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C. Morana

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Working Paper No. 15/2008
September 2008

APPLIED MATHEMATICS
WORKING PAPER SERIES

Realized Betas and the Cross-Section of Expected Returns

Claudio Morana*

September 17, 2008

Abstract

What explains the cross section of expected returns for the 25 size/value Fama-French portfolios? It is found that modelling time-varying betas is important to explain the cross-section of expected returns, as well as to comply with the time series restriction on Jensen-alpha. Support for a modified version of the conditional Jagannathan and Wang (1996) CAPM model is found, where implementation is carried out in the realized beta framework proposed in the paper. About 63% of the cross-sectional variability of the expected returns for the 25 Fama-French size and value sorted portfolios is then found to be explained by this parsimonious two-variable model.

Key words: realized regression, time-varying beta, conditional CAPM

JEL classification: G12, C22

*Department of Economic Sciences and Quantitative Methods, University of Piemonte Orientale and International Centre for Economic Research (ICER). The author is grateful to A. Beltratti for very constructive comments.

1 Introduction

Explaining the cross section of stock returns is one of the most active area in empirical finance. Financial literature has studied this problem with some characteristic methodological features that have to do with the number of factors included, assumptions about the stability over time of the relevant moments of the distribution of returns, the frequency of the observations. The Capital Asset Pricing Model (CAPM, Sharpe, 1964; Lintner, 1965) predicts a linear relation between expected returns across assets and their betas with respect to the market portfolio. After initial empirical support, failure to explain the cross-section of US stock returns since the early 1960s has been pointed out in the literature. In particular, small and value stocks have shown higher average returns than predicted on the basis of their betas. Moreover, stocks with high past beta have not shown higher average returns than stocks, with the same size, but lower past betas.[1]

Fama and French (1992) and Davis, Fama and French (2000) have shown that a three factor model is able to account for most of the cross-sectional variation in 25 size/book-to-market portfolios (FF25 thereafter). In addition to the market return, the Fama-French model includes the return on a portfolio long in small stocks and short in big stocks (SMB, size) and the return on a portfolio long in high book-to-market stocks and short in low book-to-market stocks (HML, value).

A crucial issue in the assessment of the validity of the CAPM has been the assumption about stability of the relevant moments, particularly the betas. Harvey (1989), Ferson and Harvey (1991) and Ferson and Korajczyck (1995) find that estimated betas exhibit statistically significant time variation, pointing to a conditional implementation of the model. Moreover, Jagannathan and Wang (1996) notice that when betas vary over time it is still possible to find an unconditional version of the model, but such an unconditional version implies the presence of the covariance between the time-varying beta and the time-varying market return. Such an extended model is found to be able to account for about 55% of the the cross-sectional variability of the FF25 returns, over the period 1962-1990. A similarly successful implementation of the conditional CAPM has been provided by Lettau and Ludvingson (2001), pointing to up to 70% of explained cross-sectional variation of the FF25 returns, over the period 1963-1998. In both cases the Fama-French SMB and HML factors are not anylonger statistically significant in the cross-sectional regression, suggesting that the latter may proxy for conditioning information related to the return to human capital and various sources of beta

[1]See Campbell and Vuolteenaho (2004) for an account of the initial empirical literature.

instability or to uncertainty about future investment opportunities (Petkova, 2006). More recent works also point to successful implementation of the conditional/unconditional CAPM model, once returns volatility (Ang and Chen, 2005) and learning (Adrian and Franzoni, 2005) are taken into account, or when the single market beta is divided in a cash-flow and a discount rate component (Campbell and Vuolteenaho, 2004).

Differently, Lewellen and Nagel (2005), Petkova and Zhang (2005) and Fama and French (2006), provide evidence against the conditional CAPM model, showing that a model with time-varying betas is unable to explain the book-to-market premium either because the variation in betas is not large enough or because only variation in betas related to size and value is compensated in average FF25 returns.

A possible explanation for the above contrasting evidence concerning the validity of the conditional CAPM model, can be found in the different econometric tools employed for the modelling of beta variability. For instance, in Lewellen and Nagel (2005), Petkova and Zhang (2005) and Fama and French (2006), time-varying betas are not estimated by means of an appropriate model including a dynamic equation for the beta, but use either short-window regressions or assume that betas are linear in information variables or just augment the static model by including dummy variables.

This paper moves from the above considerations, aiming at assessing the validity of the conditional CAPM model, contributing under different points of view to the current literature.

The first is the reliance on, and generalization of, recent results obtained in the high frequency literature to estimate conditional betas. These results highlight the role of realized betas and downplay the importance of specific assumptions, i.e. specific information variables or specific time series models, for explaining the dynamics of the betas. This is important in view of results both showing the sensitivity of the estimated betas to the choice of instruments used to proxy for time variation in conditional betas (Harvey, 2001), and showing the temporal instability of the equations used to project the betas on the variables in the information set (Ghysels, 1989). In particular, the realized beta estimator of Andersen et al. (2005, 2006) is generalized to the multivariate case allowing to account for non orthogonal factors.

Second, monthly realized betas are computed starting from daily data. This is of great importance and distinguishes the empirical work carried out in the paper from previous work which only uses monthly data directly. Daily data are likely to be very informative in estimating the betas because the latter are ratios of covariance and variance components. The original intuition of Merton (1980) has been developed thoroughly by the high frequency literature, which has documented the gains associated with using high frequency data for estimation of second moments.

Third, measurement error in the realized betas is taken care by means of noise filtering. The approach followed (Morana, in press) is based on flexible least squares estimation (FLS, Kalaba and Tesfatsion, 1989) of an unobserved component local level model (Harvey, 1989). The latter filter has been found to perform well independently of the persistence properties of the series to be filtered, and it is therefore particularly appropriate for the current application, as both long memory and structural change may be relevant features for the data investigated.[2]

After this introduction, the paper is organized as follows. In sections two the multivariate realized beta estimator is introduced and its applications discussed, while in section three and four the cross-sectional analysis is carried out. Finally, in section five conclusions are drawn.

2 The realized betas

Realized regression theory can be employed to estimate monthly realized betas ($\hat{\boldsymbol{\beta}}_s$) starting from daily observations. The argument, which generalizes the bivariate case studied by Andersen et al (2005, 2006)[3], is best given in continuous time. Following Andersen et al. (2001), Andersen et al. (2005, 2006) and Barndorff-Nielsen and Shephard (2002), suppose that the log $M \times 1$ vector price process, p_t, follows a multivariate continuous-time stochastic volatility diffusion

$$dp_t = \mu_t dt + \Omega_t dW_t, \tag{1}$$

where W_t denotes a standard M-dimensional Brownian motion process, and both the processes for the $M \times M$ positive definite diffusion matrix Ω_t and the M-dimensional instantaneous drift μ_t are strictly stationary and jointly independent of the W_t process. Then, conditional on the sample path realization of Ω_t and μ_t, the distribution of the continuously compounded h-period return $r_{t+h,h} = p_{t+h} - p_t$ is

$$r_{t+h,h}|\sigma\left\{\mu_{t+\tau}, \Omega_{t+\tau}\right\}_{\tau=0}^{h} \sim N(\int_0^h \mu_{t+\tau} d\tau, \int_0^h \Omega_{t+\tau} d\tau). \tag{2}$$

[2] See for instance, Lobato and Savin (1998), Beltratti and Morana (2006), Martens, van Dijk and de Pooter (2004), and Baillie and Morana (2007). See also Baillie (1996) for an introduction to long memory processes.

[3] Bollerslev and Zhang (2003) also consider a multivariate case. Yet, by assuming orthogonality of the factors, the realized betas have been computed using the standard bivariate formulas.

The integrated diffusion matrix

$$\int_0^h \Omega_{t+\tau} d\tau \tag{3}$$

can be employed as a measure of multivariate volatility.

By the theory of quadratic variation, under some weak regularity conditions,

$$\hat{\Omega}_{t+h} = \sum_{j=1,\ldots,[h/\Delta]} r_{t+j\cdot\Delta,\Delta} r'_{t+j\cdot\Delta,\Delta} \xrightarrow{p} \int_0^h \Omega_{t+\tau} d\tau, \tag{4}$$

i.e. the realized variance covariance matrix estimator is a consistent estimator, in the frequency of sampling ($\Delta \to 0$), of the integrated variance covariance matrix.

Consider then the following factor model for the generic asset q, with time t return $r^*_{q,t}$

$$r^*_{q_t} - r_{f_t} = \alpha_{q,t} + \sum_{k=1}^{K} \beta_{q,k_t} er_{k_t} + \varepsilon_{q_t}, \tag{5}$$

where r_{f_t} is the time t return on the risk-free asset, $er_{k_t} = r_{k_t} - r_{f_t}$ is the time t risk premium of the kth risk factor. The $(K \times K)$ realized variance covariance matrix for the risk factors can be written as

$$\hat{\Omega}_{(K)_{t+h}} = \sum_{j=1,\ldots,[h/\Delta]} er_{K_{t+j\cdot\Delta,\Delta}} er'_{K_{t+j\cdot\Delta,\Delta}} \xrightarrow{p} \int_0^h \Omega_{(K)_{t+\tau}} d\tau, \tag{6}$$

while the $(K \times 1)$ vector of covariances of the qth asset with each of the factors can be denoted as

$$\hat{\Omega}_{(qK)_{t+h}} = \sum_{j=1,\ldots,[h/\Delta]} er^*_{q_{t+j\cdot\Delta,\Delta}} er_{K_{t+j\cdot\Delta,\Delta}} \xrightarrow{p} \int_0^h \Omega_{(qK)_{t+\tau}} d\tau. \tag{7}$$

It then follows that

$$\hat{\beta}_{t,t+h} = \hat{\Omega}^{-1}_{(K)_{t+h}} \hat{\Omega}_{(qK)_{t+h}} \tag{8}$$

is a consistent estimator, in the frequency of sampling ($\Delta \to 0$), of $\beta_{t,t+h}$[4], i.e.

[4]This is proved by noting that $\hat{\Omega}_{(iK)_{t+h}} \xrightarrow{p} \int_0^h \Omega_{(iK)_{t+\tau}} d\tau$ and $\hat{\Omega}_{(K)_{t+h}} \xrightarrow{p} \int_0^h \Omega_{(K)_{t+\tau}} d\tau$

$$\hat{\beta}_{t,t+h} \xrightarrow{p} \beta_{t,t+h} = \left(\int_0^h \Omega_{(K)_{t+\tau}} d\tau \right)^{-1} \int_0^h \Omega_{(qK)_{t+\tau}} d\tau.^5 \tag{9}$$

Hence, realized factor betas at time (month) s can be computed as

$$\hat{\beta}_s = (X'X)_s^{-1} (X'y)_s \qquad s = 1, ..., S, \tag{10}$$

where the ijth element in the matrix $(X'X)_s$ is given by $x_{ij_s} = \sum_{h=1}^{H} er_{i_{s,h}}^{-} er_{j_{s,h}}^{-}$ $i, j = 1, ..., K$, the ith element in the vector $(X'y)_s$ is given by $xy_{i_s} = \sum_{h=1}^{H} er_{i_{s,h}}^{-} er_{q_{s,h}}^{*-}$ $i = 1, ..., k$, and the z^- variables are demeaned z variables, with $z = er_{i_{s,h}}, er_{q_{s,h}}^*$. The time-varying realized Jensen-alpha can then be computed as $\hat{\alpha}_s = \overline{er}_{q_s}^* - \sum_{i=1}^{K} \hat{\beta}_{i_s} \overline{er}_{i_s}$, where the barred variables are sample averages at time period s, i.e. $\overline{er}_{q_s}^* = \frac{1}{H} \sum_{h=1}^{H} er_{q_{s,h}}^*$ and $\overline{er}_{i_s} = \frac{1}{H} \sum_{h=1}^{H} er_{i_{s,h}}$ $i = 1, ..., k$.

The theory outlined above allows us to estimate time-varying betas with two advantages. The first, already noted, lies in the lack of specific assumptions about the dynamic model determining the beta. The second lies in the use of daily data, which are otherwise ignored in any model using monthly observations of returns to estimate the betas, as done for instance by Ferson and Harvey (1999) and Ang and Chen (2005). Another important advantage of the proposed approach is that the estimation of multi-factor models is straightforward, whereas the time-varying parameter model estimated by means of Markov Chain Monte Carlo and Gibbs sampling by Ang and Chen (2005) is more difficult to handle in a multivariate context.

Yet, when realized betas are computed starting from daily, rather than high frequency data, measurement error may affect the estimates. As the numerator and the denominator in the realized beta formula can be expected to be driven by different long memory trends, as both terms not only contain different variables, but also different transformations of the involved variables, the realized betas should be characterized by the same persistence features

and that $\hat{\Omega}_{(K)_{t+h}}^{-1} \xrightarrow{p} \left(\int_0^h \Omega_{(K)_{t+\tau}} d\tau \right)^{-1}$ using also the continuous mapping theorem.

[5] See Barndorff-Nielsen and Shephard (2002) for additionald details on the asymptotic properties of the realized regression estimator.

as the realized variances and covariances from which they are computed, i.e. long memory and structural breaks. The noise filtering method implemented in the paper (Morana, in press) is actually suited for the task, allowing to handle, differently from the one proposed in Andersen et al. (2005), both the above features.

2.1 Using the realized betas

Realized betas are interesting from several points of view. Firstly, since they are modelled as a time-varying process, their temporal dimension allow to assess how systematic risk varies over time. Secondly, the actual determinants of systematic risk can also be assessed by relying on cross-sectional approach. According to this latter strategy, the estimated realized betas can be used to test for a multifactor model of the type

$$Er_{i,t} - r_{f,t} = \alpha_i + \gamma_{Mkt}\beta_{i,Mkt,t} + \sum_{k=1}^{K}\gamma_{i,k}\beta_{i,k,t}, \tag{11}$$

by means of a standard cross-section methodology, allowing to evaluate the validity of alternative specifications. In particular, the excess returns of the test assets are regressed on the estimated betas

$$r_{i,t} - r_{f,t} = \theta + \theta_{Mkt}\hat{\beta}_{i,Mkt,t} + \sum_{i}\theta_i\hat{\beta}_{i,k,t} + \varepsilon_{i,t} \; i = 1...25 \tag{12}$$

for each observation in the sample or using averages

$$\bar{r}_i - \bar{r}_f = \theta + \theta_{Mkt}\bar{\beta}_{i,Mkt} + \sum_{i}\theta_i\bar{\beta}_{i,k} + \varepsilon_i \; i = 1...25, \tag{13}$$

as in the current application. Interesting comparisons then concern the simple CAPM, the three factor Fama-French model, the extended Fama-French model including momentum, the Petkova (2005) model allowing for innovations in the state variables, as well as the Ferson and Harvey (1999) and Jagannathan and Wang (1996) models. Estimation of the above model can be carried out by means of standard OLS or GLS regressions.

An additional test can be carried out in a time-series framework, testing for the time variability and the statistical difference from zero of the realized Jensen-alpha. This latter test can be carried out by testing H_0:$\bar{\alpha}_i = 0$, $i = 1...25$, where $\bar{\alpha}_i$ is the temporal average for the sequence of the ith realized alpha, computing the standard error $\hat{\sigma}_{\bar{\alpha}_i}$ as $1/T^{0.5}$ times the temporal standard deviation of the same sequence.

3 Estimating, filtering and smoothing realized betas

The data set investigated is composed of the 25 Fama-French size/book-to-market portfolios. Eight risk factors have been considered: the S&P500 returns (Mkt), the Fama-French size and value portfolios (HML, SMB), the momentum portfolio of Caharart (MOM), and four state variables related to the yield curve (the term spread (TS), the three-month Treasury bills rate (3M)) and the conditional distribution of asset returns (the default spread (DS) and the dividend yield (DY)). The frequency of sampling is daily, from January 4, 1965 through August 31, 2005.

Following Campbell (1996), daily innovations for the macroeconomic factors and the Fama-French-Caharart factors, have been estimated using the Choleski orthogonalized innovations from a first order VAR. Hence, the following model has been estimated

$$\begin{bmatrix} er_{Mkt_t} \\ DY_t \\ TS_t \\ DS_t \\ 3M_t \\ er_{HML_t} \\ er_{SMB_t} \\ er_{MOM_t} \end{bmatrix} = \delta + A \begin{bmatrix} er_{Mkt_{t-1}} \\ DY_{t-1} \\ TS_{t-1} \\ DS_{t-1} \\ 3M_{t-1} \\ er_{HML_{t-1}} \\ er_{SMB_{t-1}} \\ er_{MOM_{t-1}} \end{bmatrix} + v_t,$$

where er_{i_t} is the excess return on the relevant portfolio, $i = Mkt$, SMB, HML, MOM, computed relatively to the risk-free rate ($3M$), δ is the intercept component, and the orthogonalized innovations are $\hat{u}_t = \hat{H}^{-1}\hat{v}_t$, where $\hat{H}^{-1} = chol(\hat{\Sigma}_v)$ and $\hat{\Sigma}_v$ is the estimated variance covariance matrix of the reduced form disturbances $\hat{v}_t$.The shocks of interest are then the last seven components in the structural innovation vector u_t, i.e. u_t^{DY}, u_t^{TS}, u_t^{DR}, u_t^{3M}, u_t^{HML}, u_t^{SMB}, u_t^{MOM} which, still following Campbell (1996), have been scaled to match the same variance as the innovation in the excess market return.

The daily market excess return (er_{Mkt_t}) and the seven structural innovations (u_t^i) have then been employed in the computation of the monthly realized regressions, as discussed in the methodological section, leading to a total of 488 monthly observations for each of the 200 realized betas. Realized betas have also been computed using the market excess return (er_{Mkt_t}) and the (scaled) reduced form innovations (v_t^i) corresponding to the factors of interest, i.e. v_t^{DY}, v_t^{TS}, v_t^{DR}, v_t^{3M}, v_t^{HML}, v_t^{SMB}, v_t^{MOM}, in order to assess the sensitivity of the results to the identification strategy selected. It is for this latter non orthogonal case that the multivariate generalization of realized beta estimation, introduced in the paper, is actually of use.

Following the Monte Carlo results in Morana (in press), noise filtering of the estimated realized betas has been carried out by selecting a 1% significance level for the Box-Pierce test.

As shown by the empirical results, noise filtering does not affect the sample mean, but only the standard deviation, which tends to decrease sharply for almost all the betas, in general even more than 70%. Moreover, measurement error in realized betas is not negligible. In fact, the average estimated inverse signal to noise ratios are in the range 2.2 to 2.8 for the Mkt, SMB, HML, MOM factor betas and 3.5 to 5.6 for the TS, DS, 3M, DY factor betas and Jensen-alpha. Hence, most of the variation over time of the estimated realized betas is determined by measurement error (see also Figures 1 and 2). Yet, interesting slowly evolving trend dynamics can be associated with the signal component as well, implying that modelling factor betas as time-invariant parameters may be a suboptimal strategy.[6]

Concerning the average beta, a systematic pattern can be detected only for the SMB and HML factors. In fact, the average realized beta for the HML factor increases with the book-to-market ratio, while the average realized beta decreases as the size of the portfolio increases. In addition, apart from the Mkt (in the range 0.65 to 1.16), HML (in the range -0.24 to 0.44) and SMB (in the range -0.09 to 0.54) factors, the average contribution of all the other factors is very close to zero. Among the former three factors, the average beta is always largest for the Mkt factor. Moreover, the SMB factor is in general characterized by larger betas than the HML factor, apart from the value portfolio case. Figures are close to those found by Bollerslev and Zhang (2003), assuming however orthogonality of the factors and using 5-minute returns.[7]

4 The cross-section of expected returns

The cross-section of expected excess returns on the 25 Fama-French portfolios sorted by book-to-market and size has been investigated by means of the realized betas approach, using both the raw (RB) and smoothed (RBS) version. For comparison also the standard constant parameter and time-varying parameter estimation approaches used in the literature have been

[6] These findings are not peculiar to the smallest portfolio of the first Fama-French class of portfolios plotted in Figures 1 and 2, but hold for all of the estimated betas. Details are available upon request from the author.

[7] Detailed results are not reported for reasons of space.

implemented, i.e. the Black, Jensen and Sholes (BJS, 1972) and Fama and MacBeth (FMB, 1973) methods. The comparison allows to contrast the performance of the proposed time-varying beta estimation approach with the standard constant parameter and time-varying parameter approaches. Moreover, also the Ferson and Harvey (FH, 1999) approach has been implemented. The specification employed for the analysis considers the CAPM model (CAPM), the Fama-French model (FF), the Fama-French model augmented with the Caharart momentum factor (FFM), the Jagannathan and Wang (JW, 1998) model, and two models related to Petkova (2005), i.e. a state variable augmented CAPM model, including innovations to the term spread (TS), the three-month Treasury bills rate (3M)), the default spread (DS) and the dividend yield (DY) (SCAPM), and a state variable augmented Fama-French-Caharart model (SFFM).

As the cross-sectional approach to the evaluation of the pricing model is composed of two steps, in Table 4, Panel A, the results of the tests carried out on the Jensena-alphas computed from the first step time series regressions have been reported. For the time-varying parameter cases the statistical significance of the Jensen-alphas has been assessed by means of a t-ratio test, using the temporal mean and (scaled by $1/\mathrm{T}^{0.5}$) standard deviation for each of the 25 portfolios. For the constant parameter case the test is a standard t-ratio test carried out on the intercept of the time-series regression model. As shown in the table, the average Jensen-alpha over the time sample investigated is not statistically different from zero in all the cases (across portfolios and specifications), at both the 5% and 1% significance levels, only for the realized beta approach. Slightly inferior results are provided by the Ferson-Harvey approach for the CAPM specification, pointing to model failure in 36% of the cases at the 5% level and in 4% of the cases at the 1% significance level. On the other hand, the percentage of model failure for the BJS approach is very large in all the cases, with most of the values falling in the range 88% to 96%.

From the above findings a clear-cut conclusion can be drawn suggesting that modelling betas time-variability leads to superior results to neglecting this data feature. The finding also provides support to the conditional version of the CAPM model. Yet, as already found by Ghysels (1998), how beta variability is modelled can make a substantial difference, since the percentage of failures, at the 5% significance level, in the Ferson-Harvey approach is not negligible. Overall, also consistent with Petkova (2005), innovations to the term spread, the three-month Treasury bills rate, the default spread and the dividend yield, do seem to contain information useful to predict portfolios excess returns.

The results of the second step of the cross-sectional analysis are reported in Tables 4, Panel B, and 5.

In Table 4, Panel B, the actual and corrected coefficients of determination for the various cross-sectional regressions are reported. Concerning the CAPM model, the BJS, FMB and RB approaches lead to similar results, i.e. to a proportion of explained variability of about 30%. Removing observational noise from the realized betas (RBS) leads to an increase in the proportion of explained variability, getting close to 55%. Also the FH and JW[8] approaches yield a superior performance relatively to BJS, FMB and RB, explaining about 42% and 54% of the cross-sectional variability, yet still inferior to the one of the smoothed realized betas (RBS). Using the smoothed version of the realized beta approach, applied to the JW regression, yields an even larger increase relatively to BJS and FMB, pointing to up 66% of explained total variability.

Consistent with previous findings in the literature (see for instance Petkova (2005) and references therein), the inclusion of the Fama-French HML and SMB factors leads to an additional increase in the proportion of explained variance in all cases, yielding a coefficient of determination close to 70% for all the methods, apart from the RBS approach (63%). It is important to note that relatively to the smoothed realized beta version of the JW conditional CAPM model, the increase in the proportion of explained variability provided by the FF model relatively to the JW model is very small (the corrected coefficients of determination are 0.69 and 0.63, for the FF and the JW approaches, respectively. Yet, the Fama-French model clearly provides a superior performance relatively to the unconditional CAPM model. An additional small, yet significant, increase in the proportion of explained variability relatively to the FF model is provided by the momentum augmented FF model (FFM), pointing to about 76% of explained total cross-sectional variability.

Moreover, consistent with Petkova (2005), a similar performance to the FF model is achieved by the macroeconomic state variables augmented version of the CAPM model (SCAPM). The increase in the proportion of explained variability is uniform across models, being however strongest for the BJS approach (about 75%). Yet, the gap relatively to the realized beta version of the JW conditional CAPM model is very small, being the corrected coefficients of determination 0.68 and 0.63, respectively.

Finally, the inclusion of the Fama-French factors, in addition to the macroeconomic state variables, leads to a further significant increase in the

[8] The JW approach has been implemented as in Jagannathan and Wang (1996), but using the default spread innovation only as a proxy for the conditional market risk premium.

proportion of explained cross-sectional variance for all the models, which is close to 83% without degrees of freedom correction and 73% with correction, pointing to this latter model as the best model, independently of the estimation approach followed. A discrimination across models can however be carried out by considering jointly the results of the two-step analysis. In fact, on the basis of the first step time series tests carried out on the Jensen-alphas, it is however the realized beta estimation approach which should be selected, being the only one to comply with the time series restriction, and explaining an equally high proportion of cross-sectional variability (82%). The above findings partially contrast with Petkova (2005), where the macroeconomic state variables have been found to be proxy for the Fama-French HML and SMB factors, since the latter seem to provide additional explanatory power, relatively to the macro state variables. This latter result may depend on the different measurement approach for the state variable innovations followed in the current and Petkova papers, as while in Petkova (2005) the monthly innovations have been constructed using monthly data, in the current paper the monthly innovations have been constructed in a more accurate way, starting from daily observations.

Overall, support for the conditional CAPM model can be found. Once the smoothed realized beta version of the JW approach is followed, as pointed out by the corrected coefficients of determination, about 63% of total cross-section variability is explained by just two variables. On the other hand, about 75% of total variance is explained by a model including a total of eight variables (SFFM).

Further interesting evidence is reported in Table 5, where the estimated parameters (by OLS) for the various cross-sectional regression models are reported. As is shown in the Table, some similarity in the results can be noticed across specifications. Firstly, in all the cases the intercept component is statistically significant, suggesting that not all of the expected return is explained by risk exposure, as pricing theory would on the other hand require. Secondly, the market beta is an important factor in the cross-section of returns, albeit, consistent with the previous literature, takes a negative sign. As pointed out by Petkova (2005), this latter finding can be rationalized assuming that the market portfolio is an hedge for the state variables included in the specification. Yet, omitted variable bias could also explain the finding; a thorough investigation of this issue is outside the scope of the paper, but indeed deserves further study. Thirdly, the Fama-French factors do not tend to be significantly priced. In fact, albeit the impact of the HML and SMB betas in the cross-sectional regression is positive, the SMB beta is never statistically significant, while the HML beta is significant only in the BJS regression. Differently, the momentum factor does seem to be significantly priced in the cross-section. Finally, as far as the macroeconomic state variables are concerned, the evidence is mixed, with the dividend yield and the term spread betas tending to be significantly priced. Fourthly, the beta for the conditional market risk premium in the JW regression is always statistically significant, albeit its impact is negative.[9]

Overall the results of the cross-sectional analysis are clear-cut. Modelling time-varying betas is important to explain both the cross-section of expected returns, as well as to comply with the time series restriction on Jensen-alpha. When both criteria are taken into account, support for a modified version of the conditional Jagannathan and Wang (1996) CAPM model is found, where the implementation is carried out in the realized beta framework. However, even this latter model is not without drawbacks, as, likewise the other models, the market beta shows a negative sign. Further work, even in the realized beta framework, is then required.

5 Conclusions

What explains the cross-section of expected returns for the 25 size/value Fama-French portfolios? Overall the results of the paper are clear-cut. Modelling time-varying betas is important to explain both the cross-section of expected returns, as well as to comply with the time series restriction on Jensen-alpha. When both criteria are taken into account, support for a modified version of the conditional Jagannathan and Wang (1996) CAPM model is found, where implementation is carried out in the realized beta framework. About 63% of the cross-sectional variability of the expected returns for the 25 Fama-French size and value sorted portfolios is found to be explained by this parsimonious two-variable model, differently from the 75% explained by a more profligate eight-variable model, including the Fama-French value and size betas, the momentum beta and the betas for four macroeconomic state variables related to the yield curve and the conditional distribution of asset returns.

The realized beta estimator introduced in the paper generalizes previous bivariate results of Andersen et al. (2005, 2006), allowing to account for multiple non orthogonal risk factors. Relatively to previous work on time-varying betas the proposed approach has the advantage of not relying on instrumental variables to proxy the time variation in the conditional betas,

[9] The analysis has also been carried out using the GLS estimator, with and without Shanken's errors-in-variables correction for the standard errors, as well as by using the non orthogonalized residuals. Similar results have been obtained in both cases, and are available from the author upon request.

as the latter are estimated directly from returns, avoiding sensitivity to the conditioning set employed. Moreover, the estimation procedure is straightforward to implement.

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Table 1, Panel A: Time-series regressions.

	$Jensen - alpha$				$Jensen - alpha$		
5%	BJS	RB	FH	1%	BJS	RB	FH
$CAPM$	.88	.00	0.36	$CAPM$	.88	.00	0.04
FF	.96	.00		FF	.92	.00	
FFM	.96	.00		FFM	.92	.00	
JW	.88	.00		JW	.88	.00	
$MACRO$	.88	.00		$MACRO$	.88	.00	
PE	.96	.00		PE	.52	.00	

Table 1, Panel B: Cross-sectional regressions.

R^2	BJS	FMB	RB	RBS	FH	R_c^2	BJS	FMB	RB	RBS	FH
$CAPM$	.304	.344	.333	.556	.429	$CAPM$	.274	.316	.304	.537	.404
FF	.728	.705	.700	.632		FF	.689	.663	.657	.579	
FFM	.756	.685	.676	.718		FFM	.708	.622	.611	.661	
JW	.535	.588	.576	.658		JW	.493	.551	.537	.627	
$SCAPM$	.748	.723	.690	.630		$SCAPM$	.681	.650	.608	.533	
$SFFM$	.824	.831	.826	.817		$SFFM$	.736	.747	.739	.726	

In the table the proportion of violations, at the 5% and 1% significance level, of the null of zero intercept term in the time-series regressions for the Black-Jensen-Scholes (BJS), realized beta (RB) and Ferson-Harvey (FH) approaches is reported in Panel A. The acronyms CAPM, FF, FFM, JW, SCAPM, SFFM refer to the CAPM model, the Fama-French model, the Carhart momentum augmented Fama-French model, the Jagannathan-Wang model, the macroeconomic model and the Carhart momentum augmented Petkova model, respectively. The proportion of actual (R^2) and corrected (R_c^2) cross-sectional variation of the expected excess returns on 25 portfolios sorted by book-to-market and size, explained by the betas estimated by the various approaches, i.e. the Black-Jensen-Scholes (BJS), the Fama-MacBeth (FMB), realized beta (RB), smoothed realized beta (RBS) and Ferson-Harvey (FH) approaches, for the different specifications (CAPM, FF, FFM, JW, MACRO, PE) is reported in Panel B.

Table 2: Cross sectional regression

CAPM

	BJS	FM	RB	RBS	FH
	1.851	1.911	1.904	2.095	1.840
θ	(0.350)	(0.239)	(0.242)	(0.189)	(0.183)
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	−0.728	−0.789	−0.777	−1.157	−0.726
θ_{Mkt}	(0.410)	(0.227)	(0.229)	(0.215)	(0.175)
	[0.075]	[0.001]	[0.001]	[0.000]	[0.000]

JW

	BJS	FM	RB	RBS
	2.393	2.202	2.194	2.252
θ	(0.342)	(0.209)	(0.213)	(0.180)
	[0.000]	[0.000]	[0.000]	[0.000]
	−1.321	−1.155	−1.145	−1.276
θ_{Mkt}	(0.374)	(0.210)	(0.213)	(0.229)
	[0.000]	[0.000]	[0.000]	[0.000]
	−2.735	−1.433	−1.404	−5.635
θ_{prem}	(1.161)	(0.386)	(0.383)	(2.245)
	[0.019]	[0.000]	[0.000]	[0.012]

FF

	BJS	FM	RB	RBS
	1.932	2.000	1.982	2.029
θ	(0.291)	(0.438)	(0.431)	(0.292)
	[0.000]	[0.000]	[0.000]	[0.000]
	−1.018	−1.049	−1.027	−1.080
θ_{Mkt}	(0.359)	(0.246)	(0.419)	(0.286)
	[0.000]	[0.000]	[0.001]	[0.000]
	0.302	0.253	0.260	0.306
θ_{HML}	(0.267)	(0.308)	(0.259)	(0.310)
	[0.258]	[0.411]	[0.316]	[0.323]
	0.351	0.309	0.309	0.019
θ_{SMB}	(0.253)	(0.346)	(0.134)	(0.215)
	[0.165]	[0.372]	[0.021]	[0.930]

FFM

	BJS	FM	RB	RBS
	1.456	1.905	1.848	2.666
θ	(0.317)	(0.499)	(0.502)	(0.368)
	[0.000]	[0.000]	[0.000]	[0.000]
	−0.500	−0.951	−0.880	−1.726
θ_{Mkt}	(0.382)	(0.493)	(0.496)	(0.367)
	[0.190]	[0.054]	[0.076]	[0.000]
	0.643	0.318	0.362	−0.118
θ_{HML}	(0.255)	(0.303)	(0.307)	(0.333)
	[0.024]	[0.293]	[0.238]	[0.724]
	0.240	0.291	0.251	−0.297
θ_{SMB}	(0.255)	(0.184)	(0.189)	(0.243)
	[0.347]	[0.114]	[0.183]	[0.222]
	2.345	0.214	0.625	4.834
θ_{MOM}	(0.814)	(1.157)	(1.223)	(2.325)
	[0.004]	[0.853]	[0.609]	[0.038]

SCAPM

	BJS	FM	RB	RBS
	1.497	2.011	2.138	2.271
θ	(0.319)	(0.323)	(0.325)	(0.373)
	[0.000]	[0.000]	[0.000]	[0.000]
	−0.579	−1.067	−1.173	−1.282
θ_{Mkt}	(0.375)	(0.324)	(0.332)	(0.385)
	[0.123]	[0.001]	[0.000]	[0.001]
	−1.238	−0.146	−0.005	1.235
θ_{DY}	(0.775)	(0.465)	(0.484)	(3.052)
	[0.110]	[0.754]	[0.901]	[0.686]
	3.624	2.821	2.692	−1.303
θ_{TS}	(0.996)	(1.310)	(1.501)	(1.900)
	[0.000]	[0.031]	[0.073]	[0.493]
	−1.724	−1.640	−1.429	−1.693
θ_{DS}	(1.110)	(1.085)	(1.248)	(2.459)
	[0.121]	[0.130]	[0.252]	[0.491]
	−1.167	0.737	0.674	0.790
θ_{3M}	(0.880)	(1.406)	(1.526)	(2.263)
	[0.185]	[0.600]	[0.659]	[0.729]

SFFM

	BJS	FM	RB	RBS
	1.214	2.263	2.074	2.940
θ	(0.340)	(0.514)	(0.539)	(0.506)
	[0.000]	[0.000]	[0.000]	[0.000]
	−0.306	−1.351	−1.143	−1.975
θ_{Mkt}	(0.400)	(0.276)	(0.541)	(0.504)
	[0.445]	[0.000]	[0.035]	[0.000]
	−1.753	−0.702	−0.722	−2.200
θ_{DY}	(0.850)	(0.342)	(0.534)	(3.136)
	[0.039]	[0.040]	[0.176]	[0.483]
	2.030	1.615	0.975	1.845
θ_{TS}	(1.022)	(0.480)	(1.179)	(3.784)
	[0.047]	[0.001]	[0.408]	[0.626]
	0.850	1.568	1.504	−1.753
θ_{DS}	(1.285)	(0.643)	(1.320)	(2.393)
	[0.508]	[0.015]	[0.254]	[0.464]
	−1.035	0.770	0.928	4.215
θ_{3M}	(0.824)	(0.522)	(1.412)	(3.174)
	[0.209]	[0.140]	[0.511]	[0.184]
	0.589	0.150	0.280	0.034
θ_{HML}	(0.289)	(0.364)	(0.373)	(0.512)
	[0.042]	[0.681]	[0.453]	[0.946]
	0.313	0.115	0.131	−0.228
θ_{SMB}	(0.264)	(0.396)	(0.199)	(0.508)
	[0.235]	[0.771]	[0.510]	[0.654]
	2.387	1.678	2.030	6.681
θ_{MOM}	(0.806)	(0.582)	(1.166)	(2.833)
	[0.003]	[0.004]	[0.082]	[0.018]

In the table parameter estimates for the cross sectional regressions using the excess returns on 25 portfolios sorted by book-to-market and size, with OLS standard errors (in round brackets) and p-values (in square brackets), are reported. The parameters are indexed as follows: θ is the intercept component, while θ_i, $i = Mkt$, SMB, HML, MOM, DY, TS, DS, $3M$ refer to the market factor, the size factor, the value factor, the momentum factor, the dividend-yield, the term spread, the default spread and the three-month rate, respectively.

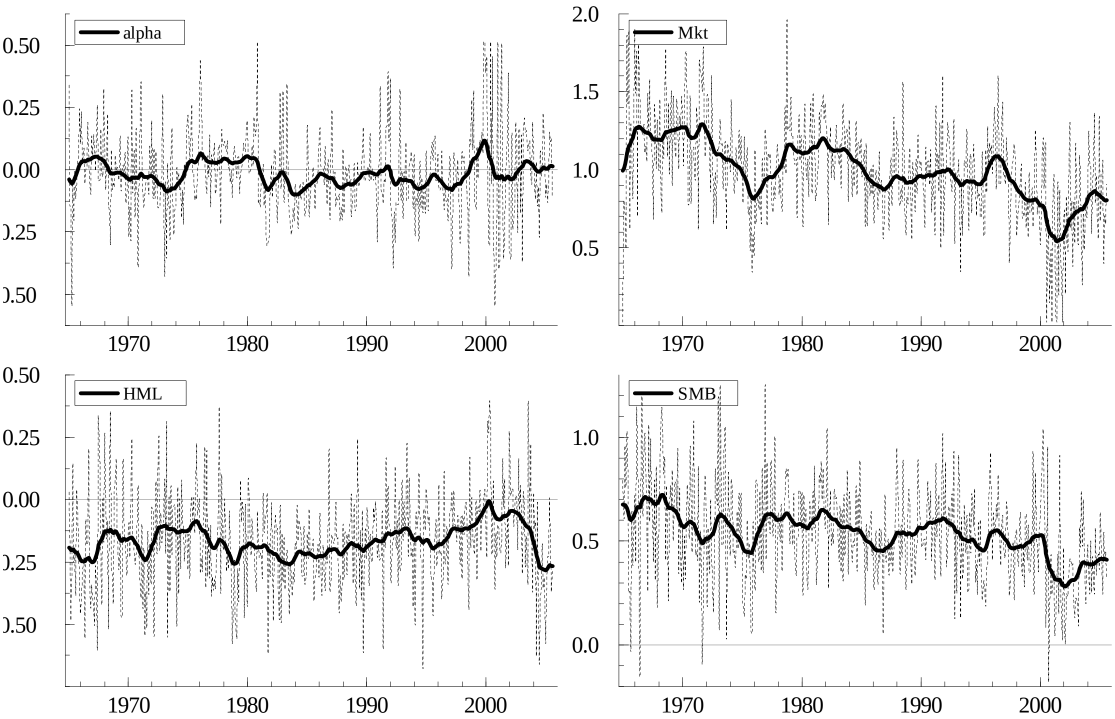

Figure 1: Filtered and actual realized factor betas for the smallest portfolios of the first Fama-French portfolio class (Jensen alpha: alpha, market: Mkt, size: SMB, value: HML).

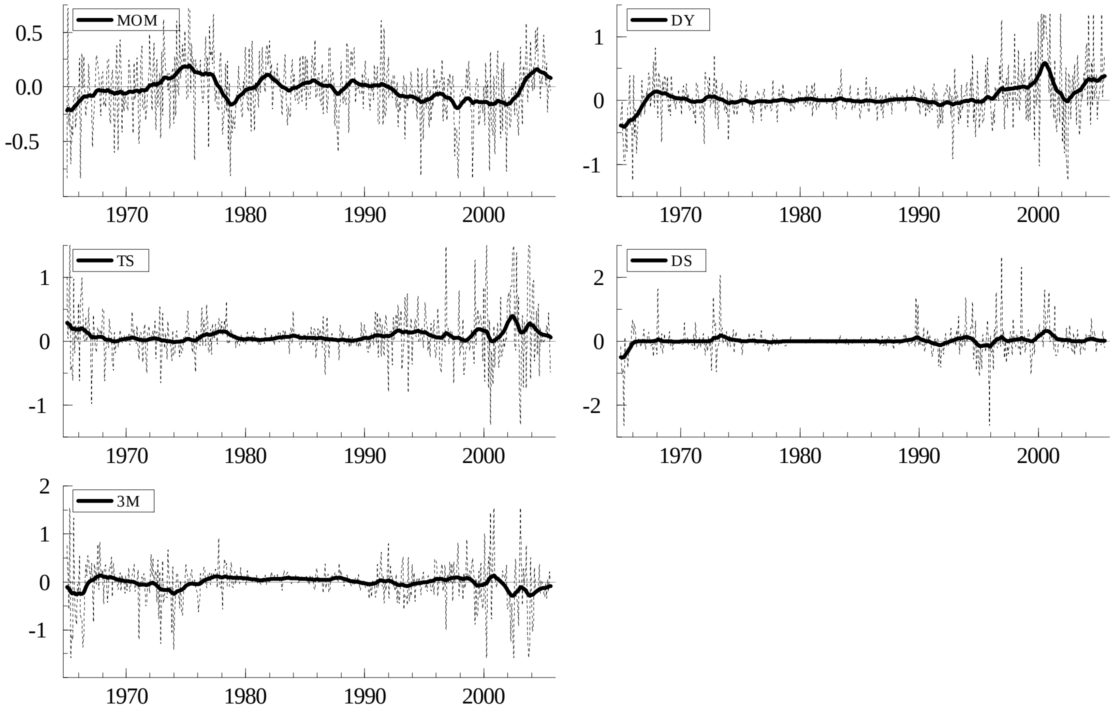

Figure 2: Filtered and actual realized factor betas for the smallest portfolios of the first Fama-French portfolio class (momentum: MOM, term structure: TS, default spread: DS, three-month rate: 3M, dividend-yield: DY).