a finite sequence of within-type progressive transfers of income.

Our last lemma will play a decisive role in the proof of our main result: it is a separation result, which states that ordered poverty gap domination can always be decomposed into headcount poverty and conditional poverty gap dominations.[13]

[13] Analogous results are well-known in the unidimensional setting where generalised Lorenz domination is decomposed into first order stochastic domination and Lorenz domination (see e.g. Marshall and Olkin (1979)).

Lemma 5.5. Let $\mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n$ $(n \geq 2)$ such that $\mathbf{a}^* = \mathbf{a}^\circ$ and $\mu(\mathbf{x}^*) = \mu(\mathbf{x}^\circ)$ and $\mathbf{s} \in \mathbb{S}_q$ $(q \geq 1)$. Then

$$(\mathbf{s}^*;\mathbf{s})^T \geq_{OPG} (\mathbf{s}^\circ;\mathbf{s})^T \tag{5.10}$$

if and only if there exist two situations $\tilde{\mathbf{s}} \in \mathbb{S}_n$ and $\hat{\mathbf{s}} \in \mathbb{S}_q$ such that

$$(\mathbf{s}^*;\mathbf{s})^T \geq_{HP} (\tilde{\mathbf{s}};\hat{\mathbf{s}})^T \geq_{CPG} (\mathbf{s}^\circ;\mathbf{s})^T . \tag{5.11}$$

Making use of Lemma 3.1, Proposition 3.1, Lemmas 4.1 and 4.2, and Lemmas 5.1 to 5.5, we obtain the following theorem, which reveals the normative foundations of Bourguignon's (1989) ordered poverty gap quasi-ordering.

Theorem 5.1. Let $\mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n$ $(n \geq 2)$ such that $\mathbf{a}^* = \mathbf{a}^\circ$ and $\mu(\mathbf{x}^*) = \mu(\mathbf{x}^\circ)$. Then statements (a), (b), (c) and (d) below are equivalent:

(a) There exists $q \geq 1$ and a situation $\mathbf{s} \in \mathbb{S}_q$ such that $(\mathbf{s}^*;\mathbf{s})^T$ is obtained from $(\mathbf{s}^\circ;\mathbf{s})^T$ by means of a finite sequence of individuals' permutations, favorable permutations and/or within-type progressive transfers.

(b) $\sum_{i=1}^n U(x_i^*, a_i^*) \geq \sum_{i=1}^n U(x_i^\circ, a_i^\circ)$, for all $U \in \mathbb{U}_2$.

(c) $U(\mathbf{x}^*;\mathbf{a}^*) \geq_{GL} U(\mathbf{x}^\circ;\mathbf{a}^\circ)$, for all $U \in \mathbb{U}_2$.

(d-1) $\mathbb{P}(\mathbf{z};(\mathbf{x}^*;\mathbf{a}^*)) \leq \mathbb{P}(\mathbf{z};(\mathbf{x}^\circ;\mathbf{a}^\circ))$, $\forall$ $\mathbf{z}$ such that $\mathbf{z}(h)$ is non-increasing in h;

(d-2) $F_2^*(h) = F_2^\circ(h)$, for all $h \in M_2(\mathbf{s}^*, \mathbf{s}^\circ)$.

Sketch proof. Invoking Lemmas 4.1 and 5.1 we deduce from statement (a) that

$$\sum_{i=1}^n U(x_i^*, a_i^*) + \sum_{i=1}^q U(x_i, a_i) \geq \sum_{i=1}^n U(x_i^\circ, a_i^\circ) + \sum_{i=1}^q U(x_i, a_i), \tag{5.12}$$

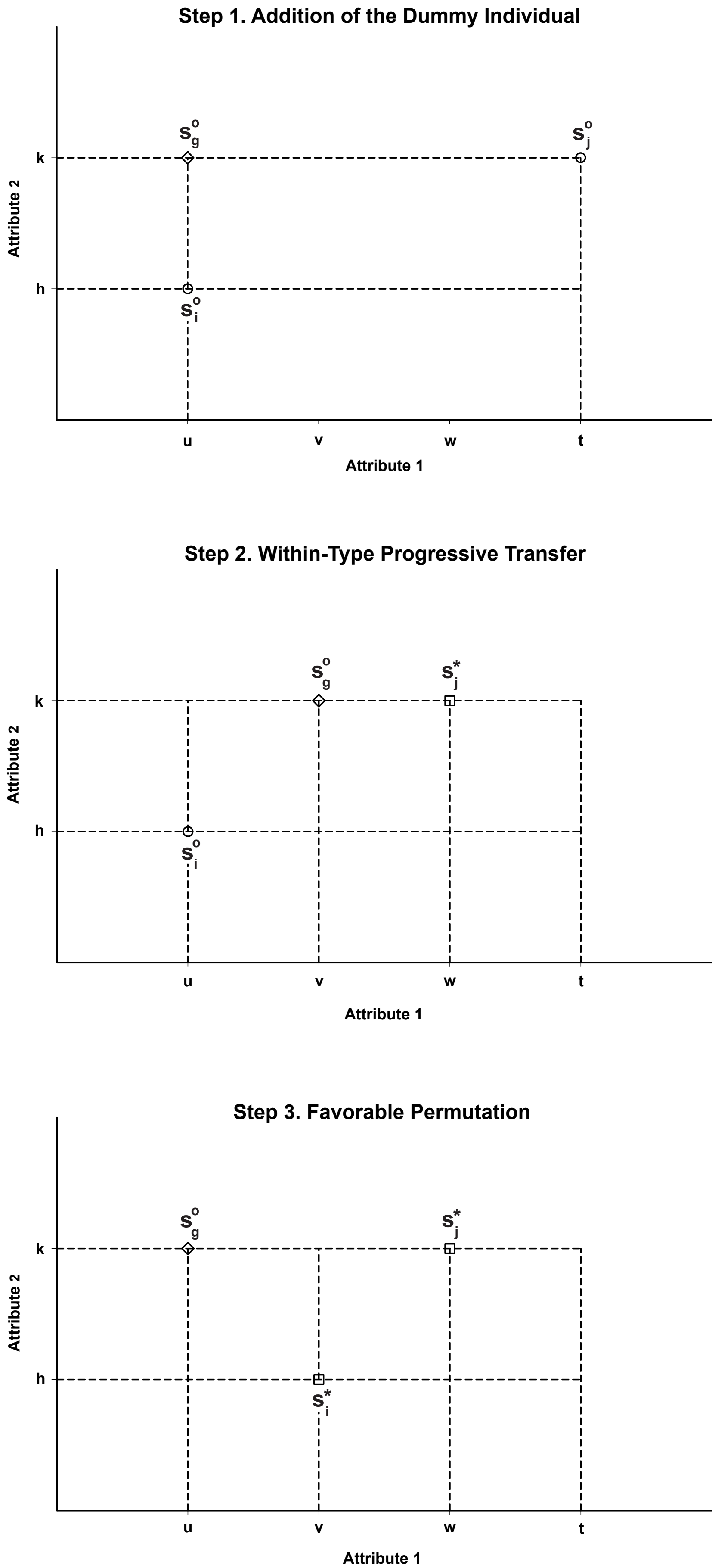
for all $U \in \mathbb{U}_2$, which by Lemma 3.1 is equivalent to statement (b). Proposition 3.1 and the fact that the class of utility functions $\mathbb{U}_2$ is closed under composition by an non-decreasing and concave function guarantee the equivalence between statements (b) and (c). The fact that (d) is necessary for (b) to hold can be easily shown by suitably choosing degenerate utility functions in the class $\mathbb{U}_2$. Invoking the separability of the ordered poverty gap dominance criterion (see Lemma 5.3) we deduce from statement (d) that

$$(\mathbf{s}^*;\mathbf{s})^T \geq_{OPG} (\mathbf{s}^\circ;\mathbf{s})^T, \ \forall\, \mathbf{s} \in \bigcup_{q=1}^{\infty} \mathbb{S}_q. \tag{5.13}$$

Using Lemma 5.5 we deduce from (5.13) that there exists a situation $(\tilde{\mathbf{s}};\hat{\mathbf{s}})^T$ such that

$$(\mathbf{s}^*;\mathbf{s})^T \geq_{HP} (\tilde{\mathbf{s}};\hat{\mathbf{s}})^T \geq_{CPG} (\mathbf{s}^\circ;\mathbf{s})^T . \tag{5.14}$$

Figure 5.3. Decomposition of a Between-Type Progressive Transfer



Finally Lemmas 4.2 and 5.4 establish that (5.14) implies statement (a), which makes the argument complete. □

The equivalence between the unanimous agreement among all utilitarian ethical observers whose marginal utility of income is non-increasing in both attributes and ordered poverty gap dominance was first established by Bourguignon (1989). However the normative meaning of this result is somewhat obscured by the fact that the nature of the underlying elementary transformations, that are needed to transform the dominated situation into the dominating one, was not revealed. Also the ingenious technique of proof employed by Bourguignon (1989) relied on the introduction of auxiliary functions whose meaning is unclear. Theorem 5.1 is motivated by these critiques but we have to admit that the answer it provides is not totally satisfactory. Indeed in order to identify the elementary transformations, which successive applications of allow one to derive the dominating situation from the dominating one, it may be the case that we have to introduce a virtual situation. Actually it is not always necessary to have recourse to such a fictitious situation to prove the equivalence between statements (a) and (d). Thanks to the separability of the different normative criteria we appeal to, the virtual situation plays only an instrumental role in the derivation of our result. There is a strong presumption that this instrumental situation actually mirrors the auxiliary functions used by Bourguignon (1989) in his proof.

Leaving aside the fact that we do not provide a means for identifying with precision the virtual situation, the main limitation of Theorem 5.1 is that it is silent as far as between-type progressive transfers are concerned. Indeed, we know from Lemma 5.2 that all utilitarian ethical observers, whose marginal utility of income is non-increasing in income and ability, will consider that a between-type progressive transfer results in improvement in social welfare. However we have not succeeded in showing that a finite sequence of such transformations enables the dominated situation to be transformed into the dominating one, nor have we been able to prove that this is impossible. All we have shown is that it is possible to transform the dominated situation into the dominating situation – where *both situations are augmented by the virtual situation* – by means of favorable permutations and within-type progressive transfers. This is a challenging question because a between-type progressive transfer can always be decomposed into a within-type progressive transfer followed by a favorable permutation provided one adds a fictitious individual endowed with the income of the beneficiary and the ability of the donor prior to the transfer. This is illustrated in Figure 5.3 which describes the three steps involved in this decomposition process. To simplify things, suppose that the population consists of two individuals i and j and that $\mathbf{s}^* = \left(\mathbf{s}_i^*, \mathbf{s}_j^*\right)^T$ is obtained from $\mathbf{s}^\circ = \left(\mathbf{s}_i^\circ, \mathbf{s}_j^\circ\right)^T$ by means of a single between-type progressive transfer so that $\mathbf{s}_i^\circ = (x_i^\circ, a_i^\circ) = (u, h)$, $\mathbf{s}_j^\circ = \left(x_j^\circ, a_j^\circ\right) = (v, k)$, $\mathbf{s}_i^* = (x_i^*, a_i^*) = (u, h)$ and $\mathbf{s}_j^* = \left(x_j^*, a_j^*\right) = (v, k)$, where $u < v < w < t$, $v - u = t - w = \Delta$ and $h < k$. Consider now an individual g whose situation is given by $\mathbf{s}_g^\circ = \left(x_g^\circ, a_g^\circ\right) = (u, k)$. Adding individual g to the initial population $\{i, j\}$, we obtain the augmented situation $\left(\mathbf{s}_i^\circ, \mathbf{s}_j^\circ; \mathbf{s}_g^\circ\right)^T$. Individuals g and j have the same

Figure 5.4.A. Ordered Poverty Gap Curves of S^1 and S^2

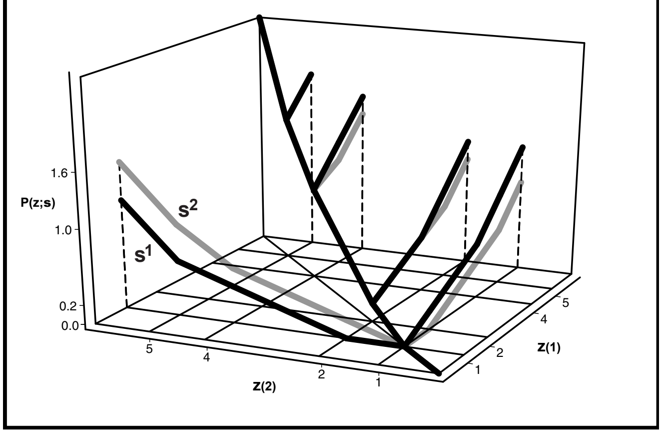


Figure 5.4.B. Ordered Poverty Gap Curves of S^1 and S^3

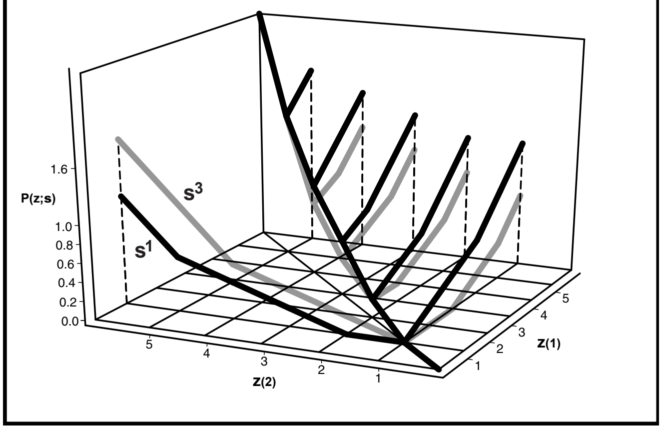


Figure 5.4.C. Ordered Poverty Gap Curves of S^1 and S^4

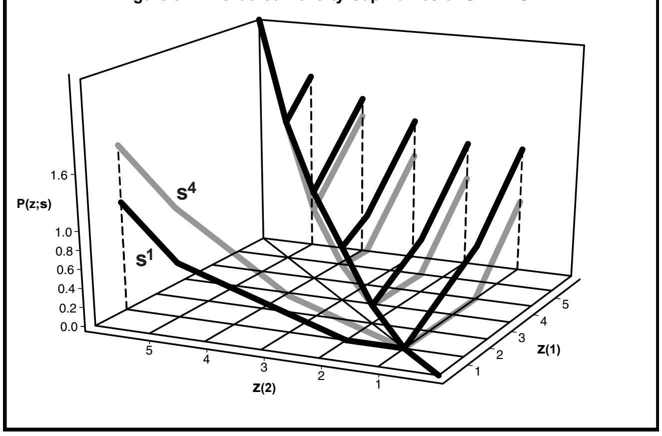


Figure 5.4.D. Ordered Poverty Gap Curves of S^2 and S^3

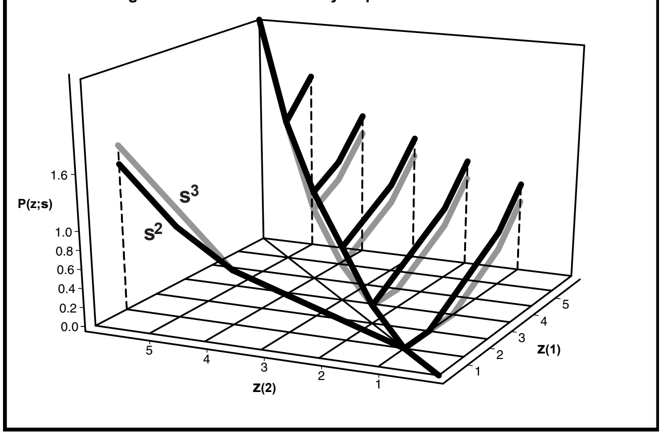


Figure 5.4.E. Ordered Poverty Gap Curves of S^2 and S^4

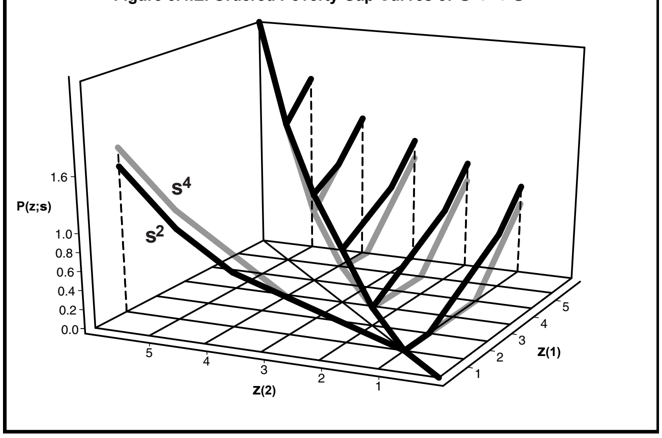
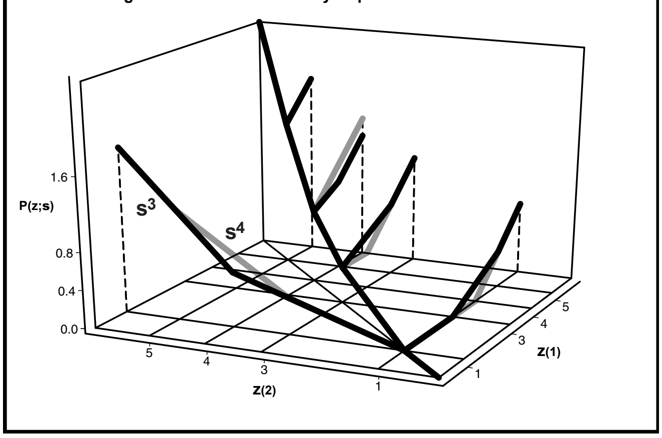


Figure 5.4.F. Ordered Poverty Gap Curves of S^3 and S^4



ability but individual j is richer than individual g. Taking an income amount $\Delta > 0$ from individual j and giving it to individual g we obtain the new situation $\left(\mathbf{s}_i^\circ, \mathbf{s}_j^* ; \mathbf{s}_g^*\right)^T$ which follows from $\left(\mathbf{s}_i^\circ, \mathbf{s}_j^\circ ; \mathbf{s}_g^\circ\right)^T$ by means of a within-type progressive transfer. Observe that individual i is deprived in both income and ability compared to individual g. Exchanging the incomes of individuals i and g we obtain the situation $\left(\mathbf{s}_i^*, \mathbf{s}_j^* ; \mathbf{s}_g^\circ\right)^T$ which follows from situation $\left(\mathbf{s}_i^\circ, \mathbf{s}_j^* ; \mathbf{s}_g^*\right)^T$ by means of a favorable permutation. Individual g is back to her initial situation so that she actually played only an instrumental part in the decomposition. We could have equally proceeded by choosing $\mathbf{s}_g^\circ = \left(x_g^\circ, a_g^\circ\right) = (v, h)$, which would have led to a within-type progressive transfer from individual g to individual i followed by a favorable permutation of income between individuals g and j.

The ordered poverty gap curves of the situations of Example 2.1 are indicated in Figure 5.4 on a pairwise basis. Application of condition (d-1) of Theorem 5.1 allows us to obtain a decisive verdict in all possible pairs of situations but one, namely $\left\{\mathbf{s}^3, \mathbf{s}^4\right\}$. That $\mathbf{s}^3 \geq_{OPG} \mathbf{s}^2$ can readily be anticipated from the fact that situation $\mathbf{s}^3$ is obtained from situation $\mathbf{s}^2$ by means of a favorable permutation involving individuals 1 and 5. Similarly it is not a surprise that $\mathbf{s}^3 \geq_{OPG} \mathbf{s}^1$ since ordered poverty gap dominance is compatible with headcount poverty dominance. On the other hand the fact that $\mathbf{s}^4 \geq_{OPG} \mathbf{s}^2$ is at first sight unexpected given the way the situations in Example 2.1 have been constructed.[14] However close inspection reveals that $\mathbf{s}^4$ can be directly obtained from $\mathbf{s}^2$ by means of a within-type progressive transfer involving individuals 1 and 3 followed by a favorable permutation of the incomes of individuals 1 and 4. This provides an instance where there is no need of introducing an auxiliary situation in order to derive the dominating situation from the dominated one by means of favorable permutations and within-type progressive transfers.

While Theorem 5.1 is concerned with situations with identical income and ability marginal distributions, it is easy to extend it to the general case of variable marginal distributions if one is willing to introduce additional restrictions on the utility functions. Specifically we consider the following class of utility function:

$$\mathbb{U}_2^* := \{U \in \mathbb{U} \mid U \text{ satisfies (C1), (C2), (C3) and (C4)}\}. \tag{5.15}$$

Dispensing with the restrictions that the situations under comparison have identical marginal distributions we obtain:

Theorem 5.2. Let $\mathbf{s}^*, \mathbf{s}^\circ \in \mathbb{S}_n$ $(n \geq 2)$. Then statements (a), (b), (c) and (d) below are equivalent:

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[14] Actually two transformations of equal magnitudes but whose impacts on social welfare go in opposite directions are needed to convert situation $\mathbf{s}^4$ into situation $\mathbf{s}^3$. The favorable permutation consisting in exchanging the incomes of individuals 1 and 4 results in a welfare improvement, while the defavorable permutation between individuals 3 and 5 of the same amounts unambiguously decreases welfare. The net effect on social welfare of these two transformations is therefore ambiguous unless one is prepared to impose additional restrictions on the utility functions.

(a) There exists $q \geq 1$ and a situation $\tilde{\mathbf{s}} \in \mathbb{S}_q$ such that $(\mathbf{s}^*; \tilde{\mathbf{s}})^T$ is obtained from $(\mathbf{s}^\circ; \tilde{\mathbf{s}})^T$ by means of a finite sequence of individuals' permutations, income increments, ability increments, favorable permutations and/or within-type progressive transfers.

(b) $\sum_{i=1}^{n} U(x_i^*, a_i^*) \geq \sum_{i=1}^{n} U(x_i^\circ, a_i^\circ)$, for all $U \in \mathbb{U}_2^*$.

(c) $U(\mathbf{x}^*; \mathbf{a}^*) \geq_{GL} U(\mathbf{x}^\circ; \mathbf{a}^\circ)$, for all $U \in \mathbb{U}_2^*$.

(d-1) $\mathbb{P}(\mathbf{z}; (\mathbf{x}^*; \mathbf{a}^*)) \leq \mathbb{P}(\mathbf{z}; (\mathbf{x}^\circ; \mathbf{a}^\circ))$, $\forall$ $\mathbf{z}$ such that $\mathbf{z}(h)$ is non-increasing in h;

(d-2) $F_2^*(h) \leq F_2^\circ(h)$, for all $h \in M_2(\mathbf{s}^*, \mathbf{s}^\circ)$.

6. Concluding Remarks

The aim of the paper was to investigate the normative foundations of two implementable criteria – the so-called headcount poverty and ordered poverty gap quasi-orderings – designed for comparing distributions of two attributes, one of which – income – is cardinally measurable while the other – ability – is ordinal. Our ambition was to provide in each case an equivalence result analogous to the celebrated Hardy-Littlewood-Polya theorem in the unidimensional framework. More precisely we wanted (i) to identify the class of utility functions such that all welfarist ethical observers, whose value judgements belong to this class, rank the situations under comparison in the same way as the dominance criterion, and (ii) to uncover the elementary transformations finite applications of which permit to derive the dominating situation from the dominated one.

A marginal utility of income that is non-increasing in ability ensures that the welfarist unanimity rule and the headcount poverty quasi-ordering rank the situations in the same way, while a favorable permutation is the transformation which, if applied a finite number of times, allows one to derive the dominating situation from the dominated one. As far as the headcount poverty – and equivalently, first order stochastic dominance – criterion is concerned Theorem 4.1 provides the desired equivalence. Things are far less satisfactory in the case of the ordered poverty gap dominance criterion since we did not manage to establish an equivalence in Theorem 5.1 without resorting to the adjunction of a virtual situation. Using this technical device we were able to show that repeated applications of favorable permutations and within-type progressive transfers allow one to transform the dominated situation into the dominating one. On the other hand, we know that all welfarist ethical observers, whose marginal utilities of income is non-increasing in both income and ability, will record as welfare-improving a between-type progressive transfer. But it is still an open question whether it is possible or not to obtain the dominating situation from the dominated one by means of between-type progressive transfers *without* resorting to dummy individuals.

Other limits of the present analysis concern the number of attributes considered and also the informational assumptions we made. Focusing on just two attributes is certainly restrictive and does not allow us to capture all the relevant dimensions of a person's well-being. The HDI is a good example of an aggregate measure focusing on three essential factors that contribute to a person's well-being: income, life expectancy and literacy. Increasing the number of dimensions is certainly one direction to go but such an extension is likely to become quickly quite involved. To give but an example, the notion of correlation, which lies at the heart of the concept of a favorable permutation, needs to be substantially reformulated when more than two attributes are considered. Suppose there are three attributes as in the HDI case: then the welfare impact of a favorable permutation involving the first and second attribute will depend on the quantity of the third attribute received by the individuals involved in the transformation.[15] Also the question arises to know if it could be possible to adapt Bourguignon's (1989) approach in order to rank situations involving only cardinal attributes as it is done in Kolm (1977) or Atkinson and Bourguignon (1982).

There is a general skepticism concerning the ability of the dominance approach to provide relevant information because of the incomplete nature of the quasi-orderings it relies on. The non-decisiveness of this approach is accentuated in the multi-attribute case and it is expected to be the more serious as the number of dimensions increases. The criteria we investigate in this paper are no exceptions and to some extent they may be considered as a first step that has to supplemented by the use of multidimensional cardinal indices, for instance like those characterized in Ebert (1995). However, to conclude on a more positive note, it is worth mentioning that the criteria we have examined provide conclusive verdicts in a non-negligible number of cases as the evidence in Gravel, Moyes and Tarroux (2008) shows.

References

Allison, R.A., and J.E. Foster, (2004): "Measuring health inequality using qualitative data", *Journal of Health Economics*, Vol. 23, pp. 505–524.

Atkinson, A.B., and F. Bourguignon, (1982): "The comparison of multidimensioned distributions of economic status", *Review of Economic Studies*, Vol. 49, pp. 183–201.

Atkinson, A.B., and F. Bourguignon, (1987): "Income distributions and differences in needs", in G.R. Feiwel (Ed.), *Arrow and the Foundation of the Theory of Economic Policy*, Macmillan, New York.

Bazen, S., and P. Moyes, (2003): "International comparisons of income distributions when population structures differ", *Research in Economic Inequality*, Vol. 9, pp. 85–104.

Berge, C., (1963): *Topological Spaces, Including a Treatment of Multi-Valued Functions, Vector Spaces and Convexity*, Oliver and Boyd, Edinburgh.

Bishop, J.A., Formby, J.P., and W.J. Smith, (1993): "International comparisons of

[15] A way of avoiding this difficulty is to make the extra value judgement that some attributes are separable from each other, which rules out any possibility of compensation between these attributes. This route is followed for instance by Muller and Trannoy (2003), who assume separability between life expectancy at birth and educational attainment but still retain the possibility of using income to compensate for poor health and low education.

welfare and poverty: Dominance orderings for ten countries", *Canadian Journal of Economics*, Vol. 26, pp. 707–726.

Blackorby, C., Bossert, W., and D. Donaldson, (2005): *Population Issues in Social Choice Theory and Ethics*, Cambridge University Press, New-York.

Bourguignon, F., (1989): "Family size and social utility: Income distribution dominance criteria", *Journal of Econometrics*, Vol. 42, pp. 67–80.

Broome, J., (1993): "A cause of preference is not an object of preference", *Social Choice and Welfare*, Vol. 10, pp. 57–68.

Dasgupta, P., Sen, A.K., and D. Starrett, (1973): "Notes on the measurement of inequality", *Journal of Economic Theory*, Vol. 6, pp. 180–187.

Ebert, U., (1995): "Income inequality and differences in household size", *Mathematical Social Sciences*, Vol. 30, pp. 37–53.

Ebert, U., (2000): "Sequential generalised Lorenz dominance and transfer principles", *Bulletin of Economic Research*, Vol. 52, pp. 113–123.

Ebert, U., (2006): "Living standards, social welfare and the redistribution of income in a heterogeneous population", Department of Economics, Universität Oldenburg.

Epstein, L.G., and S.M. Tanny, (1980): "Increasing generalized correlation: A definition and some economic consequences", *Canadian Journal of Economics*, Vol. XIII, pp. 16–34.

Fields, G.S., and J.C.H. Fei, (1978): "On inequality comparisons", *Econometrica*, Vol. 46, pp. 305–316.

Fishburn, P.C., and R.G. Vickson, (1978): "Theoretical foundations of stochastic dominance", in G.A. Whitmore and M.C. Findlay (Eds.), *Stochastic Dominance*, Lexington Books.

Fleurbaey, M., Hagneré, C., and A. Trannoy, (2003): "Welfare comparisons with bounded equivalence scales", *Journal of Economic Theory*, Vol. 110, pp. 309–336.

Foster, J.E., (1985): "Inequality measurement", in H.P. Young (Ed.), *Fair Allocation*, American Mathematical Society Proceedings of Applied Mathematics, Volume 33.

Foster, J.E., Lopez Calva, L.F., and M. Székely, (2005): "Measuring the distribution of human development: Methodology and an application to Mexico", *Journal of Human Development*, Vol. 6, pp. 5–25.

Gravel, N., and P. Moyes, (2006): "Ethically robust comparisons of distributions of two individual attributes", Idep Discussion Paper # 06-05, Cnrs-Ehess-Universités Aix-Marseille II et III.

Gravel, N., Moyes, P., and B. Tarroux, (2008): "Robust international comparisons of distributions of disposable income and access to regional public goods", *Economica*, forthcoming.

Griffin, J., (1986): *Well-Being, Its Meaning, Measurement and Moral Importance*, Clarendon Press, Oxford.

Hadar, J., and W. Russell, (1974): "Stochastic dominance in choice under uncertainty", in M.S. Bach, D.L. McFadden, and S. Wu (Eds.), *Essays on Economics Behavior under Uncertainty*, North-Holland, Amsterdam.

Hammond, P.J., (1979): "Equity in two person situations: Some consequences", *Econometrica*, Vol. 47, pp. 1127–1135.

Hardy, G.H., Littlewood, J.E., and G. Polya, (1952): *Inequalities*, Cambridge University Press, Cambridge (2nd edition).

Howe, R., (1987): "Sections and extensions of concave functions", *Journal of Mathematical Economics*, Vol. 16, pp. 53–64.

Jenkins, S.P., and P.J. Lambert, (1993): "Ranking income distributions when needs differ", *Review of Income and Wealth*, Series 39, Number 4, pp. 337–356.

Kolm, S-.C., (1969): "The optimal production of social justice", in J. Margolis and H. Guitton (Eds.), *Public Economics*, Macmillan, London, pp. 145–200.

Kolm, S-.C., (1972): *Justice et Équité*, Editions du CNRS, Paris.

Kolm, S.-C., (1977): "Multidimensional egalitarianisms", *Quarterly Journal of Economics*, Vol. XCI, pp. 1–13.

Koshevoy, G., (1995): "Multivariate Lorenz majorization", *Social Choice and Welfare*, Vol. 12, pp. 93–102.

Marshall, A.W., and I. Olkin, (1979): *Inequalities: Theory of Majorization and its Applications*, Academic Press, New-York.

Moyes, P., (1999): "Comparisons of heterogeneous distributions and dominance criteria", *Economie et Prévision*, No. 138–139, pp. 125–146 (In French).

Muller, C., and A. Trannoy, (2003): "A dominance approach to well-being inequality across countries", Idep Discussion Paper # 03-13, Cnrs-Ehess-Universités Aix-Marseille II et III.

Sen. A.K., (1985): *Commodities and Capabilities*, North-Holland, Amsterdam.

Sen, A.K., (1997): *On Economic Inequality. Expanded edition with a substantial annexe by James Foster and Amartya Sen*, Clarendon Press, Oxford.

Shorrocks, A.F., (1983): "Ranking income distributions", *Economica*, Vol. 50, pp. 3–17.

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