GREThA
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Économie Théorique et Appliquée

# Social Welfare, Inequality and Deprivation

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***Cahiers du GREThA***
**n° 2008-23**

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## Bien-Être Social, Inégalité et Privation


### Résumé

*Nous proposons une caractérisation du critère de satisfaction généralisée -- ou encore dans notre terminologie d'absence de privation -- introduit par S.R. Chakravarty (Keio Economic Studies 34 (1997), 17--32) pour effectuer des comparaisons de bien-être. Nous montrons que le quasi-ordre de non-privation vérifie une version affaiblie du principe des transferts: le bien-être social augmente seulement suite à des combinaisons spécifiques de transferts progressifs, qui imposent aux individus contribuant au transfert d'être solidaires. Nous identifions la classe des fonctions de bien-être social de Gini généralisé qui obéissent à ce principe d'équité et nous montrons que l'unanimité des points de vue au sein de cette classe conduit au même classement des distributions que le quasi-ordre de non-privation. Nous étendons notre approche à la mesure de l'inégalité à partir des indices éthiques d'inégalité relatifs et absolus.*


**Mots-clés :** Transferts Progressifs, Bien-Être Social, Inégalité, Privation, Dominance au Sens de Lorenz, Fonctions de Bien-Être Social de Gini Généralisé

## Social Welfare, Inequality and Deprivation


### Abstract

*We provide a characterization of the generalised satisfaction -- in our terminology non-deprivation -- quasi-ordering introduced by S.R. Chakravarty (Keio Economic Studies 34 (1997), 17--32) for making welfare comparisons based on the absence of deprivation. We show that the non-deprivation quasi-ordering obeys a weaker version of the principle of transfers: welfare improves only for specific combinations of progressive transfers which require that the same amount be taken from richer individuals and allocated to one arbitrary poorer individual. We identify the subclass of extended Gini social welfare functions that are consistent with this principle and we show that the unanimity of value judgements among this class is identical to the ranking of distributions implied by the non-deprivation quasi-ordering. We extend the approach to the measurement of inequality by considering the corresponding relative and absolute ethical inequality indices.*


**Keywords:** Progressive Transfers, Welfare, Inequality, Deprivation, Lorenz Dominance, Extended Gini Social Welfare Functions

**JEL : D31 ; D63**

**Reference to this paper:** Patrick MOYES, "*Bien-Être Social, Inégalité et Privation*", *Working Papers of GREThA*, n° 2008-23, http://ideas.repec.org/p/grt/wpegrt/2008-23.html.

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# Social Welfare, Inequality and Deprivation *

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* We are indebted to Stephen Bazen for very useful conversations and suggestions. We also would like to thank an associate editor and two anonymous referees whose comments have helped us to improve the paper. The constructive comments of one referee in particular have permitted to simplify greatly the exposition and clarify a couple of issues. Needless to say, the authors bear the entire responsibility for remaining errors and deficiencies.

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## 1. Introduction and Motivation

There is a widespread agreement in the literature to appeal to the generalised Lorenz dominance criterion for making welfare comparisons across societies starting with their distributions of income (see Kolm (1969), Shorrocks (1983)). Although the generalised Lorenz quasi-ordering does only provide a partial ranking of the distributions under comparison, it nevertheless constitutes a first step in the appraisal of the distribution of well-being that can later on be supplemented by the choice of particular indices in order to resolve the cases of inconclusiveness. Much of the attractiveness of the generalised Lorenz criterion – beyond its simplicity and elegance – stems from its association with the Pigou-Dalton condition. According to the latter any transfer from a richer individual to a poorer one that does not modify their respective positions on the income scale – a so-called progressive transfer – reduces inequality, and as long as total income is left unchanged also increases welfare. The generalised Lorenz quasi-ordering and its variants have been extensively used in practice for making welfare and inequality comparisons with a reasonable degree of success.

Notwithstanding its wide application in theoretical and empirical work, the generalised Lorenz criterion is not the only possibility for passing welfare judgements. There is evidence that the magnitude of the income received by an individual constitutes only part of the relevant information for the assessment of her well-being. Individuals are not living in complete isolation and they may be inclined to evaluate their life circumstances by comparison with the situations of particular reference groups of individuals. The social status of an individual, which we can assimilate in a first round with her position in the social hierarchy, is considered an important dimension of one person's self-assessment of her well-being (see e.g. Weiss and Fershtman (1998)). Similarly feelings such as envy, resentment and satisfaction may contribute to explain the way in which trends in well-being derived from subjective measures differ from those based on standard indicators focusing exclusively on the material dimensions of well-being.[1] In this respect, the notion of individual deprivation may be particularly useful for understanding the way a person's self-evaluation of her well-being departs from standard monetary measures. Indeed, according to Runciman (1966), what matters for the individual's own appraisal of well-being in a given situation is not what she gets, but rather how much she feels deprived as compared to those individuals who, she considers, are treated more favourably than she is.

The concept of deprivation has been echoed in the inequality and welfare literature even though this has been done at the cost of an oversimplification. In the economic literature the level of deprivation experienced by any individual in a given situation is associated with the average difference between her income and the incomes of the individuals richer than her (see e.g. Yitzhaki (1979), Berrebi and Silber (1985), Hey and Lambert (1980), Ebert and Moyes (2000), among others). If one believes that it is desirable to reduce deprivation, then it is natural to declare that a distribution is better than another distribution if the amount of deprivation experienced by each individual is less in the first situation than in the second. Credit has to be given to Kakwani (1984) for having made this idea precise through the introduction of his [relative] deprivation quasi-ordering. According to the latter, deprivation in a society decreases as the deprivation curve, which indicates the levels of deprivation attained by all the individuals in the population, goes down. Kakwani (1984)'s suggestion has given

[1] It is an important question from an ethical point of view to decide whether sentiments like envy and self-contentment have to be counted when comparing different social states. It is not the purpose of this paper to address this issue, which raises important and difficult philosophical questions.

rise to a considerable literature aimed at refining and extending the original deprivation quasi-ordering. Particularly important for our purpose in this strand of research are the contributions of Chakravarty (1997) and Chateauneuf and Moyes (2006).

One of the aims of Chateauneuf and Moyes (2006) is to uncover the implicit inequality value judgements embedded in Kakwani (1984)'s deprivation quasi-ordering. Because their focus is on inequality rather than on welfare, they restrict attention to income distributions with the same mean. They identify the class of extended Gini social welfare functions that are consistent with the absolute version of Kakwani (1984)'s deprivation quasi-ordering. In addition they propose an alternative to the notion of a progressive transfer, which possesses the property that, if a distribution is ranked above another by the deprivation quasi-ordering, then the former can be derived from the latter by means of such transfers.[2] A serious limitation of this work from a practical point of view is the fixed mean restriction, which prevents welfare comparisons from being made in real world situations where the income distributions typically differ in size. Inspection of Chateauneuf and Moyes (2006, Proposition 5.3) suggests however a simple method for deciding whether one distribution is welfare-superior to another distribution. The test would consist in comparing the means and the deprivation curves of the distributions under consideration: the distribution with the higher mean and the lowest deprivation curve will be ranked above the other distribution by all the extended Gini welfare functions they consider. The problem is that this procedure does not permit one to identify *all* the cases where the application of unanimity among the relevant class of extended Gini welfare functions is decisive. A criterion is missing, which would allow one to detect the distributions which are ranked in the same way by all the extended Gini welfare functions.

On the contrary Chakravarty (1997) suggested a modification of the deprivation curve that permits welfare judgements to be made when the distributions under comparison have unequal means. Building on a suggestion by Yitzhaki (1979) in a slightly different framework, Chakravarty (1997)'s proposal consists in taking the complement of a person's deprivation to mean income as a measure of her well-being. The so-called generalised satisfaction of an individual constitutes the relevant information for making welfare comparisons. A distribution is considered better than another distribution if the generalised satisfaction curve, which indicates the levels of satisfaction attained by all the individuals in the population, moves upwards. Chakravarty (1997) considers a class of welfare functions, which has as special cases the extended Gini welfare functions Chateauneuf and Moyes (2006) focused on. Therefore the extent to which both approaches fit together is not clear and this is an issue that needs clarification. Furthermore the characterization of the generalised satisfaction quasi-ordering provided by Chakravarty (1997) is not really illuminating. Admittedly it is shown that, if one distribution is ranked above another by the generalised satisfaction quasi-ordering, then the former can be obtained from the latter by means of a so-called *fair transformation*, and conversely. However it is immediately clear that both statements are just two different ways of saying the same thing, which does not add a lot to our comprehension of the complex equalising process leading to generalised satisfaction dominance.

Our paper aims at integrating the contributions of Chakravarty (1997) and Chateauneuf and Moyes (2006) in a comprehensive and consistent model. We introduce the non-deprivation quasi-ordering and we show that, if one distribution is ranked above another by this criterion,

[2] They also investigate the properties of two alternative inequality quasi-orderings related to the class of extended Gini social welfare functions. These criteria are not relevant for the present paper and we refer the interested reader to the original contribution.

then the dominating distribution can be obtained from the dominated one by successive applications of the equalising transformations considered by Chateauneuf and Moyes (2006) and/or increments, and conversely. Because our non-deprivation quasi-ordering proves to be formally identical to Chakravarty (1997)'s generalised satisfaction quasi-ordering, this result sheds light upon the precise nature of the value judgements contained implicitly in the latter criterion. We also demonstrate that the partial ordering implied by the non-deprivation quasi-ordering coincides with the way all extended Gini social welfare functions – whose weighting functions are non-decreasing and star-shaped – rank the distributions. This result extends to all monotone social welfare functions that attach a positive value to decreases in deprivation, which is actually the case considered by Chakravarty (1997). The non-deprivation – equivalently the generalised satisfaction – quasi-ordering is operationally more efficient than the two-step procedure based on comparisons of mean incomes and deprivation curves, and it has to be substituted for the latter. Previous work does not make clear the relationship between the generalised satisfaction curve and the quantile function. We show that the non-deprivation curve is but a modified version of the quantile curve where successive first differences in incomes are given decreasing weights.

We present in Section 2 our conceptual framework and we introduce the notions of social welfare functions and inequality indices that will be subsequently used. Section 3 is concerned with the standard approach to welfare and inequality measurement which builds on the principle of transfers. We recall the standard criteria of rank order dominance, generalised Lorenz dominance, relative and absolute Lorenz dominance and we present without proofs the well-known equivalences between these quasi-orderings, the underlying transformations of the distributions and the corresponding classes of welfare and inequality measures. Our main contribution is contained in Section 4, where we introduce the welfare and inequality dominance criteria that build upon the extent to which every individual feels deprived. We weaken the principle of transfers by imposing restrictions on the way the progressive transfers are combined. Next we introduce the non-deprivation quasi-ordering and we show that it is equivalent to the unanimous ranking generated by all the extended Gini social welfare functions that are consistent with our restricted principle of transfers. Appropriate normalisations of the distributions allow us to derive the corresponding relative and absolute inequality quasi-orderings. As we have insisted above our approach builds on some existing work and we therefore devote Section 5 to a discussion and clarification of the relationships between our contribution and related work in the literature. In particular we contrast our model with the contributions of Chakravarty (1997), Chateauneuf and Moyes (2006), and Hey and Lambert (1980). Our approach is illustrated in Section 6 where the non-deprivation dominance criteria are used in order to compare the distributions of income in 17 countries from the welfare and inequality point of view. Finally we summarize our results in Section 7, which also hints at some directions for future research.

## 2. Notation and Preliminary Definitions

We assume throughout that incomes are drawn from an interval $D$ of $\mathbb{R}$. An *income distribution* – or equivalently, a *situation* – for a population consisting of $n$ identical individuals ($n \geq 2$) is a list $\mathbf{x} := (x_1, x_2, \ldots, x_n)$ where $x_i \in D$ is the income of individual $i$. We suppose that incomes are arranged non-decreasingly and we use $\mathscr{Y}_n(D)$ to represent the *set of income distributions*. The arithmetic *mean* of distribution $\mathbf{x} \in \mathscr{Y}_n(D)$ is indicated by $\mu(\mathbf{x}) := \sum_{i=1}^{n} x_i/n$. We denote as $F(\,\cdot\,;\mathbf{x})$ the *cumulative distribution function* of $\mathbf{x} \in \mathscr{Y}_n(D)$ defined by $F(z;\mathbf{x}) := m(z;\mathbf{x})/n$, for all $z \in (-\infty, +\infty)$, where $m(z; \mathbf{x}) := \#\{i \in \{1, 2, \ldots, n\} \mid x_i \leq z\}$. We let $F^{-1}(\,\cdot\,; \mathbf{x})$ represent the *inverse cumulative distribution function* – or equivalently, the *quantile function* – of $\mathbf{x}$ obtained by letting $F^{-1}(0; \mathbf{x}) := x_1$ and

$$F^{-1}(p; \mathbf{x}) := \inf \Big\{ z \in (-\infty, +\infty) \,\Big|\, F(z; \mathbf{x}) \geq p \Big\}, \ \forall\, p \in (0, 1] \tag{2.1}$$

(see Gastwirth (1971)). The restrictions that the population size is fixed and that incomes are non-decreasingly arranged can be easily dispensed with by invoking respectively the *principle of population* (see Dalton (1920)) and the condition of *symmetry*.[3] Indeed, all the welfare and inequality measures – indices and quasi-orderings – considered later on satisfy these two conditions.

A *social welfare function* is a continuous mapping $W : \mathscr{Y}_n(D) \longrightarrow \mathbb{R}$ such that $W(\mathbf{x})$ measures the welfare of society in situation $\mathbf{x} \in \mathscr{Y}_n(D)$. We denote as $\mathscr{W}$ the set of social welfare functions and we take the traditional view that $W(\mathbf{x})$ represents the social welfare in situation $\mathbf{x}$ as it is conceived by an ethical observer on the basis of the information about the individuals' personal circumstances she has access to. Similarly, an *inequality index* is a continuous mapping $I : \mathscr{Y}_n(D) \longrightarrow \mathbb{R}$ such that $I(\mathbf{x})$ represents the degree of inequality in situation $\mathbf{x} \in \mathscr{Y}_n(D)$ and we denote as $\mathscr{I}$ the set of inequality indices. We restrict attention to *ethical inequality indices*, namely those indices that are derived from a social welfare function. According to the ethical approach, inequality is measured by the amount of income that would have been saved if equality had prevailed and social welfare had been the same as in the current situation (see Blackorby, Bossert, and Donaldson (1999) for a survey of the literature). Given the social welfare function $W$ we denote as $\Xi(\mathbf{x})$ the *equally distributed equivalent income* of distribution $\mathbf{x} \in \mathscr{Y}_n(D)$ defined by

$$W(\mathbf{x}) = W\left(x_1, \ldots, x_n\right) = W\left(\Xi(\mathbf{x}), \ldots, \Xi(\mathbf{x})\right). \tag{2.2}$$

The equally distributed equivalent income is uniquely defined provided that the social welfare function is increasing along the ray of equality. The *social evaluation function* $\Xi$ and the social welfare function $W$ are *ordinally equivalent* in the sense that

$$\forall\, \mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D) : W(\mathbf{x}) \geq W(\mathbf{y}) \Longleftrightarrow \Xi(\mathbf{x}) \geq \Xi(\mathbf{y}). \tag{2.3}$$

Starting with a given social welfare function, we may consider two types of inequality indices depending on the way we measure the income loss. The *relative inequality index* represents the proportion of total income that is wasted due to the presence of unequal incomes. Precisely it is defined by

$$I^R(\mathbf{x}) := 1 - \frac{\Xi(\mathbf{x})}{\mu(\mathbf{x})}, \ \forall\, \mathbf{x} \in \mathscr{Y}_n(D) \quad (D \subseteq \mathbb{R}_{++}). \tag{2.4}$$

The *absolute inequality index* represents the income that would have been saved if equality had prevailed and it is written

$$I^A(\mathbf{x}) := \mu(\mathbf{x}) - \Xi(\mathbf{x}), \ \forall\, \mathbf{x} \in \mathscr{Y}_n(D). \tag{2.5}$$

The literature has mainly focused on two general families of social welfare functions up to now. The *utilitarian approach* assumes that social welfare is simply the sum of the utilities

[3] The *principle of population* and the condition of *symmetry* require respectively that welfare and inequality do not change when the distributions are replicated and when incomes are permuted among the individuals.

achieved by the individuals and it is defined by

$$W_u(\mathbf{x}) := \frac{1}{n}\sum_{i=1}^{n} u(x_i), \ \forall\, \mathbf{x} \in \mathscr{Y}_n(D), \tag{2.6}$$

where $u \in \mathscr{U} := \{u : D \longrightarrow \mathbb{R} \mid u \text{ is continuous}\}$ is the *utility function* defined up to an increasing and affine transformation. According to the second approach – the *extended Gini* model – social welfare is given by

$$W_f(\mathbf{x}) := \sum_{i=1}^{n}\left[f\left(\frac{n-i+1}{n}\right) - f\left(\frac{n-i}{n}\right)\right] x_i, \ \forall\, \mathbf{x} \in \mathscr{Y}_n(D), \tag{2.7}$$

where $f \in \mathscr{F} := \{f : [0,1] \longrightarrow [0,1] \mid f \text{ is continuous}, f(0) = 0 \text{ and } f(1) = 1\}$ is the *weighting function* (see Weymark (1981), Yaari (1987, 1988), Ebert (1988)). The utility function and the weighting function capture the preferences of the ethical observer within the utilitarian and extended Gini models, respectively. The two former models are actually special cases of the *rank-dependent expected utility* model popularized by Quiggin (1993)[4] We indicate respectively by $I_u^R$ and $I_u^A$ the relative and absolute inequality indices derived from the utilitarian social welfare function (2.6) when the ethical observer's preferences are captured by the utility function $u$. Similarly, the relative and absolute inequality indices corresponding to the extended Gini social welfare function (2.7) are denoted by $I_f^R$ and $I_f^A$, respectively.

## 3. Lorenz Consistent Welfare and Inequality Measures

### 3.1. The Measurement of Social Welfare

Following the practice in the dominance literature we examine successively the case where the ethical observer is only concerned with efficiency and the case where equity considerations are also taken into account.

Definition 3.1. Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$, we will say that $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of an *increment* if there exist $\Delta > 0$ and an individual $i$ such that

$$x_i = y_i + \Delta \text{ and } x_h = y_h, \ \forall\, h \neq i. \tag{3.1}$$

It is usually considered that an increase in someone's income other things equal results in a social welfare improvement, hence the following condition:

Monotonicity [MON]. For all $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$, we have $W(\mathbf{x}) \geq W(\mathbf{y})$ whenever $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of an increment.

We indicate by $\mathscr{W}^1$ the set of social welfare functions that satisfy MON. One easily checks that the non-decreasingness of $u$ and $f$, respectively, ensure that the utilitarian and the extended Gini social welfare functions satisfy the above condition. We denote respectively as

$$\mathscr{U}^1 := \{u \in \mathscr{U} \mid u \text{ is non-decreasing}\} \text{ and} \tag{3.2}$$

$$\mathscr{F}^1 := \{f \in \mathscr{F} \mid f \text{ is non-decreasing}\} \tag{3.3}$$

the corresponding classes of utility and weighting functions. It is convenient for later use to introduce the following criterion:

[4] Actually a characterization of the so-called rank-dependent expected utility model is provided in Ebert (1988), where emphasis is on inequality measurement rather than on choice under uncertainty.

Definition 3.2. Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$, we will say that $\mathbf{x}$ *rank order dominates* $\mathbf{y}$, which we write $\mathbf{x} \geq_{RO} \mathbf{y}$, if and only if

$$RO\left(\frac{k}{n};\mathbf{x}\right) \geq RO\left(\frac{k}{n};\mathbf{y}\right), \ \forall\, k = 1, 2, \ldots, n, \tag{3.4}$$

where $RO(p;\mathbf{x}) := F^{-1}(p;\mathbf{x})$, for all $p \in [0, 1]$. The result below summarizes the existing results in the literature when one pays attention to efficiency considerations exclusively.

**Theorem 3.1.** *Let $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$. The following five statements are equivalent:*

(a) $\mathbf{x}$ *is obtained from* $\mathbf{y}$ *by means of a finite sequence of increments.*

(b) $W(\mathbf{x}) \geq W(\mathbf{y})$, *for all* $W \in \mathscr{W}^1$.

(b1) $W_u(\mathbf{x}) \geq W_u(\mathbf{y})$, *for all* $u \in \mathscr{U}^1$.

(b2) $W_f(\mathbf{x}) \geq W_f(\mathbf{y})$, *for all* $f \in \mathscr{F}^1$.

(c) $\mathbf{x} \geq_{RO} \mathbf{y}$.

Among other things Theorem 3.1 indicates the restrictions that have to be placed on the utility and the weighting functions for the utilitarian and the extended Gini social welfare functions to be consistent with the rank order quasi-ordering.

It is typically assumed in normative economics that inequality is reduced and welfare increased by a transfer of income from a richer individual to a poorer individual.

Definition 3.3. Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$, we will say that $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a *progressive transfer*, if there exists an income amount $\Delta > 0$ and two individuals $i, j$ such that

$$x_h = y_h, \ \forall\ h \neq i, j; \tag{3.5a}$$

$$x_i = y_i + \Delta;\ x_j = y_j - \Delta; \quad \text{and} \tag{3.5b}$$

$$\Delta \leq (y_j - y_i)/\,2. \tag{3.5c}$$

We note that, according to our definition, a progressive transfer does not reverse the relative positions of *all* the individuals in the society.

Principle of Transfers [PT]. For all $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$, we have $W(\mathbf{x}) \geq W(\mathbf{y})$ and $I(\mathbf{x}) \leq I(\mathbf{y})$ whenever $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a progressive transfer.

We let $\mathscr{W}^2$ represent the set of social welfare functions that satisfy MON and PT. It is a straightforward exercise to check that the concavity of $u$ and the convexity of $f$, respectively, guarantee that the utilitarian and the extended Gini social welfare functions obey PT. We denote as

$$\mathscr{U}^2 := \{u \in \mathscr{U} \mid u \text{ is non-decreasing and concave}\} \ \text{and} \tag{3.6}$$

$$\mathscr{F}^2 := \{f \in \mathscr{F} \mid f \text{ is non-decreasing and convex}\} \tag{3.7}$$

the corresponding classes of utility and weighting functions. The *generalised Lorenz curve* of distribution $\mathbf{x} \in \mathscr{Y}_n(D)$ is defined by

$$GL(p;\mathbf{x}) := \begin{cases} p\,x_1, & \forall\ 0 \leq p \leq \dfrac{1}{n}, \\ p\,x_1 + \displaystyle\sum_{j=2}^{k} \left(\frac{n\,p - j + 1}{n}\right)[x_j - x_{j-1}]\,, & \forall\ \dfrac{k-1}{n} < p \leq \dfrac{k}{n}, \end{cases} \tag{3.8}$$

where $k = 2, 3, \ldots, n$ (see Shorrocks (1983), Moyes (1999)).

Definition 3.4. Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$, we will say that $\mathbf{x}$ *generalised Lorenz dominates* $\mathbf{y}$, which we write $\mathbf{x} \geq_{GL} \mathbf{y}$, if and only if

$$GL\left(\frac{k}{n}; \mathbf{x}\right) \geq GL\left(\frac{k}{n}; \mathbf{y}\right), \ \forall\, k = 1, 2, \ldots, n. \tag{3.9}$$

It has long been recognized that the notion of progressive transfer is closely associated with the generalised Lorenz quasi-ordering as the following result demonstrates (see Kolm (1969), Marshall and Olkin (1979), Sen (1973), Shorrocks (1983), Foster (1985), among others):

**Theorem 3.2.** *Let* $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$. *The following five statements are equivalent:*

(a) $\mathbf{x}$ *is obtained from* $\mathbf{y}$ *by means of a finite sequence of increments and/or progressive transfers.*

(b) $W(\mathbf{x}) \geq W(\mathbf{y})$, *for all* $W \in \mathscr{W}^2$.

(b1) $W_u(\mathbf{x}) \geq W_u(\mathbf{y})$, *for all* $u \in \mathscr{U}^2$.

(b2) $W_f(\mathbf{x}) \geq W_f(\mathbf{y})$, *for all* $f \in \mathscr{F}^2$.

(c) $\mathbf{x} \geq_{GL} \mathbf{y}$.

Theorems 3.1 and 3.2 indicate that both the utilitarian and the extended Gini social welfare functions are consistent with the rank order and the generalised Lorenz quasi-orderings provided that appropriate restrictions be placed on the utility and the weighting functions. More importantly, they stress that the rankings of distributions generated by the utilitarian and the extended Gini models under the constraint of unanimity among the classes of utility and weighting functions are identical.[5] As we will see later on, this is not necessarily true when criteria different from the rank order and the generalised Lorenz quasi-orderings are considered.

### 3.2. The Measurement of Inequality

The preceding approach generalises in a straightforward way to inequality measurement by appropriate normalisations of the distributions under comparison (see e.g. Moyes (1999)).

Definition 3.5. Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$ with $D \subseteq \mathbb{R}_{++}$, we will say that $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a *scale transformation* if there exists $\lambda > 0$ such that $x_i = \lambda y_i$, for all $i = 1, 2, \ldots, n$.

Scale Invariance [SI]. For all $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$ with $D \subseteq \mathbb{R}_{++}$, we have $I(\mathbf{x}) = I(\mathbf{y})$ whenever $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a scale transformation.

We let $\mathscr{I}^2$ and $\mathscr{I}^R$ represent the set of inequality indices that satisfy conditions PT and SI, respectively. It is a straightforward exercise to check that the convexity of $f$ guarantees that $I_f^R \in \mathscr{I}^2 \cap \mathscr{I}^R$. The *relative Lorenz curve* of distribution $\mathbf{x} \in \mathscr{Y}_n(D)$ with $D \subseteq \mathbb{R}_{++}$ is defined by $RL(p; \mathbf{x}) := GL(p; \hat{\mathbf{x}})$, for all $p \in [0, 1]$, where $\hat{\mathbf{x}} := (\hat{x}_1, \ldots, \hat{x}_n)$ is the *reduced distribution* corresponding to $\mathbf{x}$ with $\hat{x}_i := x_i / \mu(\mathbf{x})$, for all $i = 1, 2, \ldots, n$.

Definition 3.6. Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$ with $D \subseteq \mathbb{R}_{++}$, we will say that $\mathbf{x}$ *relative Lorenz dominates* $\mathbf{y}$, which we write $\mathbf{x} \geq_{RL} \mathbf{y}$, if and only if

$$RL\left(\frac{k}{n}; \mathbf{x}\right) \geq RL\left(\frac{k}{n}; \mathbf{y}\right), \ \forall\, k = 1, 2, \ldots, (n-1). \tag{3.10}$$

[5] This follows from the fact that the stochastic dominance and the inverse stochastic dominance quasi-orderings coincide up to the second order.

The following result establishes the connections between relative Lorenz dominance, progressive transfers and scale transformations, and the requirement of unanimity among the ethical observers – in particular the extended Gini ones – who subscribe to PT and SI.

**Theorem 3.3.** [6] *Let* $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$ *with* $D \subseteq \mathbb{R}_{++}$*. The following four statements are equivalent:*

(a) $\mathbf{x}$ *is obtained from* $\mathbf{y}$ *by means of a scale transformation and/or a finite sequence of progressive transfers.*

(b) $I(\mathbf{x}) \leq I(\mathbf{y})$*, for all* $I \in \mathscr{I}^2 \cap \mathscr{I}^R$*.*

(b2) $I_f^R(\mathbf{x}) \leq I_f^R(\mathbf{y})$*, for all* $f \in \mathscr{F}^2$*.*

(c) $\mathbf{x} \geq_{RL} \mathbf{y}$*.*

Definition 3.7. Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$, we will say that $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a *translation transformation* if there exists $\gamma \in \mathbb{R}$ such that $x_i = y_i + \gamma$, for all $i = 1, 2, \ldots, n$.

Translation Invariance [TI]. For all $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$, we have $I(\mathbf{x}) = I(\mathbf{y})$ whenever $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a translation transformation.

We let $\mathscr{I}^A$ represent the set of inequality indices that satisfy condition TI. It is easy to verify that the convexity of $f$ guarantees that $I_f^A \in \mathscr{I}^2 \cap \mathscr{I}^A$. The *absolute Lorenz curve* of distribution $\mathbf{x} \in \mathscr{Y}_n(D)$ is defined by $AL(p; \mathbf{x}) := GL(p; \tilde{\mathbf{x}})$, for all $p \in [0, 1]$, where $\tilde{\mathbf{x}} := (\tilde{x}_1, \ldots, \tilde{x}_n)$ is the *centered distribution* corresponding to $\mathbf{x}$ with $\tilde{x}_i := x_i - \mu(\mathbf{x})$, for all $i = 1, 2, \ldots, n$ (see Moyes (1987)).

Definition 3.8. Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$, we will say that $\mathbf{x}$ *absolute Lorenz dominates* $\mathbf{y}$, which we write $\mathbf{x} \geq_{AL} \mathbf{y}$, if and only if

$$AL\left(\frac{k}{n}; \mathbf{x}\right) \geq AL\left(\frac{k}{n}; \mathbf{y}\right), \ \forall\, k = 1, 2, \ldots, (n-1). \tag{3.11}$$

The following result establishes the connections between absolute Lorenz dominance, progressive transfers and translation increments, and the requirement of unanimity among the ethical observers – were they of the extended Gini type or not – who subscribe to PT and TI.

**Theorem 3.4.** *Let* $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$*. The following four statements are equivalent:*

(a) $\mathbf{x}$ *is obtained from* $\mathbf{y}$ *by means of a translation transformation and/or a finite sequence of progressive transfers.*

(b) $I(\mathbf{x}) \leq I(\mathbf{y})$*, for all* $I \in \mathscr{I}^2 \cap \mathscr{I}^A$*.*

(b2) $I_f^A(\mathbf{x}) \leq I_f^A(\mathbf{y})$*, for all* $f \in \mathscr{F}^2$*.*

(c) $\mathbf{x} \geq_{AL} \mathbf{y}$*.*

[6] It is important to note that there is no statement analogue to statement (b2) concerning the inequality indices one derives from the utilitarian model. This originates in the fact that the unanimous agreement among all utilitarian ethical observers endowed with an Atkinson-Kolm-Sen utility function does not generally imply relative Lorenz dominance.

## 4. Social Welfare and Inequality as the Absence of Deprivation

### 4.1. The Measurement of Social Welfare

Among the criteria that have been proposed in addition to the generalised Lorenz quasi-ordering the deprivation quasi-ordering is certainly the most well-known. It builds on the idea introduced in sociology by Runciman (1966) according to which a person feels deprived if she realizes that some other persons enjoy some item – for instance social status – she does not have access to but sees no reason why she is not entitled to get it.[7] The greater the magnitude of the perceived differences between her social status and those of the persons she compares herself with, the more acute her feeling of deprivation will be. Assimilating the social status of any individual with her income, Kakwani (1984) has proposed to use the deprivation curve for comparing income distributions. For reasons that will become obvious later on, we depart slightly from Kakwani's suggestion and we introduce the *non-deprivation curve* of distribution $\mathbf{x} \in \mathscr{Y}_n(D)$ defined by

$$ND(p;\mathbf{x}) := \begin{cases} x_1, & \forall\ 0 \leq p \leq \dfrac{1}{n}, \\ x_1 + \displaystyle\sum_{j=2}^{k} \left(\frac{n-j+1}{n}\right)[x_j - x_{j-1}], & \forall\ \dfrac{k-1}{n} < p \leq \dfrac{k}{n}, \end{cases} \tag{4.1}$$

where $k = 2, 3, \dots, n$. We interpret $ND(k/n;\mathbf{x})$ as a measure of the feeling of non-deprivation of individual with rank $p = k/n$ in situation $\mathbf{x}$. In order to make the meaning of $ND(p;\mathbf{x})$ more transparent, we can develop and rearrange (4.1) when $k \geq 2$ to get

$$\begin{aligned} ND\left(\frac{k}{n};\mathbf{x}\right) &= \frac{n}{n}\,x_1 + \frac{n-1}{n}\,[x_2 - x_1] + \cdots + \frac{n-k+1}{n}\,[x_k - x_{k-1}] \\ &= \frac{1}{n}\sum_{i=1}^{k} x_i + \frac{n-k}{n}\,x_k \\ &= GL\left(\frac{k}{n};\mathbf{x}\right) + \frac{n-k}{n}\,x_k \\ &= \mu\left(\mathbf{x}^k\right), \end{aligned} \tag{4.2}$$

where $\mathbf{x}^k := \left(x_1^k, \dots, x_n^k\right)$ is the censored distribution obtained from $\mathbf{x}$ by letting $x_i^k := x_i$, for all $i = 1, 2, \dots, k$, and $x_i^k := x_k$, for all $i = k+1, k+2, \dots, n$. Manipulating (4.2) one step further we finally obtain

$$\begin{aligned} ND\left(\frac{k}{n};\mathbf{x}\right) &= \frac{1}{n}\sum_{i=1}^{k} x_i + \frac{1}{n}\sum_{i=k+1}^{n} x_i + \frac{n-k}{n}x_k - \frac{1}{n}\sum_{i=k+1}^{n} x_i \\ &= \mu(\mathbf{x}) - ADP\left(\frac{k}{n};\mathbf{x}\right), \end{aligned} \tag{4.3}$$

[7] The notion of individual deprivation comprises two dimensions: the first one is related to envy because another person has something I would like to have, while the second one stresses the injustice I feel in this situation. While there is a consensus for taking into account feelings of injustice when evaluating a person's well-being, counting for sentiments like envy or jealousy is debatable. We will not enter into this debate and we will limit ourselves with the examination of the consequences for welfare measurement of using individual deprivation as the prime information.

where

$$ADP\left(\frac{k}{n};\mathbf{x}\right) := \frac{1}{n}\sum_{j=k}^{n}\left[x_j - x_k\right], \ \forall\, k = 1, 2, \ldots, n, \tag{4.4}$$

is a measure of the *absolute deprivation* of individual $k$ in situation $\mathbf{x}$ (see Ebert and Moyes (2000), Chateauneuf and Moyes (2006)). Therefore, the non-deprivation of person $k$ is identical to the difference between the mean income and her absolute deprivation: this is actually nothing else than what Chakravarty (1997) called person's $k$ *generalised satisfaction*.[8]

Definition (4.1) makes clear that the non-deprivation curve is a step function with jumps occurring possibly at $p = k/n$, with $k = 1, 2, \ldots, n-1$. Furthermore, we have

$$ND\left(\frac{k+1}{n};\mathbf{x}\right) - ND\left(\frac{k}{n};\mathbf{x}\right) = \frac{n-k}{n}\left[x_{k+1} - x_k\right] \geq 0, \ \forall\, k = 1, 2, \ldots, n-1, \tag{4.5}$$

for all $\mathbf{x} \in \mathscr{Y}_n(D)$, which establishes that the non-deprivation curve is non-decreasing in $p \in [0,1]$. Like the quantile curve, the non-deprivation curve attains its minimum value equal to $F^{-1}(0;\mathbf{x}) = x_1$ when $p = 0$. Like the generalised Lorenz curve, it reaches its maximum value equal to mean income $\mu(\mathbf{x})$ when $p = 1$. We have represented in Figure 4.1 the rank order, the non-deprivation and the generalised Lorenz curves of distribution $\mathbf{y} := (1, 5, 6, 10)$. One observes that the non-deprivation curve lies above the generalised Lorenz curve and below the rank order curve. Definition (4.1) also draws attention to the connection with the inverse distribution function. Indeed notice that in our framework where distributions are discrete the quantile function can be written as:

$$F^{-1}(p;\mathbf{x}) := \begin{cases} x_1, & \forall\ 0 \leq p \leq \dfrac{1}{n}, \\ x_1 + \displaystyle\sum_{j=2}^{k}\left[x_j - x_{j-1}\right], & \forall\ \dfrac{k-1}{n} < p \leq \dfrac{k}{n}, \end{cases} \tag{4.6}$$

where $k = 2, 3, \ldots, n$. Comparing (4.1) and (4.6) one notices that the quantile and the non-deprivation curves only differ in the way the adjacent pairwise income differences are weighted. In the case of the quantile curve the first differences in incomes are all given the same weight equal to unity, while in the case of the non-deprivation curve the weights are decreasing at a constant rate. The impact of the weighting scheme on the shape of the quantile and non-deprivation curves is well reflected in Figure 4.1.

We follow Chakravarty (1997) and compare income distributions on the basis of the non-deprivation curves they generate. More precisely we have:

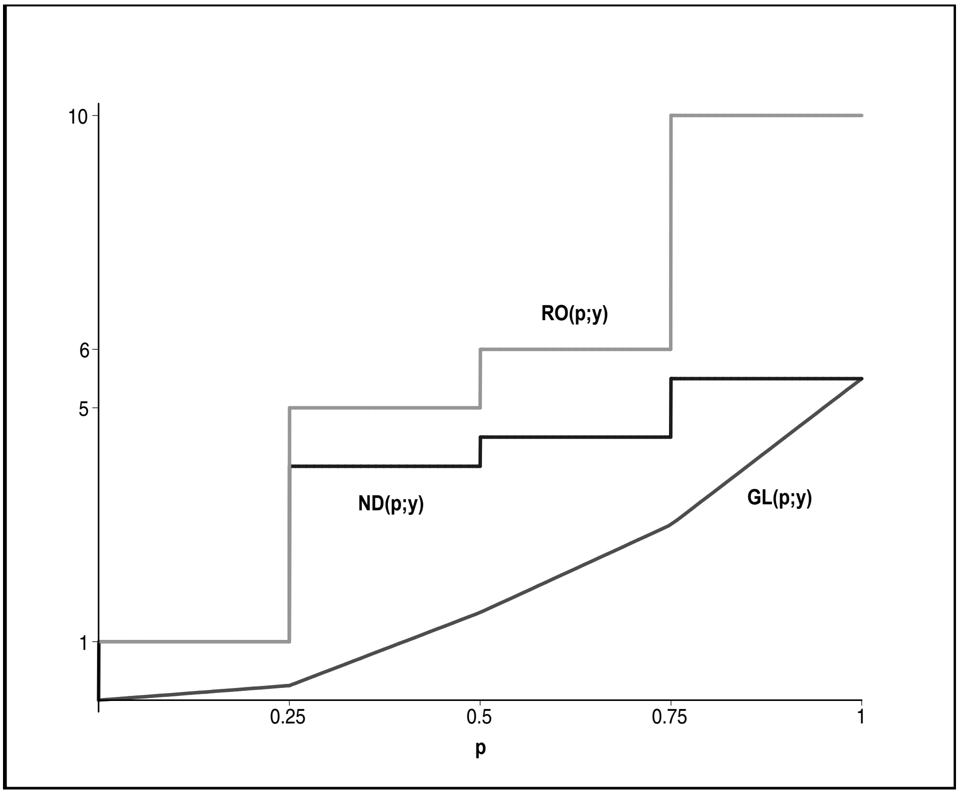
Definition 4.1. Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$, we will say that $\mathbf{x}$ *non-deprivation dominates* $\mathbf{y}$, which we write $\mathbf{x} \geq_{ND} \mathbf{y}$, if and only if

$$ND\left(\frac{k}{n};\mathbf{x}\right) \geq ND\left(\frac{k}{n};\mathbf{y}\right), \ \forall\, k = 1, 2, \ldots, n. \tag{4.7}$$

A distribution will be considered to be superior to another from a welfare point of view if the non-deprivation experienced by any individual is not smaller in the first distribution

[8] We avoid here the term of *satisfaction* because it has been already used in Chateauneuf and Moyes (2006) with a different meaning.

Figure 4.1: Rank-Order, Non-Deprivation and Generalised Lorenz Curves




than in the second distribution. An immediate question is to know how the non-deprivation quasi-ordering behaves in comparison with the rank order and the generalised Lorenz quasi-orderings. Chakravarty, Chattopadhyay, and Majumder (1995, Theorem 1) have shown that non-deprivation dominance implies generalised Lorenz dominance when the distributions under comparison have equal means. The following result dispenses with the equal mean restriction:

**Proposition 4.1.** (a) *If* $n = 2$, *then* $\geq_{RO} \subset \geq_{ND} = \geq_{GL}$. (b) *If* $n > 2$, *then* $\geq_{RO} \subset \geq_{ND} \subset \geq_{GL}$.

Proof. Because statement (a) is obvious, we only prove statement (b). It follows immediately from (4.2) that, if $x_k \geq y_k$, for all $k = 1, 2, \ldots, n$, then $\mathbf{x} \geq_{ND} \mathbf{y}$. To establish that $\mathbf{x} \geq_{ND} \mathbf{y}$ implies $\mathbf{x} \geq_{GL} \mathbf{y}$, it suffices to show that $ND(k/n; \mathbf{x}) \geq ND(k/n; \mathbf{y})$ implies $GL(k/n; \mathbf{x}) \geq GL(k/n; \mathbf{y})$, for all $k = 1, 2, \ldots, n$. Clearly, the implication is true for $k = 1$. We argue by induction and verify that, if it is true for $k$, then so it is for $k + 1$. Suppose that $ND((k+1)/n; \mathbf{x}) \geq ND((k+1)/n; \mathbf{y})$, or equivalently that:

$$GL\left(\frac{k+1}{n}; \mathbf{x}\right) + \frac{n-k-1}{n}\, x_{k+1} \geq GL\left(\frac{k+1}{n}; \mathbf{y}\right) + \frac{n-k-1}{n}\, y_{k+1}. \tag{4.8}$$

If $x_{k+1} > y_{k+1}$, then the result follows from the fact that, by assumption, $GL(k/n; \mathbf{x}) \geq GL(k/n; \mathbf{y})$. If $x_{k+1} \leq y_{k+1}$, then we deduce immediately from (4.8) that:

$$GL\left(\frac{k+1}{n}; \mathbf{x}\right) - GL\left(\frac{k+1}{n}; \mathbf{y}\right) \geq -\frac{n-k-1}{n}\,[x_{k+1} - y_{k+1}] \geq 0. \tag{4.9}$$

Finally, consider distributions $\mathbf{x} = (1, 4, 8, 9)$, $\mathbf{y} = (1, 5, 6, 10)$ and $\mathbf{z} = (1, 4, 7, 10)$. One can easily verify that $\mathbf{x} \geq_{ND} \mathbf{z}$ but $\neg\, [\mathbf{x} \geq_{RO} \mathbf{z}]$, and that $\mathbf{y} \geq_{GL} \mathbf{z}$ but $\neg\, [\mathbf{y} \geq_{ND} \mathbf{z}]$, which confirms that the inclusions in statement (b) are strict. □

The non-deprivation quasi-ordering may therefore be considered an intermediate criterion halfway between the rank order and the generalised Lorenz quasi-orderings. As a result the non-deprivation criterion will provide a more partial – or to the best no more complete – ranking of the distributions under comparison than the one implied by the generalised Lorenz quasi-ordering. From a practical point of view, this may be considered a weakness of the non-deprivation quasi-ordering as compared with the generalised Lorenz one. As we will show below the non-deprivation quasi-ordering relies on value judgements that are less demanding – and hopefully more likely to be accepted – than the generalised Lorenz quasi-ordering. The loss in the discriminatory power of the non-deprivation quasi-ordering is the price to pay for its ability to reach a larger consensus. On the other hand, the extent of the loss in the conclusive rankings one experiences, when the non-deprivation criterion is substituted for the generalised Lorenz quasi-ordering, is ultimately an empirical matter.

There is evidence that the principle of transfers, that supports the generalised Lorenz criterion, is far from being unanimously accepted, as a number of experimental studies have demonstrated (see Amiel and Cowell (1992, 1999), Ballano and Ruiz-Castillo (1993), Harrison and Seidl (1994), Gaertner and Namezie (2003), among others). However, none of these studies provides information about the subjects' ethical preferences – with the exception that these preferences are at variance with the views captured by the principle of transfers – that might explain such a rejection. Nor do they propose alternatives to the principle of transfers that might be more in line with the subjects' attitudes towards inequality. Here we propose to weaken the principle of transfers by imposing a degree of solidarity among the individuals who take part in the redistribution process. However, solidarity is restricted to the individuals who give away a fraction of their incomes: if some income is taken from a rich individual, then in order for there to be solidarity, the same amount is taken from every individual who is as rich or richer than this individual. But it is not necessary that the individuals who are poorer than the transfer recipient benefit also from an equal additional income.[9] By contrast, a progressive transfer imposes no solidarity at all among the individuals involved in the equalising transformation. It follows that the relative positions of the donors and beneficiaries of the transfers on the income scale will play a crucial role in the definition of our inequality reducing transformation. We emphasize that the idea of introducing the positions of the individuals for assessing the impact of a transfer is not new in the literature. Such an idea is at the heart of the notion of a *positional composite transfer* proposed by Chateauneuf and Wilthien (1999) and Zoli (2002) for instance.[10]

Definition 4.2. Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$, we will say that $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a *uniform on the right progressive transfer* if there exists an income amount $\Delta > 0$ and two individuals $h, k$ ($1 \leq h < k \leq n$) such that:

(4.10a) $$x_i = y_i,\ \forall\, i \in \{1, \dots, h-1\} \cup \{h+1, \dots, k-1\};$$

(4.10b) $$x_h = y_h + \Delta;$$

(4.10c) $$x_i = y_i - \frac{\Delta}{n-k+1},\ \forall\, i \in \{k, \dots, n\}.$$

[9] One may conceive of other ways of introducing solidarity among the givers and receivers, which result in different possible equalising transformations (see Chateauneuf and Moyes (2006)).

[10] The notion of a positional composite transfer generalises a suggestion of Kakwani (1980) that constitutes an alternative to the principle of diminishing transfers due to Kolm (1976).

If $k = n$, then a uniform on the right progressive transfer reduces to a usual progressive transfer, and this is actually the only case where both types of transformations coincide. Although in general uniform on the right progressive transfers and progressive transfers are different operations, the former can always be decomposed into a finite sequence of the latter, as the next result demonstrates.

**Proposition 4.2.** *Let* $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$ *and suppose that* $\mathbf{x}$ *is obtained from* $\mathbf{y}$ *by means of a single uniform on the right progressive transfer. Then* $\mathbf{x}$ *can be obtained from* $\mathbf{y}$ *by means of a finite sequence of progressive transfers.*

Proof. Suppose that $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a uniform on the right progressive transfer so that there exists $\Delta > 0$ and two individuals $h, k$ $(1 \leq h < k \leq n)$ such that conditions (4.10a), (4.10b) and (4.10c) hold. Consider the distribution $\mathbf{z}^{(1)}$ defined by

$$z_i^{(1)} = y_i = x_i, \ \forall\, i = 1, 2, \ldots, h-1, \tag{4.11a}$$

$$z_h^{(1)} = y_h + \epsilon < x_h; \tag{4.11b}$$

$$z_i^{(1)} = y_i = x_i, \ \forall\, i = h+1, \ldots, k-1, \tag{4.11c}$$

$$z_k^{(1)} = y_k - \epsilon = x_k, \tag{4.11d}$$

$$z_i^{(1)} = y_i < x_i, \ \forall\, i = k+1, k+2, \ldots, n, \tag{4.11e}$$

where $\epsilon = \Delta/(n-k+1)$. By construction we have

$$z_1^{(1)} \leq \cdots \leq z_{h-1}^{(1)} \leq z_h^{(1)} < z_{h+1}^{(1)} \leq \cdots \leq z_{k-1}^{(1)} \leq z_k^{(1)} < z_{k+1}^{(1)} \leq \cdots \leq z_n^{(1)}. \tag{4.12}$$

Therefore $\mathbf{z}^{(1)}$ is obtained from $\mathbf{y}$ by means of a progressive transfer that does not modify the positions on the income scale of all the individuals. Consider next the distribution $\mathbf{z}^{(2)}$ defined by

$$z_i^{(2)} = z_i^{(1)} = x_i, \ \forall\, i = 1, 2, \ldots, h-1, \tag{4.13a}$$

$$z_h^{(2)} = z_h^{(1)} + \epsilon < x_h, \tag{4.13b}$$

$$z_i^{(2)} = z_i^{(1)} = x_i, \ \forall\, i = h+1, \ldots, k, \tag{4.13c}$$

$$z_{k+1}^{(2)} = z_{k+1}^{(1)} - \epsilon = x_{k+1}, \tag{4.13d}$$

$$z_i^{(2)} = z_i^{(1)} < x_i, \ \forall\, i = k+2, k+3, \ldots, n. \tag{4.13e}$$

Distribution $\mathbf{z}^{(2)}$ is obtained from $\mathbf{z}^{(1)}$ by means of a rank preserving progressive transfer. Repeating this operation with $j = k+2, k+3, \ldots, n$ we finally obtain distribution $\mathbf{x}$ in $n-k+1$ steps, where each step involves a single rank preserving progressive transfer. Thus the uniform on the right progressive transfer has been decomposed into $n-k+1$ progressive transfers of an amount $\epsilon = \Delta/(n-k+1)$ as it is indicated in Table 4.1. □

It follows from Proposition 4.2 that, if a distribution is obtained from another by means of a uniform on the right progressive transfer, then the former will generalised Lorenz dominate the latter. But the fact that one distribution is ranked above another by the generalised Lorenz criterion does not imply that the former can be obtained from the latter by means of a sequence

Table 4.1: Decomposition of a Uniform on the Right Progressive Transfer

| | $\mathbf{z}^{(1)}$ | $\mathbf{z}^{(2)}$ | $\mathbf{z}^{(3)}$ | $\cdots$ | $\mathbf{z}^{(n-2)}$ | $\mathbf{z}^{(n-1)}$ | $\mathbf{z}^{(n)}$ |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | $\cdots$ | 0 | 0 | 0 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | | $\vdots$ | $\vdots$ | $\vdots$ |
| $h-1$ | 0 | 0 | 0 | $\cdots$ | 0 | 0 | 0 |
| $h$ | $+\epsilon$ | $+\epsilon$ | $+\epsilon$ | $\cdots$ | $+\epsilon$ | $+\epsilon$ | $+\epsilon$ |
| $h+1$ | 0 | 0 | 0 | $\cdots$ | 0 | 0 | 0 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | | $\vdots$ | $\vdots$ | $\vdots$ |
| $k-1$ | 0 | 0 | 0 | $\cdots$ | 0 | 0 | 0 |
| $k$ | $-\epsilon$ | 0 | 0 | $\cdots$ | 0 | 0 | 0 |
| $k+1$ | 0 | $-\epsilon$ | 0 | $\cdots$ | 0 | 0 | 0 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | | $\vdots$ | $\vdots$ | $\vdots$ |
| $n-1$ | 0 | 0 | 0 | $\cdots$ | 0 | $-\epsilon$ | 0 |
| $n$ | 0 | 0 | 0 | $\cdots$ | 0 | 0 | $-\epsilon$ |

of uniform on the right progressive transfers. Choose $\mathbf{x} = (1, 4, 4, 7)$ and $\mathbf{y} = (1, 3, 5, 7)$: clearly $\mathbf{x} \geq_{GL} \mathbf{y}$ but it is impossible to transform $\mathbf{y}$ into $\mathbf{x}$ by means of uniform on the right progressive transfers. We find convenient at this stage to introduce an intermediate result that will prove useful later on.

**Proposition 4.3.** *Let $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$. Then $\mathbf{x} \geq_{ND} \mathbf{y}$ whenever $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a uniform on the right progressive transfer.*

Proof. Suppose that $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a uniform on the right progressive transfer so that there exists $\Delta > 0$ and two individuals $h, k$ $(1 \leq h < k \leq n)$ such that conditions (4.10a), (4.10b) and (4.10c) hold. Straightforward computation yields

$$ND\left(\frac{i}{n};\mathbf{x}\right) - ND\left(\frac{i}{n};\mathbf{y}\right) = \begin{cases} 0, & 1 \leq i < h, \\ \dfrac{(n-h+1)\,\Delta}{n} > 0, & i = h, \\ \dfrac{\Delta}{n} > 0, & h < i < k, \\ 0, & k \leq i \leq n, \end{cases} \tag{4.14}$$

hence $\mathbf{x} \geq_{ND} \mathbf{y}$. □

The following condition, which requires that social welfare does not decrease as the result of a uniform on the right progressive transfer, constitutes a weakening of the usual principle of transfers.

Uniform of the Right Principle of Transfers [URPT]. For all $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$, we have $W(\mathbf{x}) \geq W(\mathbf{y})$ and $I(\mathbf{x}) \leq I(\mathbf{y})$ whenever $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a uniform on the right progressive transfer.

We indicate by $\mathscr{W}^*$ the set of social welfare functions that satisfy MON and URPT and note that $\mathscr{W}^2 \subset \mathscr{W}^*$, which actually ensures that the class $\mathscr{W}^*$ is non-empty. For instance the social welfare function $W(\mathbf{x}) := H(ND(\mathbf{x}))$, where $ND(\mathbf{x}) := (ND(1/n;\mathbf{x}), \ldots, ND(n/n;\mathbf{x}))$ is the non-deprivation profile generated by distribution $\mathbf{x}$, verifies URPT provided that $H$ be symmetric and monotone.[11] This social welfare function imposes a particular factorization process and one might be willing to consider more conventional social welfare functions such as the utilitarian and extended Gini.

It is argued in Chateauneuf and Moyes (2006) that the utilitarian principle does not permit one to distinguish between the ethical observers who subscribe to the uniform on the right principle of transfers from those who agree with the principle of transfers.[12] By contrast the extended Gini approach makes it possible to identify the ethical observers who share the views reflected by either principle. Before we examine the implications of imposing consistency with the uniform on the right principle of transfers, we have to introduce additional technicalities. We will say that $f \in \mathscr{F}$ is *star-shaped* if

$$\frac{f(q)}{q} \text{ is non-decreasing in } q, \ \forall\, q \in (0,1]. \tag{4.15}$$

It is well-known that convexity of $f \in \mathscr{F}$ is both necessary and sufficient for welfare to increase as the result of a progressive transfer when the former is evaluated by means of the extended Gini social welfare function (see e.g. Chateauneuf and Moyes (2004, Proposition 4.4)). We show below that it is possible to find a similar justification for the star-shapedness property provided one is willing to consider the impact on social welfare of particular combinations of increments.

Definition 4.3. Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$, we will say that $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a *k-uniform on the right increment* if there exists $\Delta > 0$ such that

$$x_i = y_i, \ \forall\, i = 1, 2, \ldots, k-1, \text{ and} \tag{4.16a}$$

$$x_i = y_i + \frac{\Delta}{n-k+1}, \ \forall\, i = k, k+1, \ldots, n. \tag{4.16b}$$

Clearly a $k$-uniform on the right increment results in a social welfare improvement in the extended Gini model provided that the weighting function $f$ is increasing. Consider now an arbitrary distribution $\mathbf{y}$ and let $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ be two distributions such that $\mathbf{x}^{(1)}$ is obtained from $\mathbf{y}$ by means of a $k$-uniform on the right increment equal to $\Delta > 0$ while a $(k+1)$-uniform on the right increment of the same amount transforms $\mathbf{y}$ into $\mathbf{x}^{(2)}$. Computing the corresponding welfare changes, we obtain:

$$W_f\big(\mathbf{x}^{(1)}\big) - W_f(\mathbf{y}) = f\left(\frac{n-k+1}{n}\right)\frac{\Delta}{n-k+1}, \text{ and} \tag{4.17a}$$

$$W_f\big(\mathbf{x}^{(2)}\big) - W_f(\mathbf{y}) = f\left(\frac{n-k}{n}\right)\frac{\Delta}{n-k}. \tag{4.17b}$$

The fact that $f$ is star-shaped guarantees that $W_f\big(\mathbf{x}^{(1)}\big) - W_f\big(\mathbf{y}\big) \geq W_f\left(\mathbf{x}^{(2)}\right) - W_f(\mathbf{y})$: the lower is $k$, the greater the welfare improvement caused by a $k$ uniform on the right increment of a given amount $\Delta > 0$. Observe further that, by construction, distributions $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ are such that $\mathbf{x}^{(1)}$ is derived from $\mathbf{x}^{(2)}$ by means of a uniform on the right progressive transfer

[11] There is no need to impose that the aggregation function $H$ be Schur-concave as it is done in Chakravarty and Mukherjee (1999) for social welfare as measured by $H(ND(\mathbf{x}))$ to increase as a result of an arbitrary uniform on the right progressive transfer.

[12] More precisely, concavity of the utility function is shown to be a necessary and sufficient condition for the utilitarian social welfare function to satisfy the uniform on the right principle of transfers.

of the amount $\Delta/(n-k+1) > 0$ from the individuals no poorer than $k+1$ to the individual $k$. The welfare change implied by this uniform on the right progressive transfer is equal to

$$W_f\big(\mathbf{x}^{(1)}\big) - W_f\big(\mathbf{x}^{(2)}\big) = \Big[W_f\big(\mathbf{x}^{(1)}\big) - W_f\big(\mathbf{y}\big)\Big] - \Big[W_f\big(\mathbf{x}^{(2)}\big) - W_f\big(\mathbf{y}\big)\Big] \geq 0. \tag{4.18}$$

Table 4.2, where the first row refers to the positions of the individuals on the income scale generated by distribution $\mathbf{y}$, makes the construction of distributions $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ more transparent. The above argument generalises to an arbitrary uniform on the right progressive transfer by choosing appropriately the $h$-uniform on the right and $k$-uniform on the right increments so that $h < k$. Therefore a sufficient condition for a uniform on the right progressive transfer to improve social welfare as measured by the extended Gini welfare function is that the weighting function $f$ is star-shaped.[13] Convexity is stronger than star-shapedness: the former requirement implies the latter but the converse implication is false. This is illustrated in Figure 4.2 where we have represented two different weighting functions: $f$ is star-shaped but not convex while $g$ is convex and thus star-shaped. For later reference we denote as $\mathscr{F}^*$ the set of weighting functions $f \in \mathscr{F}$ that are non-decreasing and star-shaped.

Table 4.2: Construction of Distributions $\mathbf{x}^{(1)}$, $\mathbf{x}^{(2)}$ and $\mathbf{x}^{(3)}$

| | 1 | $\cdots$ | $k-1$ | $k$ | $k+1$ | $\cdots$ | $n$ |
|---|---|---|---|---|---|---|---|
| $\mathbf{x}^{(1)} - \mathbf{y}$ | 0 | $\cdots$ | 0 | $\dfrac{+\Delta}{n-k+1}$ | $\dfrac{+\Delta}{n-k+1}$ | $\cdots$ | $\dfrac{+\Delta}{n-k+1}$ |
| $\mathbf{x}^{(2)} - \mathbf{y}$ | 0 | $\cdots$ | 0 | 0 | $\dfrac{+\Delta}{n-k}$ | $\cdots$ | $\dfrac{+\Delta}{n-k}$ |
| $\mathbf{x}^{(1)} - \mathbf{x}^{(2)}$ | 0 | $\cdots$ | 0 | $\dfrac{+\Delta}{n-k+1}$ | $\dfrac{-\Delta}{(n-k)(n-k+1)}$ | $\cdots$ | $\dfrac{-\Delta}{(n-k)(n-k+1)}$ |

We are now in a position to state the announced result, which establishes the connections between increments and uniform on the right progressive transfers, unanimity of value judgements among the ethical observers who subscribe to the uniform on the right principle of transfers, and non-deprivation domination.

**Theorem 4.1.** *Let $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$. The following four statements are equivalent:*

(a) $\mathbf{x}$ *is obtained from* $\mathbf{y}$ *by means of a finite sequence of increments and/or uniform on the right progressive transfers.*

(b) $W(\mathbf{x}) \geq W(\mathbf{y})$, *for all* $W \in \mathscr{W}^*$.

(b2) $W_f(\mathbf{x}) \geq W_f(\mathbf{y})$, *for all* $f \in \mathscr{F}^*$.

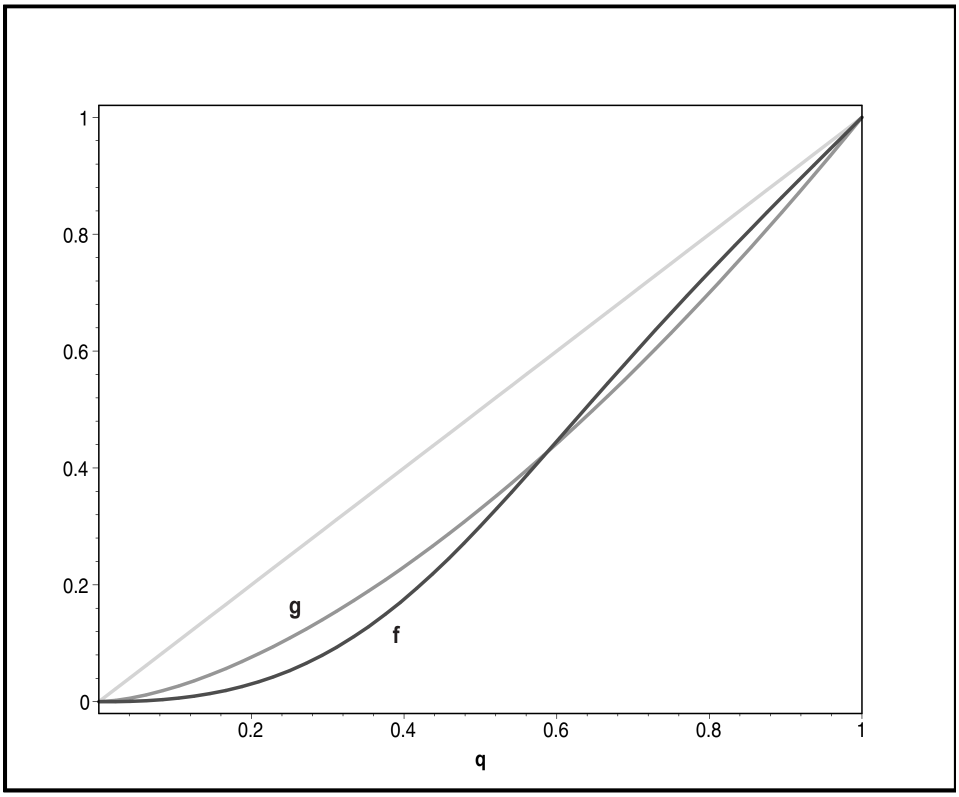
(c) $\mathbf{x} \geq_{ND} \mathbf{y}$.

Proof. Letting $q_i := (n-i+1)/n$, for all $i = 1, 2, \ldots, n$, the extended Gini social welfare function can be rewritten as

$$W_f(\mathbf{x}) = x_1 + \sum_{i=2}^{n} f\left(q_i\right)\,\left[x_i - x_{i-1}\right]. \tag{4.19}$$

[13] Imposing the further restriction that the weighting function is continuous over $[0,1)$ it can be proven that star-shapedness is also a necessary condition by making use of a replication argument. However, in this case we can no longer restrict our attention to distributions of fixed dimension and we have to allow the population to vary.

Figure 4.2: Star-Shaped and Convex Weighting Functions




Since by definition $f(q_1) = f(1) = 1$, this is equivalent to

$$W_f(\mathbf{x}) = \frac{f(q_1)}{q_1}\, q_1\, x_1 + \sum_{i=2}^{n} \frac{f(q_i)}{q_i}\, q_i\, [x_i - x_{i-1}]\,. \tag{4.20}$$

Applying Abel's decomposition rule to (4.20), we obtain

$$\begin{aligned} W_f(\mathbf{x}) = {} & \left(\frac{f(q_1)}{q_1} - \frac{f(q_2)}{q_2}\right) q_1\, x_1 \\ & + \sum_{k=2}^{n-1} \left(\frac{f(q_k)}{q_k} - \frac{f(q_{k+1})}{q_{k+1}}\right) \left[q_1\, x_1 + \sum_{i=2}^{k} q_i\, [x_i - x_{i-1}]\right] \\ & + \frac{f(q_n)}{q_n} \left[q_1\, x_1 + \sum_{i=2}^{n} q_i\, [x_i - x_{i-1}]\right], \end{aligned} \tag{4.21}$$

which, making use of (4.1), reduces finally to

$$W_f(\mathbf{x}) = \sum_{k=1}^{n-1} \left[\frac{f(q_k)}{q_k} - \frac{f(q_{k+1})}{q_{k+1}}\right] ND\left(\frac{k}{n};\mathbf{x}\right) + \frac{f(q_n)}{q_n}\, ND\left(\frac{n}{n};\mathbf{x}\right). \tag{4.22}$$

(a) $\Longrightarrow$ (b). This follows directly from the definition of the class $\mathscr{W}^*$ of social welfare functions.

(b) $\Longrightarrow$ (b2). It is a consequence of the fact that $W_f \in \mathscr{W}^*$ whenever $f \in \mathscr{F}^*$.

(b2) $\Longrightarrow$ (c). Suppose that statement (b2) holds, which upon using (4.22), is equivalent to:

$$W_f(\mathbf{x}) - W_f(\mathbf{y}) = \sum_{k=1}^{n-1} \left[\frac{f(q_k)}{q_k} - \frac{f(q_{k+1})}{q_{k+1}}\right] \left[ND\left(\frac{k}{n};\mathbf{x}\right) - ND\left(\frac{k}{n};\mathbf{y}\right)\right] + \frac{f(q_n)}{q_n}\left[ND\left(\frac{n}{n};\mathbf{x}\right) - ND\left(\frac{n}{n};\mathbf{y}\right)\right] \geq 0. \tag{4.23}$$

Let $f^{(n)}(q) = q$, for all $q \in [0,1]$, and consider the weighting function $f^{(1)}$ defined by

$$f^{(1)}(q) := \begin{cases} 0, & \forall\, 0 \leq q \leq \dfrac{n-1}{n}, \\ n\left[q - \dfrac{n-1}{n}\right], & \forall\, \dfrac{n-1}{n} < q \leq 1, \end{cases} \tag{4.24}$$

and the weighting functions $f^{(k)}$ defined by

$$f^{(k)}(q) := \begin{cases} 0, & \forall\, 0 \leq q \leq \dfrac{n-k}{n}, \\ (n-k+1)\left[q - \dfrac{n-k}{n}\right], & \forall\, \dfrac{n-k}{n} < q \leq \dfrac{n-k+1}{n}, \\ q, & \forall\, \dfrac{n-k+1}{n} < q \leq 1, \end{cases} \tag{4.25}$$

where $k = 2, 3, \ldots, n-1$. One can easily check that $f^{(k)} \in \mathscr{F}^*$, for $k = 1, 2, \ldots, n$. Upon substitution into (4.23), we obtain

$$W_{f^{(k)}}(\mathbf{x}) - W_{f^{(k)}}(\mathbf{y}) = ND\left(\frac{k}{n};\mathbf{x}\right) - ND\left(\frac{k}{n};\mathbf{y}\right) \geq 0,\ \forall\, k = 1, 2, \ldots, n, \tag{4.26}$$

from which we conclude that $\mathbf{x} \geq_{ND} \mathbf{y}$.

(c) $\Longrightarrow$ (a). Suppose that $\mathbf{x} \geq_{ND} \mathbf{y}$ in which case there are two possibilities.

Case 1: $\mu(\mathbf{x}) = \mu(\mathbf{y})$. Using (4.3), it follows that $\mathbf{x} \geq_{ND} \mathbf{y}$ is equivalent to

$$ADP\left(\frac{k}{n};\mathbf{x}\right) \leq ADP\left(\frac{k}{n};\mathbf{y}\right),\ \forall k = 1, 2, \ldots, n-1, \tag{4.27}$$

and we deduce from Chateauneuf and Moyes (2006, Theorem 2) that distribution $\mathbf{x}$ can be obtained from distribution $\mathbf{y}$ by means of a finite sequence of uniform on the right progressive transfers.

Case 2: $\mu(\mathbf{x}) > \mu(\mathbf{y})$. Consider the distribution $\mathbf{z} := (z_1, \ldots, z_n)$ such that $z_i = y_i$, for all $i = 1, 2, \ldots, n-1$, and $z_n = y_n + n[\,\mu(\mathbf{x}) - \mu(\mathbf{y})\,]$. Distribution $\mathbf{z}$ is obtained from distribution $\mathbf{y}$ by means of a single increment, hence $\mathbf{z} \geq_{RO} \mathbf{y}$ and thus $\mathbf{z} \geq_{ND} \mathbf{y}$ by application of Proposition 4.1. Furthermore we have

$$ND\left(\frac{k}{n};\mathbf{z}\right) = ND\left(\frac{k}{n};\mathbf{y}\right) \leq ND\left(\frac{k}{n};\mathbf{x}\right),\ \forall k = 1, 2, \ldots, n-1, \text{ and} \tag{4.28a}$$

$$ND\left(\frac{n}{n};\mathbf{z}\right) = \mu(\mathbf{z}) = \mu(\mathbf{x}) = ND\left(\frac{n}{n};\mathbf{x}\right), \tag{4.28b}$$

hence $\mathbf{x} \geq_{ND} \mathbf{z}$ and $\mu(\mathbf{z}) = \mu(\mathbf{x})$. We are back to the previous case and we conclude that distribution $\mathbf{x}$ can be derived from distribution $\mathbf{z}$ by means of a finite sequence of uniform on the right progressive transfers. □

According to Theorem 4.1, if the non-deprivation curves of the distributions under comparison do not intersect, then there is no room for disagreement among ethical observers provided they subscribe to MON and URPT. More precisely, the distribution whose non-deprivation curve is located above that of the other distribution will be socially preferred from a welfare point of view. Requiring unanimity of value judgements over the subset of those ethical observers who make comparisons on the basis of the extended Gini social welfare function does not make a difference. The only restriction then will be that the weighting function, which captures the ethical observer's attitude towards inequality, must be star-shaped. A contrario, if the non-deprivation curves associated with two distributions intersect, then it is always possible to find two social welfare functions in the class $\mathscr{W}^*$ – and therefore two star-shaped weighting functions – that will rank these distributions in the opposite way. More importantly, Theorem 4.1 identifies the equalizing process that leads to dominance according to the non-deprivation quasi-ordering. If one distribution is ranked higher than another by the non-deprivation quasi-ordering, then it is always possible to obtain the dominating distribution from the dominated one by successive applications of uniform on the right progressive transfers and/or increments.

The non-deprivation quasi-ordering does not exhaust all the possibilities for checking whether one distribution will be judged as better than another distribution by all social welfare functions in the class $\mathscr{W}^*$. Chakravarty (1997) has already noted that the comparisons of the means of the censored distributions provide an alternative and equivalent test. Indeed, making use of (4.2), condition (c) in Theorem 4.1 reduces to

$$\mu\left(\mathbf{x}^k\right) \geq \mu\left(\mathbf{y}^k\right), \ \forall\, k = 1, 2, \ldots, n. \tag{4.29}$$

We know from Proposition 4.1 that the non-deprivation quasi-ordering is more incomplete than the generalised Lorenz quasi-ordering, which actually means that the former criterion is more demanding than the latter. Substituting (4.2) into condition (c) in Theorem 4.1, we obtain

$$GL\left(\frac{k}{n};\mathbf{x}\right) - GL\left(\frac{k}{n};\mathbf{y}\right) + \frac{n-k}{n}\left[x_k - y_k\right] \geq 0, \ \forall\, k = 1, 2, \ldots, n, \tag{4.30}$$

which indicates what is needed in addition to generalised Lorenz dominance for a distribution to be ranked above another by the non-deprivation quasi-ordering. Non-deprivation dominance necessitates that the vertical distance at any $p = k/n$ between the generalised Lorenz curves of the distributions under comparison be greater than a minimal value as the following result indicates:

**Proposition 4.4.** *Let $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$. Then $\mathbf{x} \geq_{ND} \mathbf{y}$ if and only if:*

$$GL\Big(\frac{k}{n};\mathbf{x}\Big) - GL\Big(\frac{k}{n};\mathbf{y}\Big) \geq \max\Big\{0, -\frac{n-k}{n}\Big[x_k - y_k\Big]\Big\}, \tag{4.31}$$

*for all $k = 1, 2, \ldots, n$.*

Proof. In the light of (4.30), one can immediately check that condition (4.31) implies that $\mathbf{x} \geq_{ND} \mathbf{y}$. To show the converse implication suppose that there exists $k \in \{1, 2, \ldots, n\}$ such that

$$GL\Big(\frac{k}{n};\mathbf{x}\Big) - GL\Big(\frac{k}{n};\mathbf{y}\Big) < \max\Big\{0, -\frac{n-k}{n}\Big[x_k - y_k\Big]\Big\}. \tag{4.32}$$

Case 1: $\max\left\{0, -\frac{n-k}{n}[x_k - y_k]\right\} = -\frac{n-k}{n}[x_k - y_k]$. Then

$$GL\left(\frac{k}{n};\mathbf{x}\right) - GL\left(\frac{k}{n};\mathbf{y}\right) < -\frac{n-k}{n}[x_k - y_k], \tag{4.33}$$

which appealing to (4.30) implies that $\neg[\mathbf{x} \geq_{ND} \mathbf{y}]$.

Case 2: $\max\left\{0, -\frac{n-k}{n}[x_k - y_k]\right\} = 0$. Then

$$GL\left(\frac{k}{n};\mathbf{x}\right) - GL\left(\frac{k}{n};\mathbf{y}\right) < 0, \tag{4.34}$$

which, upon invoking Proposition 4.1 implies that $\neg[\mathbf{x} \geq_{ND} \mathbf{y}]$, which makes the proof complete. □

### 4.2. The Measurement of Inequality

It is typically assumed when making welfare comparisons that efficiency and equity considerations must be accounted for. An implication of the way such considerations are incorporated in most welfare criteria is that efficiency is given a primer as compared to equity. The non-deprivation quasi-ordering is no exception: it always ranks a distribution above another distribution if the former is obtained from the latter by means of an increment. Efficiency considerations are eliminated in inequality measurement where the focus of interest is on the way income is distributed among the individuals. We are interested in the implications for inequality measurement of requiring that inequality does not increase as the result of a uniform on the right progressive transfer and we denote as $\mathscr{I}^*$ the set of ethical inequality indices that satisfy URPT. The extended Gini social evaluation function $\Xi_f$ inherits all the properties of the extended Gini social welfare function: in particular, it satisfies URPT if and only if the weighting function $f$ is star-shaped. A direct consequence is that star-shapedness of $f$ is also a necessary and sufficient condition for $I_f^R$ and $I_f^A$ to fulfil URPT.

The non-deprivation curve can be easily adapted in order to capture the relative inequality dimension of the distribution. The *relative non-deprivation curve* of distribution $\mathbf{x} \in \mathscr{Y}_n(D)$ with $D \subseteq \mathbb{R}_{++}$ is defined by $RND(p;\mathbf{x}) := ND(p;\hat{\mathbf{x}})$, for all $p \in [0,1]$. Hence the relative non-deprivation curve is nothing else than the non-deprivation curve of the reduced distribution.

Definition 4.4. Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$ with $D \subseteq \mathbb{R}_{++}$, we will say that $\mathbf{x}$ *relative non-deprivation dominates* $\mathbf{y}$, which we write $\mathbf{x} \geq_{RND} \mathbf{y}$, if and only if

$$RND\left(\frac{k}{n};\mathbf{x}\right) \geq RND\left(\frac{k}{n};\mathbf{y}\right), \ \forall\, k = 1, 2, \ldots, (n-1). \tag{4.35}$$

The following result, which establishes the connections between relative non-deprivation dominance, uniform on the right progressive transfers and scale transformations, and the requirement of unanimity among the ethical observers – in particular those of the extended Gini type – who subscribe to URPT and SI, follows from Theorem 4.1.

**Theorem 4.2.** *Let $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$ with $D \subseteq \mathbb{R}_{++}$. The following four statements are equivalent:*

(a) $\mathbf{x}$ *is obtained from* $\mathbf{y}$ *by means of a scale transformation and/or a finite sequence of uniform on the right progressive transfers.*
(b) $I^R(\mathbf{x}) \leq I^R(\mathbf{y})$, *for all* $I^R \in \mathscr{I}^* \cap \mathscr{I}^R$.
(b2) $I_f^R(\mathbf{x}) \leq I_f^R(\mathbf{y})$, *for all* $f \in \mathscr{F}^*$.
(c) $\mathbf{x} \geq_{RND} \mathbf{y}$.

Proof.

(a) $\Longrightarrow$ (b). Suppose first that $\mathbf{x} = \lambda\mathbf{y}$ for some $\lambda > 0$. Then, we have $\mu(\mathbf{x}) = \mu(\lambda\mathbf{y}) = \lambda\,\mu(\mathbf{y})$ and $\Xi_f(\mathbf{x}) = \Xi_f(\lambda\mathbf{y}) = \lambda\,\Xi_f(\mathbf{y})$, which follows from the fact that the mean and the equally distributed equivalent income of the extended Gini social welfare function are homogenous of degree one. Upon substitution into the definition of $I^R$, we obtain

$$I^R(\mathbf{x}) = 1 - \frac{\Xi_f(\lambda\mathbf{y})}{\mu(\lambda\mathbf{y})} = 1 - \frac{\lambda\Xi_f(\mathbf{y})}{\lambda\mu(\mathbf{y})} = I^R(\mathbf{y}). \tag{4.36}$$

Next, if $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a single uniform on the right transfer, then $\mu(\mathbf{x}) = \mu(\mathbf{y})$ and $W(\mathbf{x}) \geq W(\mathbf{y})$. Using (2.4) this implies that

$$I^R(\mathbf{x}) = 1 - \frac{\Xi_f(\mathbf{x})}{\mu(\mathbf{x})} \leq 1 - \frac{\Xi_f(\mathbf{y})}{\mu(\mathbf{y})} = I^R(\mathbf{y}). \tag{4.37}$$

The result generalises to an arbitrary sequence of uniform on the right progressive transfers by transitivity.

(b) $\Longrightarrow$ (b2). It is a consequence of the fact that $I_f^R \in \mathscr{I}^* \cap \mathscr{I}^R$ whenever $f \in \mathscr{F}^*$.

(b2) $\Longrightarrow$ (c). Suppose that statement (b2) holds, which upon using scale invariance, is equivalent to:

$$I_f^R(\mathbf{x}) = 1 - \frac{\Xi_f(\mathbf{x})}{\mu(\mathbf{x})} = 1 - \frac{\Xi_f(\hat{\mathbf{x}})}{\mu(\hat{\mathbf{x}})} \leq 1 - \frac{\Xi_f(\hat{\mathbf{y}})}{\mu(\hat{\mathbf{y}})} = 1 - \frac{\Xi_f(\mathbf{y})}{\mu(\mathbf{y})} = I_f^R(\mathbf{y}), \tag{4.38}$$

for all $f \in \mathscr{F}^*$. Since $\mu(\hat{\mathbf{x}}) = \mu(\hat{\mathbf{y}}) = 1$, condition (4.38) is equivalent to $\Xi_f(\hat{\mathbf{x}}) \geq \Xi_f(\hat{\mathbf{y}})$. Making use of (2.4) we conclude that $W_f(\hat{\mathbf{x}}) \geq W_f(\hat{\mathbf{y}})$ that is:

$$\sum_{i=1}^{n} \left[ f\left(\frac{n-i+1}{n}\right) - f\left(\frac{n-i}{n}\right)\right] \hat{x}_i \geq \sum_{i=1}^{n} \left[ f\left(\frac{n-i+1}{n}\right) - f\left(\frac{n-i}{n}\right)\right] \hat{y}_i. \tag{4.39}$$

Consider the weighting functions $f^{(k)} \in \mathscr{F}^*$ used in the proof of Theorem 4.1. Upon substitution into (4.39), we obtain

$$W_{f^{(k)}}(\hat{\mathbf{x}}) = RND\left(\frac{k}{n};\mathbf{x}\right) \geq RND\left(\frac{k}{n};\mathbf{y}\right) = W_{f^{(k)}}(\hat{\mathbf{y}}),\ \forall\, k = 1, 2, \ldots, (n-1), \tag{4.40}$$

from which we conclude that $\mathbf{x} \geq_{RND} \mathbf{y}$.

(c) $\Longrightarrow$ (a). Suppose that $\mathbf{x} \geq_{RND} \mathbf{y}$, in which case there are three possibilities. If $\mu(\mathbf{x}) = \mu(\mathbf{y})$, then this is equivalent to $\mathbf{x} \geq_{ND} \mathbf{y}$ and it follows from Theorem 4.1 that $\mathbf{x}$ can be obtained from $\mathbf{y}$ by means of a finite sequence of uniform on the right progressive transfers. If $\mu(\mathbf{x}) > \mu(\mathbf{y})$, then choose $\lambda = \mu(\mathbf{x})/\mu(\mathbf{y}) > 1$. We have $\mathbf{x} \geq_{ND} \lambda\mathbf{y} \sim_{RND} \mathbf{y}$ and $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a finite sequence of uniform on the right progressive transfers and a scale transformation. Finally if $\mu(\mathbf{x}) < \mu(\mathbf{y})$, then we choose $\lambda = \mu(\mathbf{y})/\mu(\mathbf{x}) > 1$ and we have

$\mathbf{x} \sim_{RND} \lambda\mathbf{x} \geq_{ND} \mathbf{y}$. Again $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of one scale transformation and a finite sequence of uniform on the right progressive transfers. □

Turning next our attention to absolute inequality we define the *absolute non-deprivation curve* of distribution $\mathbf{x} \in \mathscr{Y}_n(D)$ by $AND(p; \mathbf{x}) := ND(p; \tilde{\mathbf{x}})$, for all $p \in [0,1]$. The absolute non-deprivation curve obtains by application of the non-deprivation curve to the centered distribution.

Definition 4.5. Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$, we will say that $\mathbf{x}$ *absolute non-deprivation dominates* $\mathbf{y}$, which we write $\mathbf{x} \geq_{AND} \mathbf{y}$, if and only if

$$AND\left(\frac{k}{n}; \mathbf{x}\right) \geq AND\left(\frac{k}{n}; \mathbf{y}\right), \ \forall\, k = 1, 2, \ldots, (n-1). \tag{4.41}$$

The following result constitutes the counterpart of Theorem 4.2 when one is interested in absolute inequality.

**Theorem 4.3.** *Let $\mathbf{x}, \mathbf{y} \in \mathscr{Y}_n(D)$. The following four statements are equivalent:*
(a) *$\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a translation transformation and/or a finite sequence of uniform on the right progressive transfers.*
(b) *$I^A(\mathbf{x}) \leq I^A(\mathbf{y})$, for all $I^A \in \mathscr{I}^* \cap \mathscr{I}^A$.*
(b2) *$I^A_f(\mathbf{x}) \leq I^A_f(\mathbf{y})$, for all $f \in \mathscr{F}^*$.*
(c) *$\mathbf{x} \geq_{AND} \mathbf{y}$.*

Proof.
(a) $\Longrightarrow$ (b2). Suppose first that $\mathbf{x} = \mathbf{y} + \gamma\mathbf{1}$ for some $\gamma \in \mathbb{R}$, where $\mathbf{1} := (1, \ldots, 1)$. Then we have $\mu(\mathbf{x}) = \mu(\mathbf{y} + \gamma\mathbf{1}) = \mu(\mathbf{y}) + \gamma$ and $\Xi_f(\mathbf{x}) = \Xi_f(\mathbf{y} + \gamma\mathbf{1}) = \Xi_f(\mathbf{y}) + \gamma$, which follows from the fact that the mean and the equally distributed equivalent income of the extended Gini social welfare function are translatable of degree one. Upon substitution into the definition of $I^A$, we obtain that

$$I^A(\mathbf{x}) = \mu(\mathbf{y}) + \gamma - \Xi_f(\mathbf{y}) - \gamma = \mu(\mathbf{y} + \gamma\mathbf{1}) - \Xi_f(\mathbf{y} + \gamma\mathbf{1}) = I^A(\mathbf{y} + \gamma\mathbf{1}). \tag{4.42}$$

Next if $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a single uniform on the right progressive transfer, then $\mu(\mathbf{x}) = \mu(\mathbf{y})$ and $W(\mathbf{x}) \geq W(\mathbf{y})$. Appealing to (2.5) this implies that:

$$I^A(\mathbf{x}) = \mu(\mathbf{x}) - \Xi_f(\mathbf{x}) \leq \mu(\mathbf{y}) - \Xi_f(\mathbf{y}) = I^A(\mathbf{y}). \tag{4.43}$$

The result generalises to an arbitrary sequence of uniform on the right progressive transfers by transitivity.

(b) $\Longrightarrow$ (b2). It is a consequence of the fact that $I^A_f \in \mathscr{I}^* \cap \mathscr{I}^A$ whenever $f \in \mathscr{F}^*$.

(b2) $\Longrightarrow$ (c). Suppose that statement (b2) holds, which upon using translation invariance, is equivalent to:

$$I^A_f(\mathbf{x}) = \mu(\mathbf{x}) - \Xi_f(\mathbf{x}) = \mu(\tilde{\mathbf{x}}) - \Xi_f(\tilde{\mathbf{x}}) \leq \mu(\tilde{\mathbf{y}}) - \Xi_f(\tilde{\mathbf{y}}) = \mu(\mathbf{y}) - \Xi_f(\mathbf{y}) = I^A_f(\mathbf{y}), \tag{4.44}$$

for all $f \in \mathscr{F}^*$. Since $\mu(\tilde{\mathbf{x}}) = \mu(\tilde{\mathbf{y}}) = 0$, condition (4.44) is equivalent to $\Xi_f(\tilde{\mathbf{x}}) \geq \Xi_f(\tilde{\mathbf{y}})$. Making use of (2.5) we conclude that $W_f(\tilde{\mathbf{x}}) \geq W_f(\tilde{\mathbf{y}})$ that is:

$$\sum_{i=1}^{n} \left[ f\left(\frac{n-i+1}{n}\right) - f\left(\frac{n-i}{n}\right) \right] \tilde{x}_i \geq \sum_{i=1}^{n} \left[ f\left(\frac{n-i+1}{n}\right) - f\left(\frac{n-i}{n}\right) \right] \tilde{y}_i. \tag{4.45}$$

Consider the weighting functions $f^{(k)} \in \mathscr{F}^*$ used in the proof of Theorem 4.1. Upon substitution into (4.45), we obtain

$$(4.46)\qquad W_{f^{(k)}}(\tilde{\mathbf{x}}) = AND\left(\frac{k}{n};\mathbf{x}\right) \geq AND\left(\frac{k}{n};\mathbf{y}\right) = W_{f^{(k)}}(\tilde{\mathbf{y}}),\ \forall\ k = 1, 2, \ldots, (n-1),$$

from which we conclude that $\mathbf{x} \geq_{AND} \mathbf{y}$.

(c) $\Longrightarrow$ (a). Suppose that $\mathbf{x} \geq_{AND} \mathbf{y}$, in which case there are three possibilities. If $\mu(\mathbf{x}) = \mu(\mathbf{y})$, then this is equivalent to $\mathbf{x} \geq_{ND} \mathbf{y}$ and it follows from Theorem 4.1 that $\mathbf{x}$ can be obtained from $\mathbf{y}$ by means of a finite sequence of uniform on the right progressive transfers. If $\mu(\mathbf{x}) > \mu(\mathbf{y})$, then choose $\gamma = \mu(\mathbf{x}) - \mu(\mathbf{y}) > 0$. We have $\mathbf{x} \geq_{ND} \mathbf{y} + \gamma\mathbf{1} \sim_{AND} \mathbf{y}$ and $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a finite sequence uniform on the right progressive transfers and a translation transformation. Finally if $\mu(\mathbf{x}) < \mu(\mathbf{y})$, then we choose $\gamma = \mu(\mathbf{y}) - \mu(\mathbf{x}) > 0$ and we have $\mathbf{x} \sim_{AND} \mathbf{x}+\gamma\mathbf{1} \geq_{ND} \mathbf{y}$. Again $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of one translation transformation and a finite sequence of uniform on the right progressive transfers. □

We note that no further restriction in addition to star-shapedness is needed for the relative (resp. absolute) inequality indices inherited from the extended Gini social welfare function to be consistent with the relative (resp. absolute) non-deprivation quasi-ordering.[14] Because URPT is a weaker condition than the usual PT, there is a high presumption that the relative and absolute non-deprivation quasi-orderings will provide far more incomplete rankings than the ones generated by the relative and absolute Lorenz quasi-orderings. As we will see later on the application of our criteria to the comparison of a range of countries confirms this conjecture.

## 5. Discussion of Related Results in the Literature

An examination of the literature suggests that our approach and results overlap to a certain degree with already published work mainly by Chakravarty (1997) on the one hand and Chateauneuf and Moyes (2006) on the other hand. We certainly do not claim for originality but we would like to point at a number of differences with these existing works and clarify some points. An alternative approach for measuring deprivation has been proposed by Yitzhaki (1979) and subsequently refined by Hey and Lambert (1980). We devote some space to a discussion of their approach emphasizing the main differences with ours.

As we mentioned above what we call the non-deprivation curve in this paper has already been known for some time in the literature as the generalised satisfaction curve introduced by Chakravarty (1997). However, starting from its definition, it is not completely clear what the generalised satisfaction curve looks like. In particular, examination of Chakravarty *et al.* (1995, Figure 1) suggests that the satisfaction curve is continuously increasing over the interval $[0, 1]$, while our definition of the non-deprivation curve insists that it is a stepwise function with a finite number of discontinuities. Actually, the non-deprivation curve is a modified version of the quantile curve, where the first differences in incomes are given different weights, which are decreasing at a constant rate with the individuals' ranks. From a practical point of view, the interpolation introduced implicitly by Chakravarty *et al.* (1995) in their definition of the generalised satisfaction curve does not pose a problem as long as one is interested in the

[14] This may be contrasted with the utilitarian framework where the consistency of the relative and absolute inequality indices with relative and absolute Lorenz dominance implies restrictions on the utility function that go far beyond concavity (see e.g. Ebert (1988)).

comparisons of distributions for populations of *fixed* size. Things happen to be very different when the populations involved have different sizes, in which case Chakravarty (1997)'s generalised satisfaction criterion and our non-deprivation criterion may lead to different rankings of the distributions under comparison.

Up to the restriction that the distributions have the same means, our Theorem 4.1 looks at first sight quite similar to Theorem 3 in Chakravarty (1997). However, there is a major difference that concerns the procedure used in order to convert the dominated distribution into the dominating one: a *fair transformation* in Chakravarty (1997) and a sequence of *uniform on the right progressive transfer*s in our case. Following Chakravarty (1997), we will say that distribution $\mathbf{x}$ is obtained from distribution $\mathbf{y}$ by means of a fair transformation if there exists a vector $\mathbf{u} := (u_1, \dots, u_n)$ such that $x_i = y_i + u_i$, for all $i = 1, 2, \dots, n$, and

$$u_k \geq \frac{1}{n-k} \sum_{j=k+1}^{n} u_j, \; \forall\, k = 1, 2, \dots, n-1. \tag{5.1}$$

Using the fact that $\mathbf{u} = \mathbf{x} - \mathbf{y}$ and upon substitution, condition (5.1) reduces to

$$x_k - y_k \geq \frac{1}{n-k} \sum_{j=k+1}^{n} [x_j - y_j]\,, \; \forall\, k = 1, 2, \dots, n-1, \tag{5.2}$$

which, upon rearranging terms, proves finally to be equivalent to

$$ADP\left(\frac{k}{n}; \mathbf{x}\right) \leq ADP\left(\frac{k}{n}; \mathbf{y}\right), \; \forall\, k = 1, 2, \dots, n-1, \tag{5.3}$$

hence $\mathbf{x} \geq_{ADP} \mathbf{y}$. Invoking (4.3), we obtain that this is equivalent to $\mathbf{x} \geq_{ND} \mathbf{y}$ when $\mu(\mathbf{x}) = \mu(\mathbf{y})$. To conclude: requiring that distribution $\mathbf{x}$ is obtained from distribution $\mathbf{y}$ by means of a fair transformation is but a different way of imposing that $\mathbf{x} \geq_{ND} \mathbf{y}$. Therefore, the notion of a fair transformation adds little to our understanding of the implicit equalising process that leads to non-deprivation dominance. Although a uniform on the right progressive transfer is admittedly a more complex operation than a progressive transfer, it is far more elementary than a fair transformation. For this reason it is believed to be more informative than a fair transformation, making it easier to comprehend what is precisely meant by inequality reduction.

To contrast our results with those of Chateauneuf and Moyes (2006), we start with the definition of the extended Gini social welfare function given in (2.7). Letting $q_i := (n-i+1)/n$, for all $i = 1, 2, \dots, n$, and upon straightforward manipulation, we obtain

$$W_f(\mathbf{x}) = \mu(\mathbf{x}) - \sum_{i=1}^{n-1} \left[\frac{f(q_i)}{q_i} - \frac{f(q_{i+1})}{q_{i+1}}\right] ADP\left(\frac{i}{n}; \mathbf{x}\right), \tag{5.4}$$

where we have made use of the fact that $ADP(1; \mathbf{x}) = 0$ (for details see Chateauneuf and Moyes (2006)). If the weighting function $f$ is star-shaped, then it is sufficient for welfare to improve that mean income increases and that the absolute deprivation curve moves downwards. This suggests a two-stage procedure for evaluating income distributions: first, the absolute deprivation curves of the distributions are compared and, if they do not cross, then the means are computed in a second stage. The distribution that comes with the higher mean and the lower absolute deprivation curve is considered better from a social welfare point of view. Formally this criterion can be written as:

$$\mu(\mathbf{x}) \geq \mu(\mathbf{y}) \text{ and } ADP\left(\frac{k}{n}; \mathbf{x}\right) \leq ADP\left(\frac{k}{n}; \mathbf{y}\right), \; \forall\, k = 1, 2, \dots, (n-1). \tag{5.5}$$

This is reminiscent of the procedure based on the comparisons of the relative Lorenz curves and mean incomes that constituted the starting point of Shorrocks (1983). The problem with this way of proceeding – as Shorrocks (1983) emphasized – is that the distribution with the higher Lorenz curve has also generally the lowest mean, which makes the conclusion of this two-stage evaluation process ambiguous. A similar observation can be made here even though it remains to be investigated whether the distributions with higher means are also those where the feelings of deprivation are the more exacerbated among the population. Then the fact that two distributions do not pass the test described by (5.5) does not preclude the possibility that one distribution be ranked above the other one by all social welfare functions in the class $\mathscr{W}^*$. Obviously, if $W_f(\mathbf{x}) \geq W_f(\mathbf{y})$, for all $f \in \mathscr{F}^*$, then $\mu(\mathbf{x}) \geq \mu(\mathbf{y})$. But it may well be case that the absolute deprivation curves of $\mathbf{x}$ and $\mathbf{y}$ intersect – or what is even worse that the absolute deprivation curve of $\mathbf{x}$ is everywhere above that of $\mathbf{y}$ – making it impossible to rank distributions $\mathbf{x}$ and $\mathbf{y}$. Consider for instance the distributions $\mathbf{x} = (1, 4, 5, 8)$ and $\mathbf{y} = (1, 2, 6, 7)$, and note that $\mathbf{x}$ and $\mathbf{y}$ cannot be ordered by the rank order criterion. One can easily verify that the absolute deprivation curves of distributions $\mathbf{x}$ and $\mathbf{y}$ intersect while $\mu(\mathbf{x}) > \mu(\mathbf{y})$, making it impossible to conclude whether $\mathbf{x}$ will be ranked above $\mathbf{y}$ or not by all ethical observers who subscribe to URPT and MON. However, the non-deprivation curve of $\mathbf{x}$ is everywhere located above that of $\mathbf{y}$, and Theorem 4.1 confirms that $W(\mathbf{x}) \geq W(\mathbf{y})$, for all $W \in \mathscr{W}^*$.[15]

Finally we would like to contrast our approach with the alternative method proposed by Yitzhaki (1979) and further developed by Hey and Lambert (1980) for measuring overall deprivation in the society. The concepts of *individual* deprivation referred to in the two above papers and in ours are identical: an individual's deprivation is associated with the aggregate gap between her income and those of the individuals richer than her, up to a scale factor equal to the population size. It follows that the measures of individual non-deprivation give the same value for a given individual even though, in Yitzhaki (1979) an individual is associated with her income, while in our case she is identified by her relative position on the income scale. However, these two approaches differ in the way the individuals' deprivation levels are aggregated over the population in order to derive the overall deprivation in the society. While Hey and Lambert (1980) compare the individual non-deprivation profiles at each income level, we make this comparison for individuals located at the same position on the income scale. Under the condition that the distributions under comparison have equal means, Hey and Lambert (1980)'s dominance criterion reduces to

$$(5.6) \qquad \frac{1}{n} \sum_{i=1}^{m(z;\mathbf{x})} (z - x_i) \leq \frac{1}{n} \sum_{i=1}^{m(z;\mathbf{y})} (z - y_i)\,,\ \forall\, z \in \mathbb{R},$$

where $m(z;\mathbf{x})$ and $m(z;\mathbf{y})$ represent the number of individuals with an income no greater than $z$ in situations $\mathbf{x}$ and $\mathbf{y}$, respectively. In other words, poverty in situation $\mathbf{x}$ must not exceed

[15] Actually Chakravarty (1997, Theorem 9) has identified the class of social welfare functions with the property that, if distribution $\mathbf{x}$ is unanimously preferred to distribution $\mathbf{y}$, then condition (5.5) will be met, and conversely. Monotonicity has to be replaced by the weaker requirement of *incremental improvement* according to which welfare increases as the result of equal additions to all incomes (see Kolm (1976), Shorrocks (1983)). This equivalence does not hold when one restricts attention to the extended Gini social welfare functions even though by definition they satisfy the incremental improvement condition. Indeed, we know from Theorem 4.1 that the ranking of distributions generated by unanimous agreement among all extended Gini ethical observers subscribing to MON and URPT proves to be identical to that implied by the non-deprivation criterion. However, non-deprivation domination of one distribution by another is compatible with the fact that their absolute deprivation curves cross, as distributions $\mathbf{x}$ and $\mathbf{y}$ above illustrate.

poverty in situation $\mathbf{y}$ for all possible poverty lines $z$, which appealing to Foster and Shorrocks (1988, Theorem 2) is equivalent to $\mathbf{x} \geq_{GL} \mathbf{y}$. Proposition 4.1 makes clear that our approach is less decisive than that of Hey and Lambert (1980) because non-deprivation dominance implies generalised Lorenz dominance but not the converse. On the other hand it is based on value judgements that might be considered less controversial than the ones implicitly adopted by Hey and Lambert (1980).

## 6. Welfare and Inequality Comparisons Across 17 Countries

The generalised Lorenz and the non-deprivation quasi-orderings capture different features of the distributions under comparison. On the other hand, Proposition 4.1 indicates that non-deprivation dominance implies generalised Lorenz dominance. It is then natural to question how far away the ranking generated by the non-deprivation criterion will be from that implied by the application of the generalised Lorenz test. Similar questions arise for the relative and absolute non-deprivation criteria as compared with the relative and absolute Lorenz ones. In order to contrast the rankings of distributions our new criteria generate with those obtained in the traditional framework, we have chosen to measure well-being and inequality in 17 countries using income data from the Luxembourg Income Study (LIS).

Table 6.1: The Countries Under Comparison

| No | Country | Year | DHI per Capita | Sample Size |
|---|---|---|---|---|
| 1 | Mexico | 2000 | 5 788.32 | 10 072 |
| 2 | Poland | 1999 | 6 330.86 | 30 812 |
| 3 | Hungary | 1999 | 6 513.81 | 1 927 |
| 4 | Greece | 2000 | 13 598.82 | 3 873 |
| 5 | Spain | 2000 | 17 627.86 | 4 761 |
| 6 | Finland | 2000 | 18 152,60 | 10 419 |
| 7 | Sweden | 2000 | 18 356.47 | 14 471 |
| 8 | Netherlands | 1999 | 19 249.84 | 4 331 |
| 9 | Germany | 2000 | 20 123.77 | 10 982 |
| 10 | United Kingdom | 1999 | 20 733.02 | 24 830 |
| 11 | Austria | 2000 | 20 945.18 | 2 329 |
| 12 | Belgium | 2000 | 21 177.11 | 2 080 |
| 13 | Canada | 2000 | 23 583.00 | 28 902 |
| 14 | Norway | 2000 | 24 603.77 | 12 870 |
| 15 | Switzerland | 2000 | 25 825.90 | 3 627 |
| 16 | USA | 2000 | 29 028.81 | 49 294 |
| 17 | Luxembourg | 2000 | 29 504.36 | 2 415 |

For each household, the LIS database indicates the disposable household income (DHI), that is its total income after taxation and transfer payments, and the household type determined by its composition and size. Incomes are provided in local currencies in order to facilitate comparisons within a country over time. These figures have been converted using the purchasing power parities (PPP) proposed by the OECD in order to make them comparable across countries. To take family needs into account we have chosen to adjust household incomes by means of equivalence scales. Although we are aware that there are good reasons that militate for the avoidance of this method of adjustment, we have adopted this procedure for simplicity to the extent that this exercise is purely illustrative. Taking the single adult as the reference type, the household's equivalent income is obtained by deflating the household's total income by the OECD equivalence scale equal to an isoelastic transformation of family size. More precisely, the equivalent income of a household with income $y$ and size $m$ is given by $E(y; m) = y/m^\rho$, for all $m \geq 1$ and all $y \in D$, where $\rho \in [0, 1]$ is the scale elasticity. When $\rho = 0$, no adjustment is made for family needs and the household's equivalent income is equal to household's income. When $\rho = 1$, the household's equivalent income is equal to per capita income. In general one chooses an intermediate value of the scale elasticity and we have followed this practice by setting $\rho = .5$ as it is done for instance in Atkinson, Rainwater, and Smeeding (1995). Table 6.1 gives the list of countries we have retained and indicates for each of these the year when the data were collected, the corresponding adjusted DHIs per capita, and the number of households in the samples.

Table 6.2: Ranking of Countries by the Rank Order Criterion (IU)

| | | Country $j$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No | Country $i$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 1 | Mexico | # | # | # | 0 | 0 | 0 | # | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Poland | | # | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | Hungary | | | # | # | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | Greece | | | | # | 0 | 0 | # | 0 | 0 | 0 | 0 | 0 | 0 | 0 | # | 0 |
| 5 | Spain | | | | | # | # | # | # | 0 | # | # | 0 | 0 | 0 | # | 0 |
| 6 | Finland | | | | | | # | # | # | # | # | # | # | # | # | # | 0 |
| 7 | Sweden | | | | | | | # | # | # | # | # | # | 0 | 0 | # | 0 |
| 8 | Netherlands | | | | | | | | # | # | # | # | # | # | # | # | 0 |
| 9 | Germany | | | | | | | | | # | # | # | # | 0 | 0 | # | 0 |
| 10 | United Kingdom | | | | | | | | | | # | # | # | # | 0 | # | # |
| 11 | Austria | | | | | | | | | | | # | # | # | 0 | # | 0 |
| 12 | Belgium | | | | | | | | | | | | # | # | # | # | # |
| 13 | Canada | | | | | | | | | | | | | # | 0 | # | # |
| 14 | Norway | | | | | | | | | | | | | | # | # | # |
| 15 | Switzerland | | | | | | | | | | | | | | | # | # |
| 16 | USA | | | | | | | | | | | | | | | | # |
| 17 | Luxembourg | | | | | | | | | | | | | | | | |

"#" means that countries $i$ and $j$ are not comparable.
"1" means that country $i$ dominates country $j$.
"0" means that country $j$ dominates country $i$.
"≈" means that countries $i$ and $j$ are equivalent.

For each country, the individuals' equivalent incomes are aggregated in centiles, where each of these is assigned the average income of the individuals belonging to it. It would have been possible to adopt alternative groupings, for instance deciles as in Atkinson *et al.* (1995). While this way of proceeding is computationally less time consuming, it results in an artificial smoothing of the distribution of incomes and may cause a substantial loss of information about the shape of the distributions. On the contrary, it is expected that the division of the population into centiles is sufficiently detailed to preserve most on the information contained in the LIS database. As we have seen, our dominance criteria amount to comparing particular modifications of the quantile curves of the distributions. It follows that each of our criteria will require a finite number of comparisons to be made at each abscissa $k/K$, for $k = 1, 2, \ldots, K$, where $K = 100$, for deciding whether one distribution dominates another or not. In order to take into account the fact that the distributions under comparisons are samples drawn from larger populations, we have implemented statistical inference using suc-

Table 6.3: Ranking of Countries by the Generalised Lorenz Criterion (IU)

| | | Country $j$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No | Country $i$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 1 | Mexico | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Poland | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | Hungary | | | # | # | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | Greece | | | | # | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | # | 0 |
| 5 | Spain | | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | # | 0 |
| 6 | Finland | | | | | | # | # | # | # | # | # | # | # | # | # | 0 |
| 7 | Sweden | | | | | | | # | # | # | 0 | # | # | 0 | 0 | # | 0 |
| 8 | Netherlands | | | | | | | | # | # | # | # | # | # | # | # | 0 |
| 9 | Germany | | | | | | | | | # | # | # | # | 0 | 0 | # | 0 |
| 10 | United Kingdom | | | | | | | | | | # | # | # | 0 | 0 | # | 0 |
| 11 | Austria | | | | | | | | | | | # | # | # | 0 | # | 0 |
| 12 | Belgium | | | | | | | | | | | | # | # | 0 | # | 0 |
| 13 | Canada | | | | | | | | | | | | | 0 | 0 | # | 0 |
| 14 | Norway | | | | | | | | | | | | | | # | # | 0 |
| 15 | Switzerland | | | | | | | | | | | | | | | # | 0 |
| 16 | USA | | | | | | | | | | | | | | | | 0 |
| 17 | Luxembourg | | | | | | | | | | | | | | | | |

"#" means that countries $i$ and $j$ are not comparable.
"1" means that country $i$ dominates country $j$.
"0" means that country $j$ dominates country $i$.
"≈" means that countries $i$ and $j$ are equivalent.

cessively the *intersection-union* (IU) method advocated by Howes (1994) and Kaur, Prakasao, and Singh (1994), and the *union-intersection* (UI) method introduced by Bishop and Formby (1999).[16] The general strategy consists in viewing the set of inequalities corresponding to a domination relationship as a statistical hypothesis and it is described in Appendix A. In this illustration we have performed both tests for each dominance criterion, at the 95 % confidence level. In order to save space, we only present in the text the tables giving the results of the pairwise comparisons obtained under the restrictive IU method for each criterion, and we relegate to Appendix B the corresponding tables for the UI method. Substituting the more liberal UI method for the IU one results in an appreciable gain in decisiveness, as it is shown in Table 6.9, but it does not affect the general conclusions drawn from the application of the dominance criteria.

Considering the welfare ranking of countries we have applied successively the rank order, the generalised Lorenz and the non-deprivation quasi-orderings to the distributions of adjusted household incomes. The most striking result is the rather good performance of rank order dominance, which allows us to rank conclusively 65 out of 136 pairs of countries hence a success rate of 47.79 %. We have indicated in Table 6.2 the results of the pairwise comparisons according to rank order dominance. The 17 countries are listed in order of adjusted DHI per capita ranging from the lowest (Mexico) to the highest (Luxembourg) figures. The table allows us to distinguish two broad groups of countries, where the first group consisting of developed countries clearly dominates the second one composed of Mexico, Poland, Hungary

[16] Actually, most of the microdata sets provided by the member countries to the LIS are generally not randomly drawn from the national populations. The potential biases in the representativeness of the national data sets are corrected by means of the introduction of appropriate household weights.

Table 6.4: Ranking of Countries by the Non-Deprivation Criterion (IU)

| | | Country $j$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No | Country $i$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 1 | Mexico | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Poland | | # | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | Hungary | | | # | # | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | Greece | | | | # | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | # | 0 |
| 5 | Spain | | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | # | 0 |
| 6 | Finland | | | | | | # | # | # | # | # | # | # | # | # | # | 0 |
| 7 | Sweden | | | | | | | # | # | # | # | # | # | 0 | 0 | # | 0 |
| 8 | Netherlands | | | | | | | | # | # | # | # | # | # | # | # | 0 |
| 9 | Germany | | | | | | | | | # | # | # | # | 0 | 0 | # | 0 |
| 10 | United Kingdom | | | | | | | | | | # | # | # | 0 | 0 | # | 0 |
| 11 | Austria | | | | | | | | | | | # | # | # | 0 | # | 0 |
| 12 | Belgium | | | | | | | | | | | | # | # | # | # | 0 |
| 13 | Canada | | | | | | | | | | | | | 0 | 0 | # | 0 |
| 14 | Norway | | | | | | | | | | | | | | # | # | 0 |
| 15 | Switzerland | | | | | | | | | | | | | | | # | 0 |
| 16 | USA | | | | | | | | | | | | | | | | 0 |
| 17 | Luxembourg | | | | | | | | | | | | | | | | |

"#" means that countries $i$ and $j$ are not comparable.
"1" means that country $i$ dominates country $j$.
"0" means that country $j$ dominates country $i$.
"≈" means that countries $i$ and $j$ are equivalent.

and Greece. Application of generalised Lorenz dominance permits to reach definite conclusions in 87 out of 136 cases, which gives a success rate of 63.97%. Our empirical findings confirm the theoretical result that the ranking generated by the generalised Lorenz criterion is finer than the one implied by the rank order criterion. What is perhaps surprising is the magnitude of the increase in the success rate implied by the substitution of the generalised Lorenz for rank order dominance: 16% of the success rate. The results of the pairwise comparisons of countries on the basis of generalised Lorenz dominance are indicated in Table 6.3. Typically, the ranking of countries by the generalised Lorenz criterion appears to be closely related to the ordering one would get on the basis on per capita income. While the success rates we obtain are quite satisfactory, they are far below those obtained for instance by Bishop, Formby, and Thistle (1991). Applying the UI method these authors found that rank order dominance was decisive in 75% of the cases, while the application of generalised Lorenz dominance led to unambiguous rankings in 84% of the cases. Substituting the UI method for the IU one results in significant increases in our success rates more in line with those obtained by Bishop *et al.* (1991): 61.76% for rank order dominance and 76.47% for generalised Lorenz dominance.

We know from Proposition 4.1 that the non-deprivation quasi-ordering is less powerful than the generalised Lorenz quasi-ordering since the latter is implied by the former. One may therefore expect that the success rate of the non-deprivation criterion will fall below that achieved by the generalised Lorenz criterion in practice. The results of the comparisons based on the non-deprivation quasi-ordering are indicated in Table 6.4. The striking thing is that the success rate of the non-deprivation quasi-ordering is comparable to the one attained by the generalised Lorenz quasi-ordering. Indeed, the non-deprivation quasi-ordering allows to rank conclusively 84 out of 136 pairs of countries, which means that this criterion is decisive

Table 6.5: Ranking of Countries by the Relative Lorenz Criterion (IU)

| | | Country $j$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No | Country $i$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 1 | Mexico | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | # | 0 | 0 | 0 | # | 0 |
| 2 | Poland | | # | # | # | # | # | 0 | # | # | 0 | # | # | # | # | 1 | 0 |
| 3 | Hungary | | | # | # | # | # | # | # | 1 | # | # | # | # | # | 1 | 0 |
| 4 | Greece | | | | # | # | # | 0 | # | # | 0 | # | # | # | # | 1 | 0 |
| 5 | Spain | | | | | # | # | 0 | 0 | # | 0 | # | # | # | # | 1 | 0 |
| 6 | Finland | | | | | | # | # | # | 1 | # | 1 | 1 | # | 1 | 1 | # |
| 7 | Sweden | | | | | | | # | # | 1 | # | # | 1 | # | # | 1 | # |
| 8 | Netherlands | | | | | | | | # | 1 | # | # | 1 | # | # | 1 | # |
| 9 | Germany | | | | | | | | | 1 | # | # | 1 | # | # | 1 | 0 |
| 10 | United Kingdom | | | | | | | | | | # | # | # | # | 0 | # | 0 |
| 11 | Austria | | | | | | | | | | | # | 1 | # | # | 1 | # |
| 12 | Belgium | | | | | | | | | | | | # | # | # | # | 0 |
| 13 | Canada | | | | | | | | | | | | | # | 0 | 1 | 0 |
| 14 | Norway | | | | | | | | | | | | | | # | # | # |
| 15 | Switzerland | | | | | | | | | | | | | | | 1 | 0 |
| 16 | USA | | | | | | | | | | | | | | | | 0 |
| 17 | Luxembourg | | | | | | | | | | | | | | | | |

"#" means that countries $i$ and $j$ are not comparable.
"1" means that country $i$ dominates country $j$.
"0" means that country $j$ dominates country $i$.
"≈" means that countries $i$ and $j$ are equivalent.

Table 6.6: Ranking of Countries by the Relative Non-Deprivation Criterion (IU)

| | | Country $j$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No | Country $i$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 1 | Mexico | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | # | 0 | 0 | 0 | # | 0 |
| 2 | Poland | | # | # | # | # | # | 0 | # | # | 0 | # | # | # | # | # | 0 |
| 3 | Hungary | | | # | # | # | # | # | # | 1 | # | # | # | # | # | 1 | 0 |
| 4 | Greece | | | | # | # | # | 0 | # | # | # | # | # | # | # | 1 | 0 |
| 5 | Spain | | | | | # | # | 0 | # | # | 0 | # | # | # | # | # | 0 |
| 6 | Finland | | | | | | # | # | # | 1 | # | 1 | 1 | # | 1 | # | # |
| 7 | Sweden | | | | | | | # | # | 1 | # | # | 1 | # | # | # | # |
| 8 | Netherlands | | | | | | | | # | 1 | # | # | 1 | # | # | 1 | # |
| 9 | Germany | | | | | | | | | 1 | # | # | 1 | # | # | 1 | # |
| 10 | United Kingdom | | | | | | | | | | # | # | # | # | 0 | # | 0 |
| 11 | Austria | | | | | | | | | | | # | 1 | # | # | 1 | # |
| 12 | Belgium | | | | | | | | | | | | # | # | # | # | 0 |
| 13 | Canada | | | | | | | | | | | | | # | # | # | 0 |
| 14 | Norway | | | | | | | | | | | | | | # | # | # |
| 15 | Switzerland | | | | | | | | | | | | | | | # | 0 |
| 16 | USA | | | | | | | | | | | | | | | | 0 |
| 17 | Luxembourg | | | | | | | | | | | | | | | | |

"#" means that countries $i$ and $j$ are not comparable.
"1" means that country $i$ dominates country $j$.
"0" means that country $j$ dominates country $i$.
"≈" means that countries $i$ and $j$ are equivalent.

in 61.76% of the cases. Contrary to what one might have anticipated, the substitution of the non-deprivation quasi-ordering for the generalised Lorenz one does not result in a significant decrease in the number of pairs of countries that can be ranked. Invoking again Proposition 4.1, this means that the rankings of countries provided by the generalised Lorenz and the non-deprivation quasi-orderings are almost identical, something that can be checked by comparing Tables 6.3 and 6.4. Actually, the only difference concerns three pairs of countries that are ordered by the generalised Lorenz criterion while they are not comparable under the non-deprivation criterion: Poland and Hungary, Sweden and Austria, and Belgium and Switzerland. As it is expected, the non-deprivation quasi-ordering performs better when the more liberal UI method is used instead of the IU method. In this case one obtains a conclusive verdict in 101 out of 136 possible pairs of countries, hence a success rate of 74.26% to be contrasted with the 61.76% score attained under the IU method. But here again the substitution of the generalised Lorenz criterion for the non-deprivation one does not result in a substantial gain in decisiveness. There are only three pairs of countries for which the generalised Lorenz criterion is decisive under the UI method while the non-deprivation is not: Sweden and Germany, Netherlands and Austria, Germany and Belgium. However, while the gain in decisiveness under both inference methods are identical, we note that the pairs of countries concerned are distinct. To sum up, in the case of the distributions examined here, a switch from the usual generalised Lorenz criterion – or equivalently second degree stochastic dominance – to the non-deprivation criterion does not result in a dramatic change in the picture of the distribution of well-being across countries. But the non-deprivation criterion tells us much more about the equalising process that leads to generalised Lorenz dominance. It indicates the nature of the equalising transformations that were needed in addition to increments for deriving the dominating distribution from the dominated one. Actually, these transformations are not arbitrary progressive transfers – as the rankings of the countries by the generalised Lorenz quasi-ordering might have suggested – but uniform on the right progressive transfers.[17] Although it is certainly premature to generalise, this example provides a response to a serious objection that could be made against the non-deprivation criteria: its lower degree of decisiveness as compared with the generalised Lorenz criterion.

The application of the standard Lorenz tests and of the non-deprivation based criteria to the comparisons of inequality results in more contrasted pictures. Tables 6.5 and 6.6 indicate the results of the pairwise comparisons of countries on the basis of the relative Lorenz and of the relative non-deprivation quasi-orderings. Similarly, the rankings of countries generated by the absolute Lorenz and by the absolute non-deprivation quasi-orderings are summarized in Tables 6.7 and 6.8, respectively. A first remark is that, on the whole, the inequality quasi-orderings are far less discriminatory than the corresponding welfare quasi-orderings. This originates in part in the fact that the size effect due to differences in disposable per capita income, which explains much of the rankings of countries obtained on the basis of the welfare criteria, is eliminated by means of the normalisation procedures implicit in the definition of our inequality criteria. Now it is the way the aggregate adjusted income is split between the households that plays a role in the determination of the rankings of the distributions. A second remark is that the inequality quasi-orderings rooted in the notion of non-deprivation are less powerful than those derived from the generalised Lorenz curve, something which is more in

[17] We know from the proof of Theorem 4.1 that, if country $i$ non-deprivation dominates country $j$, then the distribution of income in country $i$ can be obtained from that of country $j$ by (i) giving an additional income to the richest individual in country $j$ that makes the mean incomes of the countries equal, and (ii) performing a finite number of uniform on the right progressive transfers.

Table 6.7: Ranking of Countries by the Absolute Lorenz Criterion (IU)

| | | Country $j$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No | Country $i$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 1 | Mexico | # | # | # | 1 | 1 | 1 | # | 1 | 1 | # | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | Poland | | # | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | Hungary | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | Greece | | | | 1 | # | # | # | 1 | 1 | # | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | Spain | | | | | # | # | # | # | 1 | # | # | 1 | 1 | 1 | 1 | # |
| 6 | Finland | | | | | | # | # | # | 1 | # | 1 | 1 | 1 | 1 | 1 | 1 |
| 7 | Sweden | | | | | | | # | 1 | 1 | # | 1 | 1 | 1 | 1 | 1 | 1 |
| 8 | Netherlands | | | | | | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 9 | Germany | | | | | | | | | 1 | # | # | 1 | 1 | 1 | 1 | 1 |
| 10 | United Kingdom | | | | | | | | | | # | # | # | # | # | 1 | # |
| 11 | Austria | | | | | | | | | | | # | 1 | 1 | 1 | 1 | 1 |
| 12 | Belgium | | | | | | | | | | | | # | # | # | # | # |
| 13 | Canada | | | | | | | | | | | | | # | # | 1 | # |
| 14 | Norway | | | | | | | | | | | | | | # | 1 | # |
| 15 | Switzerland | | | | | | | | | | | | | | | 1 | # |
| 16 | USA | | | | | | | | | | | | | | | | 0 |
| 17 | Luxembourg | | | | | | | | | | | | | | | | |

"#" means that countries $i$ and $j$ are not comparable.
"1" means that country $i$ dominates country $j$.
"0" means that country $j$ dominates country $i$.
"≈" means that countries $i$ and $j$ are equivalent.

Table 6.8: Ranking of Countries by the Absolute Non-Deprivation Criterion (IU)

| | | Country $j$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No | Country $i$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 1 | Mexico | # | # | # | 1 | 1 | # | # | 1 | 1 | # | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | Poland | | # | 1 | 1 | 1 | 1 | # | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | Hungary | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | Greece | | | | 1 | # | # | # | 1 | 1 | # | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | Spain | | | | | # | # | # | # | 1 | # | # | 1 | # | 1 | 1 | # |
| 6 | Finland | | | | | | # | # | # | 1 | # | 1 | 1 | 1 | 1 | 1 | # |
| 7 | Sweden | | | | | | | # | # | 1 | # | 1 | 1 | 1 | 1 | 1 | # |
| 8 | Netherlands | | | | | | | | 1 | 1 | # | 1 | 1 | 1 | 1 | 1 | 1 |
| 9 | Germany | | | | | | | | | 1 | # | # | 1 | 1 | 1 | 1 | # |
| 10 | United Kingdom | | | | | | | | | | # | # | # | # | # | # | # |
| 11 | Austria | | | | | | | | | | | # | 1 | 1 | 1 | 1 | 1 |
| 12 | Belgium | | | | | | | | | | | | # | # | # | # | # |
| 13 | Canada | | | | | | | | | | | | | # | # | 1 | # |
| 14 | Norway | | | | | | | | | | | | | | # | # | # |
| 15 | Switzerland | | | | | | | | | | | | | | | # | # |
| 16 | USA | | | | | | | | | | | | | | | | 0 |
| 17 | Luxembourg | | | | | | | | | | | | | | | | |

"#" means that countries $i$ and $j$ are not comparable.
"1" means that country $i$ dominates country $j$.
"0" means that country $j$ dominates country $i$.
"≈" means that countries $i$ and $j$ are equivalent.

line with what one would expect. Indeed, as we have already insisted, the non-deprivation inequality criteria impose restrictions on the way progressive transfers are combined and this results in more demanding conditions to be met for obtaining dominance. This is particularly the case for relative inequality where the non-deprivation approach permits only to rank 46 pairs of countries out of 136, while the relative Lorenz quasi-ordering is able to establish a conclusive verdict in 56 cases. As a consequence the success rate falls from 41.18% to 33.82% when the relative non-deprivation quasi-ordering is substituted for the relative Lorenz quasi-ordering. A similar situation obtains in the case of absolute inequality: application of the absolute Lorenz criterion allows to rank 93 pairs of countries (68.38% of the cases), while this figure falls only to 82 (60.29% of the cases) when one appeals to the absolute non-deprivation quasi-ordering. Applying the less conservative UI method rather than the IU method results in a substantial increase of the success rates for relative inequality, which move up to 63.24% for the relative non-deprivation quasi-ordering and to 74.26% for the relative Lorenz quasi-ordering. We also witness increases in the success rates for absolute inequality though these are less marked: 78.68% for the absolute non-deprivation quasi-ordering and 86.03% for the absolute Lorenz quasi-ordering. A third remark is that more pairs of countries can be ranked when one considers the absolute criteria rather than the relative ones, something which is consistent with related findings in the literature. Tables 6.7 and 6.8 suggest that the absolute inequality rankings of countries are close to the ordering one would get if the countries were ranked in decreasing order of per capita income.[18]

Table 6.9: Success Rates of the Welfare and Inequality Quasi-Orderings

| | | Social Welfare | | | Inequality | | | |
|---|---|---|---|---|---|---|---|---|
| | | RO | ND | GL | RND | RL | AND | AL |
| IU Test | Nb | 65 | 84 | 87 | 46 | 56 | 82 | 93 |
| | (%) | (47.79) | (61.76) | (63.97) | (33.82) | (41.18) | (60.29) | (68.38) |
| UI Test | Nb | 84 | 101 | 104 | 86 | 101 | 107 | 117 |
| | (%) | (61.76) | (74.26) | (76.47) | (63.24) | (74.26) | (78.68) | (86.03) |

Finally, let us come back to the two-stage procedure originating in Chateauneuf and Moyes (2006) that consists in declaring that one distribution generates higher welfare than another distribution if it has a higher mean and a lower absolute deprivation curve. Under the conservative IU method, this procedure succeeds in being decisive in only one case: Luxembourg and the USA, the former dominating the latter! This particularly bad score does not come as a surprise given the almost perfect inverse correlation that exists between the ordering of countries on the basis of mean income and the ranking obtained by resorting to the absolute deprivation criterion as Table 6.8 makes clear. When the less conservative UI method is substituted for the IU method, this procedure produces conclusive verdicts in 8 additional cases (namely, Mexico and Hungary, Poland and Hungary, Spain and Finland, Spain and Sweden, Spain and the Netherlands, Sweden and the Netherlands, Germany and Austria, United Kingdom and Austria), which amounts to a success rate of slightly less than 7%. Contrasting these figures with those obtained when the non-deprivation criterion is used – whatever the statistical inference method – gives an idea of the inefficiency of this procedure for approximating

[18] This is a mechanical consequence of the very definition of the absolute measures, where the mean is subtracted from each individual income, and of the fact that we have listed countries in increasing order of average equivalent household disposable income.

the ranking of countries implied by unanimous agreement among all ethical observers who subscribe to the MON and URPT.

## 7. Concluding Comments

Building on Chakravarty (1997) and Chateauneuf and Moyes (2006), we have proposed in this paper a method for making welfare and inequality comparisons based on the absence of deprivation. This method constitutes a natural alternative to the standard approach in normative economics, which consists in comparing the – generalised, relative and absolute – Lorenz curves of the distributions. The Lorenz criteria are all consistent with the principle of transfers, which requires that inequality decreases and welfare increases as the result of an *arbitrary* progressive transfer. The criteria we propose actually obey a weaker version of the principle of transfers: inequality decreases only for some *specific* combinations of the progressive transfers, where the positions of the donor(s) and the beneficiary of the transfer play a crucial role. Furthermore these criteria are related to the notion of deprivation that arises from the comparison for each individual of her situation with those of the individuals she considers as better-off.

Contrary to what might have been expected, preliminary empirical investigations suggest that the non-deprivation criterion performs rather well as compared with the generalised Lorenz criterion. Although it is premature to generalise at this stage, this result is nevertheless worth noting given the general skepticism one may face when proposing criteria ethically more demanding than the generalised Lorenz quasi-ordering. Things are more contrasted as far as inequality comparisons are concerned, but still there the non-deprivation based inequality quasi-orderings do not perform too badly by comparison with the standard relative and absolute Lorenz criteria. The application of the non-deprivation quasi-ordering may provide additional information about the nature of the equalising process that leads to the domination of a distribution by another. This is indisputable when the substitution of the non-deprivation quasi-ordering for the generalised Lorenz quasi-ordering does not affect the ranking of the distributions. For in this case, we get additional information concerning the structure of the modifications of the distributions that give rise to generalised Lorenz domination: these must be uniform on the right progressive transfers.

The criteria we have proposed do only generate partial rankings of the situations under comparison. While this might be considered satisfactory in some cases – for instance for the design of tax reforms – it generally provides insufficient grounds for making decisions. These criteria must therefore be considered a first round approach, which should be supplemented by the use of particular indices in the general classes we have identified. An avenue for future research would be to characterise by means of additional conditions particular social welfare functions and inequality indices among those that are consistent with the non-deprivation quasi-ordering.

The approach developed in this paper was partly motivated by the observation that the principle of transfers, which supports the generalised Lorenz criterion, does not achieve a consensus among those individuals having taken part in the questionnaire studies. It would therefore be interesting to check if the non-deprivation quasi-ordering rooted in the uniform on the right principle of transfers is more favourably accepted by the individuals participating in such studies (see Magdalou (2006) for a preliminary investigation).

## A. The Statistical Inference Methods

We are interested in the comparisons in terms of welfare and inequality of the income distributions of 17 selected countries. The *actual income distribution* of country $i$ is indicated by $X^i$ and it is not observable. Since we do not have access to the actual income distributions, we will base our comparisons on the sample data provided by the Luxembourg Income Study (LIS) database. For each country $i$ and each household $h$, the LIS database reports the disposable income $y_h^i$ and the number of persons $m_h^i$ in the sample. In order to correct for biases in the representativeness of the national samples, the LIS assigns to each household a weight $v_h^i$. The equivalent income of household $h$ is given by $x_h^i = y_h^i \,/\sqrt{m_h^i}$ and we let $w_h^i = m_h^i\, v_h^i$ be the corresponding weight. We therefore apply the standard procedure which consists in weighting the household income by the number of persons, but we acknowledge the fact that there are other possibilities (see e.g. Ebert (1997)). The *sample adjusted income distribution* of country $i$ is indicated by $(\mathbf{x}^i \,|\, \mathbf{w}^i) := \left(x_1^i, \ldots, x_{n_i}^i \,\middle|\, w_1^i, \ldots, w_{n_i}^i\right)$, where $n_i$ is the sample size. We derive the *sample cumulative distribution function* $F\left(z; (\mathbf{x}^i \,|\, \mathbf{w}^i)\right)$, which upon inversion gives the *sample quantile function* $F^{-1}\left(p; (\mathbf{x}^i \,|\, \mathbf{w}^i)\right)$. We consider $K$ quantiles and we indicate by $\mu_k\left(\mathbf{x}^i \,|\, \mathbf{w}^i\right)$ the *sample quantile mean income* for country $i$, where $k = 1, 2, \ldots, K$ (see e.g. Beach, Chow, Formby, and Slotsve (1994)). By definition each quantile $k$ contains $\sum_{h=1}^{n_i} w_h^i \,/K$ individuals and

(A.1)
$$\mu_k\left(\mathbf{x}^i \,\middle|\, \mathbf{w}^i\right) := \left(\sum_{h=1}^{n_i} w_h^i \,/\, K\right)^{-1} \int_{(k-1)/K}^{k/K} F^{-1}\left(p; \left(\mathbf{x}^i \,\middle|\, \mathbf{w}^i\right)\right) dp,$$

for $k = 1, 2, \ldots, K$. Therefore, for each country $i$ in the database we have an ordered list of $K$ sample quantile means $\mu_1\left(\mathbf{x}^i \,|\, \mathbf{w}^i\right), \mu_2\left(\mathbf{x}^i \,|\, \mathbf{w}^i\right), \ldots, \mu_K\left(\mathbf{x}^i \,|\, \mathbf{w}^i\right)$, where $K = 100$.

All our dominance criteria require that $K$ inequalities be satisfied for one distribution to be ranked above another one. More precisely, distribution $X$ is ranked above distribution $Y$ by the dominance criterion $\eta$ if

(A.2)
$$\eta\left(\frac{k}{K}; X\right) - \eta\left(\frac{k}{K}; Y\right) \geq 0, \ \forall\ k = 1, 2, \ldots, K,$$

where $\eta \in \{RO, ND, GL, RND, RL, AND, AL\}$. We appeal to two methods in order to compare the distributions: the intersection-union (IU) method and the union-intersection (UI) method. For either inference rule, we need to construct test statistics for the seven different point measures $\eta_k^i \equiv \eta\left(k/K; X^i\right)$ we consider in the paper. Let the statistic

(A.3)
$$T_k = \frac{\widehat{\eta}_k^A - \widehat{\eta}_k^B}{\left(\frac{\widehat{\omega}_{kk}^A}{n^A} + \frac{\widehat{\omega}_{kk}^B}{n^B}\right)^{\frac{1}{2}}},$$

where $\widehat{\eta}_k^i = \eta\left(k/K; (\mathbf{x}^i \,|\, \mathbf{w}^i)\right)$ is the sample estimate of $\eta_k^i$, and $\widehat{\omega}_{kk}^i$ is the sample estimate of the variance of $\eta_k^i$. The covariance matrix $\widehat{\mathbf{\Omega}} = [\widehat{\omega}_{hk}]$ of the estimates of the generalised Lorenz curve of distribution $X$ is defined by

(A.4)
$$\widehat{\omega}_{hk} = \tfrac{h}{K}\left[\widehat{\lambda}_h^2 + \left(1 - \tfrac{k}{K}\right)\left(\widehat{\mu}_h - \widehat{\gamma}_h\right)\left(\widehat{\mu}_k - \widehat{\gamma}_k\right) + \left(\widehat{\mu}_h - \widehat{\gamma}_h\right)\left(\widehat{\gamma}_k - \widehat{\gamma}_h\right)\right],$$

for $h \leq k$ and $k = 1, 2, \ldots, K$, where $\widehat{\mu}_k = \mu_k\left(\mathbf{x} \,|\, \mathbf{w}\right)$ is the estimate of the quantile mean, $\widehat{\gamma}_k = \gamma_k\left(\mathbf{x} \,|\, \mathbf{w}\right)$ the estimate of the conditional mean, and $\widehat{\lambda}_h^2 = \lambda_k^2\left(\mathbf{x} \,|\, \mathbf{w}\right)$ the estimate of the conditional variance of the first $k$ incomes in the sample $(\mathbf{x} \,|\, \mathbf{w})$ (see Beach and Davidson (1983)). To implement the tests we only need the variance estimates

$$\widehat{\omega}_{kk} = \tfrac{k}{K}\left[\widehat{\lambda}_k^2 + \left(1 - \tfrac{k}{K}\right)(\widehat{\mu}_k - \widehat{\gamma}_k)^2\right]. \tag{A.5}$$

In order to derive the corresponding variance estimates for the quantile curve and the non-deprivation curve, we use the fact that both curves are linear transformations of the generalised Lorenz curve. Given the sample distribution $(\mathbf{x} \,|\, \mathbf{w})$, we indicate by

$$RO\,(\mathbf{x} \,|\, \mathbf{w}) := \Big(RO\big(\tfrac{1}{K};(\mathbf{x} \,|\, \mathbf{w})\big), RO\big(\tfrac{2}{K};(\mathbf{x} \,|\, \mathbf{w})\big), \ldots, RO\big(\tfrac{K}{K};(\mathbf{x} \,|\, \mathbf{w})\big)\Big), \text{ and} \tag{A.6a}$$

$$ND\,(\mathbf{x} \,|\, \mathbf{w}) := \Big(ND\big(\tfrac{1}{K};(\mathbf{x} \,|\, \mathbf{w})\big), ND\big(\tfrac{2}{K};(\mathbf{x} \,|\, \mathbf{w})\big), \ldots, ND\big(\tfrac{K}{K};(\mathbf{x} \,|\, \mathbf{w})\big)\Big) \tag{A.6b}$$

the corresponding quantile and non-deprivation curves. Then, one can easily check that $RO\,(\mathbf{x} \,|\, \mathbf{w})^T = \boldsymbol{M}\,GL\,(\mathbf{x} \,|\, \mathbf{w})^T$ and $ND\,(\mathbf{x} \,|\, \mathbf{w})^T = \boldsymbol{R}\,GL\,(\mathbf{x} \,|\, \mathbf{w})^T$, where the $K \times K$ matrices $\boldsymbol{M}$ and $\boldsymbol{R}$ are respectively defined by

$$\boldsymbol{M} := \begin{bmatrix} K & 0 & 0 & \cdots & 0 & 0 & 0 \\ -K & K & 0 & \cdots & 0 & 0 & 0 \\ 0 & -K & K & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & K & 0 & 0 \\ 0 & 0 & 0 & \cdots & -K & K & 0 \\ 0 & 0 & 0 & \cdots & 0 & -K & K \end{bmatrix} \text{ and} \tag{A.7}$$

$$\boldsymbol{R} := \begin{bmatrix} K & 0 & 0 & \cdots & 0 & 0 & 0 \\ -(K-2) & (K-1) & 0 & \cdots & 0 & 0 & 0 \\ 0 & -(K-3) & (K-2) & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 3 & 0 & 0 \\ 0 & 0 & 0 & \cdots & -1 & 2 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix}. \tag{A.8}$$

It follows from Rao (1965, page 321), that the covariance matrices of the estimates for the quantile curve and the non-deprivation curve are given by $\boldsymbol{M}\,\widehat{\boldsymbol{\Omega}}\,\boldsymbol{M}^T$ and $\boldsymbol{R}\,\widehat{\boldsymbol{\Omega}}\,\boldsymbol{R}^T$, respectively, from which we derive the variance of the estimates. Appropriate normalisations of the sample distribution $(\mathbf{x} \,|\, \mathbf{w})$ permit to obtain the variances of the estimates for the relative and absolute Lorenz curves and for the relative and absolute non-deprivation curves.

Given two distributions $X$ and $Y$, the IU method involves the following two hypotheses:

$$H_0(X,Y)\text{:} \quad \exists k \in \{1,2,\ldots,K\} \mid \eta(k/K;X) - \eta(k/K;Y) < 0, \text{ and}$$

$$H_1(X,Y)\text{:} \quad \eta(k/K;X) - \eta(k/K;Y) \geq 0, \ \forall\, k \in \{1,2,\ldots,K\}.$$

We first test $H_0(X,Y)$ against $H_1(X,Y)$: *either* we accept $H_1(X,Y)$ in which case distribution $X$ weakly dominates distribution $Y$, *or* we reject $H_1(X,Y)$ and we move to the second stage where $H_1(Y,X)$ is tested against $H_0(Y,X)$. Then, *either* we accept $H_1(Y,X)$ and distribution $Y$ weakly dominates distribution $X$, *or* we reject $H_1(Y,X)$ and we conclude that distributions

$X$ and $Y$ are non-comparable. The sequence of tests for the IU method is summarized in Figure A.1. To sum up, given two countries A and B, the IU rule allows us to conclude that:

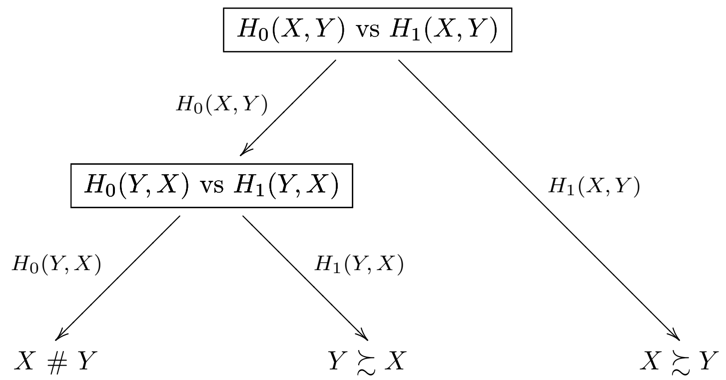
(A.9a) A weakly dominates B iff $+Z_\alpha < \min\{T_1, \ldots, T_K\}$,

(A.9b) B weakly dominates A iff $\max\{T_1, \ldots, T_K\} < -Z_\alpha$,

(A.9c) A and B are non-comparable otherwise,

where $Z_\alpha$ is the critical value for a significance level of $\alpha$ derived from Student's $t$-distribution. We note that this method does not allow for equivalence: *either* one country weakly dominates the other one, *or* the two countries are non-comparable.

Figure A.1: Test Procedure Under the IU Method




Given two distributions $X$ and $Y$, the UI method involves the following two hypotheses:

$$H_0(X,Y){:}\quad \eta\,(k/K;X) - \eta\,(k/K;Y) = 0,\ \forall\ k \in \{1,2,\ldots,K\},$$

$$H_1(X,Y){:}\quad \exists k \in \{1,2,\ldots,K\} \mid \eta\,(k/K;Y) - \eta\,(k/K;X) > 0,$$

and we note that $\neg\, H_0(X,Y) = H_1(X,Y) \vee H_1(Y,X)$. In a first step, we test $H_0(X,Y)$ against $H_1(X,Y)$ and there are two possibilities. If we accept $H_0(X,Y)$, then we test in a second stage $H_0(Y,X)$ against $H_1(Y,X)$: *either* we accept $H_0(Y,X)$, in which case we conclude that distributions $X$ and $Y$ are equivalent, *or* we accept $H_1(Y,X)$ and we conclude that distribution $Y$ strongly dominates distribution $X$. If in the first step we accept $H_1(X,Y)$, then we test in a second stage $H_0(Y,X)$ against $H_1(Y,X)$: *either* we accept $H_0(Y,X)$, in which case we conclude that distribution $X$ strongly dominates distribution $Y$, *or* we accept $H_1(Y,X)$ and we conclude that distributions $X$ and $Y$ are non-comparable. We have represented in Figure A.2 the test procedure for the UI method.
To sum up, given two countries A and B, the UI rule allows us to conclude that:

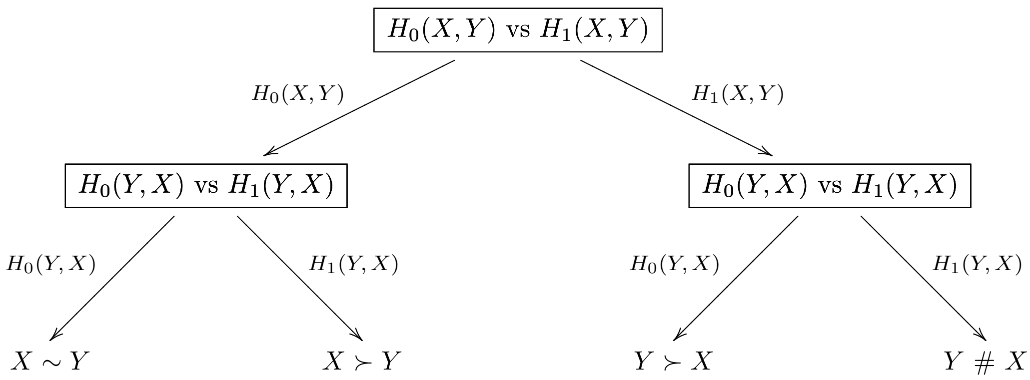
(A.10a) A strongly dominates B iff $-C_\alpha < \min\{T_1, \ldots, T_K\}$ and $C_\alpha < \max\{T_1, \ldots, T_K\}$,

(A.10b) B strongly dominates A iff $\min\{T_1, \ldots, T_K\} < -C_\alpha$ and $\max\{T_1, \ldots, T_K\} < C_\alpha$,

(A.10c) A is equivalent to B iff $-C_\alpha < \min\{T_1, \ldots, T_K\}$ and $\max\{T_1, \ldots, T_K\} < C_\alpha$,

(A.10d) A and B are non-comparable otherwise,

Figure A.2: Test Procedure Under the UI Method




where $C_\alpha$ is the critical value for a significance level of $\alpha$ determined from the Student Maximum Modulus (SMM) distribution provided by Stoline and Ury (1979).

Comparison of (A.9a) and (A.10a) makes clear that it is more difficult with the IU method to conclude that country A dominates country B than with the UI method. This results in a substantial loss in the discriminatory power of the IU method as compared with the UI method. This is particularly true in our illustration where the success rates attained under the IU method fall on average 10 points below those obtained under the UI method (see Table 6.9). In addition, it must be noted that the IU method does not allow for the possibility that two distributions be considered equivalent according to our welfare criteria.

## B. Ranking of Countries under the Union-Intersection Rule

We present for completeness the tables giving the results of the pairwise comparisons obtained under the UI method at the 95% confidence level. While the substitution of the more liberal UI method for the IU one results in an appreciable gain in decisiveness, it does not affect the general conclusions drawn from the application of the dominance criteria.

A *curiosum* is worth noting: Tables B.1, B.2 and B.3 indicate that Austria dominates Belgium according to the rank order, the generalised Lorenz and the non-deprivation criteria under the UI method while the former country has a lower mean than the latter. This comes in contradiction with the theory according to which, if a distribution rank order dominates another distribution, then it has also a higher mean. A possible explanation of this puzzle might originate in the statistical inference method used for deciding when one distribution dominates another. In the present case, we observe that the centile means for Austria are always greater than those for Belgium with the exception of the first three and last centiles where the opposite situation occurs. It is the huge gap in the last centile – the centile mean for Belgium is about three times that of Austria – that explains the fact that the mean income of Belgium is greater than the mean income of Austria. Actually, application of the UI method indicates that the first, second, third and last centile means for Austria are not significantly different from those for Belgium, while at the same time there is a centile mean – distinct from these – for which Austria is significantly better than Belgium. The fact that the difference between the last centile means of Belgium and Austria are not found to be statistically significant is somewhat surprising given the extent of the means difference. By definition, the test statistics defined by (A.4) involves the conditional variances of the sample

Table B.1: Ranking of Countries by the Rank Order Criterion (UI)

| | | Country $j$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No | Country $i$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 1 | Mexico | # | # | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Poland | | # | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | Hungary | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | Greece | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | Spain | | | | | # | # | # | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | Finland | | | | | | # | # | # | # | # | # | # | # | # | # | 0 |
| 7 | Sweden | | | | | | | # | # | # | # | 0 | # | 0 | 0 | # | 0 |
| 8 | Netherlands | | | | | | | | # | # | # | # | # | 0 | 0 | # | 0 |
| 9 | Germany | | | | | | | | | # | # | # | # | 0 | 0 | # | 0 |
| 10 | United Kingdom | | | | | | | | | | # | # | # | # | 0 | # | # |
| 11 | Austria | | | | | | | | | | | 1 | # | 0 | 0 | # | 0 |
| 12 | Belgium | | | | | | | | | | | | # | 0 | 0 | # | 0 |
| 13 | Canada | | | | | | | | | | | | | # | 0 | # | # |
| 14 | Norway | | | | | | | | | | | | | | # | # | # |
| 15 | Switzerland | | | | | | | | | | | | | | | # | # |
| 16 | USA | | | | | | | | | | | | | | | | # |
| 17 | Luxembourg | | | | | | | | | | | | | | | | |

"#" means that countries $i$ and $j$ are not comparable.
"1" means that country $i$ dominates country $j$.
"0" means that country $j$ dominates country $i$.
"≈" means that countries $i$ and $j$ are equivalent.

Table B.2: Ranking of Countries by the Generalised Lorenz Criterion (UI)

| | | Country $j$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No | Country $i$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 1 | Mexico | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Poland | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | Hungary | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | Greece | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | Spain | | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | Finland | | | | | | # | # | # | # | # | # | # | # | # | # | 0 |
| 7 | Sweden | | | | | | | 0 | 0 | # | 0 | 0 | # | 0 | 0 | # | 0 |
| 8 | Netherlands | | | | | | | | # | # | 0 | # | # | 0 | 0 | # | 0 |
| 9 | Germany | | | | | | | | | # | 0 | 0 | # | 0 | 0 | # | 0 |
| 10 | United Kingdom | | | | | | | | | | 0 | # | # | 0 | 0 | # | 0 |
| 11 | Austria | | | | | | | | | | | 1 | # | 0 | 0 | # | 0 |
| 12 | Belgium | | | | | | | | | | | | # | 0 | 0 | # | 0 |
| 13 | Canada | | | | | | | | | | | | | 0 | 0 | # | 0 |
| 14 | Norway | | | | | | | | | | | | | | # | # | 0 |
| 15 | Switzerland | | | | | | | | | | | | | | | # | 0 |
| 16 | USA | | | | | | | | | | | | | | | | 0 |
| 17 | Luxembourg | | | | | | | | | | | | | | | | |

"#" means that countries $i$ and $j$ are not comparable.
"1" means that country $i$ dominates country $j$.
"0" means that country $j$ dominates country $i$.
"≈" means that countries $i$ and $j$ are equivalent.

Table B.3: Ranking of Countries by the Non-Deprivation Criterion (UI)

| No | Country $i$ | Country $j$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 1 | Mexico | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | Poland | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | Hungary | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | Greece | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | Spain | | | | | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | Finland | | | | | | # | # | # | # | # | # | # | # | # | # | 0 |
| 7 | Sweden | | | | | | | 0 | # | # | 0 | 0 | # | 0 | 0 | # | 0 |
| 8 | Netherlands | | | | | | | | # | # | # | # | # | 0 | 0 | # | 0 |
| 9 | Germany | | | | | | | | | # | 0 | # | # | 0 | 0 | # | 0 |
| 10 | United Kingdom | | | | | | | | | | 0 | # | # | 0 | 0 | # | 0 |
| 11 | Austria | | | | | | | | | | | 1 | # | 0 | 0 | # | 0 |
| 12 | Belgium | | | | | | | | | | | | # | 0 | 0 | # | 0 |
| 13 | Canada | | | | | | | | | | | | | 0 | 0 | # | 0 |
| 14 | Norway | | | | | | | | | | | | | | # | # | 0 |
| 15 | Switzerland | | | | | | | | | | | | | | | # | 0 |
| 16 | USA | | | | | | | | | | | | | | | | 0 |
| 17 | Luxembourg | | | | | | | | | | | | | | | | |

"#" means that countries $i$ and $j$ are not comparable.
"1" means that country $i$ dominates country $j$.
"0" means that country $j$ dominates country $i$.
"$\approx$" means that countries $i$ and $j$ are equivalent.

Table B.4: Ranking of Countries by the Relative Lorenz Criterion (UI)

| No | Country $i$ | Country $j$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 1 | Mexico | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | # | 0 |
| 2 | Poland | | 0 | # | # | 0 | 0 | 0 | 0 | # | 0 | 0 | # | # | 0 | 1 | 0 |
| 3 | Hungary | | | 1 | 1 | 0 | # | 0 | # | 1 | 0 | $\approx$ | 1 | # | # | 1 | 0 |
| 4 | Greece | | | | $\approx$ | 0 | 0 | 0 | 0 | # | 0 | 0 | # | # | # | 1 | 0 |
| 5 | Spain | | | | | 0 | 0 | 0 | 0 | # | 0 | 0 | 0 | # | 0 | 1 | 0 |
| 6 | Finland | | | | | | # | # | # | 1 | # | 1 | 1 | 1 | 1 | 1 | # |
| 7 | Sweden | | | | | | | 0 | # | 1 | 0 | 1 | 1 | # | # | 1 | # |
| 8 | Netherlands | | | | | | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | # |
| 9 | Germany | | | | | | | | | 1 | 0 | $\approx$ | 1 | # | # | 1 | 0 |
| 10 | United Kingdom | | | | | | | | | | 0 | 0 | # | # | 0 | # | 0 |
| 11 | Austria | | | | | | | | | | | 1 | 1 | 1 | 1 | 1 | 0 |
| 12 | Belgium | | | | | | | | | | | | 1 | 0 | $\approx$ | 1 | 0 |
| 13 | Canada | | | | | | | | | | | | | # | 0 | 1 | 0 |
| 14 | Norway | | | | | | | | | | | | | | # | # | # |
| 15 | Switzerland | | | | | | | | | | | | | | | 1 | 0 |
| 16 | USA | | | | | | | | | | | | | | | | 0 |
| 17 | Luxembourg | | | | | | | | | | | | | | | | |

"#" means that countries $i$ and $j$ are not comparable.
"1" means that country $i$ dominates country $j$.
"0" means that country $j$ dominates country $i$.
"$\approx$" means that countries $i$ and $j$ are equivalent.

Table B.5: Ranking of Countries by the Relative Non-Deprivation Criterion (UI)

| | | Country $j$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No | Country $i$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 1 | Mexico | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | # | 0 |
| 2 | Poland | | 0 | # | # | 0 | 0 | 0 | 0 | # | 0 | 0 | # | # | 0 | 1 | 0 |
| 3 | Hungary | | | 1 | 1 | # | # | 0 | # | 1 | 0 | ≈ | 1 | # | # | 1 | 0 |
| 4 | Greece | | | | 0 | # | # | 0 | 0 | # | 0 | 0 | # | # | # | 1 | 0 |
| 5 | Spain | | | | | 0 | 0 | 0 | 0 | # | 0 | 0 | # | # | # | 1 | 0 |
| 6 | Finland | | | | | | # | # | # | 1 | # | 1 | 1 | # | 1 | # | # |
| 7 | Sweden | | | | | | | 0 | # | 1 | # | 1 | 1 | # | # | # | # |
| 8 | Netherlands | | | | | | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | # |
| 9 | Germany | | | | | | | | | 1 | 0 | # | 1 | # | # | 1 | 0 |
| 10 | United Kingdom | | | | | | | | | | 0 | 0 | # | # | 0 | # | 0 |
| 11 | Austria | | | | | | | | | | | 1 | 1 | # | 1 | 1 | # |
| 12 | Belgium | | | | | | | | | | | | 1 | 0 | 0 | 1 | 0 |
| 13 | Canada | | | | | | | | | | | | | # | # | # | 0 |
| 14 | Norway | | | | | | | | | | | | | | # | # | # |
| 15 | Switzerland | | | | | | | | | | | | | | | # | 0 |
| 16 | USA | | | | | | | | | | | | | | | | 0 |
| 17 | Luxembourg | | | | | | | | | | | | | | | | |

"#" means that countries $i$ and $j$ are not comparable.
"1" means that country $i$ dominates country $j$.
"0" means that country $j$ dominates country $i$.
"≈" means that countries $i$ and $j$ are equivalent.

Table B.6: Ranking of Countries by the Absolute Lorenz Criterion (UI)

| | | Country $j$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No | Country $i$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 1 | Mexico | # | 0 | 1 | 1 | 1 | 1 | # | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | Poland | | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | Hungary | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | Greece | | | | 1 | 1 | 1 | # | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | Spain | | | | | 0 | 0 | 0 | # | 1 | # | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | Finland | | | | | | # | # | 1 | 1 | # | 1 | 1 | 1 | 1 | 1 | 1 |
| 7 | Sweden | | | | | | | 0 | 1 | 1 | # | 1 | 1 | 1 | 1 | 1 | 1 |
| 8 | Netherlands | | | | | | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 9 | Germany | | | | | | | | | 1 | 0 | ≈ | 1 | 1 | 1 | 1 | 1 |
| 10 | United Kingdom | | | | | | | | | | 0 | ≈ | # | # | # | 1 | # |
| 11 | Austria | | | | | | | | | | | ≈ | 1 | 1 | 1 | 1 | 1 |
| 12 | Belgium | | | | | | | | | | | | 1 | 1 | 1 | 1 | 1 |
| 13 | Canada | | | | | | | | | | | | | # | # | 1 | # |
| 14 | Norway | | | | | | | | | | | | | | # | 1 | # |
| 15 | Switzerland | | | | | | | | | | | | | | | 1 | # |
| 16 | USA | | | | | | | | | | | | | | | | 0 |
| 17 | Luxembourg | | | | | | | | | | | | | | | | |

"#" means that countries $i$ and $j$ are not comparable.
"1" means that country $i$ dominates country $j$.
"0" means that country $j$ dominates country $i$.
"≈" means that countries $i$ and $j$ are equivalent.

Table B.7: Ranking of Countries by the Absolute Non-Deprivation Criterion (UI)

| | | Country $j$ | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| No | Country $i$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 1 | Mexico | # | 0 | # | 1 | 1 | 1 | # | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | Poland | | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | Hungary | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | Greece | | | | 1 | # | 1 | # | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 5 | Spain | | | | | 0 | 0 | 0 | # | 1 | # | 1 | 1 | 1 | 1 | 1 | 1 |
| 6 | Finland | | | | | | # | # | # | 1 | # | 1 | 1 | 1 | 1 | 1 | # |
| 7 | Sweden | | | | | | | 0 | # | 1 | # | 1 | 1 | 1 | 1 | 1 | # |
| 8 | Netherlands | | | | | | | | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 9 | Germany | | | | | | | | | 1 | 0 | 1 | 1 | 1 | 1 | 1 | # |
| 10 | United Kingdom | | | | | | | | | | 0 | ≈ | # | # | # | # | # |
| 11 | Austria | | | | | | | | | | | 1 | 1 | 1 | 1 | 1 | 1 |
| 12 | Belgium | | | | | | | | | | | | 1 | 1 | 1 | 1 | 1 |
| 13 | Canada | | | | | | | | | | | | | # | # | 1 | # |
| 14 | Norway | | | | | | | | | | | | | | # | # | # |
| 15 | Switzerland | | | | | | | | | | | | | | | # | # |
| 16 | USA | | | | | | | | | | | | | | | | 0 |
| 17 | Luxembourg | | | | | | | | | | | | | | | | |

"#" means that countries $i$ and $j$ are not comparable.
"1" means that country $i$ dominates country $j$.
"0" means that country $j$ dominates country $i$.
"≈" means that countries $i$ and $j$ are equivalent.

distributions under comparison. It is therefore possible that the difference in the last centile conditional variances of the distributions for Austria and Belgium is sufficiently large to offset the differences in the corresponding mean incomes. To some extent that might be considered a drawback of the UI statistical inference method as compared with the IU one.

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*La coordination scientifique des Cahiers du GREThA est assurée par Sylvie FERRARI et Vincent FRIGANT. La mise en page est assurée par Dominique REBOLLO.*